Polarized 3-parton production
in
future DIS experiments at small x

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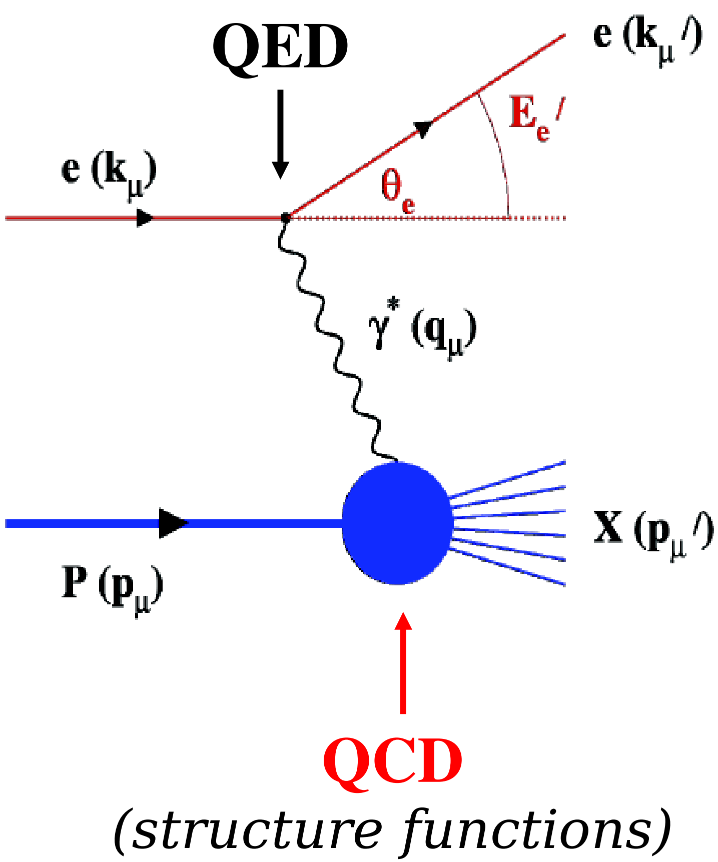
GDR QCD 2016
8-10 November, 2016
IPN Orsay

Deeply Inelastic Scattering (DIS)

probing hadron structure



Kinematic Invariants

$$Q^2 = -q^2 = -(k_\mu - k_\mu')^2$$

$$Q^2 = 4E_e E_e' \sin^2\left(\frac{\theta_e'}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E_e'}{E_e}\cos^2\left(\frac{\theta_e'}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

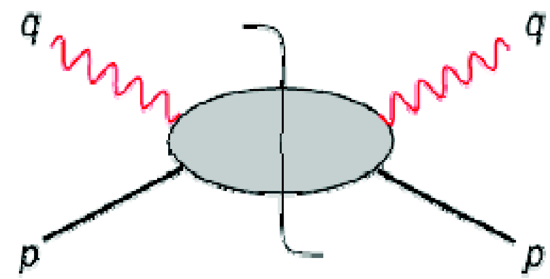
Measure of momentum fraction of struck quark

$$\mathbf{s} \equiv (\mathbf{p} + \mathbf{k})^2$$

Deeply Inelastic Scattering

Strong interactions: contained in the hadronic tensor $\mathbf{W}_{\mu\nu}(\mathbf{p},\mathbf{q})$

to all orders in the strong interaction $W_{\mu\nu}$ is given by the square of $\gamma^*(q)\, h(p) \to X$

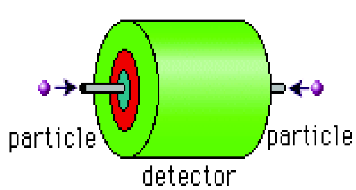


symmetries (parity, Lorentz), hermiticity & current conservation tell us that $W^{\nu\mu} = W^{\mu\nu *}$ $\quad q_\mu W^{\mu\nu} = 0$

$$W_{\mu\nu}(p,q) = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right)\frac{1}{p\cdot q} F_2(x,Q^2)$$

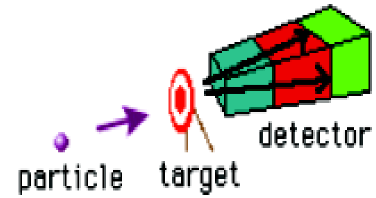
structure functions

- Collider experiment: Electron-Proton collisions at HERA (DESY, Hamburg, Germany)

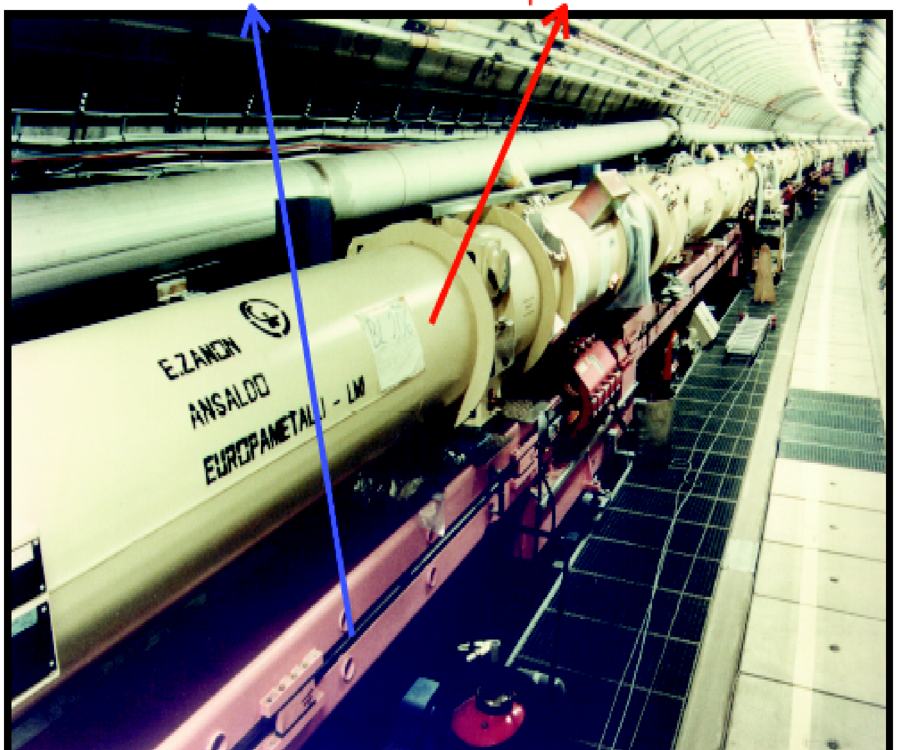


Equivalent to fixed target of

E_e = 50600 GeV:

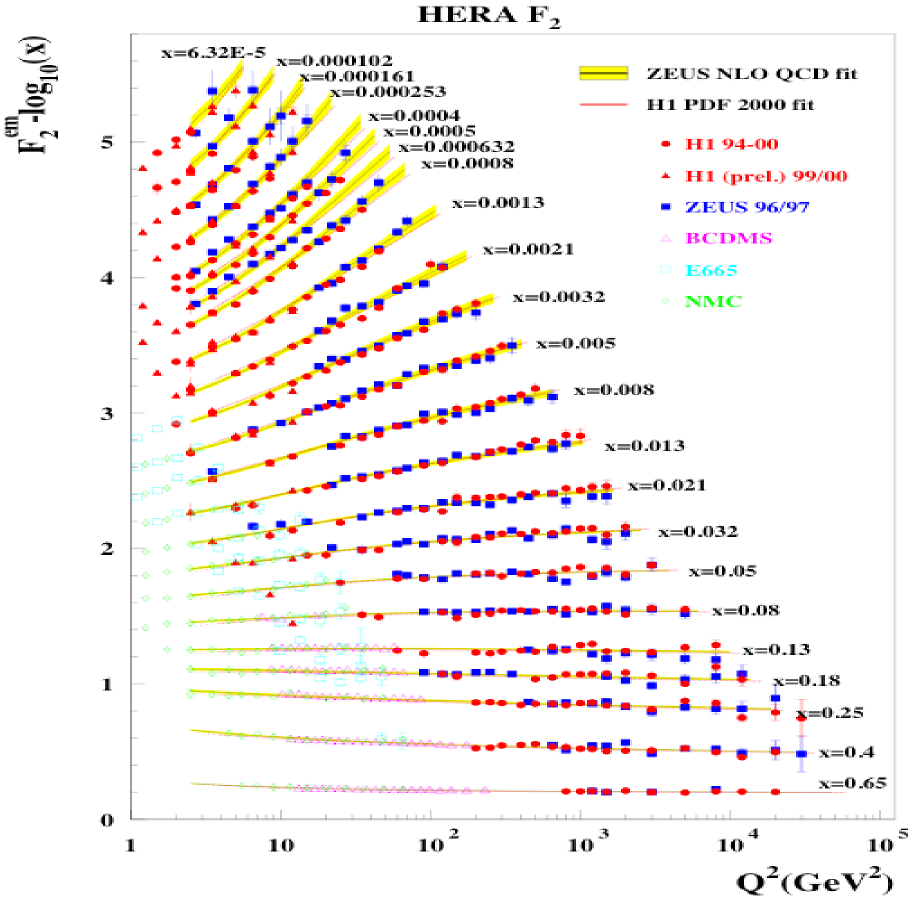


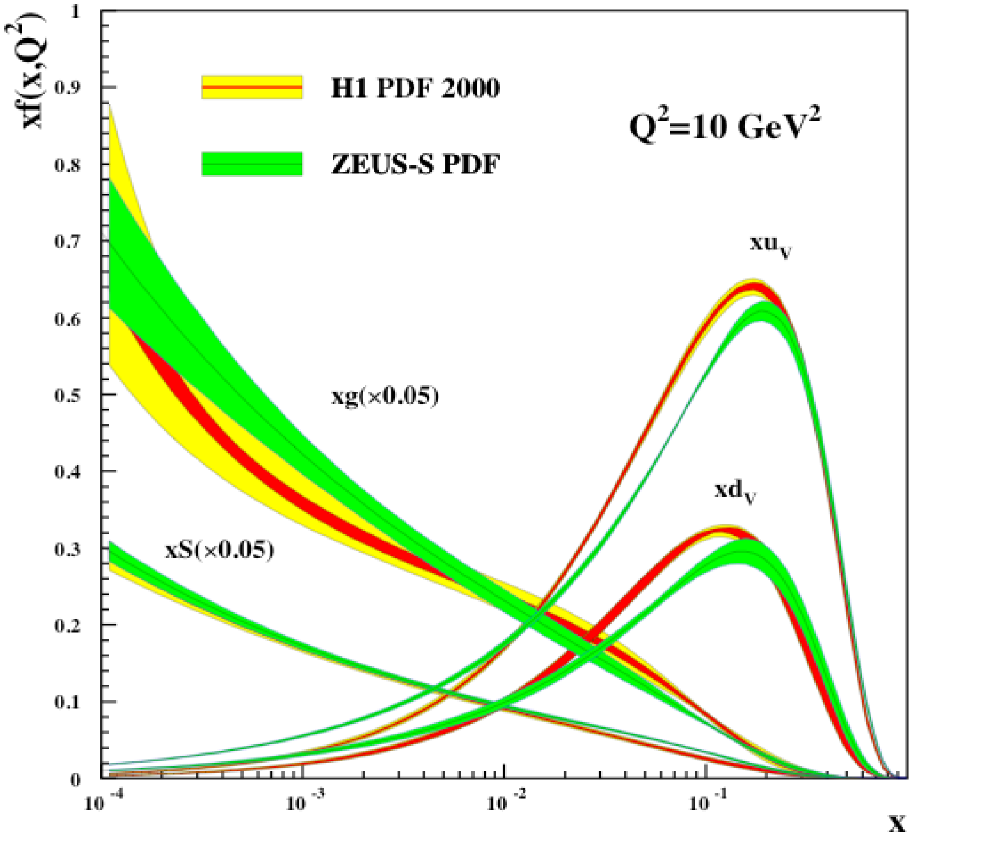
E_e = 27.5 GeV E_p = 920 GeV



QCD structure functions

parton distributions





What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_S{}^0)$ $(x \neq 1)$

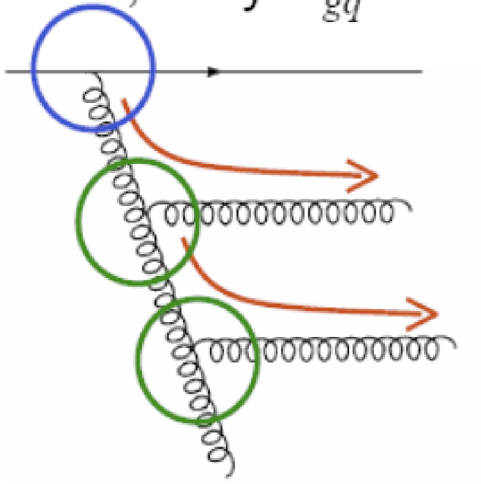
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A \left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]$$

At small x, only P_{gq} and P_{gg} are relevant.



