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Strong three-meson couplings in charmonia

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Purpose

Calculation of strong three-meson coupling constants, involving charmed or charmonium particles. Cannot be measured in hadronic decays, because of lack of phase space, but enter in the description of properties of these hadrons and in the calculation of form factors. They appear in the residues of poles in the timelike region.

$$\langle P'(p_2)|j_\mu|P(p_1)\rangle = F_+^{P\to P'}(q^2)\,(p_1+p_2)_\mu + \cdots\ .$$

The form factor F_+ has a vector meson pole at mass M_{V_R} (widths neglected):

$$F_+^{P\to P'}(q^2)\Big|_{\rm pole} = \frac{g_{PP'V_R}\,f_{V_R}}{2M_{V_R}(1-q^2/M_{V_R}^2)}\ ,$$

where f_V is the vector meson decay constant:

$$\langle 0|j_\mu|V(p)\rangle = f_V M_V \varepsilon_\mu(p)\ .$$

Similarly

$$\langle V(p_2)|j_\mu|P(p_1)\rangle = \frac{2V^{P\to V}(q^2)}{(M_P+M_V)}\,\epsilon_{\mu\nu\rho\sigma}\,\varepsilon^{*\nu}(p_2)p_1^\rho p_2^\sigma\ ,$$

$$V^{P\to V}(q^2)\Big|_{\rm pole} = \frac{(M_V+M_P)\,g_{PVV_R}\,f_{V_R}}{2M_{V_R}(1-q^2/M_{V_R}^2)}\,,$$

$$\langle V(p_2)|j_\mu^5|P(p_1)\rangle = \mathrm{i}\,q_\mu\,(\varepsilon^*(p_2).p_1)\,\frac{2M_V}{q^2}\,A_0^{P\to V}(q^2)+\cdots\ ,$$

$$A_0^{P\to V}(q^2)\Big|_{\rm pole} = \frac{g_{PP_RV}\,f_{P_R}}{2M_V(1-q^2/M_{P_R}^2)}.$$

f_P is the meson decay constant of the pseudoscalar meson:

$$\langle 0|j_\mu^5|P(p)\rangle = \mathrm{i}f_P p_\mu\ .$$

Relativistic constituent quark model

Relativistic formalism adapted for the calculation of form factors by Anisovitch, Melikhov, et al. (1992). (D. Melikhov, arXiv:hep-ph/0110087, Eur. Phys. J. direct 4 C2 (2002).)

Approximation of internal quark lines in Feynman diagrams by the two-body intermediate states or two-body on-mass shell elastic scattering amplitude. One transforms Feynman diagram loop calculations into three-dimensional integrals and one ends up with dispersion relation type representations, which possess specific analyticity properties.

In the application domain of the model (form factors, decay constants), concerning mainly S-wave states or ground states, the detailed shape of the wave functions does not play an important role; only the spatial extension of the bound state, or its radius, is important; approximate guesses of the wave functions, with one or two parameters, is widely sufficient.

Another problem is the relationship between current and constituent quarks, which appears at the level of currents. In terms of constituent quarks (Q), one has

$$j_\mu = g_V \bar{Q}_1 \gamma_\mu Q_2 + \text{other Lorentz structures} \, ,$$

$$j_\mu^5 = g_A \bar{Q}_1 \gamma_\mu \gamma_5 Q_2 + \text{other Lorentz structures} \, .$$

When the currents contain at least one heavy quark, the choice $g_V = g_A = 1$, neglecting other terms, provides satisfactory results.

When the currents contain only light quarks, g_V and g_A may be different from 1 and more complicated structures might be needed. (Pham, (1982).)

Typical expressions of decay constants and form factors are the following:

$$f_i = \int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s\, \phi_i(s)\, \rho_i(s)\ , \qquad i = P, V.$$

$$\mathcal{F}_i(q^2) = \int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s_2\, \phi_2(s_2) \int_{s_1^-(s_2,q^2)}^{s_1^+(s_2,q^2)} \mathrm{d}s_1\, \phi_1(s_1)\ \Delta_i(s_1, s_2, q^2)\ .$$

$\rho_i(s)$ and $\Delta_i(s_1, s_2, q^2)$ are single and double spectral densities, calculated from the two-body quark kinematics.

Representation of $\mathcal{F}_i$ is valid in general for $-\infty \leq q^2 \leq (m_1 - m_2)^2$.

