Numerical study of ghost-ghost-gluon vertex in Landau gauge

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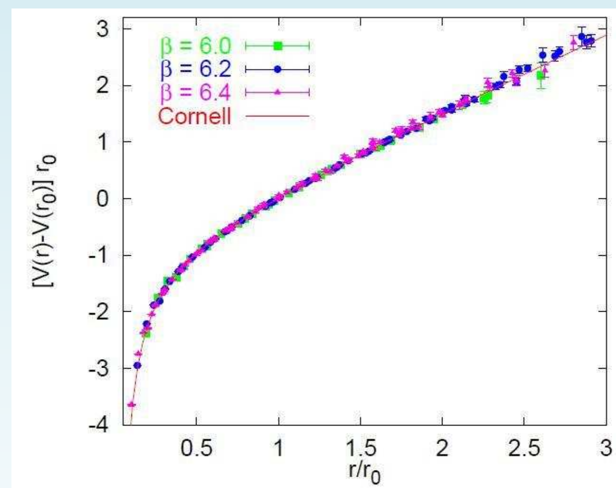
Color Confinement

Millennium Prize Problems by the Clay Mathematics Institute (US$1,000,000): Yang-Mills Existence and Mass Gap: Prove that, for any compact simple gauge group G, a non-trivial quantum Yang-Mills Theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$.

Heavy-Quark Potential

- How does linearly rising potential come about?

$$V(R) = -\frac{\alpha}{R} + \frac{R}{a^2}$$

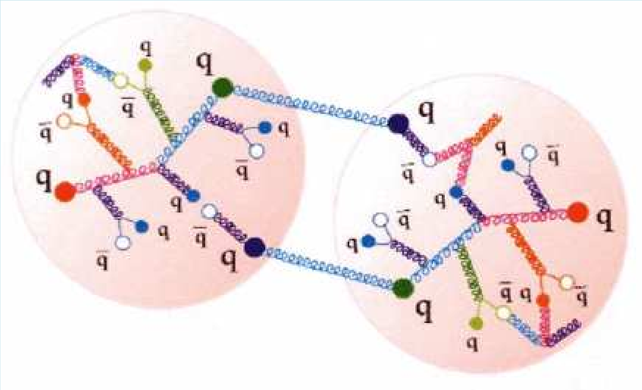


- One could have

$$V(R) \sim \int \frac{d^3k}{k^4} e^{ik\cdot R} \sim R$$

Pathways to Confinement

- Green's functions carry all information of a QFT's physical and mathematical structure.



- Gluon propagator (two-point function) as the most basic quantity of QCD.

- Confinement given by behavior at large distances (small momenta) $\Rightarrow$ nonperturbative study of IR gluon propagator. Proposal by Mandelstam (1979) linking linear potential to infrared behavior of gluon propagator as $1/k^4$.

Ongoing Project

The building blocks of QCD (in a given gauge) are:

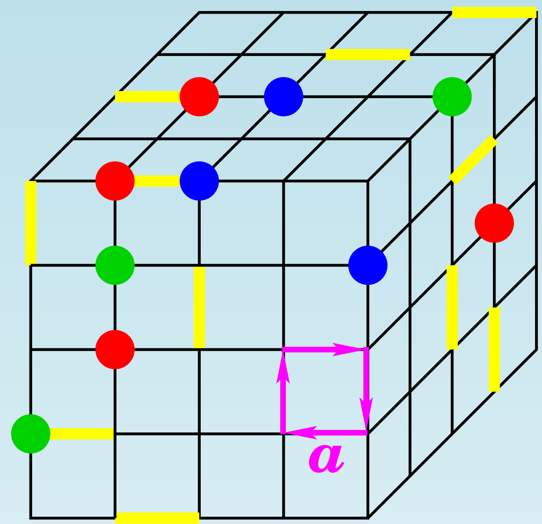
- Propagators: gluon, quark, ghost
- Vertices: three-gluon, four-gluon, ghost-gluon, quark-gluon

Project: evaluate all propagators and vertices in Landau gauge $\Longrightarrow$
By-Product: running coupling constant α_s

Challenges:

- infinite-volume limit
- breaking of rotational symmetry and discretization effects
- renormalization and continuum limit
- physical limit (quark masses)

Lattice QCD



In the lattice formulation the theory is rewritten in discrete space-time (subtle→ must preserve the gauge symmetry) and the fields may be thought of as elements of a thermodynamical system at a fixed temperature.