→ Gluon dominant at small *x*!

The double log approximation (DLA) of DGLAP is easily solved.

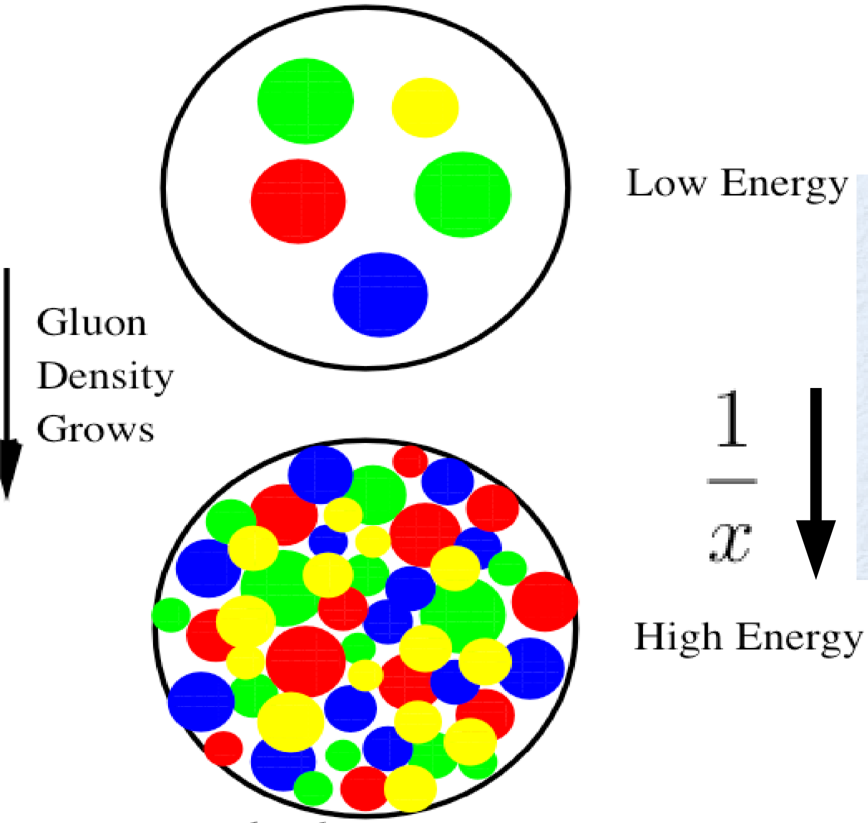
-- increase of gluon distribution at small x

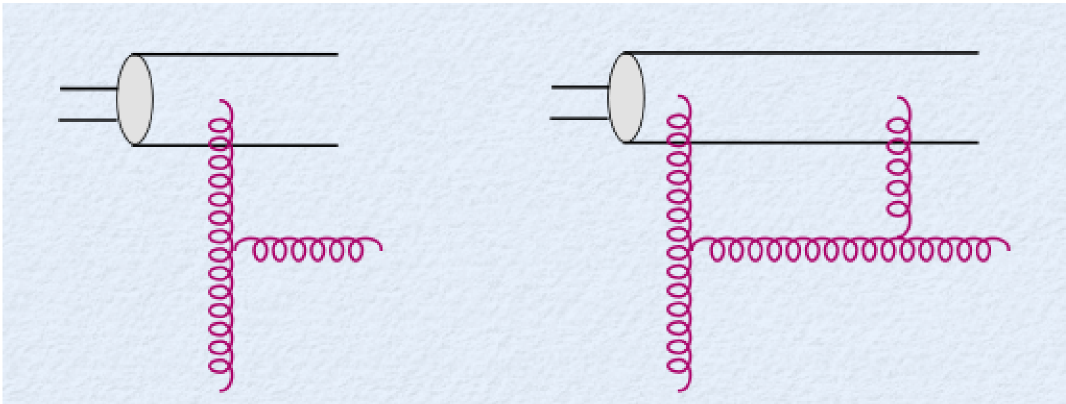
$$\mathbf{xg(x, Q^2) \sim e^{\sqrt{\alpha_s (\log 1/x)(\log Q^2)}}}$$

Gluon saturation

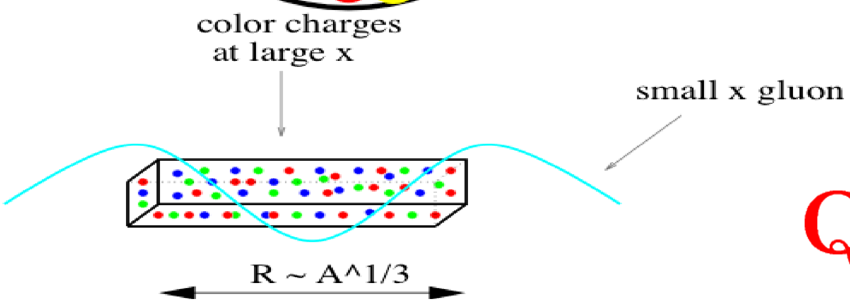
Gribov-Levin-Ryskin
Mueller-Qiu

"attractive" bremsstrahlung vs.
"repulsive" recombination





$$\frac{\alpha_s}{Q^2} \frac{xG(x, b_t, Q^2)}{S_\perp} \sim 1$$



$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

Signatures?

