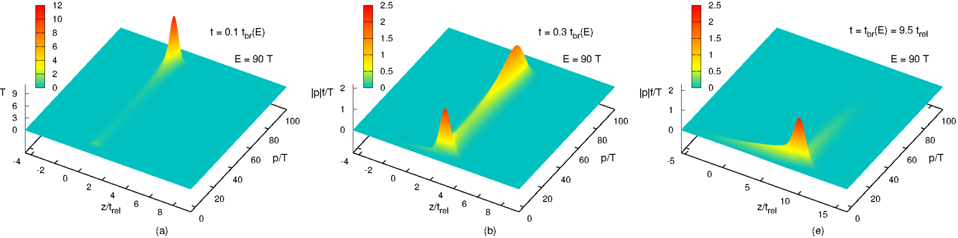
# *Jets in a dense QCD medium*

**Edmond Iancu**
IPhT Saclay & CNRS

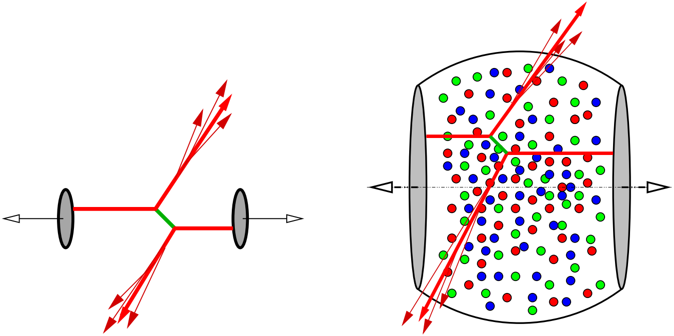
based on recent work by the Saclay collaboration

