Perturbative aspects of the QCD phase diagram

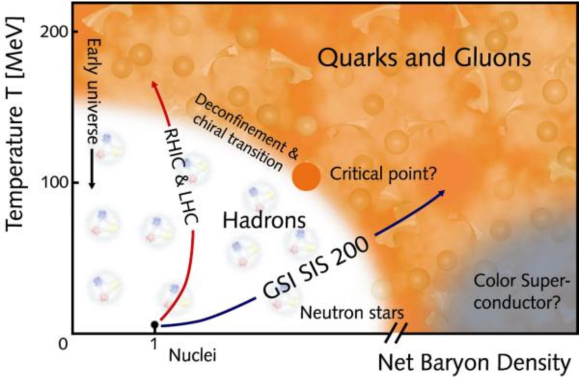
Urko Reinosa*

(based on collaborations with J. Serreau, M. Tissier and N. Wschebor)

*Centre de Physique Théorique, Ecole Polytechnique, CNRS.

GDR-QCD, November 10, 2016, IPNO.

Theoretical approaches to the QCD phase diagram



Non-exhaustive list:

- finite-temperature lattice QCD
- Schwinger-Dyson equations
- functional renormalization group

} non-perturbative

Perturbation theory?

Outline

I. A modified perturbation theory in the infrared?

(Vacuum, Landau gauge).

II. Application to the QCD phase structure.

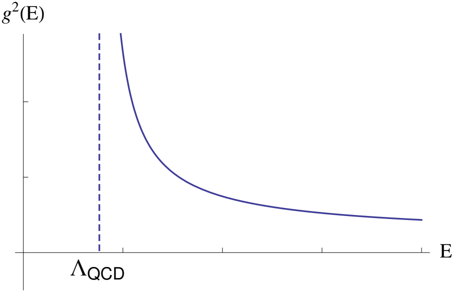
(Finite T, Landau-DeWitt gauge).

A modified perturbation theory in the infrared

Perturbation theory: common wisdom

The QCD running coupling decreases at high energies: asymptotic freedom.

As a counterpart, the (perturbative) running coupling increases when the energy is decreased, and even diverges at a Landau pole known as Λ_{QCD}:

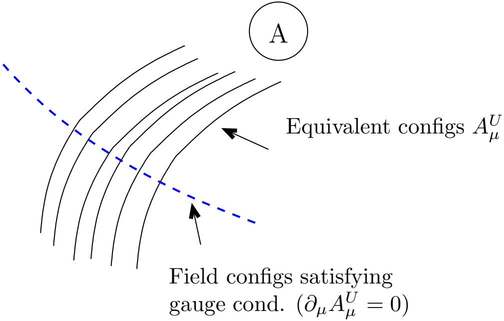


Yes but ... perturbation theory is based on the Faddeev-Popov procedure which is at best valid at high energies and should be modified at low energies.

Perturbation theory: (not so) common wisdom

Perturbation theory defined only within a given gauge-fixing (ex: $\partial_\mu A^a_\mu = 0$).

The gauge-fixing is based on the Faddeev-Popov approach:



$$\Rightarrow \mathcal{L} = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a (D_\mu c)^a + i h^a \partial_\mu A^a_\mu$$

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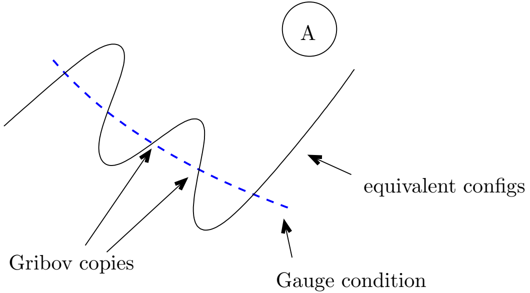

$$\Rightarrow \mathcal{L} = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a (D_\mu c)^a + i h^a \partial_\mu A^a_\mu$$

Valid at best at high energies!

Perturbation theory: (not so) common wisdom

Perturbation theory defined only within a given gauge-fixing (ex: $\partial_\mu A_\mu^a = 0$).

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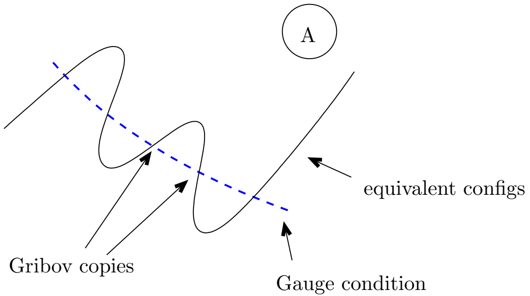


$$\Rightarrow \mathcal{L} = \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + \partial_\mu \bar{c}^a (D_\mu c)^a + i h^a \partial_\mu A_\mu^a + \boxed{\mathcal{L}_{\text{Gribov}} \leftarrow \text{not known so far}}$$

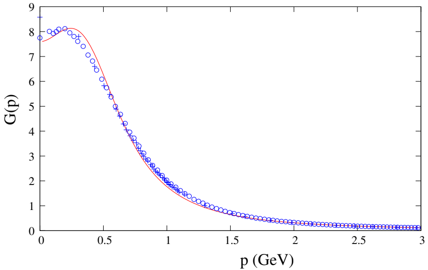
Could $\mathcal{L}_{\text{Gribov}}$ restore the applicability of perturbation theory in the IR?

