High Precision Nuclear Beta Spectroscopy

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Introduction

Main goal: Understand Standard Model & Go Beyond

Where to look for it? Weak Interaction!

How? Nuclear β decay, because it's

- Experimentally 'easy'
- Wealth of different transitions
- Many available observables

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Complex system $\rightarrow$ need accurate theoretical predictions from different areas of physics

Introduction

General Hamiltonian

$$\mathcal{H} = \sum_{j=V,A,S,P,T} \langle f| \mathcal{O}_j |i\rangle \langle e| \mathcal{O}_j [C_j + C_j^{'} \gamma_5] |\nu\rangle + h.c.$$

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In Standard Model only V-A $\rightarrow$ where are the others?

QCD influences $\rightarrow$ *induced* currents, what about nuclear structure?

Beta Spectrum Shape

Exploring the Standard Model and Beyond via the β spectrum shape:

$$\frac{dN}{dE_e} \propto 1 + b_{\mathrm{Fierz}} \gamma \frac{m_e}{E_e} + b_{WM} E_e$$

b_{Fierz}: Proportional to scalar (Fermi) and tensor (Gamow-Teller) couplings

b_{WM}: Weak Magnetism (main induced current), poorly known for $A > 60$, forbidden decays

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This requires knowledge of the theoretical spectrum shape to $\leq 10^{-3}$ level!

General description

General matrix element: Combination of $\mathcal{H}_\beta$ and Coulomb effects.

$$M_{fi} = -2\pi i \delta(E_f - E_i) \langle f | \, T \left[\exp\left(-i \int_0^\infty dt \mathcal{H}^Z(t) \right) \right] \times \mathcal{H}_\beta(0) T \left[\exp\left(-i \int_{-\infty}^0 dt \mathcal{H}^{Z'}(t) \right) \right] |i\rangle$$

Immediately two main parts:

1. Electromagnetic corrections
2. Nuclear & recoil corrections

Specifically...

Expanding slightly:

1. Electromagnetic corrections
   - Fermi function
   - Radiative corrections
   - Atomic effects
   - Molecular effects
2. Nuclear & recoil corrections
   - Finite nuclear size & mass
   - Nuclear structure

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Different formalisms: Behrens-Bühring, Holstein, Wilkinson, ...
$\rightarrow$ Problems with double counting, rigour, accessibility

Behrens-Bühring, Clarendon Press, Oxford, 1982; B. Holstein, RMP **46**, 789; D. Wilkinson, NIM A **335**, 172; etc..

Quick overview

Our Goal: Fully analytical description to 10^{-4} precision

Result:

$$\begin{aligned} N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3} & \; F_0(Z,W)\; L_0(Z,W)\; U(Z,W)\; R_N(W,W_0,M) \\ & \times \; Q(Z,W,M)\; R(W,W_0)\; S(Z,W)\; X(Z,W)\; r(Z,W) \\ & \times \; C(Z,W)\; pW(W_0-W)^2\; dW \\ \equiv \frac{G_V^2 V_{ud}^2}{2\pi^3} & \; K(Z,W,W_0,M)\; C(Z,W)\; pW(W_0-W)^2\; dW. \end{aligned}$$

Where K considers electromagnetic and kinematic effects, and C contains nuclear structure info.

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$$N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3} F_0(Z,W)\, L_0(Z,W)\, U(Z,W)\, R_N(W,W_0,M)$$
$$\times\; Q(Z,W,M)\, R(W,W_0)\, \boxed{S(Z,W)\, X(Z,W)\, r(Z,W)}$$
$$\times\; \boxed{C(Z,W)}\, pW(W_0-W)^2\, dW$$
$$\equiv \frac{G_V^2 V_{ud}^2}{2\pi^3} K(Z,W,W_0,M)\, C(Z,W)\, pW(W_0-W)^2\, dW$$

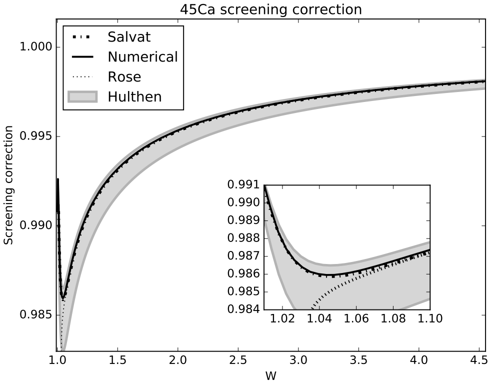
Corrections and improvements:

Atomic effects: Screening, exchange, atomic mismatch, shake-up & shake-off, molecular & chemical effects

Nuclear effects: Spatial variation of wave functions & nuclear structure

Atomic Screening Corrections

Final state interactions with atomic electrons.
Powerful analytical treatment by Bühring, coupled with accurate atomic potential



Greatly reduced theoretical uncertainty!

