

From DIS to pA at small x :
Azimuthal angular correlations

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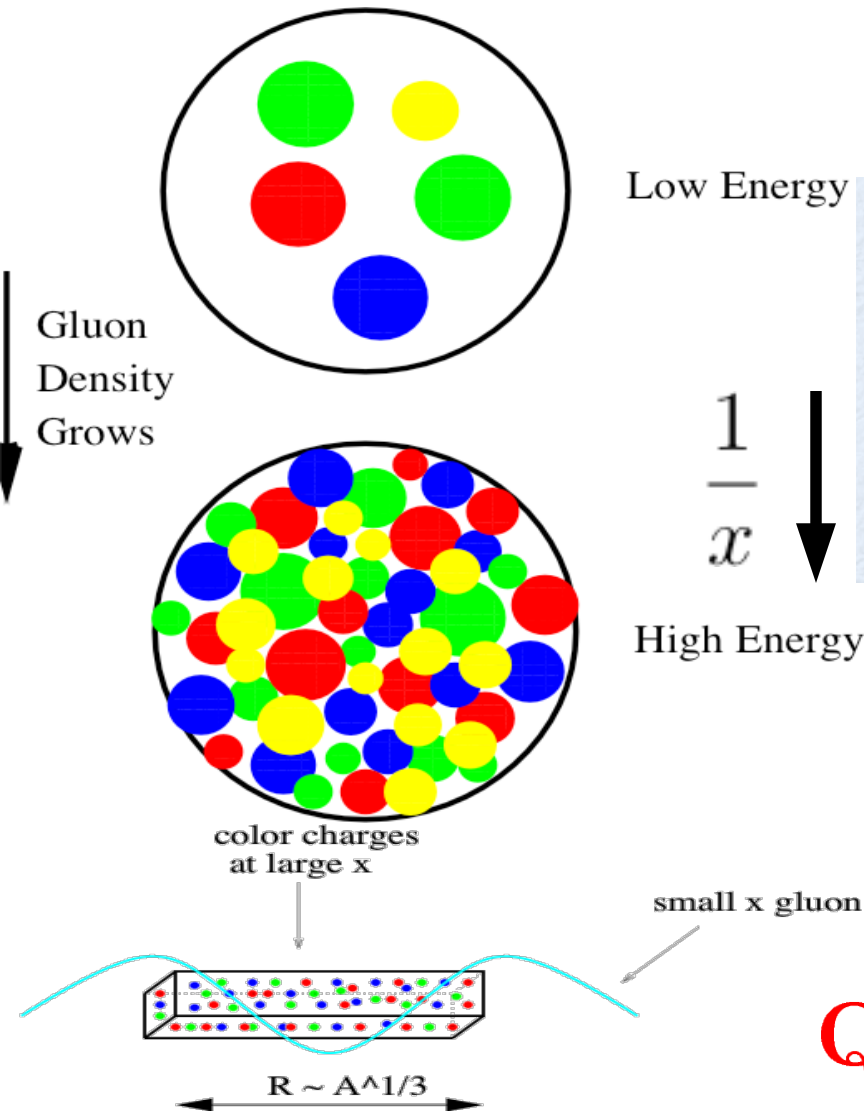
and

Ecole Polytechnique, Palaiseau

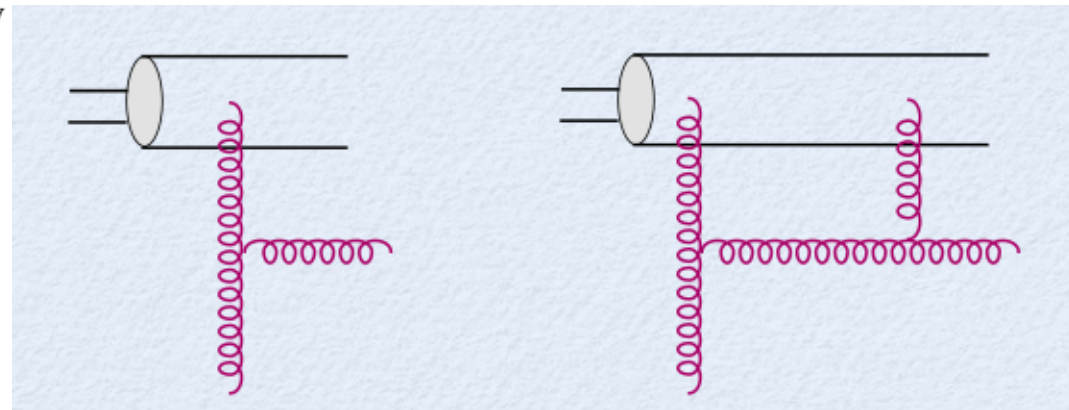
GDR QCD 2016
8-10 November, IPN Orsay

Gluon saturation

*Gribov-Levin-Ryskin
Mueller-Qiu*



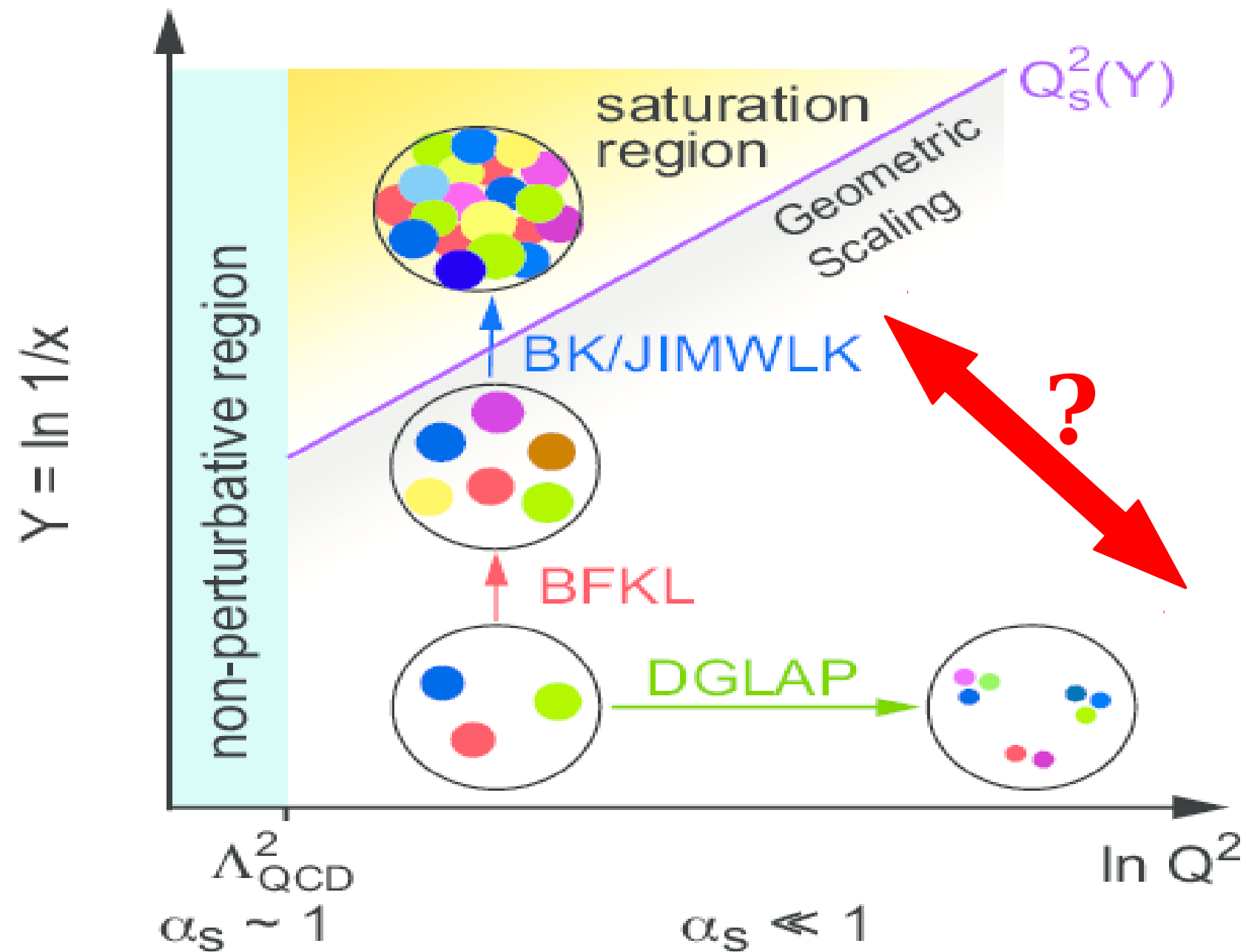
“attractive” bremsstrahlung vs.
“repulsive” recombination



$$\frac{\alpha_s}{Q^2} \frac{xG(x, b_t, Q^2)}{S_\perp} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