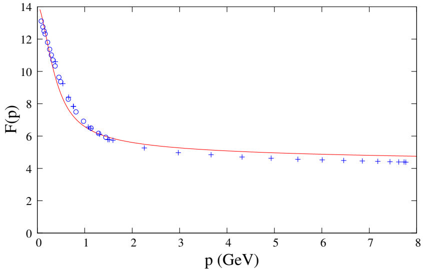
How to constrain $\mathcal{L}_{\text{Gribov}}$?

(Landau gauge) lattice simulations [Cucchieri and Mendes; Bogolubsky *et al*; Dudal *et al*; . . .]
are crucial: ⋆ free from the Gribov ambiguity;
⋆ provide valuable information to construct models for $\mathcal{L}_{\text{Gribov}}$.

In particular, they predict:

- ⋆ a gluon propagator $G(p)$ that behaves like a massive one at $p = 0$;
- ⋆ a ghost propagator $F(p)/p^2$ that remains massless.





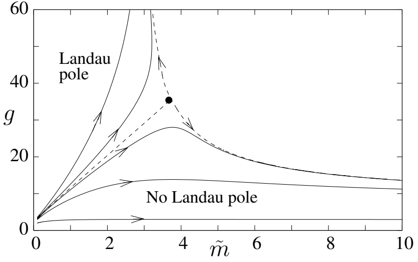
Our model

As the dominant contribution to $\mathcal{L}_{\text{Gribov}}$, we propose a gluon mass term:

$$\mathcal{L} = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a (D_\mu c)^a + i h^a \partial_\mu A^a_\mu + \frac{1}{2} m^2 A^a_\mu A^a_\mu$$

Particular case of the Curci-Ferrari (CF) model. Most appealing features:

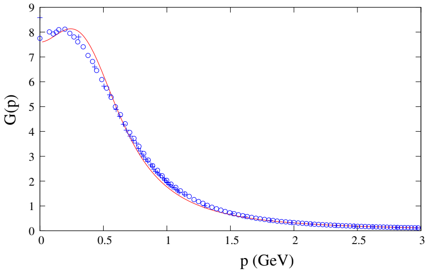
- ⋆ Just one additional parameter (simple modification of the Feynman rules).
- ⋆ Perturbatively renormalizable and ∃ RG trajectories without Landau pole:

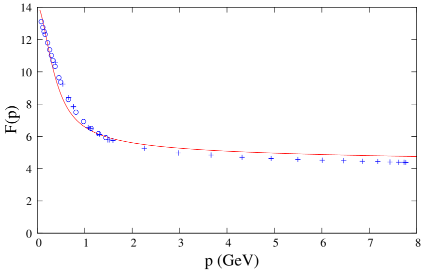


⇒ perturbative calculations may be pushed down to the IR!

Vacuum correlators

Amazing agreement between LO perturbation theory in the CF model and Landau gauge lattice vacuum correlators [with $m \simeq 500\,\mathrm{MeV}$ for SU(3)]:



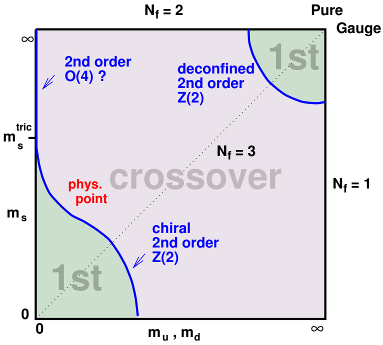


[Tissier, Wschebor, Phys.Rev. D84 (2011)]

What are the predictions of the model at finite temperature?

QCD phase structure

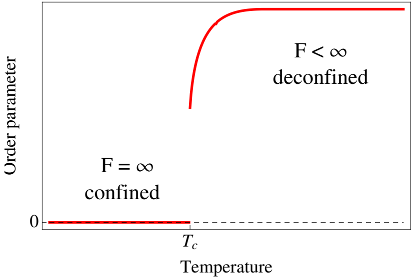
Back to the QCD phase diagram



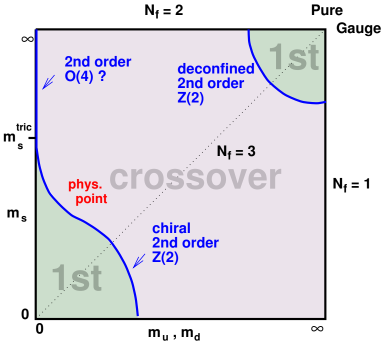
Order parameter(s): Polyakov loop(s)

$$\ell \equiv \frac{1}{3}\left\langle \mathrm{tr}\, \mathcal{P}\, e^{ig\int_0^{1/T} d\tau\, A_0} \right\rangle \propto e^{-\beta F_{\mathrm{quark}}}$$

$$\bar{\ell} \equiv \frac{1}{3}\left\langle \mathrm{tr} \left(\mathcal{P}\, e^{ig\int_0^{1/T} d\tau\, A_0} \right)^{\dagger} \right\rangle \propto e^{-\beta F_{\mathrm{antiquark}}}$$



Back to the QCD phase diagram



Underlying symmetry: center symmetry

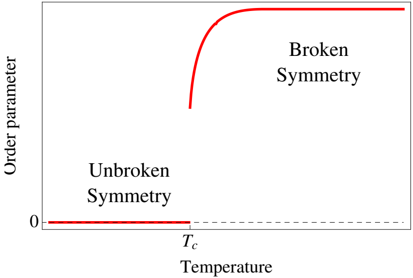
$$A_\mu^U = U A_\mu U^\dagger - i U \partial_\mu U^\dagger$$

$$U(\tau + 1/T, \vec{x}) = U(\tau, \vec{x})\, e^{i\frac{2\pi}{3}k} \quad (k = 0, 1, 2)$$

Under such a transformation: $\ell \to e^{i2\frac{\pi}{3}k}\ell$

Deconfinement transition
≡ SSB of center symmetry.

