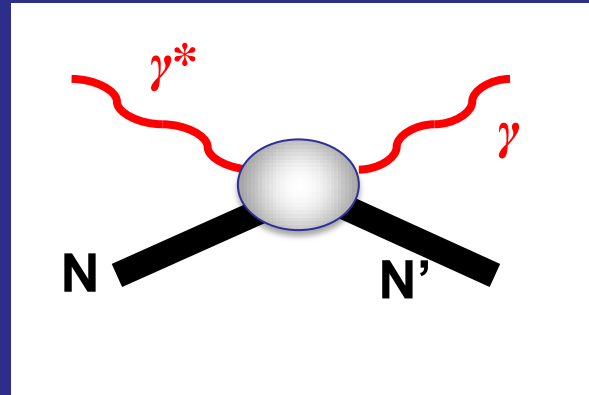


Virtual Compton Scattering on the proton at low energy



MERIEM BENALI

November 09, 2016

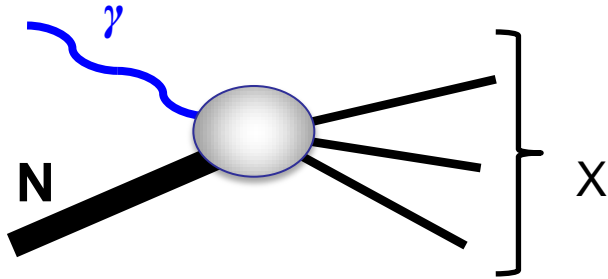
LPC-Clermont-Ferrand
GDR-QCD

Plan

- **Generalized Polarizabilities (GPs) of the proton**
- **Extraction methods of GPs at $Q^2=0.45 \text{ GeV}^2$:**
 - **Low Energy eXpansion approach (LEX)**
 - **Dispersion Relations model (DR)**
- **Conclusion**
- **VCS perspectives**

Why use EM probes?

- Powerful tool to study the internal structure of the nucleon
- An elementary and understood probe (QED)

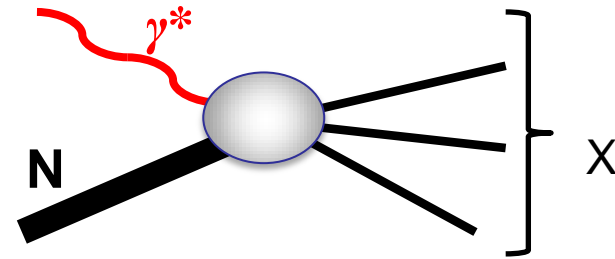


Real Photon

Momentum = energy

($q = v$)
Photon is massless

$$Q^2 = |q|^2 - v^2 = 0$$



Virtual Photon

Non-zero and negative squared mass

$$(q \neq v)$$

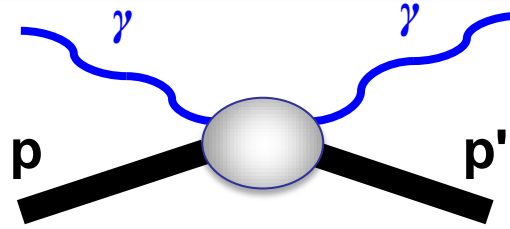
$$\text{Virtuality } Q^2 = -\underline{q}^2 \neq 0$$

= momentum transfer to the nucleon

=

Resolution of the probe $\lambda = 1/\sqrt{Q^2}$

Proton Scalar Polarizabilities



Real Compton Scattering : **RCS**

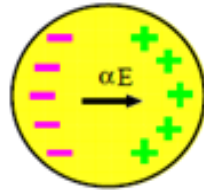
Polarizabilities: measures the response of the proton to an applied electromagnetic field.

Gives access to the **electric** and **magnetic** polarizabilities

α_E et β_M at $Q^2=0 \text{ GeV}^2$

Electric polarizability

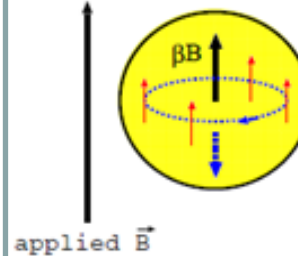
$$\vec{d}_E = \alpha_E \vec{E}$$



$$\alpha_E = (12.1 \pm 0.3_{\text{stat}} \mp 0.4_{\text{syst}}) 10^{-4} \text{ fm}^3$$

Magnetic polarizability

$$\vec{d}_M = (\beta_{\text{para}} + \beta_{\text{dia}}) \vec{B} = \beta_M \vec{B}$$



$$\beta_M = (1.6 \pm 0.4_{\text{stat}} \pm 0.4_{\text{syst}}) 10^{-4} \text{ fm}^3$$

(V.Olmos de Leon et al, Eur.Phys. J. A 10 (2001) 207)

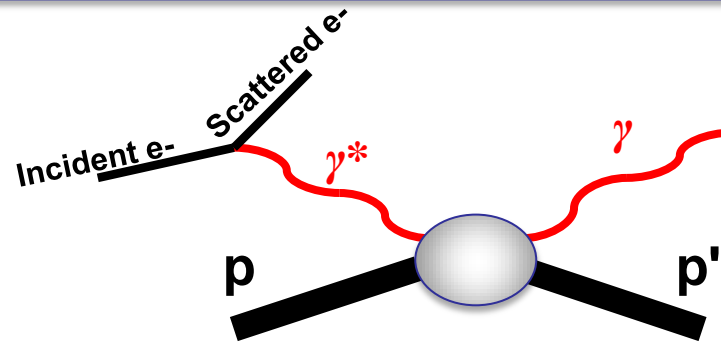
✓ α_E and β_M in RCS are the integral of local deformation in different points in the proton

To probe **locally** the polarizabilities



Virtual Compton Scattering (VCS)

From RCS to VCS



Virtual **C**ompton **S**cattering : **VCS**

$$\gamma^* p \rightarrow \gamma p$$



VCS gives access to **Generalized Polarizabilities GPs** ($Q^2 \neq 0$) $\alpha_E(Q^2)$ et $\beta_M(Q^2)$

Electric Form Factor G_E



(Fourier transform)

Spatial density of electric charge

Generalized polarizabilities



(Fourier transform)

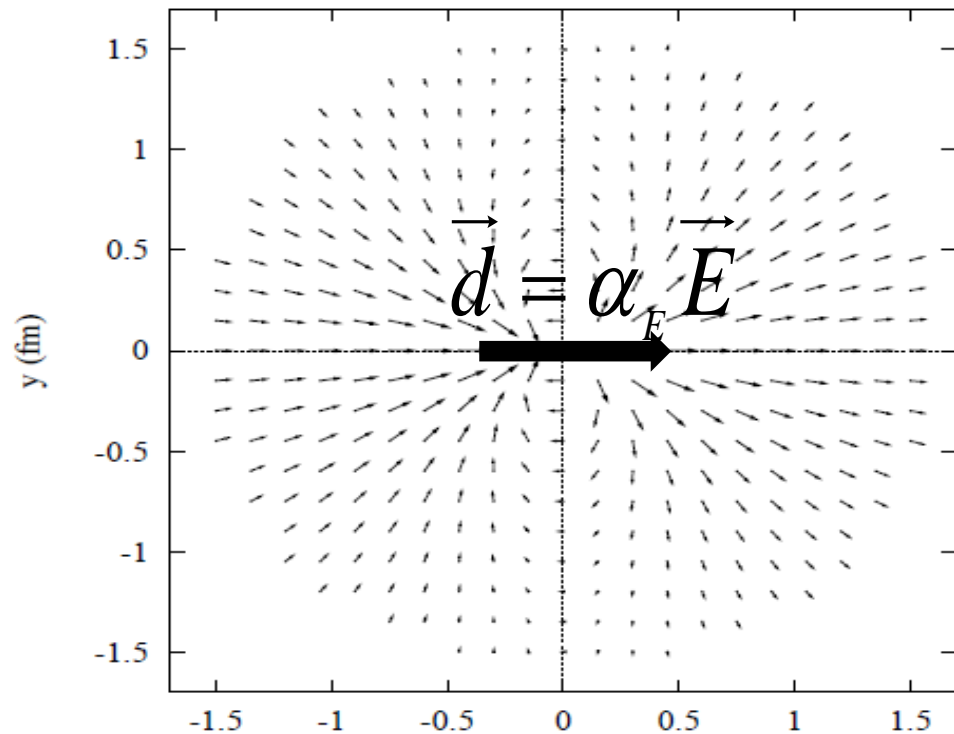
spatial density of polarization locally inside the deformed proton

$$\alpha_E(Q^2) \text{ and } \beta_M(Q^2) \xrightarrow{Q^2 \rightarrow 0} \alpha_E \text{ and } \beta_M \text{ in RCS}$$

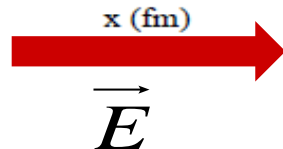
Generalized Polarizabilities GPs

- Theoretical picture obtained in the heavy-baryon chiral perturbation theory :
- ❖ Mapping the spatial distribution of the deformation (small dipoles) in the proton
 - ❖ Parameterize **the local response** of a proton in an external field.