QCD at high energy: *saturation*

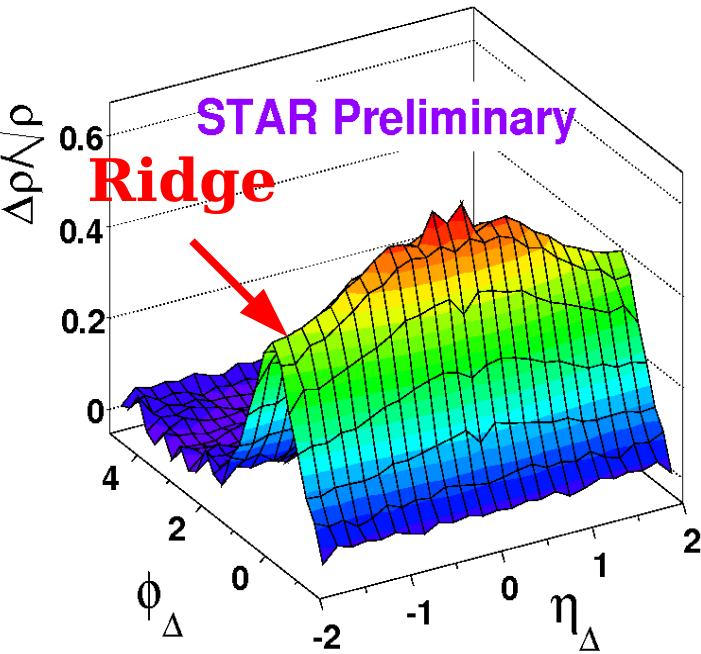


Probing saturation: di-hadron correlations

polar angle (rapidity correlations)

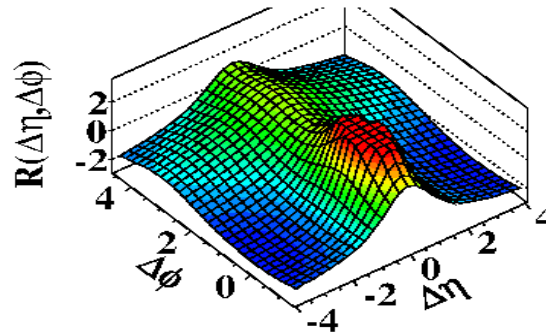
azimuthal angle (back to back)

long-range rapidity correlations: the ridge

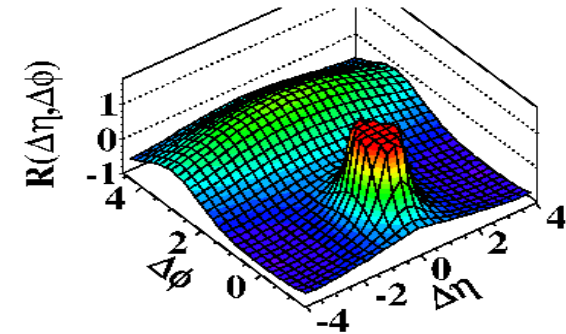


AA at RHIC

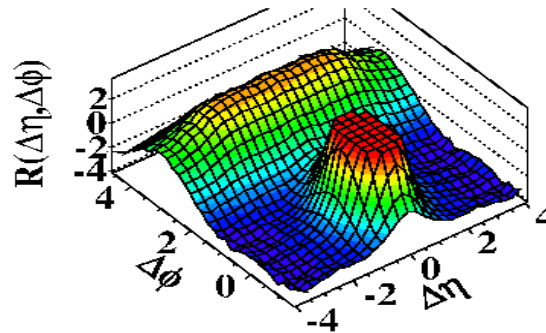
(a) CMS MinBias, $p_T > 0.1 \text{ GeV}/c$



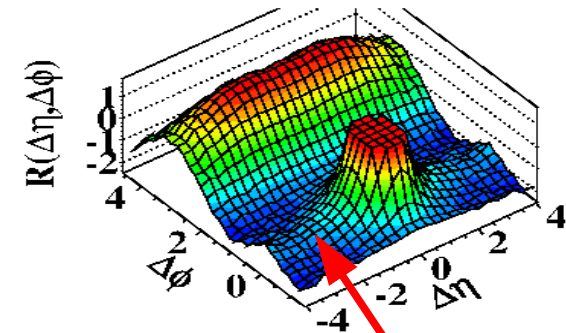
(b) CMS MinBias, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



(c) CMS $N \geq 110$, $p_T > 0.1 \text{ GeV}/c$



(d) CMS $N \geq 110$, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



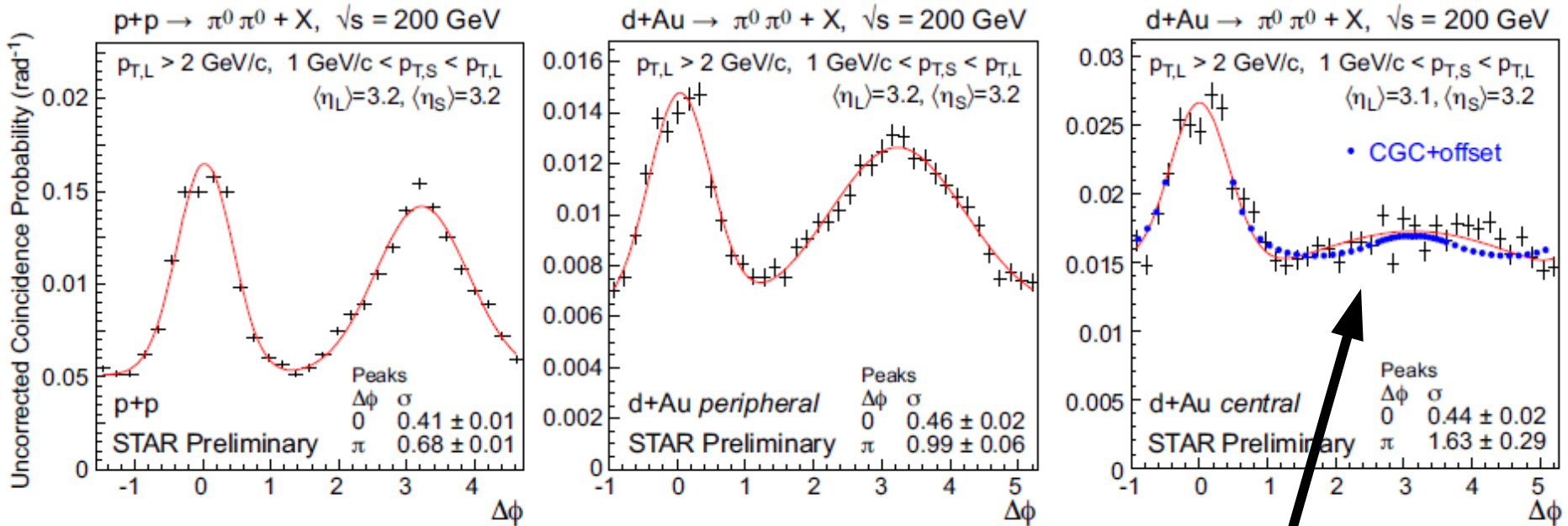
PP at LHC

Ridge

Initial state vs final state ?
if final state, early or late times?

di-hadron correlations in pA

Recent STAR measurement (arXiv:1008.3989v1):



Marquet, NPA (2007), Albacete + Marquet, PRL (2010)

Tuchin, NPA846 (2010)