Problem: center symmetry is not manifest in the Landau gauge.



Changing to the Landau-DeWitt gauge

We move from the Landau gauge to the Landau-DeWitt gauge:

$$0 = \partial_\mu A_\mu^a \;\rightarrow\; 0 = (\bar{D}_\mu a_\mu)^a \qquad \left\{ \begin{array}{rcl} \bar{D}_\mu^{ab} & \equiv & \partial_\mu \delta_{ab} + f^{acb} \bar{A}_\mu^c \\ a_\mu^a & \equiv & A_\mu^a - \bar{A}_\mu^a \end{array} \right.$$

with $\bar{A}$ a given background field configuration.

Faddeev-Popov gauge-fixed action + phenomenological $\mathcal{L}_{\mathrm{Gribov}}$:

$$\mathcal{L}_{\bar{A}} = \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + \underbrace{(\bar{D}_\mu \bar{c})^a (D_\mu c)^a + ih^a (\bar{D}_\mu a_\mu)^a + \frac{1}{2} m^2 a_\mu^a a_\mu^a}_{\text{Landau gauge} + \partial_\mu \to \bar{D}_\mu \text{ and } A_\mu \to a_\mu}$$

Background and symmetries

In practice, one chooses $\bar{A}$ complying with the symmetries at finite T:

$$\bar{A}_\mu(\tau,\vec{x}) = T\delta_{\mu 0}\left(r_3\frac{\lambda^3}{2} + r_8\frac{\lambda^8}{2}\right) \in \text{Cartan subalgebra}$$

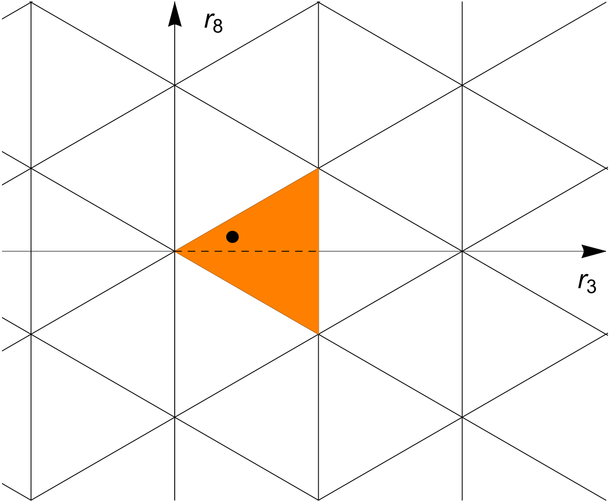
It is also convenient to work with self-consistent backgrounds: $0 = \langle a_\mu\rangle_{\bar{A}}$.

- ⋆ shown to be the minima of a certain Gibbs potential $\tilde{\Gamma}[\bar{A}] \equiv \Gamma_{\bar{A}}[a=0]$.
- ⋆ this functional is center-symmetric:

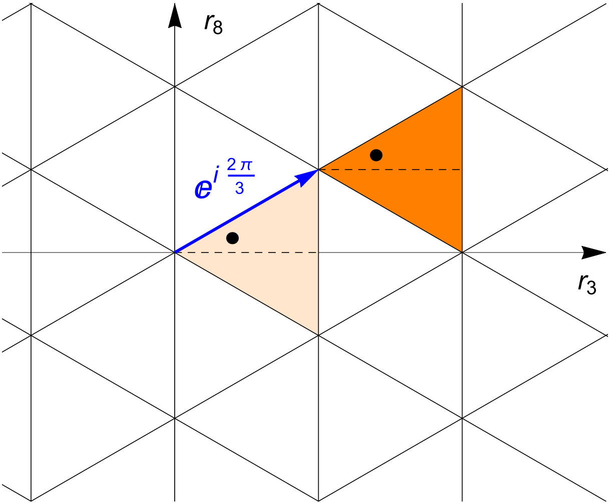
$$\boxed{\tilde{\Gamma}[\bar{A}^U] = \tilde{\Gamma}[\bar{A}],\ \forall\ U(\tau+1/T) = e^{i\frac{2\pi}{3}k}\,U(\tau)}$$

As in the textbook situation (ex: Ising-model), SSB occurs when the ground state of the Gibbs potential moves from a symmetric state to a symmetry breaking one. But what is the (center-)symmetric state in the present case?