HBC χ PT $O(p^3)$: Electric polarization in the nucleon induced by the field E_x

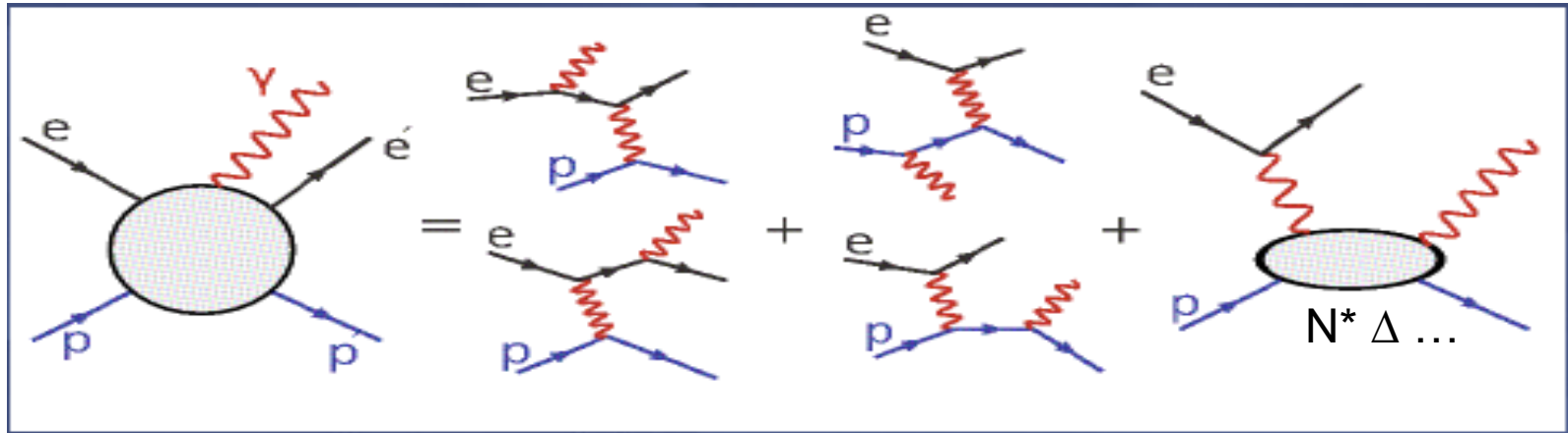


$$\vec{d} = \sum_i \vec{d}_i = \sum_i \alpha_i \vec{E}$$



Measurement Methods

The VCS amplitude is a coherent sum of the **Bethe-Heitler**, **Born** and **Non-Born** contributions:



Bethe-Heitler

VCS Born

VCS Non-Born



Known term
and
Calculated with elastic form factor

Unknown term
and
At low energy **parametrized by**
Generalized Polarisabilities GPs

Measurement Methods

Cross section $d^5\sigma(ep \rightarrow e'p'\gamma)$

Low Energy Theorem (LEX)

Extract the GPs in a model-independent way

P.Guichon et al, NPA 591 (1995) 606

Dispersion Relations model (DR)

Model-dependent approach

B.Pasquini et al, EPJA 11 (2001) 185

Structure functions
(linear combinations of GPs)

Structure functions
(linear combinations of GPs)

Need to fix the spin-flip GPs

Scalar GPs of the proton electric $\alpha_E(Q^2)$ and magnetic $\beta_M(Q^2)$

Measurement Methods

1- Low Energy Theorem (LEX)

- ❖ Unpolarized cross section below threshold pion production:

expansion in powers of q'

$$d\sigma(ep \rightarrow e' p' \gamma) = d\sigma(BH + Born) + \Phi q' \left[v_{LL} \left(P_{LL} - \frac{P_{TT}}{\epsilon} \right) + v_{LT} (P_{LT}) \right] + O(q'^2)$$

new information on the structure of the proton

negligible term at low q'

- q' = momentum of real photon

{P.A.M. Guichon et al. Nucl. Phys. (1995)}

- ❖ 2 structure functions (linear combinations of GPs) :

$$P_{LL} - \frac{P_{TT}}{\epsilon} = (\dots) \times \alpha_E(Q^2) + [\text{Spin flip GPs}]$$

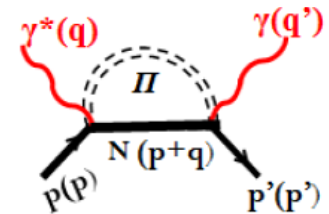
$$P_{LT} = -(\dots) \times \beta_M(Q^2) + [\text{Spin flip GPs}]$$

- ❖ For extraction of the scalar GPs, $\alpha_E(Q^2)$ and $\beta_M(Q^2)$, spin-flip GPs need to be fixed using some theoretical model (example: **using DR Model**)

Measurement Methods

2- Dispersion Relations Model (DR)

- VCS amplitude is computed through dispersion integrals calculations by using MAID model for pion production by real or virtual photon



$$d\sigma(ep \rightarrow e' p' \gamma) = d\sigma(BH + Born) + [\dots]$$

{B. Pasquini et al. EPJA 11 (2001) 185.}

includes all orders in q'

Advantages of DR model :

- Model is valid over a wide range in Q^2
- The calculation is valid above the pion production threshold
- Spin GPs are fixed by Dispersion Relations
- $\alpha_E(Q^2)$ and $\beta_M(Q^2)$ are directly parametrized by 2 free parameters Λ_α and Λ_β



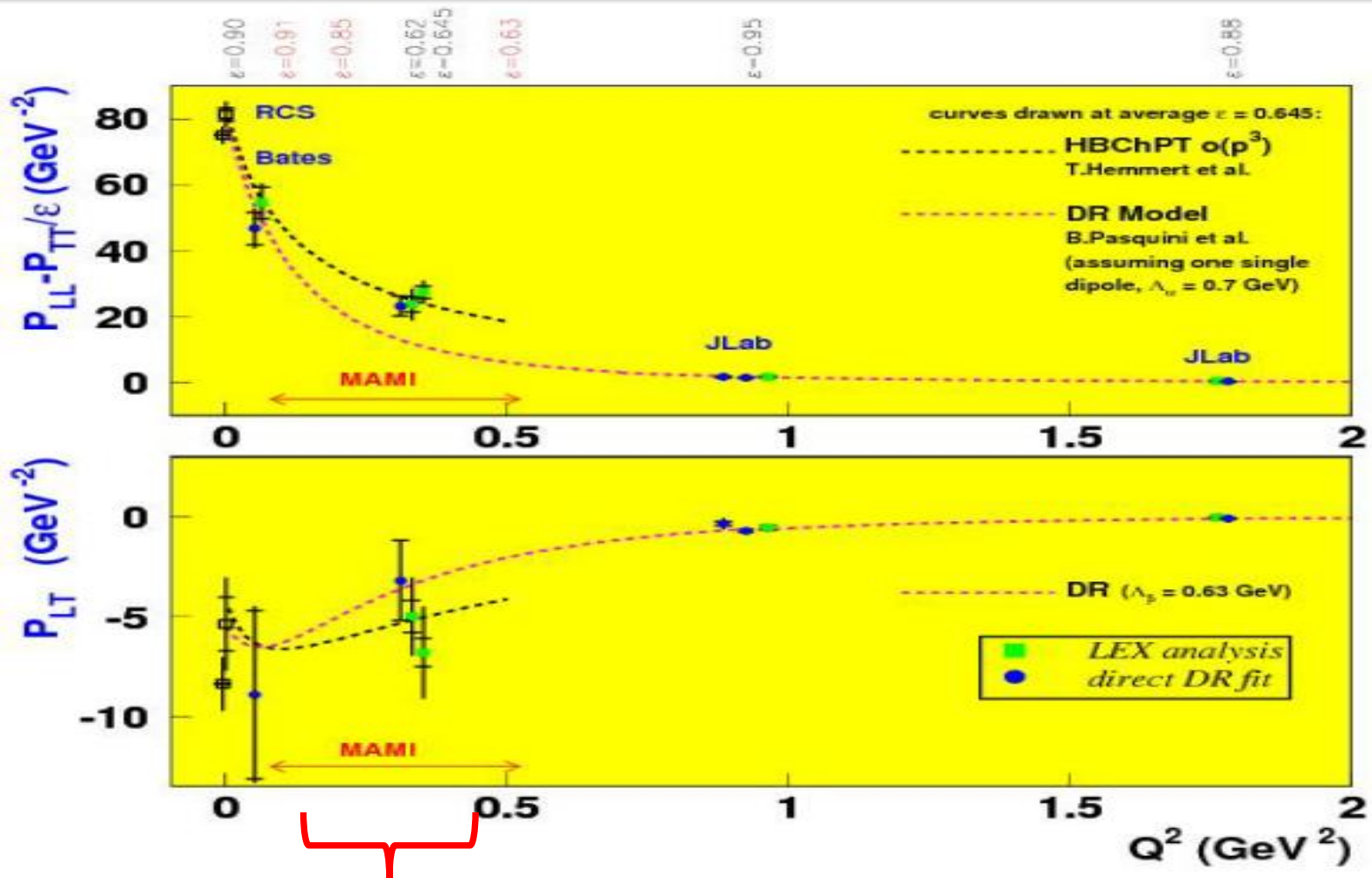
$$\alpha_E(Q^2) = \alpha_E^{\pi N}(Q^2) + \frac{(\alpha_E^{\text{exp}} - \alpha_E^{\pi N})_{Q^2=0}}{\left(1 + \frac{Q^2}{\Lambda_\alpha^2}\right)^2}$$

Asymptotic part unconstrained by DR

$$\beta_M(Q^2) = \beta_M^{\pi N}(Q^2) + \frac{(\beta_M^{\text{exp}} - \beta_M^{\pi N})_{Q^2=0}}{\left(1 + \frac{Q^2}{\Lambda_\beta^2}\right)^2}$$

Λ_α et Λ_β obtained by adjusting the $d^5\sigma^{\text{exp}}$ by $d^5\sigma^{\text{DR}}$ (predicted by the DR model)

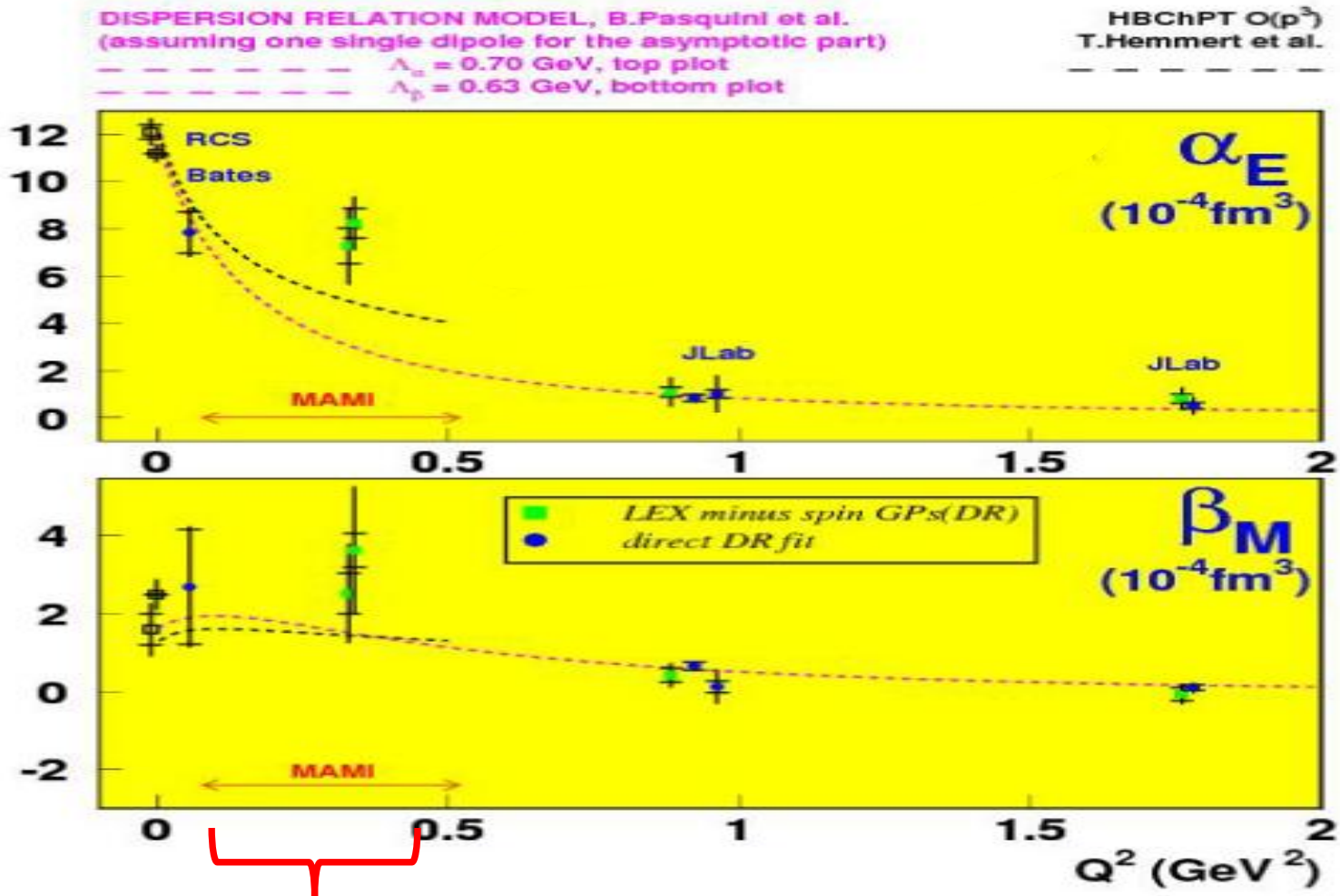
World data on the VCS Structure Functions