The wave functions ϕ_i have the following structure:

$$\phi_i(s) = \frac{\pi}{s^{3/4}}\sqrt{\frac{s^2-(m_1^2-m^2)^2}{2\,[s-(m_1-m)^2]}}\,w_i(k^2),$$

with

$$k^2 = \frac{(s-m_1^2-m_2^2)^2-4\,m_1^2\,m_2^2}{4\,s}\,.$$

The function w_i satisfies the normalization condition

$$\int \mathrm{d}k\,k^2\,w_i^2(k^2) = 1\,.$$

For the present applications, the detailed shape of the wave function ϕ_i is not crucial. It is mainly its spatial extension (or radius) that is important. w_i is chosen here as a gaussian function (for S-states):

$$w_i(k^2) = A_i\exp\left(-\frac{k^2}{2\,\beta_i^2}\right)\,.$$

The previous representations (in the region $q^2 \leq 0$ for the form factors) are equivalent to the light-front representations of the relativistic constituent quark model (Jaus (1991), Schlumpf (1994), Cardarelli *et al.* (1994)).

Advantage of the present representation: its analytic validity in the extended region $0 \leq q^2 \leq (m_1 - m_2)^2$. Important for numerical extrapolation to the resonance regions.

Numerical values of parameters

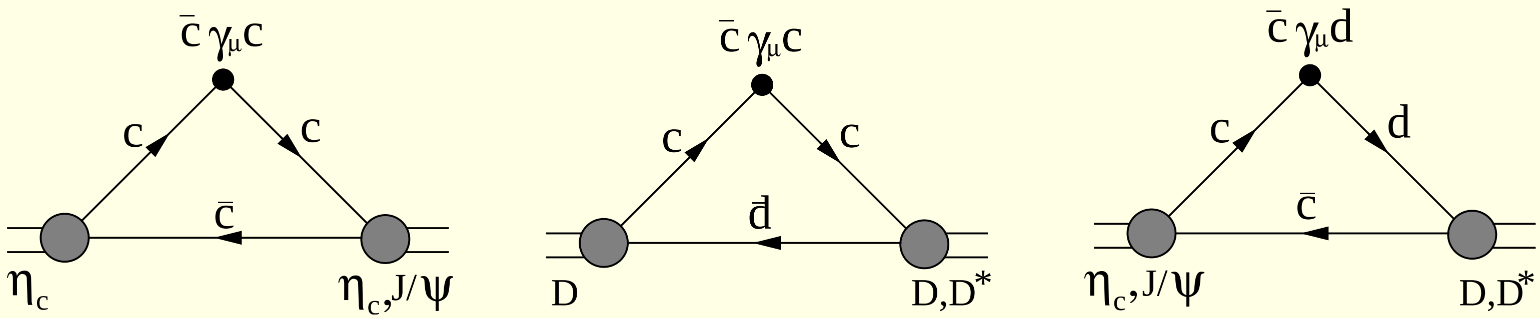
Quark flavour Q	u	d	s	c
Quark mass m_Q (GeV)	0.23	0.23	0.35	1.45

Meson	D	D^*	D_s	D_s^*	η_c	J/ψ
M (GeV)	1.87	2.010	1.97	2.11	2.980	3.097
f (MeV)	206 ± 8	260 ± 10	248 ± 2.5	311 ± 9	394.7 ± 2.4	405 ± 7
β (GeV)	0.475	0.48	0.545	0.54	0.77	0.68

Form factors

$M_1 \to M_2$ meson–meson transitions for $M_{1,2} = \eta_c, J/\psi, D, D^*$, D_s, D_s^*, mediated by vector or axial-vector currents.

Examples of Feynman graphs, with vector currents, yielding the one-loop contributions to the double spectral density $\Delta(s_1, s_2, q^2)$:



Extrapolation to the resonance region

Once the slopes β_i are fixed from the values of the decay constants, the form factors $\mathcal{F}_i$ are known in their analytic representation region $-\infty \leq q^2 \leq (m_1 - m_2)^2$ (for fixed values of the constituent quark masses, already given).

The form factors have resonance poles in the timelike region of q^2 at mass values M_R ($R = P_R, V_R$). The analytic expressions of $\mathcal{F}$ should provide a basis for an extrapolation to the first resonance region.

To implement the extrapolation, a four-parameter fit is adopted for $\mathcal{F}$:

$$\mathcal{F}(q^2) = \frac{\mathcal{F}(0)}{\left(1 - \sigma_1\, q^2/M_R^2 + \sigma_2\,(q^2)^2/M_R^4\right)} \frac{1}{(1 - q^2/M_R^2)}, \qquad R = P_R, V_R\,.$$

(For conserved vector currents $\mathcal{F}(0) = 1$.)