two main effects:

multiple scatterings
evolution with x (rapidity)

dense-dense (AA, pA, pp) collisions

initial conditions

dilute-dense (pA, forward pp) collisions

multiplicities
p_t spectra
angular correlations

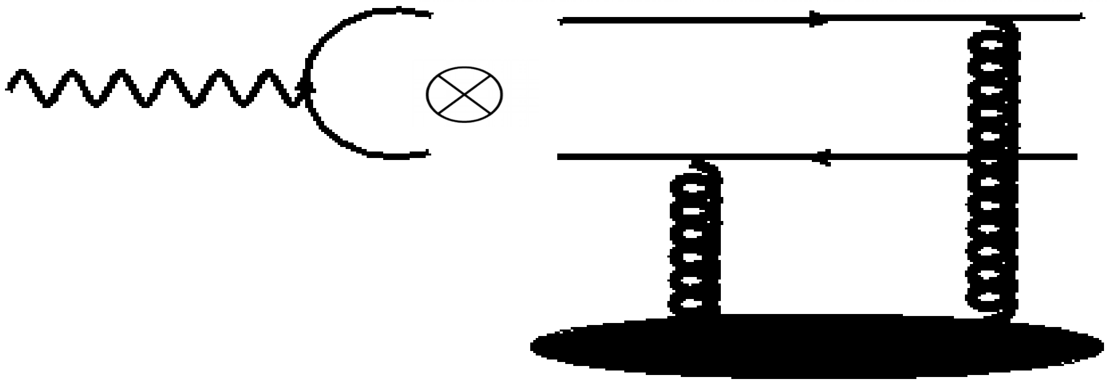
DIS

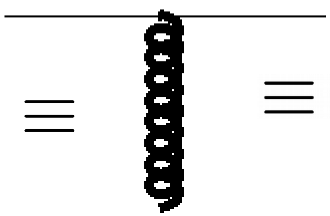
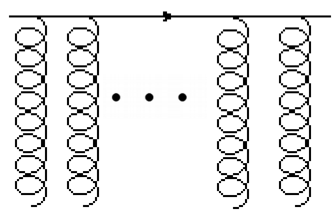
structure functions *(diffraction)*
NLO di-hadron correlations
3-hadron/jet azimuthal angular correlations

DIS total cross section (F_2, F_L)

$$\sigma^{total}_{DIS} = 2\int_0^1 dz \int d^2x_t\, d^2y_t\, \left|\Psi(k^{\pm}, k_t | z, x_t, y_t)\right|^2 T(x_t, y_t)$$

***dipole* cross section** $\quad T(x_t, y_t) \equiv \frac{1}{N_c} \mathrm{Tr} \left\langle 1 - V(x_t) V^{\dagger}(y_t) \right\rangle$



$V(x_t) \equiv$  $\equiv$  $\sim 1 + O(g\, A) + O(g^2\, A^2)$

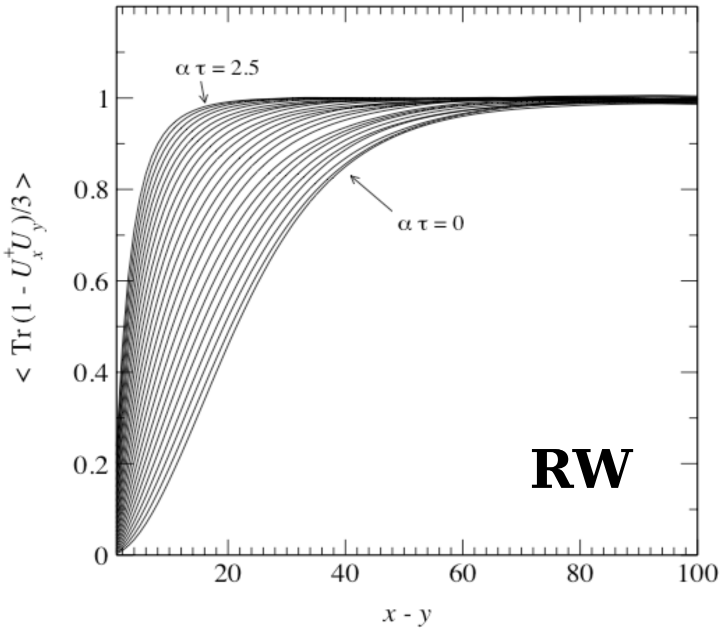
multiple scatterings *encoded in Wilson line V*

energy (rapidity or x) dependence *via JIMWLK evolution of correlators of V's*

Dipoles at large N_c : BK eq.

$$\frac{d}{dy}T(x_t - y_t) = \frac{\bar{\alpha}_s}{2\pi}\int d^2z_t\,\frac{(x_t - y_t)^2}{(x_t - z_t)^2(y_t - z_t)^2}\times$$

$$\left[T(x_t - z_t) + T(z_t - y_t) - T(x_t - y_t) - T(x_t - z_t)T(z_t - y_t)\right]$$



$\tilde{T}(p_t) \to \log\left[\frac{Q_s^2}{p_t^2}\right]$ **saturation region**

$\tilde{T}(p_t) \to \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^{\gamma}$ **extended scaling region**