A. Stasto + B-W. Xiao + F. Yuan, PLB716 (2012)

T. Lappi + H. Mantysaari, NPA908 (2013)

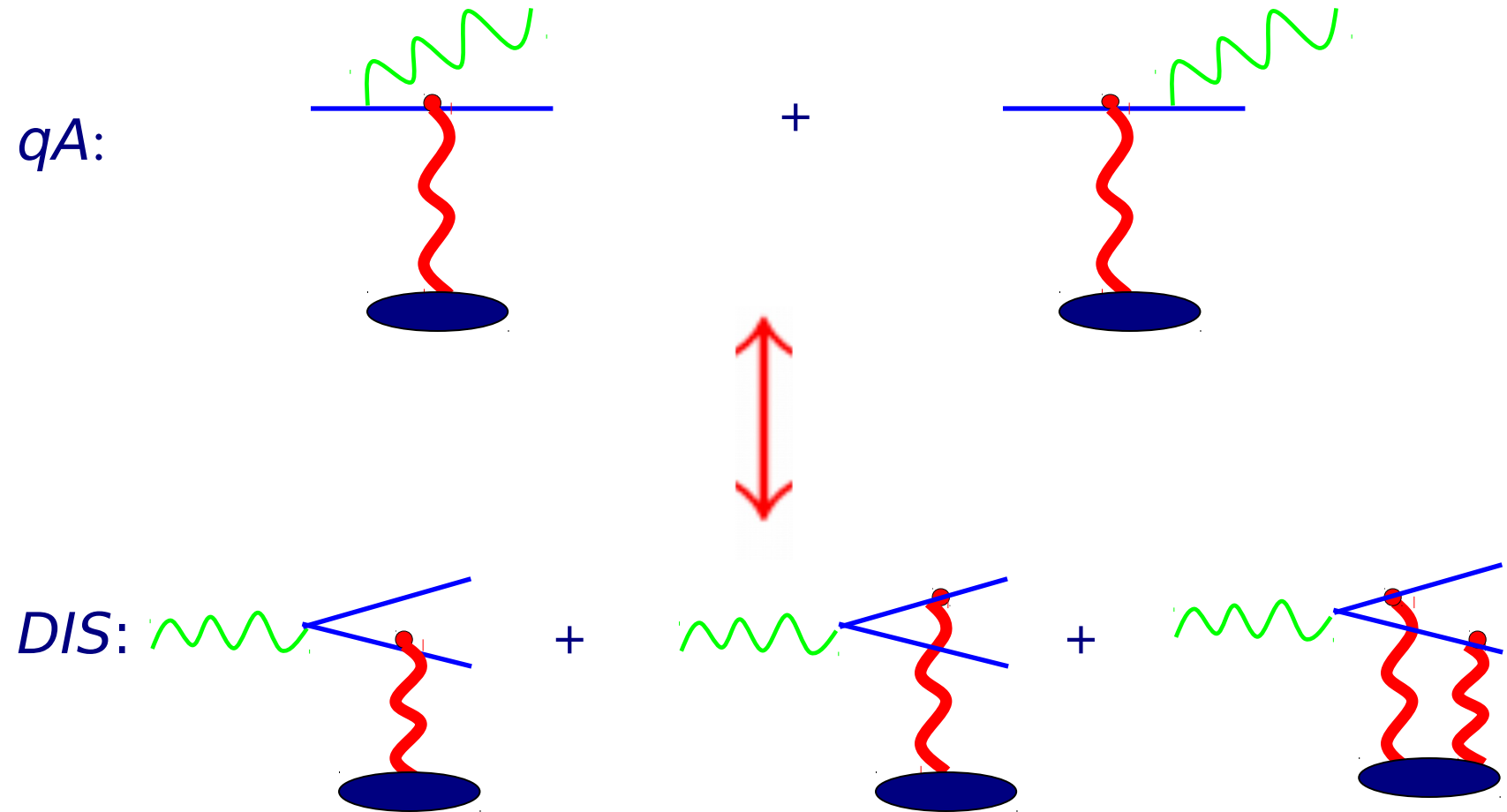
**saturation effects
de-correlate
the hadrons**

shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

DIS \longleftrightarrow **pA**
crossing symmetry

From DIS to DY: crossing symmetry (LO)

$$\gamma^* A \rightarrow q \bar{q} X \quad \leftrightarrow \quad q A \rightarrow q \gamma^* X$$

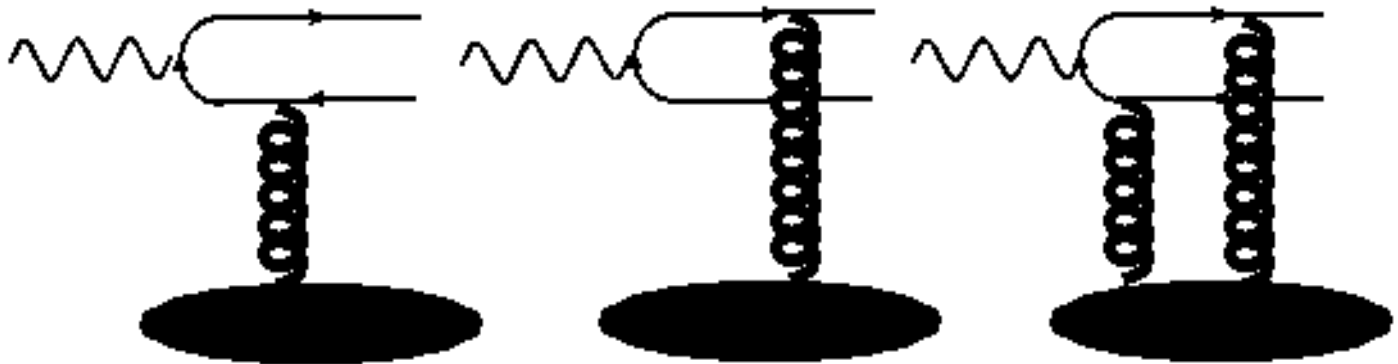


di-hadron (azimuthal) angular correlations in DIS

DIS total cross section (F_L, F_2): **dipoles** $\langle \text{Tr } V V^\dagger \rangle$

di-jet production in DIS: **quadrupoles** $\langle \text{Tr } V V^\dagger V V^\dagger \rangle$

$$\text{LO: } \gamma^* T \rightarrow q \bar{q} X$$



di-hadron production in DIS

$$\gamma^*(\mathbf{k}) \mathbf{p} \rightarrow \mathbf{q}(\mathbf{p}) \bar{\mathbf{q}}(\mathbf{q}) \mathbf{X}$$

$$\begin{aligned} \mathcal{A}^\mu(k, q, p) = & \frac{i}{2} \int \frac{d^2 l_\perp}{(2\pi)^2} d^2 x_\perp d^2 y_\perp e^{i(p_\perp + q_\perp - k_\perp - l_\perp) \cdot y_\perp} \\ & e^{i l_\perp \cdot x_\perp} \bar{u}(q) \Gamma^\mu(k^\pm, k_\perp, q^-, p^-, q_\perp - l_\perp) v(p) \\ & [V(x_\perp) V^\dagger(y_\perp) - 1] \end{aligned}$$

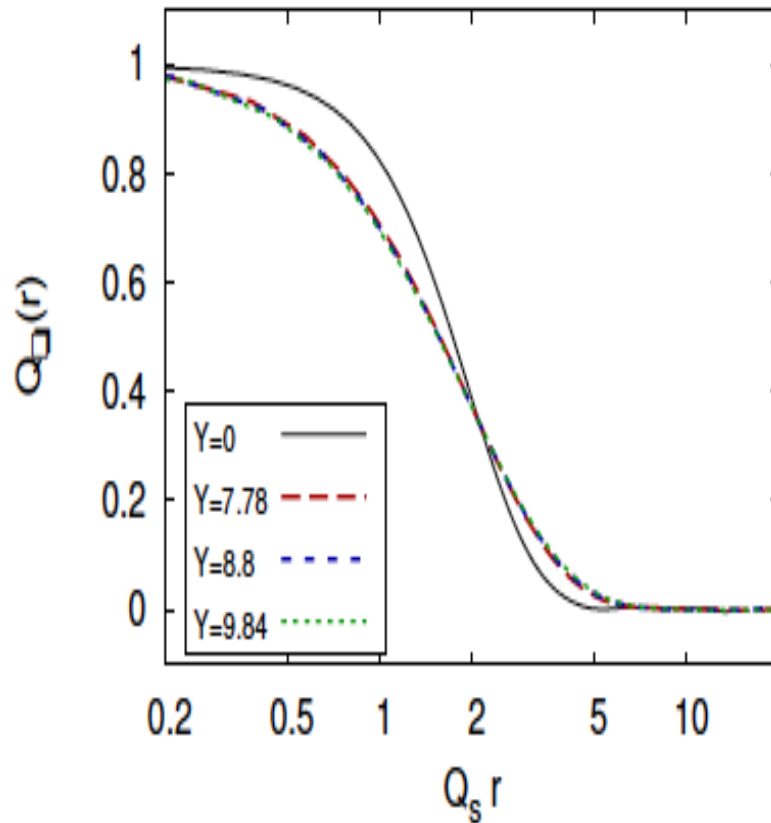
with

$$\begin{aligned} \Gamma^\mu \equiv & \\ & \frac{\gamma^- (\not{q} - \not{l} + m) \gamma^\mu (\not{q} - \not{k} - \not{l} + m) \gamma^-}{p^- [(q_\perp - l_\perp)^2 + m^2 - 2q^- k^+] + q^- [(q_\perp - k_\perp - l_\perp)^2 + m^2]} \end{aligned}$$

