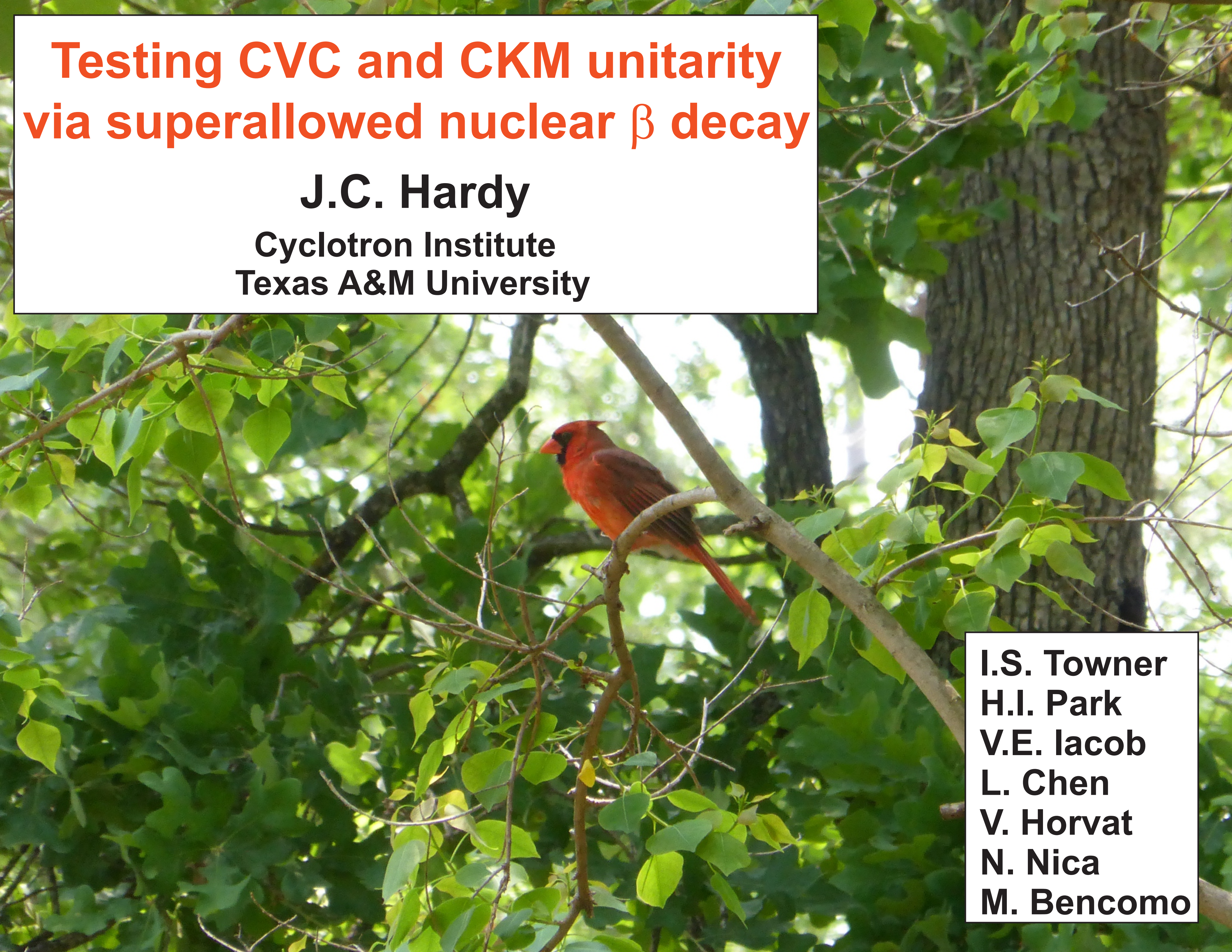
Testing CVC and CKM unitarity via superallowed nuclear β decay

J.C. Hardy

Cyclotron Institute
Texas A&M University



I.S. Towner
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L. Chen
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N. Nica
M. Bencomo

SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

BASIC WEAK-DECAY EQUATION

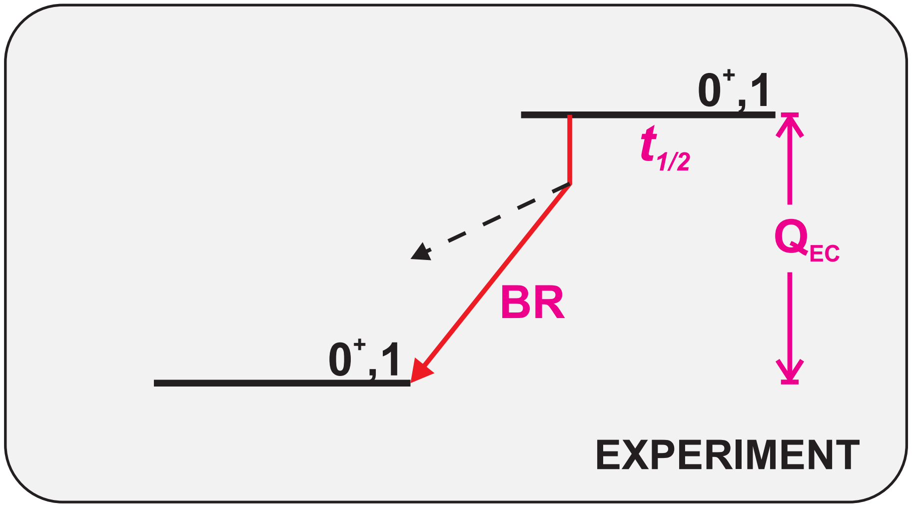
$$ft = \frac{K}{G_V^2 \langle \tau \rangle^2}$$

f = statistical rate function: $f(Z, Q_{EC})$

t = partial half-life: $f(t_{1/2}, BR)$

G_V = vector coupling constant

$\langle \tau \rangle$ = Fermi matrix element



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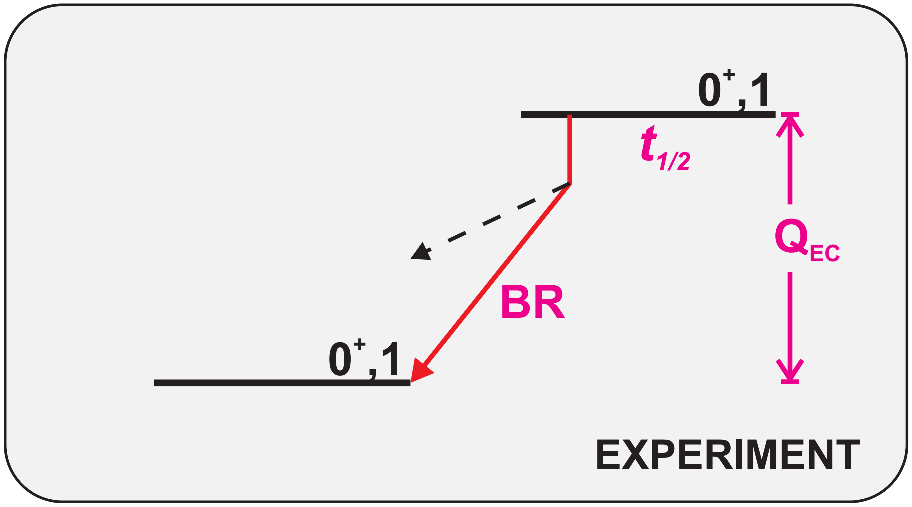

INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS

$$\mathcal{F}t = ft\,(1 + \delta_R')\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

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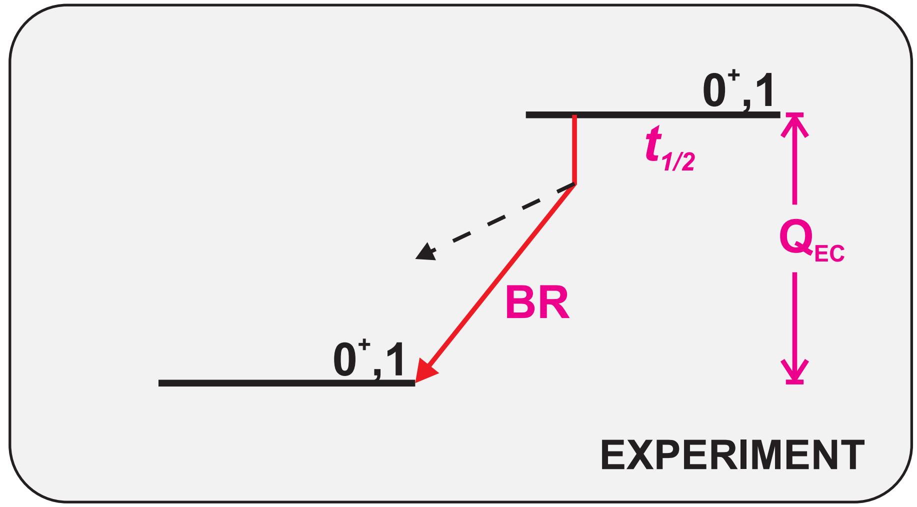

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$f(Z, Q_{EC})$
~1.5%

f(nuclear structure)
0.3-1.5%

f(interaction)
~2.4%

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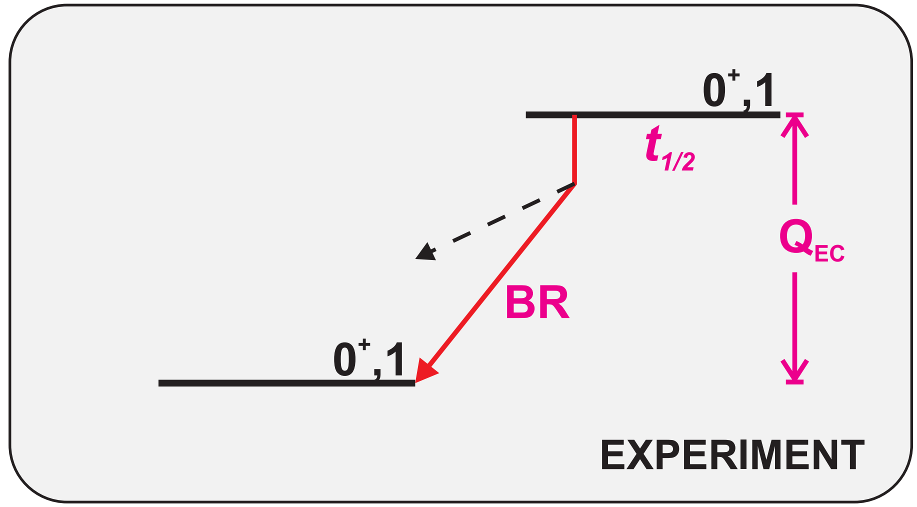

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$$\mathcal{F}t = ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

δ'_R: $f(Z, Q_{EC})$ ~1.5%

$\delta_C - \delta_{NS}$: f(nuclear structure) 0.3-1.5%

Δ_R: f(interaction) ~2.4%

THEORETICAL UNCERTAINTIES

0.05 – 0.10%

WHAT CAN WE LEARN?

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FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta'_R)\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

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$$\mathcal{F}t = ft\,(1 + {}'_R)[1 - ({}_C - {}_{NS})] = \frac{K}{2G_V^2\,(1 + {}_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate the correction terms

Test for presence of a Scalar current

$\mathcal{F}t$ values constant

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WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates — Cabibbo Kobayashi Maskawa (CKM) matrix — mass eigenstates

Obtain precise value of $G_V^2(1 + \ _R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G^{\ 2}$$

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Obtain precise value of $G_V^2(1 + {}_R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G^2$$

Test CKM unitarity

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

WHAT CAN WE LEARN?

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Experimentally determine $G_V^2(1 + {}_R)$

$$\mathcal{F}t = ft\,(1 + {}'_R)[1 - ({}_C - {}_{NS})] = \frac{K}{2G_V^2\,(1 + {}_R)}$$

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weak eigenstates

Cabibbo Kobayashi Maskawa (CKM) matrix

mass eigenstates

Obtain precis... (1 + ${}_R$)

Determin...

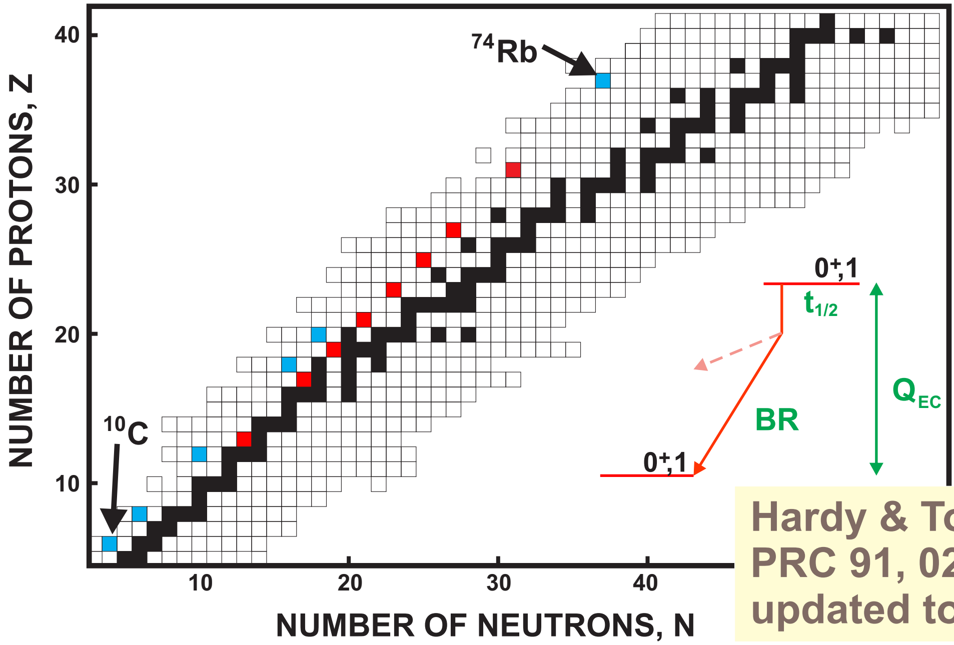
...unitarity

ONLY POSSIBLE IF PRIOR CONDITIONS SATISFIED

$$V_{ud}^2 = G_V^2/G^2$$

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

WORLD DATA FOR $0^+ \rightarrow 0^+$ DECAY, 2016



Hardy & Towner
PRC 91, 025501 (2015)
updated to 2016

- 8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.
- ~220 individual measurements with compatible precision

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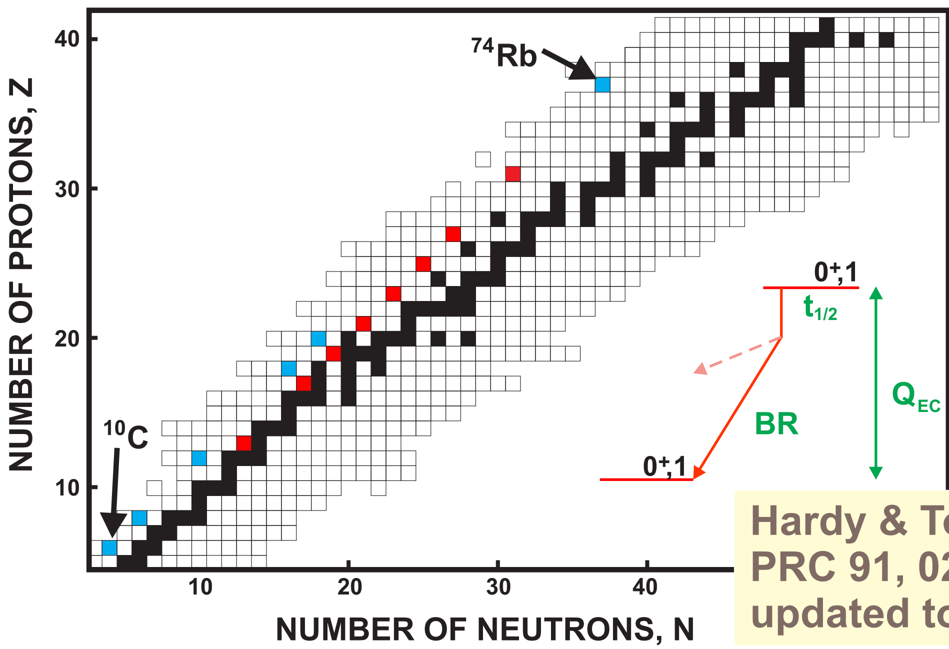