As a first test, the above parametrization is confronted with the analytic expression of $\mathcal{F}(q^2)$ in the region $-M_R^2 \leq q^2 \leq 0$.

The best fits produce the value of M_R, with the appropriate quantum numbers, very close to the physical mass of the resonance (within a few percent accuracy). This step confirms the soundness of the parametrization for the extrapolation to the timelike region near the resonance mass.

As a second step, the parametrization, with the mass M_R fixed now at its experimental value, is used in the timelike resonance region. The residue at the resonance pole is

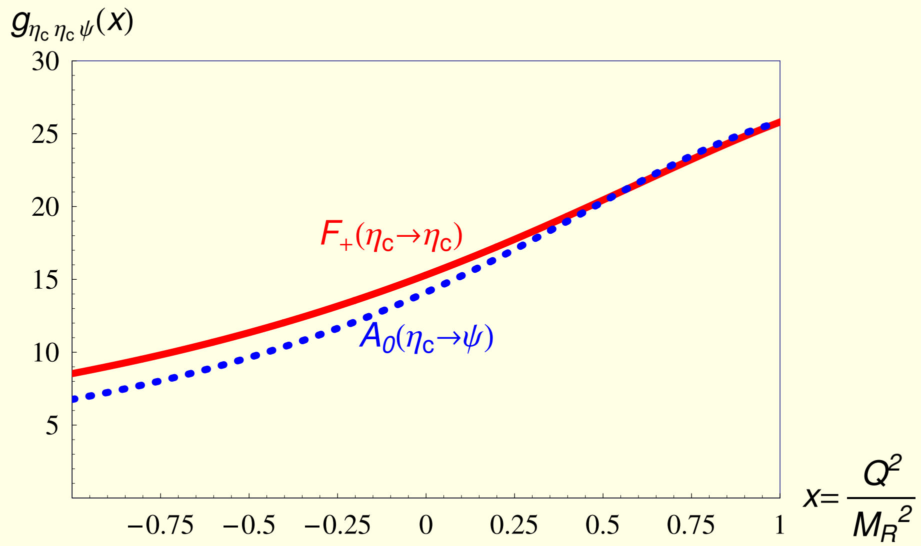
$$\mathrm{Res}\,\mathcal{F}(q^2 = M_R^2) = \frac{\mathcal{F}(0)}{(1 - \sigma_1 + \sigma_2)} = g_{PPV_R} f_{V_R}/(2M_{V_R})\,.$$

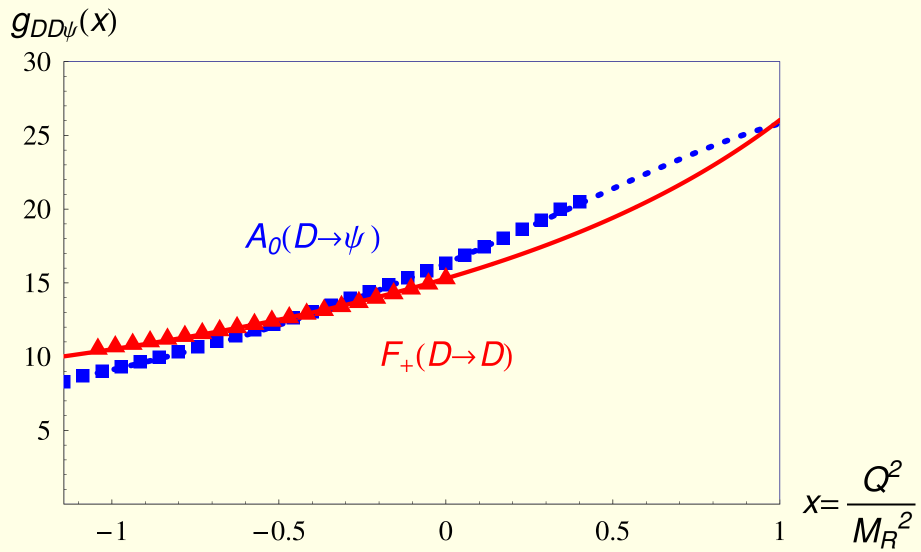
(A particular channel has been considered in the right.)

The same coupling constant can appear in different form factors (vector, axial). This imposes additional consistency checks.