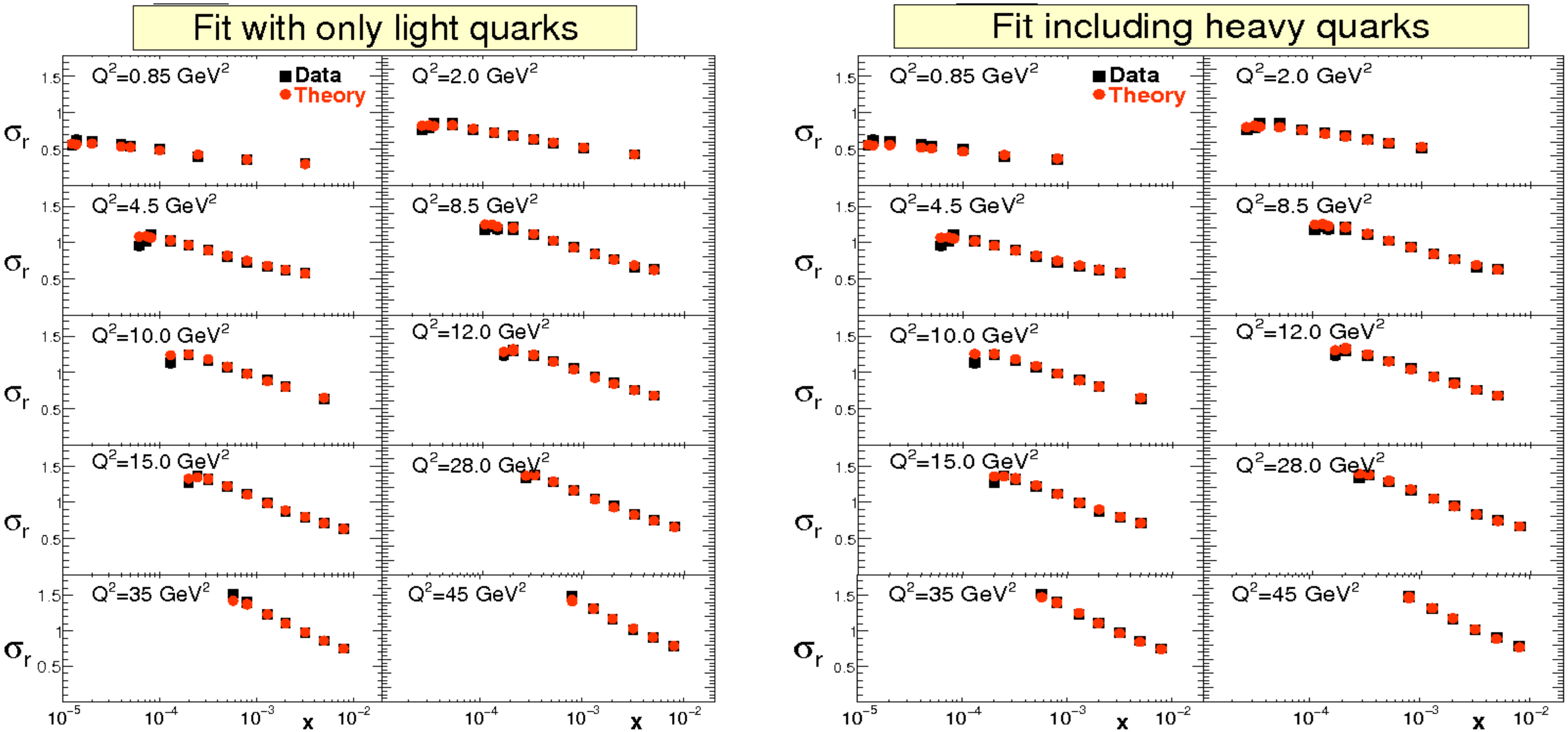
$\tilde{T}(p_t) \to \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]$ **pQCD region**

R*ummukainen-****W****eigert, NPA739 (2004) 183*

NLO: *Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)*

Structure functions at HERA

AAMQS(2010)



$$\mathbf{Q_s^2 = 1\,GeV^2[x_0/x]^\lambda}$$

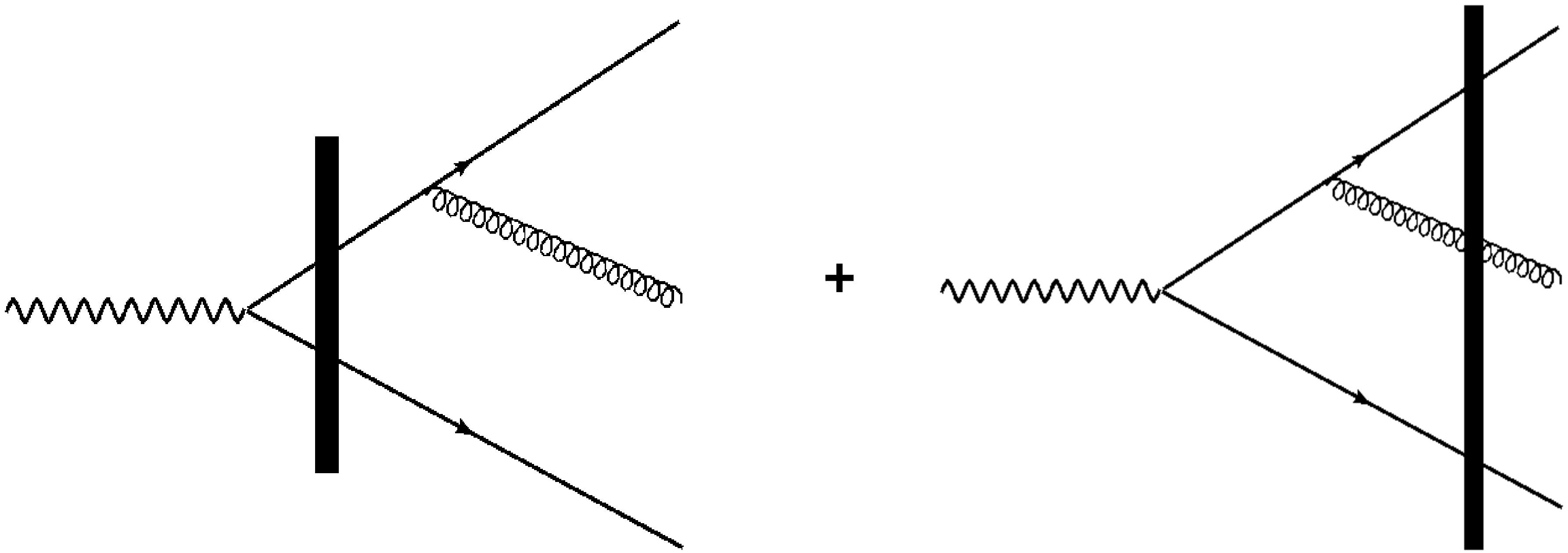
$$\lambda \sim \mathbf{0.25 - 0.3}$$

$$\mathbf{x_0 = 3X10^{-4}}$$

something with more discriminating power

angular correlations in 3-parton production in DIS

$$\gamma^\star \mathbf{T} \to \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$$