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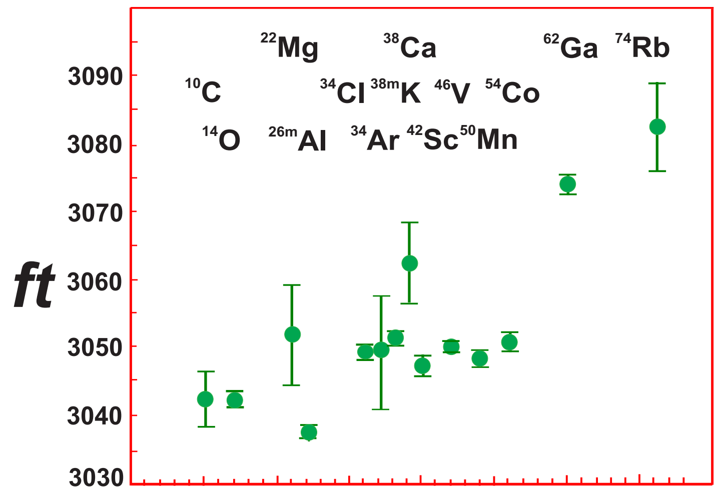

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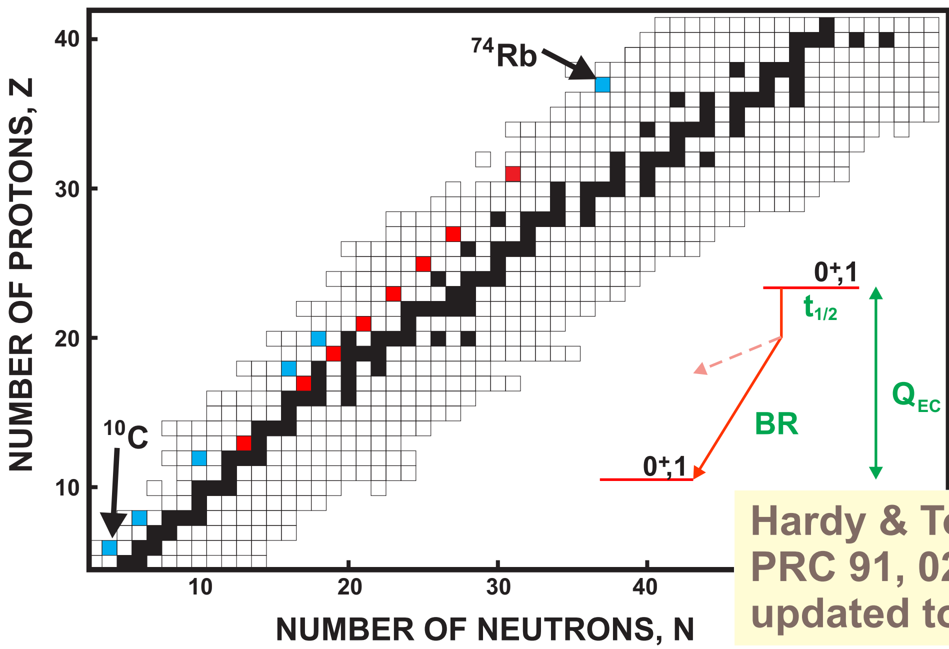

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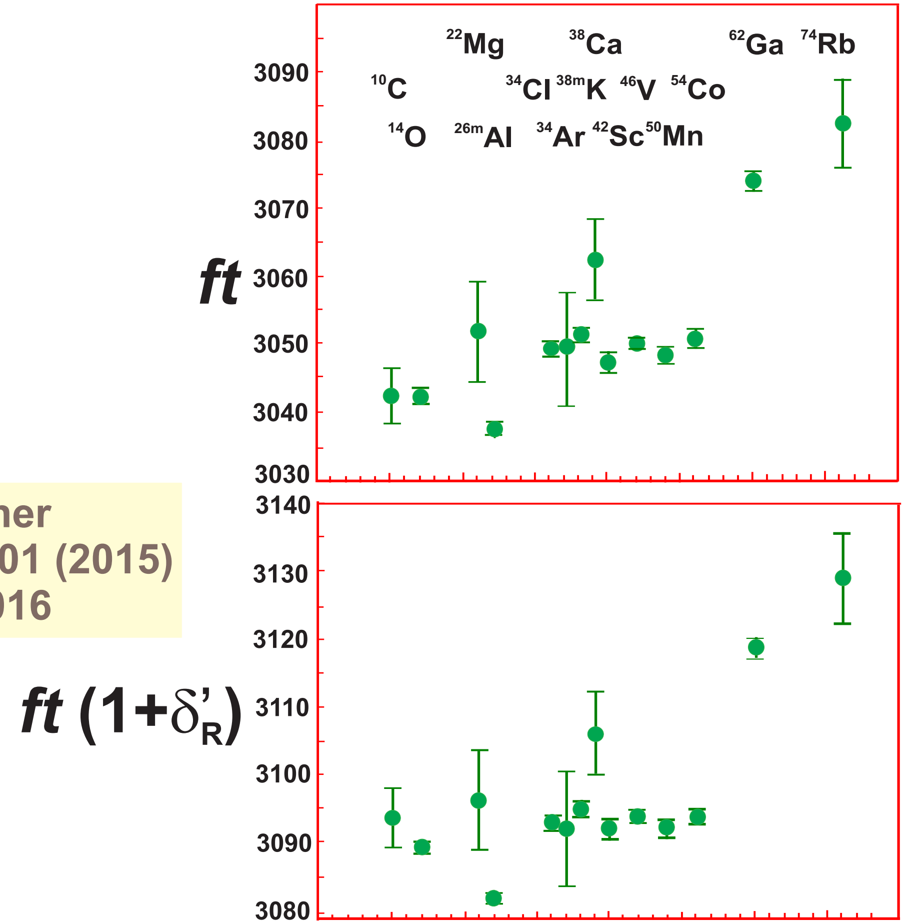

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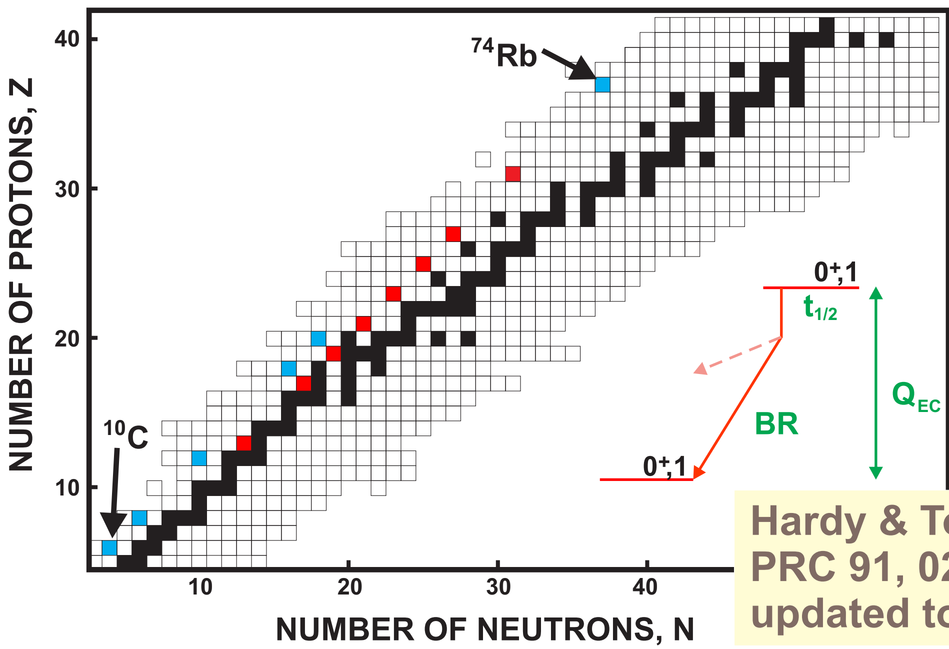

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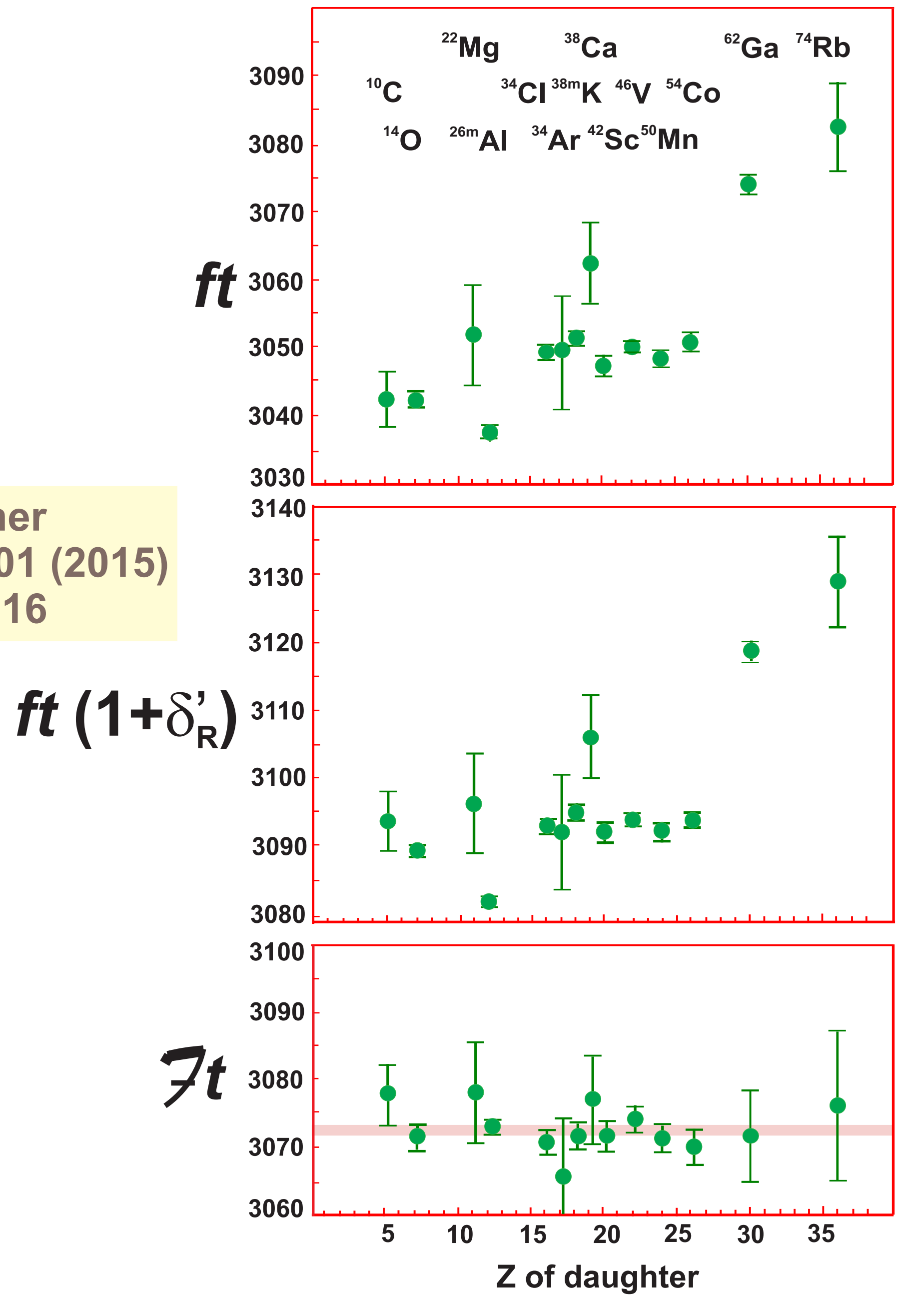

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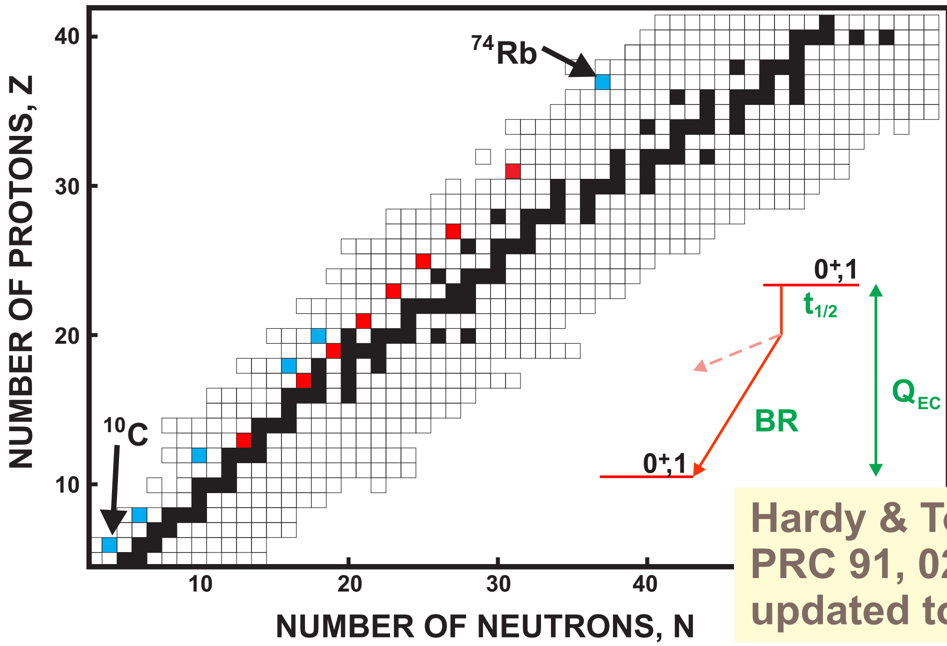

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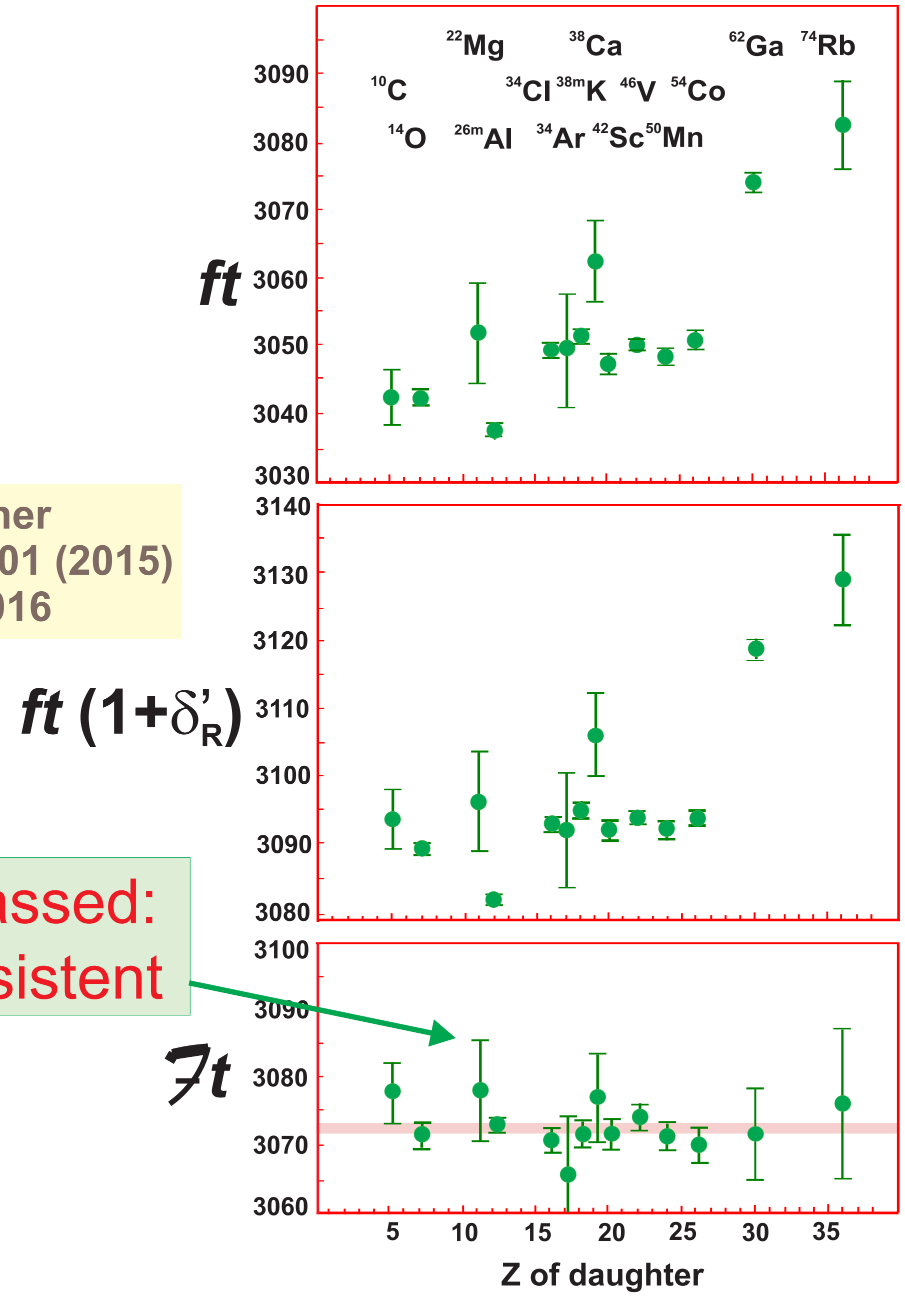

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Critical test passed:
$\mathcal{F}t$ values consistent

$$\mathcal{F}t = ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

CALCULATED CORRECTIONS TO $0^+ \rightarrow 0^+$ DECAYS

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1. Radiative corrections

$${}'_R = \frac{\;}{2}[g(E_m) + {}_2 + {}_3 + \dots]$$

$${}_R = \frac{\;}{2}[4\ln(m_Z/m_p) + \ln(m_p/m_A) + 2C_{Born} + \dots]$$

$_{NS}$

2. Isospin symmetry-breaking corrections

$_C$

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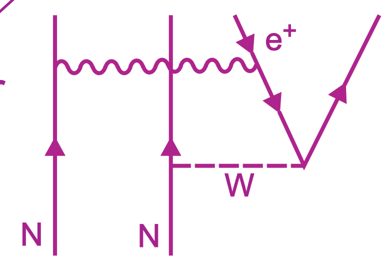
Order- axial-vector universal photonic contributions



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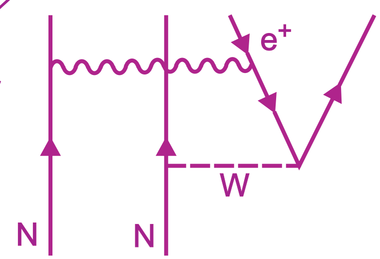

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Charge-dependent mismatch between parent and daughter analog states (members of the same isospin triplet).