W. Bühring, Nucl. Phys. A **430**, 1 (1984) & F. Salvat *et al.*, PRA **36**, 467 (1987)

Atomic exchange

Exchange: Probability of decaying into bound state with emission of bound e^-

$$X(E) = 1 + \sum_n \eta_{ex}^{ns}(E)$$

where

$$\eta_{ex}^{ns}(E) \propto \langle Es' | ns \rangle$$

spatial overlap between continuum and bound wave functions

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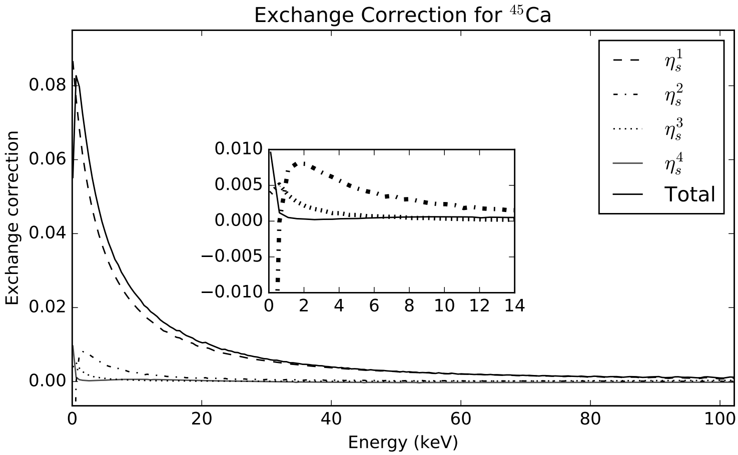
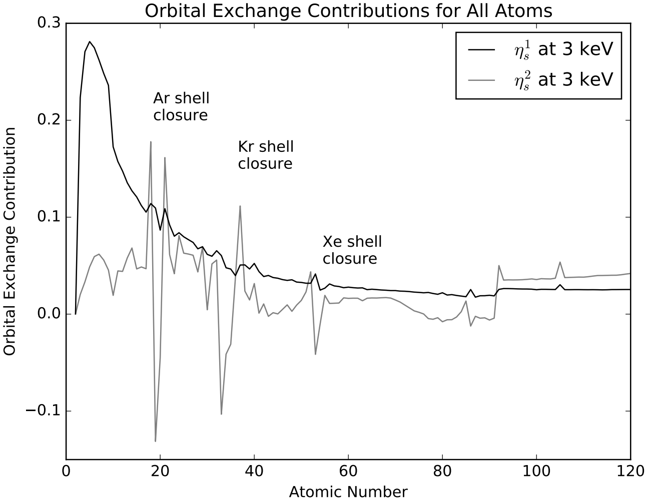
Need accurate wave functions for arbitrary potentials over the entire space $\rightarrow$ numerical!

Atomic Exchange



Atomic Exchange

Contributions from different orbitals → sensitive to *atomic* physics!



Nuclear Structure & Convolution

Combination of two effects:

- Spatial variation of the leptonic wave functions
- Nuclear structure and induced currents (weak magnetism)

and we write

$$C(Z,E) \equiv {}^{NS}C(Z,E)^{LC}C(Z,E)$$

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and we write

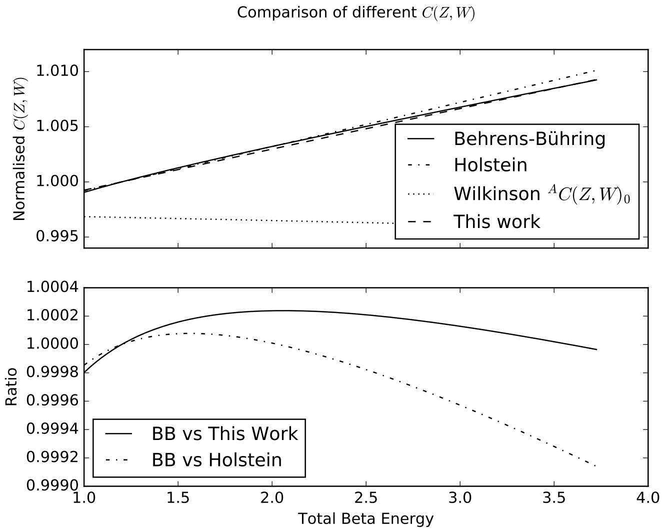
$$C(Z,E) \equiv {}^{NS}C(Z,E)^{LC}C(Z,E)$$