J.-P. Blaizot, F. Dominguez, M. Escobedo, Y. Mehtar-Tani, B. Wu



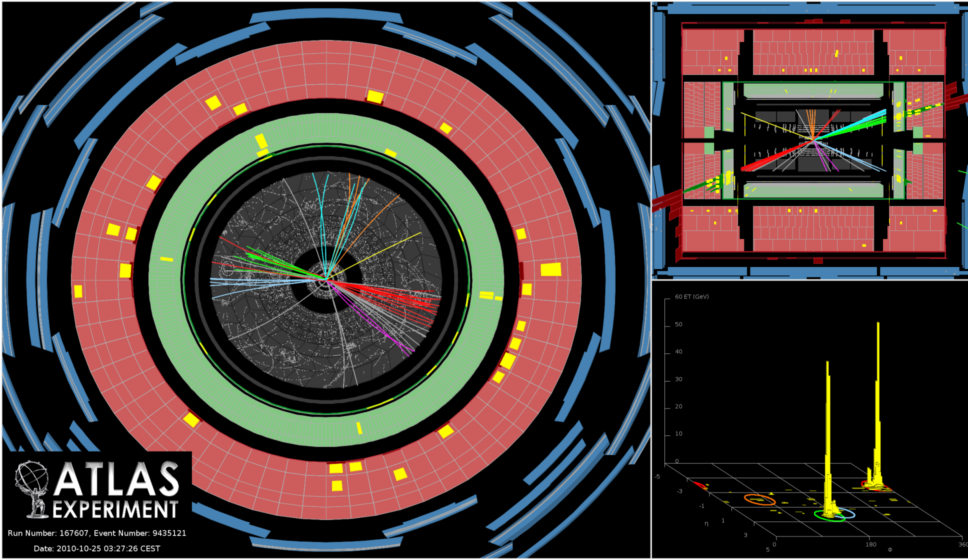
# Jet quenching in heavy ion collisions

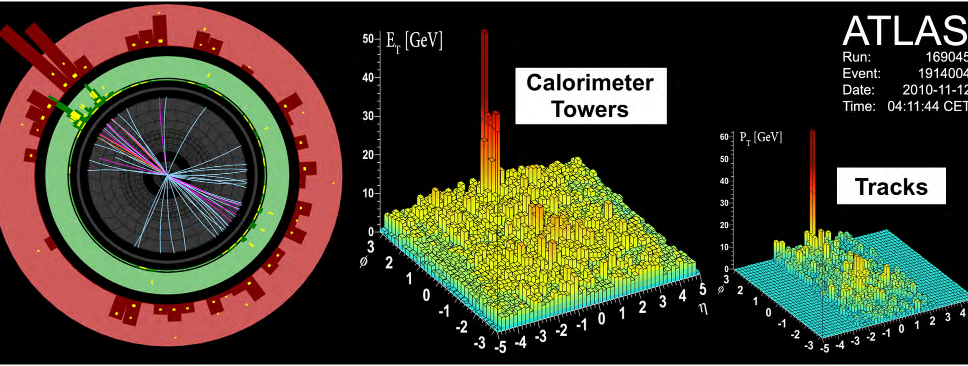
- Hard processes in QCD typically create pairs of partons which propagate back–to–back in the transverse plane
- In the vacuum, this leads to a pair of symmetric jets
- In a dense medium, the two jets can be differently affected by their interactions with the surrounding medium: 'di-jet asymmetry'



- The ensemble of medium-induced modifications: 'jet quenching'

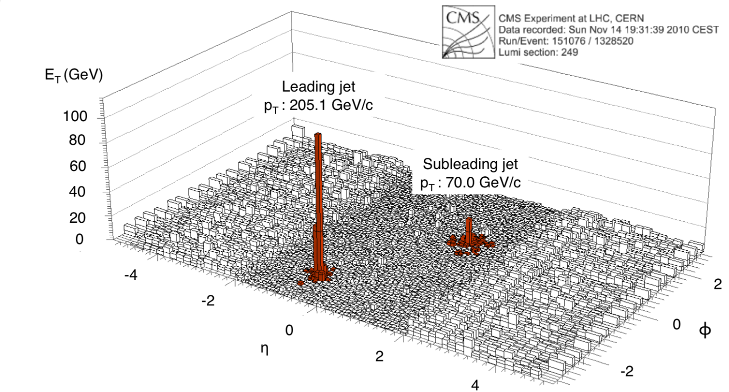
# From di–jets in p+p collisions ...





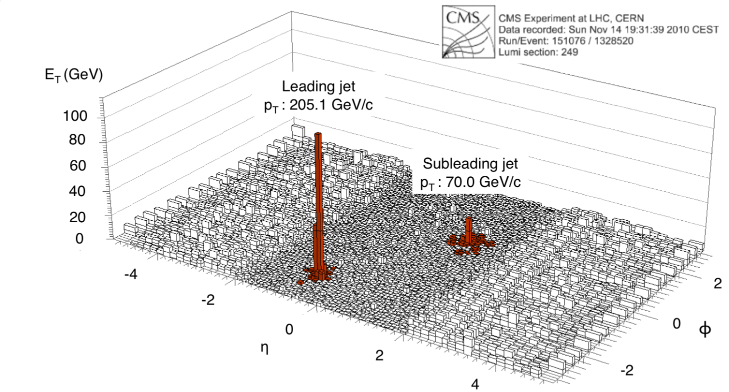

- Central Pb+Pb: 'mono–jet' events
- The secondary jet can barely be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

# Di–jet asymmetry *(CMS)*




- A di-jet asymmetry $E_1 - E_2$ already exists in p+p collisions:
  - 3-jet events, fluctuations in the branching process ...
- Additional energy imbalance as compared to p+p : 20 to 30 GeV
  - attributed to interactions in the surrounding medium
- Large compared to the medium 'temperature' $T \sim 1$ GeV (average $p_\perp$)

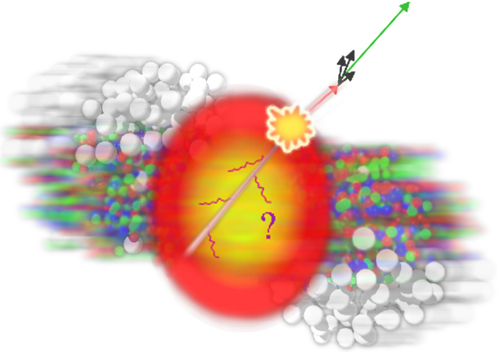
# Di–jet asymmetry *(CMS)*




- The 'missing energy' is actually found in the underlying event:
  - many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles
- Very different from the usual jet fragmentation pattern in the vacuum
  - bremsstrahlung favors collinear splittings $\Rightarrow$ jets are collimated
- Can we understand this difference within pQCD ?