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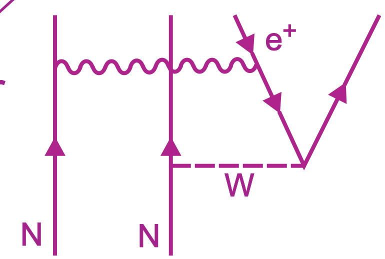


$_{NS}$ ← Order- axial-vector universal photonic contributions

2. Isospin symmetry-breaking corrections

$_C$ Charge-dependent mismatch between parent and daughter analog states (members of the same isospin triplet).

Dependent on nuclear structure

ISOSPIN SYMMETRY BREAKING CORRECTIONS

$$\delta_C = \delta_{CM} + \delta_{RO}$$

δ_{CM}

Difference in configuration mixing between parent and daughter.

- Shell-model calculation with well-established two-body matrix elements.
- Charge dependence tuned to known single-particle energies and to measured IMME coefficients.
- Results also adjusted to measured 0^+ state energies.

0.01 – 0.3 %

δ_{RO}

Mismatch in radial wave function between parent and daughter.

- Full-parentage Saxon-Woods wave function matched to known binding energy and charge radius from electron scattering.
- Core states included based on measured spectroscopic factors.

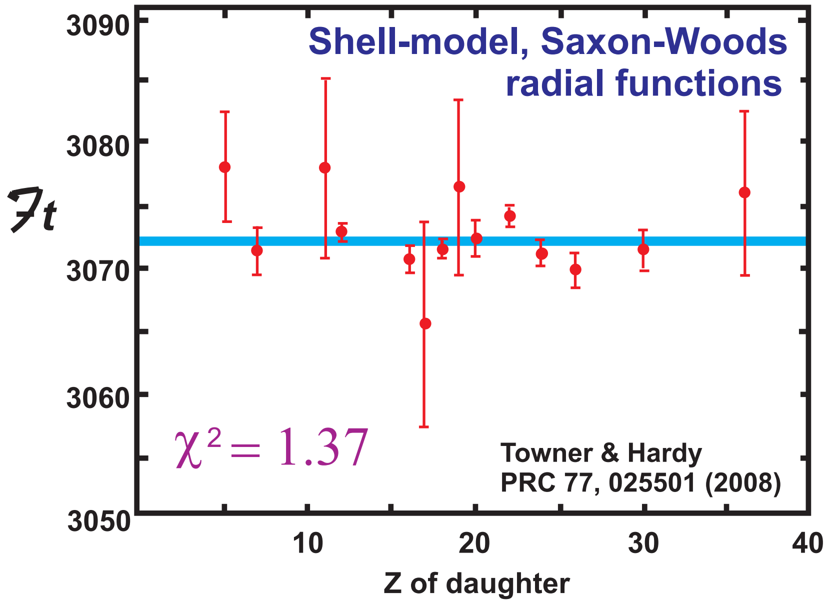
0.4 – 1.5 %

TESTS OF δ_c CALCULATIONS

A. Agreement with CVC:

$\mathcal{F}t$ values have been calculated with different models for δ_c, then tested for consistency. Normalized χ^2 and confidence levels are shown.

Model	χ^2/N	CL(%)
SM-SW	1.37	17



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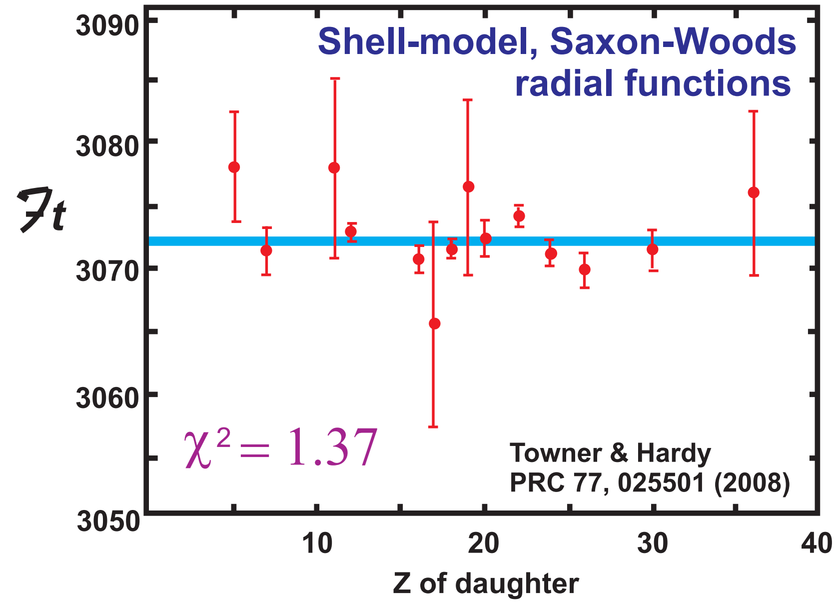

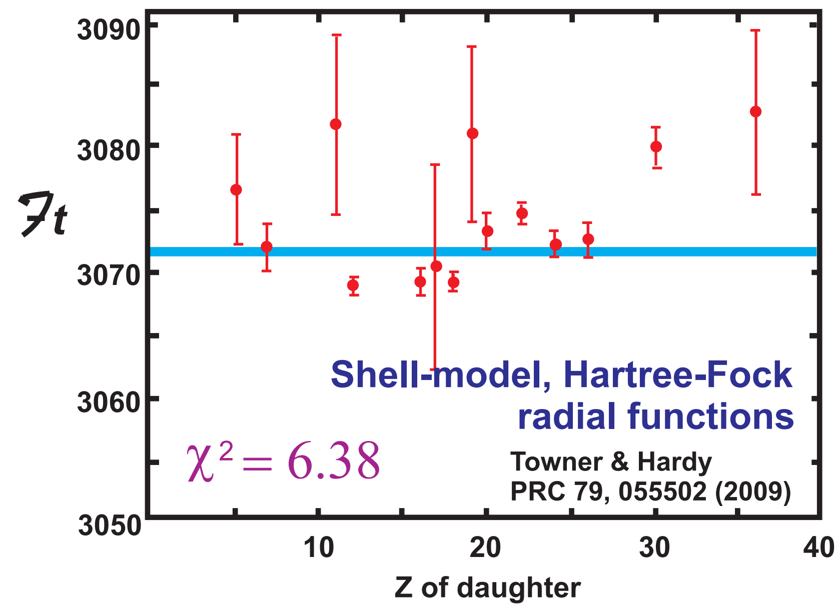

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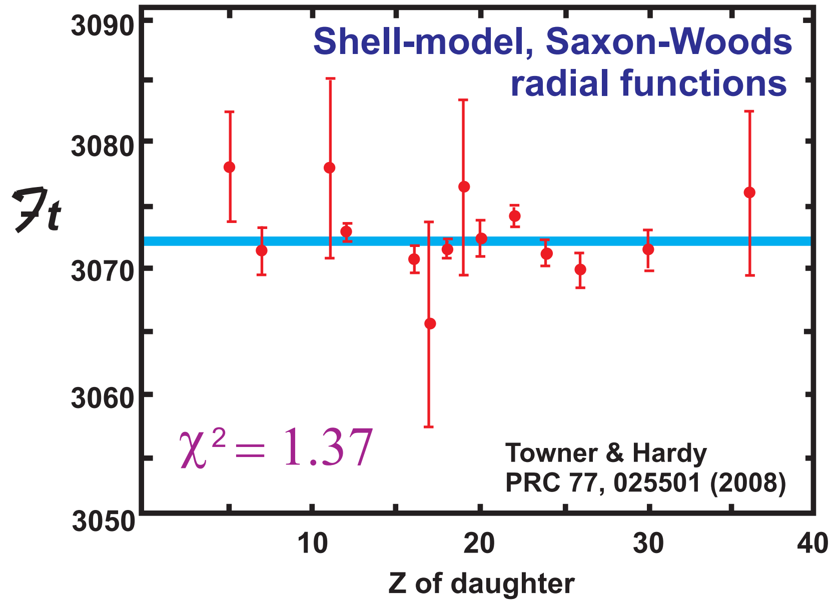

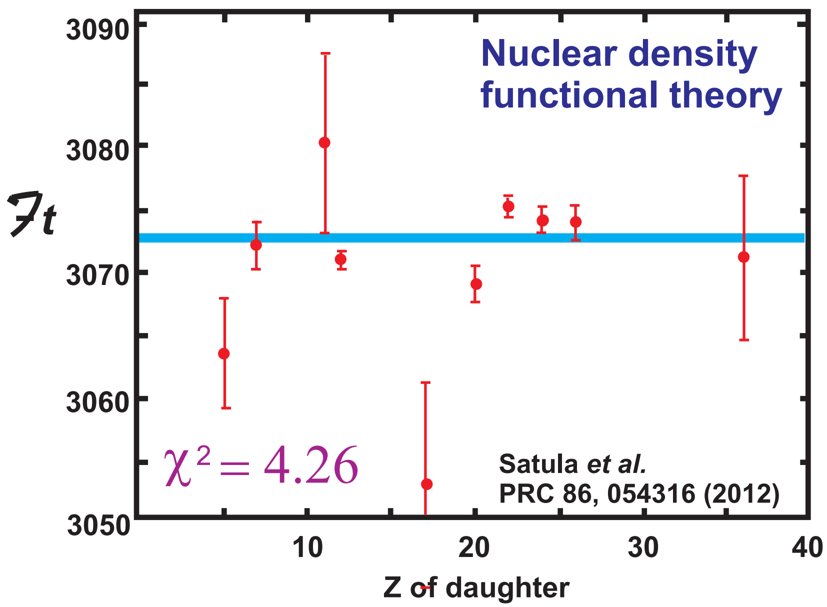

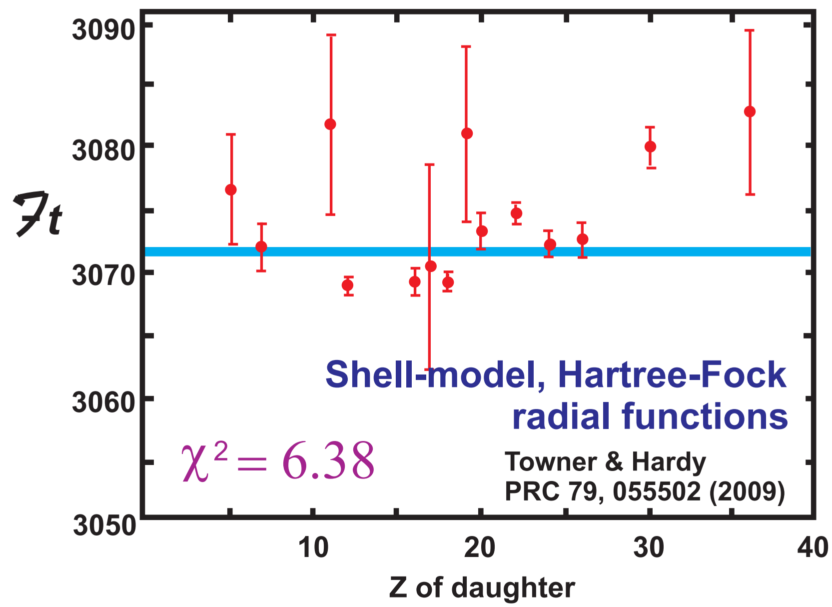

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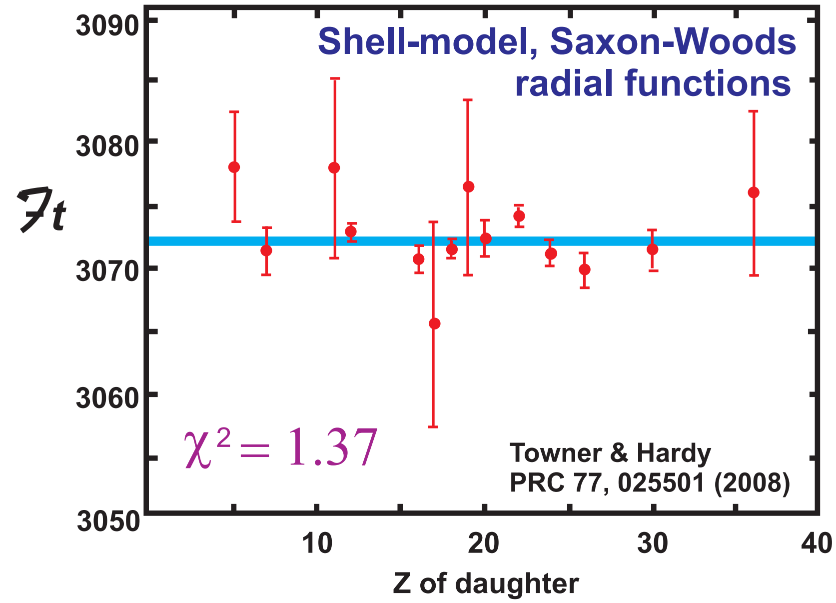
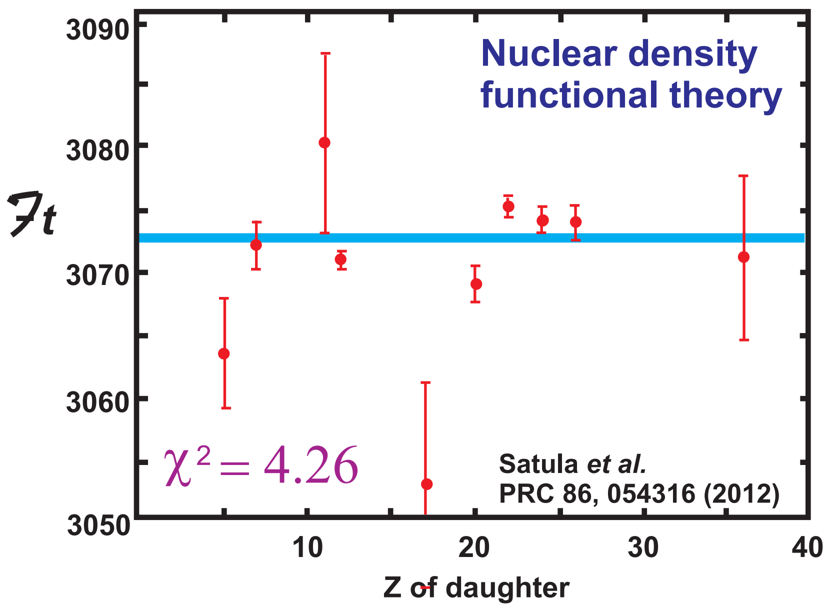
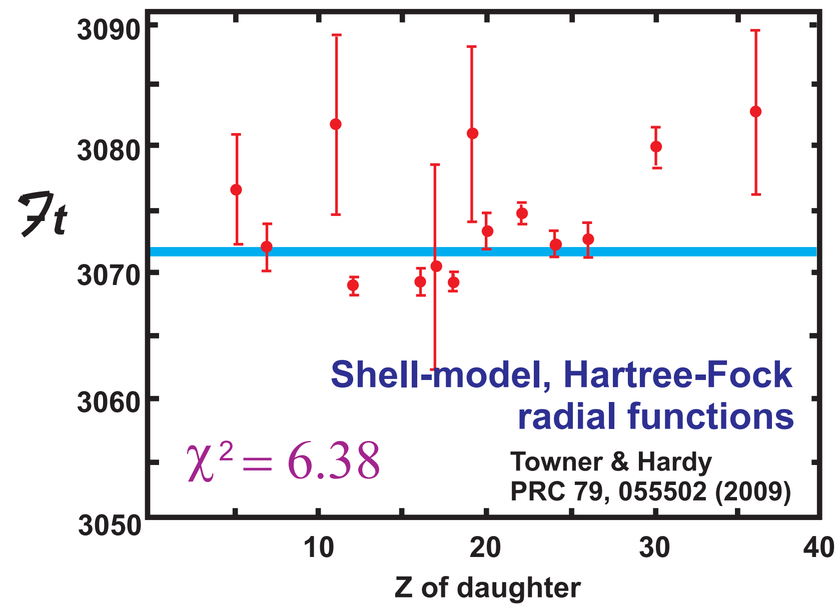
RHF-RPA	4.91	0
RH-RPA	3.68	0