Different formalisms, different strengths $\rightarrow$ rigorous connection combining a 'best of':

- Rigorous treatment of lepton wave functions
- Nuclear structure in single ratio b/Ac of matrix elements (b weak magnetism; c Gamow-Teller)

e.g. B. R. Holstein, RMP **46**, 789 (1974) & H. Behrens and W. Bühring, Clarendon Press, Oxford (1982)

Nuclear Structure & Convolution



Included Corrections

Item	Effect	Formula
1	Phase space factor	$pW(W_0 - W)^2$
2	Neutrino mass	Negligible
3	Forbidden decays	Not incorporated
4	Traditional Fermi function	F_0
5	Finite size of the nucleus	L_0
6	Diffuse nuclear surface	U
7	Recoiling nucleus	R_N
8	Distorted Coulomb potential due to recoil	Q
9	Radiative corrections	R
10	Atomic screening	S
11	Atomic exchange	X
12	Shake-Up	See item 14
13	Shake-Off	See item 14 & χ_{ex}^{cont}
14	Atomic mismatch	r
15	Bound state β decay	Γ_b/Γ_c
16	Molecular screening	ΔS_{Mol}
17	Molecular exchange	Case by case
18	Shape factor	C

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L. Hayen *et al.*, Invited for submission to Rev. Mod. Phys. (Soon on arXiv)

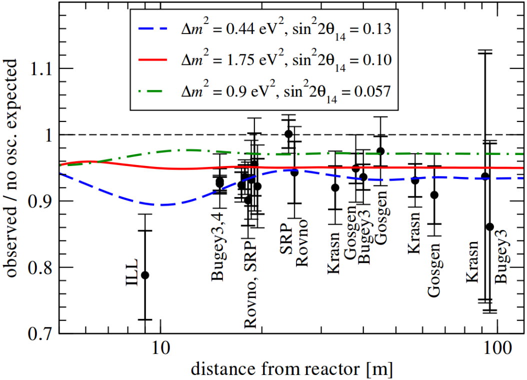
Beta Spectrum Generator

All corrections are implemented in C++ generator program. (Still come up with nice acronym)

Will be freely available for use and download, together with custom event generator CRADLE++

L. Hayen *et al.*, technical paper to be published.

Intermezzo: Reactor Antineutrino Anomaly

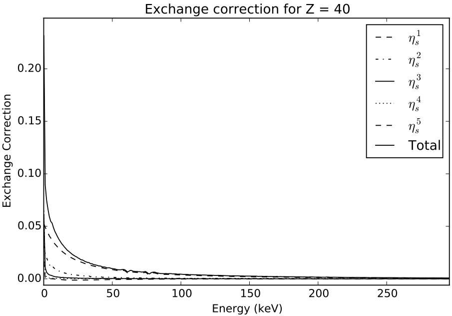


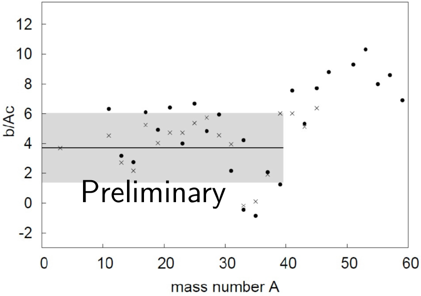
Influence from all β spectrum corrections $\rightarrow$ understanding of atomic corrections & weak magnetism is crucial!

A. Hayes et al., PRL 112, 202501 (2014) & J. Kopp et al., JHEP 05 (2013) 050

Corrections to RNA Analysis

Several influences currently insufficiently/not included in analysis!



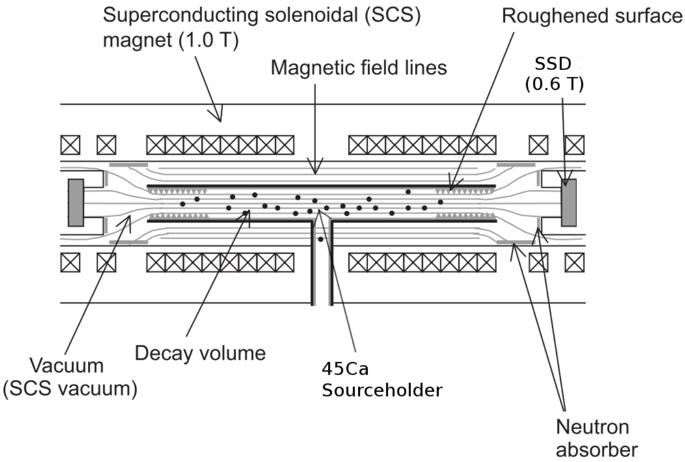


Low energy β corrections $\rightarrow$ high energy $\bar{\nu}_e$!