# The generally expected picture

- "One jet crosses the medium along a distance longer than the other"




- Implicit assumption: fluctuations in energy loss are small
  - "the energy loss is always the same for a fixed medium size"

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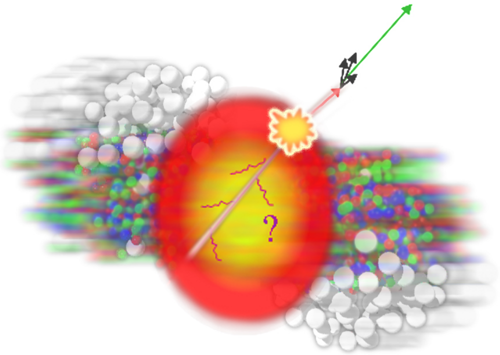
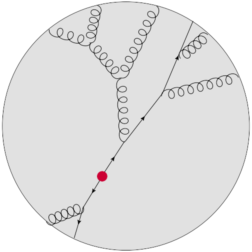


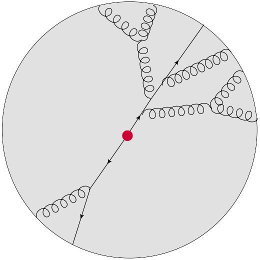

- Implicit assumption: fluctuations in energy loss are small
  - "the energy loss is always the same for a fixed medium size"
- Fluctuations are known to be important for a branching process

# The role of fluctuations

- Different path lengths



- Fluctuations in the branching pattern



- Fluctuations in the energy loss are as large as the average value

  *(M. Escobedo and E.I., arXiv:1601.03629 & 1609.06104)*

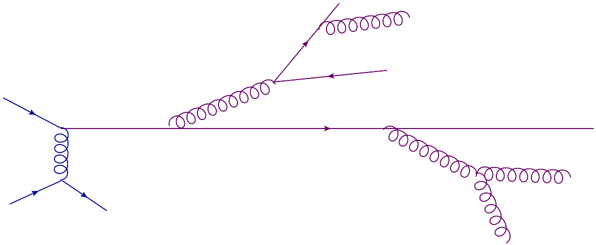
- Confirmed by a recent Monte-Carlo study using JEWEL

  *(Milhano and Zapp, arXiv:1512.08107)*

- One needs a better understanding of the in-medium jet dynamics

# Medium-induced jet evolution

- The leading particle (LP) is produced by a hard scattering
- It subsequently evolves via radiation (branchings) ...



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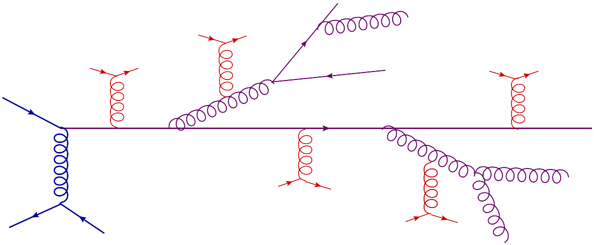

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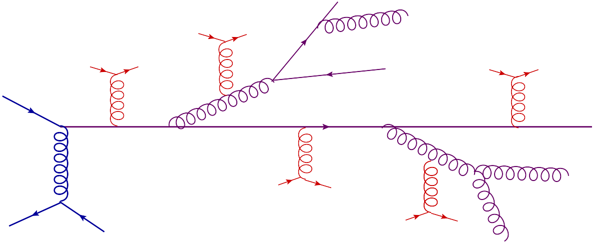
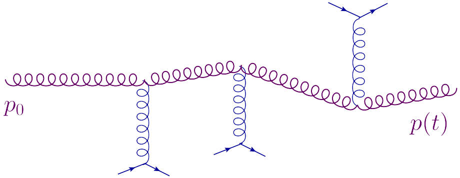