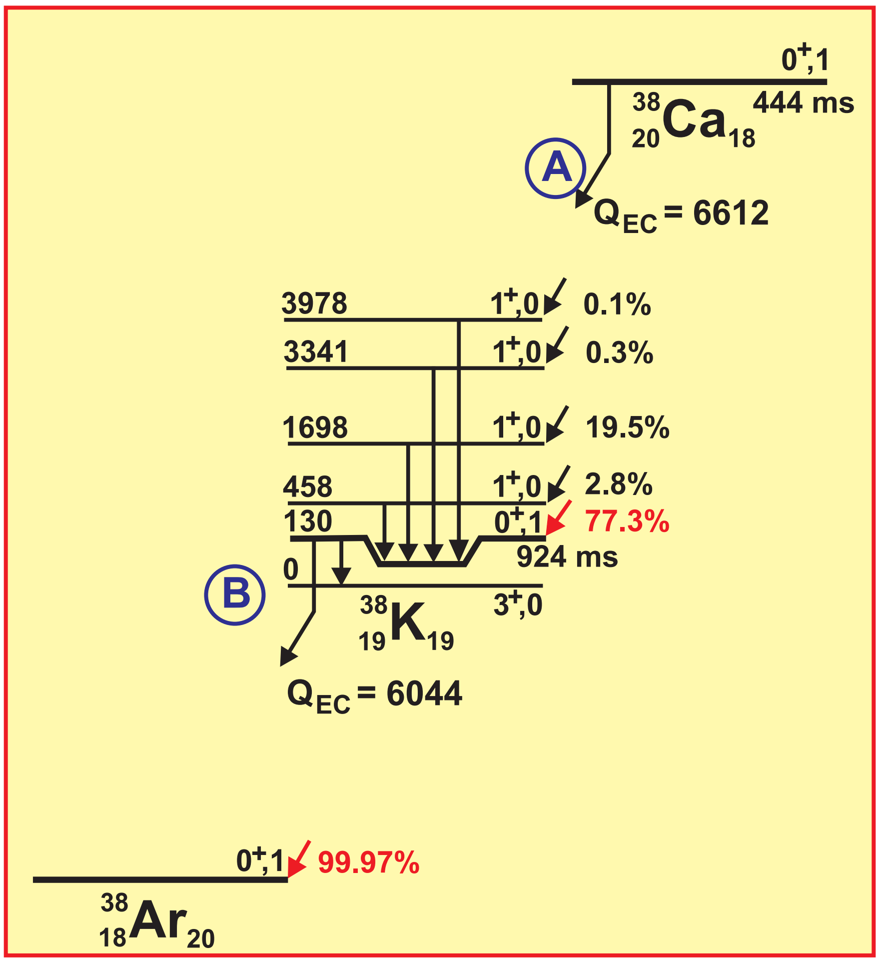






TESTS OF δ_c CALCULATIONS

B. Measurements of mirror superallowed transitions:



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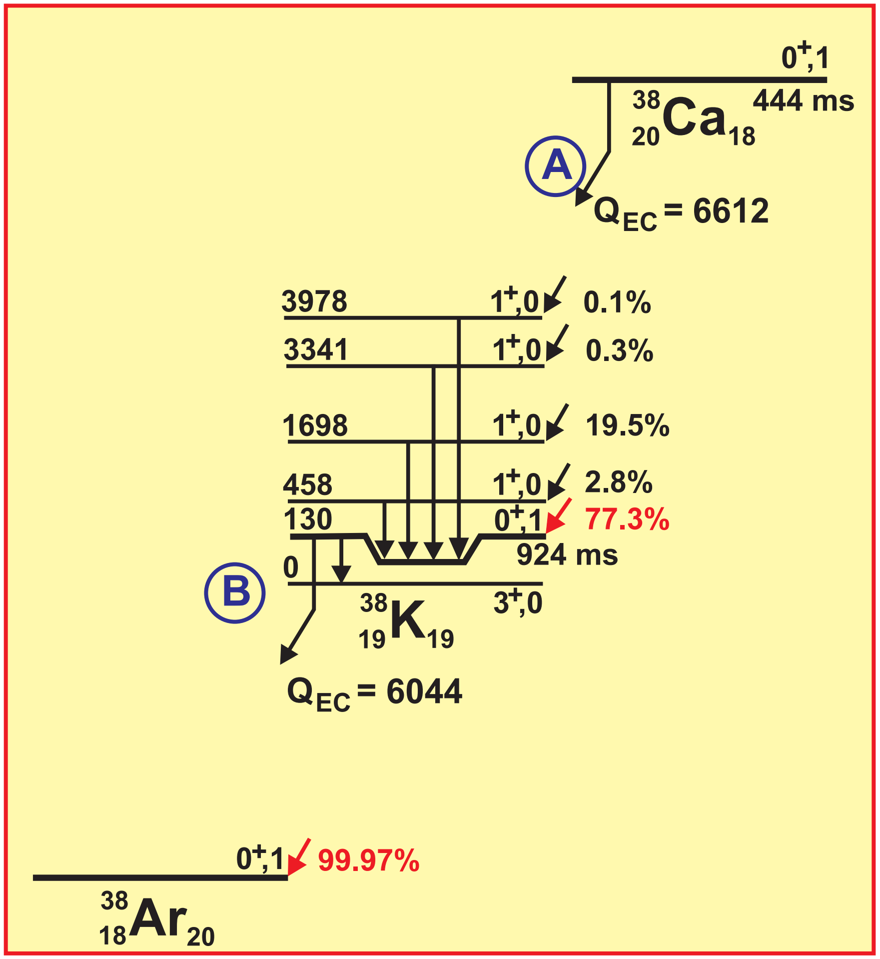
$$\mathcal{F}t = ft\,(1 + \delta_R')\left[1 - (\delta_C - \delta_{NS})\right]$$



$$\frac{ft_A}{ft_B} = \frac{(1 + \delta_R'^{B})[1 - (\delta_C^B - \delta_{NS}^B)]}{(1 + \delta_R'^{A})[1 - (\delta_C^A - \delta_{NS}^A)]}$$

$$= 1 + (\delta_R'^{B} - \delta_R'^{A}) + (\delta_{NS}^B - \delta_{NS}^A) - (\delta_C^B - \delta_C^A)$$

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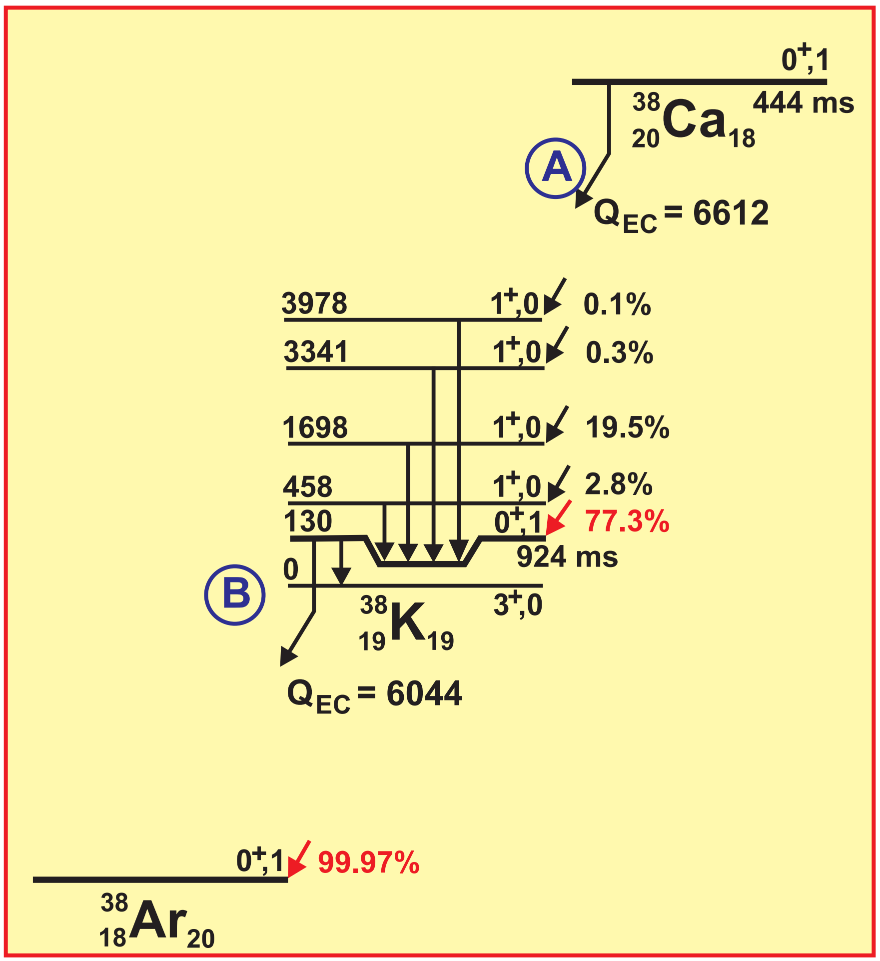
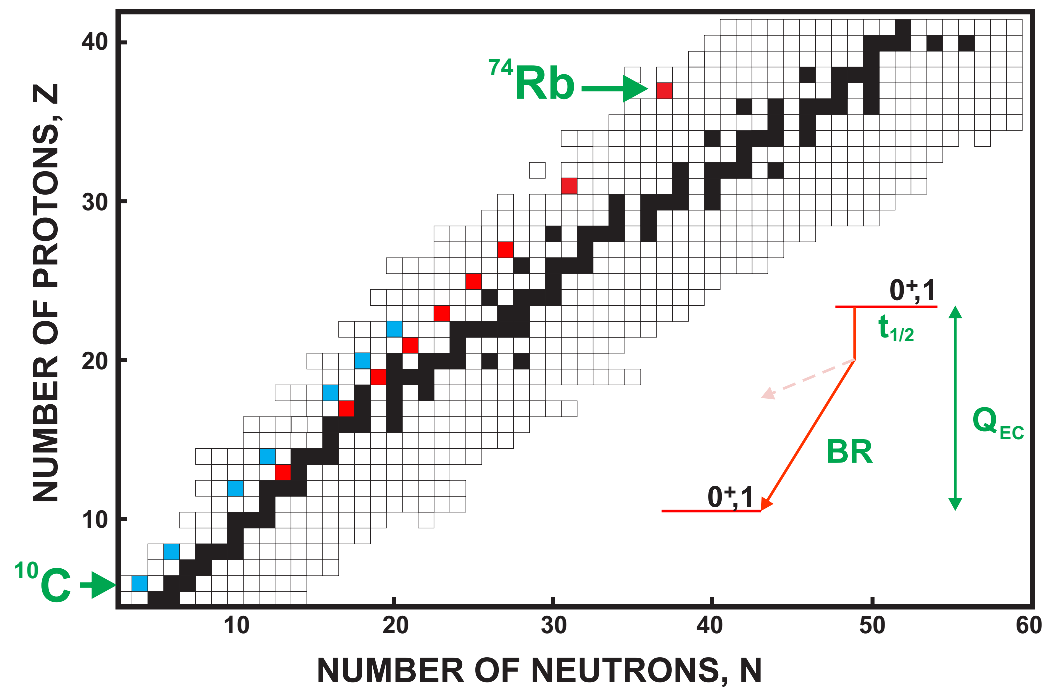
$$= 1 + (\delta_R'^B - \delta_R'^A) + (\delta_{NS}^B - \delta_{NS}^A) - (\delta_C^B - \delta_C^A)$$





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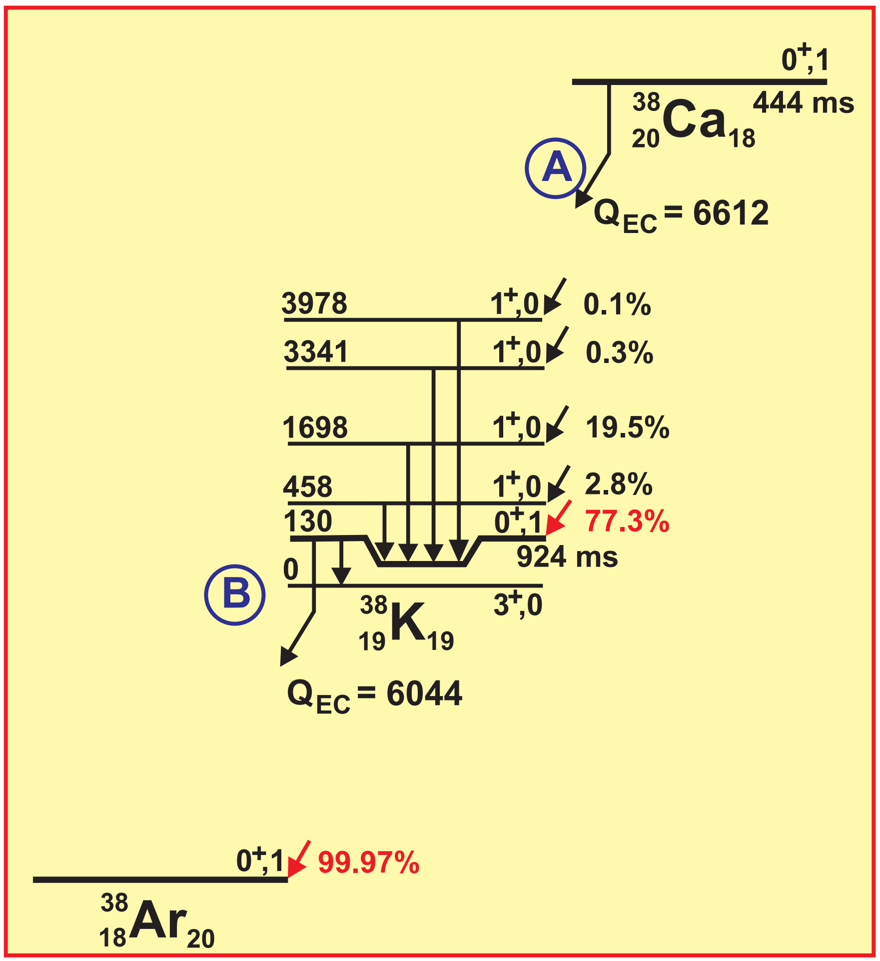
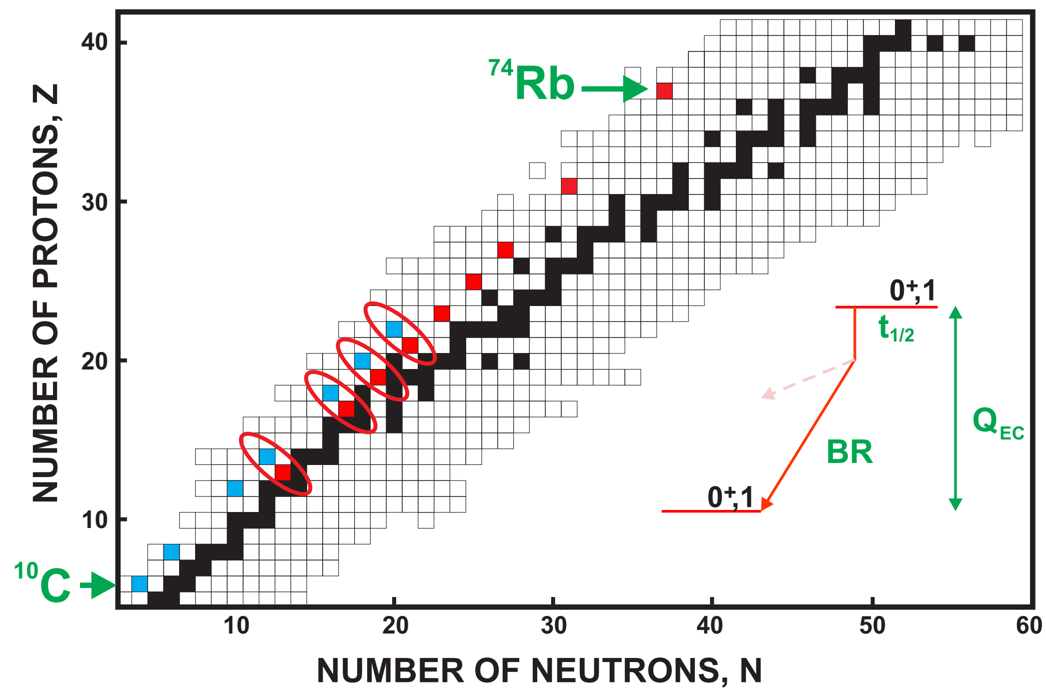
$$= 1 + (\delta'^B_R - \delta'^A_R) + (\delta^B_{NS} - \delta^A_{NS}) - (\delta^B_C - \delta^A_C)$$