VCS experiment at MAMI-A1 (VCSq2)

3 new values of $Q^2 = 0.1, 0.2, 0.45 \text{ GeV}^2$

World data on the VCS GPs



VCS experiment at MAMI-A1 (VCSq2)

3 new values of $Q^2 = 0.1, 0.2, 0.45$ GeV²

(Figures constructed by H. FONVIEILLE)

Goal

Goal of VCSq2 experiment:

Measure the GPs of the proton at $Q^2=0.1, 0.2$ and 0.45 GeV^2 below pion threshold

My thesis Goal (at $Q^2=0.45 \text{ GeV}^2$)

- ❖ Measure the 2 structure functions (using LEX approach and DR model):
 $(P_{LL}-P_{TT}/\epsilon)$ and P_{LT}
- ❖ Measure the electric $\alpha_E(Q^2)$ and magnetic $\beta_M(Q^2)$ GPs

Experimental Facility

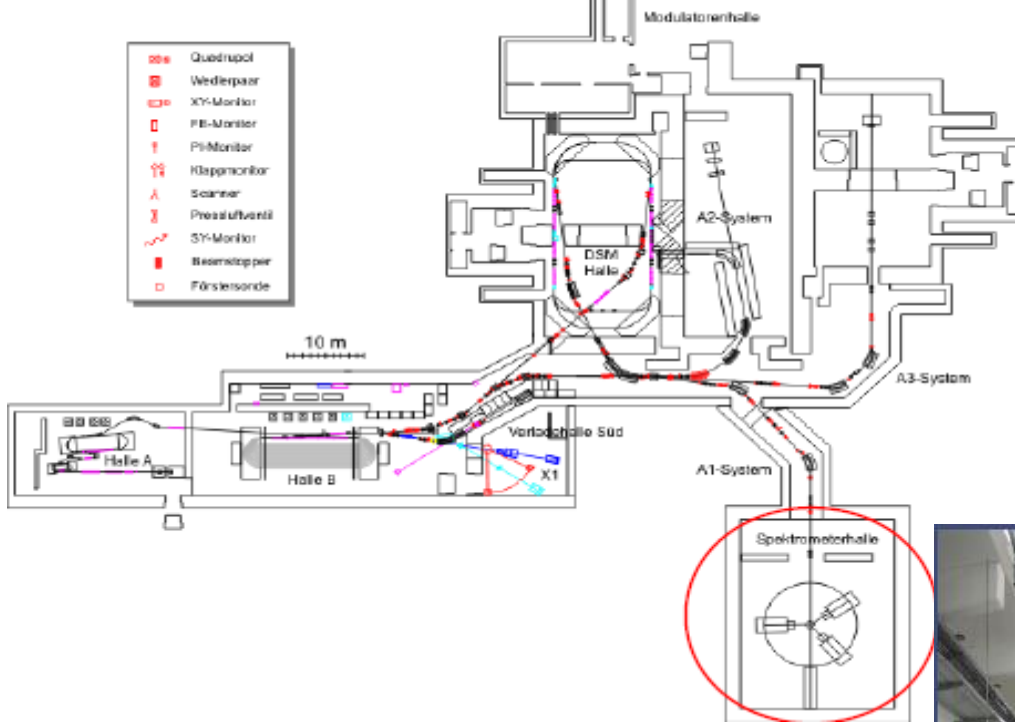
The VCS experiment was performed at **MAMI-A1** from 2011 to 2015

Mainz Microtron MAMI

❖ Electron accelerator

- ✓ Polarized electron source
- ✓ Linac
- ✓ 4 microtrons
- ✓ $E_{max}=1.6$ GeV
- ✓ $I_{max}=100$ μ A

❖ 4 experimentals Halls (**A1 electron scattering**, A2 tagged photons, A4 parity violation, X1 X-rays)



Hall A1 (3 spectrometers setup)