- ... and via collisions off the medium constituents
- Collisions can have two main effects
  - trigger additional radiation ('medium-induced branching')
  - thermalize the products of this radiation
- BDMPSZ mechanism *(Baier, Dokshitzer, Mueller, Peigné, Schiff; Zakharov)*
  - gluon emission is linked to transverse momentum broadening

## Transverse momentum broadening

- Independent multiple scattering $\Longrightarrow$ a random walk in $p_\perp$

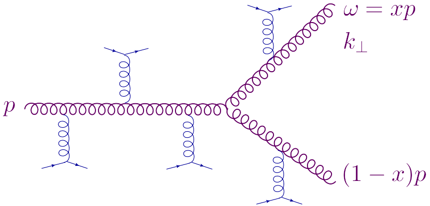



$$\langle p_\perp^2 \rangle \simeq \hat{q}\,\Delta t$$

$$\hat{q} \simeq \frac{m_D^2}{\lambda} \sim \alpha_s^2 T^3 \ln \frac{1}{\alpha_s}$$

$$\hat{q} \simeq 1\,\mathrm{GeV}^2/\mathrm{fm}$$

- Collisions destroy quantum coherence and thus trigger emissions




formation time

$$t_\mathrm{f} \simeq \frac{1}{\Delta E} \simeq \frac{\omega}{k_\perp^2}$$

- During formation, the gluon acquires a momentum $k_\perp^2 \sim \hat{q} t_\mathrm{f}$

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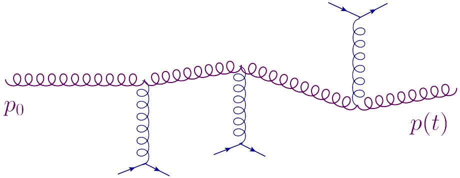


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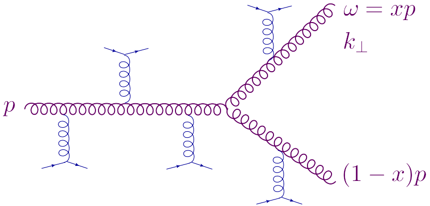


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# Multiple branchings

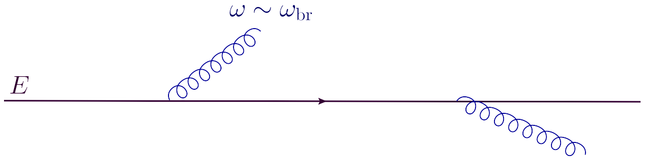
- Probability for emitting a gluon with energy $\geq \omega$ during a time $L$

$$\mathcal{P}(\omega, L) \simeq \alpha_s \frac{L}{t_{\rm f}(\omega)} \simeq \alpha_s L \sqrt{\frac{\hat{q}}{\omega}}$$

- When $\mathcal{P}(\omega, L) \sim 1$, multiple branching becomes important

$$\omega \lesssim \omega_{\rm br} \equiv \alpha_s^2 \hat{q} L^2 \quad : \text{ the fundamental scale for what follows}$$

- LHC: the leading particle has $E \sim 100\,\text{GeV} \gg \omega_{\rm br} \sim 5 \div 10\,\text{GeV}$




- In a typical event, the LP emits ...
  - a number of $\mathcal{O}(1)$ of gluons with $\omega \sim \omega_{\rm br}$

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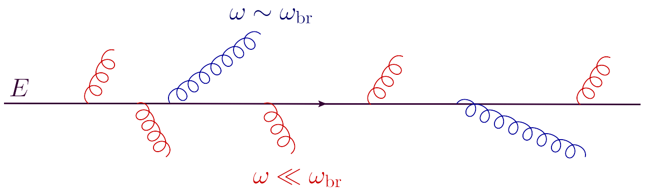


- In a typical event, the LP emits ...
  - a large number of softer gluons with $\omega \ll \omega_{\rm br}$