- The only first-principles, nonperturbative way to study QCD with no uncontrolled approximations.
- Lattice is an UV cutoff which goes to zero in the continuum (= physical) limit: rigorous formulation of quantum field theory.
- Application of statistical-mechanics techniques — such as Monte Carlo simulation, study of critical phenomena — to quantum field theories.
- Grand Challenge Problem (report to the US President, 2005, “Computational Science: ensuring America’s competitiveness”).

Ghost Fields on the Lattice

When we are interested in gauge-dependent quantities we consider the following steps:

1. Choose an initial configuration $\mathcal{C}_0 = U_\mu(x) \in SU(N)$
2. Thermalize the initial configuration (heat-bath, etc.) $\mathcal{C}_0 \to \mathcal{C}_1$
3. Fix the gauge for the configuration $\mathcal{C}_i$ with $i = 1, 2, \ldots$
4. Evaluate (gauge-dependent) quantities using the configuration $\mathcal{C}_i$
5. Produce a new (independent) configuration $\mathcal{C}_i \to \mathcal{C}_{i+1}$
6. Go back to step 3

We do not need to simulate anti-commuting variables or to evaluate the determinant of the Faddeev-Popov matrix!

Gluon Propagator on the Lattice

Define the lattice gluon field $A^b_\mu(x)$ from $U_\mu(x) = \exp\left[\, i\, a\, g_0\, \mathcal{A}_\mu(x)\,\right]$ so that

$$A^b_\mu(x) \,=\, a\, g_0\, \mathcal{A}^b_\mu(x)\,[\,1 \,+\, \mathcal{O}(a^m)\,]$$

Evaluate the lattice gluon propagator (in some gauge) in momentum space

$$D(k) \propto \sum_{\mu,b} \langle\, \widetilde{A}^b_\mu(k)\, \widetilde{A}^b_\mu(-k)\,\rangle$$

$$\widetilde{A}^b_\mu(k) = \sum_x A^b_\mu(x)\, \exp\left[2\pi i\,(k\cdot x + k_\mu/2)\,/\,N\right]$$

as a function of

$$k^2 \,=\, 4\sum_{\mu=1}^{d} \sin^2\left(\frac{\pi\, k_\mu}{N}\right)$$

with $k_\mu = 0, 1, \ldots, N-1$.

Momenta on the Lattice

The components of the lattice momentum are given by

$$k_\mu \;=\; 2\,\sin\left(\frac{\pi\, k_\mu}{N}\right) \qquad\qquad k_\mu = 0, 1, \ldots, N/2$$

For a given lattice side N and lattice spacing a we have (in $4d$)