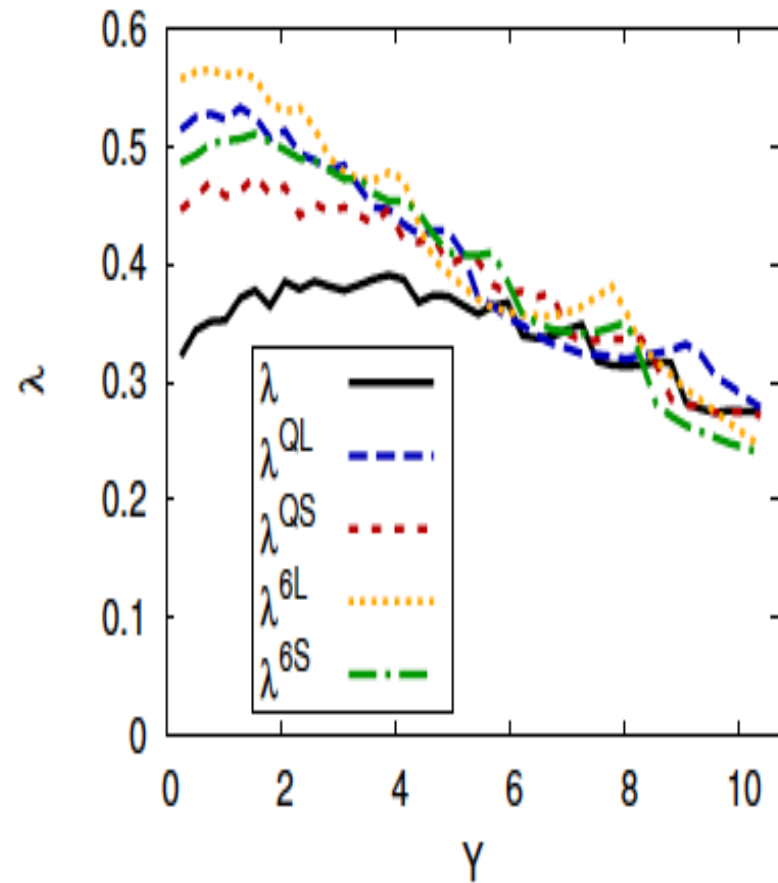
F. Gelis and J. Jalilian-Marian, PRD67 (2003) 074019

Zheng + Aschenauer + Lee + Xiao, PRD89 (2014)7, 074037

Quadrupole evolution: JIMWLK



Geometric scaling also present in quadrupoles



Energy dependence of saturation scale

quadrupole: limits

$$\langle Q(r, \bar{r}, \bar{s}, s) \rangle \equiv \frac{1}{N_c} \langle \text{Tr} V(r) V^\dagger(\bar{r}) V(\bar{s}) V^\dagger(s) \rangle$$

line config.: $r = \bar{s}, \bar{r} = s, z \equiv r - \bar{r}$

square config.: $r - \bar{s} = \bar{r} - s = r - \bar{r} = \dots \equiv z$

“naive” Gaussian: $Q = S^2$

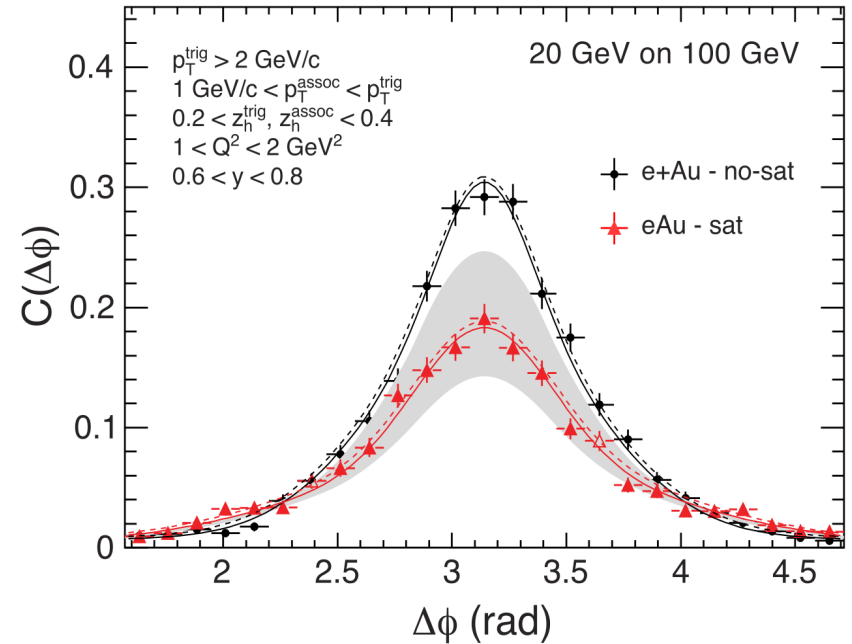
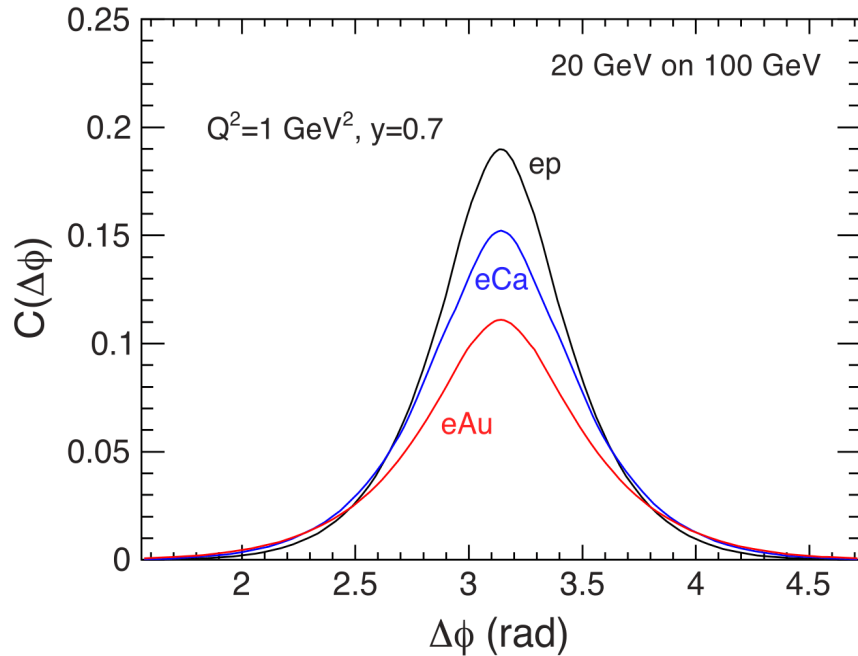
Gaussian $Q_{|}(z) \approx \frac{N_c + 1}{2} [S(z)]^{2\frac{N_c+2}{N_c+1}} - \frac{N_c - 1}{2} [S(z)]^{2\frac{N_c-2}{N_c-1}}$

$$Q_{sq}(z) = [S(z)]^2 \left[\frac{N_c + 1}{2} \left(\frac{S(z)}{S(\sqrt{2}z)} \right)^{\frac{2}{N_c+1}} - \frac{N_c - 1}{2} \left(\frac{S(\sqrt{2}z)}{S(z)} \right)^{\frac{2}{N_c-1}} \right]$$