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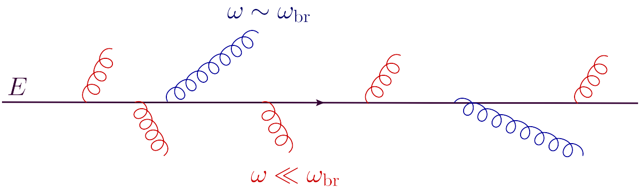
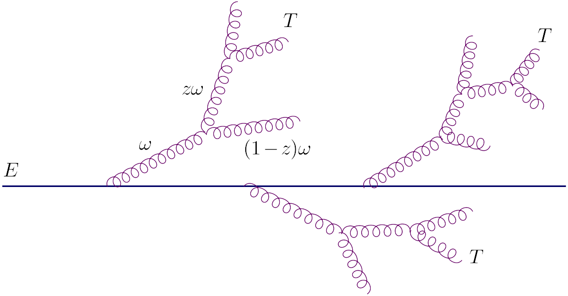



- The energy loss is controlled by the hardest primary emissions

# Democratic branchings

*J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)*

- The primary gluons generate 'mini-jets' via democratic branchings
  - daughter gluons carry comparable energy fractions: $z \sim 1 - z \sim 1/2$
  - contrast to asymmetric splittings in the vacuum: $z \ll 1$




- Via successive branchings, the energy is efficiently transmitted to softer and softer gluons, down to $\omega \sim T$
  - the soft gluons thermalize *(E.I. and Bin Wu, arXiv:1506.07871)*

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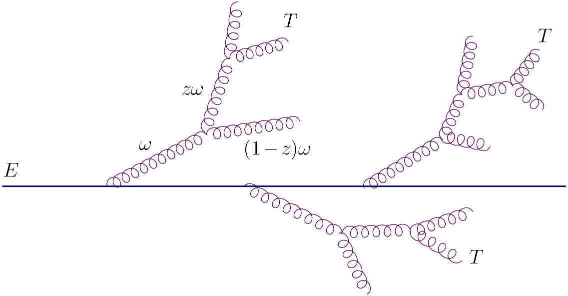



- Energy appears in many soft quanta propagating at large angles ✓
- What is the average energy loss and its fluctuations ?

# Probabilistic picture

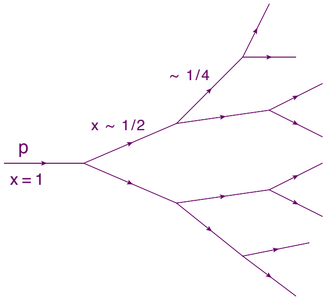
- Medium-induced jet evolution $\approx$ a Markovien stochastic process
  - successive branchings are independent from each other
  - interference phenomena could complicate the picture ...
    *(in the vacuum, interference leads to angular ordering)*
  - ... but they are suppressed by rescattering in the medium
    *Blaizot, Dominguez, E.I., Mehtar-Tani (2012)*
- Infinite hierarchy of equations for $n$-point correlation functions

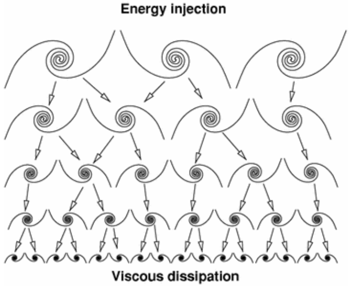
$$D(x,t) \equiv x \left\langle \frac{\mathrm{d}N}{\mathrm{d}x}(t) \right\rangle , \qquad D^{(2)}(x,x',t) \equiv xx' \left\langle \frac{\mathrm{d}N_{\rm pair}}{\mathrm{d}x\,\mathrm{d}x'}(t) \right\rangle$$

- Exact analytic solutions
  *Blaizot, E. I., Mehtar-Tani (2013); M. Escobedo and E.I. (2016)*
- Interesting new phenomena:
  - wave turbulence, KNO scaling, large fluctuations

# Wave turbulence

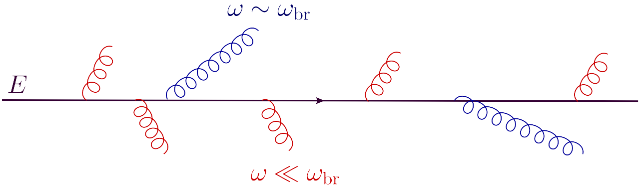
- Democratic branchings lead to wave turbulence
  - energy flows from one parton generation to the next one, at a rate which is independent of the generation
  - it eventually dissipates into the medium, via thermalization
  - mathematically: a fixed point $D(x) = \frac{1}{\sqrt{x}}$ (Kolmogorov spectrum)