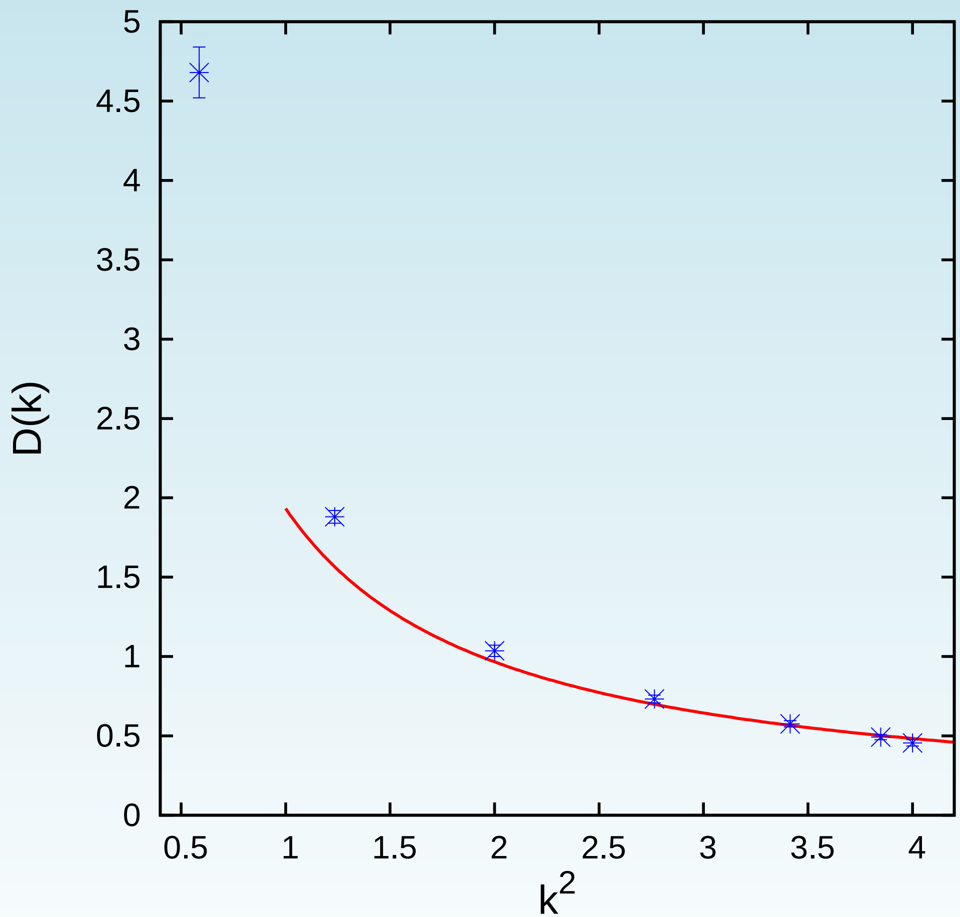
$$k_{min} \;=\; \frac{2}{a}\,\sin\left(\frac{\pi}{N}\right) \qquad\qquad k_{max} \;=\; \frac{4}{a}\,\sin\left(\frac{\pi}{2}\right)$$

- Relatively small lattice volume $\rightarrow\; N \sim 20$
- Usual values for the lattice coupling $\beta\;\rightarrow\; a \sim 0.2 - 1.0$ GeV^{-1}

$$\Longrightarrow \left\{ \begin{array}{l} k_{min} \;\sim\; 0.3 - 1.5\ \text{GeV} \\ k_{max} \;\sim\; 4.0 - 20.0\ \text{GeV} \end{array} \right.$$

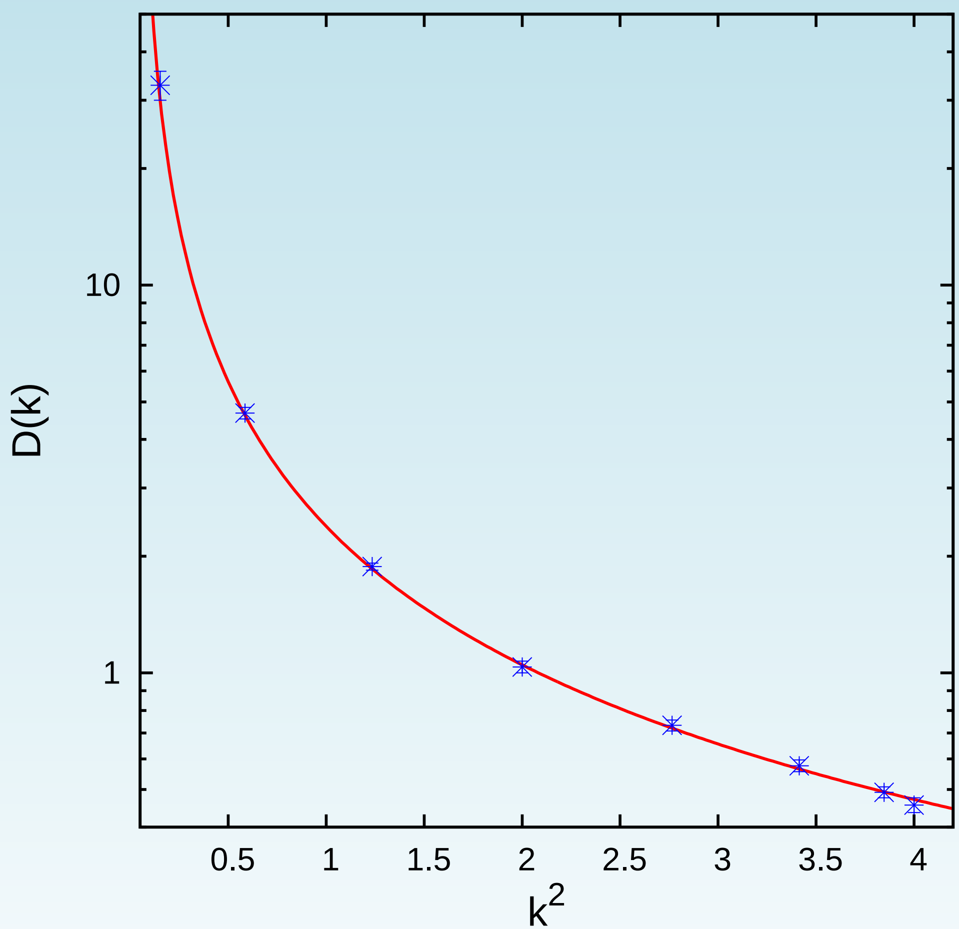
Do We Really Know what We Are Doing?