Gaussian + large N_c $Q_{|}(z) \rightarrow S^2(z)[1 + 2 \log[S(z)]]$

$$Q_{sq}(z) = \left[1 + 2 \ln \left(\frac{S(z)}{S(\sqrt{2}z)} \right) \right]$$

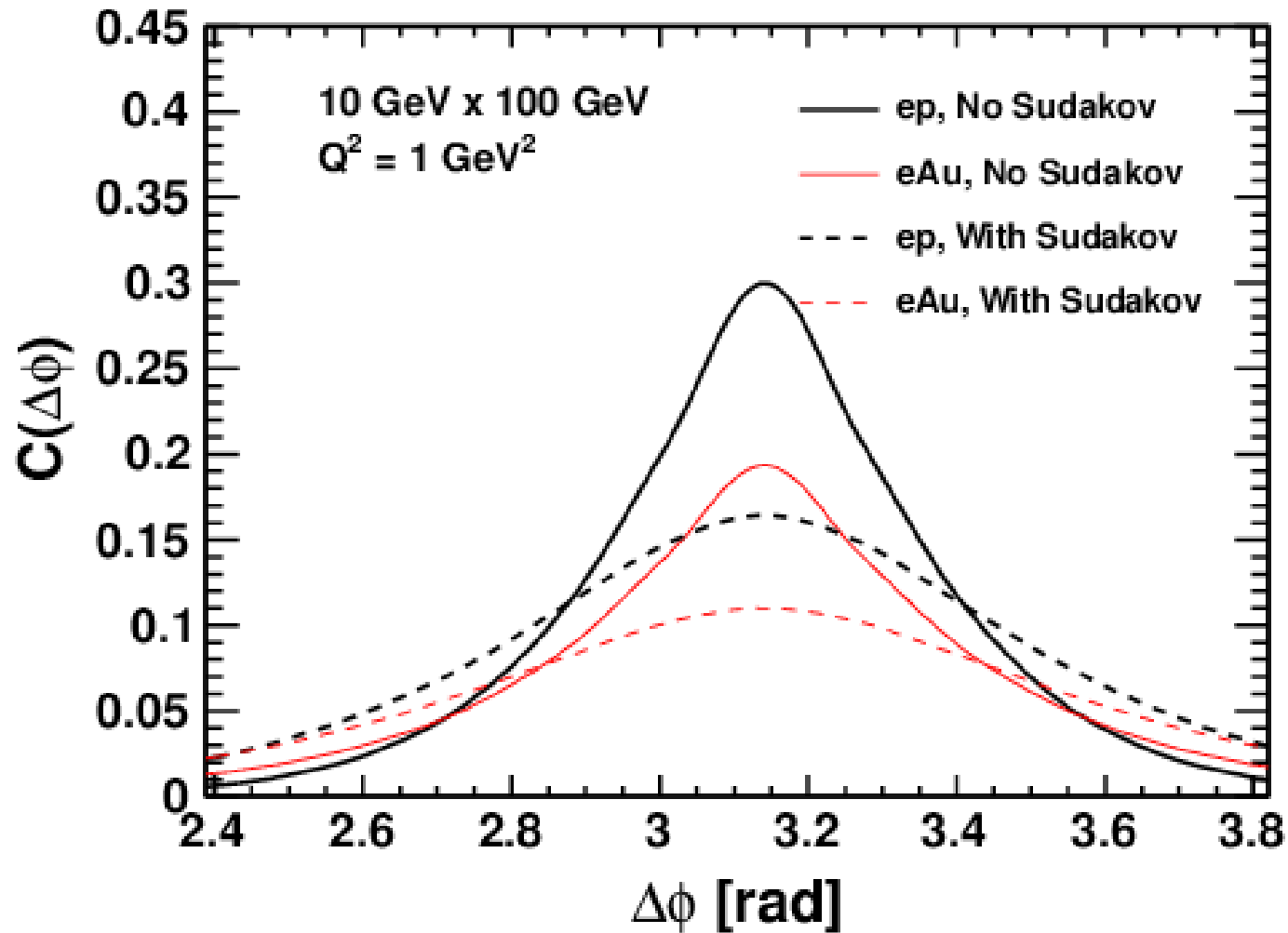
Di-hadron azimuthal correlations in DIS



Electron Ion Collider...., A. Accardi et al., arXiv:1212.1701

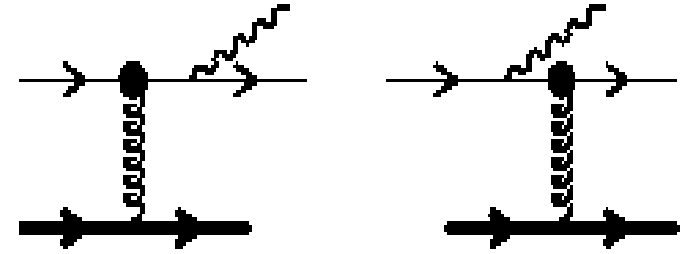
Zheng-Aschenauer-Lee-Xiao, PRD89 (2014)7, 074037

Azimuthal correlations in DIS



DY in pA

$$q T \rightarrow q \gamma^* X$$



$$\begin{aligned} M^\mu(\mathbf{p}; \mathbf{k}, \mathbf{q}) &= i \int d^2 \mathbf{x}_t e^{i(\mathbf{q}_t + \mathbf{k}_t - \mathbf{p}_t) \cdot \mathbf{x}_t} \bar{u}(\mathbf{q}) \bar{\Gamma}^\mu(\mathbf{k}; \mathbf{q}, \mathbf{p}) u(\mathbf{p}) [V(\mathbf{x}_t) - 1] \\ &= \frac{i}{2} \int \frac{d^2 l_t}{(2\pi)^2} d^2 x_t d^2 y_t e^{i l_t \cdot x_t} e^{i(q_t + k_t - p_t - l_t) \cdot y_t} \bar{u}(q) \Gamma^\mu(-k; q, -p) u(p) \\ &\quad [V(x_t) V^\dagger(y_t) - 1] \underbrace{V(y_t)} \end{aligned}$$

extra: unitary matrix

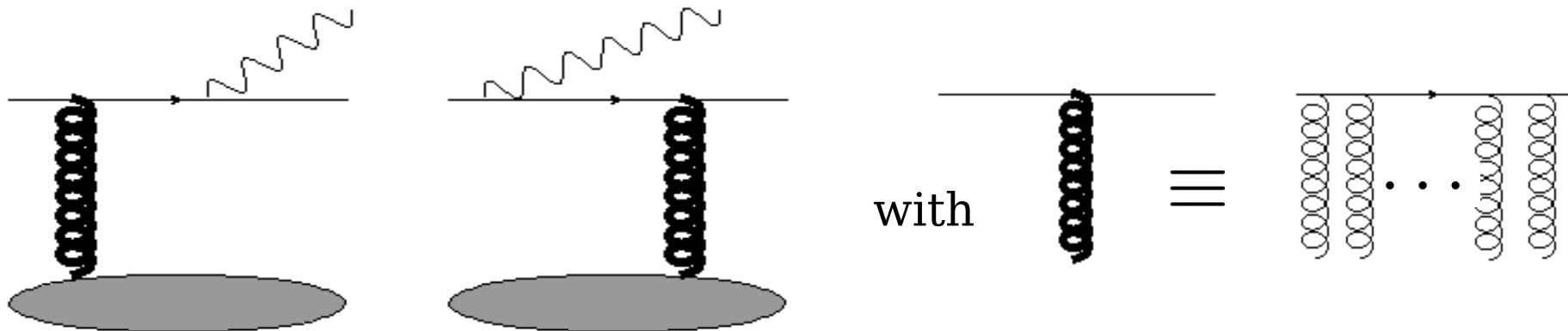
← same as DIS

cross section

$$\begin{aligned} \frac{d\sigma}{dz d^2 k_t d \log M^2 d^2 b_t} &= \frac{2\alpha_{em}^2}{3\pi} \int \frac{d^2 l_t}{(2\pi)^4} d^2 r_t e^{i l_t \cdot r_t} T(x_g, b_t, r_t) \left\{ \right. \\ &\quad \left[\frac{1 + (1-z)^2}{z} \right] \frac{z^2 l_t^2}{[k_t^2 + (1-z)M^2][(k_t - z l_t)^2 + (1-z)M^2]} \\ &\quad \left. - z(1-z)M^2 \left[\frac{1}{[k_t^2 + (1-z)M^2]} - \frac{1}{[(k_t - z l_t)^2 + (1-z)M^2]} \right]^2 \right\} \end{aligned}$$