Weyl chambers and center symmetry



Weyl chambers and center symmetry



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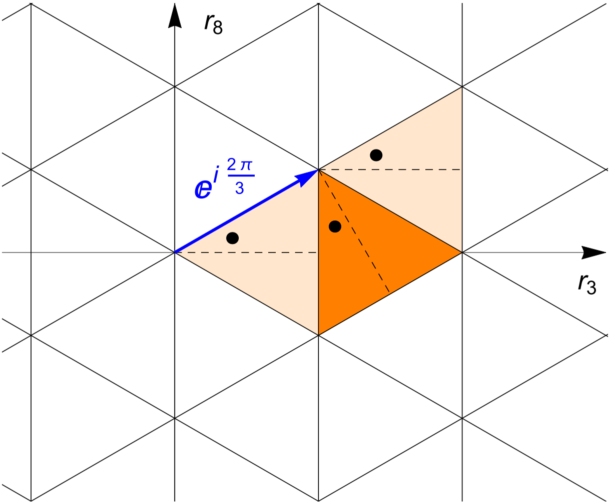

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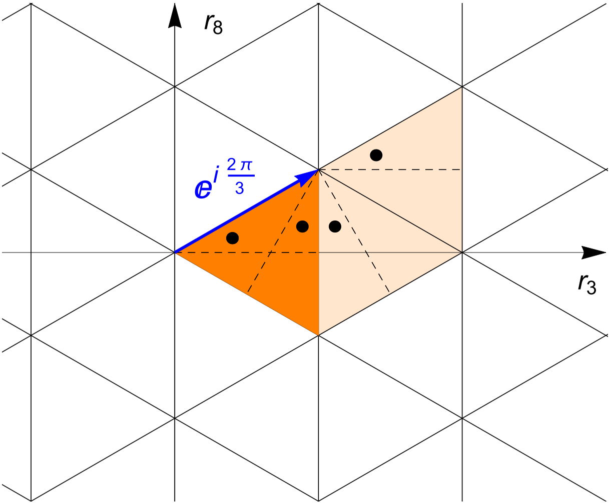

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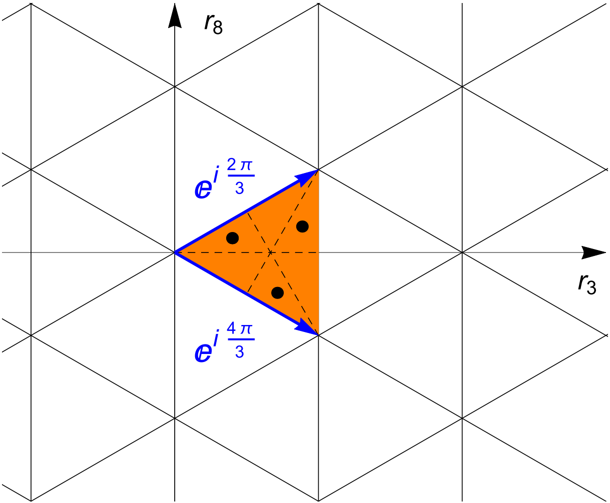
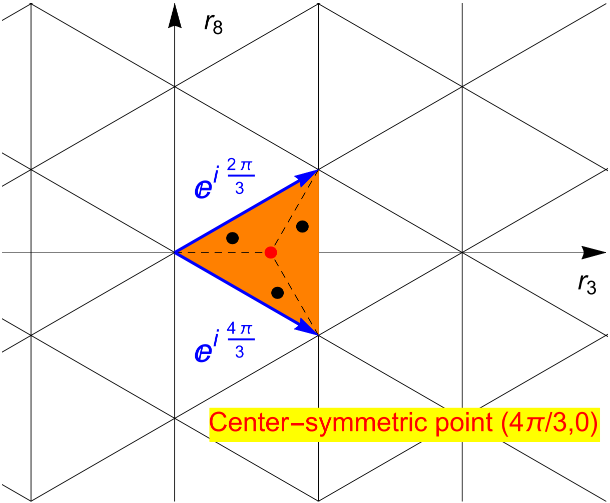
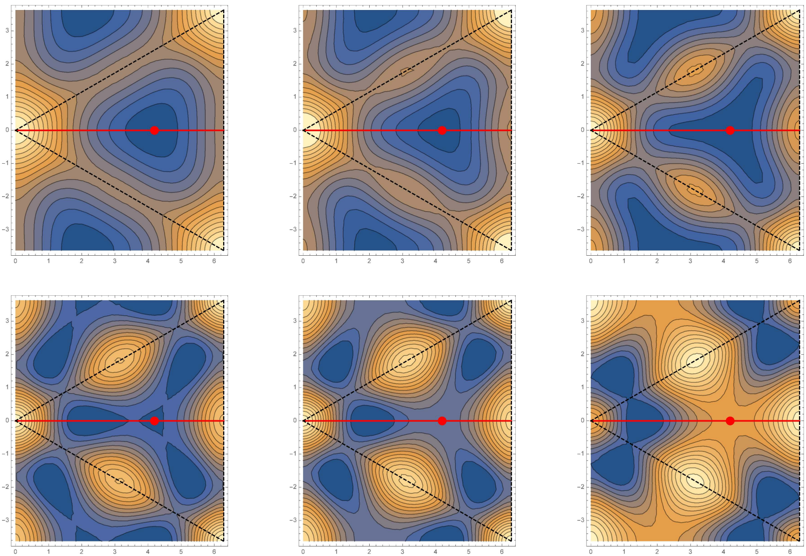