Large variations for $A < 60$ $\rightarrow$ uncertain extrapolation, look at DFT

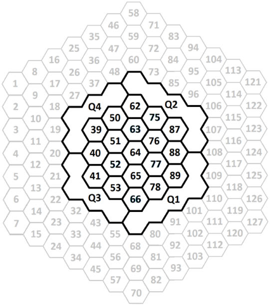
^{45}Ca Runs @ LANL (May 2016)

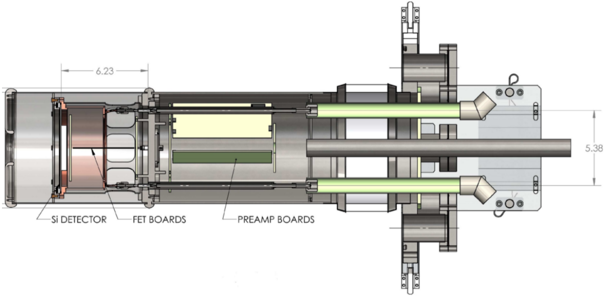
Use UCNA set-up, put Segmented Si Detector (SSD) instead



Collaboration with A. Young and parts of UCNB & Nab groups

Si Detector



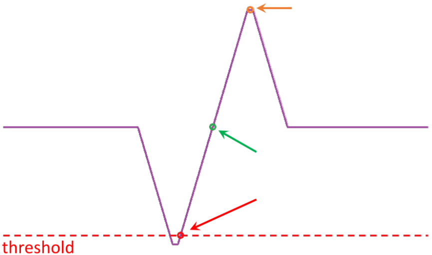


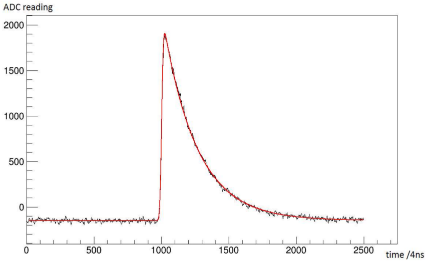
A lot of effort into performance, 3 keV FHWM & 7 keV threshold, 4 ns timing resolution

Waveform analysis

Waveform trace

1. Online using double trapezoid filter $\rightarrow$ timestamp + energy
2. 10 μs trace for offline analysis (52 kB)





Courtesy of A. Sprow

e.g. Jordanov *et al.*, NIM A **353**, 261; A. Sprow, To be published

Systematics

Most important systematics

- Backscattering
- Pile-up
- Missed events
- Magnetic reflection
- Foil losses

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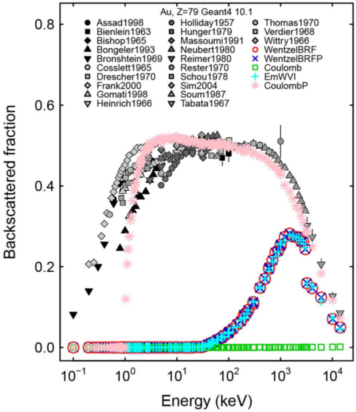
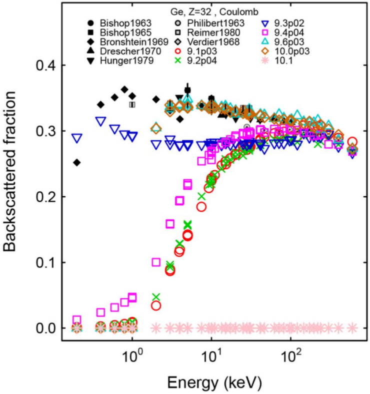
Require complete Geant4 simulation, however...

Geant4 Multiple Scattering

Geant4 Multiple Coulomb Scattering has not been performing in a stable way, nor correct



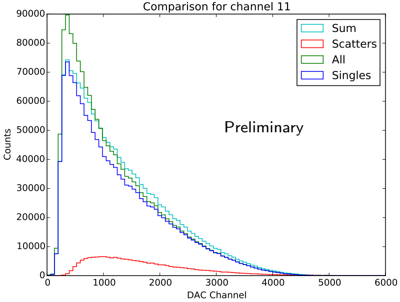


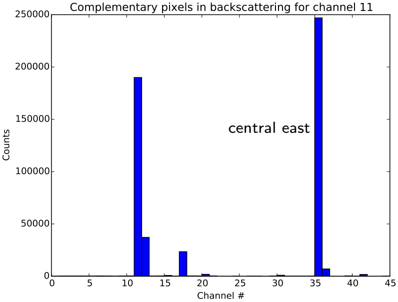
S. Kim *et al.*, IEEE Trans. Nucl. Sc. **62**, 451

Dealing with scattering

Geant4 uncertainty of MCS is significant: deal with it or lose!