We used 2 Spectrometers A and B

- ✓ momentum resolution $\leq 10^{-4}$
- ✓ Angular resolution ≤ 3 mrad

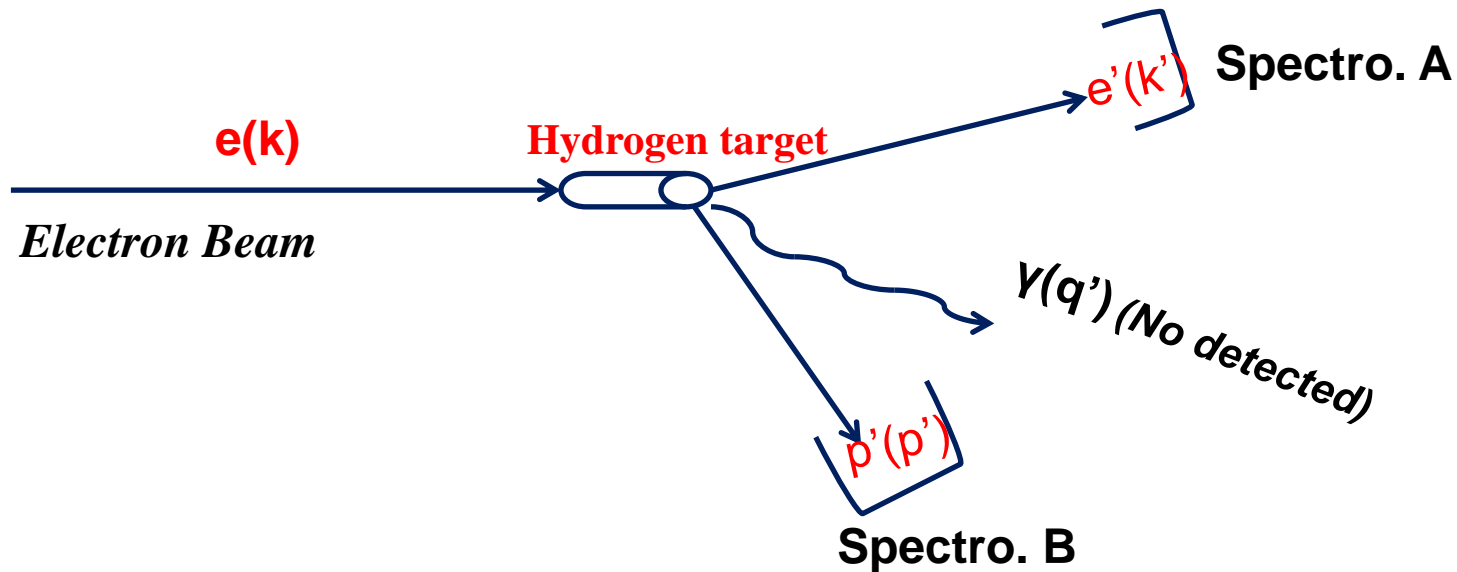
Composed by

- ✓ 4 VDC planes
- ✓ 2 scintillators planes
- ✓ Cerenkov detector



Hall A1

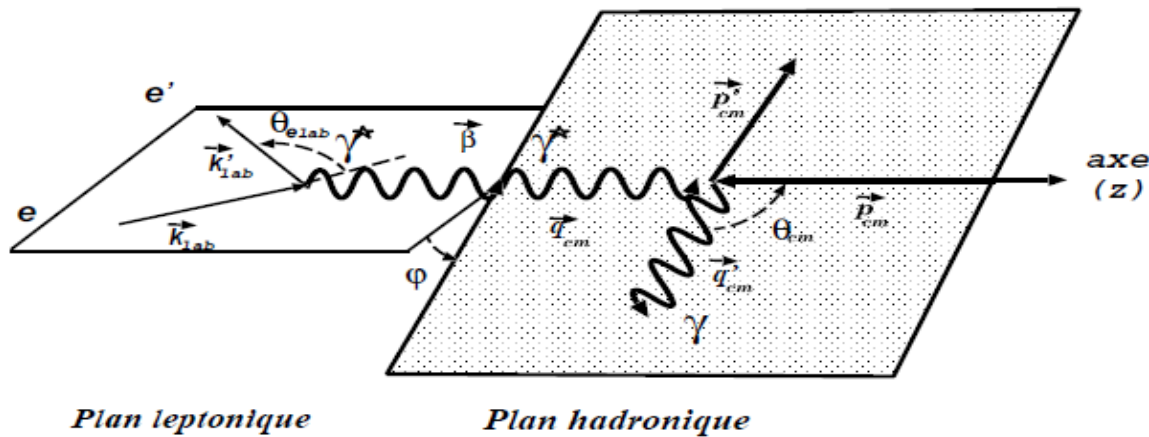
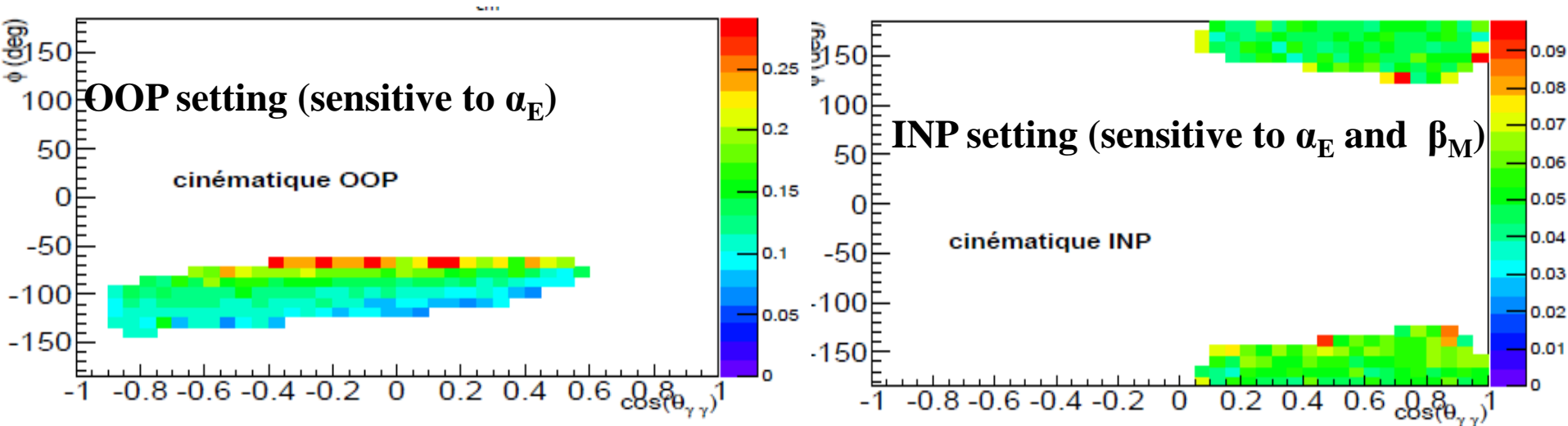
- In our experiment we detect the scattered **electron** in spectrometer B and the recoil **proton** in spectrometer A
- The emitted real photon is signed by a zero missing mass:



- ✓ Beam energy $\approx 1\text{ GeV}$.
- ✓ $Q^2=0.45\text{ GeV}^2$
- ✓ $q'_{\text{cm}}=0.104\text{ GeV}$

Kinematics of the experiment

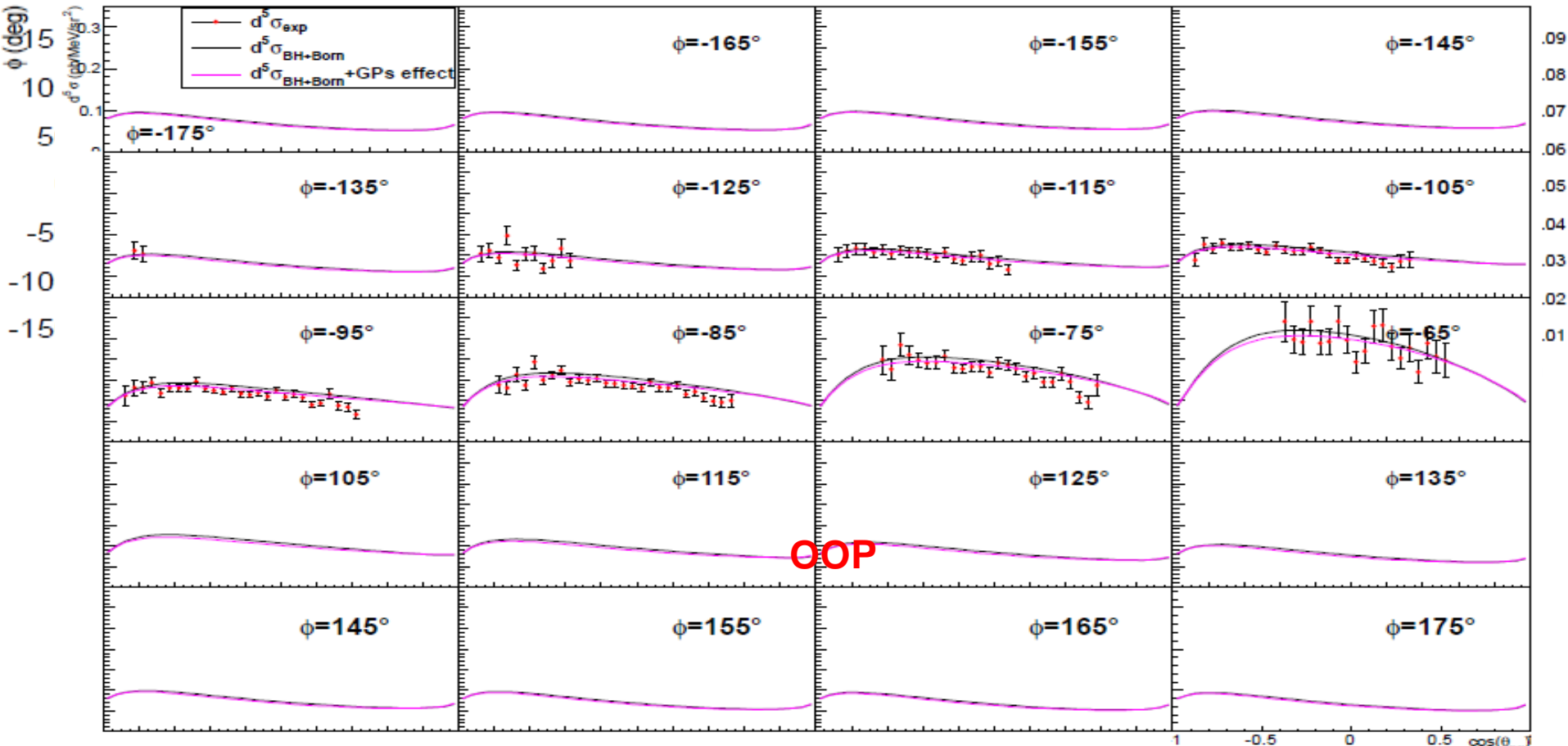
Our analyzed data are a combination out-of-Plane and In-Plane data, covering two different angular regions in $(\phi, \cos(\theta))$



Kinematics of the experiment

Our analyzed data are a combination out-of-Plane and In-Plane data, covering two different angular regions in $(\phi, \cos(\theta))$

— $\sigma(\text{exp})$ — $\sigma(\text{BH+Born})$: No GPs effect — $\sigma(\text{BH+Born+GPs effect})$

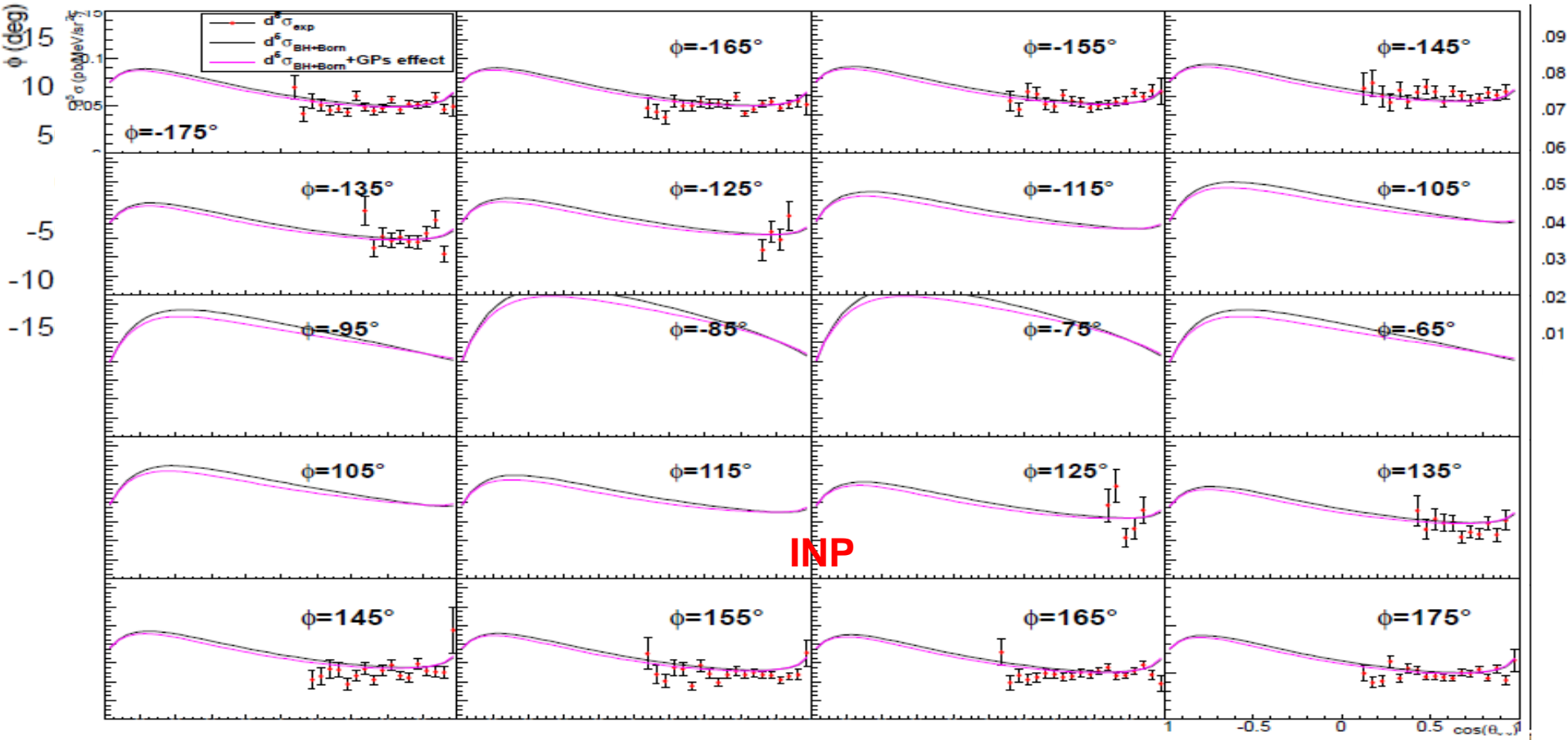


We measure the small deviation between $\sigma(\text{BH+Born})$ and $\sigma(\text{BH+Born+GPs})$ in order to extract the GPs

Kinematics of the experiment

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We measure the small deviation between $\sigma(\text{BH+Born})$ and $\sigma(\text{BH+Born+GPs})$ in order to extract the GPs