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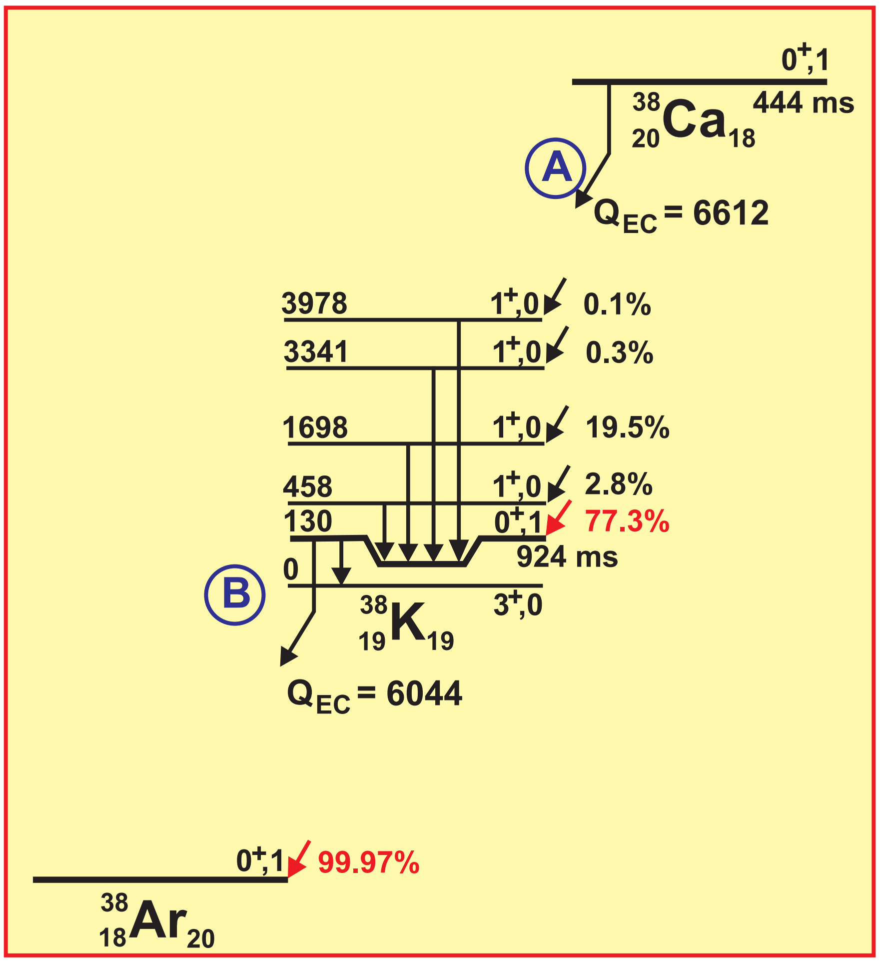
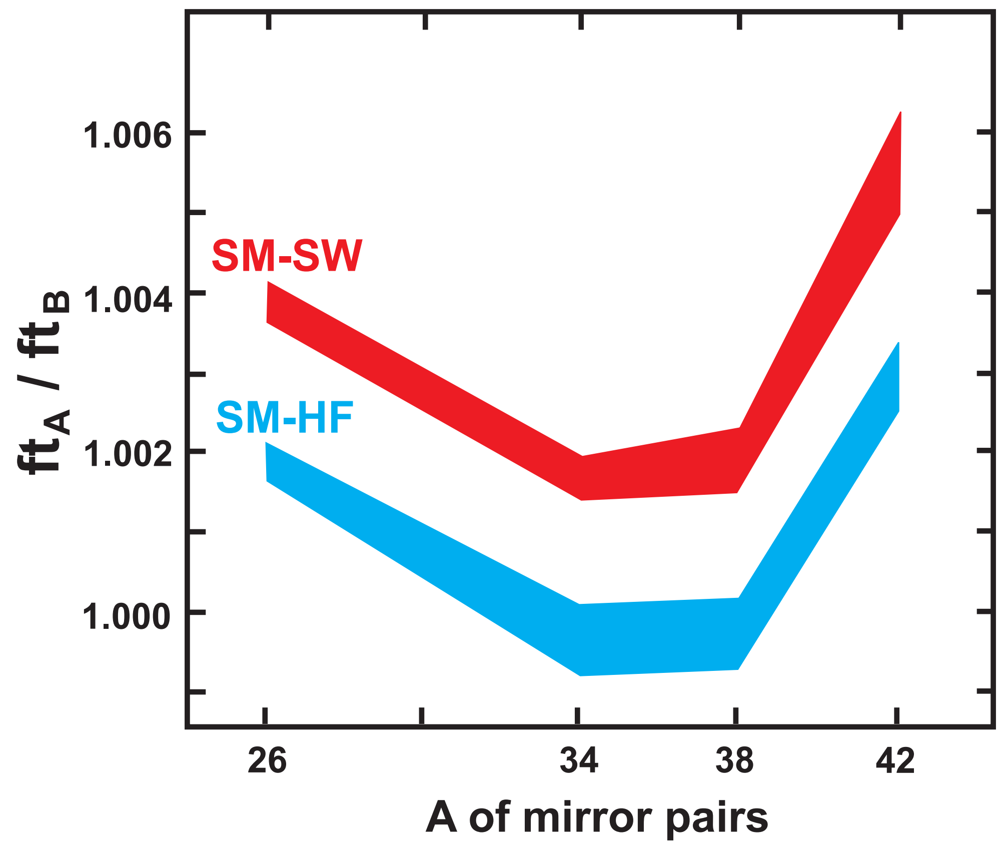
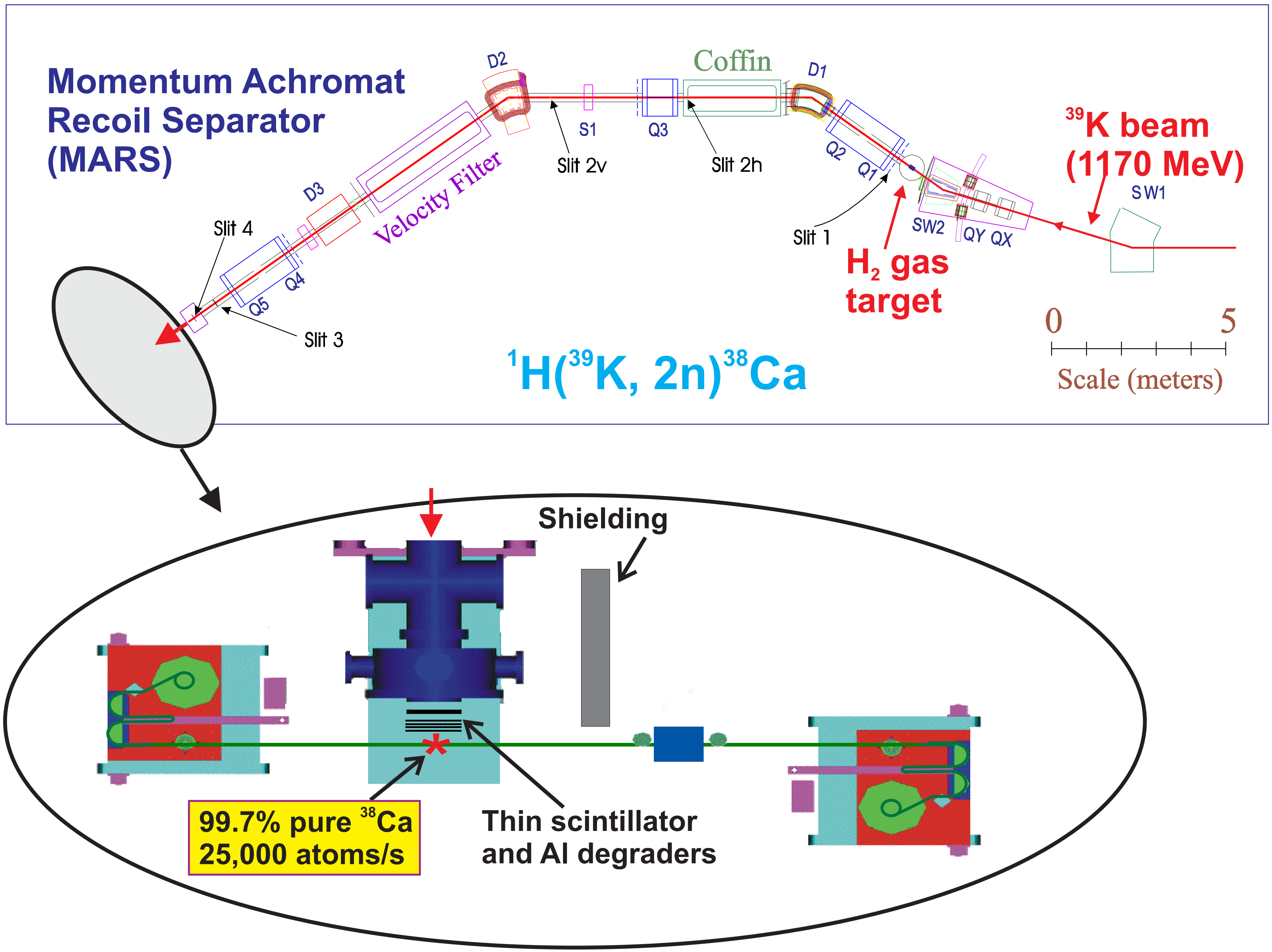


$$\frac{ft_A}{ft_B} = \frac{(1 + \delta_R'^B)[1 - (\delta_C^B - \delta_{NS}^B)]}{(1 + \delta_R'^A)[1 - (\delta_C^A - \delta_{NS}^A)]}$$

$$= 1 + (\delta_R'^B - \delta_R'^A) + (\delta_{NS}^B - \delta_{NS}^A) - (\delta_C^B - \delta_C^A)$$



PRECISION DECAY MEASUREMENTS AT TAMU



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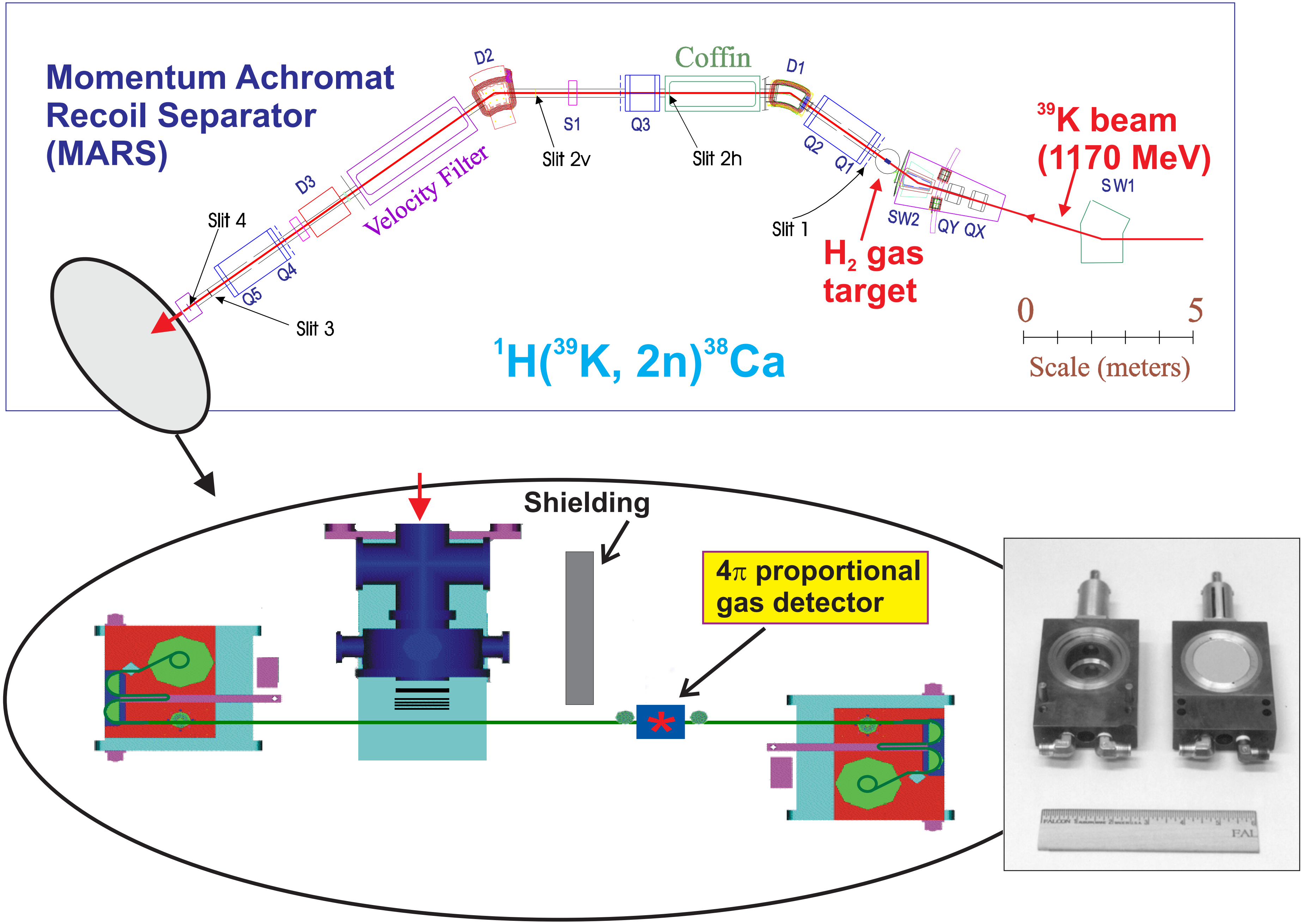