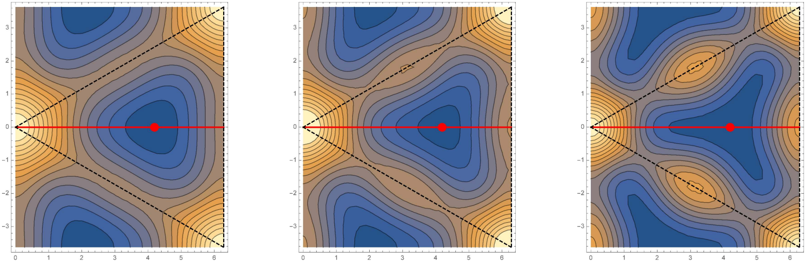
Weyl chambers and center symmetry



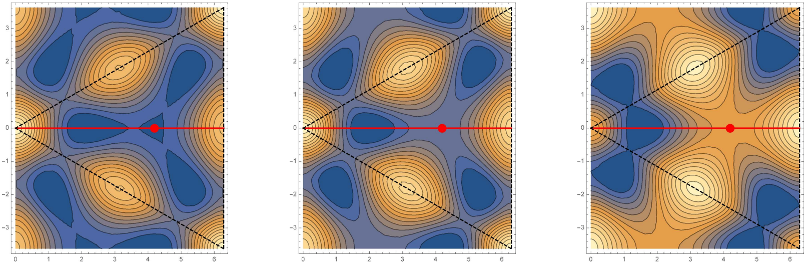
One-loop result - SU(3)



One-loop result - SU(3)

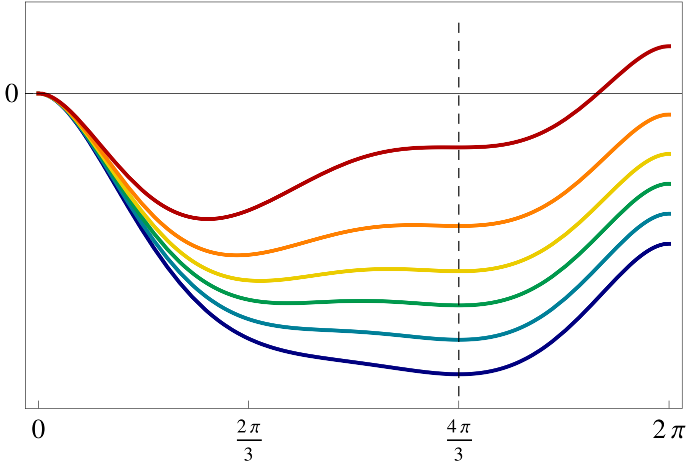


$$T^{-4}\tilde{\Gamma}[\bar{A}] = 3F(r_3, r_8, m/T) - F(r_3, r_8, 0) \sim \begin{cases} -F(r_3, r_8) \text{ if } T \ll m \\ 2F(r_3, r_8) \text{ if } T \gg m \end{cases}$$



One-loop result - SU(3)

We obtain a first order transition in agreement with lattice predictions:



[UR, J. Serreau, M. Tissier and N. Wschebor, Phys.Lett. B742 (2015) 61-68]

Summary of one-loop results

order	lattice	fRG	our model at 1-loop
SU(2)	2nd	2nd	2nd
SU(3)	1st	1st	1st
SU(4)	1st	1st	1st
Sp(2)	1st	1st	1st

T_c (MeV)	lattice	fRG(*)	our model at 1-loop(**)
SU(2)	295	230	238
SU(3)	270	275	185

(*) L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010.

(**) UR, J. Serreau, M. Tissier and N. Wschebor, Phys.Lett. B742 (2015) 61-68.

Summary of two-loop results

order	lattice	fRG	our model at 1-loop	our model at 2-loop
SU(2)	2nd	2nd	2nd	2nd
SU(3)	1st	1st	1st	1st
SU(4)	1st	1st	1st	1st
Sp(2)	1st	1st	1st	1st

T_c (MeV)	lattice	fRG[(*)]	our model at 1-loop	our model at 2-loop[(**)]
SU(2)	295	230	238	284
SU(3)	270	275	185	254

[(*)] L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010.

[(**)] UR, J. Serreau, M. Tissier and N. Wschebor, Phys.Rev. D93 (2016) 105002.

Adding (fundamental) quarks

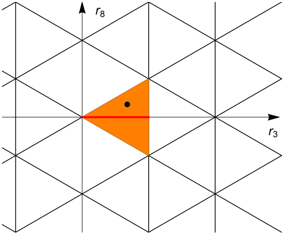
We add N_f (heavy) quarks in the fundamental representation:

$$\mathcal{L} = \mathcal{L}_{\rm LdW} + \mathcal{L}_{\rm Gribov} + \sum_{f=1}^{N_f} \left\{ \bar{\psi}_f (\partial\!\!\!/ - ig A\!\!\!/^a t^a + M_f + \mu\gamma_0)\psi_f \right\}$$

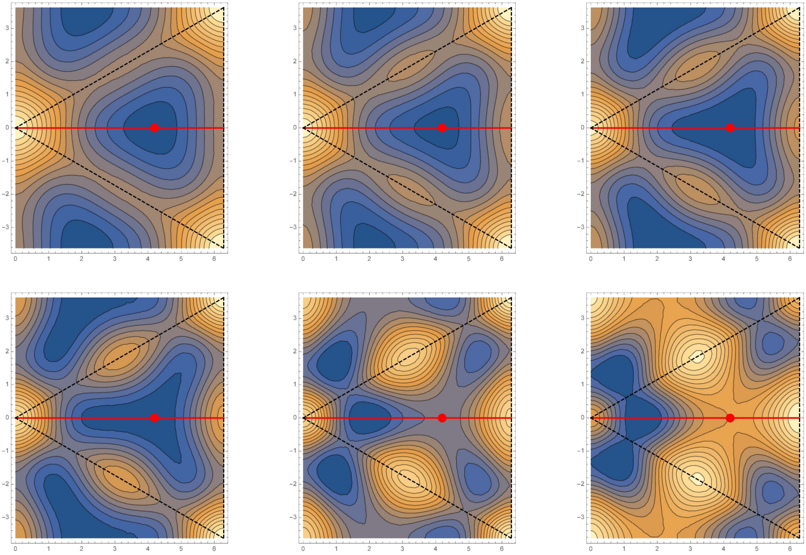
Center symmetry is explicitly broken by the boundary conditions:

$$\left.\begin{aligned} \psi(1/T,\vec{x}) &= -\psi(0,\vec{x}) \\ U(1/T,\vec{x}) &= e^{-i2\pi/3}U(0,\vec{x}) \end{aligned}\right\} \Rightarrow \psi'(1/T,\vec{x}) = -e^{-i2\pi/3}\psi'(0,\vec{x})$$

C-symmetry is also broken as soon as $\mu \neq 0$:

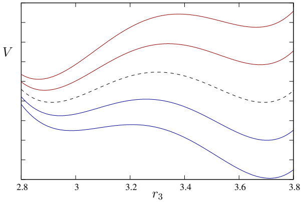


$\mu = 0$: Phase transition



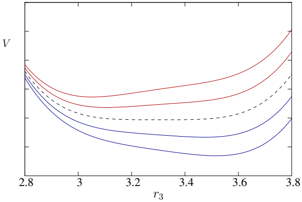
$\mu = 0$: Dependence on the quark masses

$M > M_c$:



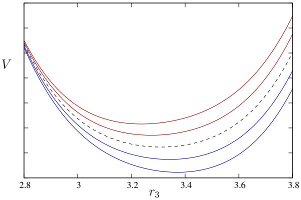
first order

$M = M_c$:



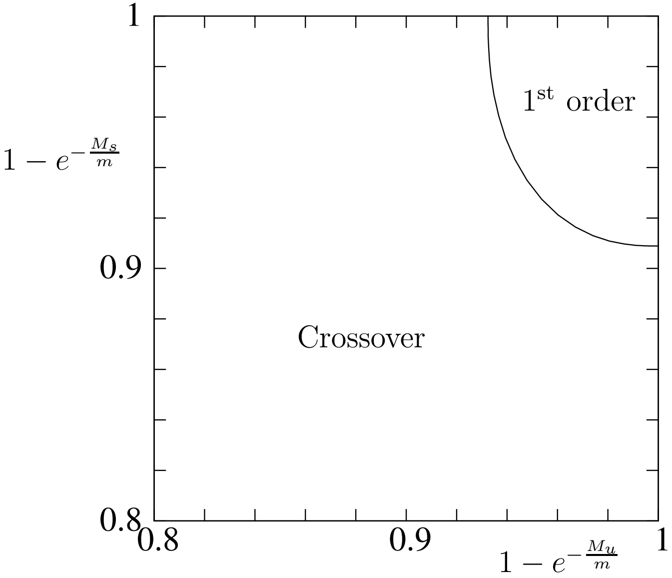
second order

$M < M_c$:



crossover

$\mu = 0$: Columbia plot



$\mu = 0$: Columbia plot

N_f	$(M_c/T_c)^{\text{our model (*)}}$	$(M_c/T_c)^{\text{lattice (**)}}$	$(M_c/T_c)^{\text{matrix (***)}}$	$(M_c/T_c)^{\text{SD (****)}}$
1	6.74	7.22	8.04	1.42
2	7.59	7.91	8.85	1.83
3	8.07	8.32	9.33	2.04

(*) UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015).

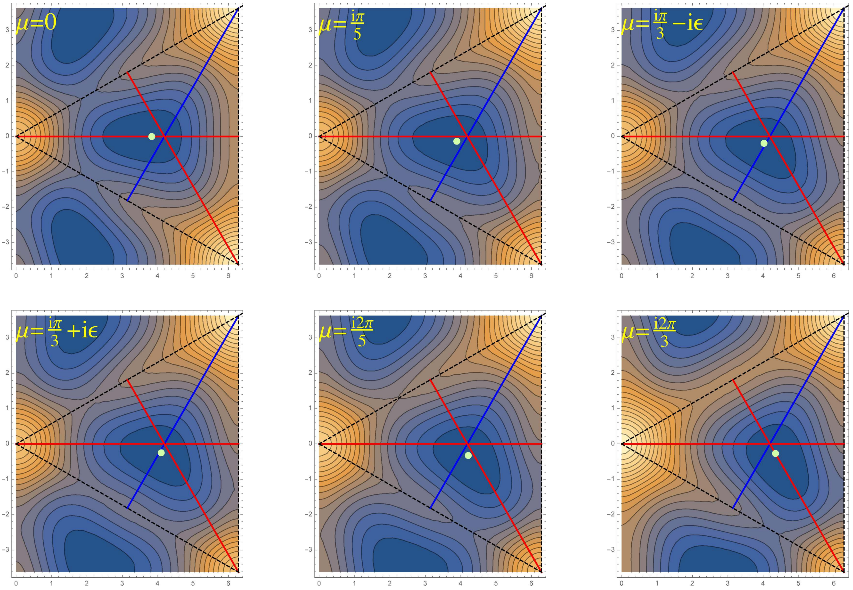
(**) M. Fromm, J. Langelage, S. Lottini and O. Philipsen, JHEP 1201 (2012) 042.