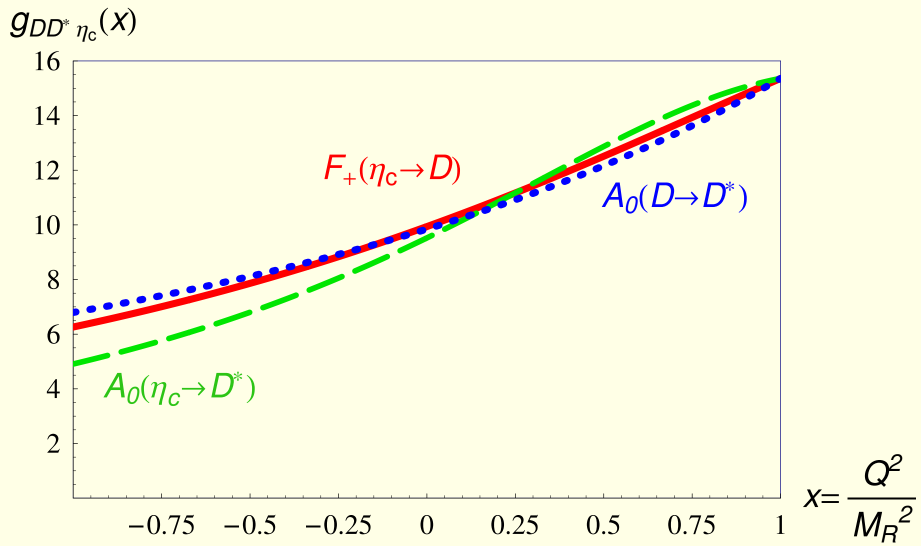
The matching between the three parameter fit ($\mathcal{F}(0),\ \sigma_1,\ \sigma_2$) and the analytic expression of $\mathcal{F}(q^2)$ in the spacelike region is realized within a 0.2% accuracy.

The figure below displays the dependence on $x \equiv \frac{q^2}{M_R^2}$ of the "off-shell strong coupling" $g_{\eta_c\eta_c\psi}(x)$, from the transition $\eta_c \to \eta_c$ (resonance $R = J/\psi$, red) and from the transition $\eta_c \to J/\psi$ (resonance $R = \eta_c$, blue), respectively. The two curves converge smoothly to the same value at $q^= m_R^2$.





$g_{DD\psi}(x)$
30
25
20
15
10
5
$A_0(D\to\psi)$
$F_+(D\to D)$
−1
−0.5
0
0.5
1
$x=\frac{Q^2}{M_R^2}$



$g_{DD^*\eta_c}(x)$
16
14
12
10
8
6
4
2
$F_+(\eta_c\to D)$
$A_0(D\to D^*)$
$A_0(\eta_c\to D^*)$
−0.75
−0.5
−0.25
0
0.25
0.5
0.75
1
$x=\frac{Q^2}{M_R^2}$

$PP'V$ coupling	$g_{PP'V}$
$\eta_c \eta_c J/\psi$	25.8 ± 1.7
DDJ/ψ	26.04 ± 1.43
$DD^* \eta_c$	15.51 ± 0.45
$D_s D_s J/\psi$	23.83 ± 0.78
$D_s D_s^* \eta_c$	14.15 ± 0.52

$PV'V$ coupling	$g_{PV'V}$ (GeV^{-1})
$\eta_c J/\psi J/\psi$	10.6 ± 1.5
$DD^* J/\psi$	10.7 ± 0.4
$D^* D^* \eta_c$	9.76 ± 0.32
$D_s D_s^* J/\psi$	9.6 ± 0.8
$D_s^* D_s^* \eta_c$	8.27 ± 0.37

Uncertainties do not include systematic uncertainties related to the model. Comparisons of predictions for other channels with experimetal data and lattice calculations indicate that they are not worse than **15-20%**.

Comparison with other approaches

QCD sum rules (Matheus *et al.* (2005), Rodrigues *et al.* (2014,2015)).

Strong coupling	$g_{DD\psi}$	$g_{DD^*\psi}$(GeV^{-1})	$g_{D_sD_s\psi}$	$g_{D_sD_s^*\psi}$(GeV^{-1})
This investigation	$\mathbf{26.04 \pm 1.43}$	$\mathbf{10.7 \pm 0.4}$	$\mathbf{23.83 \pm 0.78}$	$\mathbf{9.6 \pm 0.8}$
QCD sum rules	$\mathbf{11.6 \pm 1.8}$	$\mathbf{4.0 \pm 0.6}$	$\mathbf{11.96 \pm 1.34}$	$\mathbf{4.30 \pm 1.53}$

In spite of similarities of approaches, a factor of two difference between both sets of results.

More refined calculations require inclusion of perturbative contributions of gluon radiative corrections.