The QCD Quark Condensate from RG Optimized Spectral Density

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mostly based on J.-L. K., A. Neveu, PRD 92 (2015)

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1. Introduction/Motivations

Specifically: unconventional resummation of perturbative expansions

Versatile: relevant both at $T = 0$ or $T \neq 0$ (and/or finite density)

Here, addressing only $T = 0$ QCD:
previous context: estimate with our approach the order parameter $F_\pi(m_q = 0)/\Lambda^{\rm QCD}_{\overline{MS}}$:
$F_\pi \simeq 92.2\text{MeV} \to F_\pi(m_q = 0) \to \Lambda^{n_f=3}_{\overline{MS}} \to \alpha_S^{\overline{MS}}(\mu = m_Z)$.

N^3LO: $F_\pi^{m_q=0}/\Lambda^{n_f=3}_{\overline{MS}} \simeq 0.25 \pm .01 \to \alpha_S(m_Z) \simeq 0.1174 \pm .001 \pm .001$
(JLK, A.Neveu, PRD88 (2013))
(compares well with latest (2016) α_S lattice and world average values [PDG2016])

Here: applied to $\langle \bar{q}q \rangle$ at N^3LO (using spectral density of Dirac operator):
$\langle \bar{q}q \rangle^{1/3}_{m_q=0}(2\,\mathrm{GeV}) \simeq -(0.84 \pm 0.01)\Lambda_{\overline{MS}}$ (JLK, A.Neveu, PRD 92 (2015))

Chiral Symmetry Breaking Order parameters

-Well-known facts:
1. $\langle \bar{q}q \rangle^{1/3}$, F_π,... $\sim \mathcal{O}(\Lambda_{QCD}) \simeq 330$ MeV
$\rightarrow$ large α_S at very low scale $\rightarrow$ invalidates perturbative expansion

2. F_π, $\langle \bar{q}q \rangle$,... anyway vanishing in standard perturbation:
e.g. $\langle \bar{q}q \rangle_{pert} \sim m_q^3 \sum_{n,p} \alpha_s^n \ln^p(m_q) \rightarrow 0$ for $m_q \rightarrow 0$
at any perturbative order (trivial chiral limit)

$\rightarrow$ CSB parameters are "intrinsically NON perturbative"

-Optimized perturbation (OPT): basically an (old) trick to circumvent 2.:
gives a nontrivial result for $m_q \rightarrow 0$, starting from perturbative content.

-Our more recent RG(OPT): reconciles OPT with RG invariance;
+ appears to partly circumvents 1.

$\langle \bar{q}q \rangle$: indirect determination from Gell-Mann Oakes Renner (GMOR) relation: $F_\pi^2 m_\pi^2 = -(m_u + m_d)\langle \bar{q}q \rangle + \mathcal{O}(m^2)$; ($m_{u,d}$ from lattice or spectral sum rules).
Or directly in simplified effective models (Nambu-Jona-Lasinio, approximated Schwinger-Dyson Eqs.,...)
or directly, on the lattice (many works, most recent: Engel et al '14)

2. (Variationally) Optimized Perturbation (OPT)

Trick: add and subtract a mass, consider $m\,\delta$ as interaction:

$$\mathcal{L}_{QCD}(g,m) \to \mathcal{L}_{QCD}(\delta\, g, m(1-\delta)) \ \ \text{(in QCD } g \equiv 4\pi\alpha_S)$$

where $0 < \delta < 1$ interpolates between $\mathcal{L}_{free}$ and *massless* $\mathcal{L}_{int}$;
e.g. (quark) mass $m_q \to m$: arbitrary trial parameter

• Take any standard (renormalized) QCD pert. series, expand in δ *after:*
$m_q \to m\,(1-\delta)$; $\quad g \to \delta\, g$
then take $\delta \to 1$ (to recover original massless theory):