Recover backscattered events in offline analysis





Central pixel on ‘west’ detector

Dealing with scattering

Combination of geometry & DAQ allows independent of study of MCS:

Get individual and summed energies, AND:

1. Initial pixel hit → approximate $E_{\parallel}^{i}$ & θ^{i}
2. Time difference → approximate $E_{\parallel}^{1}$ & θ_{out}^{1} & θ_{in}^{2}

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2. Time difference $\rightarrow$ approximate $E_{\parallel}^{1}$ & θ_{out}^{1} & θ_{in}^{2}

Several calibration sources + 1/4 T field $\rightarrow$ larger pixel spread, more precise step 1

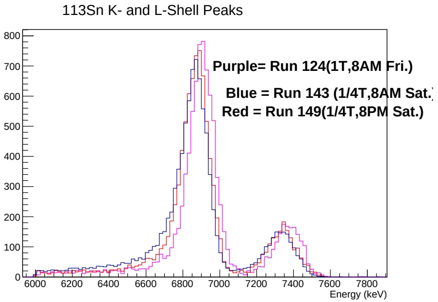
Statistics and stability

Statistics:

- 1h run: $\approx 5 \times 10^6$ events
- In total 2 weeks of data, collected about 5×10^8 at 1T, same for 1/4T

Stability:

- Calibration runs every 8 hours
- ^{139}Ce, ^{113}Sn, ^{133}Ba, ^{207}Bi
- $>$ 15 calibration peaks, $\lesssim 10^{-3}$ linearity
- Average gain drifts of $\sim 0.04\%/h$



Current status

Analysis has started, working on comparison of waveform and online data

Custom GPU code to quickly analyze waveforms near completion

Complete Geant4 simulation up and running

Currently investigating

- Charge sharing
- Cross-talk
- Gain stability
- ...

Aim is to obtain $\sim 10^{-3}$ b_{Fierz}, ^{45}Ca is feasibility case (Later e.g. ^{32}P (1.7 MeV))

Conclusion & Outlook

We have constructed a fully analytical β spectrum, combining

- Atomic & molecular corrections
- Electromagnetic & kinematical corrections
- Nuclear structure effects in a 'best of' way

for the first time, and accurate to a few 10^{-4}. Will be published soon(ish).
C++ code will be made freely available, to be published.

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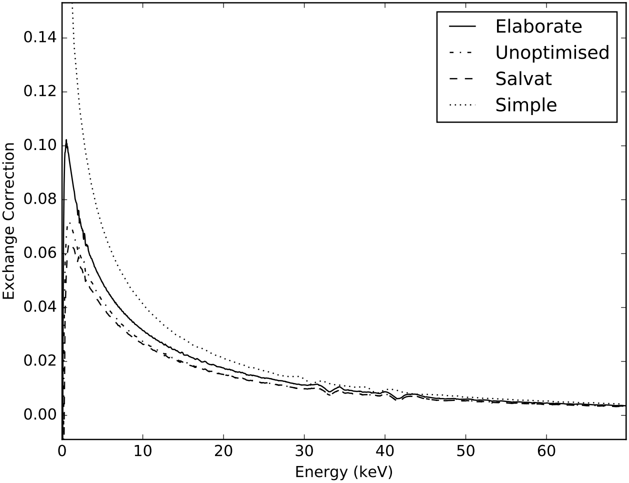
Effects of β spectrum shape on Reactor Neutrino Anomaly are being looked into, looks promising

Numerical calculations are non-trivial, require careful optimisation and not 'user-friendly'



Atomic Screening Correction

Formula used by X. Mougeot based upon exchange results by Pyper & Harston

$$S(E) = \frac{\int f_c^2(r)\ dr d\Omega}{\int (f_c^2(r) + g_c^2(r))\ dr d\Omega} \tag{1}$$

Important: Spatial average $\Leftrightarrow$ amplitude at origin

Not theoretically sound! Currently working on *ab initio* approach based on Behrens & Bühring approach