# The average energy loss

- Recall: energy loss is controlled by the primary emissions with $\omega \sim \omega_{\rm br}$




  - softer emissions ($\omega \ll \omega_{\rm br}$) carry very little energy

- Confirmed by an exact calculation *(Blaizot, E. I., Mehtar-Tani, 2013)*

$$\langle \Delta E \rangle = E \left[ 1 - {\rm e}^{-\pi \frac{\omega_{\rm br}}{E}} \right] \simeq \pi \omega_{\rm br}$$

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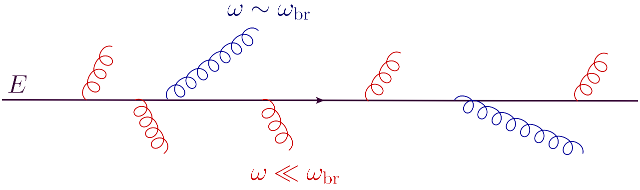


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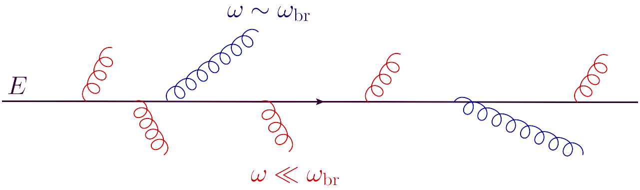
$$\langle \Delta E \rangle = E\left[1 - {\rm e}^{-\pi\frac{\omega_{\rm br}}{E}}\right] \simeq \pi\omega_{\rm br} = \pi\alpha_s^2\hat{q}L^2$$

  - independent of the energy $E$ of the leading particle
  - rapidly increasing with the medium size $\propto L^2$

# Fluctuations in the energy loss at large angles

- Recall: the probability for a primary emissions with $\omega \sim \omega_{\rm br}$ is of $\mathcal{O}(1)$




  - the average number of such emissions is of $\mathcal{O}(1)$ (indeed, it is $\pi$)
  - successive such emissions are quasi-independent $(E \gg \omega_{\rm br})$

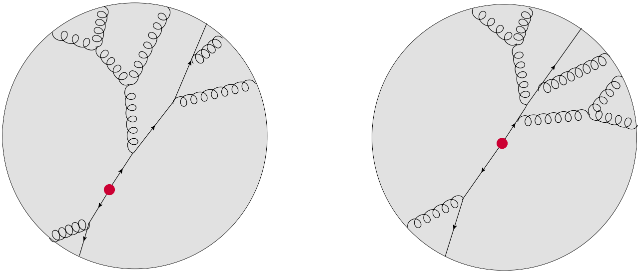
- Fluctuations in the number of such emissions must be of $\mathcal{O}(1)$ as well

- Confirmed by exact calculations *(M. Escobedo and E. I., arXiv:1601.03629)*

$$\sigma^2 \equiv \langle \Delta E^2 \rangle - \langle \Delta E \rangle^2 \simeq \frac{\pi^2}{3}\,\omega_{\rm br}^2 = \frac{1}{3}\langle \Delta E \rangle^2$$

- Variance is comparable to expectation value: large fluctuations

# Di-jet asymmetry from fluctuations



- Average asymmetry is controlled by the difference in path lengths

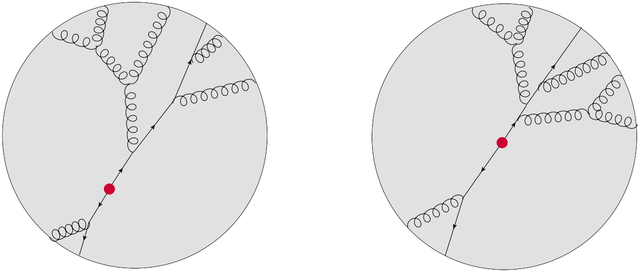
$$\langle E_1 - E_2 \rangle = \langle \Delta E_2 - \Delta E_1 \rangle \propto \langle L_2^2 - L_1^2 \rangle$$