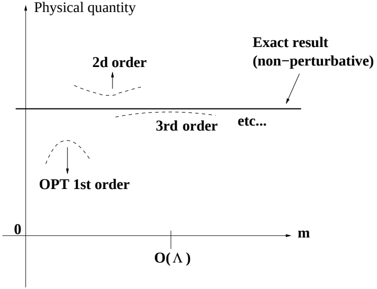
•BUT a m-dependence remains at any finite δ^k-order:
fixed typically by optimization (OPT):
$\frac{\partial}{\partial m}$(physical quantity) $= 0$ for $m = \tilde{m}_{opt}(g) \neq 0$:

•Exhibits *dimensional transmutation*: $\tilde{m}_{opt} \sim \mu\, e^{-1/(\beta_0 g)}$

•At $T \neq 0$ same basic idea dubbed "screened perturbation" (SPT), or "hard thermal loop resummation",...

But does this 'cheap trick' always work? and why?

Expected behaviour (Ideally)



But not quite what happens... (except in simple oscillator model)
Most calculations (e.g $T \neq 0$) (very) difficult beyond first order:
$\rightarrow$ what about convergence?

Main pb at higher order: OPT: $\partial_m(\ldots) = 0$ has multi-solutions (some complex!), how to choose right one, if no nonperturbative "insight"?

Simpler model's support + properties

•Convergence proof of this procedure for $D = 1$ $\lambda\phi^4$ oscillator
(cancels large pert. order factorial divergences!) (Guida et al '95)

particular case of 'order-dependent mapping' (Seznec +Zinn-Justin '79)
(exponentially fast convergence for ground state energy $E_0 = const.\lambda^{1/3}$;
good to % level at second δ-order)

•In renormalizable Field Theories: applied on V_{eff}^{1-loop}, first δ-order
equivalent to large N approximation.
Also, produces factorial damping at large pert. orders (JLK, Reynaud '02)

•Flexible approach: many variants exist, specially at $T \neq 0$:
'screened perturbation', 'hard thermal loop', ...

•At $T \neq 0$ our recent, RG-compatible approach, sensibly improves the
generically unstable + badly scale-dependent thermal perturbative
expansions (JLK, M.B Pinto, 1507.03508 PRL 116, 1508.02610)

3. RG compatible OPT ($\equiv$ RGOPT)

Our main additional ingredient to OPT (JLK, A. Neveu 2010):

Consider a "physical" quantity $P(m,g)$ (i.e. perturbatively RG invariant)
(in present context $P(m,g) \equiv m\langle\bar{q}q\rangle(m,g)$):

in addition to OPT Eq: $\frac{\partial}{\partial m}P^{(k)}(m,g,\delta=1)|_{m\equiv\tilde{m}} \equiv 0$,
Require (δ-modified!) series at order δ^k to satisfy a standard (perturbative) Renormalization Group (RG) equation:

$$\mathrm{RG}\left(P^{(k)}(m,g,\delta=1)\right) = 0$$

with standard RG operator ($g \equiv 4\pi\alpha_S$ for QCD):

$$\mathrm{RG} \equiv \mu\frac{d}{d\,\mu} = \mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma_m(g)\,m\frac{\partial}{\partial m}$$

$\beta(g) \equiv -2b_0 g^2 - 2b_1 g^3 + \cdots$, $\gamma_m(g) \equiv \gamma_0 g + \gamma_1 g^2 + \cdots$

$\rightarrow$ Additional nontrivial constraint (even if started from RG invariant standard perturbation)

RG compatible OPT (RGOPT)

$\rightarrow$ Combined with OPT, RG Eq. reduces to massless form:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right]P^{(k)}(m,g,\delta=1)=0$$

Note: using OPT AND RG completely fix $m\equiv\tilde{m}$ and $g\equiv\tilde{g}$.

But $\Lambda_{\overline{MS}}(g)$ satisfies by def.:
$[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}]\,\Lambda_{\overline{MS}}\equiv 0$ consistently at a given pert. order for $\beta(g)$.