Fit of structure functions – LEX

LEX cross section: $\sigma^{LEX} = \sigma(BH + Born) + \Phi q' \Psi_0 + O(q'^2)$

Adjustment : $\chi^2 = \sum_i \left[\frac{\sigma_i^{exp} - \sigma_i^{LEX}}{\Delta \sigma_i^{exp}} \right]^2$

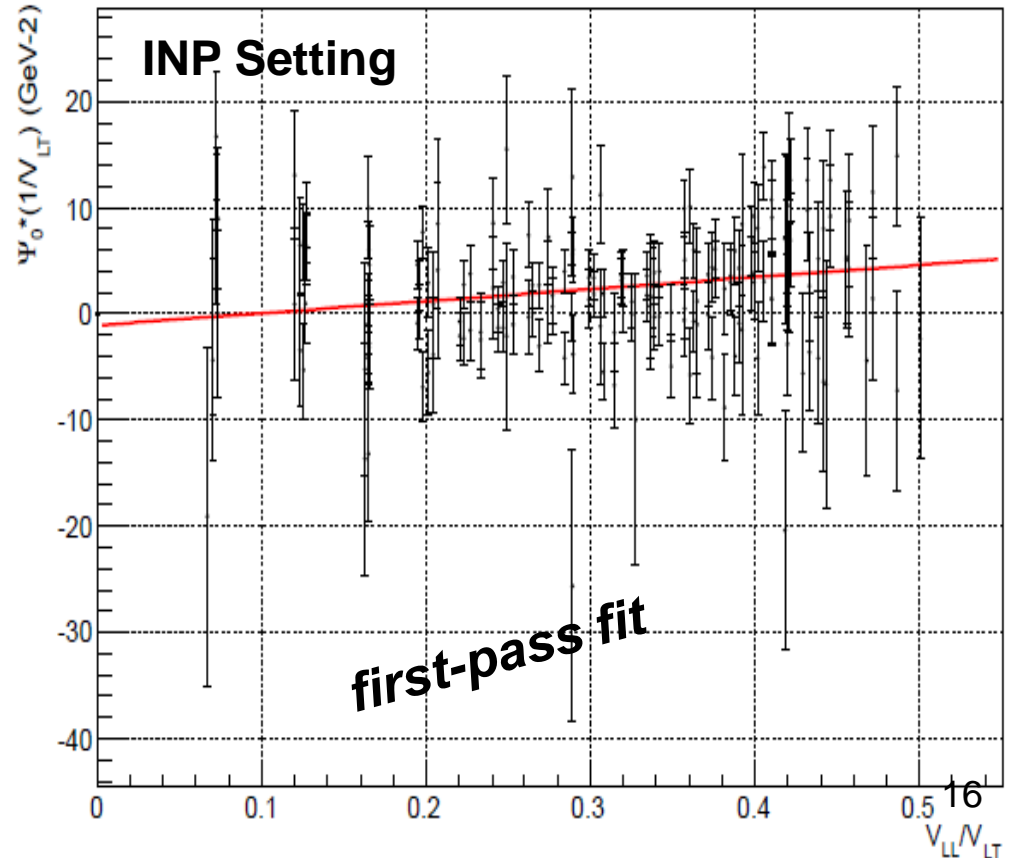
$$\Psi_0 = v_{LL} \left(P_{LL} - \frac{P_{TT}}{\epsilon} \right) + v_{LT} P_{LT}$$

At low q' :

$$\frac{\Psi_0}{v_{LT}} = \frac{\sigma^{exp} - \sigma(BH + Born)}{\Phi q' v_{LT}}$$

$$= \frac{v_{LL}}{v_{LT}} \left(P_{LL} - \frac{P_{TT}}{\epsilon} \right) + P_{LT}$$

slope
intercept



Normalization step

Interest: At low q' (**37.5 MeV/c** in our case) the experimental cross section is dominated by the (BH+Born) cross section :

$$\sigma^{LEX} = \sigma^{BH+B} + q'[\dots] \xrightarrow{\text{Low energy theorem}} \left[\begin{array}{l} q' \rightarrow 0 \\ \sigma^{\text{exp}} \rightarrow \sigma^{BH+B} \end{array} \right]$$

- **GPs effect at low q' ~ 1 %**
- **Renormalize the σ^{exp} on σ^{LEX} at low q' (χ^2 minimization)**
- **Renormalization is Form factors depends**

($P_{LL} - P_{TT} / \epsilon$) (GeV^{-2})

Form factors	Fnorm	before normalisation	After normalisation
Bernauer Phys.Rev.Lett. 105:242001, 2010	1.052	15.29 ± 0.96	7.39 ± 1.02
F-Walcher Eur.Phys, J.A17 :607 2003	1.026	10.62 ± 0.96	6.88 ± 0.98
Arrington Phys.Rev.C76:035205, 2007	1.032	11.80 ± 0.96	7.14 ± 1.00

Result fit LEX

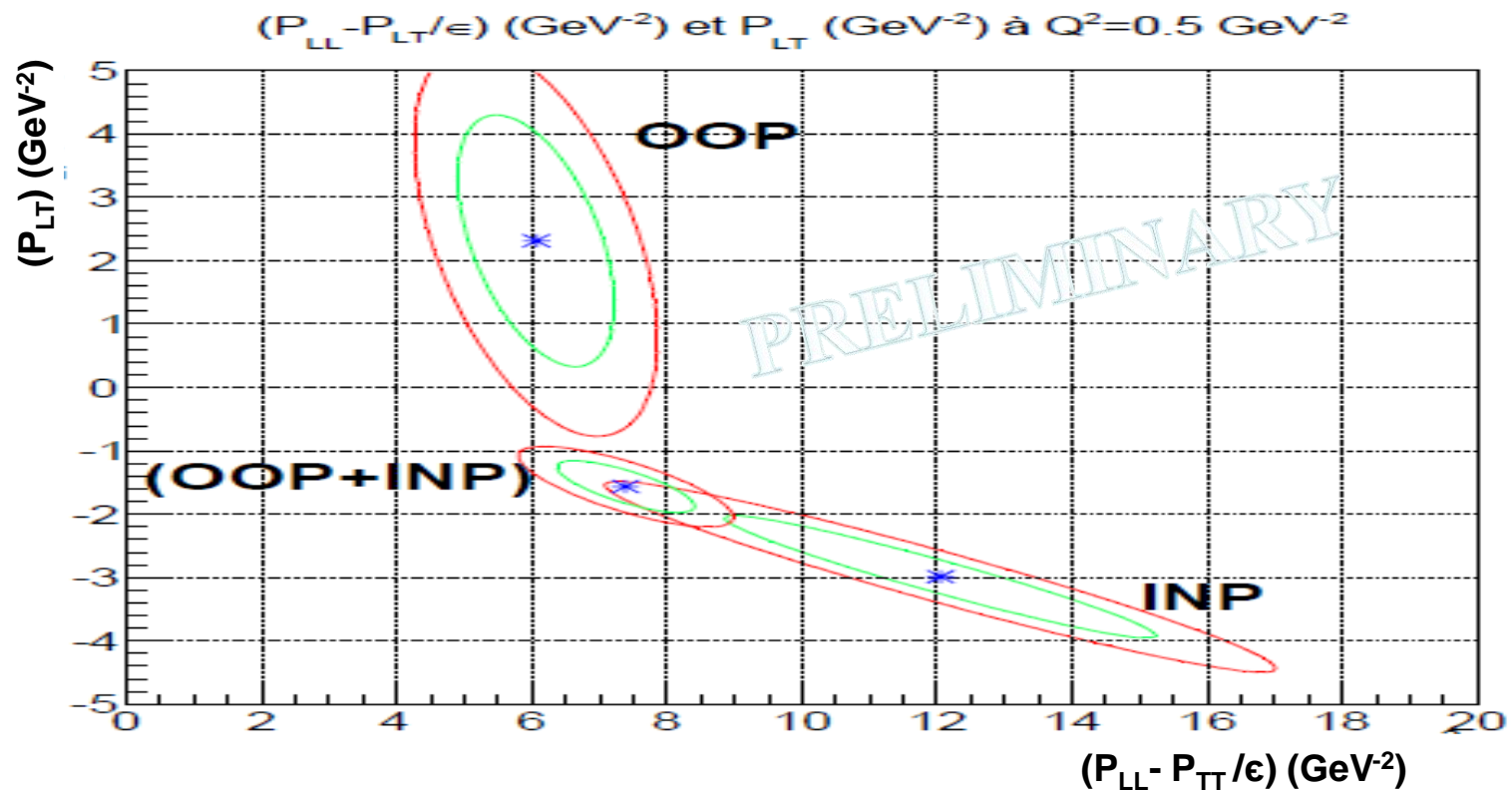
Preliminary results of structure functions extracted with LEX approach,

(The systematic error is obtained from 2% overall error on the cross section)

$$P_{LL}-P_{TT}/\epsilon = 7.39 (\pm 1.02)_{stat} (\pm 2.52)_{syst} \text{ GeV}^{-2}$$

$$P_{LT} = -1.56 (\pm 0.41)_{stat} (\pm 0.04)_{syst} \text{ GeV}^{-2}$$

$$X^2_{reduced} = 1.35$$



Generalized polarizabilities fit – DR

Method:

$$\chi_{DR}^2 = \sum_i \left[\frac{\sigma_i^{\text{exp}} - \sigma_i^{\text{DR}}(\alpha_E, \beta_M)}{\Delta \sigma_i^{\text{exp}}} \right]^2$$