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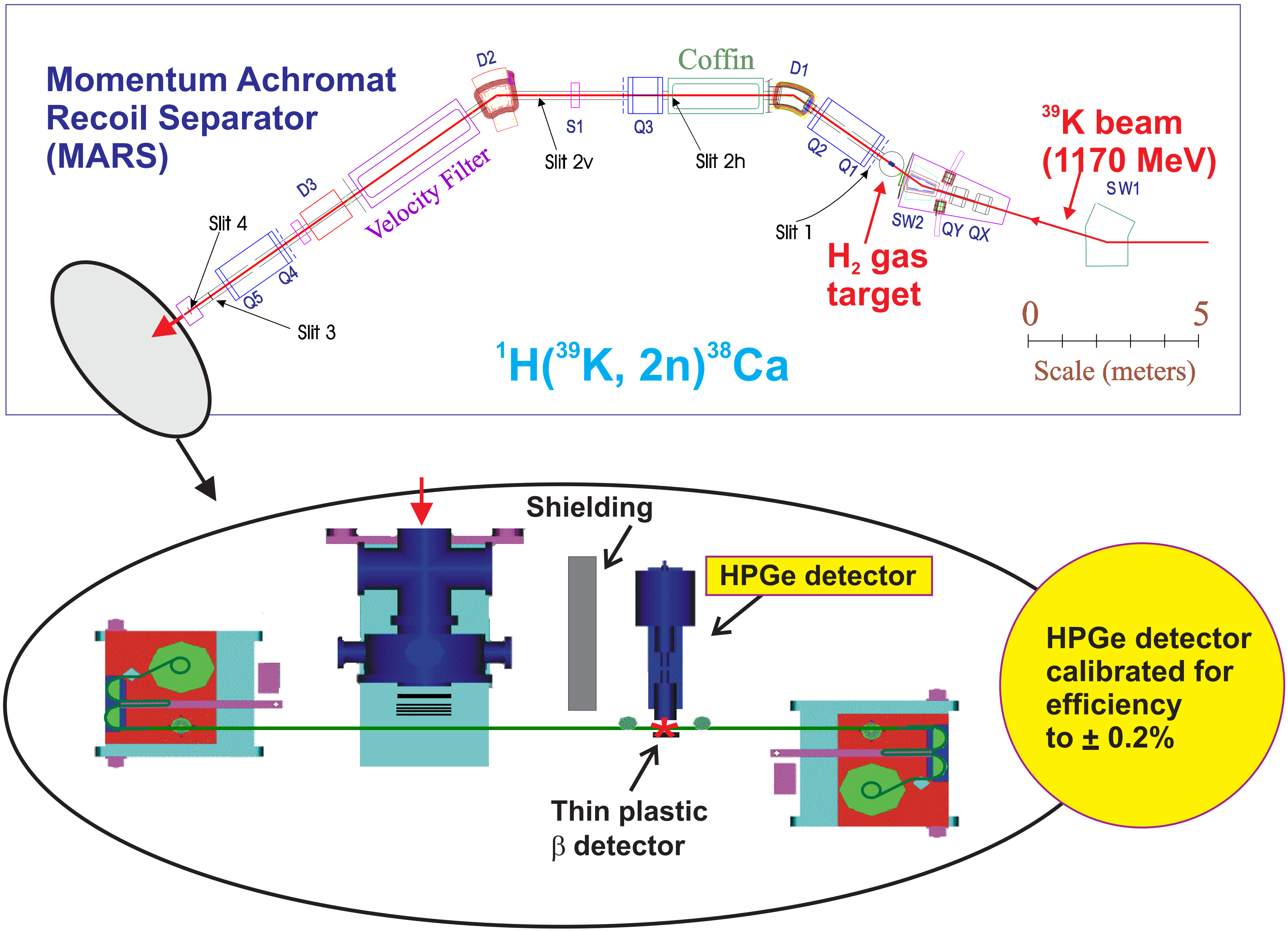
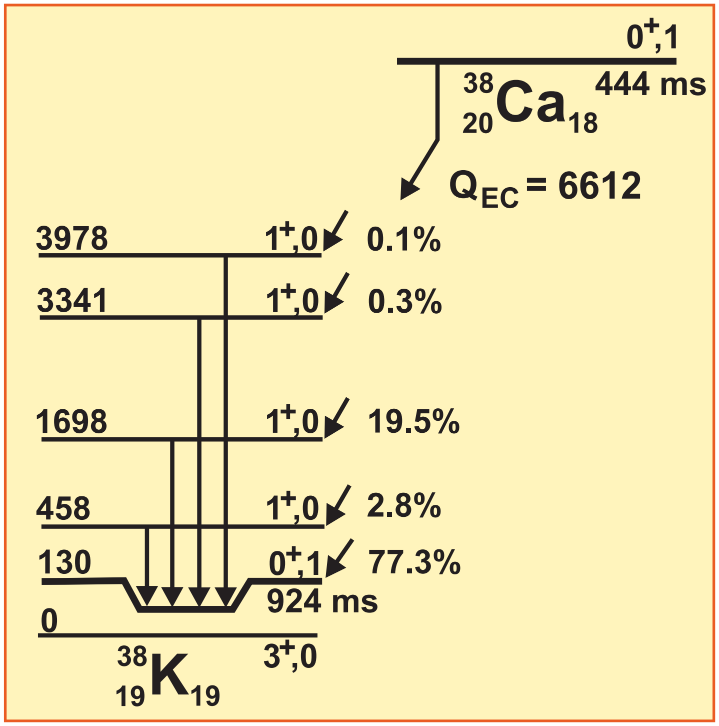
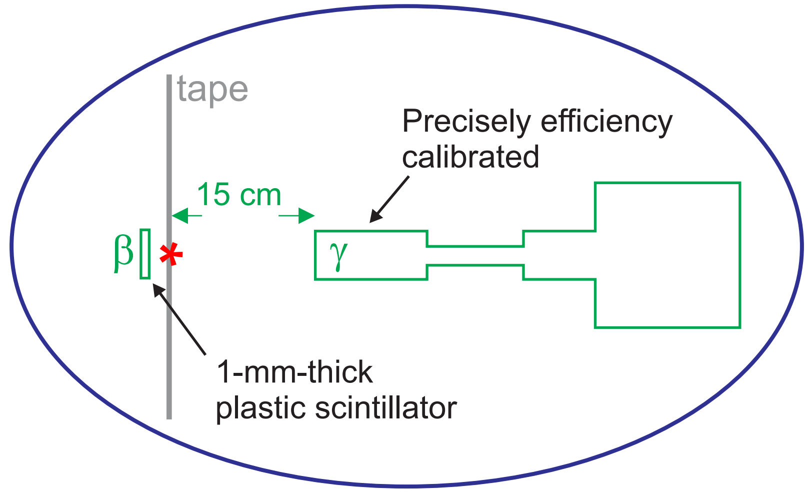
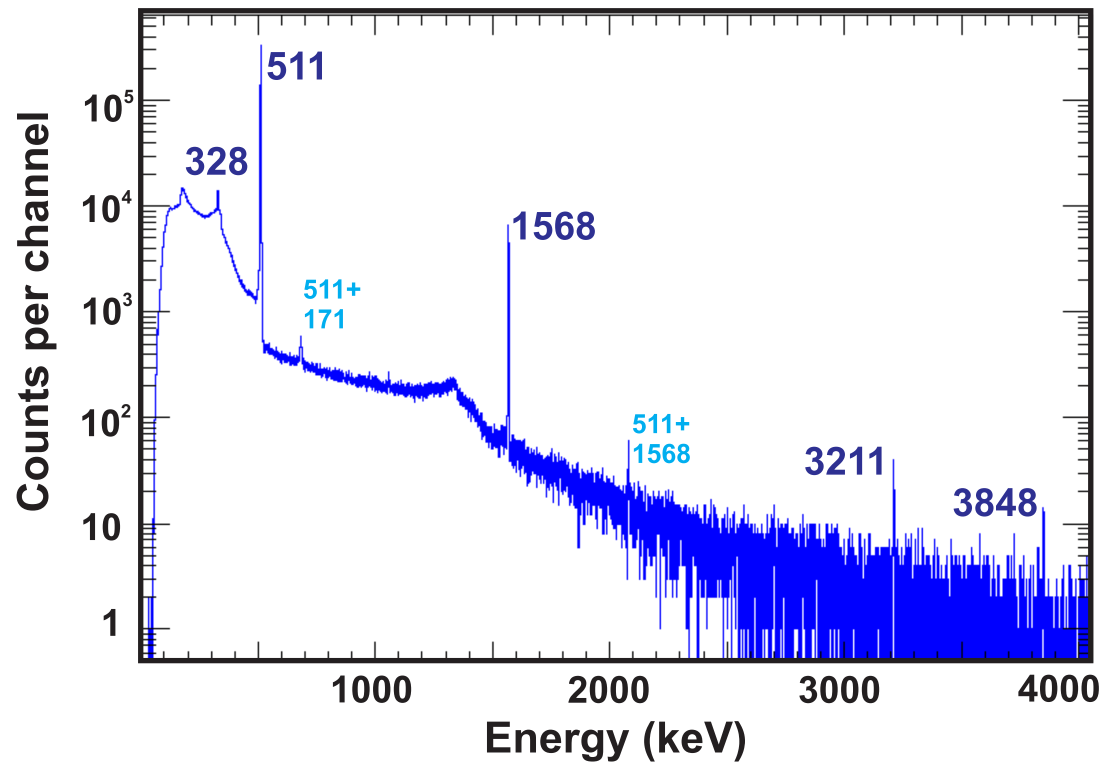


BETA-DECAY BRANCHING OF ^{38}Ca







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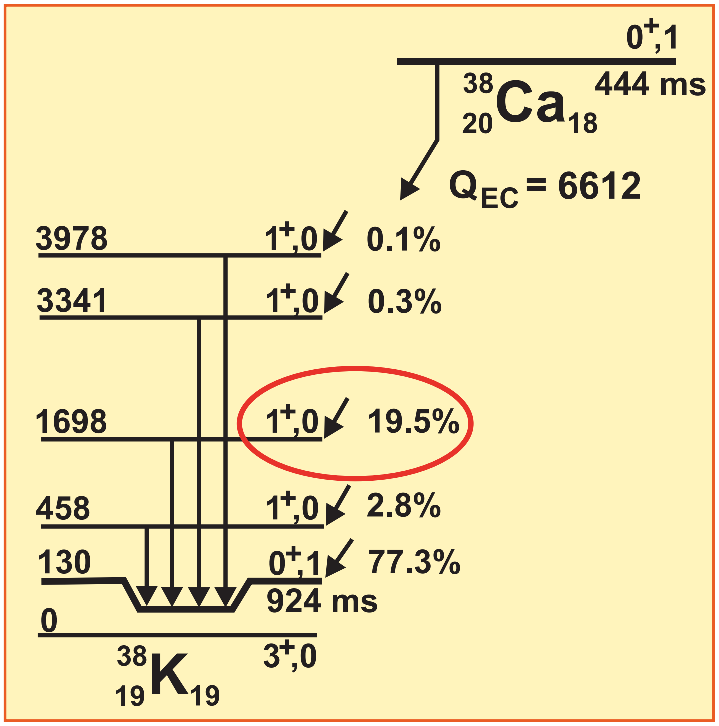

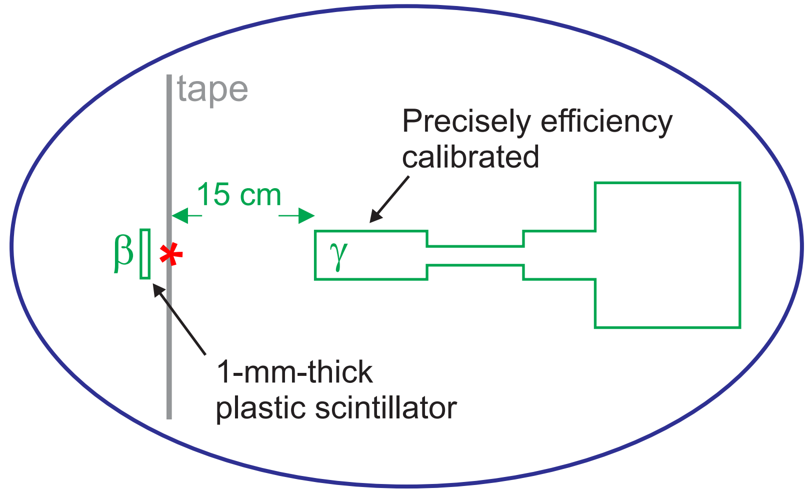

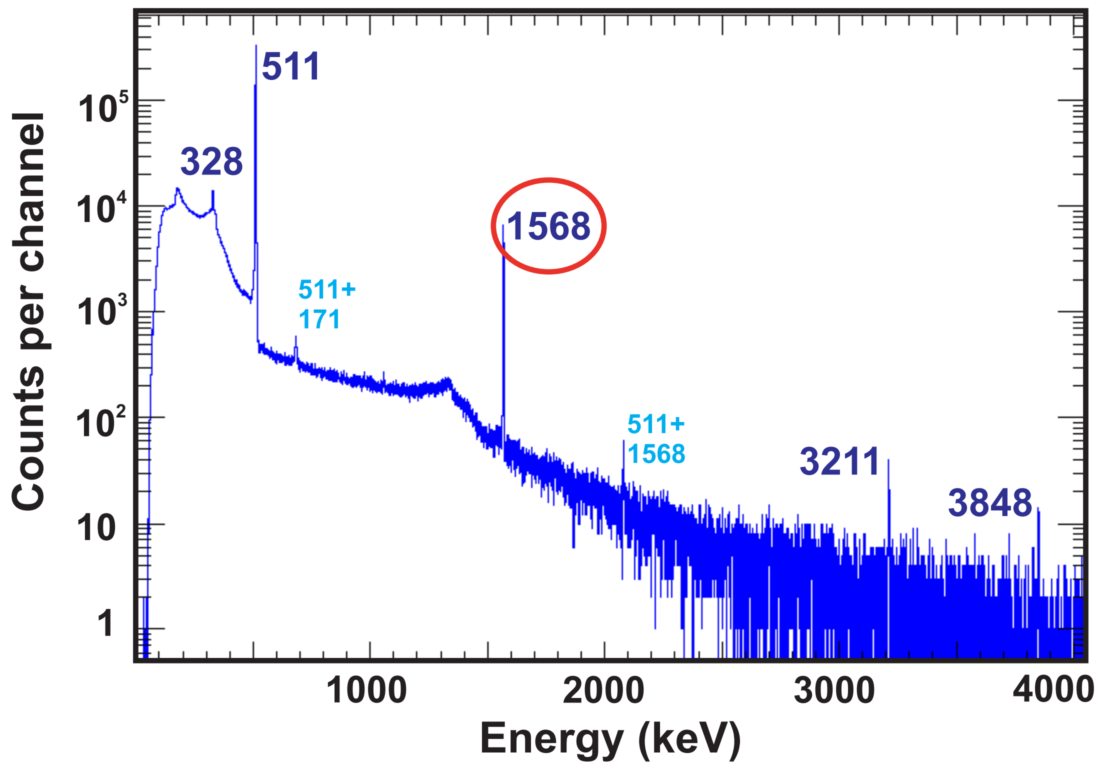

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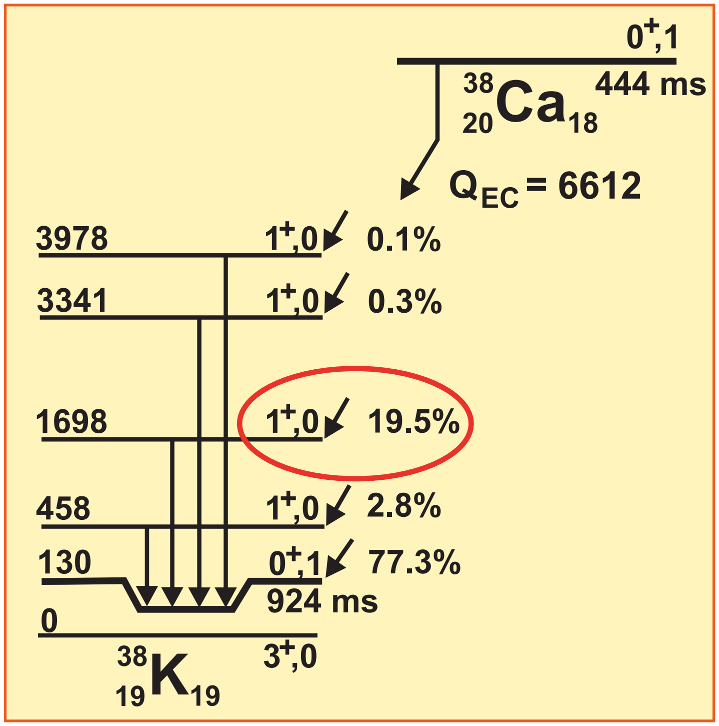
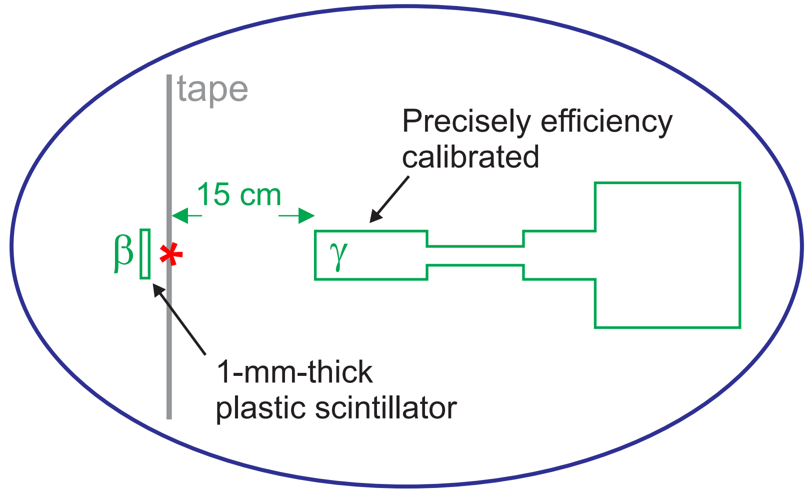
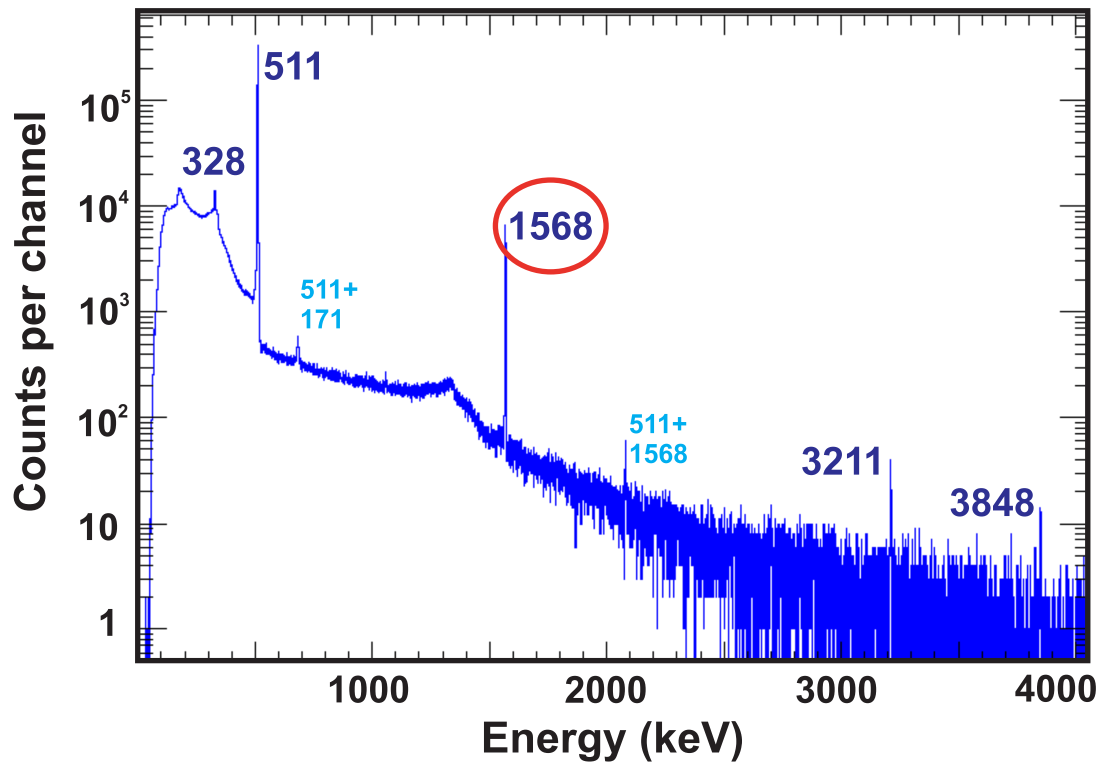




$$\frac{N_{\gamma_1\beta}}{N_\beta} = \frac{N_o R_1 \varepsilon_{\beta_1} \varepsilon_{\gamma_1}}{N_o \varepsilon_{\beta_{tot}}}$$