Thus equivalent to:

$$\frac{\partial}{\partial\,m}\left(\frac{P^k(m,g,\delta=1)}{\Lambda_{\overline{MS}}(g)}\right)=0\ ;\quad \frac{\partial}{\partial\,g}\left(\frac{P^k(m,g,\delta=1)}{\Lambda_{\overline{MS}}(g)}\right)=0 \text{ for } \tilde{m},\tilde{g}$$

Sort of "virtual" (variational) fixed point (but with $\beta(g)\neq 0$!)
Optimal $\tilde{m},\tilde{g}=4\pi\tilde{\alpha}_S$ unphysical: final (physical) result from $P(\tilde{m},\tilde{g})$

It reproduces at first order exact nonperturbative results in simpler models [e.g. Gross-Neveu model]

OPT + RG = RGOPT main new features

•Previous works: embarrassing a priori freedom in interpolating form:
why not $m \to m(1-\delta)^a$, with arbitrary a?
Most previous works: linear case $a=1$ invoked for simplicity
but (we have shown) $a=1$ generally spoils RG invariance!

•OPT,RG Eqs: many solutions (often complex) at increasing δ^k-orders

→ Our approach restores RG, + forces OPT, RG sol. to match standard perturbation (e.g. Asymptotic Freedom for QCD): $\alpha_S \to 0$, $\mu \to \infty$:
$\tilde{g} = 4\pi\tilde{\alpha}_S \sim \frac{1}{2b_0 \ln \frac{\mu}{\tilde{m}}} + \cdots$

→ At arbitrary order, AF-compatible RG + OPT branch, often unique, *only* appear for a critical universal exponent a:
$m \to m(1-\delta)^{\frac{\gamma_0}{2b_0}}$ (e.g. $\frac{\gamma_0}{2b_0}(\mathrm{QCD}, n_f = 3) = \frac{4}{9}$)

→ Goes beyond simple "add and subtract mass" trick
\+ Removes spurious solutions incompatible with AF
− But, does not always avoid complex solutions
(if such (perturbative artifacts) occur, they are possibly cured by renormalization scheme change [JLK, Neveu '13])

Digression: pre-QCD guidance: Gross-Neveu model

•$D = 2$ $O(2N)$ GN model shares many properties with QCD (asymptotic freedom, (discrete) chiral sym., mass gap,..)

$$\mathcal{L}_{GN} = \bar{\Psi} i \not{\partial} \Psi + \frac{g_0}{2N} (\sum_1^N \bar{\Psi}\Psi)^2 \; (massless)$$

Standard mass-gap (massless, large N approx.):

work out $V_{eff}(\sigma \sim \langle \bar{\Psi}\Psi \rangle) \sim \frac{\sigma^2}{2g} + Tr \ln(i \not{\partial} - \sigma)$;

$\frac{\partial V_{eff}}{\partial \sigma} = 0$: $\quad \to \sigma \equiv M = \mu e^{-\frac{2\pi}{g}} \equiv \Lambda_{\overline{MS}}$

•Mass gap also known exactly for any N:

$$\frac{M_{exact}(N)}{\Lambda_{\overline{MS}}} = \frac{(4e)^{\frac{1}{2N-2}}}{\Gamma[1 - \frac{1}{2N-2}]}$$

(From $D = 2$ integrability: Bethe Ansatz) Forgacs et al '91

Massive (large N) GN model

$M(m,g) \equiv m(1+g\ln\frac{M}{\mu})^{-1}$: Resummed mass $(g/(2\pi) \to g)$
$= m(1 - g\ln\frac{m}{\mu} + g^2(\ln\frac{m}{\mu} + \ln^2\frac{m}{\mu}) + \cdots)$ (pert. re-expanded)

- Only fully resummed $M(m,g)$ gives right result, upon:

-identifying $\Lambda \equiv \mu e^{-1/g}$; $\to M(m,g) = \frac{m}{g\ln\frac{M}{\Lambda}} \equiv \frac{\hat{m}}{\ln\frac{M}{\Lambda}}$;
-taking *reciprocal*: $\hat{m} = M\,\ln\frac{M}{\Lambda} \to M(\hat{m} \to 0) \sim \frac{\hat{m}}{\hat{m}/\Lambda + \mathcal{O}(\hat{m}^2)} = \Lambda$

never seen in standard perturbation: $M_{pert}(m \to 0) \to 0!$

- Now (RG)OPT gives $M = \Lambda$ at *first* (and any) δ-order!
(at any order, OPT sol.: $\ln\frac{\tilde{m}}{\mu} = -\frac{1}{\tilde{g}}$, RG sol.: $\tilde{g} = 1$)

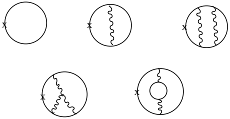
- At δ^2-order (2-loop), RGOPT $\sim 1-2\%$ from $M_{exact}(\text{any}N)$

- Not specific to GN model: generalize to any model:
RG, OPT solutions at first (and all) orders:
$\ln\frac{\tilde{m}}{\mu} = -\frac{\gamma_0}{2b_0}$; $\tilde{g} = \frac{1}{\gamma_0}$ correctly resums pure RG LL, NLL,... (as far as b_0, γ_0 dependence concerned).

4. Perturbative QCD quark condensate

Chiral symmetry breaking order parameter:
$SU(n_f)_L \times SU(n_f)_R \to SU(n_f)_{L+R}$, n_f massless quarks. ($n_f = 2, 3$)

Perturbative result known to 3 loops (Chetyrkin et al '94; Chetyrkin +Maier, private comm.)



$$m\,\langle\bar{q}q\rangle(m,g)_{\overline{MS}} = 3\frac{m^4}{2\pi^2}\left[\frac{1}{2} - L_m + \frac{g}{\pi^2}(L_m^2 - \frac{5}{6}L_m + \frac{5}{12})\right.$$
$$\left.+(\frac{g}{16\pi^2})^2[f_{30}(n_f)L_m^3 + f_{31}(n_f)L_m^2 + f_{32}(n_f)L_m + f_{33}(n_f)]\right]$$

$(L_m \equiv \ln\frac{m}{\mu})$
NB: finite part (after mass + coupling renormalization) not separately RG-inv: (i.e. $m\langle\bar{q}q\rangle$ mixes with m^4 1 operator: related to vacuum energy anomalous dimension)

First attempt: direct RGOPT of $m\langle \bar{q}q \rangle$?

In principle one may apply RGOPT directly on the (RG-invariant) expression $m\langle \bar{q}q \rangle(m, g)$:

first order (one-loop): no nontrivial common OPT +RG solution...

Higher RGOPT orders (2- and 3-loops): right order of magnitude, but ambiguous: plagued by large, unphysical, imaginary parts
$\rightarrow$ no conclusive stability/convergence trend (appears slow at best)

Problems traced to strong sensitivity to (vacuum energy) anomalous dimensions, related to original quadratic divergences of the condensate:

with a cutoff the (dominant) one-loop quadratic divergence has correct (negative) sign (pillar of the success of Nambu-Jona-Lasinio model!)
but sign flips in dimensional regularization + $\overline{\text{MS}}$

Yet important to keep benefits of $\overline{\text{MS}}$: high order true QCD perturbative calculations available: crucial for stability/convergence check.

$\rightarrow$ Like with any other variational methods, sensible to start from a suitable quantity to optimize: here the spectral density of the Dirac operator, intimately related to $\langle \bar{q}q \rangle$.