Photon-hadron correlation in p(d)A

$$q(p) T \rightarrow q(l) \gamma(k) X$$



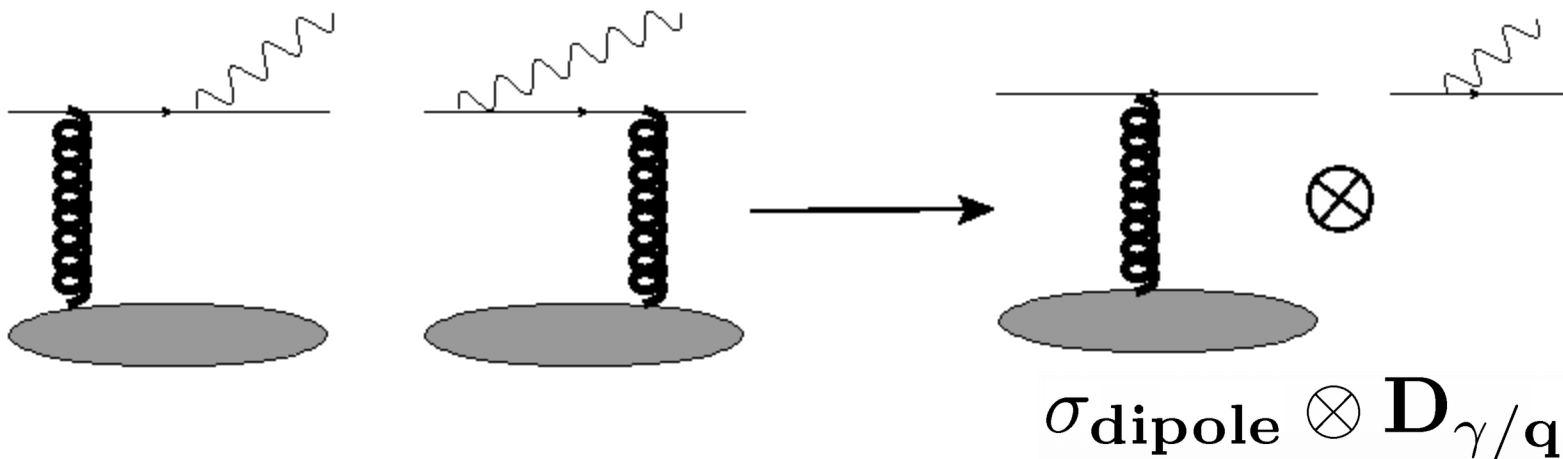
$$\frac{d\sigma^{q(p) T \rightarrow q(l) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dl_t^2 dy_\gamma dy_l d\theta} = \frac{e_q^2 \alpha_{em}}{\sqrt{2}(2\pi)^3} \frac{k^-}{k_t^2 \sqrt{S}} \frac{1 + \left(\frac{l^-}{p^-}\right)^2}{[k^- \vec{l}_t - l^- \vec{k}_t]^2}$$

$$\delta\left[x_q - \frac{l_t}{\sqrt{S}} e^{y_l} - \frac{k_t}{\sqrt{S}} e^{y_\gamma}\right] \left[2l^- k^- \vec{l}_t \cdot \vec{k}_t + k^- (p^- - k^-) l_t^2 + l^- (p^- - l^-) k_t^2\right]$$

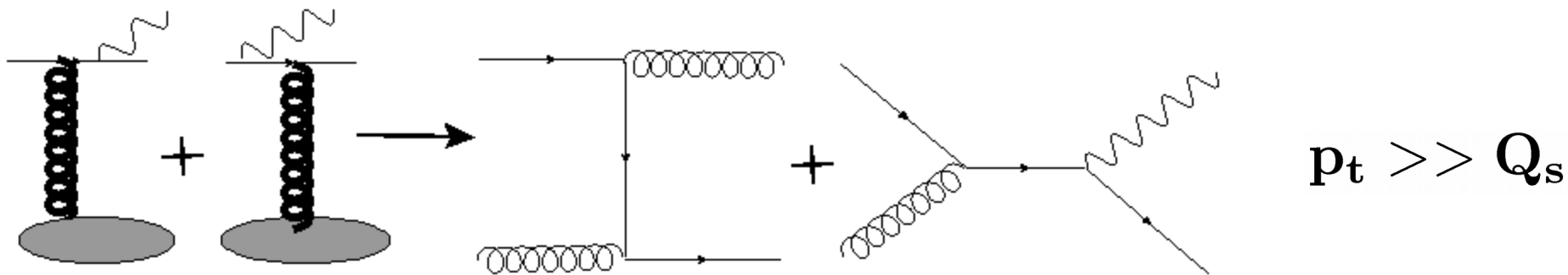
$$\int d^2\vec{r}_t e^{i(\vec{l}_t + \vec{k}_t) \cdot \vec{r}_t} N_F(b_t, r_t, x_g)$$

pQCD limits

near side: collinear divergence $\theta \rightarrow 0$



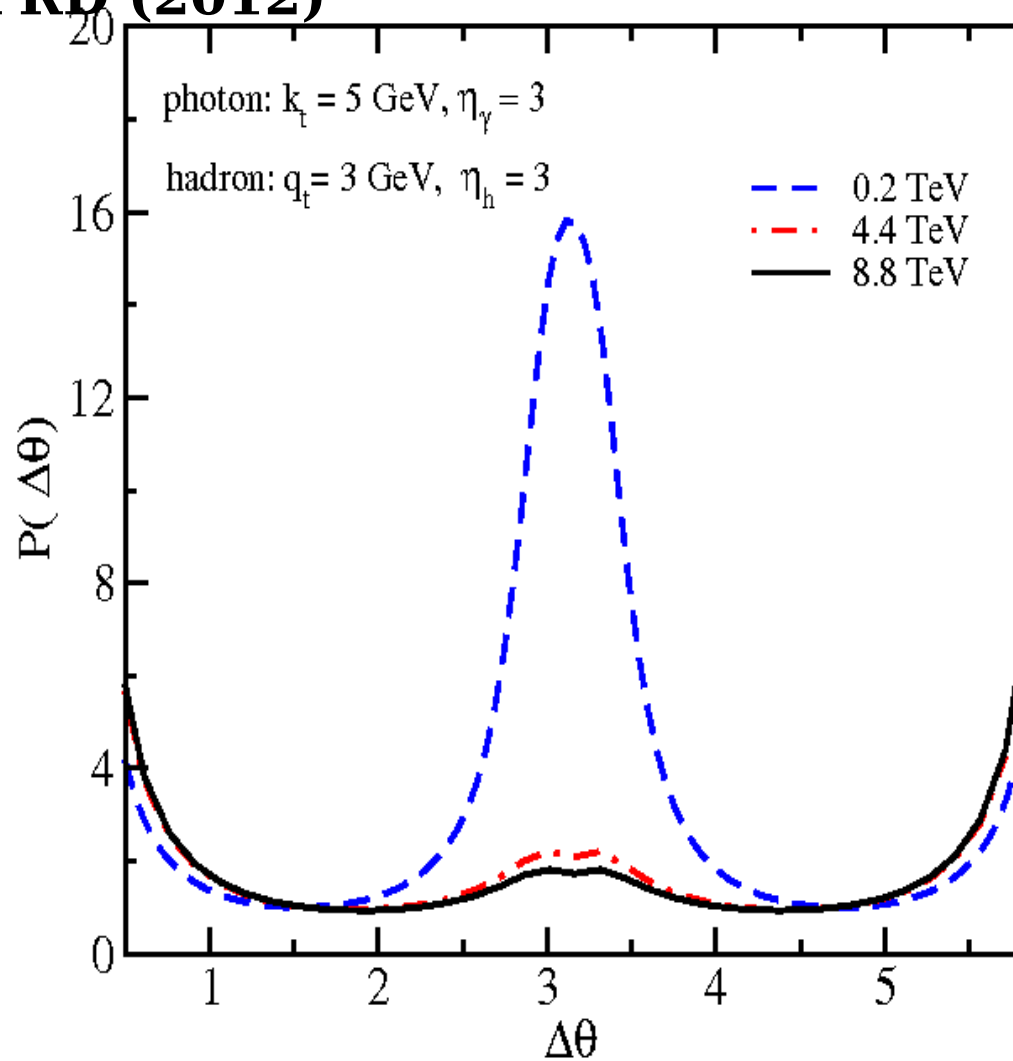
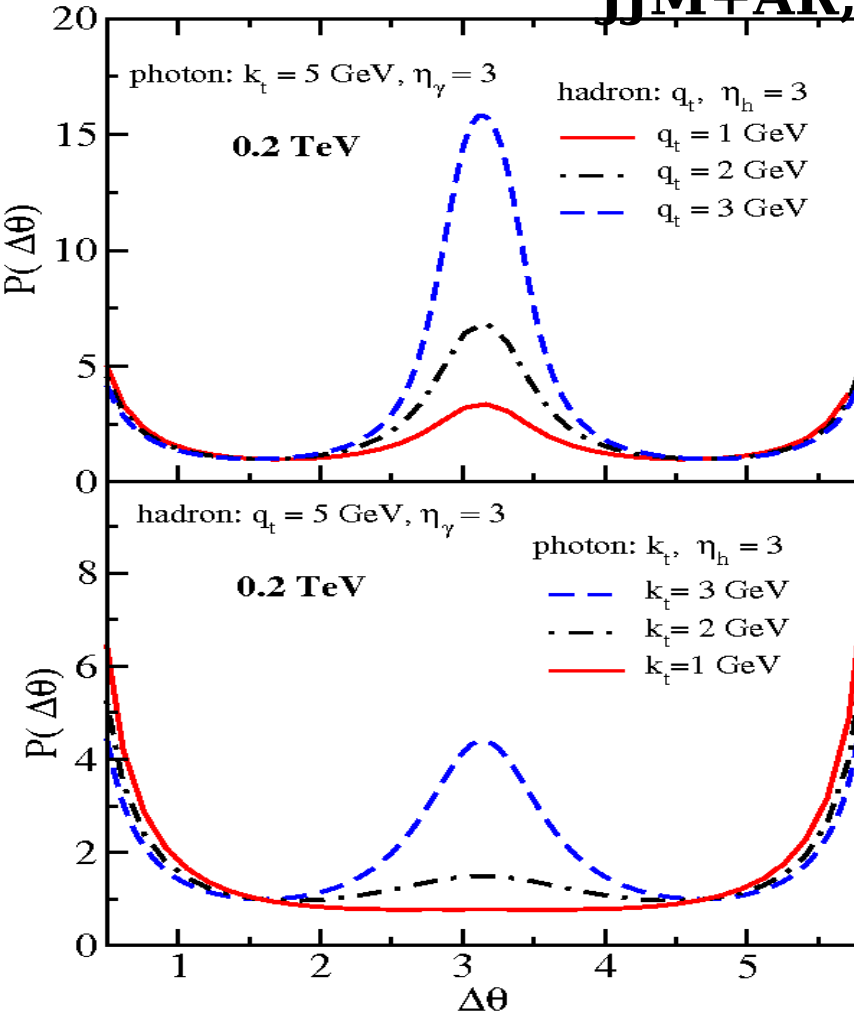
away side: $\theta \rightarrow \pi$