Comparison with Perturbation Theory



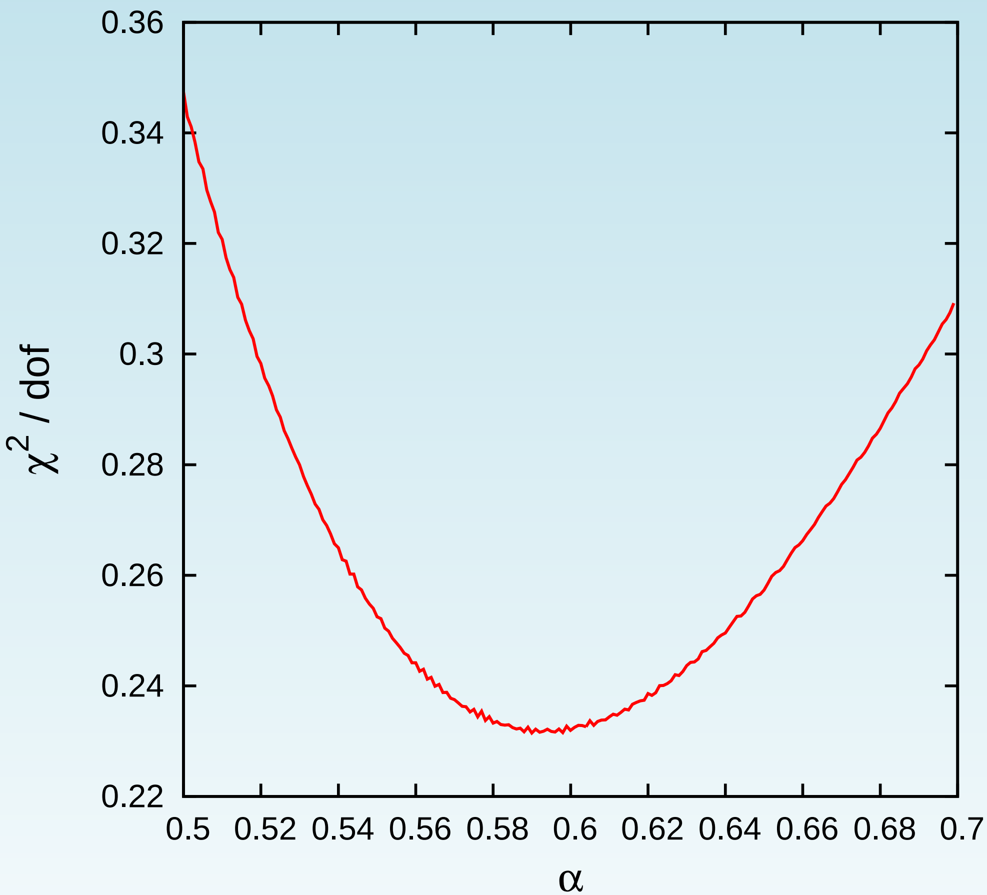
Fit of the Landau gluon propagator $D(k)$ using $c/p^2(k)$ in the SU(2) case at $\beta = 2.7$ and $V = 16^4$.