4. $\langle \bar{q}q \rangle$ and Spectral density $\rho(\lambda)$

Euclidean Dirac operator:
$i \not{D}\, u_n(x) = \lambda_n\, u_n(x)$; $\not{D} \equiv \not{\partial} + g \not{A}$;
NB $i \not{D}\, (\gamma_5 u_n(x)) = -\lambda_n\, (\gamma_5 u_n(x))$

On a lattice: $\rho(\lambda) \equiv \frac{1}{V} \langle \sum_n \delta(\lambda - \lambda_n^{[A]}) \rangle_A$

$V \to \infty$: spectrum becomes dense, and
$\langle \bar{q}q \rangle \equiv \frac{1}{V}\, \mathrm{Tr} \frac{1}{m + \not{D}} \to \langle \bar{q}q \rangle_{V \to \infty}(m) \equiv -2m \int_0^\infty d\lambda \frac{\rho(\lambda)}{\lambda^2 + m^2}$

$\rho(\lambda)$: spectral density of the (euclidean) Dirac operator.

Banks-Casher relation (1980): $\langle \bar{q}q \rangle(m \to 0) \equiv -\pi \rho(0)$
(using e.g. $\lim_{m \to 0} \frac{1}{m - i\lambda} = i\, PV(\frac{1}{\lambda}) + \pi \delta(\lambda)$)

'Washes out' large λ problems (e.g. quadratic UV divergences)

Conversely: $-\rho(\lambda) = \frac{1}{2\pi} \left(\langle \bar{q}q \rangle(i\lambda + \epsilon) - \langle \bar{q}q \rangle(i\lambda - \epsilon) \right) |_{\epsilon \to 0}$
i.e. $\rho(\lambda)$ determined by discontinuities of $\langle \bar{q}q \rangle(m)$ across imaginary axis.

Perturbative expansion: $\to \ln(m \to i\lambda)$ discontinuities
$\to$ no contributions from divergence and non-log terms (like anom. dim.)

Adapting OPT and RG Eqs. to spectral density

- Perturbative logarithmic discontinuities simply from

$$\ln^n\left(\frac{m}{\mu}\right) \to \frac{1}{2i\pi}\left[\left(\ln\frac{|\lambda|}{\mu}+i\frac{\pi}{2}\right)^n - \left(\ln\frac{|\lambda|}{\mu}-i\frac{\pi}{2}\right)^n\right] \qquad (1)$$

i.e. $\ln\left(\frac{m}{\mu}\right) \to 1/2;\ \ \ln^2\left(\frac{m}{\mu}\right) \to \ln\frac{|\lambda|}{\mu};\ \ \ln^3\left(\frac{m}{\mu}\right) \to \frac{3}{2}\ln^2\frac{|\lambda|}{\mu} - \frac{\pi^2}{8};\cdots$

- Modified perturbation: intuitively λ plays the role of m, so:

$$\rho_{pert}(\lambda, g) \to \rho_{opt}(\lambda(1-\delta)^{\frac{4}{3}\frac{\gamma_0}{2b_0}}, \delta g)\,;\ \text{expand in }\delta\,;\ \ \delta\to 1 \qquad (2)$$

- OPT Eq.: $$\frac{\partial}{\partial\lambda}\rho_{opt}(g,\lambda) = 0 \text{ for } \lambda = \tilde\lambda_{opt}(g) \neq 0 \qquad (3)$$

- Using $\frac{\partial}{\partial m}\frac{m}{\lambda^2+m^2} = -\frac{\partial}{\partial\lambda}\frac{\lambda}{\lambda^2+m^2}$, one finds $\rho(\lambda)$ obeys RG eq.:

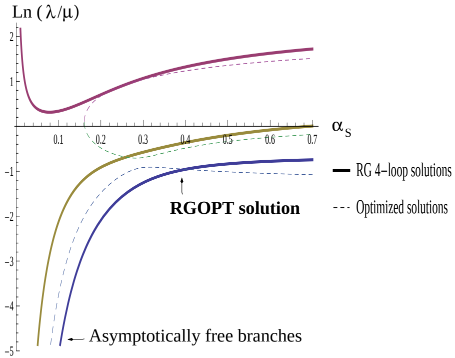
$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma_m(g)\,\lambda\frac{\partial}{\partial\lambda} - \gamma_m(g)\right]\rho(g,\lambda) = 0 \qquad (4)$$

$\to$ well-defined RGOPT recipe: -$\langle\bar q q\rangle_{pert}(m, g) \to \rho_{pert}(\lambda, g)$ from (1); -perform (2); -solve (3), (4) for optimal $\tilde\lambda$, $\tilde g$;

then $\rho(\tilde\lambda, \tilde g) \simeq \rho(0) \equiv -\langle\bar q q\rangle(m_q = 0)/\pi$.

