Multiparton interactions

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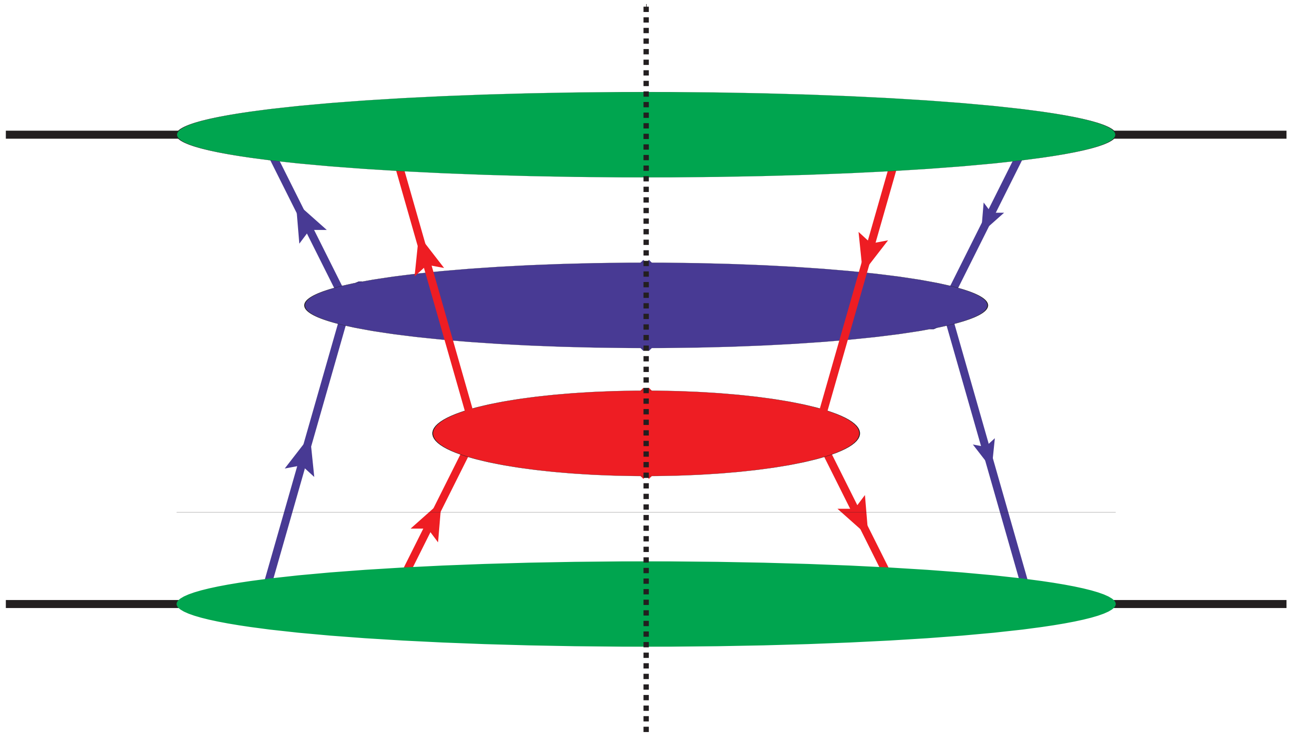
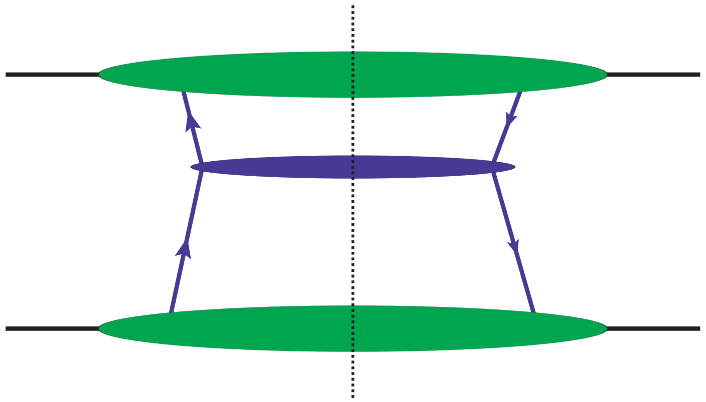
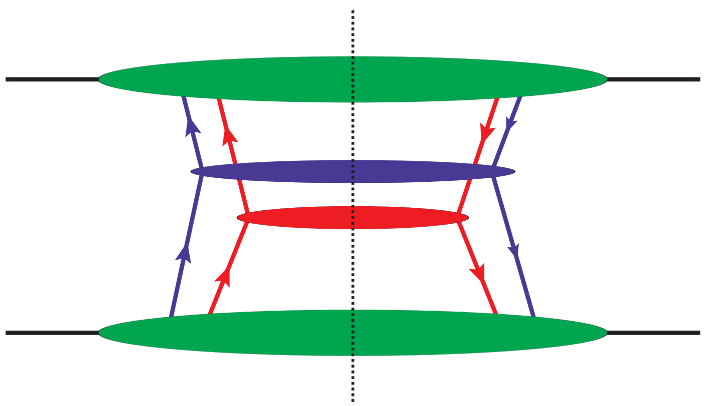


GDR QCD - Orsay, September 29, 2016

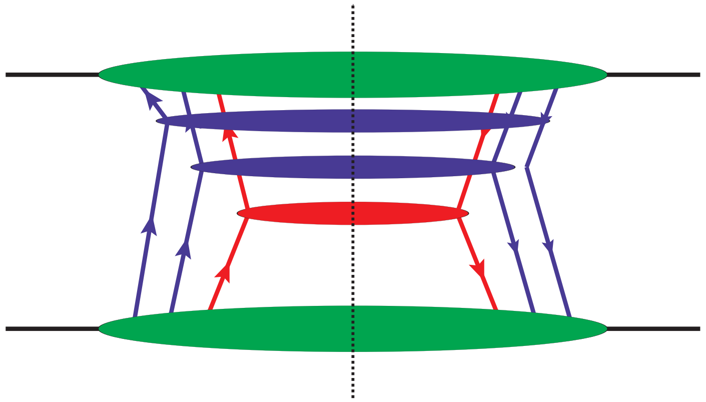
What is multiparton interactions..

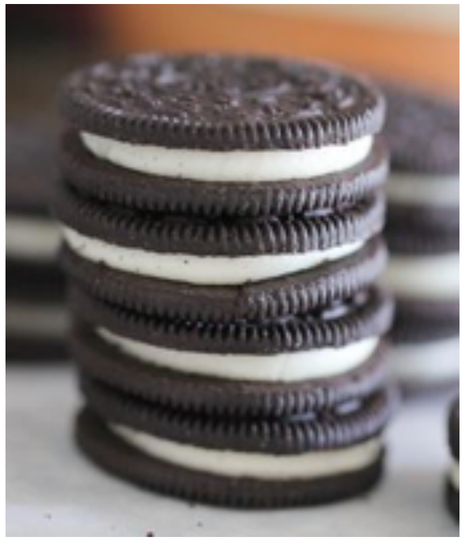

What is multiparton interactions..

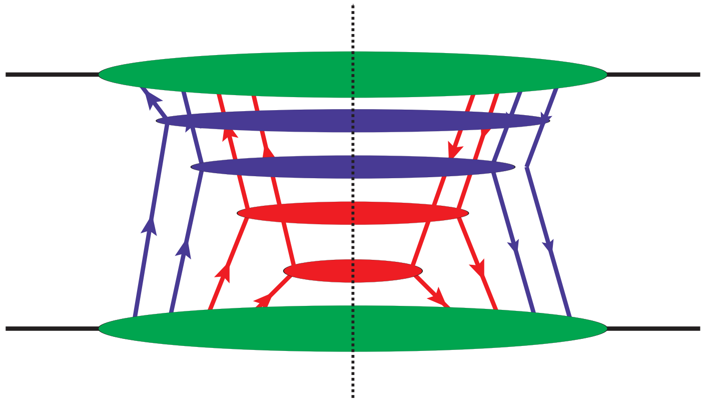


What is multiparton interactions..



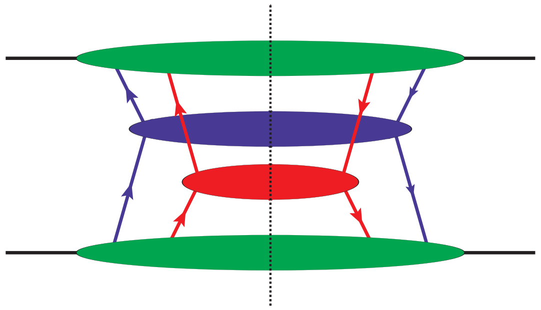
What is multiparton interactions..



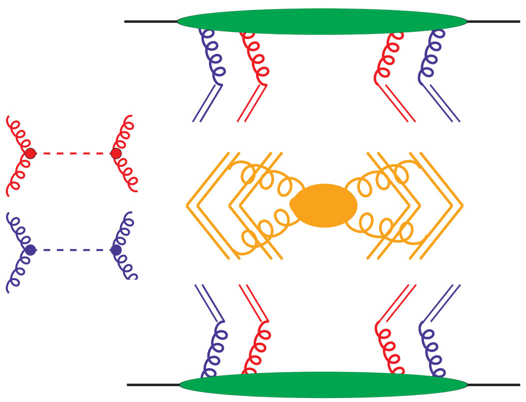


Outline

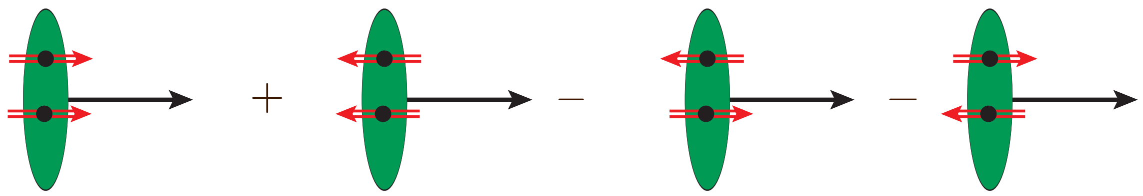
- Introduction to MPI



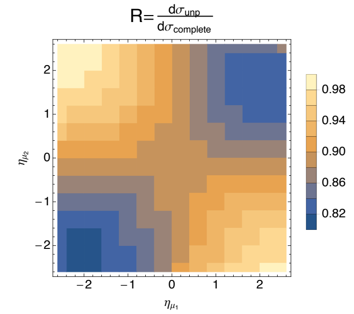
- Theoretical status



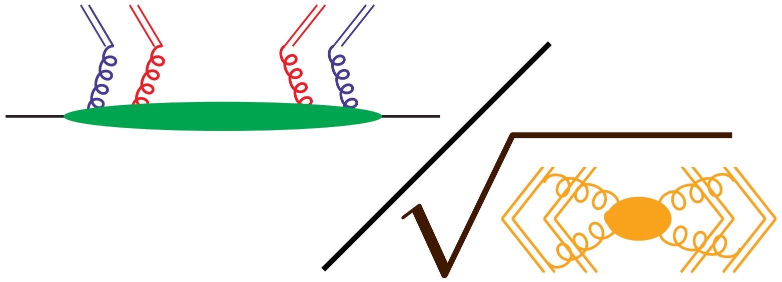
- Correlations



  - in cross sections: double D0 and double W



- Transverse momentum



MPI in hadron-hadron collisions

- Cross section from factorization

$$\text{cross section} = \text{parton distribution} \times \text{partonic cross section}$$

  - single parton example: $pp \to Z + X \to l^+l^- + X$

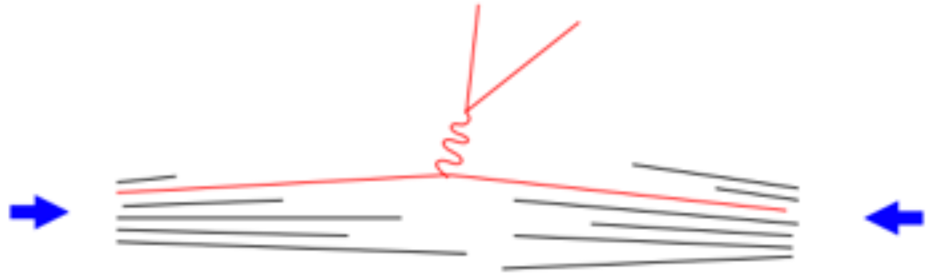
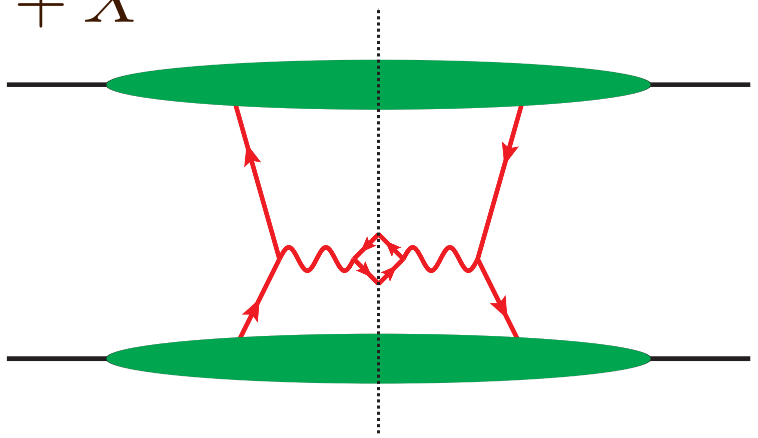


figure from M. Diehl, QCD Evolution 2014



- Total cross section $\sigma = \hat{\sigma}_{ab\to Y} \otimes f_a(x_1) \otimes f_b(\bar{x}_1)$
  - (collinear) parton distributions: PDFs
  - Y produced in partonic scattering (specified)
  - X everything else (summed over fully inclusive)
- Measured net transverse momenta $d\sigma \propto \int d^2\boldsymbol{b} e^{i\boldsymbol{q}\boldsymbol{b}} f_a(x_1, \boldsymbol{b}) f_b(\bar{x}_1, \boldsymbol{b})$
  - Transverse momentum dependent parton distributions, TMDs

MPI in hadron-hadron collisions

- Cross section from factorization

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  - single parton example: $pp \to Z + X \to l^+ l^- + X$

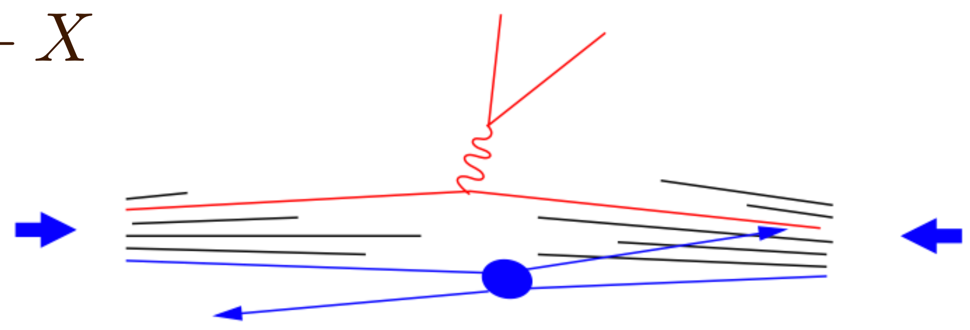


figure from M. Diehl, QCD Evolution 2014

- Spectator-spectator interactions
  - cancel in inclusive cross sections (unitarity)
  - affects final state X
- Ask questions about X ⇨ sensitivity to additional interaction
- predominantly at low transverse momenta: underlying event
- high collision energies (e.g. LHC) can be hard: multiple hard scattering
  - theory: from the 80s, current increase of attention
  - experiment: since ISR, many recent measurements at Tevatron and LHC
  - Modelled in event generators: Pythia, Herwig++, Sherpa etc.