(***) K. Kashiwa, R. D. Pisarski and V. V. Skokov, Phys.Rev. D85 (2012) 114029.

(****) C. S. Fischer, J. Luecker and J. M. Pawlowski, Phys.Rev. D91 (2015) 1, 014024.

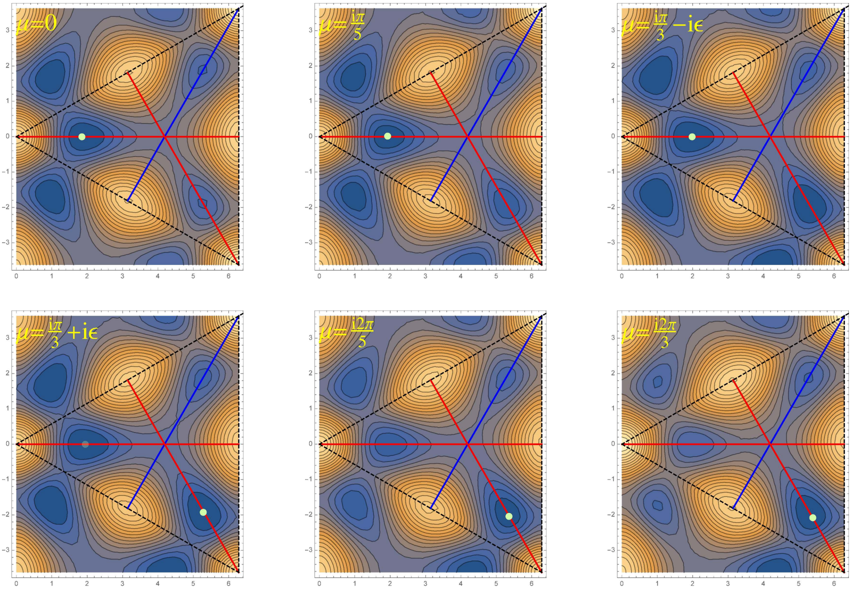
$\mu \in i\mathbb{R}$: Phase transition

Low T

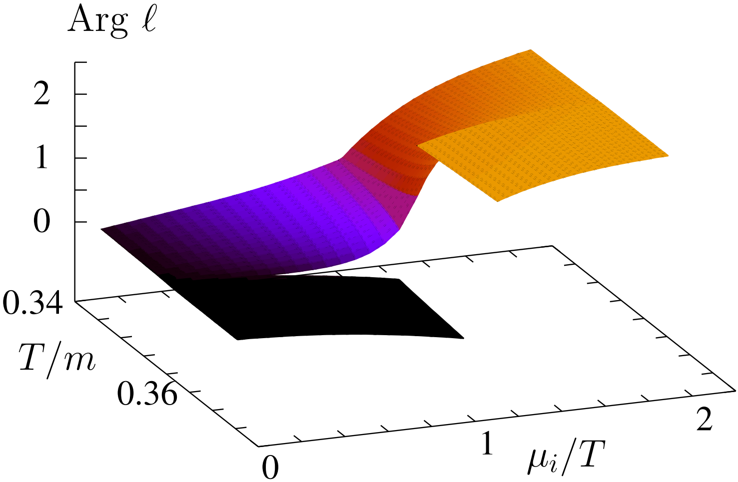


$\mu \in i\mathbb{R}$: Phase transition

High T

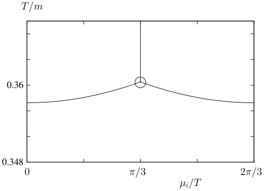


$\mu \in i\mathbb{R}$: Roberge-Weiss transition

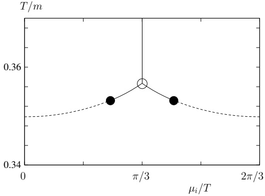


$\mu \in i\mathbb{R}$: Dependence on the quark masses

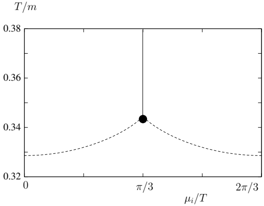
$M > M_c(0)$:



$M \in [M_c(i\pi/3), M_c(0)]$:

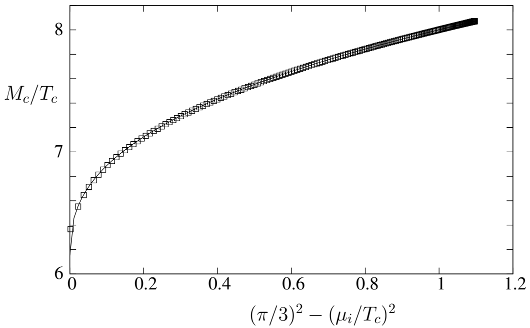


$M = M_c(i\pi/3)$:



Same structure as in the lattice study of [P. de Forcrand, O. Philipsen, Phys.Rev.Lett. 105 (2010)]

$\mu \in i\mathbb{R}$: Tricritical scaling



$$\frac{M_c}{T_c} = \frac{M_{\text{tric.}}}{T_{\text{tric.}}} + K\left[\left(\frac{\pi}{3}\right)^2 + \left(\frac{\mu}{T}\right)^2\right]^{2/5}$$

	our model(*)	lattice(**)	SD(***)
K	1.85	1.55	0.98
$\frac{M_{\text{tric.}}}{T_{\text{tric.}}}$	6.15	6.66	0.41

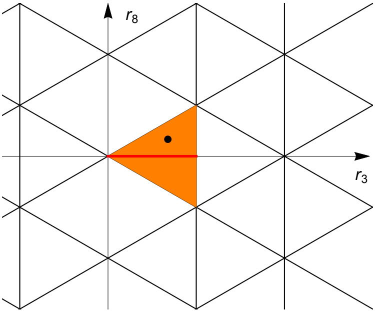
(*) UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015).