Radiative corrections

Seminal work by Sirlin, Zucchini & Marciano. Split into *inner* (nucleus-independent) and *outer* (nucleus-dependent). Focus on *outer*:

$$R(W, W_0) = 1 + \delta_1 + \delta_2 + \delta_3$$

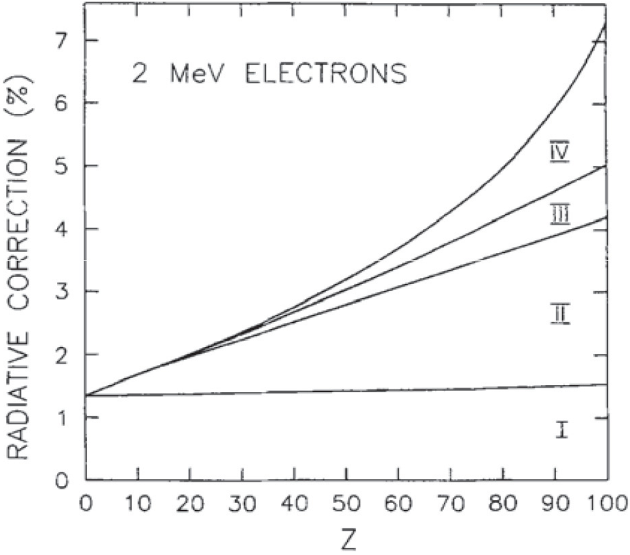
Higher order corrections estimate

$$\delta_{higher} \approx \sum_{n=3}^{n=\infty} \delta_{Z^n\alpha^{n+1}} = \delta_{Z^3\alpha^4}/(1 - Z\alpha)\ .$$

D. H. Wilkinson, Nucl. Instr. & Methods A **365**, 497 (1995) & **401**, 275 (1997)

I. S. Towner and J. C. Hardy, Phys. Rev. C **77**, 012501 (2014)

Radiative Corrections



Atomic Screening Corrections

Final state interactions with atomic electrons.

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Final state interactions with atomic electrons.

Change free lepton spinors by Dirac spinors in Coulomb field in matrix element $\mathcal{H}_\beta$

$$\int d^3r\ \bar{\Psi}_e(\mathbf{r},\mathbf{p})\gamma_\mu(1+\gamma_5)v(l)\int \frac{d^3k}{2\pi^3}e^{i\mathbf{r}\cdot\mathbf{k}}\langle f|\ V^\mu + A^\mu\ |i\rangle$$

Ψ_e not analytically solvable for anything but pure Coulomb $\frac{\alpha Z}{r}$

Effective Lagrangian for β decay

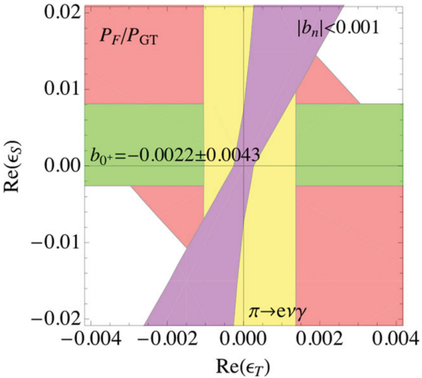
$$
\begin{aligned}
\mathscr{L}_{\text{eff}} = & -\frac{G_F V_{ud}}{\sqrt{2}} \left[1 + \text{Re}\left(\epsilon_L + \epsilon_R\right)\right] \\
& \times \left\{ \bar{e}\gamma_\mu (1-\gamma_5)\nu_e \cdot \bar{u}\gamma^\mu \left[1 - (1-2\epsilon_R)\gamma_5\right] d \right. \\
& + \epsilon_S \; \bar{e}(1-\gamma_5)\nu_e \cdot \bar{u}d \\
& \left. + \epsilon_T \; \bar{e}\sigma_{\mu\nu}(1-\gamma_5)\nu_e \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma_5) d \right\} + \text{h.c.},
\end{aligned}
$$

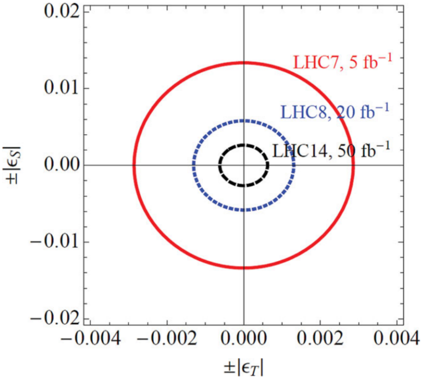
Neglecting pseudoscalar contributions + right-handed neutrino's

T. Bhattacharya et al., PRD 85, 054512 (2012)

V. Cirigliano et al., JHEP 1302, 046 (2013)

Current limits on Exotic Currents





Low and High energy physics are **competitive**

O. Naviliat-Cuncic and M. Gonzalez-Alonso, Ann. Phys. 525 (2013) 600

Fermi function

Influence on the spectrum because of Coulomb field of daughter nucleus. Generally

$$F(Z,W) = \lim_{r\to 0} \frac{f_1^2(r) + g_{-1}^2(r)}{2p^2} \ .$$

with

$$\Psi_\kappa(\hat{r}) = \begin{pmatrix} g_\kappa(r) \sum_\mu \chi_\kappa^\mu \\ if_\kappa(r) \sum_\mu \chi_{-\kappa}^\mu \end{pmatrix}$$

solution of the radial Dirac equation with some Coulomb field.