Unlike χ_{LEX}^2 , the χ_{DR}^2 is not linear with α_E et β_M



- Construct a grid in the plan (α_E, β_M)
- Calculate χ^2 for each bin



Find the χ_{\min}^2 using paraboloid fit

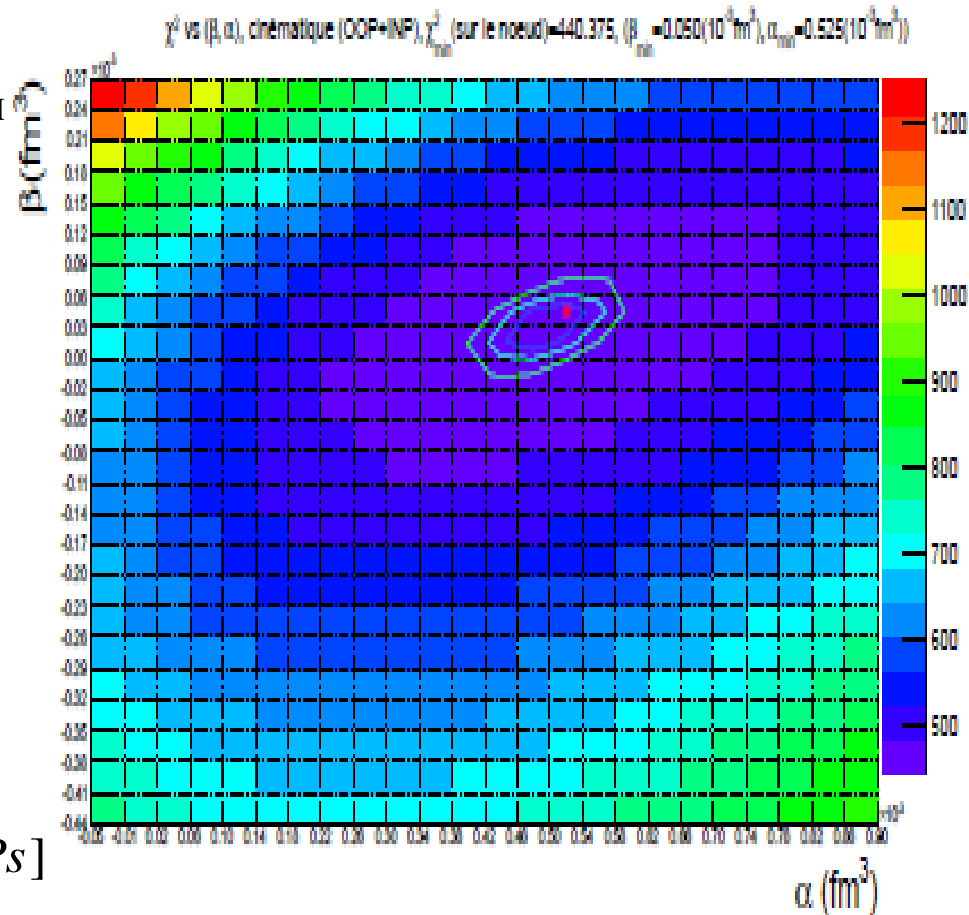


Optimal values of α_E and β_M

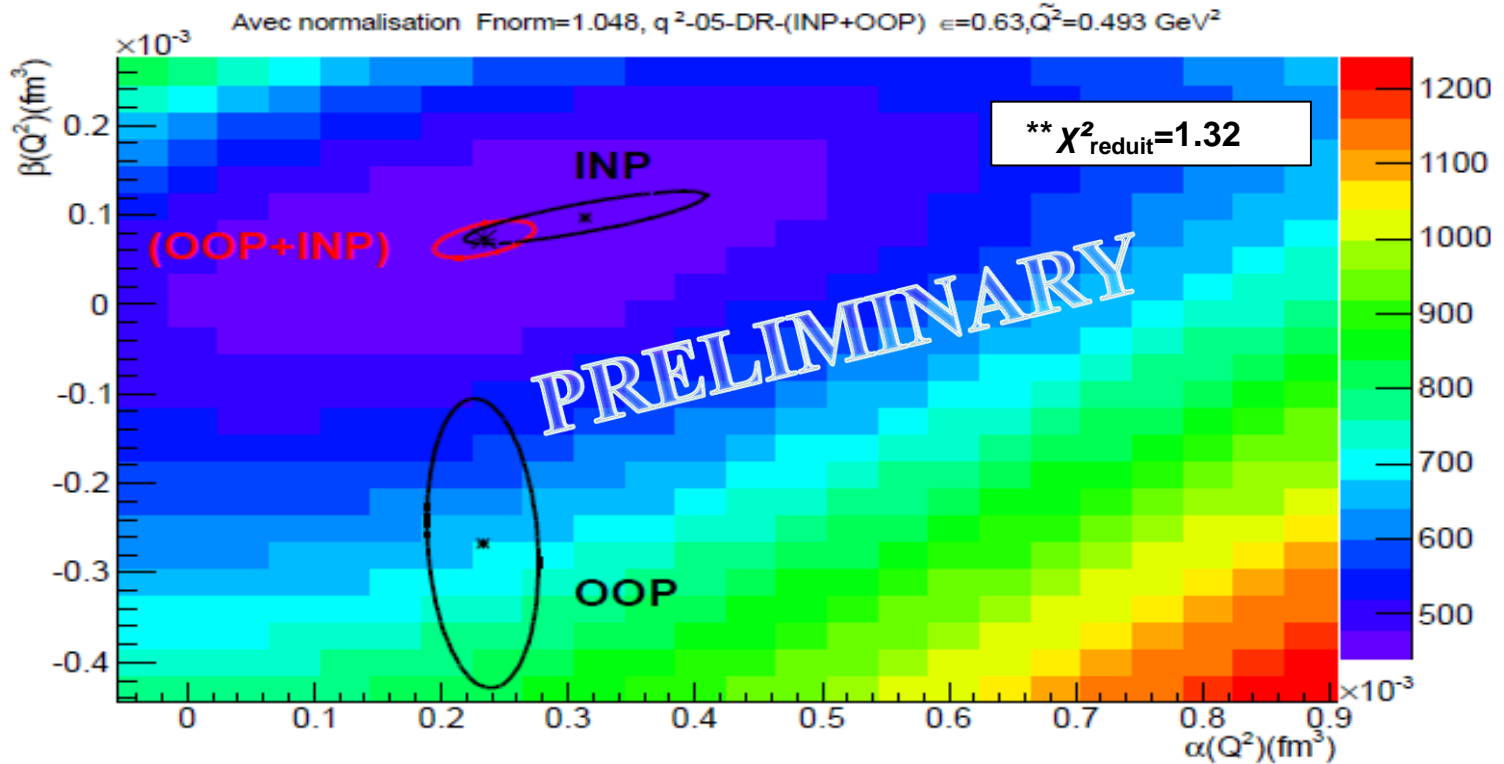


$$P_{LL} - \frac{P_{TT}}{\epsilon} = (\dots) \times \alpha_E(Q^2) + [Spin_flip_GPs]$$

$$P_{LT} = -(\dots) \times \beta_M(Q^2) + [Spin_flip_GPs]$$



Generalized polarizabilities fit – DR



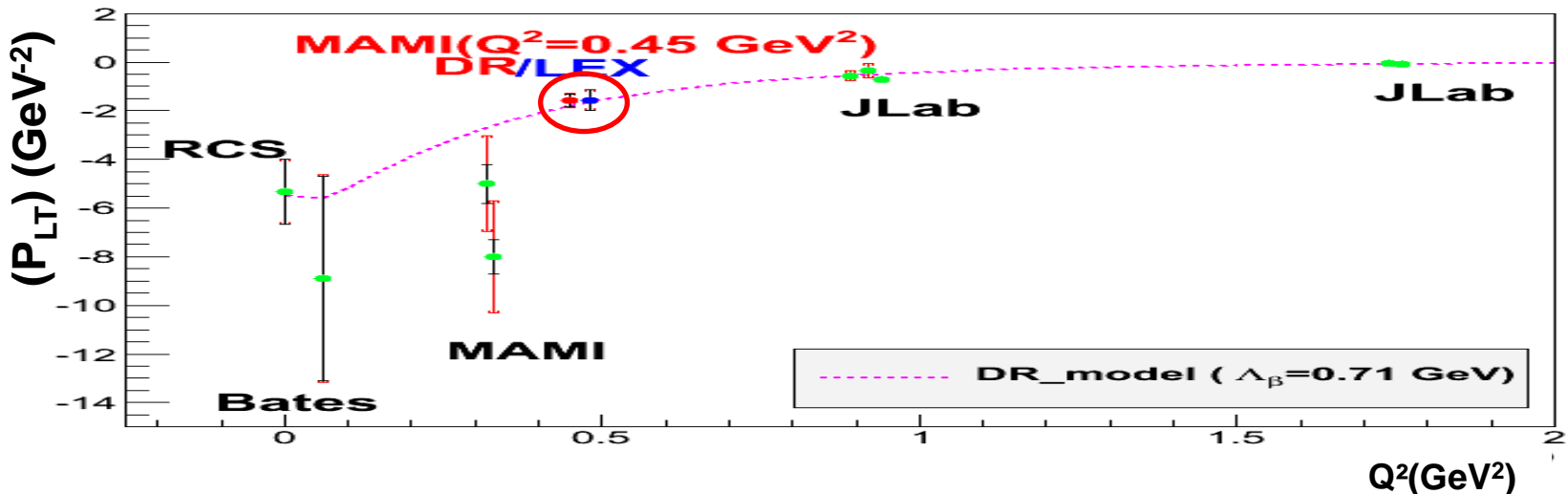
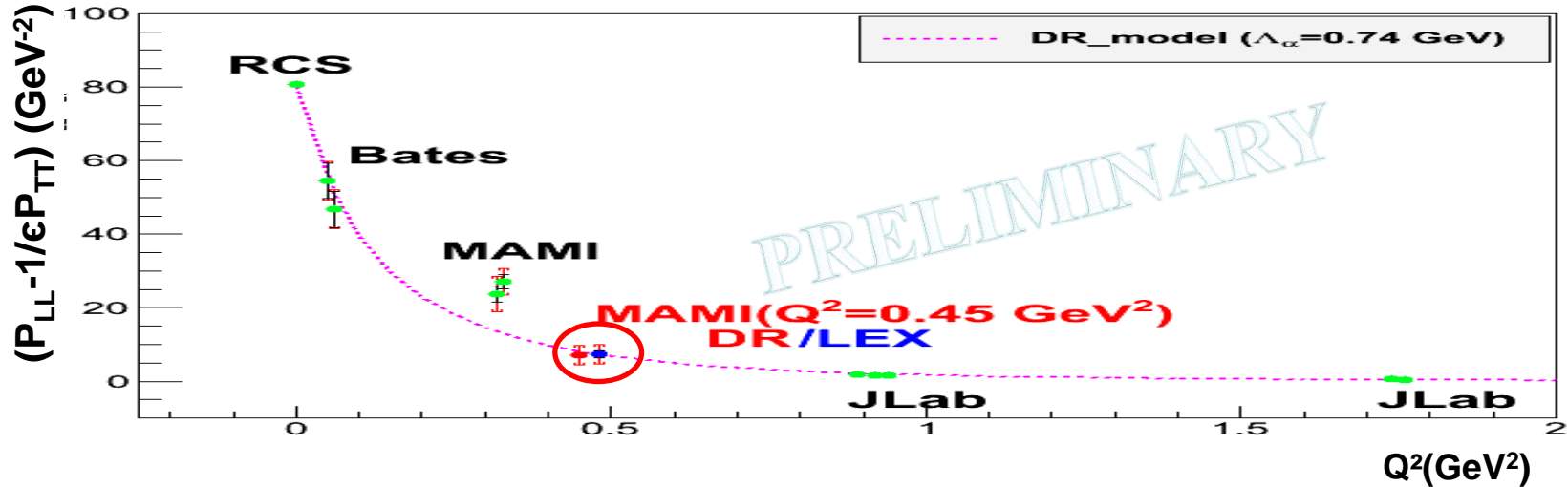
- Contour at $(\chi^2_{min}+1)$, 70% probability that each parameter is inside separately

- Systematic errors obtained from $\pm 2\%$ normalization error on the cross section.