photon-hadron azimuthal correlations

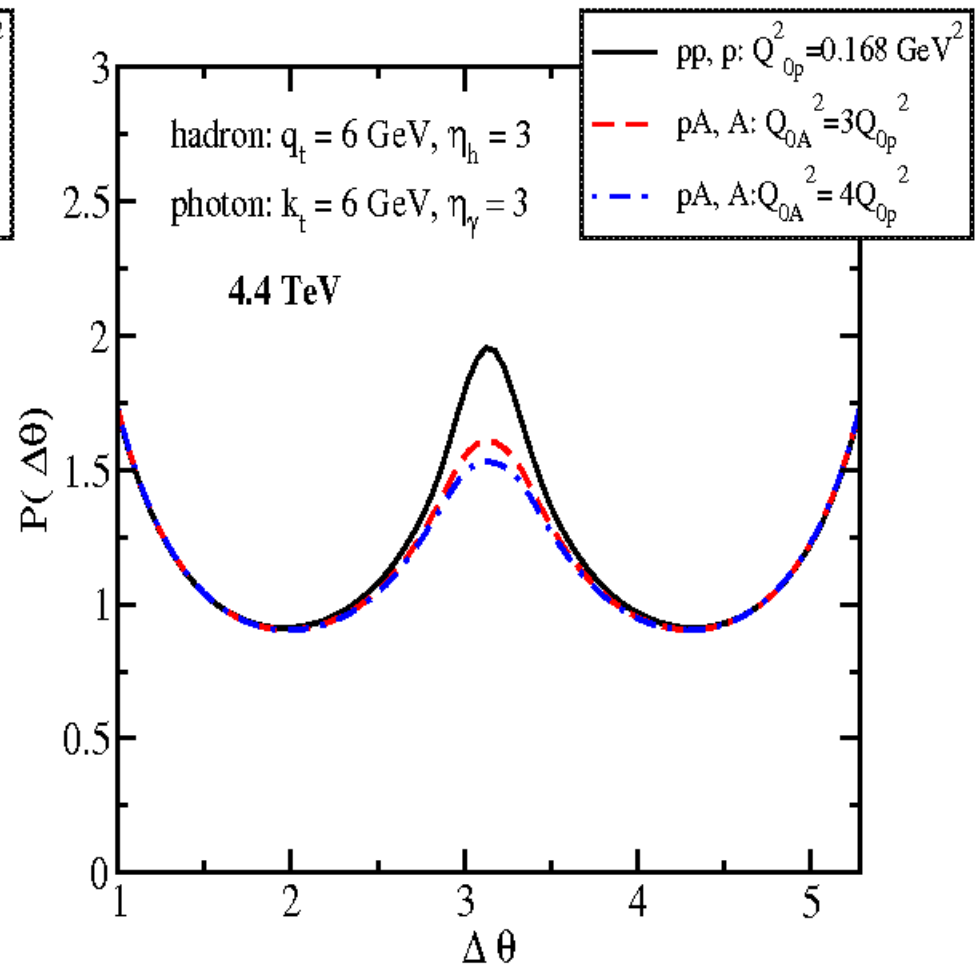
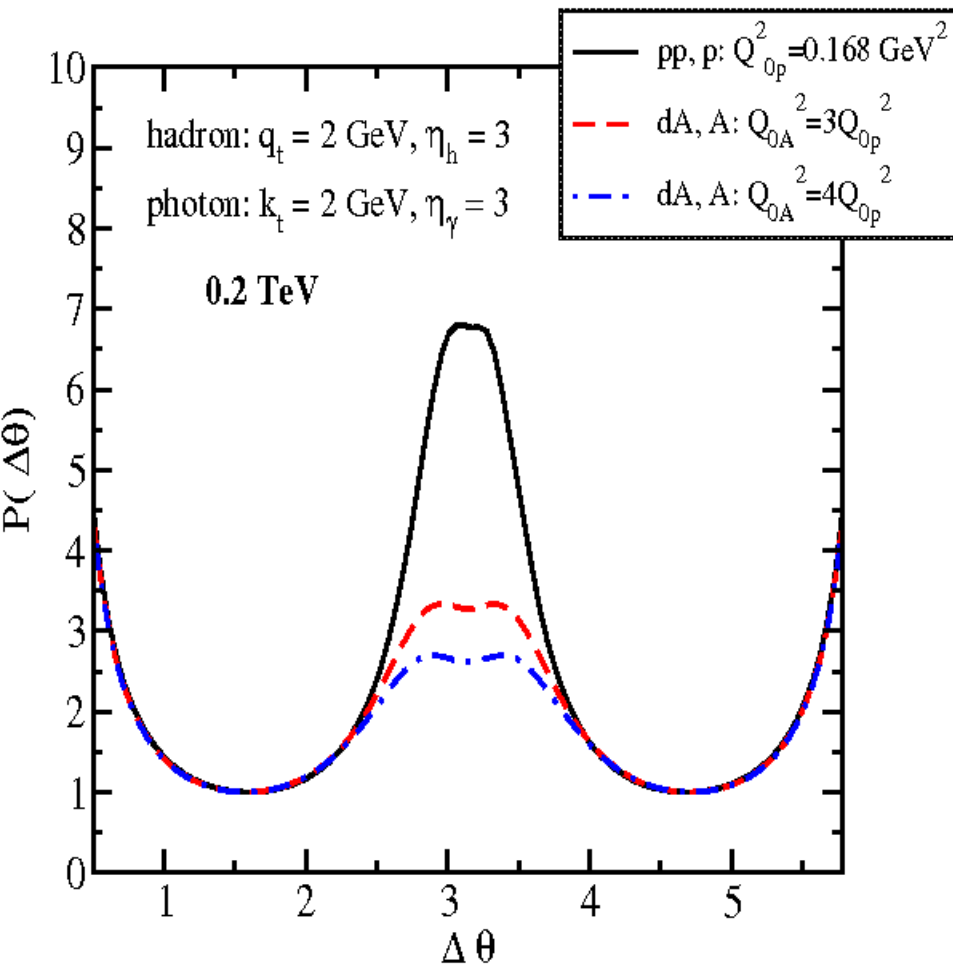
$$P(\Delta\theta) = \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\Delta\theta] / \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\theta = \theta_c]$$

JJM+AR, PRD (2012)



Centrality dependence

$$P(\Delta\theta) = \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\Delta\theta] / \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\theta = \theta_c]$$