MPI in hadron-hadron collisions

- Multiple hard interactions

  cross section = multiple parton distribution × partonic cross section

  Paver, Treleani 1982, Mekhfi 1985, ..., Diehl, Ostermeier, Schäfer 2011

- Second interaction hard — Double Parton Scattering (DPS)
  example: $pp \to Z + b\bar{b} + X$

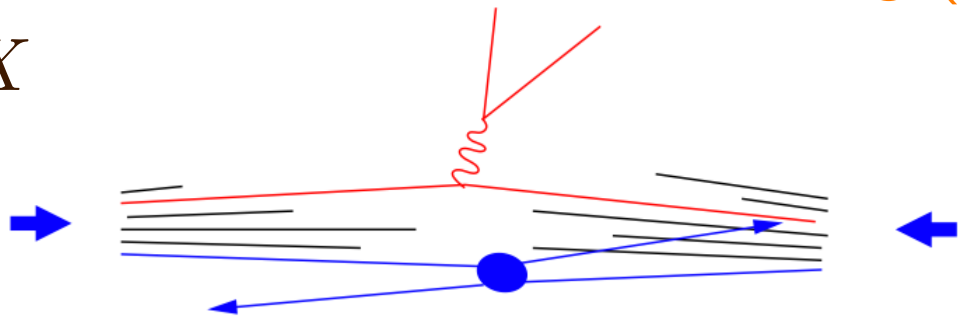


figure from M. Diehl, QCD Evolution 2014

  - Most frequent type of MPI, first step towards complete description

- DPS cross section:

$$\sigma_{\rm DPS} = \hat{\sigma}_{ab}\hat{\sigma}_{cd} \int d^2\boldsymbol{y} \otimes f_{ac}(x_1, x_2, \boldsymbol{y}) \otimes f_{bd}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

  - in terms of double parton distributions (DPDs)

- Here, focus on DPS

Signal and background

- Double parton scattering contribute both to signal and background
  - $pp \to H + Z + X \to b\bar{b} + \mu^+\mu^- + X$ Del Fabbro, Treleani, 1999

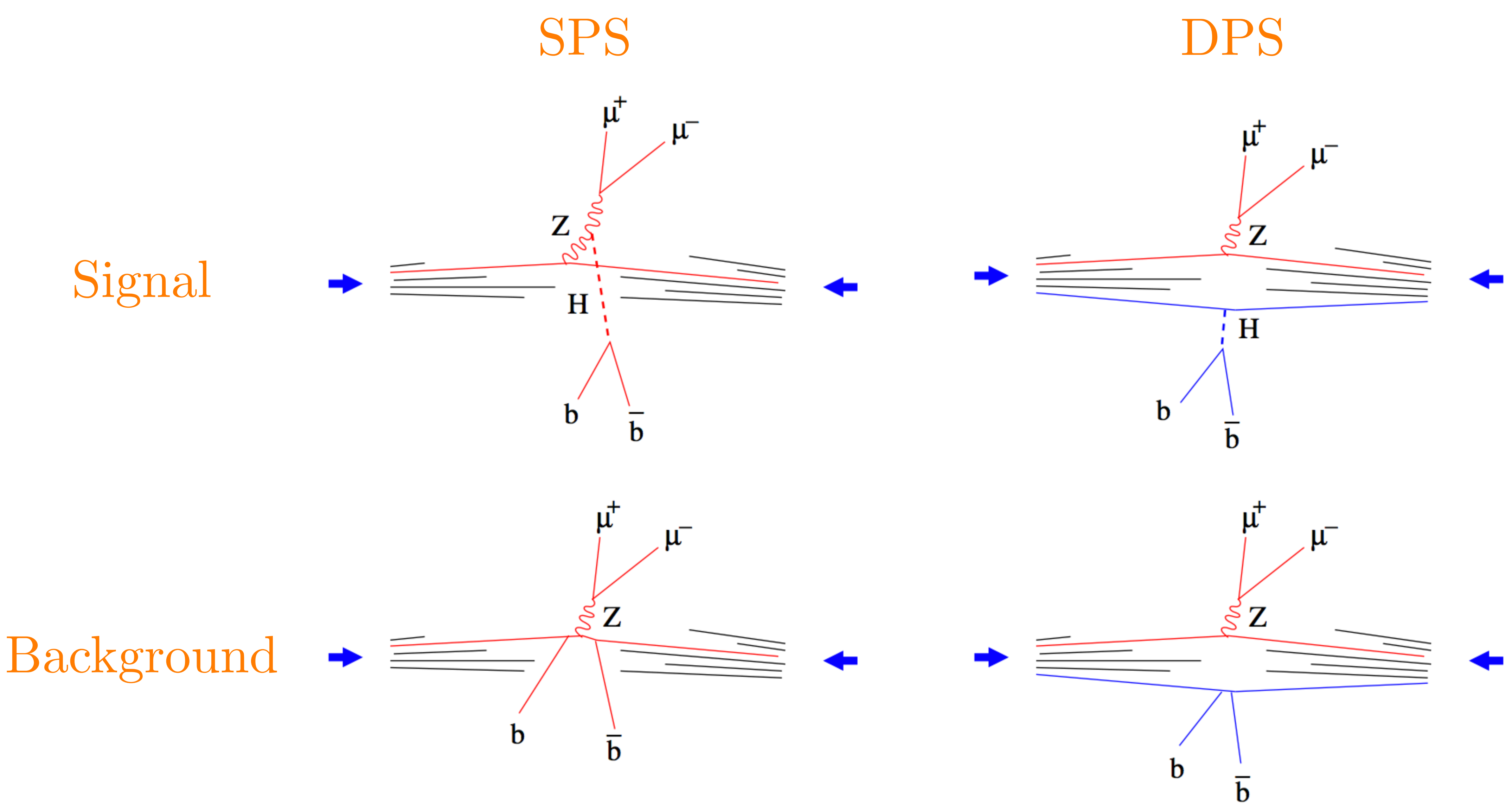
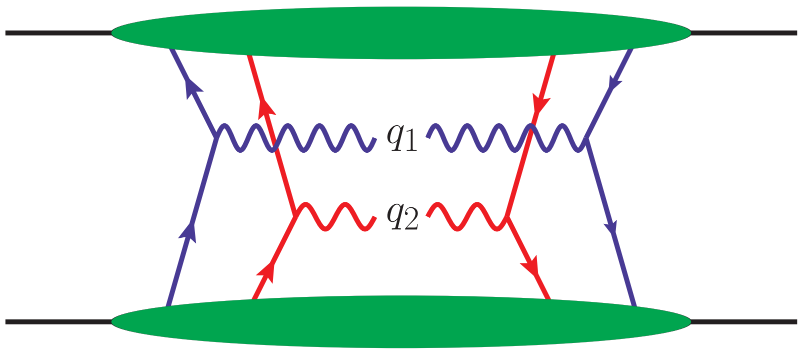
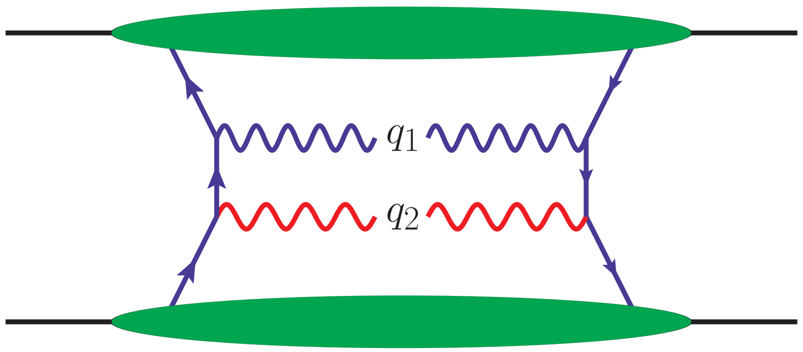


figure from Diehl, QCD Evolution 2014

Double vs single hard scattering





- Inclusive cross section

$$\sigma_{\text{DPS}}/\sigma_{\text{SPS}} \sim \frac{\Lambda^2}{Q^2}$$

- DPS populates final state phase space in a different way than SPS

$$|\boldsymbol{q}_1|,\ |\boldsymbol{q}_2| \sim \Lambda << Q : \quad \frac{d\sigma_{SPS}}{d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} \sim \frac{d\sigma_{DPS}}{d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4\Lambda^2}$$

  DPS same power as SPS

- Large parton density ⇨ enhanced DPS $\sigma_{DPS} \sim (\text{parton density})^4$
- DPS cross section from region of small(ish) momentum fractions

When should one care about DPS?

- Rule of thumb:
  - Several final state particles (typically 4 or more)
  - High energy hadron collisions
    — low momentum fractions are probed (low x)
  - SPS suppressed — two single production cross sections large compared to their "combination"
- Conditions are often fulfilled for processes studied at the LHC
- A few examples:
  - 2x same sign W's (small cross section but very clean) Gaunt, Kom, Kulesza, Stirling, 2010
  - Double open charm production (D0D0)
    - Double dominates single parton scattering? Hameren, Maciula, and Szczurek, 2014,...; Echevarria, TK, Mulders, Pisano, 2015
  - Double quarkonia production, Lansberg, Shao, 2015; Kom, Kulesza, Stirling, 2011
  - W+b (rough estimates about 20% DPS) ATLAS Collaboration, 2013
  - H+W Bandurin, Golovanov, Skachkov, 2011
  - double meson productions, W+bbar, 4 jets, photon + 3 jets, etc. etc.

Double vs single hard scattering

- Size of DPS cross sections?
  - If (!?) no partonic correlations, all partons have the same transverse profile etc. etc.

    $\Rightarrow$ DPS cross section:

$$\sigma_{DPS} \sim \frac{\sigma_1 \sigma_2}{\sigma_{\text{eff}}}$$

$$\sigma_{\text{eff}} \sim 15\text{mb}$$

Pocket formula gives order of magnitude estimates of DPS cross section

- Where is DPS important?
- What is the uncertainty of this approach? 10%? 100%? 1000%? ...
- Where does it break down?