(**) Fromm et.al., JHEP 1201 (2012) 042.

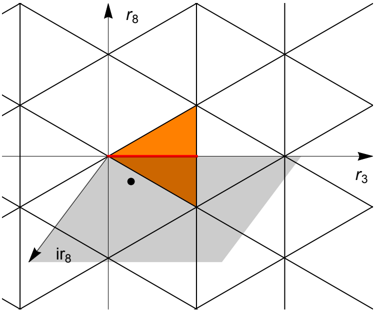
(***) Fischer et.al., Phys.Rev. D91 (2015) 1, 014024.

$\mu \in \mathbb{R}$: Complex backgrounds

We find that, for $\mu \in \mathbb{R}$, the background r_8 should be chosen imaginary:



$\mu \in i\,\mathbb{R}$, $r_8 \in \mathbb{R}$



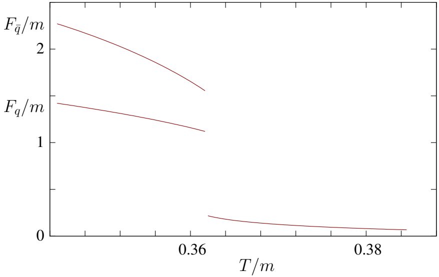
$\mu \in \mathbb{R}$, $r_8 \in i\,\mathbb{R}$

$\mu \in \mathbb{R}$: Complex backgrounds

The choice $r_8 = i\tilde{r}_8$, $\tilde{r}_8 \in \mathbb{R}$ is crucial for the interpretation of ℓ and $\bar{\ell}$ in terms of F_q and $F_{\bar{q}}$ to hold true [UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015)]:

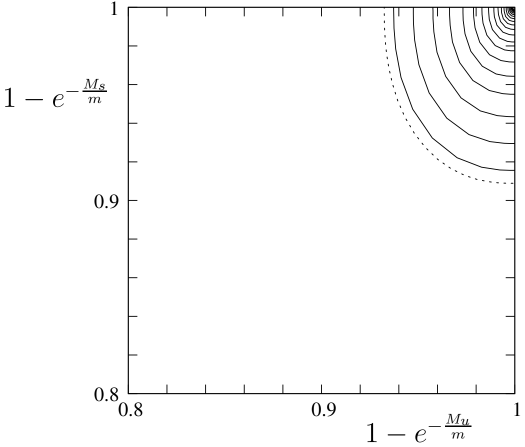
$$\ell(\mu) = e^{-\beta F_q} = \frac{e^{\frac{\tilde{r}_8}{\sqrt{3}}} + 2e^{-\frac{\tilde{r}_8}{2\sqrt{3}}}\cos(r_3/2)}{3}$$

$$\bar{\ell}(\mu) = e^{-\beta F_{\bar{q}}} = \frac{e^{-\frac{\tilde{r}_8}{\sqrt{3}}} + 2e^{\frac{\tilde{r}_8}{2\sqrt{3}}}\cos(r_3/2)}{3}$$



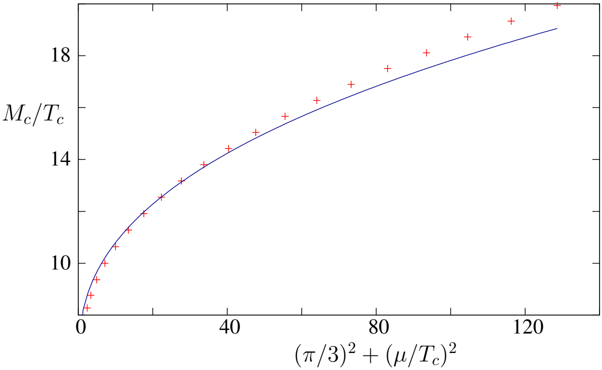
$\mu \in \mathbb{R}$: Columbia plot

The critical surface moves towards larger quark masses:



$\mu \in \mathbb{R}$: tricritical scaling

The tricritical scaling survives deep in the $\mu^2 > 0$ region:



Conclusions

Simple one-loop calculations in a model for a Gribov completion of the Landau-DeWitt gauge account for qualitative and quantitative features of the QCD phase diagram in the heavy quark limit:

- ⋆ Correct account of the order parameter in the quenched limit;
- ⋆ Critical line of the Columbia plot at $\mu = 0$;
- ⋆ Roberge-Weiss phase diagram and its mass dependence for $\mu \in i\mathbb{R}$.

The case $\mu \in \mathbb{R}$ requires the use of complex backgrounds.

Certain aspects require the inclusion of two-loop corrections.

TODO:

- ⋆ Chiral phase transition? Critical end-point in the (T, μ) diagram?
- ⋆ Hadronic bound states? Glueballs?
- ⋆ How to define the physical space?