Photon-hadron correlations

*new processes to probe the dynamics
of high energy QCD*

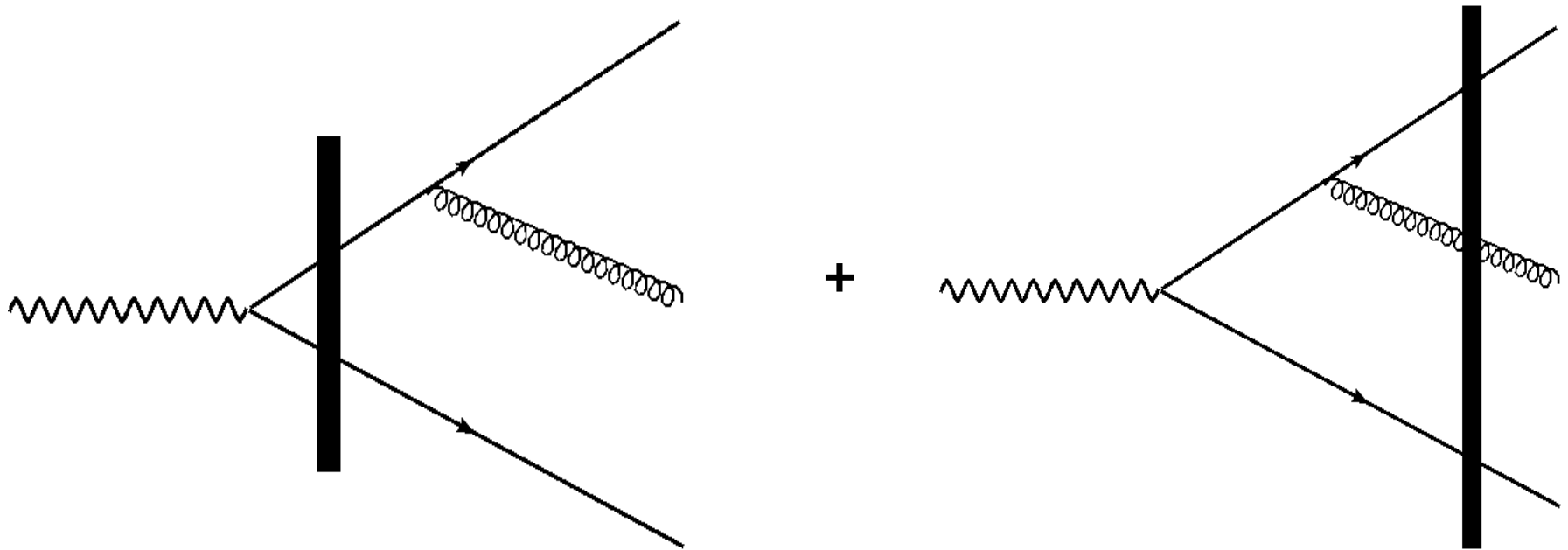
suppression of prompt photon
spectrum in forward rapidity in $p(d)A$

disappearance of the away side peak in
photon-hadron azimuthal correlations
in $p(d)A$

need to measure these at RHIC/LHC

3-parton production in DIS

$$\gamma^* \mathbf{T} \rightarrow q \bar{q} g \mathbf{X}$$



+ radiation from anti-quark

crossing symmetry

3-jet production in DIS

$$\gamma^{(*)} \longrightarrow q \bar{q} g \longleftrightarrow \left\{ \begin{array}{l} q \longrightarrow q g \gamma^{(*)} \\ \bar{q} \longrightarrow \bar{q} g \gamma^{(*)} \\ g \longrightarrow q \bar{q} \gamma^{(*)} \end{array} \right\}$$

2-jet + photon/DY production in pA collisions

(collinear factorization in proton?)

$$pA \longrightarrow h_1 h_2 \gamma^{(*)} X$$

MPI (in proton)

SUMMARY

Azimuthal angular correlations offer a unique probe of CGC

Di-hadrons, hadron-photon/dilepton, 3-hadrons,...

Forward-forward correlations probe small x

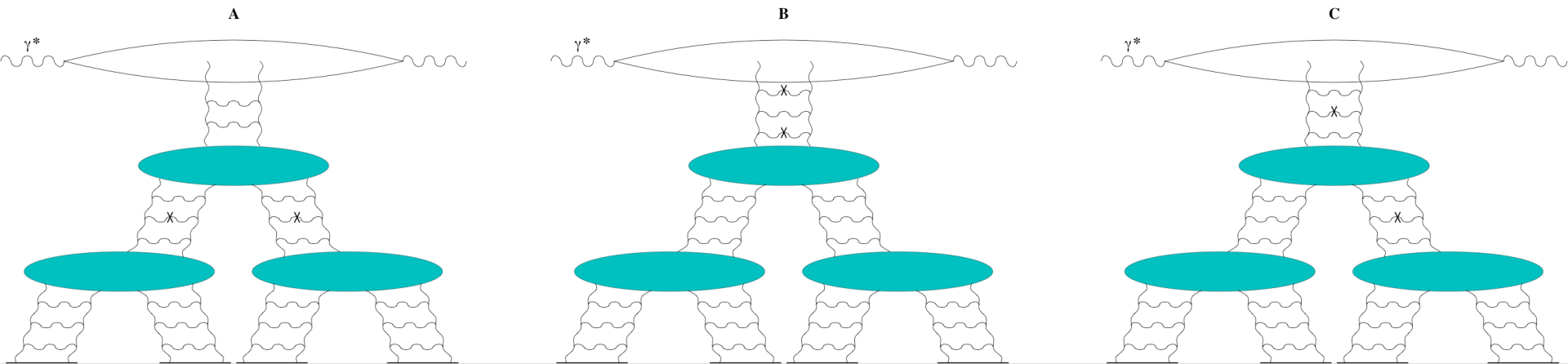
Vary nuclei, transverse momentum, rapidity,...

Azimuthal correlations in DIS

di-jet production in DIS: quadrupoles $\langle \text{Tr } \mathbf{V} \mathbf{V}^\dagger \mathbf{V} \mathbf{V}^\dagger \rangle$

LO: $\gamma^* \mathbf{T} \rightarrow \mathbf{g} \mathbf{g} \mathbf{X}$ JJM+ Y. Kovchegov, PRD (2004)

gluons widely separated in rapidity



spinor helicity methods

Review:
L. Dixon, hep-ph/9601359

massless quarks: helicity eigenstates

$$u_{\pm}(k) \equiv \frac{1}{2} (1 \pm \gamma_5) u(k)$$

$$\overline{u_{\pm}(k)} \equiv \overline{u(k)} \frac{1}{2} (1 \mp \gamma_5)$$

$$v_{\mp}(k) \equiv \frac{1}{2} (1 \pm \gamma_5) v(k)$$

$$\overline{v_{\mp}(k)} \equiv \overline{v(k)} \frac{1}{2} (1 \mp \gamma_5)$$

helicity operator $h \equiv \vec{\Sigma} \cdot \hat{p}$

$$\vec{\Sigma} \cdot \hat{p} U_{\pm}(p) = \pm U_{\pm}(p)$$

$$-\vec{\Sigma} \cdot \hat{p} V_{\pm}(p) = \pm V_{\pm}(p)$$

$$u_+(k) = v_-(k) = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \\ \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \end{bmatrix}$$

$$u_-(k) = v_+(k) = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{k^-} e^{-i\phi_k} \\ -\sqrt{k^+} \\ -\sqrt{k^-} e^{-i\phi_k} \\ \sqrt{k^+} \end{bmatrix}$$

with $e^{\pm i\phi_k} \equiv \frac{k_x \pm ik_y}{\sqrt{k^+ k^-}}$

and $k^{\pm} = \frac{E \pm k_z}{\sqrt{2}}$

spinor helicity methods

notation:

$$|i^\pm\rangle \equiv |k_i^\pm\rangle \equiv U_\pm(k_i) = V_{\mp}(k_i) \quad \langle i^\pm| \equiv \langle k_i^\pm| \equiv \bar{U}_\pm(k_i) = \bar{V}_\mp(k_i)$$

basic spinor products:

$$\begin{aligned} \langle ij \rangle &\equiv \langle i^- | j^+ \rangle = \bar{U}_-(k_i) U_+(k_j) = \sqrt{|s_{ij}|} e^{i\phi_{ij}} & \cos \phi_{ij} &= \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}| k_i^+ k_j^+}} \\ [ij] &\equiv \langle i^+ | j^- \rangle = \bar{U}_+(k_i) U_-(k_j) = -\sqrt{|s_{ij}|} e^{-i\phi_{ij}} & \sin \phi_{ij} &= \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}| k_i^+ k_j^+}} \end{aligned}$$

with

$$\begin{aligned} s_{ij} &= (k_i + k_j)^2 = 2k_i \cdot k_j \\ &= -\langle ij \rangle [ij] \end{aligned}$$

and

$$\begin{aligned} \langle ii \rangle &= [ii] = 0 \\ \langle ij \rangle &= [ij] = 0 \end{aligned}$$

any off-shell momentum $k^\mu \equiv \bar{k}^\mu + \frac{k^2}{2k^+} n^\mu$

where \bar{k}^μ is on-shell $\bar{k}^2 = 0$

any on-shell momentum $\not{p} = |p^+\rangle \langle p^+| + |p^-\rangle \langle p^-|$

work with a given helicity state