Comparison with RG-Improved PT



Fit of the Landau gluon propagator $D(k)$ using $c\left[\log\left(p^2(k)/\Lambda^2\right)\right]^{-\alpha}/p^2(k)$ in the SU(2) case at $\beta = 2.7$ and $V = 16^4$. Here $\alpha = 2\,c_3/b_0 = 13/22$, where $b_0 = 11n/(3 \times 16\pi^2)$ is (minus) the first coefficient of the beta function, and $c_3 = 13n/(12 \times 16\pi^2)$ is (minus) the first coefficient of the anomalous dimension of the gluon field.

Anomalous Dimension of the Gluon Field



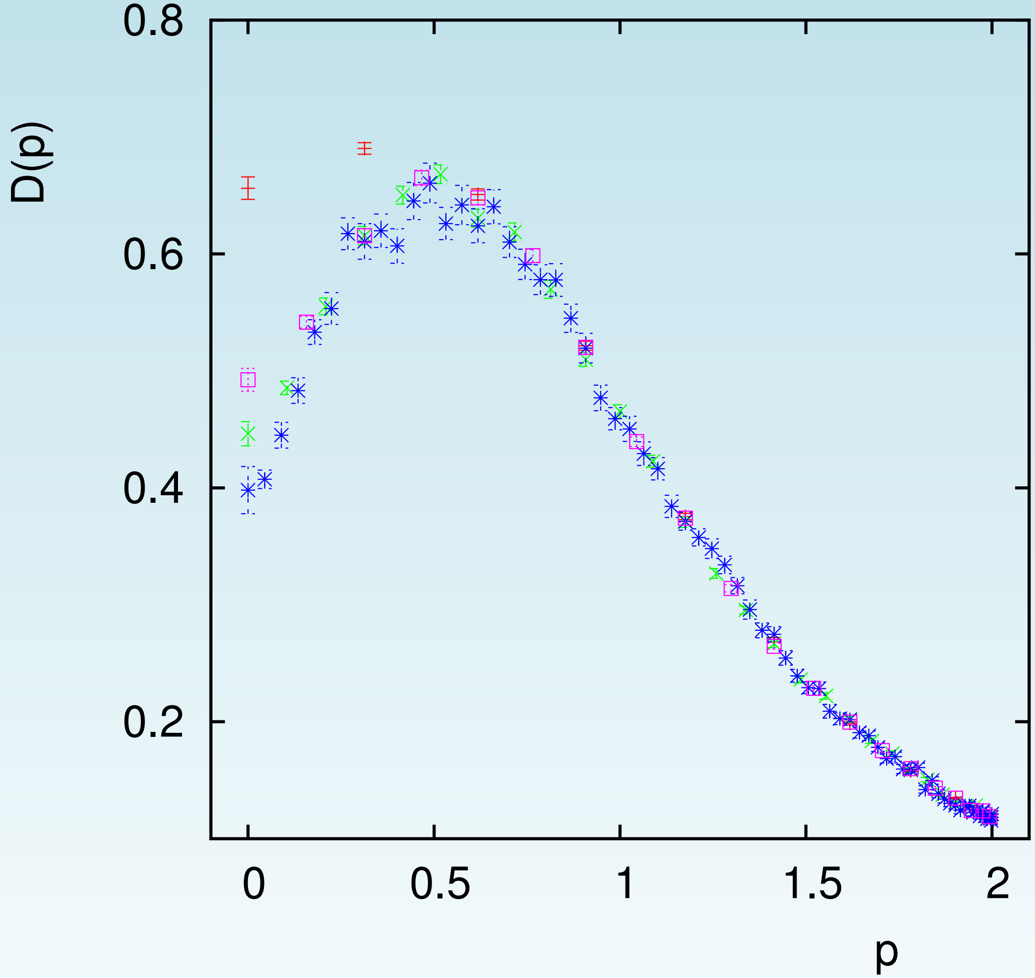
Numerical evaluation of α considering the best χ^2/dof

Theoretical value: 0.5909.