Road to the pocket formula

- What approximations goes into σ_{eff}

$$\sigma_{\mathrm{DPS}} = \hat{\sigma}_{ab}\hat{\sigma}_{cd} \int d^2\boldsymbol{y} \otimes f_{ac}(x_1, x_2, \boldsymbol{y}) \otimes f_{bd}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

- Approximations step 1: Ignore quantum correlations (to be explained)
- Approximations step 2: Separation of transverse dependence

$$f_{ab}(x_1, x_2, \boldsymbol{y}; \mu) = f_{ab}(x_1, x_2; \mu) G(\boldsymbol{y})$$

- Approximations step 3: Separation of longitudinal dependence

$$f_{ab}(x_1, x_2) = f_a(x_1) f_b(x_2)$$

- Results in the (in)famous pocket formula:

$$\sigma_{DPS} \sim \frac{\sigma_1 \sigma_2}{\sigma_{\mathrm{eff}}}$$

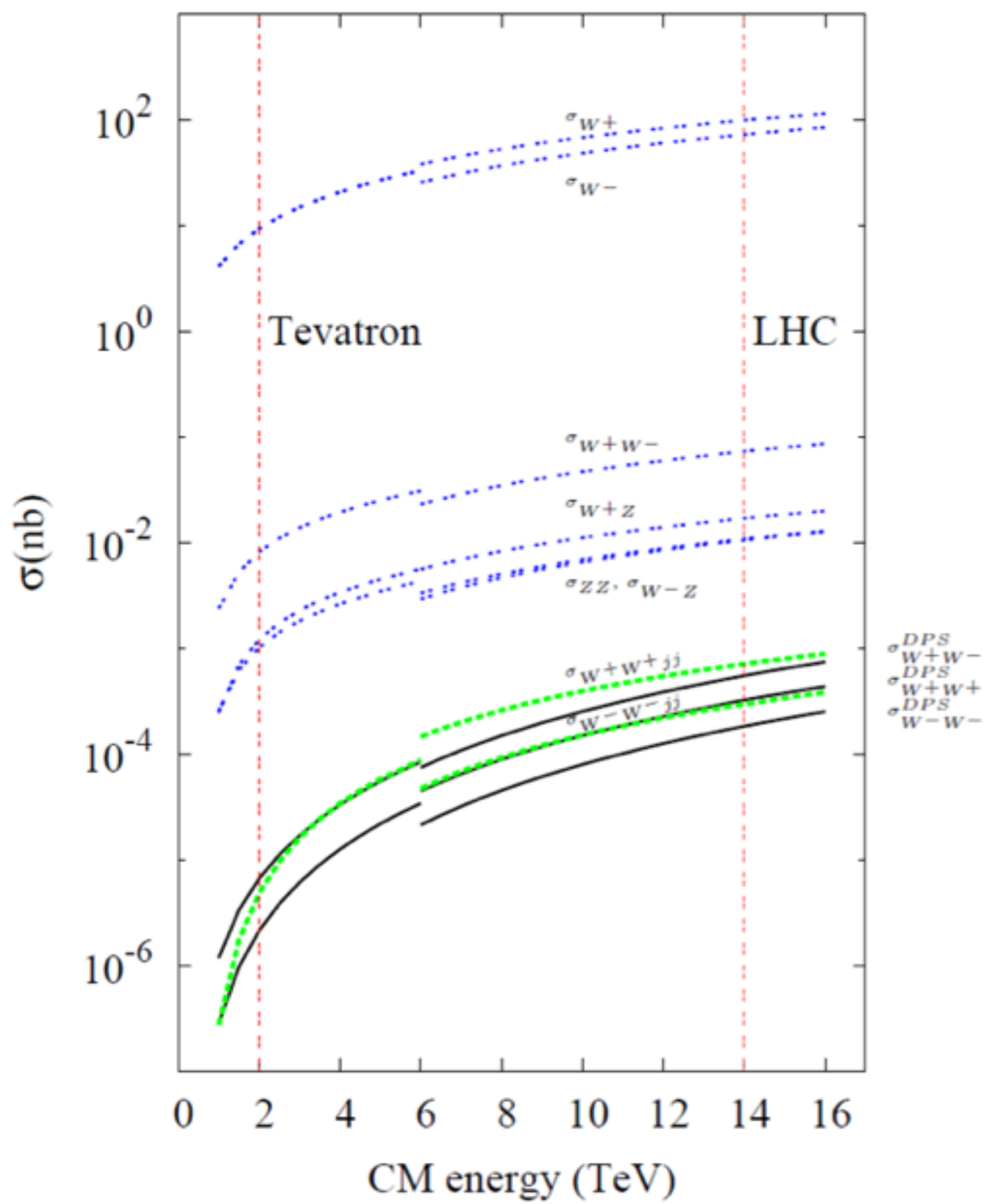
- All steps problematic and difficult to control or systematize

Cross section estimates

- Example: double same-sign W
  - small cross section but very clean
  - single parton scattering suppressed by α_s^2

    $$qq \to qq + W^+W^+$$

    and can be suppressed experimentally



Gaunt, Kom, Kulesza, and Stirling. 2010

Experimental status

- Extractions of σ_{eff} , under assumption $\sigma_{DPS} \sim \dfrac{\sigma_1 \sigma_2}{\sigma_{\text{eff}}}$
- Additional measurements in hadronic final states by LHCb in similar range
- Compared with a grain of salt (differences in assumptions on SPS etc.)

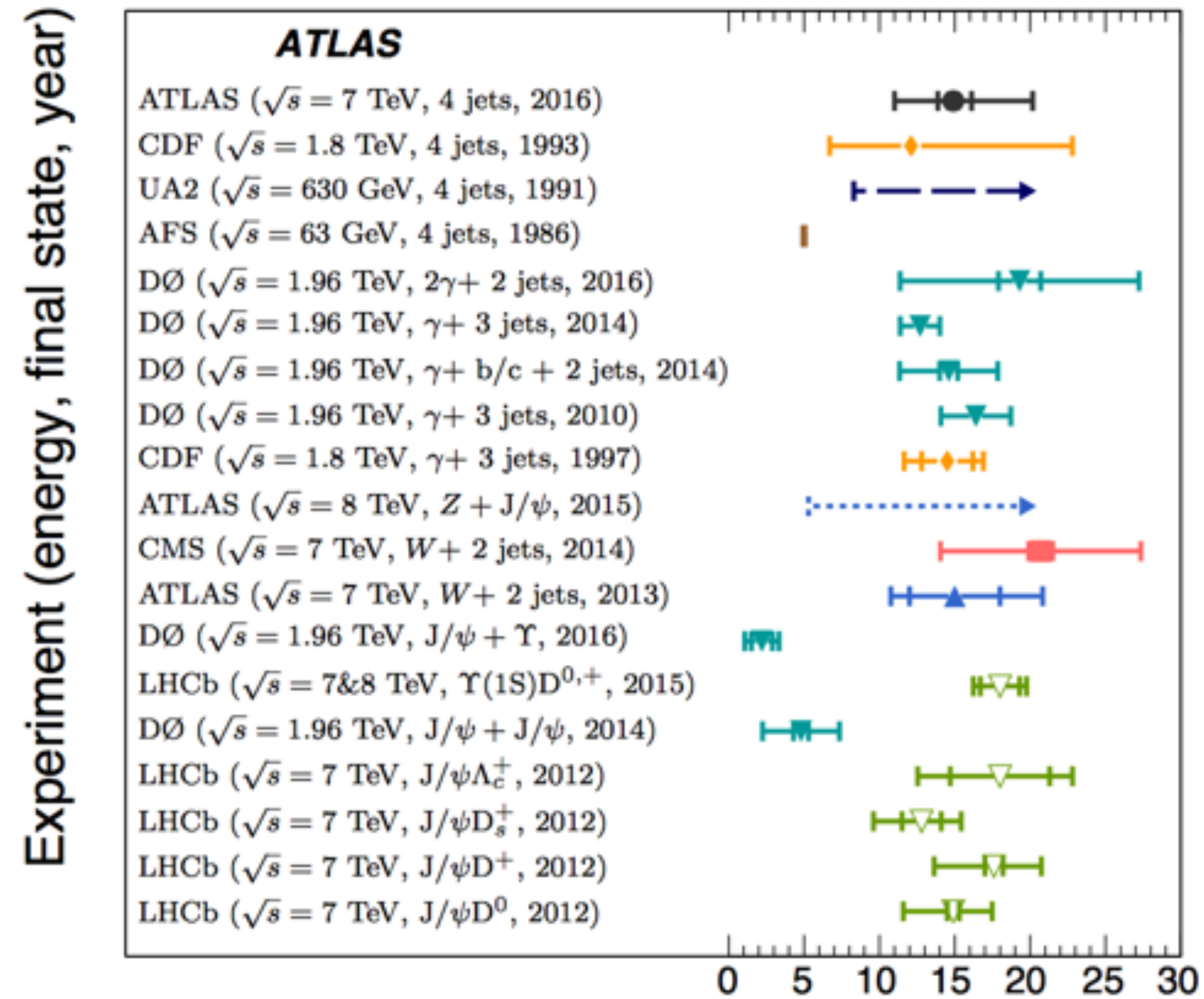


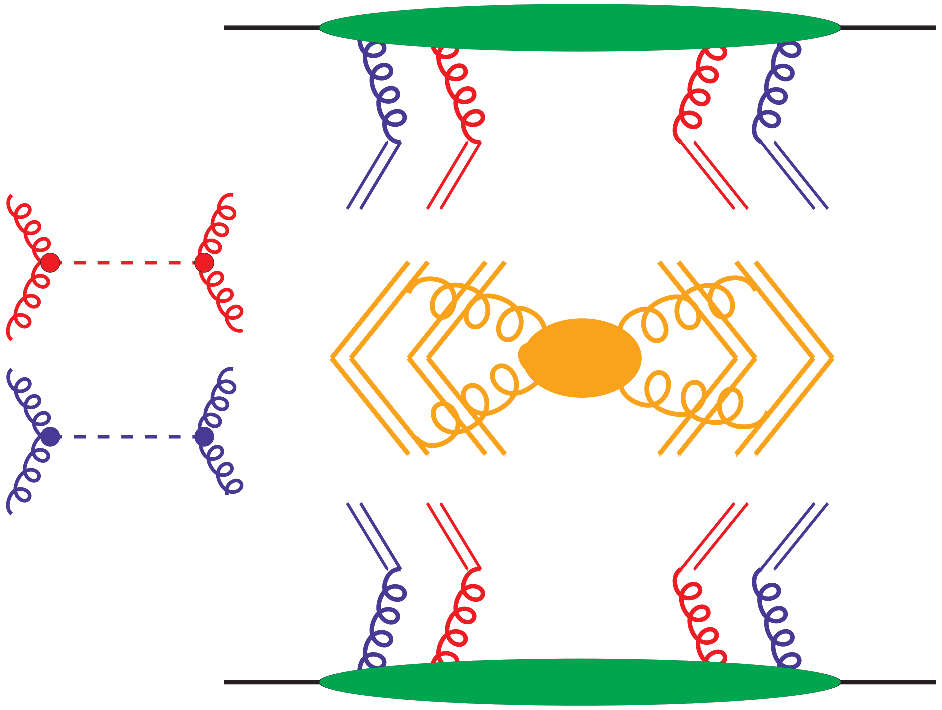
figure from ATLAS, 2016

- No (a-priori) reason to be the same
- Neglecting parton correlations, gives $\sigma_{\text{eff}} \sim 40 \text{ mb}$
  - Much larger than experimental measurements of 5-20 mb

  $\Rightarrow$ complete independence between partons disfavored

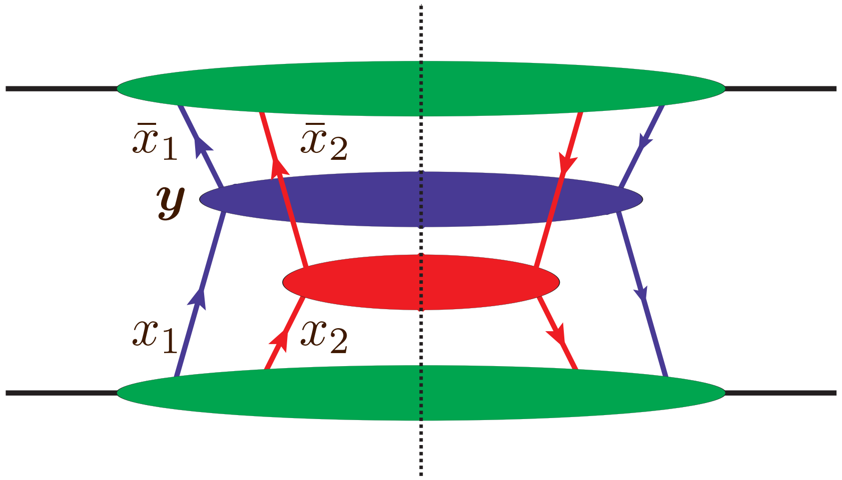
see Calucci, Treleani 1999; Frankfurt, Strikman, Weiss 2003; Blok et al 2013

MPI theory status



Cross section and DPDs

- DPS cross-section



- QCD requires inclusion of the transverse separation between hard scatterings

Paver, Treleani, 1982; Mekhfi, 1985;
Diehl, Ostermeier, Schäfer, 2011

$$d\sigma_{DPS} \sim d\sigma_1 d\sigma_2 \int d^2\boldsymbol{y} \left[f_{qq}(x_1, x_2, \boldsymbol{y}) f_{\bar{q}\bar{q}}(\bar{x}_1, \bar{x}_2, \boldsymbol{y}) + \ldots \right]$$

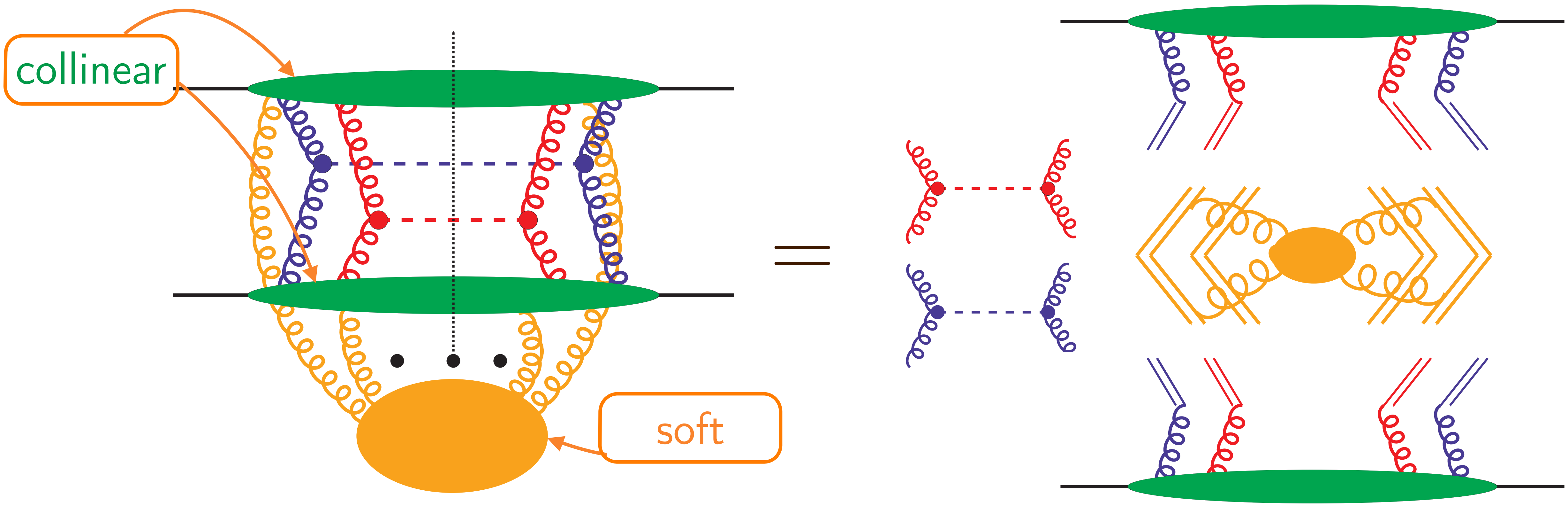
- + New phenomena !?!