- In experiments though, one rather measures $|E_1 - E_2|$

$$\langle (E_1 - E_2)^2 \rangle - \langle E_1 - E_2 \rangle^2 = \sigma_1^2 + \sigma_2^2 \propto \langle L_2^2 + L_1^2 \rangle$$

- Fluctuations dominate whenever $L_1 \sim L_2$ (the typical situation)

# Di-jet asymmetry from fluctuations



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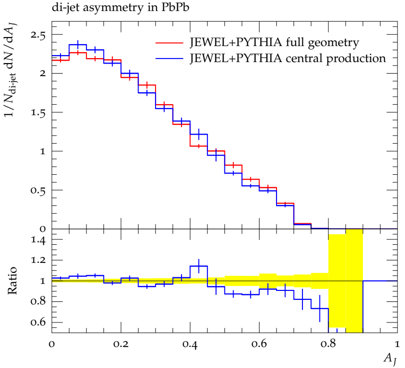
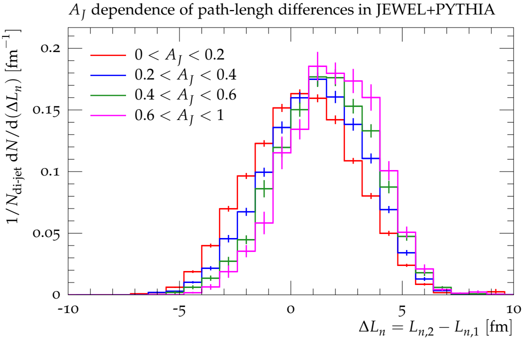
- Difficult to check: no direct experimental control of $L_1$ and $L_2$

# Monte-Carlo generated events (central collisions)

*(Milhano and Zapp, JEWEL, arXiv:1512.08107)* $A_{\rm J} = \frac{E_1 - E_2}{E_1 + E_2} > 0$



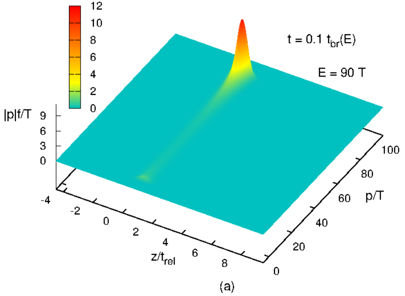




- Left: Central production $(L_1 = L_2)$ vs. randomly distributed production points ("full geometry")
- Right: Distribution of $\Delta L \equiv L_1 - L_2$ for di-jet events in different classes of asymmetry $(A_J)$
  - the width of the distribution is a measure of fluctuations

# Jet thermalization

*E.I. and Bin Wu, arXiv:1506.07871*

- The soft gluons $(\omega \sim T)$ thermalize via elastic collisions
- Gluon distribution in energy $(p)$ and longitudinal coordinate $(z)$
- Initial condition at $t = z = 0$: $E \;=\; 90\,T$




- $t_{\rm br}(E)$ : the lifetime of the leading particle
- the time before the LP undergoes a first democratic branching

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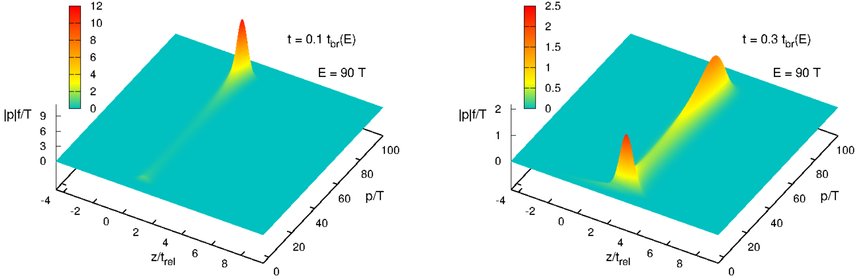
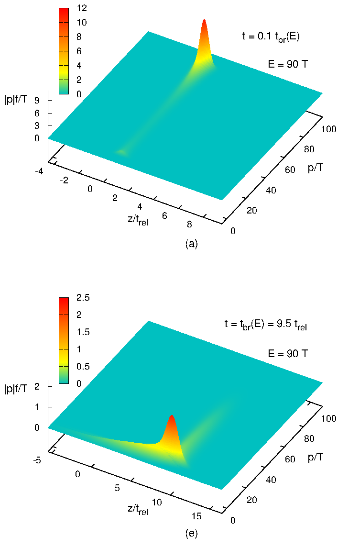