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solution of the radial Dirac equation with some Coulomb field.

Problem: Only solvable for point charge $\rightarrow$ split into

$$F(Z, W) = F_0 L_0$$

with L_0 calculated numerically (tabulated by Wilkinson).

D. H. Wilkinson, Nucl. Instr. & Meth. A **335**, 203 (1990)

Atomic Screening Corrections

Look at modification of Fermi function, only **local** change.

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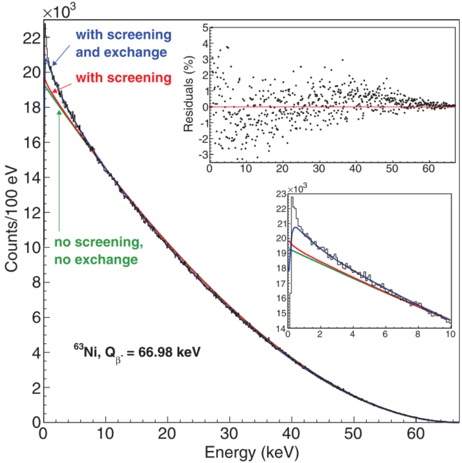
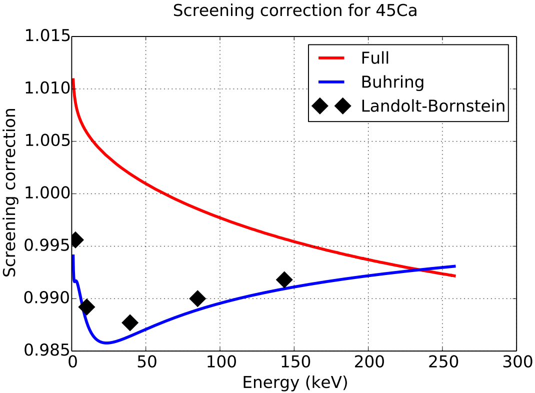
Analytically by Behrens & Bühring, however experimental disagreement?



Atomic Screening Corrections

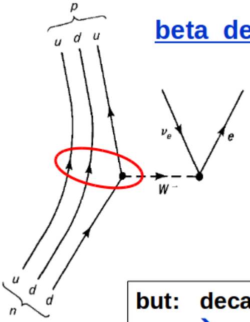
Influence of atomic screening on β spectrum shape via Fermi function



'Full': X. Mougeot and C. Bisch, PRA 90, 012501 (2014)

'Buhring': W. Bühring, Nucl. Phys. A 430 (1984) 1-20

'Landolt-Börnstein': Behrens and Jänecke, Landolt-Börnstein Tables, Springer (1969)



beta decay: $H = G_F \langle \psi_f | V_\mu(0) + A_\mu(0) | \psi_i \rangle l^\mu$

with $l^\mu = \bar{e}(p)\gamma^\mu(1+\gamma_5)v(k)$

free quark:

$$V_\mu(q^2) = \bar{u}[g_V(q^2)\gamma_\mu]d \,,$$
$$A_\mu(q^2) = \bar{u}[g_A(q^2)\gamma_\mu\gamma_5]d$$

but: decaying quark is not free but bound in a nucleon
→ extra terms induced by strong interaction

neutron decay:

weak magnetism

$$V_\mu(q^2) = \bar{p}[g_V(q^2)\gamma_\mu + \boxed{g_M(q^2)\sigma_{\mu\nu}\frac{q_\nu}{2M}} + ig_S(q^2)\frac{q_\mu}{m_e}]n$$

$$A_\mu(q^2) = \bar{p}[g_A(q^2)\gamma_\mu\gamma_5 + g_T(q^2)\sigma_{\mu\nu}\gamma_5\frac{q_\nu}{2M} + ig_P(q^2)\frac{q_\mu}{m_e}\gamma_5]n$$