Numerical value:

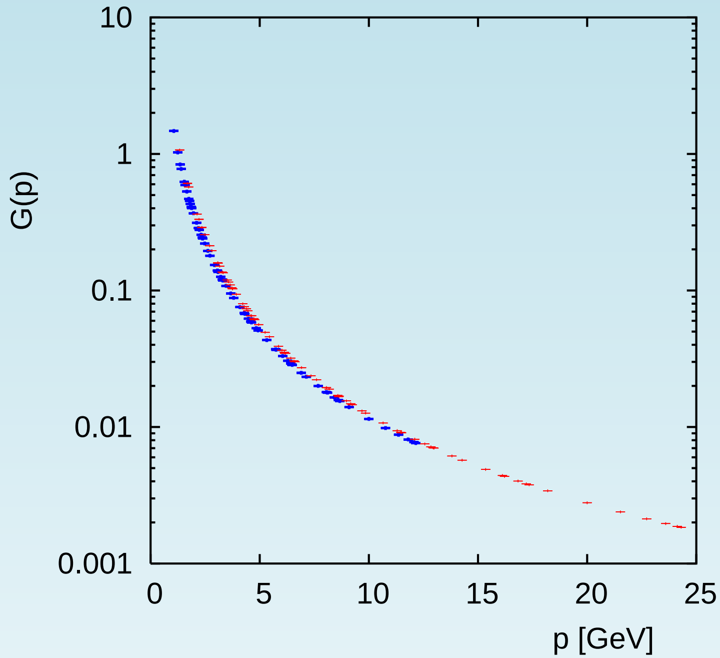
0.590 ± 0.001!

Gluon Propagator: Infinite-Volume Limit

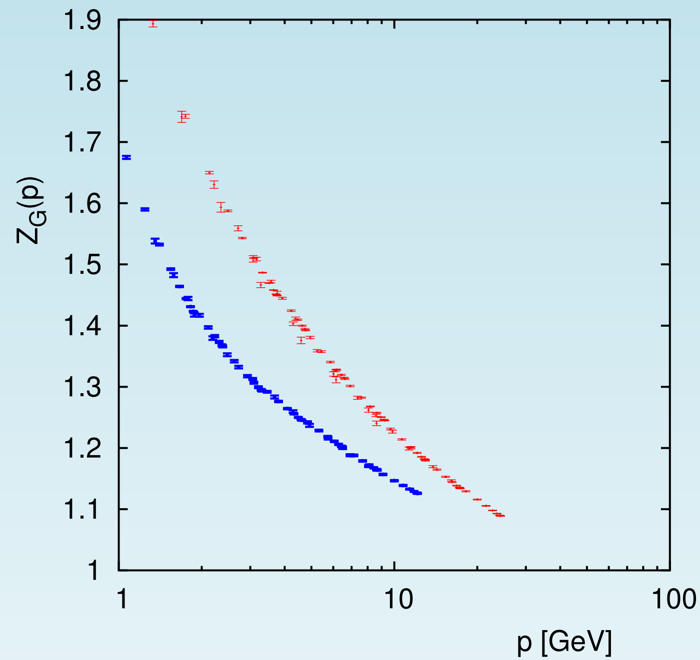


Gluon propagator as a function of the lattice momentum p for lattice volumes $V = 20^3$, 40^3, 60^3 and 140^3 at $\beta = 3.0$ About 100 days using a 13 Gflops PC cluster (2003).

Breaking of rotational invariance



Ghost propagator (in Landau gauge) for two different sets of momenta [$V = 26^4$, SU(2) case].



Ghost propagator (in Landau gauge) multiplied by p^2 for two different sets of momenta [$V = 26^4$, SU(2) case].

Ghost-Gluon Vertex (I)

One can consider the 3-point function

$$V_\mu^{abc}(x,y,z) \;=\; \langle\, A_\mu^a(x)\,\eta^b(y)\,\bar{\eta}^c(z)\,\rangle$$

In momentum space we can write

$$V_\mu^{abc}(q,s;k) \;=\; (2\pi)^4\;\delta^4(k+q-s)\;G_\mu^{abc}(k,q)$$

Then, the ghost-gluon vertex function is given by

$$\Gamma_\mu^{abc}(q,s;k) \;=\; \frac{G_\mu^{abc}(q,s;k)}{D(k^2)\,G(q^2)\,G(s^2)}\;,$$

where $s = k + q$ and $D(k^2)$ and $G(q^2)$ are, respectively, the gluon and ghost propagators. At tree level (in the continuum) one obtains

$$\Gamma_\mu^{abc}(q,s;k) \;=\; i\,g_0\,f^{abc}\,q_\mu\;.$$

Ghost-gluon vertex (II)