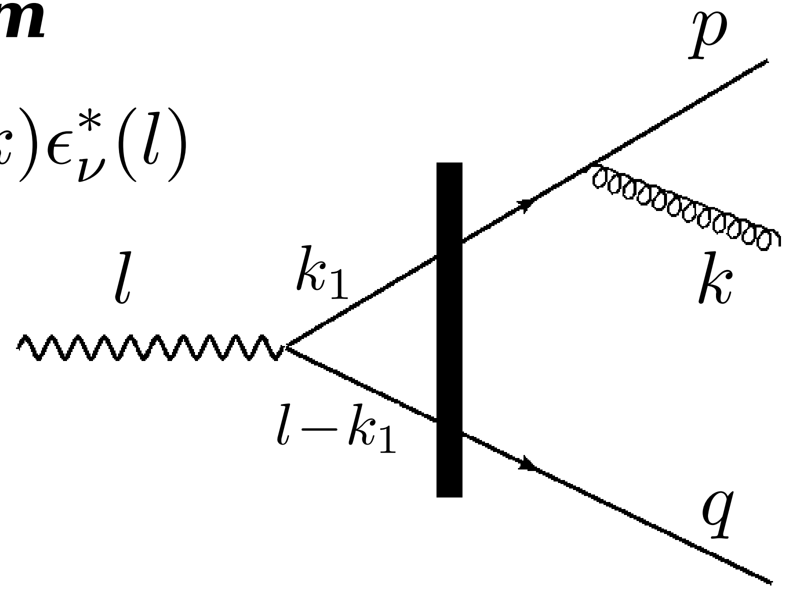


+ radiation from anti-quark

Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans; PLB761 (2016) 229

1^{st} diagram

$$\mathcal{A} \equiv -eg\,\bar{u}(p)\,[A]^{\mu\nu}\,v(q)\,\epsilon_\mu\,(k)\epsilon^*_\nu(l)$$



$$A_1^{\mu\nu} = \gamma^\mu\, t^a\, S_F^0(p+k)\,\tau_F(p+k,k_1)\,S_F^0(k_1)\,\gamma^\nu\,S_F^0(l-k_1)\,\tau_F(l-k_1,q)\,\frac{d^4k_1}{(2\pi)^4}$$

$$= \frac{i}{2\,l^-}\frac{\delta(l^- - p^- - q^- - k^-)}{(p+k)^2}\int d^2x_t\,d^2y_t\,e^{-i(p_t+k_t)\cdot x_t}\,e^{-iq_t\cdot y_t}$$
$$\gamma^\mu\,t^a\,i(\not{p}+\not{k})\gamma^-\,i\not{k}_1\,\gamma^\nu\,i(\not{l}-\not{k}_1)\,\gamma^-\,K_0\,[L\,(x_t-y_t)]$$
$$V(x_t)\,V^\dagger(y_t)$$

with

$$L^2 = \frac{q^-(p^-+k^-)}{l^-\,l^-}Q^2 \qquad k_1^- = p^- - k^- \qquad k_1^+ = \frac{k_{1t}^2 - i\epsilon}{2(p^-+k^-)} \qquad k_{1t} = -i\,\partial_{x_t-y_t}$$

spinor helicity methods

Review:
L. Dixon, hep-ph/9601359

massless quarks: helicity eigenstates

$$u_\pm(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, u(k) \qquad \overline{u_\pm(k)} \equiv \overline{u(k)}\, \frac{1}{2}(1 \mp \gamma_5)$$

$$v_\mp(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, v(k) \qquad \overline{v_\mp(k)} \equiv \overline{v(k)}\, \frac{1}{2}(1 \mp \gamma_5)$$

helicity operator $h \equiv \vec{\Sigma} \cdot \hat{p}$

$$\vec{\Sigma} \cdot \hat{p}\, U_\pm(p) = \pm U_\pm(p)$$
$$-\vec{\Sigma} \cdot \hat{p}\, V_\pm(p) = \pm V_\pm(p)$$

$$u_+(k) = v_-(k) = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \\ \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \end{bmatrix} \qquad u_-(k) = v_+(k) = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{k^-} e^{-i\phi_k} \\ -\sqrt{k^+} \\ -\sqrt{k^-} e^{-i\phi_k} \\ \sqrt{k^+} \end{bmatrix}$$

with $e^{\pm i\phi_k} \equiv \dfrac{k_x \pm i k_y}{\sqrt{k^+ k^-}}$ and $k^\pm = \dfrac{E \pm k_z}{\sqrt{2}}$

spinor helicity methods

notation:

$$|i^{\pm}> \equiv |k_i^{\pm}> \equiv U_{\pm}(k_i) = V_{\mp}(k_i) \qquad < i^{\pm}| \equiv < k_i^{\pm}| \equiv \overline{U}_{\pm}(k_i) = \overline{V}_{\mp}(k_i)$$

basic spinor products:

$$< i\, j > \;\equiv\; < i^-|j^+> = \overline{U}_-(k_i)\, U_+(k_j) \;=\; \sqrt{|s_{ij}|}e^{i\phi_{ij}}$$

$$[i\, j] \;\equiv\; < i^+|j^-> = \overline{U}_+(k_i)\, U_-(k_j) \;=\; -\sqrt{|s_{ij}|}e^{-i\phi_{ij}}$$

$$\cos\phi_{ij} = \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

$$\sin\phi_{ij} = \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

with

$$s_{ij} = (k_i + k_j)^2 = 2k_i \cdot k_j$$
$$= -< ij > [ij]$$

and

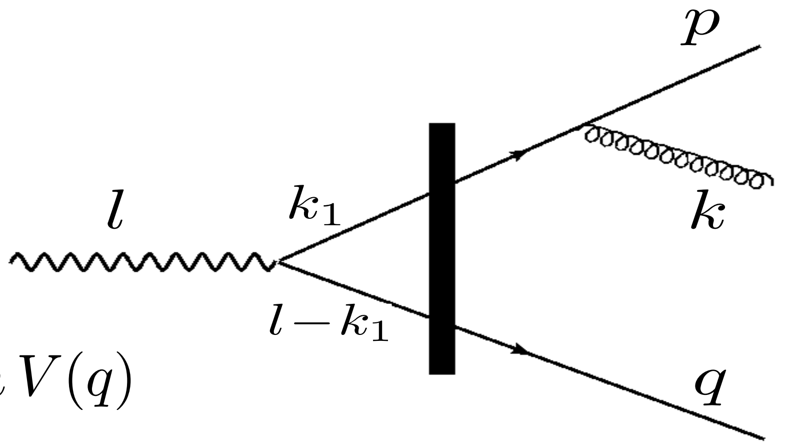
$$< ii > = [ii] = 0$$
$$< ij] = [ij >= 0$$