RG and OPT solutions

NB $\langle \bar{q} q \rangle_{pert}$ exactly known at present up to 3-loop α_S^2 order.
But 1) RG properties determine next (4-loop) $\alpha_S^3 \ln^p(m/\mu)$ coefficients,
2) by def. non-logarithmic terms do not contribute to spectral density
$\rho_{pert}(\lambda)$: $\rightarrow$ we obtain $\rho_{pert}(\lambda)$ exactly to 4-loop!



5. RGOPT 2,3,4-loop results for $\langle \bar{q} q \rangle$ ($n_f = 2, 3$)

Real, unique AF-compatible solutions are obtained:

$n_f = 2$:

δ^k, RG order	$\ln \frac{\tilde{\lambda}}{\mu}$	$\tilde{\alpha}_S$	$\frac{-\langle \bar{q} q \rangle^{1/3}}{\bar{\Lambda}_2}(\tilde{\mu})$	$\frac{\tilde{\mu}}{\bar{\Lambda}_2}$	$\frac{-\langle \bar{q} q \rangle^{1/3}_{RGI}}{\bar{\Lambda}_2}$
δ, RG 2-loop	-0.45	0.480	0.822	2.8	0.821
δ^2, RG 3-loop	-0.703	0.430	0.794	3.104	0.783
δ^3, RG 4-loop	-0.820	0.391	0.796	3.446	0.773

$n_f = 3$:

δ^k order	$\ln \frac{\tilde{\lambda}}{\mu}$	$\tilde{\alpha}_S$	$\frac{-\langle \bar{q} q \rangle^{1/3}}{\bar{\Lambda}_3}(\tilde{\mu})$	$\frac{\tilde{\mu}}{\bar{\Lambda}_3}$	$\frac{-\langle \bar{q} q \rangle^{1/3}_{RGI}}{\bar{\Lambda}_3}$
δ, RG 2-loop	-0.56	0.474	0.799	3.06	0.789
δ^2, RG 3-loop	-0.788	0.444	0.780	3.273	0.766
δ^3, RG 4-loop	-0.958	0.400	0.773	3.700	0.744

NB: $\langle \bar{q} q \rangle_{RGI} = \langle \bar{q} q \rangle(\mu)\,(2 b_0\, g)^{\frac{\gamma_0}{2 b_0}} \left(1 + (\frac{\gamma_1}{2 b_0} - \frac{\gamma_0\, b_1}{2 b_0^2})\, g + \cdots \right)$

- stability/convergence exhibited;
- already realistic at first nontrivial (2-loop) order

Evolution to (standard) reference scale $\mu = 2$ GeV

$\langle \bar{q} q \rangle(\mu' = 2 GeV) = \langle \bar{q} q \rangle(\tilde{\mu}) \exp[\int_{g(\tilde{\mu})}^{g(2GeV)} dg \frac{\gamma_m(g)}{\beta(g)}]$

(equivalently extract from $\langle \bar{q} q \rangle_{RGI}$ with $\alpha_S(2\mathrm{GeV}) \simeq 0.305 \pm 0.004$)
(NB for $n_f = 3$ account for $\alpha_S(\mu \sim m_c)$ threshold effects)

$$-\langle \bar{q} q \rangle^{1/3}_{n_f=2}(2\mathrm{GeV}) = (0.833_{(4-loop)} - 0.845_{(3-loop)})\bar{\Lambda}_2$$
$$-\langle \bar{q} q \rangle^{1/3}_{n_f=3}(2\mathrm{GeV}) = (0.814_{(4-loop)} - 0.838_{(3-loop)})\bar{\Lambda}_3$$