Double Parton Distributions (DPDs)

- To be added to SPS to obtain total cross section, $\sigma = \sigma_{\text{SPS}} + \sigma_{\text{DPS}}$

Double parton scattering - factorization

- Factorization theorem (largely) proven for color singlet final states

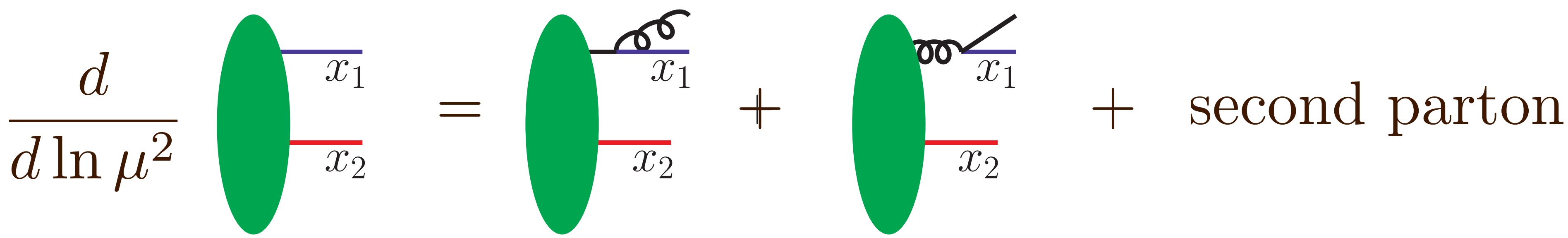


- Glauber gluons cancel for both collinear and TMD factorization
- Leading regions:
  - Hard, $n-$collinear, $\bar{n}-$collinear and soft regions
- Factorize into Hard part, Soft and Collinear matrix elements

Diehl, Gaunt, Ostermeier, Plößl, Schäfer, 2015;
Manohar and Waalewijn, 2012;
Diehl, Ostermeier, Schäfer, 2011

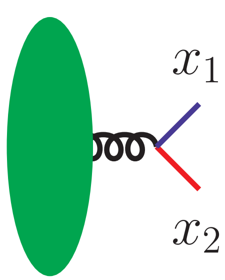
Evolution of DPDs

- Evolution of double PDFs (DPDFs) at $|\boldsymbol{y}| \neq 0$



$$\frac{d}{d\ln\mu^2} F_{ab}(x_1, x_2, \boldsymbol{y}) = \sum_c P_{b/c}(x_1') \underset{x_1}{\otimes} F_{cb}(x_1', x_2, \boldsymbol{y}) + \text{second parton}$$

- DGLAP splitting kernels for each of the two partons
- Evolve in separate branches
- The two partons can be generated from a perturbative splitting



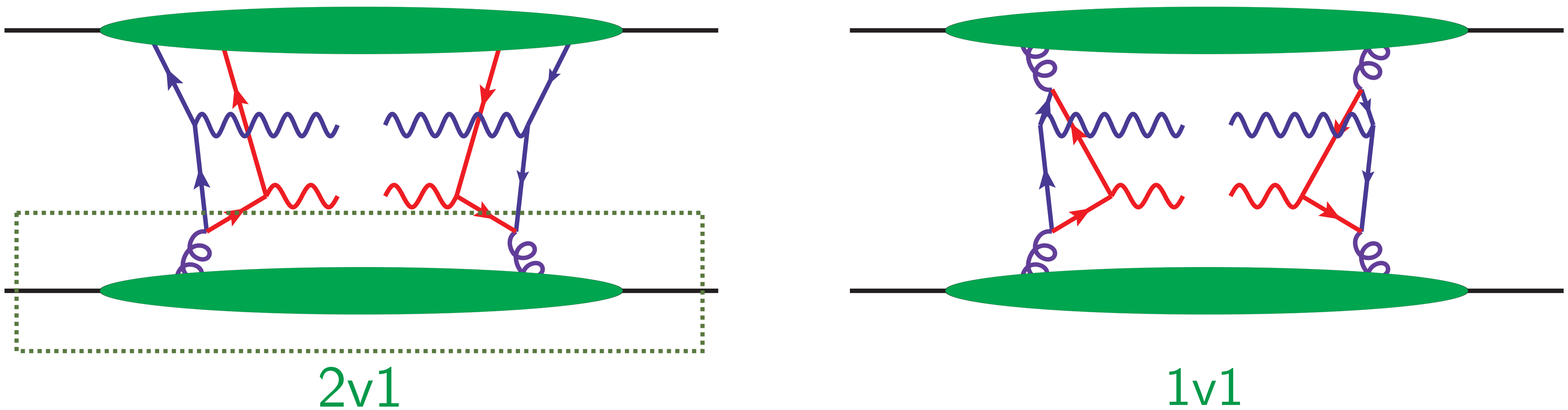
which serves as a feed in to the evolution at scale $\dfrac{1}{|\boldsymbol{y}|}$

Contribution has been under intense study and debate

Diehl, Ostermeier, Schafer, 2012; Manohar, Waalewijn, 2012; Gaunt, Stirling, 2011; Blok et al., 2012; Ryskin, Snigirev, 2011; Cacciari, Salam, Sapeta, 2010; etc.

Double or single?

- Double or single — and not to count double



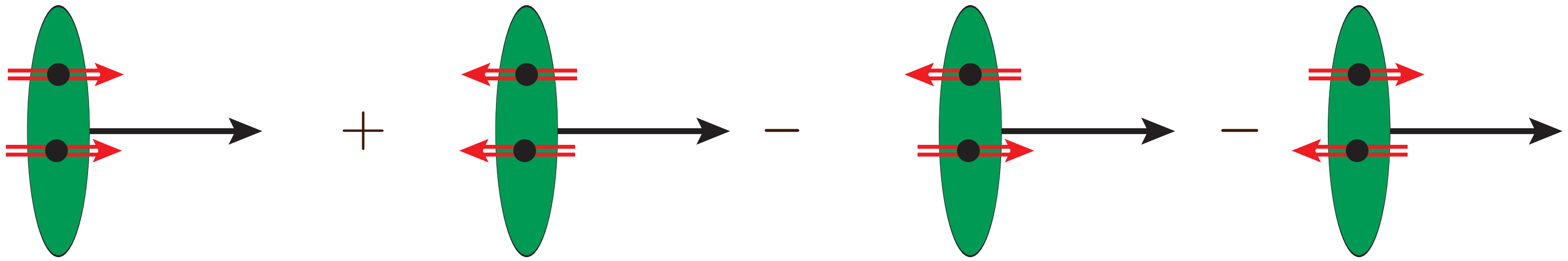
- Small $\boldsymbol{y}$: $\frac{1}{\boldsymbol{y}}$ = perturbative scale: $f_{ab}(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2}\left[T_{c\to ab} \otimes f_c(x_1 + x_2)\right]$

- Naive 1v1 cross section: $\implies \sigma \propto \int d^2\boldsymbol{y} \left(\frac{1}{\boldsymbol{y}^2}\right)^2 \to \text{UV divergent!}$

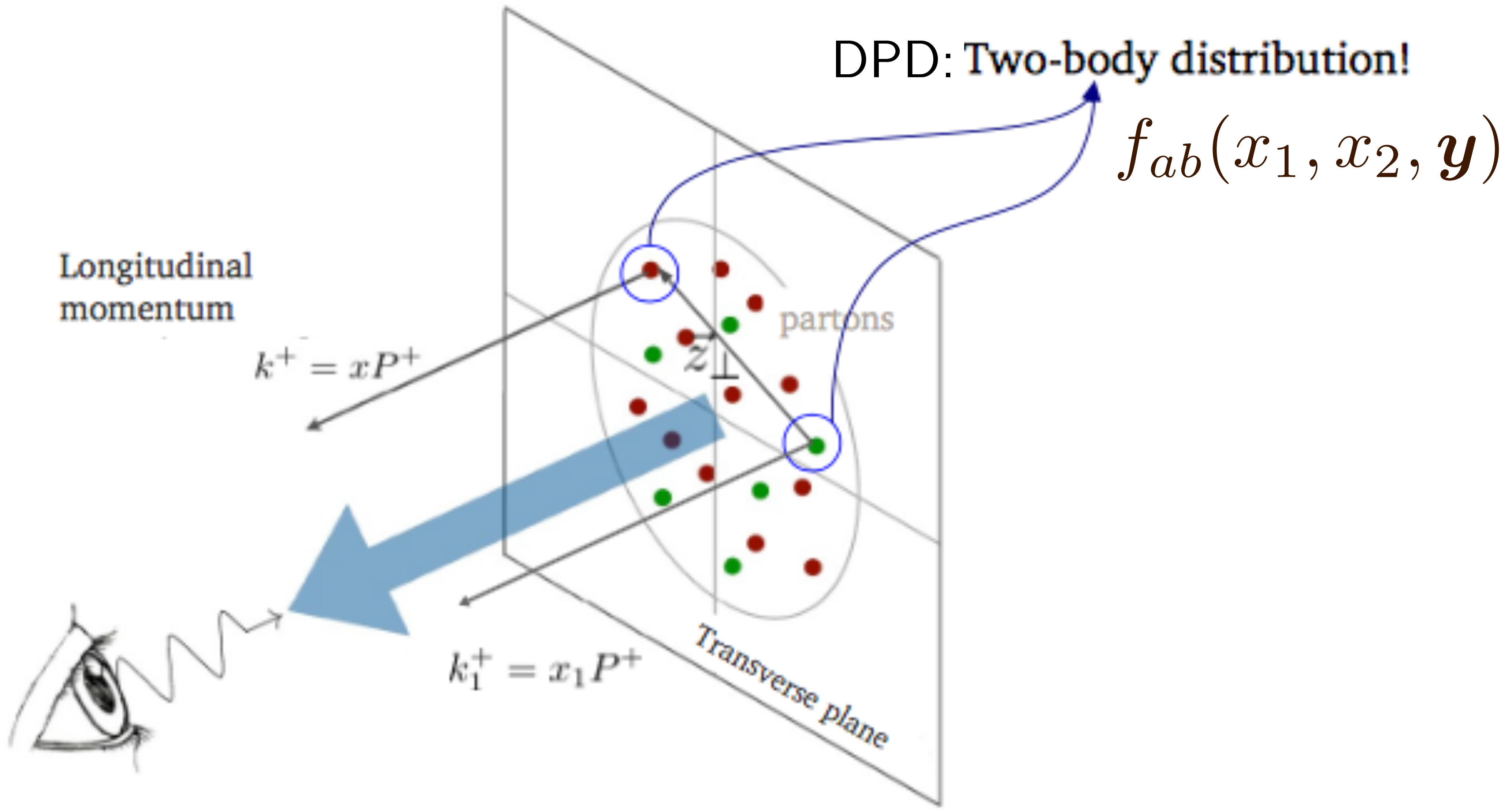
  power divergence in naive DPS including pert. splitting
  (= "leaking" of leading power SPS into DPS)

- Solution: DPD includes splitting, regulate small y limit of cross section and subtract to avoid double counting, $\sigma = \sigma_{DPS} - \sigma_{sub} + \sigma_{SPS}$ Diehl, Gaunt, 2016

DPS correlations



Double parton distributions (DPDs)

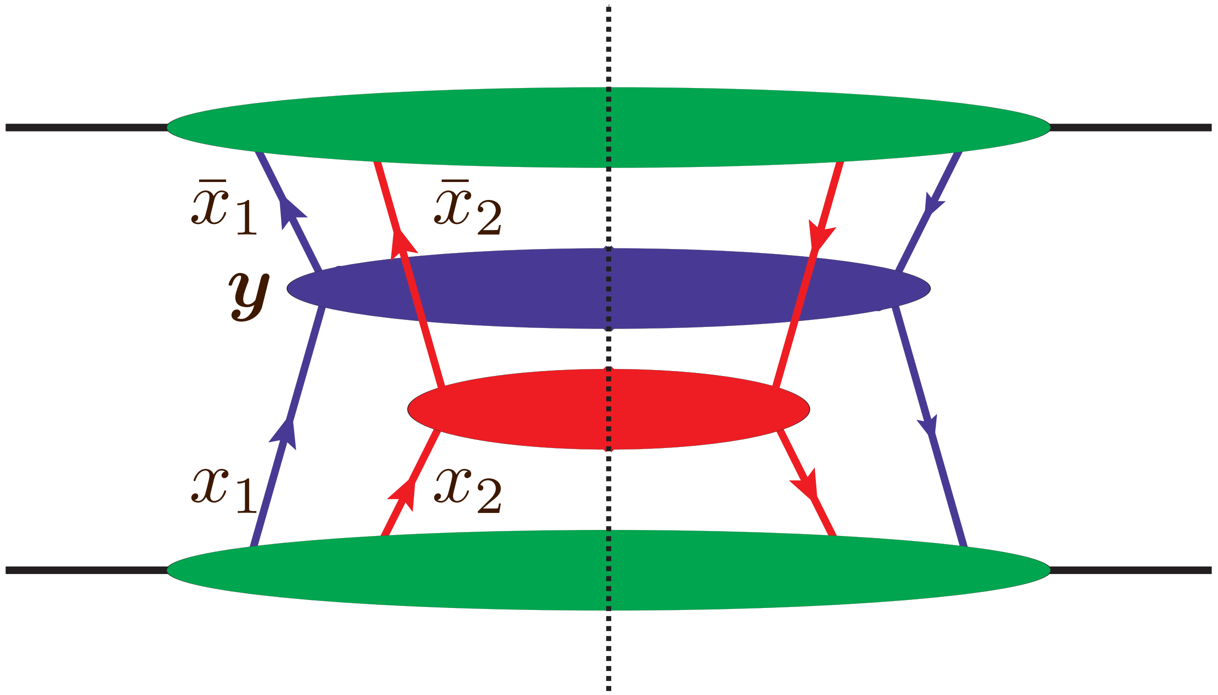


New way to access information on the non-perturbative structure of the PROTON!