any off-shell momentum $k^\mu \equiv \bar{k}^\mu + \frac{k^2}{2k^+}\, n^\mu$ where $\bar{k}^\mu$ is on-shell $\bar{k}^2 = 0$

any on-shell momentum $\not{p} = |p^+ >< p^+| + |p^- >< p^-|$

work with a given helicity state

Diagram A1



Numerator: Dirac Algebra

$$a_1 \equiv \overline{U}(p)\,\not\epsilon^\star(k)\,(\not p + \not k)\,\not n\,\not k_1\,\not\epsilon(l)\,(\not k_1 - \not l)\,\not n\,V(q)$$

longitudinal photons

quark anti-quark gluon helicity: + - +

$$\not l = l^+ \not{\overline{n}} - \frac{Q^2}{2l^+}\not n$$

$$a_1^{L;+-+} \;=\; -\frac{\sqrt{2}}{[n\,k]}\frac{Q}{l^+}\,[n\,p] < k\,p > [n\,p] < n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] < n\,q >$$

$$\left(< n\,\overline{k}_1 > \left[n\,\overline{k}_1\right] - l^+ < n\,\overline{n} > [n\,\overline{n}]\right)$$

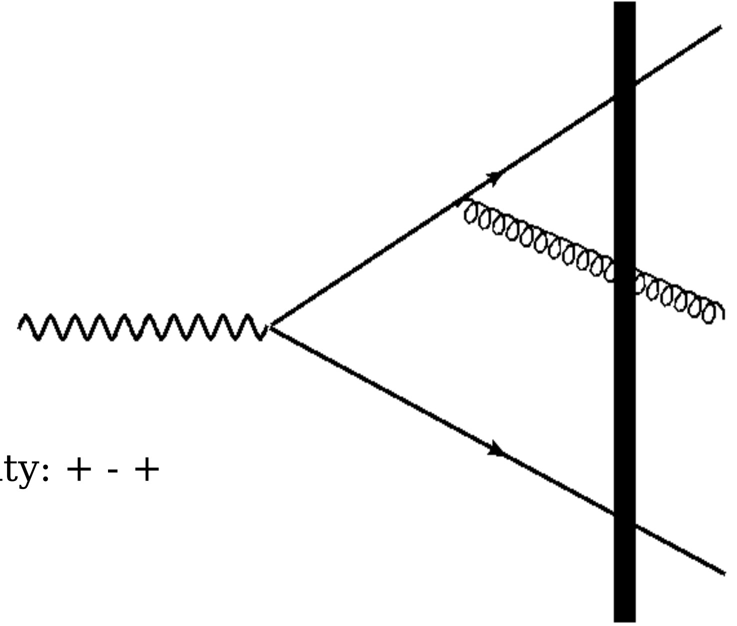
with

$$< np >= -[np] = \sqrt{2p^+}$$

transverse photons: +

$$a_1^{\perp=+;+-+} = -\frac{\sqrt{2}}{[nk]}[pn] < kp > [pn] < nk_1 > [k_1 n] < \bar{n} k_1 > [k_1 n] < nq >$$

Diagram A3



Numerator: Dirac Algebra

longitudinal photons

quark anti-quark gluon helicity: + - +

$$a_3^{L;+-+} = \frac{\sqrt{2}Q}{l^+ [n\bar{k}_2]} [pn] \left(< n\bar{k}_1 > [\bar{k}_1 n] - < n\bar{k}_2 > [\bar{k}_2 n]\right) < \bar{k}_2 \bar{k}_1 > [\bar{k}_1 n]$$

$$\left(< n\bar{k}_1 > [\bar{k}_1 n] - l^+ < n\bar{n} > [\bar{n}n]\right) < nq >$$

$$= -2^4 Q (l^+)^2 \frac{(z_1 z_2)^{3/2}}{z_3} \left[z_3\, k_{1\,t} \cdot \epsilon - (z_1 + z_3) k_{2\,t} \cdot \epsilon\right]$$

the rest is some standard integrals

add up the amplitudes, add, square..