On the lattice, the ghost-ghost-gluon vertex at tree level is given by

$$\Gamma_\mu^{abc}(q,s;k) \;=\; i\,g_0\,f^{abc} q_\mu \cos\left(\frac{\hat{s}_\mu a}{2}\right) \;,$$

where a is the lattice spacing and $\hat{s}_\mu$ is a momentum component in physical units. Then, the vertex renormalization function in momentum-subtraction scheme is given by

$$\widetilde{Z}_1^{-1}(p^2) \;=\; \frac{-i}{g_0\,N_c\,(N_c^2\,-\,1)}\,\sum_\mu\,\frac{p_\mu}{p^2\,\cos(p_\mu/2)}\,\sum_{a,b,c} f^{abc}\,\Gamma_\mu^{abc}(0,p)\;,$$

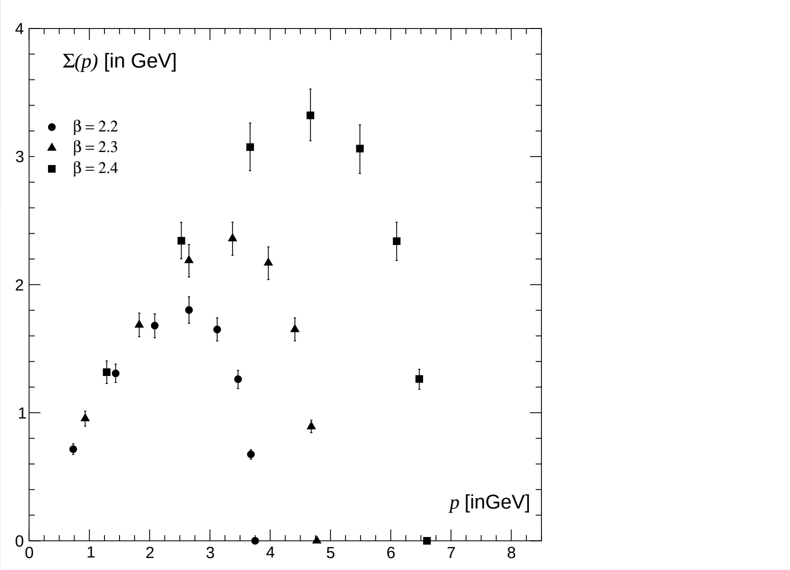
where N_c is the number of colors and we consider the asymmetric point with zero momentum for the gluon ($k=0$), implying $s=q$.

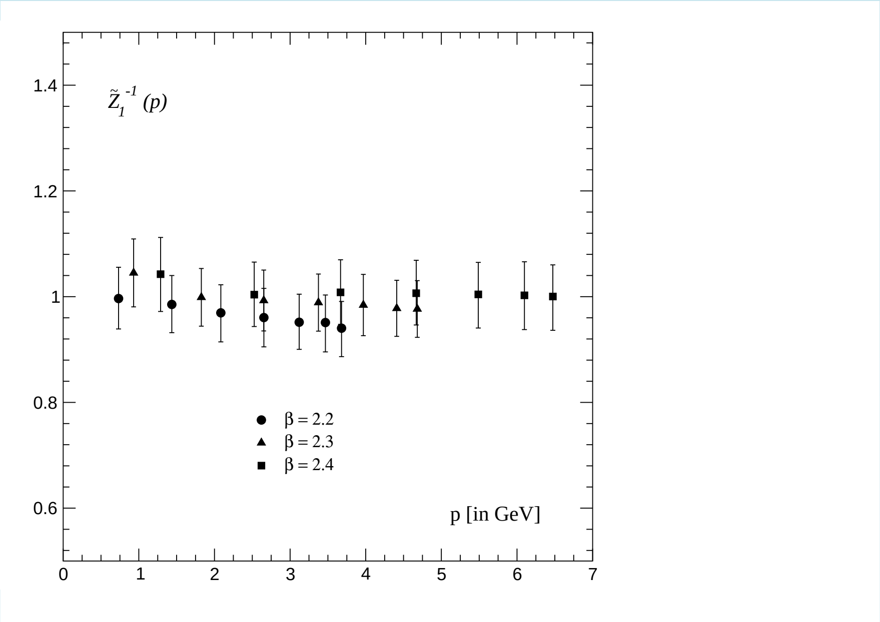
If we consider momenta with $p_\mu = 2\pi n/N$ and $n = 1, \ldots, N/2$, then we can write