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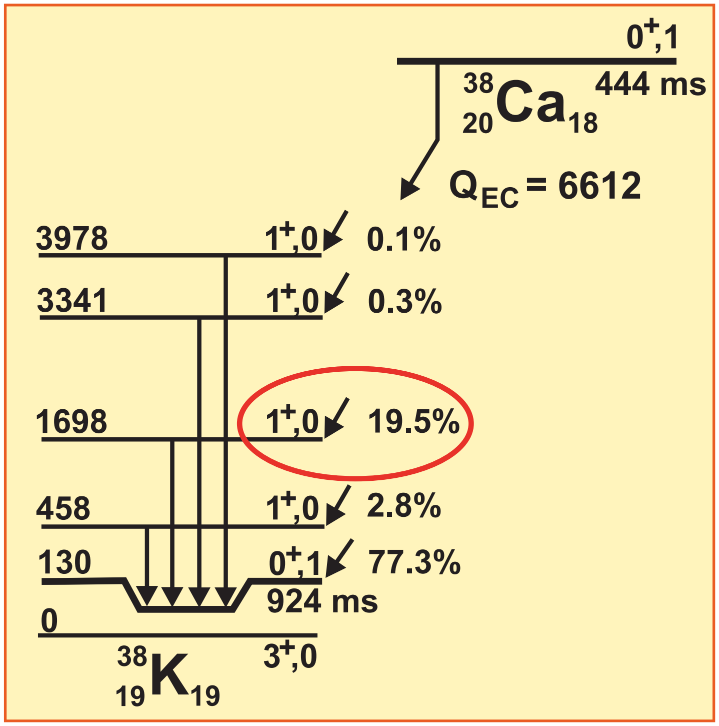
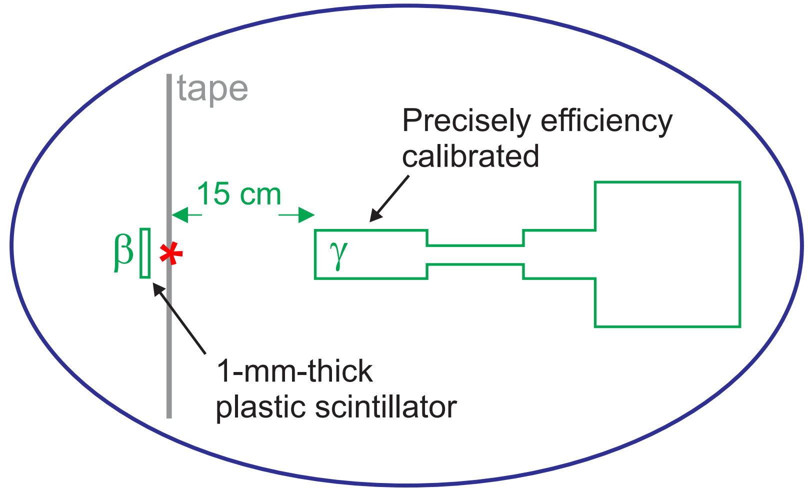
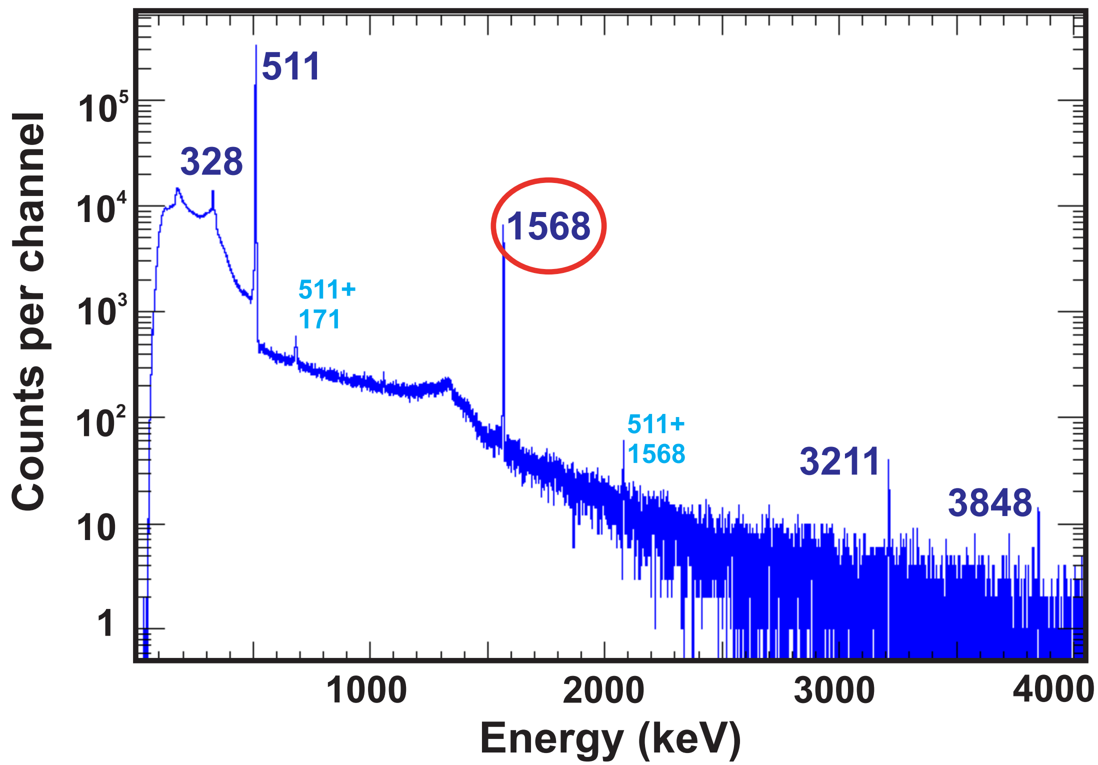


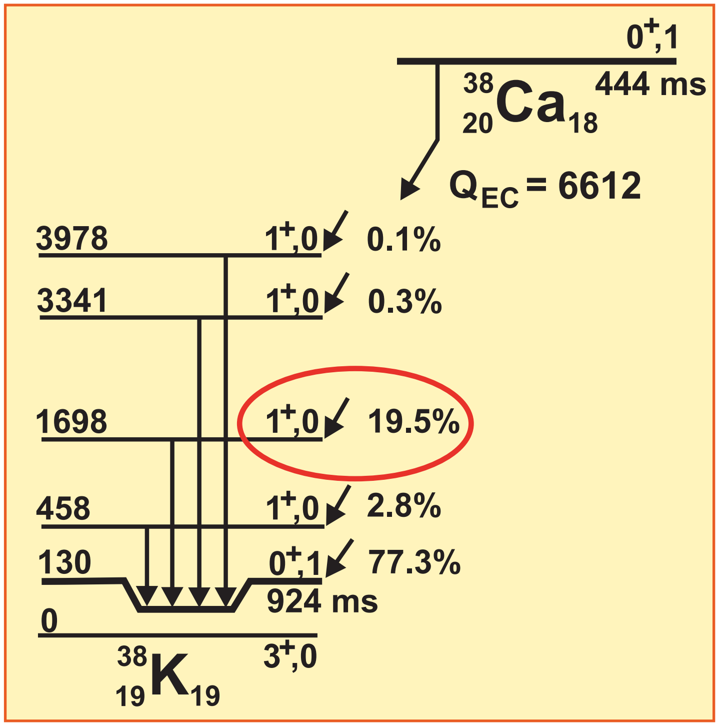


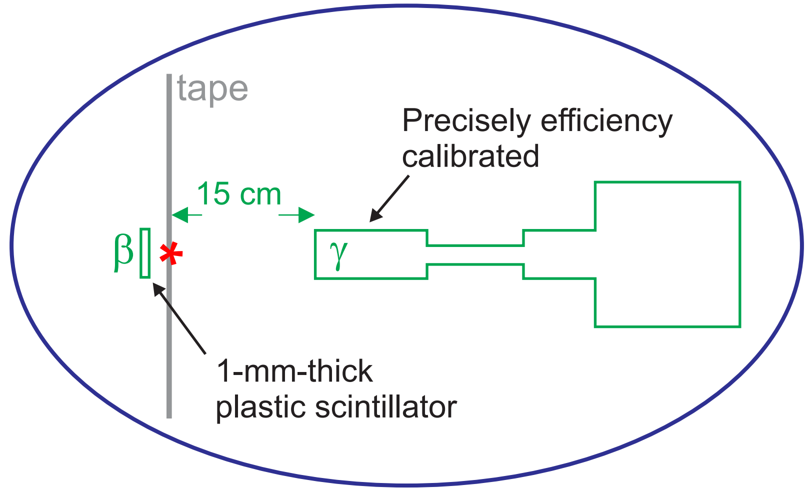
$$\frac{N_{\gamma_1\beta}}{N_\beta} = \frac{N_0 R_1 \varepsilon_{\beta_1}\varepsilon_{\gamma_1}}{N_0 \varepsilon_{\beta_{tot}}}$$

$$R_1 = \frac{N_{\gamma_1\beta}}{N_\beta \varepsilon_{\gamma_1}} \frac{\varepsilon_{\beta_{tot}}}{\varepsilon_{\beta_1}} k$$



BETA-DECAY BRANCHING OF ^{38}Ca





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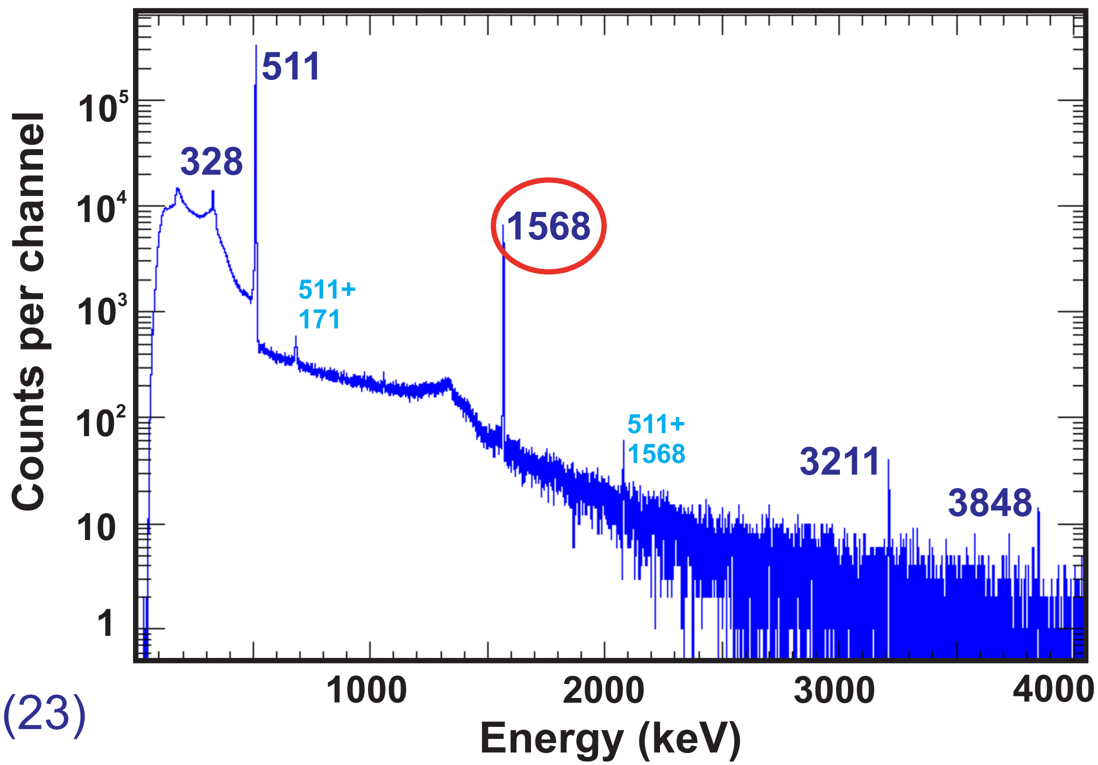
$k = f$(dead-time, pile-up, coincidence summing) = 1.0391(23)



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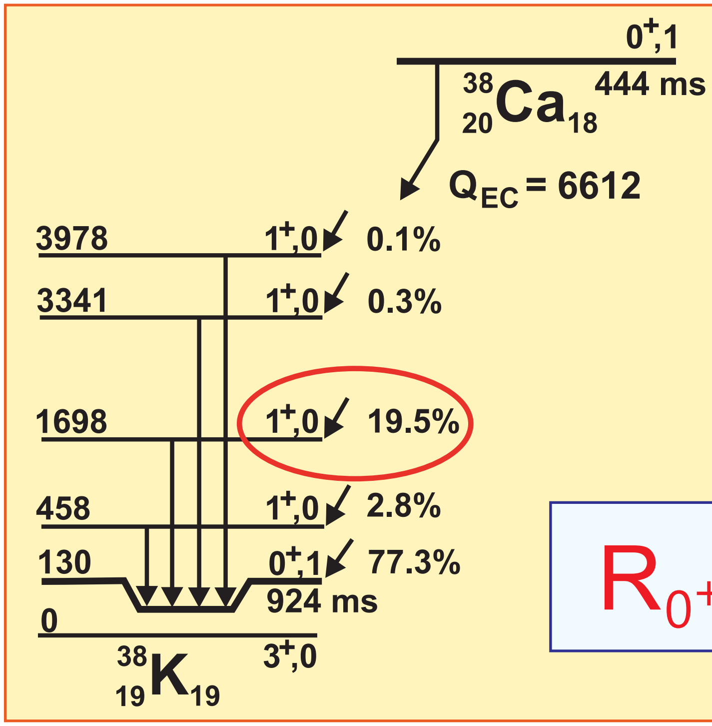

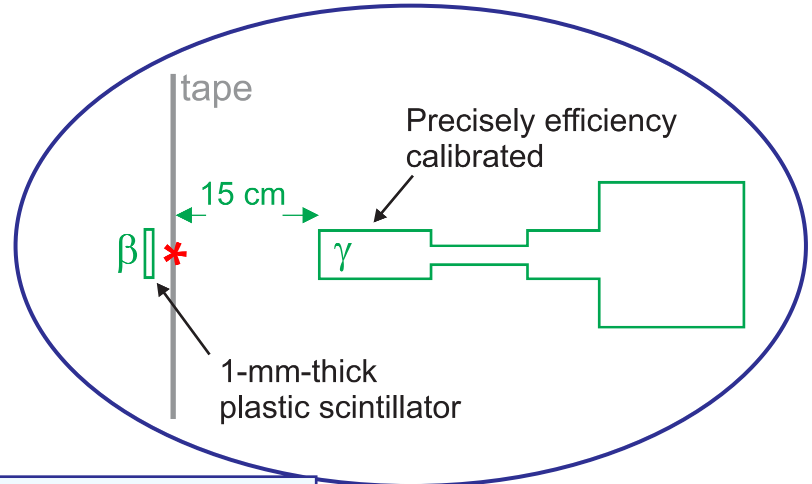

$$R_{0^+} = 77.28(16)\%$$

$$\frac{N_{\gamma_1\beta}}{N_\beta} = \frac{N_0 R_1 \varepsilon_{\beta_1}\varepsilon_{\gamma_1}}{N_0 \varepsilon_{\beta_{tot}}}$$