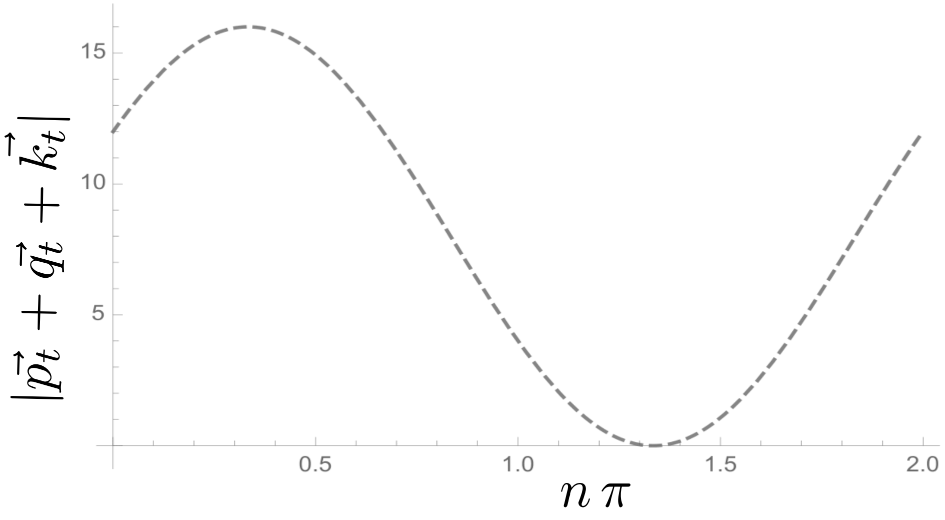
3-parton production

<u>linear regime</u>: <u>*use ugd's*</u>

$$z_1 = z_2 = 0.2, z_3 = 0.6$$

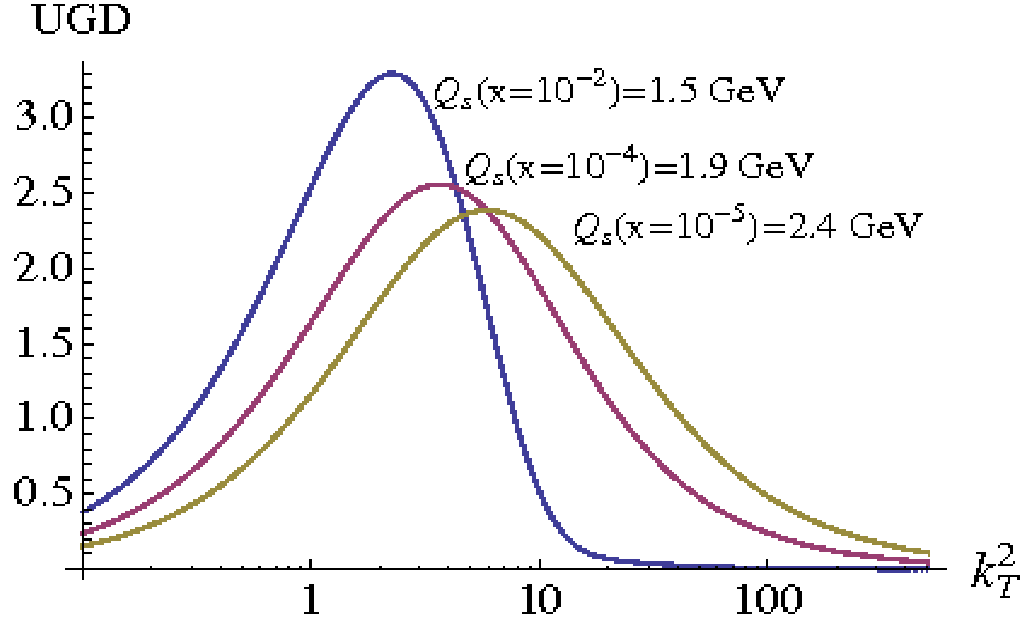
$$p_t = q_t = k_t = 4\,GeV$$

$$Q^2 = 16\,GeV^2$$



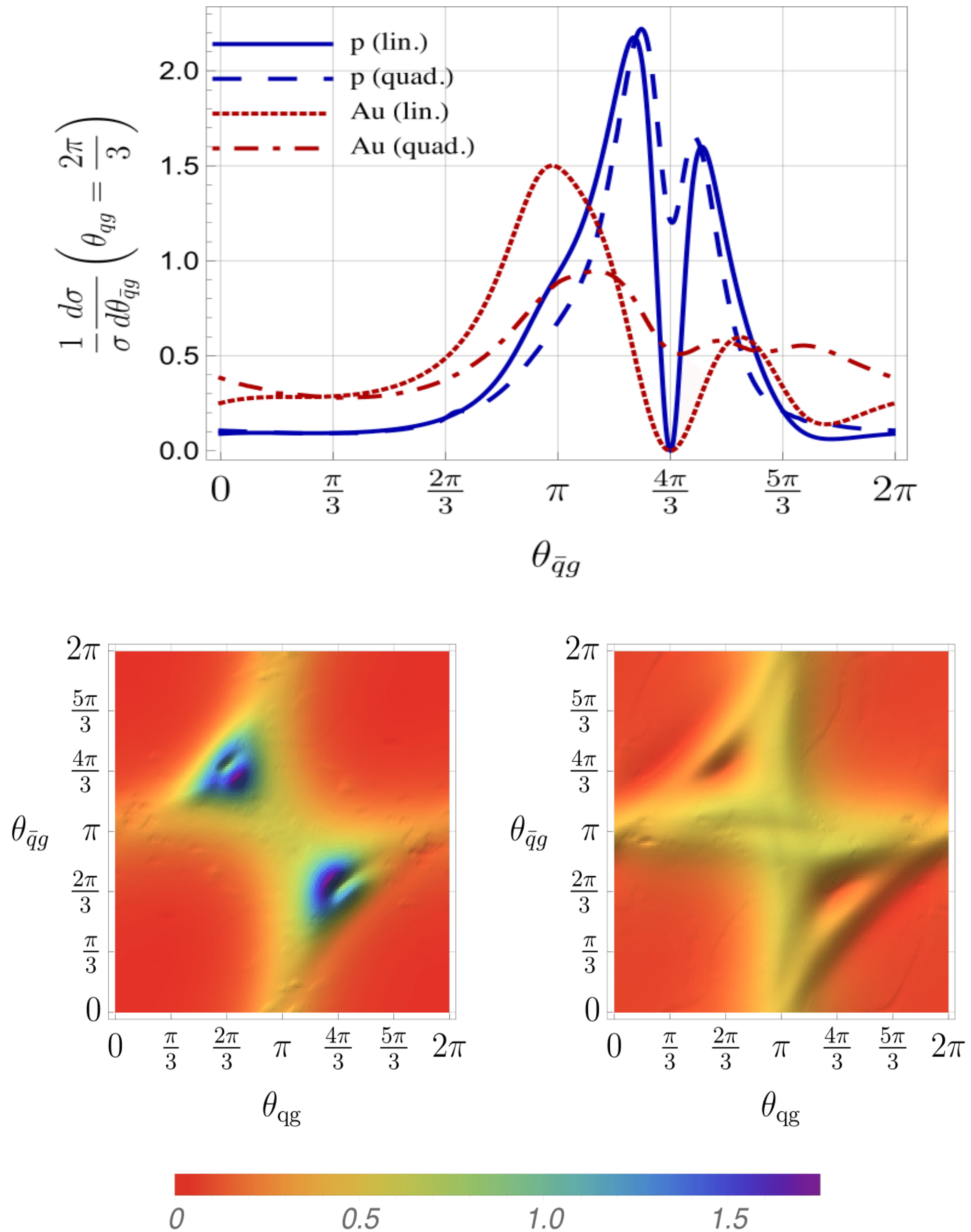
$$\Delta\phi_{12} = \frac{2\pi}{3} \qquad \text{vary} \quad \Delta\phi_{13}$$

$$k_t^2\,\tilde{T}(k_t)$$



multiple scattering: broadening of the peak
x-evolution: reduction of magnitude

3-parton azimuthal angular correlations



SUMMARY

CGC is a systematic approach to high energy collisions

Leading Log CGC works (too) well

it has been used to fit a wealth of data; ep, eA, pp, pA, AA

need to eliminate/minimize late time/hadronization effects

Precision (NLO) studies of less-inclusive observables are needed

Azimuthal angular correlations offer a unique probe of CGC

3-hadron/jet correlations should be even more discriminatory

DIS is the ideal ground for precision CGC studies

Large Hadron Collider (LHC)