from Matteo Rinaldi, MPI@LHC 2015

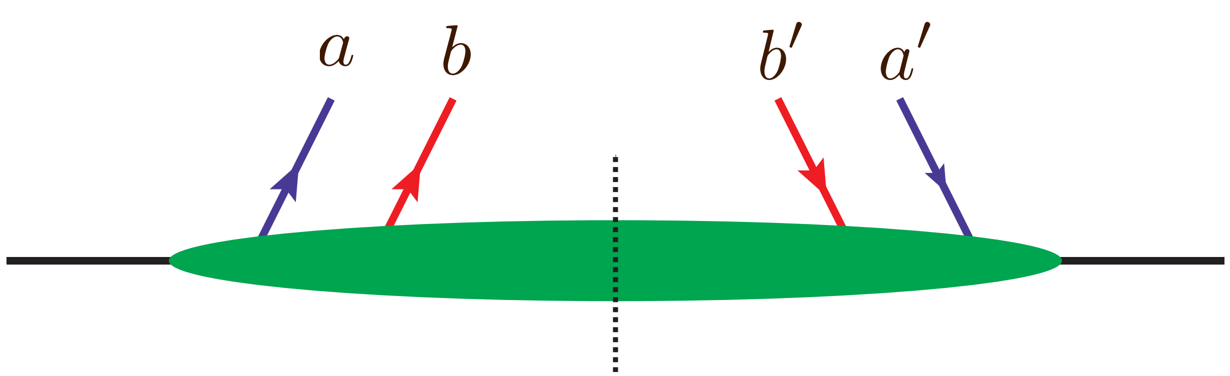
Double parton scattering

- DPS cross section:



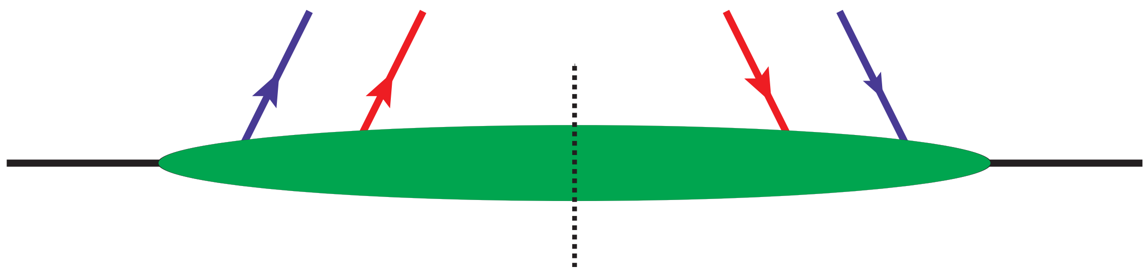
$$\frac{d\sigma_{DPS}}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} = \frac{1}{C}\hat{\sigma}_1\hat{\sigma}_2 \int d^2\boldsymbol{y} F(x_1, x_2, \boldsymbol{y}) F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

- Correlations encoded in the double parton correlator



$$(a+b) = (a'+b') \not\Leftrightarrow \begin{cases} a = a' \\ b = b' \end{cases}$$

Correlations in DPS



- Color
- Fermion number interference
- Spin (polarization)
    - longitudinal
    - transverse/linear
- Flavor interference
- Between $\boldsymbol{y}$ and $\boldsymbol{x}$'s
- Parton type and $\boldsymbol{y}$
- Between $\boldsymbol{x}$'s

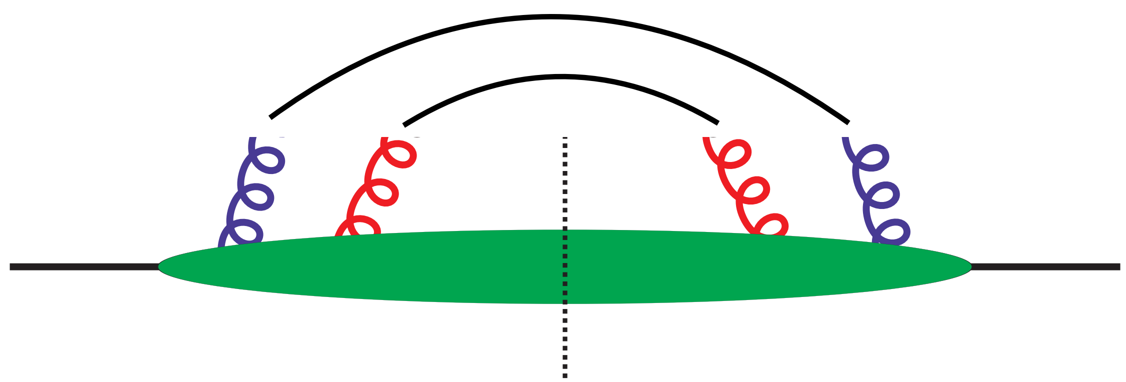
- DPS cross section:

Depend on x_1, x_2, $\boldsymbol{y}$, spin, flavor, color and scale

$$\frac{d\sigma}{dx_1\, dx_2\, dx_3\, dx_4} = \frac{1}{C} \sum_{p_1, p_2, p_3, p_4} \hat{\sigma}_{p_1 p_3}(x_1, x_3)\, \hat{\sigma}_{p_2 p_4}(x_2, x_4)$$
$$\times \int d^2\boldsymbol{y}\, F_{p_1 p_2}(x_1, x_2, \boldsymbol{y})\, F_{p_3 p_4}(x_3, x_4, \boldsymbol{y})$$
$$+ \ \{\text{color, flavour and fermion number interference terms}\}$$

Color structure

- Color structure for double gluon distribution:



- Coupling the gluons with their partners in the conjugate amplitude into:

$$8 \otimes 8 = 1 \oplus 8^A \oplus 8^S \oplus 10 \oplus \bar{10} \oplus 27$$

- Results in 5 independent double gluon distributions with different color structures - which all do contribute to the cross sections

$${}^{1}F_{gg},\ {}^{8^S}F_{gg},\ {}^{8^A}F_{gg},\ {}^{10+\bar{10}}F_{gg},\ {}^{27}F_{gg}$$

Diehl, Schäfer, Ostermeier, 2011;
TK, Mulders, 2014;

Color correlations

- Color singlet and octet distributions

$${}^{1}F_{q_1 q_2} \to (\bar{q}_2 \mathbb{1} q_2)(\bar{q}_1 \mathbb{1} q_1) \qquad {}^{8}F_{q_1 q_2} \to (\bar{q}_2 t^a q_2)(\bar{q}_1 t^a q_1)$$

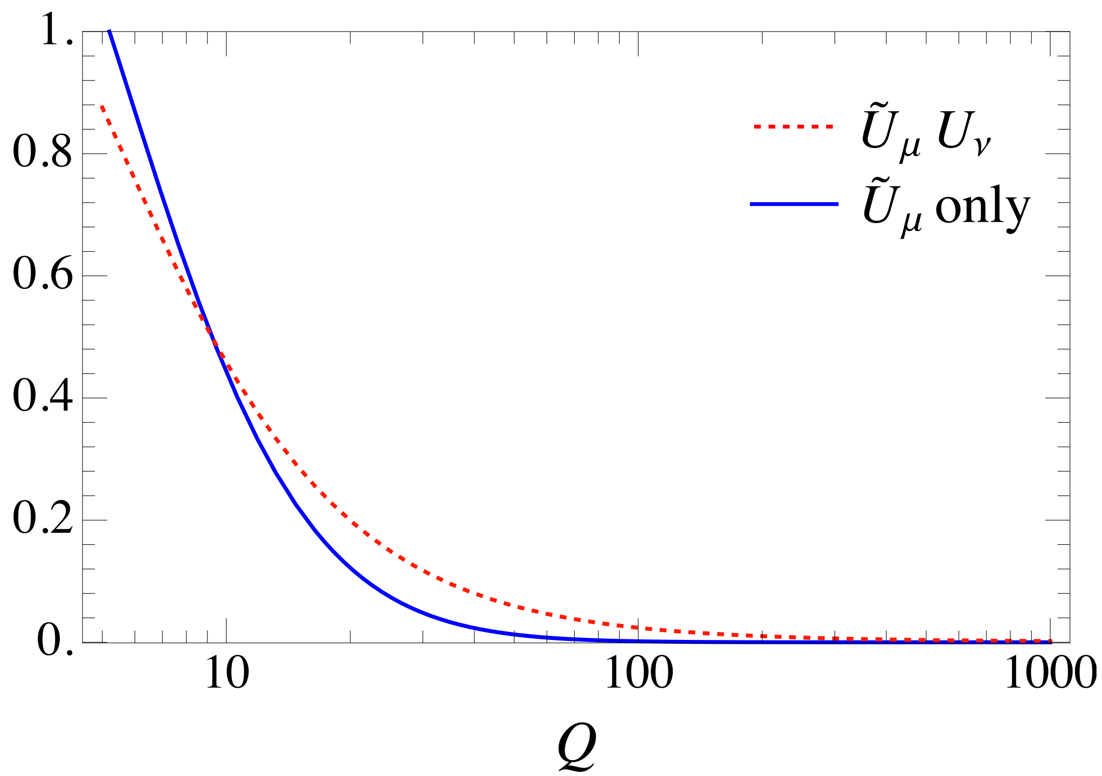
- Color correlations enter cross section weighted by a Sudakov factor

  $\Rightarrow$ Suppressed at large Q

Manohar, Waalewijn, 2012;
Mekhfi, Artru, 1985

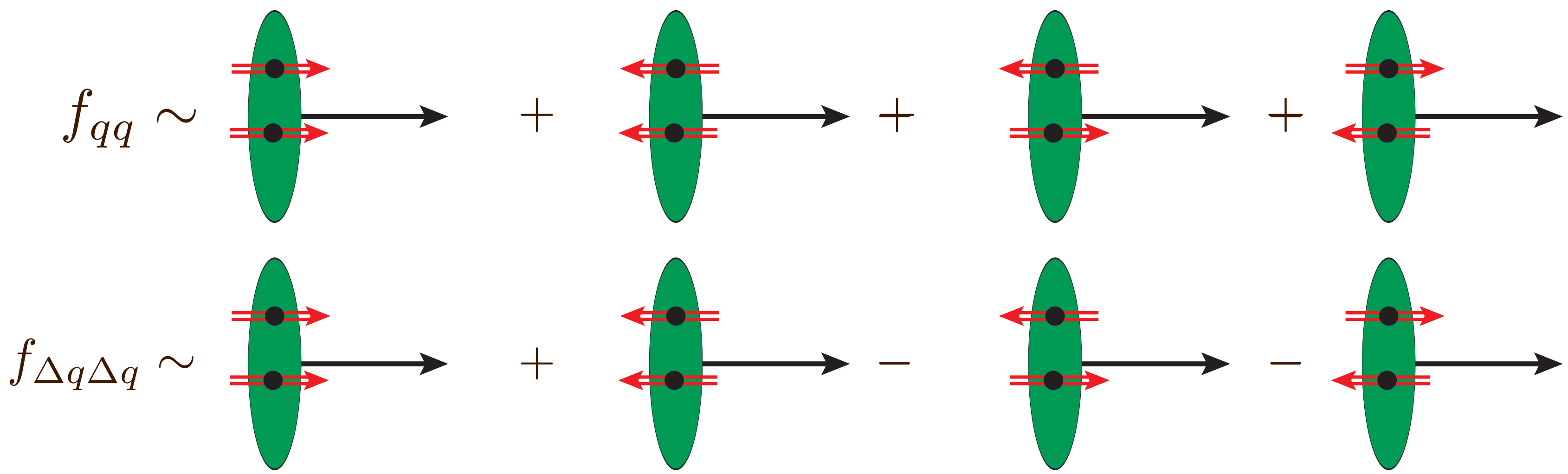
$$\tilde{U}_\mu(\Lambda, Q) = \exp\left[-\frac{\alpha_s C_A}{2\pi} \ln^2 \frac{Q^2}{\Lambda^2}\right]$$

- Color correlations should not be relevant at large scales.
- Interpretation:
  Transportation of color over hadronic distance



Manohar, Waalewijn, 2012

Polarization



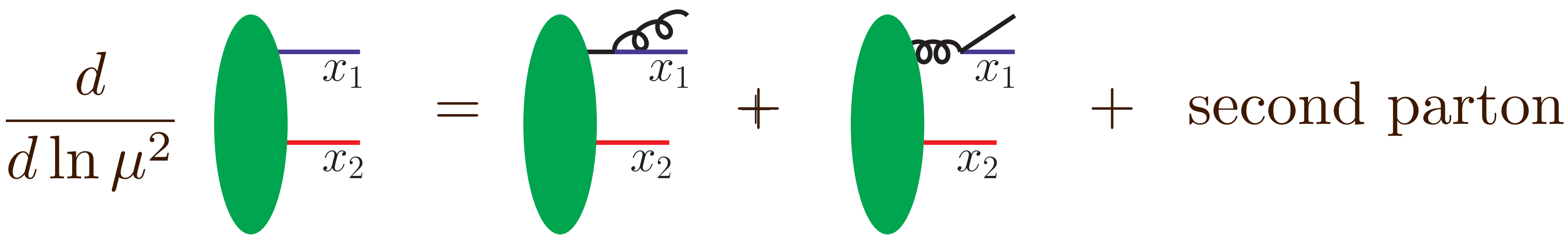
- Two partons in an unpolarized proton can each be unpolarized, longitudinally polarized and linearly/transversely polarized,
  - Correlations between spin, transverse momenta and separation of the two partons