Weak Magnetism: β decay

nuclear β decay:

form factor	formula Imp. App.
Vector type	
a	$a \cong g_V M_F$
e	$e \cong g_V (M_F \pm A g_S)$
b	$b \cong A(g_M M_{GT} + g_V M_L)$
f	$f \cong g_V \sqrt{\frac{2}{3}} M \frac{\Delta}{\hbar c^2} M_Q$
g	$g \cong -\frac{4}{3} M^2 g_V \frac{M_Q}{\hbar c^2}$
Axial vector type	
c	$c \cong g_A M_{GT}$
d	$d \cong A(g_A M_{\sigma L} \pm g_{II} M_{GT})$
h	$h \cong \frac{-2}{\sqrt{10}} M^2 g_A \frac{M_{1y}}{\hbar c^2} - A^2 g_P M_{GT}$
j_2	$j_2 \cong \frac{-2}{3} M^2 g_A M_{2y}$
j_3	$j_3 \cong \frac{-2}{3} M^2 g_A M_{3y}$

Matrix element	Operator form
M_F	$\langle \beta \| \Sigma \tau_i^{\pm} \| \alpha \rangle$
M_{GT}	$\langle \beta \| \Sigma \tau_i^{\pm} \vec{\sigma}_i \| \alpha \rangle$
M_L	$\langle \beta \| \Sigma \tau_i^{\pm} \vec{l}_i \| \alpha \rangle$
$M_{\sigma r^2}$	$\langle \beta \| \Sigma \tau_i^{\pm} \vec{\sigma}_i r_i^2 \| \alpha \rangle$
$M_{\sigma L}$	$\langle \beta \| \Sigma \tau_i^{\pm} i \vec{\sigma}_i \times \vec{l}_i \| \alpha \rangle$
M_Q	$\left(\frac{4\pi}{5}\right)^{\frac{1}{2}} \langle \beta \| \Sigma \tau_i^{\pm} r_i^2 Y_2(\hat{r}_i) \| \alpha \rangle$
M_{ky}	$\left(\frac{16\pi}{5}\right)^{\frac{1}{2}} \langle \beta \| \Sigma \tau_i^{\pm} \sigma_i^2 C_{12k}^{nn'k} \sigma_{in} Y_2^{n'}(\hat{r}_i) \| \alpha \rangle$

B. R. Holstein, Rev. Mod. Phys. 46 (1974) 789

F.P. Calaprice et al., Phys. Rev. C 15 (1977) 2178

Weak Magnetism: β decay

for a pure GT transition,
and neglecting terms ∝ 1/M² and ∝ m_e^2/E :

$$H_0(E) = c^2 - \frac{2}{3}\frac{E_0}{M}c(c+d\pm b) + \frac{2}{3}\frac{E}{M}c(5c\pm 2b)$$

→ $$H_0(E) = f_1 + f_2 E$$

→ $$S(E) \equiv \frac{H_0(E)}{H_0(E=0)}$$

$$S(E) \approx 1 + \frac{2}{3M}\left(5 \pm 2\frac{b}{c}\right)E_e$$

Bhattacharya formalism

The full correction goes like

$$\frac{dN}{dt} \propto 1 + c_0 + c_1 \frac{E_e}{M_N} + \frac{m_e}{E_e} \bar{b}$$

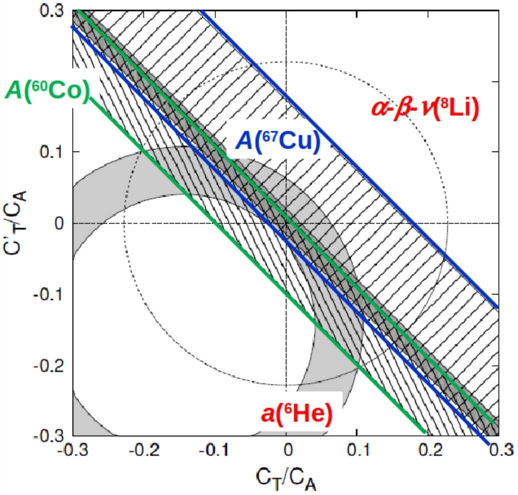
where

$$c_0 = -\frac{2\lambda(\lambda + \mu_V)}{1 + 3\lambda^2} \frac{E_0}{M_N}$$

$$c_1 = \frac{3 + 4\lambda\mu_V + 9\lambda^2}{1 + 3\lambda^2}$$

$$b = -\frac{m_e}{M_N} \frac{1 + 2\mu_V\lambda + \lambda^2}{1 + 3\lambda^2}$$

Tensor constraints



***a*(^{6}He)**
C. Johnston et al.,
PR 132 (1963) 1149

***A*(^{60}Co)**
F. Wauters, N.S. et al.,
PR C 82 (2010) 055502

***α-β-ν*(^{8}Li)**
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PRL 110 (2013) 082502

***A*(^{67}Cu)**
G. Soti, N.S. et al., (2013) submitted

black band: P_F / P_{GT}
A.S. Carnoy et al.
PR C 43 (1991) 2825

Atomic Screening