$\Lambda\alpha$ (GeV)	$0.74 (\pm 0.06)_{stat} (\pm 0.16)_{syst}$
$\Lambda\beta$ (GeV)	$0.71 (\pm 0.04)_{stat} (\pm 0.02)_{syst}$
$\alpha_E (10^{-4} \text{ fm}^3)$	$2.34 (\pm 0.40)_{stat} (\pm 1.07)_{syst}$
$\beta_M (10^{-4} \text{ fm}^3)$	$0.73 (\pm 0.20)_{stat} (\pm 0.13)_{syst}$
$P_{LL}-P_{TT}/\epsilon$ (GeV ²)	$7.08 (\pm 0.90)_{stat} (\pm 2.40)_{syst}$ ₂₀
P_{LT} (GeV ²)	$-1.58 (\pm 0.25)_{stat} (\pm 0.16)_{syst}$

LEX and DR comparison

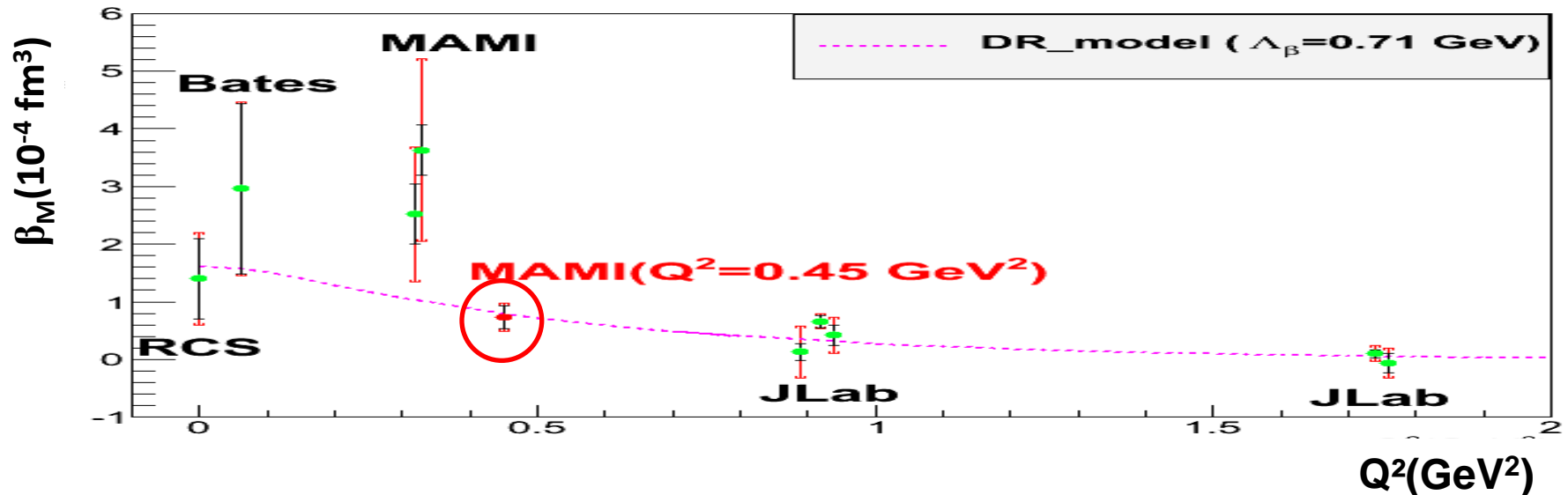
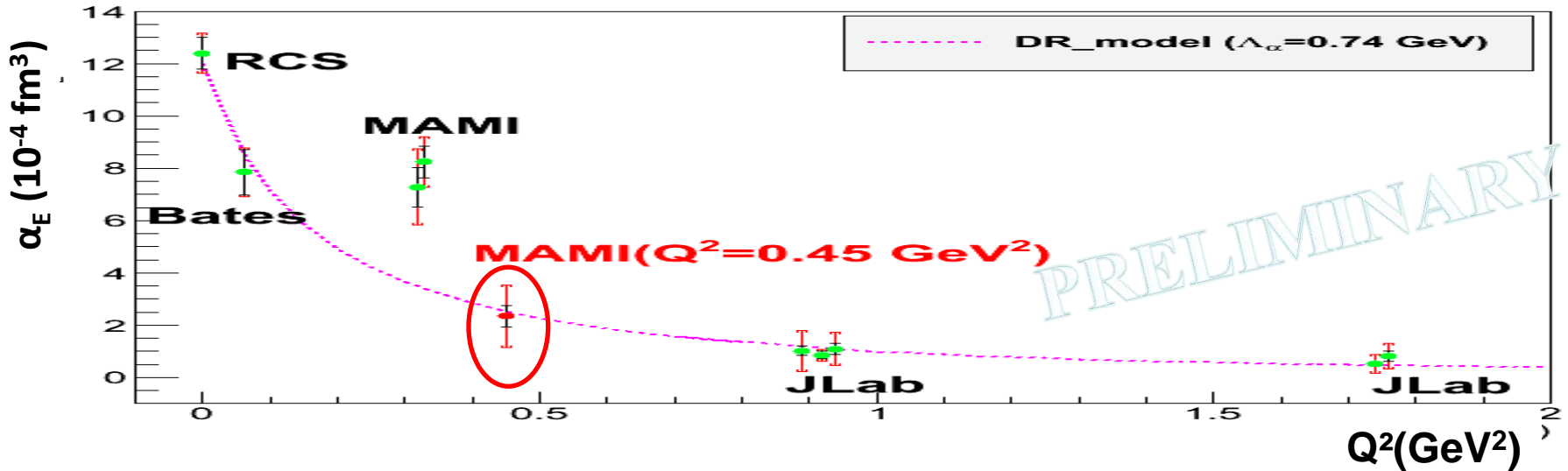
- A good agreement between the **LEX** and **DR Fit** at $Q^2=0.45 \text{ GeV}^2$



DR model does NOT predict the structure functions, the “DR curve” includes another assumption: fixing the two free parameters to a constant, independently of Q^2

GPs DR extraction at $Q^2=0.45 \text{ GeV}^2$

World data on the VCS Generalized Polarizabilities (GPs)

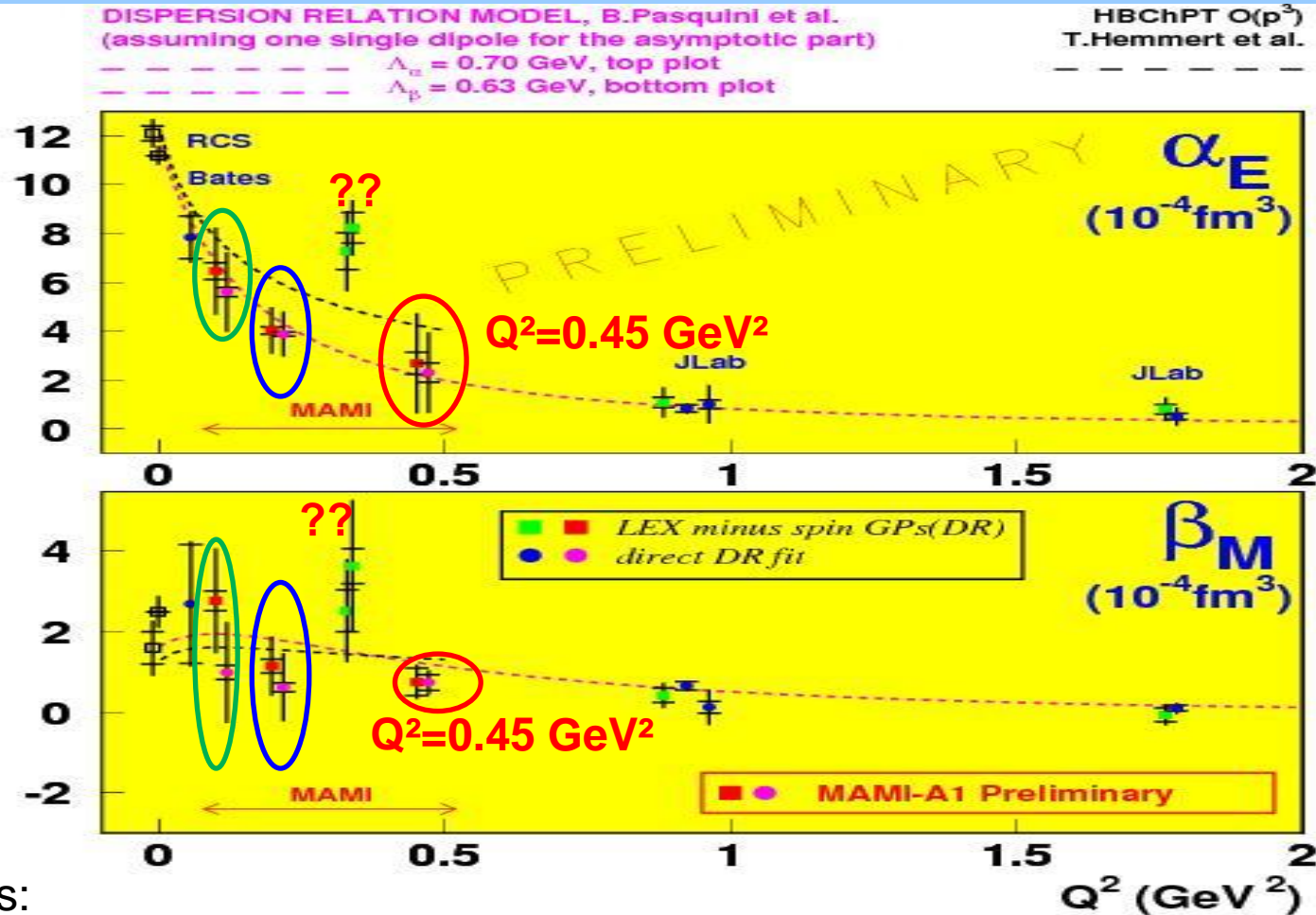


→ More complete picture on the GPs repartition inside the proton

Conclusion

- Fit of the **electric (α_E) and magnetic (β_M)** Generalised Polarizabilities at ($Q^2=0.45\text{GeV}^2$) via the Dispersion Relations model and **deduce the structure functions values P_{LL} - P_{TT} / ϵ and P_{LT} .**
- Extracting the **same two structure functions** via LEX approach
 - A good agreement between the results of the two extractions
 - New constraint on nucleon structure at low energy