    Mekhfi, 1985; Diehl, Schäfer, 2011; Diehl, Ostermeier, Schäfer, 2011
  - Several polarized DPDs which contribute to DPS cross sections
  - Large in model calculations

    Chang, Manohar, Waalewijn, 2011; Rinaldi, Scopetta, Traini, Vento, 2014; TK, Mukherjee, 2016;
- Changes total cross sections, distributions of final state particles and cause azimuthal asymmetries/spin asymmetries

  Manohar, Waalewijn, 2011; Diehl, TK, 2012; Echevarria, TK, Mulders, Pisano 2015

Evolution of polarization



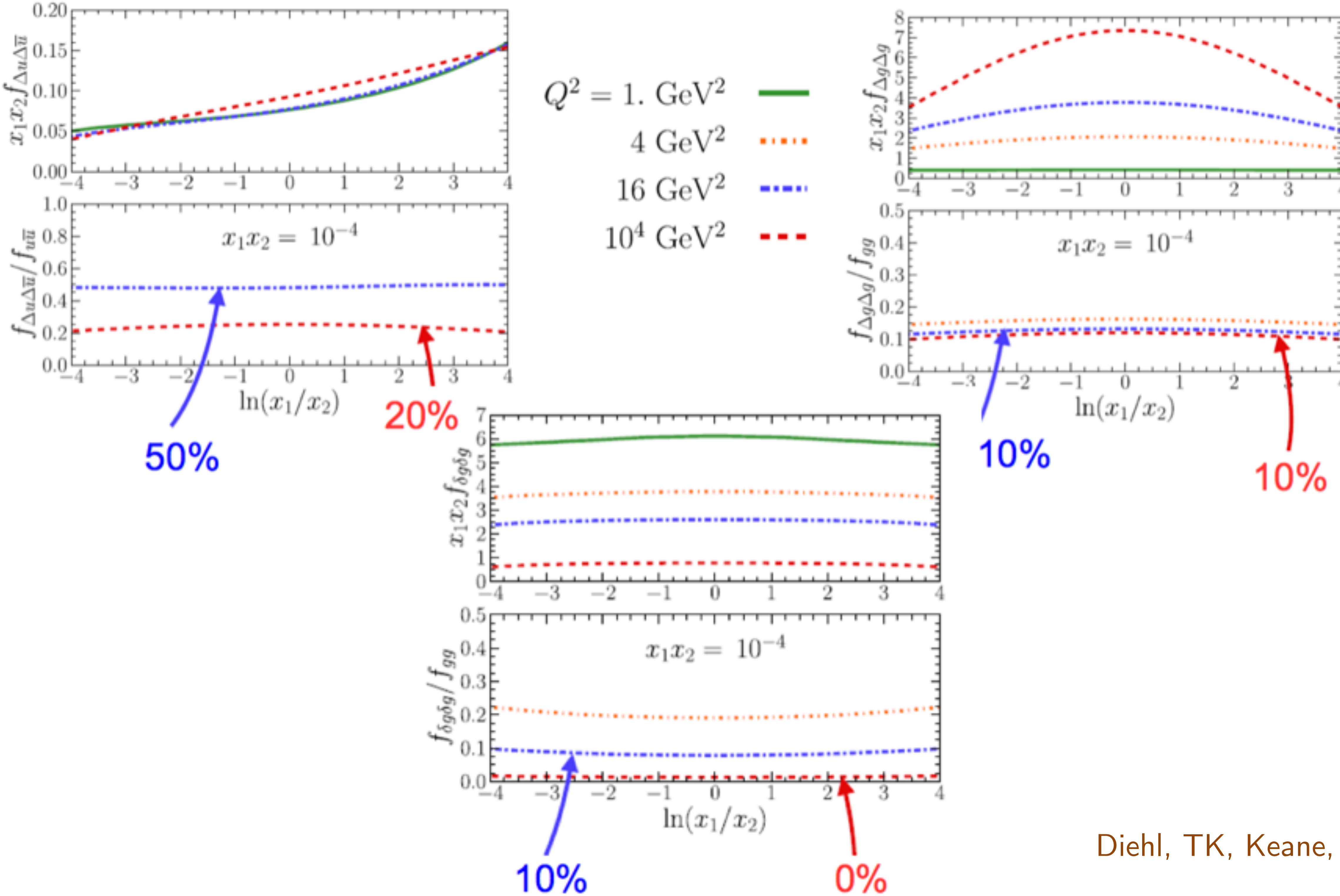
- DGLAP splitting kernels (for color singlet unpolarized DPDs)
- Separate branchings - expect evolution to wash out correlations
- Evolution + positivity bounds -upper limits on correlations

Diehl, TK, 2013; TK, Mulders, 2014

- At medium to large scales and small momentum fractions - gluon polarization suppressed
  - Because of rapid increase of unpolarized distribution

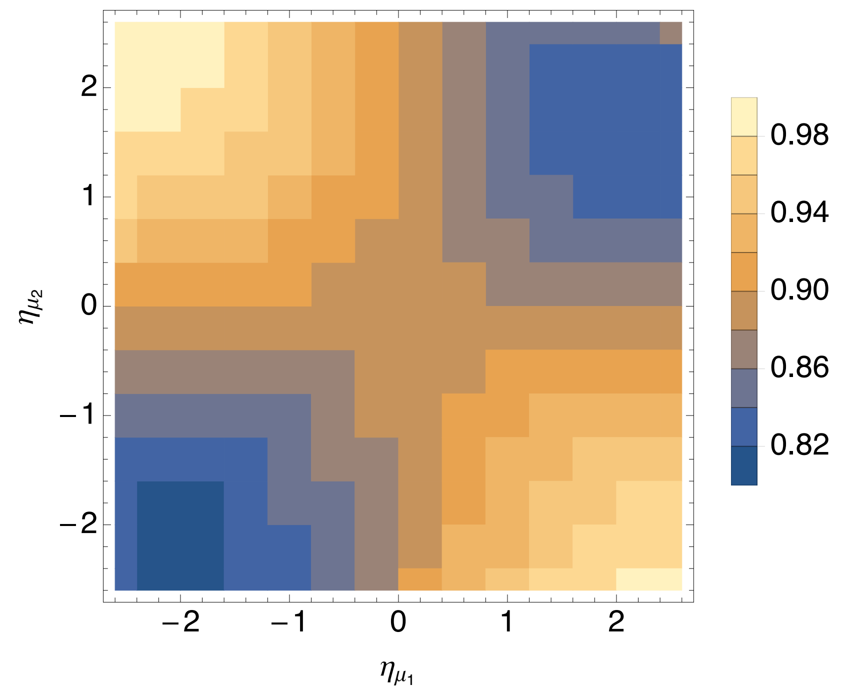
Diehl, TK, 2014

Evolution of polarization



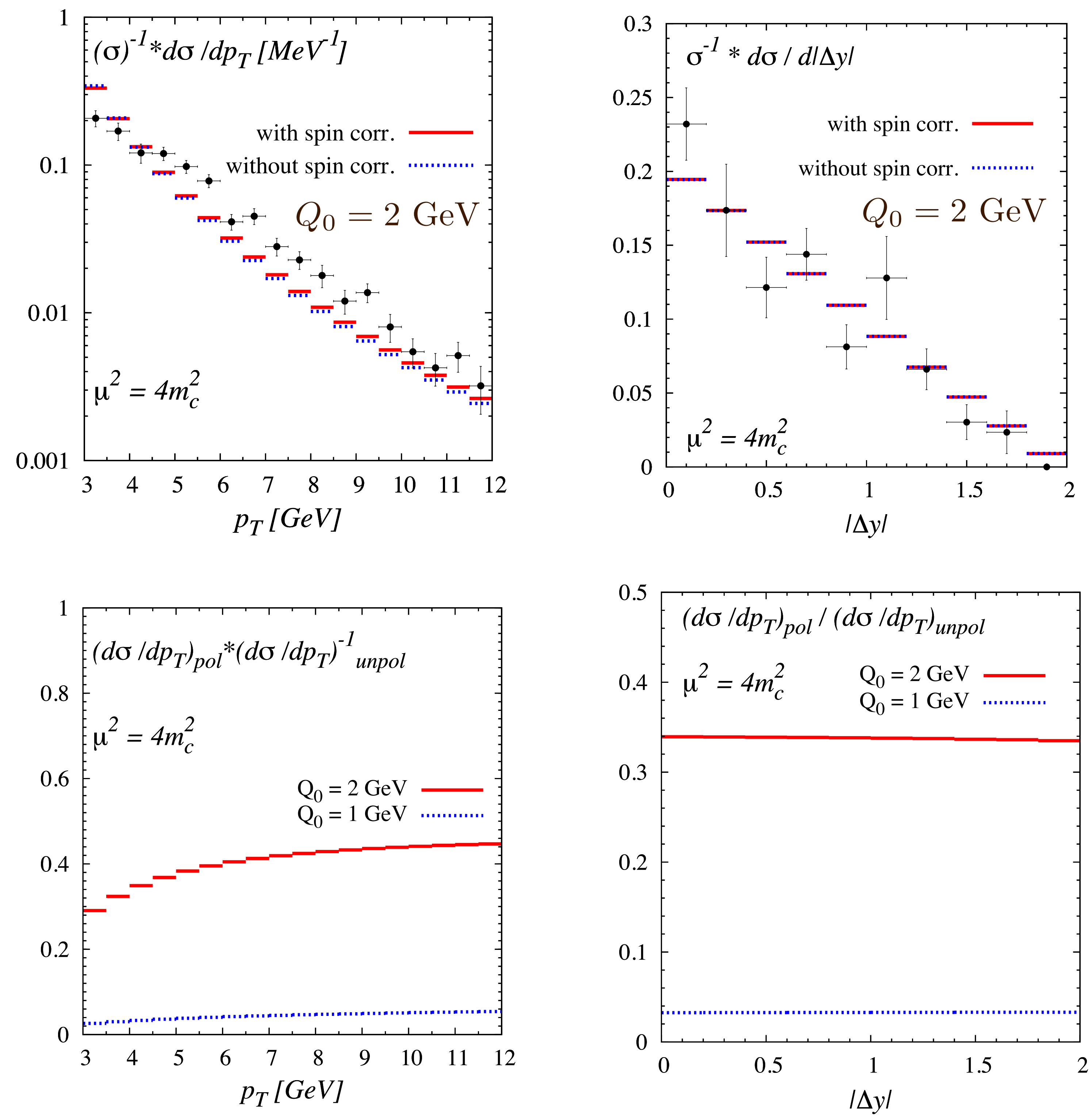
Diehl, TK, Keane, 2014

Parton correlations and cross sections



Cross section vs transverse momentum

- $D^0 D^0$ data from LHCb
- Starting from maximal polarization at low scale
- Polarization can give large contributions, both to magnitude and shape
- Strong dependence on input scale



Echevarria, TK, Mulders, Pisano 2015

Double (same sign) W:

- Small cross section but very clean (measurable in LHC run 2)
- Ansatz for the initial DPDs at $Q_0 = 1\ \mathrm{GeV}$
  For unpolarized DPDs $f_{p_1p_2}(x_1, x_2, \boldsymbol{y}; Q_0) = \tilde{f}_{p_1p_2}(x_1, x_2; Q_0)\, G(\boldsymbol{y})\,,$
  with
  Alt 1: $\tilde{f}_{ab}(x_1, x_2; Q_0) = f_a(x_1; Q_0)\, f_b(x_2; Q_0)\,,$
  Alt 2:
  $$\tilde{f}_{ab}(x_1, x_2; Q_0) = \frac{(1-x_1-x_2)^2}{(1-x_1)^2(1-x_2)^2} f_a(x_1; Q_0)\, f_b(x_2; Q_0)\,,$$
- Single PDFs from MSTW 2008

Martin, Stirling, Thorne, Watt, 2009;