$$\widetilde{Z}_1^{-1}(p^2) = \frac{\Sigma(p^2)}{2\sin(\pi n/N)\cos(\pi n/N)} = \frac{\Sigma(p^2)}{\sin(2\pi n/N)}$$

Ghost-gluon vertex (III)

Numerical results (A.C., T.Mendes & A.Mihara, JHEP 2004):



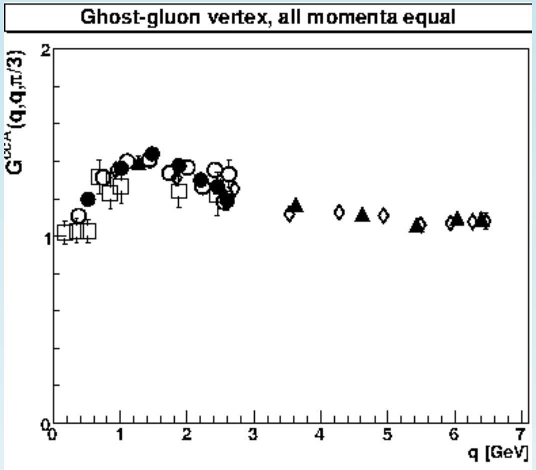


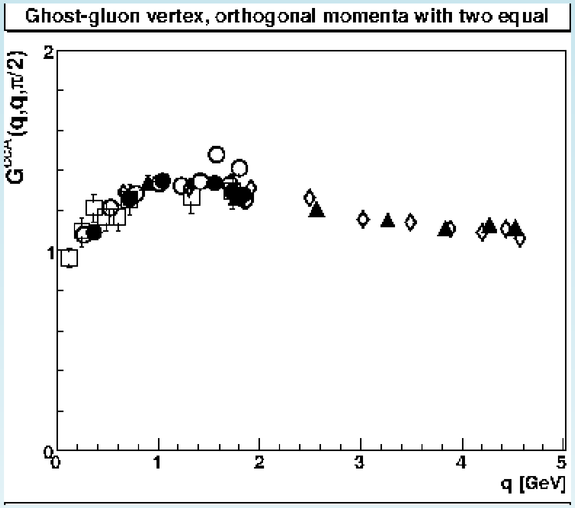
$\Sigma(p^2)$ (in Landau gauge) for the lattice volume $V = 16^4$ [SU(2) case] and three different lattice spacings.

$\widetilde{Z}_1^{-1}(p^2)$ (in Landau gauge) for the lattice volume $V = 16^4$ [SU(2) case] and three different lattice spacings.

Ghost-gluon vertex (IV)

Numerical results (A.C., A.Maas & T.Mendes, PRD 2008):





The ghost-gluon vertex (in Landau gauge) for the several lattice volumes V [SU(2) case] and different lattice spacings, with three equal momenta.

The ghost-gluon vertex (in Landau gauge) for the several lattice volumes V [SU(2) case] and different lattice spacings, with two orthogonal momenta.

Conclusions

Lattice numerical simulations allow a precise non-perturbative determination of QCD propagators and vertices, once the systematic effects are under control.

Challenge: what can one say about the usual continuum parametrization (J.S.Ball & T.-W.Chiu, 1980)

$$\Gamma_\mu^{abc}(q,s;k) = i g_0 f^{abc} q_\nu \Gamma_{\nu\mu}(q,s;k)$$

with

$$\Gamma_{\nu\mu}(q,s;k) = \delta_{\nu\mu} a(q,s;k) - k_\nu\, s_\mu\, b(q,s;k) + q_\nu\, k_\mu\, c(q,s;k)$$
$$+ k_\nu\, q_\mu\, d(q,s;k) + q_\nu\, q_\mu\, e(q,s;k)\ ?$$

MERCI!