(a) (b)

- With increasing time, the jet substructure is softening and broadening

# Jet thermalization




- $t = t_{\rm br}(E)$ : the leading particle disappears (democratic branching)

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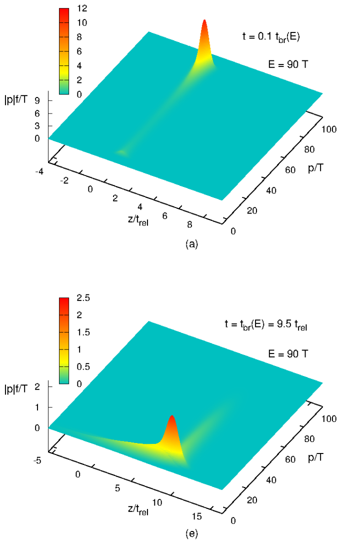


- $t = 1.5\, t_{\rm br}(E)$ : the jet is fully quenched

# Conclusions

- Effective theory and physical picture for jet quenching from pQCD
  - event-by-event physics: multiple branching
  - democratic branchings leading to wave turbulence
  - thermalization of the soft branching products with $p \sim T$
  - efficient transmission of energy to large angles
  - wide probability distribution, strong fluctuations, KNO scaling
- Fluctuations compete with the difference in path lengths in determining the di-jet asymmetry
- Qualitative and semi-quantitative agreement with the phenomenology of di-jet asymmetry at the LHC
- Important dynamical information still missing: vacuum-like radiation (parton virtualities), medium expansion ...

# Gluon multiplicities

- Average number of gluons with $\omega \geq \omega_0$
  - $\omega_0 \ll E$ is the 'resolution scale'

$$\langle N(\omega_0) \rangle = \int_{\omega_0}^{E} \mathrm{d}\omega \, \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq 1 + 2 \left[ \frac{\omega_{\rm br}}{\omega_0} \right]^{1/2} \quad \text{(LP + radiation)}$$

  - would be divergent if $\omega_0 \to 0$
  - in experiments too, one has a non-zero resolution scale
  - independent of the energy $E$ of the LP
  - $\langle N(\omega_0) \rangle \simeq 1$ when $\omega_0 \gg \omega_{\rm br}$ : just the leading particle
  - $\langle N(\omega_0) \rangle \gg 1$ when $\omega_0 \ll \omega_{\rm br}$ : multiple branching
  - amusingly enough: $\langle N(\omega_{\rm br}) \rangle = 3 \simeq \pi$
- Multiplicities are high for soft gluons: $\omega_0 \ll \omega_{\rm br}$

# Koba-Nielsen-Olesen scaling

- One has similarly computed all the higher moments $\langle N^p \rangle$ with $p \geq 2$ *(M. Escobedo and E. I., arXiv:1609.06104)*
- For soft gluons, $\omega_0 \ll \omega_{\rm br}$, they are all determined by the 1-point function:

$$\frac{\langle N^2 \rangle}{\langle N \rangle^2} \simeq \frac{3}{2}, \qquad \frac{\langle N^p \rangle}{\langle N \rangle^p} \simeq \frac{(p+1)!}{2^p}$$

- KNO scaling : the reduced moments are pure numbers
  - independent of any of the physical parameters of the problem
- A special negative binomial distribution (parameter $r = 2$)
  - huge fluctuations (say, as compared to a Poissonian distribution)

$$\frac{\sigma_N}{\langle N \rangle}\bigg|_{\rm KNO} = \frac{1}{\sqrt{2}} \qquad \text{vs.} \qquad \frac{\sigma_N}{\langle N \rangle}\bigg|_{\rm Poisson} = \frac{1}{\sqrt{\langle N \rangle}}$$

- KNO scaling also holds for a jet in the vacuum ...
- ... but the medium-induced distribution is much wider !