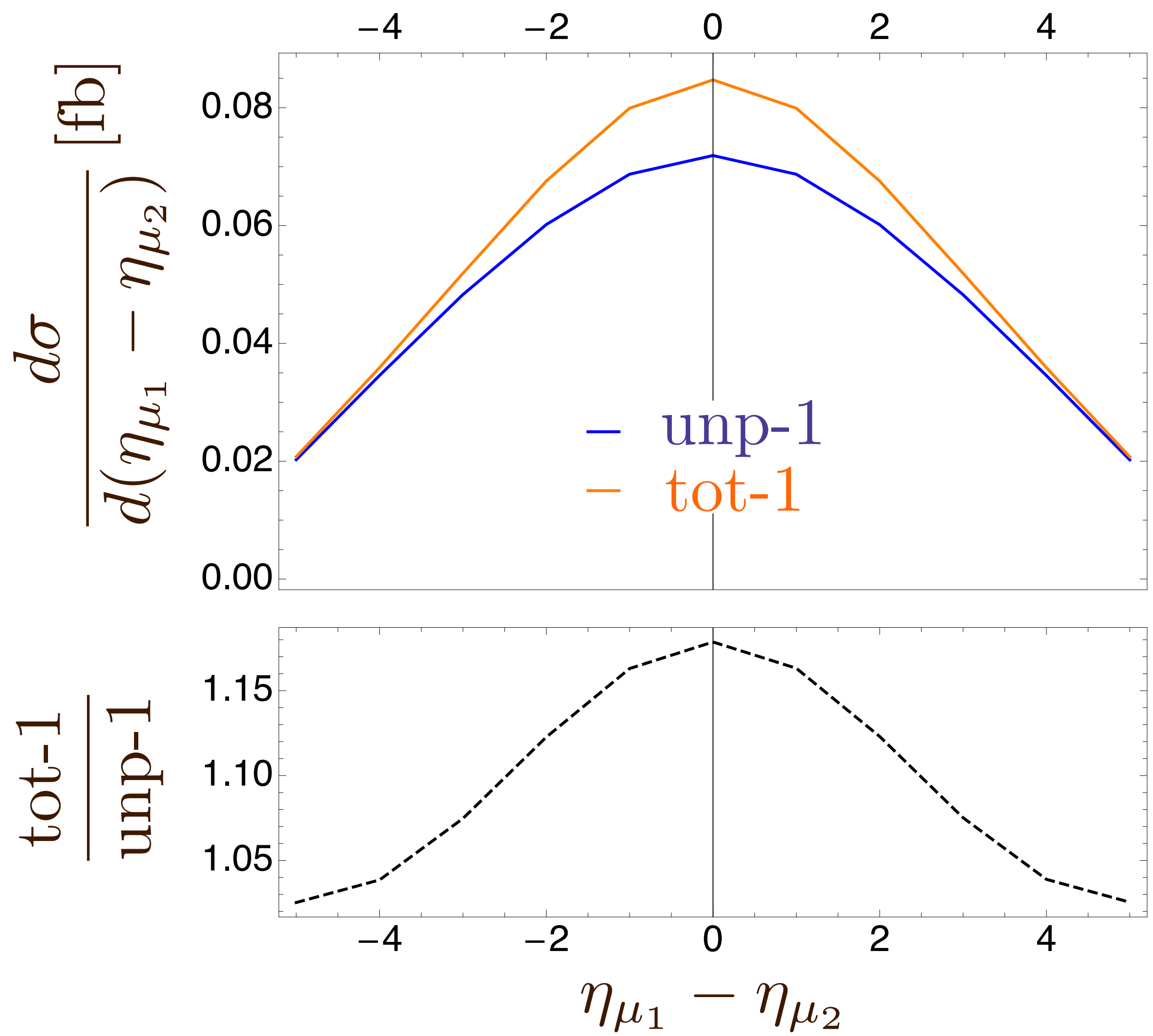
- Set, $\sigma_{\text{eff}} = 1/\int d^2\boldsymbol{y}\, G^2(\boldsymbol{y}) \approx 15\ \mathrm{mb}$
- Evolve with double DGLAP to hard scale
- Polarized DPDs:
  - saturate positivity bound at input scale. Evolve with polarized version of double DGLAP.

Preliminary

Results for double W:

- DPS cross section results at LO for two $W^+ \to \mu^+ \nu_\mu$ at $\sqrt{s} = 7$ TeV
  - cuts: muon rapidity $-2.5 \leq \eta_{\mu_i} \leq 2.5$
  - Absolute size proportional to $1/\sigma_{\text{eff}}$ can easily vary by factor of 2

	$\sqrt{s} = 7$ TeV [fb]
$\sigma_{\text{unp-1}}$	0.59
$\sigma_{\text{tot-1}}$	0.66
σ_{indep}	0.61
$\sigma_{\text{unp-2}}$	0.44
$\sigma_{\text{tot-2}}$	0.48
	Ratios
$\sigma_{\text{tot-1}}/\sigma_{\text{unp-1}}$	1.11
$\sigma_{\text{indep}}/\sigma_{\text{unp-1}}$	1.03
$\sigma_{\text{unp-2}}/\sigma_{\text{unp-1}}$	0.74
$\sigma_{\text{tot-2}}/\sigma_{\text{tot-1}}$	0.73
$\sigma_{\text{tot-2}}/\sigma_{\text{unp-2}}$	1.10



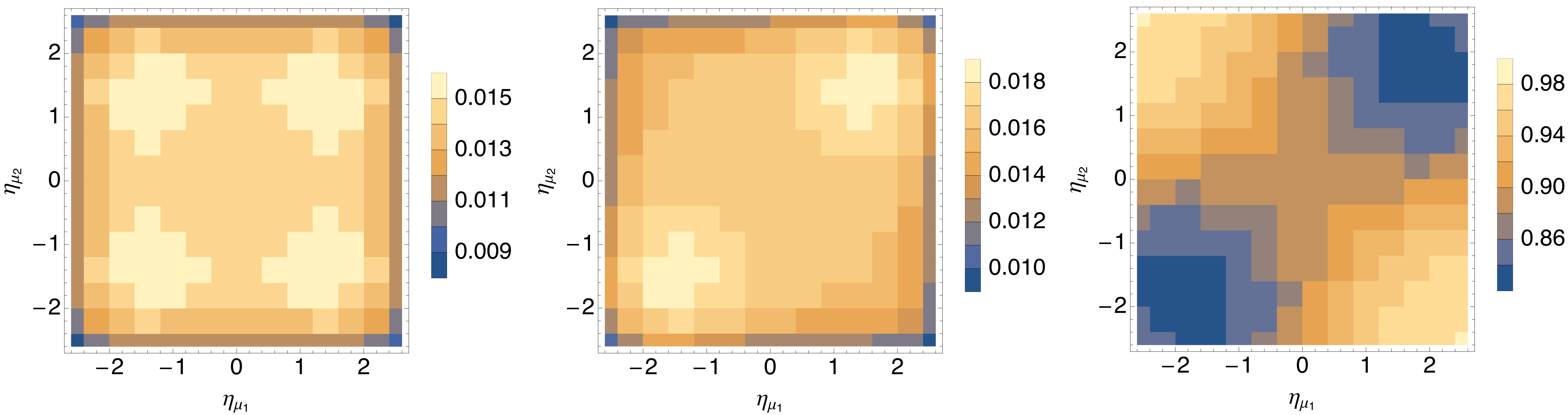
work in progress with S. Cotogno

Preliminary

Results for double W:

- Double differential in muon rapidities

$d\sigma_{\text{unp-1}} / d\eta_{\mu_1} d\eta_{\mu_2} [fb]$ $d\sigma_{\text{tot-1}} / d\eta_{\mu_1} d\eta_{\mu_2} [fb]$ unp-1/tot-1



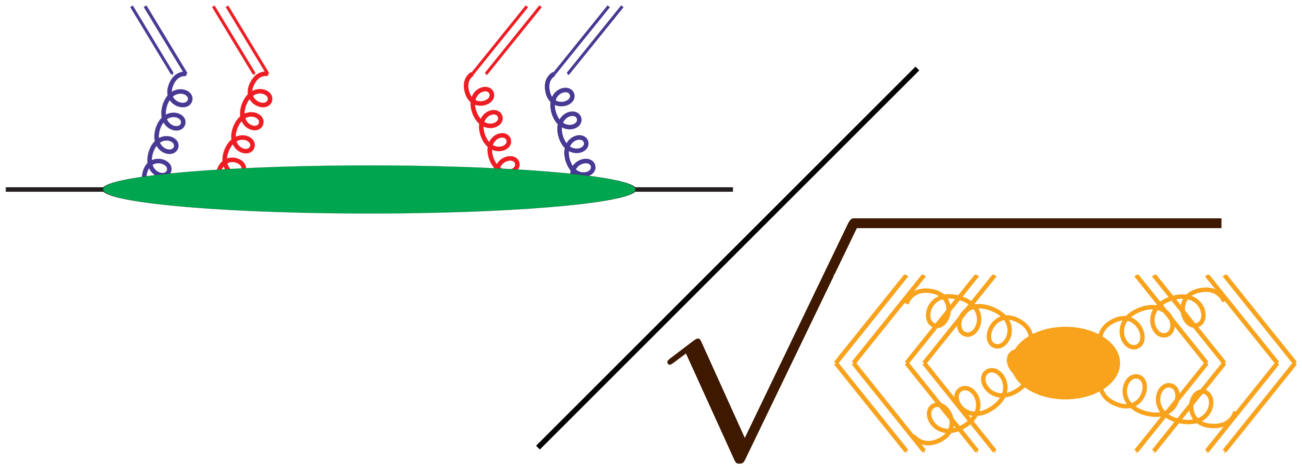
- Same / opposite hemisphere

	$\frac{\sigma(\text{same})}{\sigma(\text{opposite})}$, $\sqrt{s} = 7$ TeV
indep	1.00
unp-1	1.00
unp-2	0.98
tot-1	1.09
tot-2	1.13

= promising variable to detect correlations

work in progress with S. Cotogno

Transverse momentum in DPS



work (in progress) with Maarten Buffing and Markus Diehl

Transverse momentum dependence in DPS

- Most of single parton scattering at low transverse momentum
  - Accurately describing this region including non-perturbative effects
  - Factorization: for color singlet production in proton collisions (i.e. Drell-Yan, Higgs boson in gluon fusion)

    Collins, Soper Sterman, 80s; ...Collins, 2011
- Combine collinear and soft: TMDPDFs free from rapidity divergencies
  - Depend on two scales: renormalization and rapidity reg.

    Collins, 2011; Echevarria, Idilbi, Scimemi, 2011; Echevarria, TK, Mulders, Pisano, 2015
- Production of two color singlets (in DPS)
  - For example double Higgs, vector bosons, color singlet quarkonia etc.
- Small pT region: DPS contribute at leading power

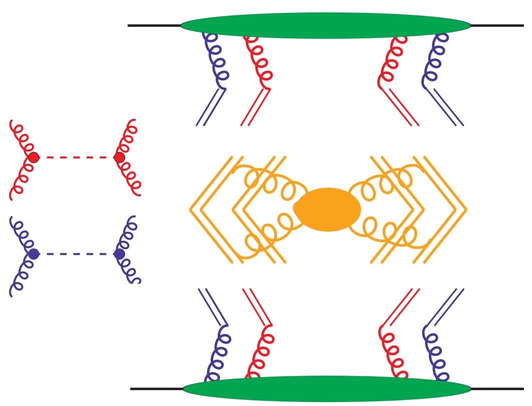
  Diehl, Ostermeier, Schäfer, 2011

  $\Rightarrow$ Low pT region, of twofold importance for DPS
- Theoretically well grounded treatment of double TMDPDFs (DTMDs)

Transverse momentum dependence in DPS

- Set up the theoretical (DTMD) framework, within QCD
  - As few assumptions as possible
  - As much perturbative input as possible, to enhance predictive power
- Provide the basis, correctly including and treating the different effects.
  - Once set up in place, can introduce modeling and approximations to connect with experiments
- Additional difficulties compared to TMDs for SPS
  - Different regions which require different matchings
  - Color (and polarization) structure
  - etc.
- Compared to the pocket formula, it represents the other end of DPS research

Soft and collinear functions

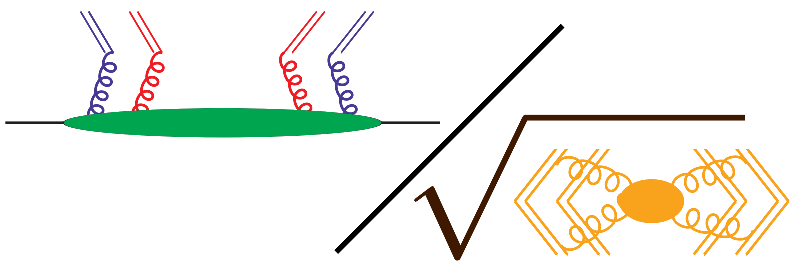


- DPS cross section proportional to

$$F^T_{\mathrm{us},gg}\, S^{-1/2}_{gg}\, S^{-1/2}_{gg}\, F_{\mathrm{us},gg} = F^T_{gg}\, F_{gg}$$

- We define rapidity divergency free DTMDs as

$$F_{gg}(v_C) = \lim_{y_L^2 \to -\infty} S^{-1/2}_{gg}[2(y_C - y_L)]\, F_{\mathrm{us},gg}(v_L)\,,$$



  - Collinear matrix element

$$F_{\mathrm{us},gg}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}) \sim \int \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi}\, dy^-\, e^{-i(x_1 z_1^- + x_2 z_2^-)p^+} \times \langle p|\, \mathcal{O}_g(0, z_2)\, \mathcal{O}_g(y, z_1)\, |p\rangle\,,$$

Diehl, Schäfer, Ostermeier, 2011

operators dressed by Wilson lines (adjoint rep.)

$$\mathcal{O}_{g_i}(y, z_i) = g_{T\mu\nu}\, \mathcal{W}^\dagger G^{+\nu}\, \mathcal{W} G^{+\mu}\Big|_{z_i^+ = y^+ = 0},$$

  - Soft function

$$S_{gg} \sim \langle 0\, |\mathcal{W}\mathcal{W}^\dagger\, \mathcal{W}\mathcal{W}^\dagger \mathcal{W}\mathcal{W}^\dagger\, \mathcal{W}\mathcal{W}^\dagger|\, 0\rangle$$

DTMD cross section

- For color singlet production (photon, z, Higgs etc.) at $|\boldsymbol{q}_{1,2}| \sim q_T \ll Q$

$$\frac{d\sigma_{\mathrm{DPS}}}{dx_1\, dx_2\, d\bar{x}_1\, d\bar{x}_2\, d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} = \frac{1}{C} \sum_{a_1,a_2,b_1,b_2} \hat{\sigma}_{a_1 b_1}(q_1^2, \mu_1^2)\, \hat{\sigma}_{a_2 b_2}(q_2^2, \mu_2^2)$$

$$\times \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2} \frac{d^2\boldsymbol{z}_2}{(2\pi)^2}\, d^2\boldsymbol{y}\; e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2}\, W_{a_1 a_2 b_1 b_2}(\bar{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \nu)$$

with:

$$W = \Phi(\nu \boldsymbol{y}_+)\, \Phi(\nu \boldsymbol{y}_-) \sum_R {}^{R}F_{b_1 b_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \bar{\zeta})\, {}^{R}F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta)$$

$\Phi(\nu \boldsymbol{y}_\pm)$ removes UV region $\boldsymbol{y}_\pm \ll 1/\nu$. Choose $\nu \sim Q$. $\boldsymbol{y}_\pm = \boldsymbol{y} \pm \frac{1}{2}(\boldsymbol{z}_1 - \boldsymbol{z}_2)$

Φ dependence cancelled by subtraction

- Double TMDs (DTMDs) ${}^{R}F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta)$ depend on:

$R = 1, 8, \ldots$ color label, $a_{1,2}$, $b_{1,2}$ = parton and polarization label

$x_{1,2}$ = momentum fractions

$\boldsymbol{y}$, $\boldsymbol{z}_{1,2}$ = transverse distances

$\mu_{1,2}$ = UV renormalization scales

ζ = rapidity regularization scale, $\zeta\bar{\zeta} = Q_1^2 Q_2^2$

Scale evolution

- UV and rapidity scale

$$\frac{\partial}{\partial \log \mu_1} {}^{R}F_{a_1a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta) = \gamma_{F,a_1}(\mu_1, x_1\zeta/x_2)\, {}^{R}F_{a_1a_2}$$

$$\frac{\partial}{\partial \log \zeta} {}^{R}F_{a_1a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta) = \frac{1}{2}\, {}^{RR'}K_{a_1a_2}(\boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})\, {}^{R'}F_{a_1a_2}$$

- Complicated functions (3 transverse vectors!), little predictive power
- When $\Lambda \ll q_T \ll Q$: $|\boldsymbol{q}_1| \sim |\boldsymbol{q}_2| \sim |\boldsymbol{q}_1 \pm \boldsymbol{q}_2| \sim q_T$

$$\int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2} \frac{d^2\boldsymbol{z}_2}{(2\pi)^2}\, d^2\boldsymbol{y}\, e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2}\, W_{a_1a_2b_1b_2}(\bar{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_1, \mu_2, \nu)$$

then region of perturbative $|\boldsymbol{z}_i| \sim 1/q_T$ dominates result

- But what about the size of $\boldsymbol{y}$
— can be either small $|\boldsymbol{y}| \sim 1/q_T$ or large $|\boldsymbol{y}| \sim 1/\Lambda$

Region or large $\boldsymbol{y}$

- Scalings $|\boldsymbol{z}_i| \sim \frac{1}{q_T}, |\boldsymbol{y}| \sim \frac{1}{\Lambda}$ $\qquad \Lambda \ll q_T \ll Q$
- Match DTMDs onto the DPDFs

$$ {}^{R}F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}) = \sum_{b_1 b_2} {}^{R}C_{f,a_1 b_1}(x_1', \boldsymbol{z}_1) \underset{x_1}{\otimes} {}^{R}C_{f,a_2 b_2}(x_2', \boldsymbol{z}_2) \underset{x_2}{\otimes} {}^{R}F_{b_1 b_2}(x_i', \boldsymbol{y}) $$

  - Mixing between quark and gluon distributions
  - Combine ${}^{RR}S_{qq}$ and ${}^{R}F_{\mathrm{us},a_1 b_1}$ into subtracted DTMD possible since ${}^{RR}S_{qq}(\boldsymbol{y}) = {}^{RR}S_{gg}(\boldsymbol{y})$ (independent of parton type)
- We calculate soft function and matching coefficients at one-loop order (all parton types, polarizations and color representations, CSS and SCET)
  - Coefficients equal to TMDs — PDFs matching coeffs. appart from:
    1) Color factors for non-singlet
    2) Different vector dependence, since DTMDs and DPDs are parametrized in terms of same distance between partons
    3) additional polarizations possible

Region or large $\boldsymbol{y}$

- Rapidity evolution kernel simplifies considerably

$$^{RR'}K_{a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y}; \mu_i) = \delta_{RR'} \left[{}^{R}K_{a_1}(\boldsymbol{z}_1; \mu_1) + {}^{R}K_{a_2}(\boldsymbol{z}_2; \mu_2) + {}^{R}J(\boldsymbol{y}; \mu_i) \right]$$

  - Diagonal in color, distance dependence separated
    ${}^{1}K_{a_1}(\boldsymbol{z}_1; \mu_1)$ usual Collins-Soper kernel
  - ${}^{R}J(\boldsymbol{y}; \mu_i)$ remains for DPDFs (rapidity scale evolution for collinear func.)

- Solution to evolution equations:

$$\begin{aligned}
&{}^{R}F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta) \\
&\quad = \sum_{b_1 b_2} {}^{R}\Big[C_{a_1 b_1}(\boldsymbol{z}_1) \underset{x_1}{\otimes} C_{a_2 b_2}(\boldsymbol{z}_2) \underset{x_2}{\otimes} F_{b_1 b_2}(x_i', \boldsymbol{y}; \mu_{0i}, \zeta_0) \Big] \\
&\qquad \times \exp\Bigg\{ \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu} \left[\gamma_{F,a_1} - \gamma_{K,a_1} \log \frac{\sqrt{x_1 \zeta / x_2}}{\mu} \right] + {}^{R}K_{a_1}(\boldsymbol{z}_1) \log \frac{\sqrt{x_1 \zeta / x_2}}{\mu_{01}} \\
&\qquad\quad + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu} \left[\gamma_{F,a_2} - \gamma_{K,a_2} \log \frac{\sqrt{x_2 \zeta / x_1}}{\mu} \right] + {}^{R}K_{a_2}(\boldsymbol{z}_2) \log \frac{\sqrt{x_2 \zeta / x_1}}{\mu_{02}} \\
&\qquad\quad + {}^{R}J(\boldsymbol{y}) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \Bigg\}
\end{aligned}$$

Region or large $\boldsymbol{y}$

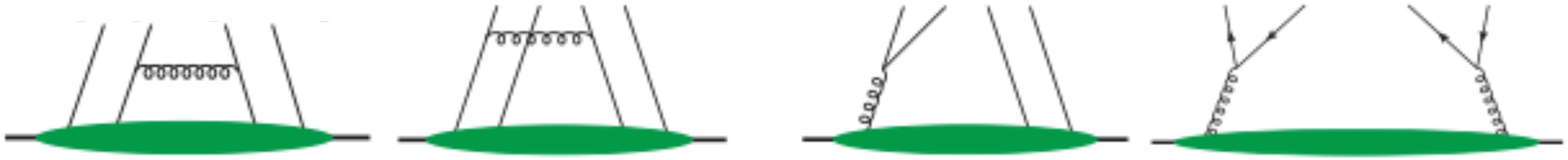
- Cross section for large y:

$$W_{\text{large } \boldsymbol{y}} = \sum_{c_1 c_2 d_1 d_2,\, R} \left[\Phi(\nu \boldsymbol{y})\right]^2 \exp\left[{}^R J(\boldsymbol{y}, \mu_{0i}) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0}\right]$$
$$\times \exp\left\{ \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu} \left[\gamma_{F,a_1}(\mu,\mu^2) - \gamma_{K,a_1}(\mu) \log \frac{q_1^2}{\mu^2}\right] + {}^R K_{a_1}(\boldsymbol{z}_1, \mu_{01}) \log \frac{q_1^2}{\mu_{01}^2} \right.$$
$$\left. + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu} \left[\gamma_{F,a_2}(\mu,\mu^2) - \gamma_{K,a_2}(\mu) \log \frac{q_2^2}{\mu^2}\right] + {}^R K_{a_2}(\boldsymbol{z}_2, \mu_{02}) \log \frac{q_2^2}{\mu_{02}^2} \right\}$$
$$\times {}^R\!\left[C_{b_1 d_1}(\boldsymbol{z}_1) \underset{\bar{x}_1}{\otimes} C_{b_2 d_2}(\boldsymbol{z}_2) \underset{\bar{x}_2}{\otimes} F_{c_1 c_2}(x_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)\right]$$
$$\times {}^R\!\left[C_{a_1 c_1}(\boldsymbol{z}_1) \underset{x_1}{\otimes} C_{a_2 c_2}(\boldsymbol{z}_2) \underset{x_2}{\otimes} F_{d_1 d_2}(\bar{x}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)\right]$$

- Non-perturbative input — collinear DPDFs, only one transverse distance and several model calculations available
  - at large scales, color singlet distributions dominate
  - ideal future, measured distributions — still a long way to go

Region or small $\boldsymbol{y}$

- Scaling: $\boldsymbol{y} \sim 1/q_T \sim \boldsymbol{z}_i$
- Soft function perturbatively calculable ${}^{RR'}S_{a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y}) = {}^{RR'}C_{s,a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y})$
- Expand on collinear distributions (all fields at same position)



$$F_{\text{intr}} = G + C \otimes G \sim \Lambda^2,\ G = \text{twist 4},\ C \propto \alpha_s$$
$$F_{\text{tw3}}, \quad \text{only chiral odd, discard}$$
$$F_{\text{spl}} \sim \frac{\boldsymbol{y}_+}{\boldsymbol{y}_+^2}\frac{\boldsymbol{y}_-}{\boldsymbol{y}_-^2} T \cdot f(x_1 + x_2) \sim q_T^2,\ f = \text{PDF},\ T \propto \alpha_s$$

- ${}^{R}F = {}^{R}F_{\text{split}} + {}^{R}F_{\text{intr}}$

- Size of the contributions

$$\int d^2\boldsymbol{y} W(\boldsymbol{z}_i, \boldsymbol{y})\Big|_{\text{small } \boldsymbol{y}} \sim \begin{cases} \alpha_s^2 q_T^2 & \text{from } F_{\text{split}} \times F_{\text{split}}\ (1\text{vs}1) \\ \alpha_s \Lambda^2 & \text{from } F_{\text{split}} \times F_{\text{intr}}\ (1\text{vs}2) \\ \Lambda^4/q_T^2 & \text{from } F_{\text{intr}} \times F_{\text{intr}}\ (2\text{vs}2) \end{cases}$$

Region or small $\boldsymbol{y}$

- DPS cross section contribution

$$W_{\text{small } \boldsymbol{y}} = \exp\left\{ \int_{\mu_0}^{\mu_1} \frac{d\mu}{\mu} \left[\gamma_{F,a_1} - \gamma_{K,a_1} \log \frac{q_1^2}{\mu^2}\right] + {}^1K_{a_1}(\boldsymbol{z}_1, \mu_0) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0} + \int_{\mu_0}^{\mu_2} \frac{d\mu}{\mu} \left[\gamma_{F,a_2} - \gamma_{K,a_2} \log \frac{q_2^2}{\mu^2}\right] + {}^1K_{a_2}(\boldsymbol{z}_2, \mu_0) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0} \right\}$$

$$\times \sum_{RR'} \left[{}^{R}F_{\text{spl+int},\, b_1 b_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)\right]^{RR'} \exp\left[M_{a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y}) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0}\right]$$

$$\times \left[{}^{R'}F_{\text{spl+int},\, a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)\right]$$

- Non-perturbative input: PDF and twist-four collinear (all color representations)
- Twist-four contribution can (at leading order) be modelled through DPDFs (part of both DTMDs and DPDFs in this region can be matched onto twist four distributions)

Combine regions

- Contributions from the two regions:

$$W_{\text{large } \boldsymbol{y}} = \left[\Phi(\nu \boldsymbol{y})\right]^2 \sum_R \exp\left\{ {}^R S(\boldsymbol{z}_1) + {}^R S(\boldsymbol{z}_1) \right\} \exp\left[{}^R J(\boldsymbol{y}, \mu_{0i}) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0} \right]$$

$$\times\, {}^R\!\left[C(\boldsymbol{z}_1) \underset{\bar{x}_1}{\otimes} C(\boldsymbol{z}_2) \underset{\bar{x}_2}{\otimes} F(\bar{x}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0) \right] {}^R\!\left[C(\boldsymbol{z}_1) \underset{x_1}{\otimes} C(\boldsymbol{z}_2) \underset{x_2}{\otimes} F(x_i, \boldsymbol{y}; \mu_{0i}, \zeta_0) \right]$$

$$W_{\text{small } \boldsymbol{y}} = \Phi(\nu \boldsymbol{y}_+)\Phi(\nu \boldsymbol{y}_-) \exp\left\{ {}^1 S(\boldsymbol{z}_1) + {}^1 S(\boldsymbol{z}_2) \right\} \sum_{RR'} \left[{}^R F_{\text{spl+int}}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0) \right]$$

$$\times\, {}^{RR'} \exp\left[M(\boldsymbol{z}_i, \boldsymbol{y}) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0} \right] \left[{}^{R'} F_{\text{spl+int}}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0) \right]$$

- Combine large and small y:

$$W = W_{\text{large } \boldsymbol{y}} - W_{\text{subt}} + W_{\text{small } \boldsymbol{y}}$$

  - Collins subtraction formalism $W_{\text{subt}} = W_{\text{large } \boldsymbol{y}}\big|_{|\boldsymbol{y}| \ll 1/\Lambda}$ or $W_{\text{subt}} = W_{\text{small } \boldsymbol{y}}\big|_{|\boldsymbol{z}_i| \ll |\boldsymbol{y}|}$, equal up to differences in scale choice (beyond accuracy for suitable choices)

Use and relation to most naive DPS

- Back to order 0 (pocket formula):
  - Model DPDs as product of single PDFs divided by σ_{eff} (as I showed earlier)
  - Neglect color correlations, take only R=1
  - Neglect spin correlations
  - Neglect explicit treatment of small y region (assume that it is absorbed by σ_{eff})
  - etc. etc.
- Provides us with formalism which tells us what we neglect, and which allows us to (for example) do targeted studies of the different effects
- Without much increase in unknown input, can get a lot more out

Summary

- DPS start to be on a rather solid theoretical ground
  - but (naturally) still places which require further development
- Experimental side advancing (many more measurements in last years)
  - Luminosity is getting close to measure DPS in double same sign W
- Phenomenology, many recent studies:
  - Most neglect the parton correlations
- Promising opportunities to nail down DPS correlations
  — for example by hemisphere asymmetry in double W
  - Double quarkonia/heavy meson production promising, DPS cross section are large or even dominant.
- Large fraction of DPS at low/intermediate transverse momenta
- Development for DTMD framework: definitions of DTMDs, their evolution and matching in different regimes
  - phenomenology
  - connect with experiments
  - useful input to MC generators