• Discrepancy between 3- and 4-loop results define our 'intrinsical' (RGOPT) theoretical error, $\sim 1 - 2\%$

Comparison with other nonperturbative results

$n_f = 2$: using most precise $\bar\Lambda_2$ lattice result: $\bar\Lambda_2 = 331 \pm 21$
(quark potential, Karbstein et al '14):
$-\langle \bar q q\rangle^{1/3}_{n_f=2}(2GeV) \simeq 278 \pm 2(rgopt) \pm 18(\bar\Lambda_2)\ MeV$

•compares rather well (within uncertainties) with latest:
-lattice (Engel et al '14, from spectral density):
$-\langle \bar q q\rangle^{1/3}_{n_f=2}(\mu = 2\mathrm{GeV}) = 261 \pm 6 \pm 8$ MeV
-spectral sum rules (latest, Narison '14): $-\langle \bar u u\rangle^{1/3} \sim 276 \pm 7$ MeV
(but sum rules indirect: determine $m_{u,d}$, then use GMOR relation
$F_\pi^2 m_\pi^2 = -(m_u + m_d)\langle \bar q q\rangle$)

•$n_f = 3$: using 2016 worl average (PDG):
$\bar\alpha_S(m_Z) = 0.118 \pm 0.0013 \rightarrow \bar\Lambda_3^{wa} \simeq 330 \pm 20 MeV$:
$-\langle \bar q q\rangle^{1/3}_{n_f=3}(2GeV, \bar\Lambda_3^{wa}) \simeq 273 \pm 4(rgopt) \pm 16(\bar\Lambda_3)\ MeV$

Alternatively we also obtain parameter-free RG-invariant results:

$$\frac{-\langle \bar q q\rangle^{1/3}_{RGI,n_f=2}}{F} = 3.25 \pm 0.02^{+0.35}_{-0.24}; \quad \frac{-\langle \bar q q\rangle^{1/3}_{RGI,n_f=3}}{F_0} = 3.04 \pm 0.04^{+0.14}_{-0.07},$$

$$\frac{\langle \bar q q\rangle^{1/3}_{RGI,n_f=3}}{\langle \bar q q\rangle^{1/3}_{RGI,n_f=2}} \simeq (0.97 \pm 0.01)\,\frac{\bar\Lambda_3}{\bar\Lambda_2} \simeq (0.94 \pm 0.01 \pm 0.12)\,\frac{F_0}{F}$$

Conclusion, prospects

•OPT gives a simple procedure to resum perturbative expansions, using only perturbative information.

•Our RGOPT version includes 2 major differences w.r.t. most previous similar approaches:

1) OPT+ RG minimizations fix optimized $\tilde{m}$ *and* $\tilde{g} = 4\pi\tilde{\alpha}_S$
2) Requiring AF-compatible solutions uniquely fixes the basic interpolation $m \to m(1-\delta)^{\gamma_0/(2b_0)}$: discards spurious solutions and accelerates convergence.

•Application to spectral density: $\pi\rho(\lambda = 0) \equiv -\langle\bar{q}q\rangle_{m\to 0}$:

•Intrinsical RGOPT theoretical error (3-4 loop): $\lesssim 2\%$

•We find a moderate reduction of $n_f = 3$ $|\langle\bar{q}q\rangle|$ w.r.t. $n_f = 2$

•final accuracy only limited due to not so precise $\bar{\Lambda}_2$ (mostly), $\bar{\Lambda}_3$

•Prospects: $T = 0$: extend our approach to calculate other order parameters of chiral sym. breaking (coefficients of chiral PT typically), or at $T \neq 0$: applications to Quark Gluon Plasma (works in progress)