GPs from the VCSq2 MAMI experiment at $Q^2=0.1, 0.2$ and 0.45 GeV^2



GPs:

at $Q^2=0.1 \text{ GeV}^2$, J.Bericic, PhD student, Ljubljana university, Slovenia
at $Q^2=0.2 \text{ GeV}^2$, L.Correa, PhD student (LPC Clermont Ferrand, France)

- Understand the region around $Q^2=0.33 \text{ GeV}^2$

Perspectives in VCS

JLab VCS proposal in Hall C to measure the electric (α_E) and magnetic (β_M) at $Q^2 = 0.3 \text{ GeV}^2$ to 0.75 GeV^2 (+ **one more measurement at $Q^2 = 0.33 \text{ GeV}^2$**):

- Using asymmetry of the ($ep \rightarrow ep\gamma$) cross section in the $\Delta(1232)$ region
- Using the DR model

N.Sparveris, M.Paolone, A.Camsonne, M.Jones et al (2016)

**Measurement of the Generalized Polarizabilities of the Proton
in Virtual Compton Scattering**

Proposal to Jefferson Lab PAC-44

*H. Atac, H. Banjade, A. Blomberg, S. Joosten, Z.E. Meziani,
M. Paolone (spokesperson), N. Sparveris (spokesperson / contact person)
Temple University, Philadelphia, PA, USA*

*A. Camsonne (spokesperson), J.P. Chen, M. Jones (spokesperson)
Thomas Jefferson National Accelerator Facility, Newport News, VA, USA*

*J. Badman, S. Li, E. Long, K. McCarty, C. Meditz, M. O'Meara, R. Paremuzyan,
S. Santiesteban, P. Solvignon-Slifer, K. Slifer, B. Yale, R. Zielinski
University of New Hampshire, Durham NH, 03824*

K Allada A Bernstein S Gilad

THANK YOU FOR YOUR ATTENTION

Back-up

Normalization step of the $d\sigma^{\text{exp}}$ cross section

Interest: At low q' (37.5 MeV/c in our case) the experimental cross section is dominated by the (BH+Born) cross section :

$$\sigma^{\text{exp}} = \sigma^{\text{BH+B}} + q'[\dots] \xrightarrow{\text{Low energy theorem}} \begin{cases} q' \rightarrow 0 \\ \sigma^{\text{exp}} \rightarrow \sigma^{\text{BH+B}} \end{cases}$$

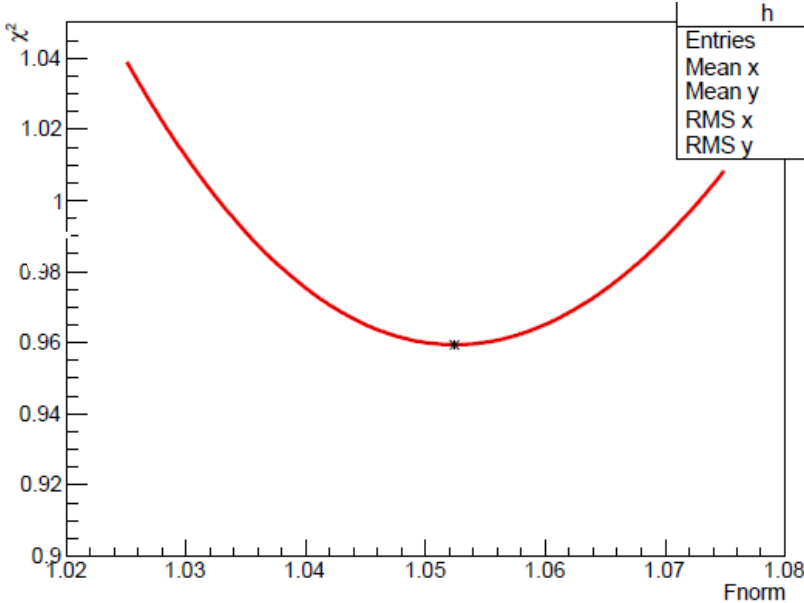
At low q' , the **GPs effect** accounts for about ~ 1.0% (but at high $q'=104$ MeV/c \rightarrow **GPs effect ~ 6%**) of the cross section. Using the value of the structure functions from our first-pass fit \rightarrow we can basically **FIX** the term [...] and test if our measured cross section needs to be renormalized globally:

Method:

$$\chi^2 = \sum_i \left[\frac{F_{\text{norm}} \times \sigma_i^{\text{exp}} - \sigma_{(\text{low } q')}^{\text{LEX}}}{F_{\text{norm}} \times \Delta \sigma_i^{\text{exp}}} \right]^2$$

↓

$$F_{\text{norm}} = 1.052 \quad \text{Using Bernauer Form factors}$$



\rightarrow this is of the order of an GPs effect at important q' !!!

Our hypothesis: this percentage (5.2%) depends on the choice of proton form factors, because the (BH+Born) cross section depends on this FF choice.

Normalization step of the $d\sigma^{\text{exp}}$ cross section

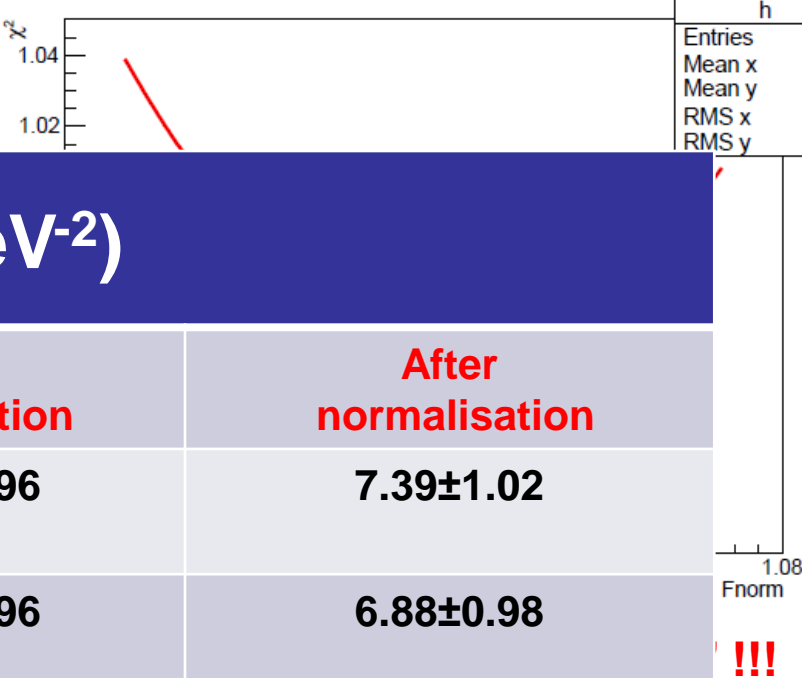
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Method:

$(P_{\text{LL}} - \frac{P_{\text{TT}}}{\epsilon})$ et P_{LL} de (OOP + INP) (From first pass fit)



$(P_{\text{LL}} - P_{\text{TT}} / \epsilon) (\text{GeV}^{-2})$

Form factors	Fnorm	before normalisation	After normalisation
Bernauer Phys.Rev.Lett. 105:242001, 2010	1.052	15.29±0.96	7.39±1.02
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Normalization step of the $d\sigma^{\text{exp}}$ cross section

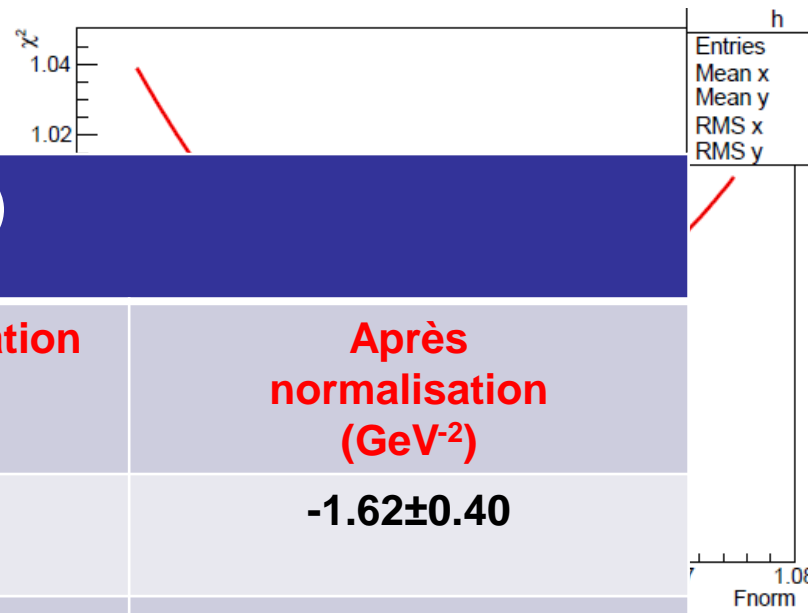
Interest: At low q' (37.5 MeV/c in our case) the experimental cross section is dominated by the (BH+Born) cross section :

$$\sigma^{\text{exp}} = \sigma^{\text{BH+B}} + q'[\dots] \xrightarrow{\text{Low energy theorem}} \begin{cases} q' \rightarrow 0 \\ \sigma^{\text{exp}} \rightarrow \sigma^{\text{BH+B}} \end{cases}$$

At low q' , the **GPs effect** accounts for about $\sim 1.0\%$ (but at high $q'=104$ MeV/c \rightarrow **GPs effect** $\sim 6\%$) of the cross section. Using the value of the structure functions from our first-pass fit \rightarrow we can basically **FIX** the term [...] and test if our measured cross section needs to be renormalized globally:

Method:

$(P_{LL} - \frac{P_{TT}}{2})$ et P_{de} de (OOP + INP) (From first pass fit)



P_{LT} (GeV⁻²)

	Facteurs de forme	Fnorm	Avant normalisation (GeV ⁻²)	Après normalisation (GeV ⁻²)
χ^2	Bernauer	1.052	-1.57±0.39	-1.62±0.40
	Phys.Rev.Lett. 105:242001, 2010			
	F-Walcher	1.026	-1.74±0.38	-1.76±0.40
	Eur.Phys. J.A17 :607 2003			
	Arrington	1.032	-1.65±0.38	-1.68±0.40
	Phys.Rev.C76:035205, 2007			

q' !!!