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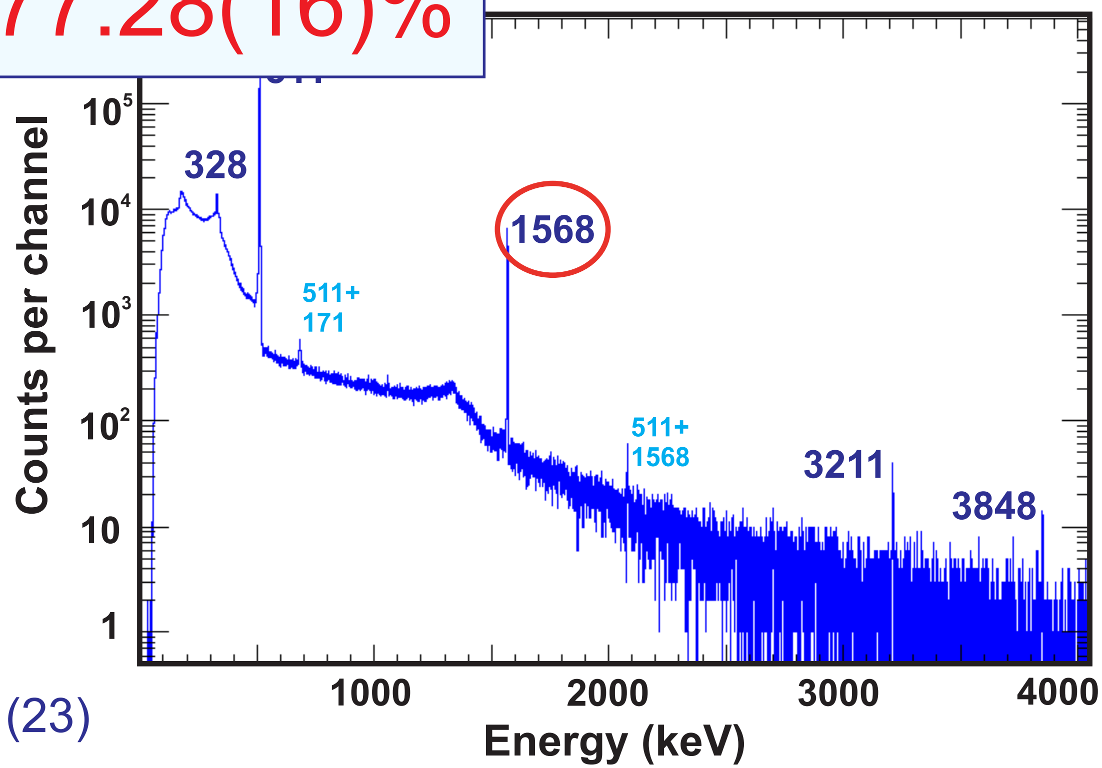

TESTS OF δ_C CALCULATIONS

B. Measurements of mirror superallowed transitions:

$$\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})]$$

$$\frac{ft_A}{ft_B} = \frac{(1 + \delta_R'^B)[1 - (\delta_C^B - \delta_{NS}^B)]}{(1 + \delta_R'^A)[1 - (\delta_C^A - \delta_{NS}^A)]}$$

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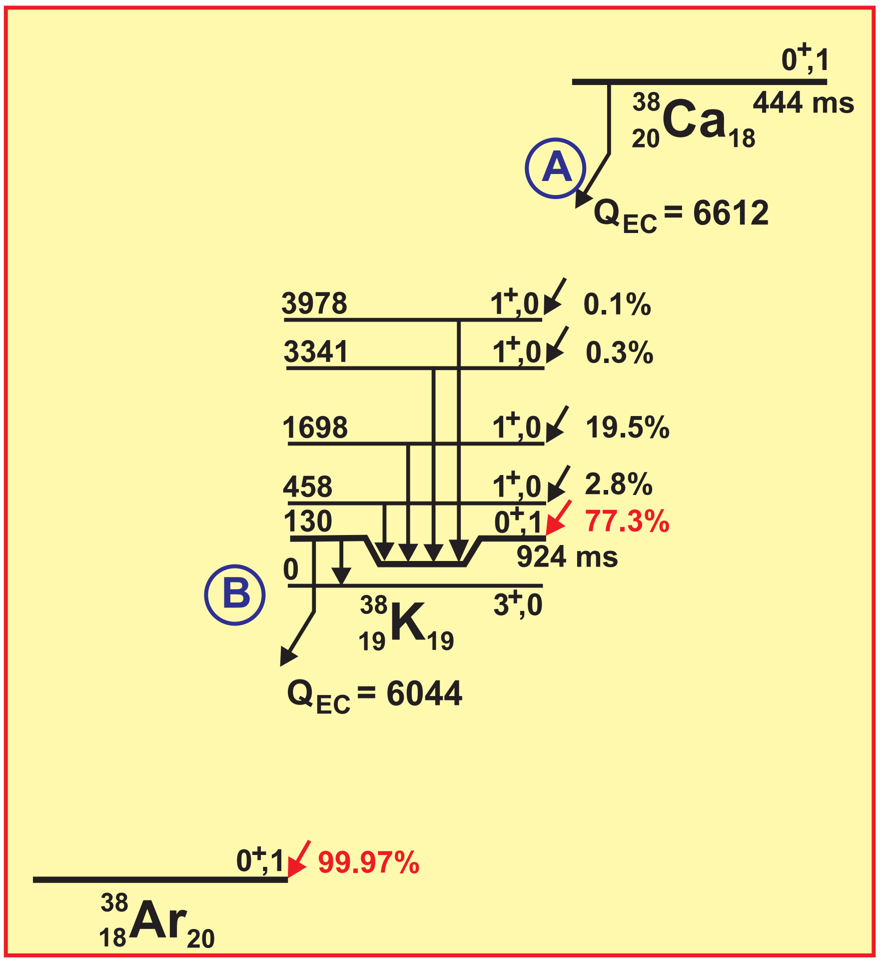
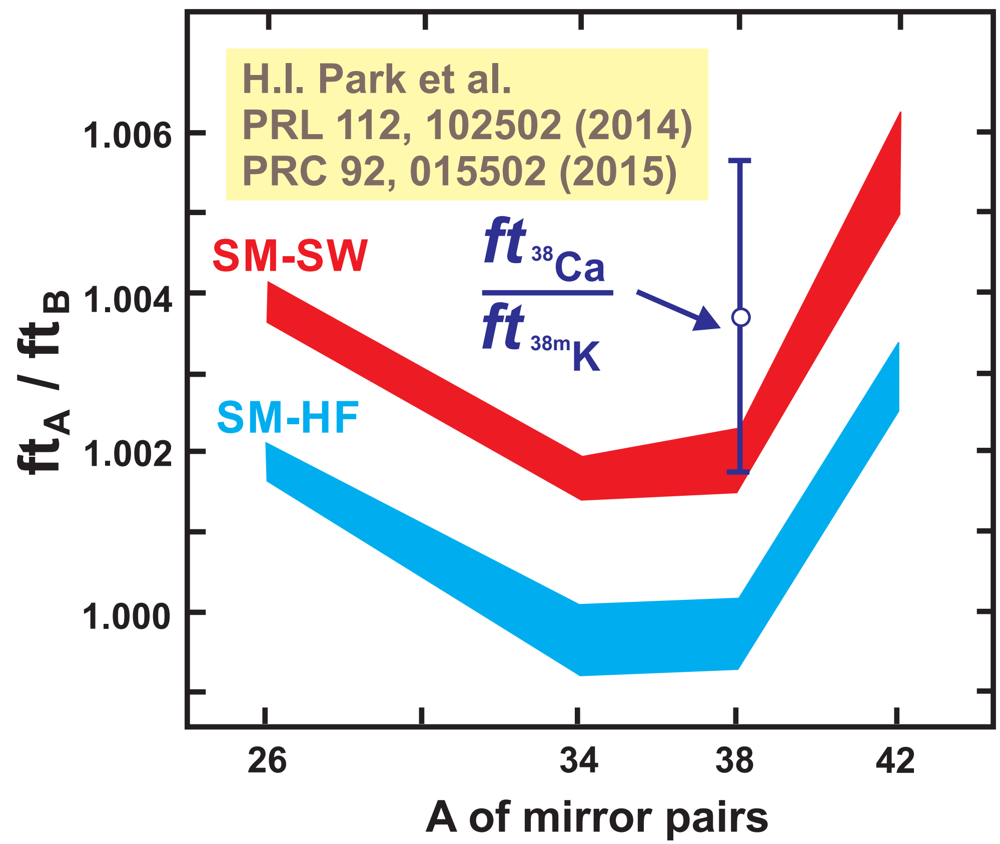




RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2 (1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta_R')\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

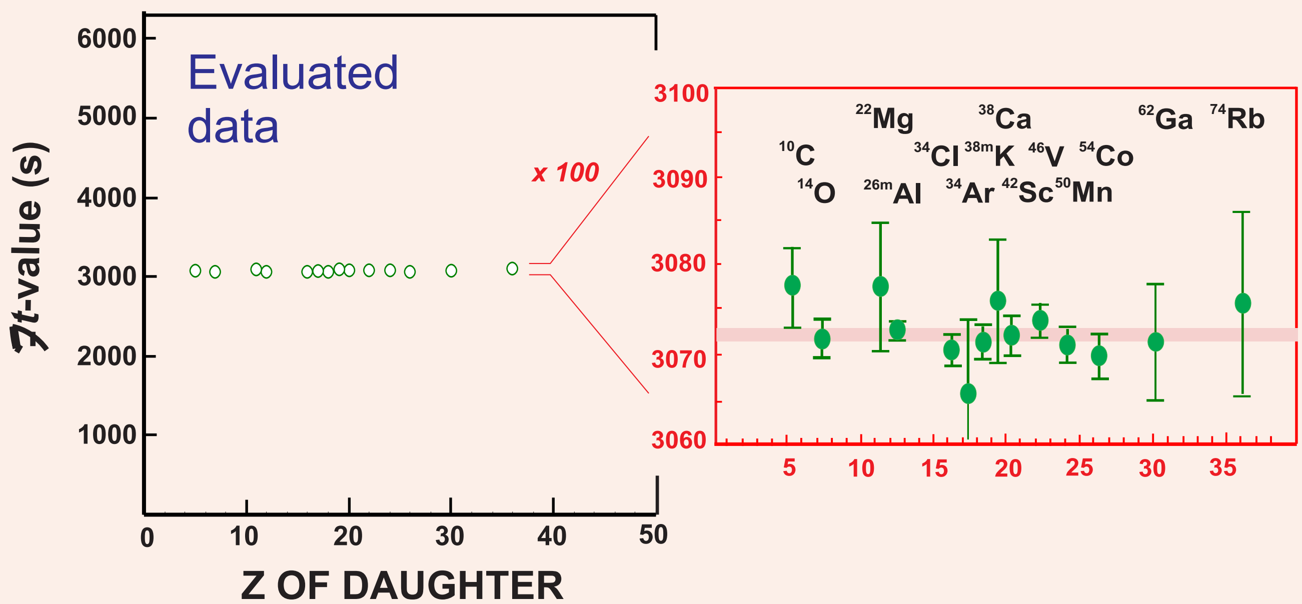
Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

G_V constant to ± 0.011%



$\overline{\mathcal{F}t}$ = 3072.1(7)

$G_V(1+\Delta_R)^{1/2}/(\hbar c)^3$ = 1.14962(13) x10^{-5} GeV^{-2}

$\chi^2/\nu = 0.6$

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2 (1 + \Delta_R)$

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms

G_V constant to ± 0.011%

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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Experimentally determine $G_V^2(1 + \Delta_R)$

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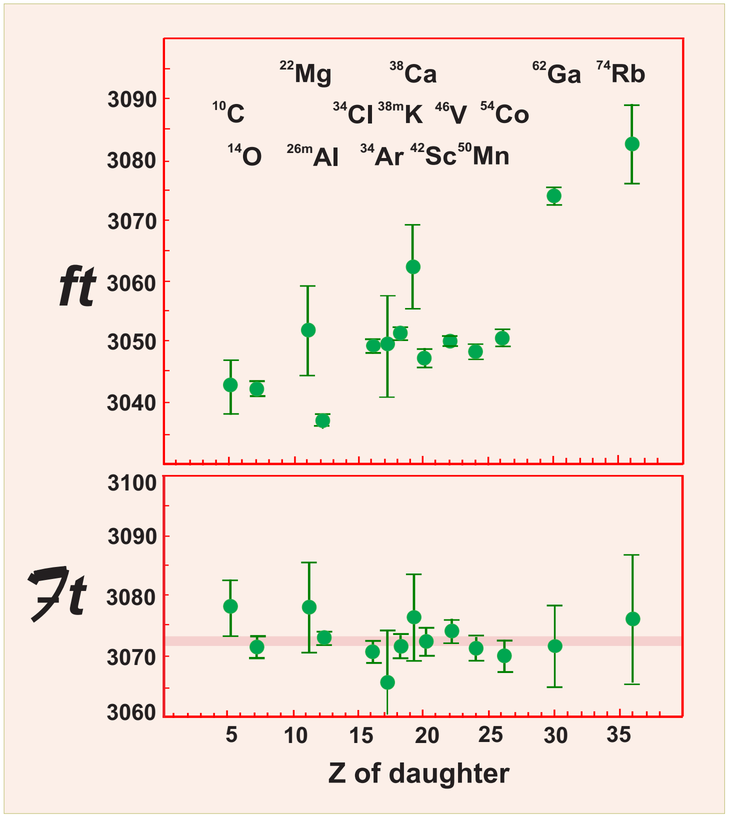

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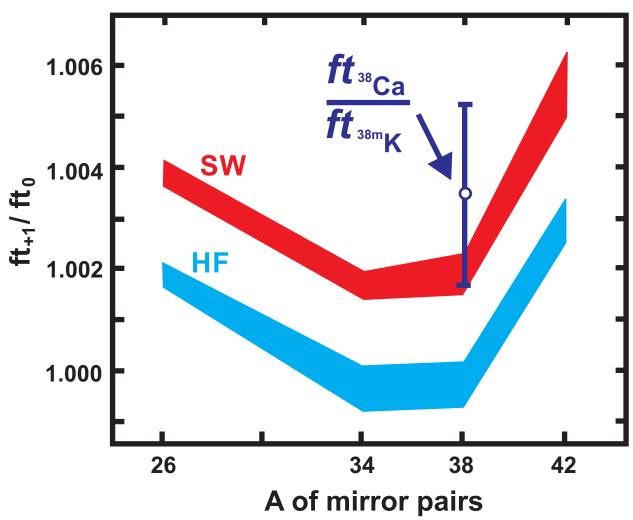
FROM MANY TRANSITIONS

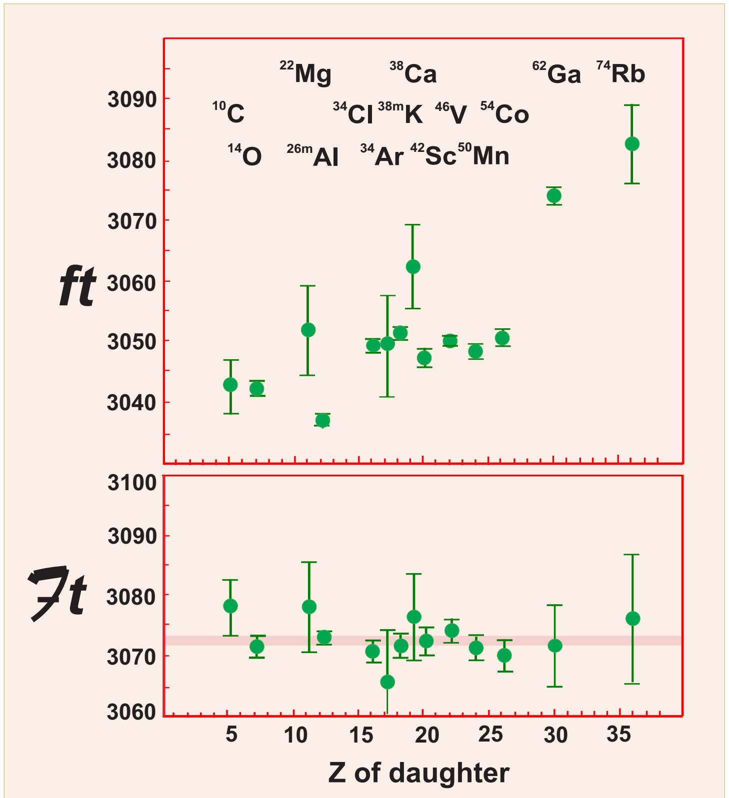
Test Conservation of the Vector current (CVC)

Validate correction terms ✔

G_V constant to ± 0.011%

Model	χ^2/N	CL(%)
SM-SW	1.37	17
SM-HF	6.38	0
DFT	4.26	0
RHF-RPA	4.91	0
RH-RPA	3.68	0





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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

Test for Scalar current

G_V constant to ± 0.011%

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

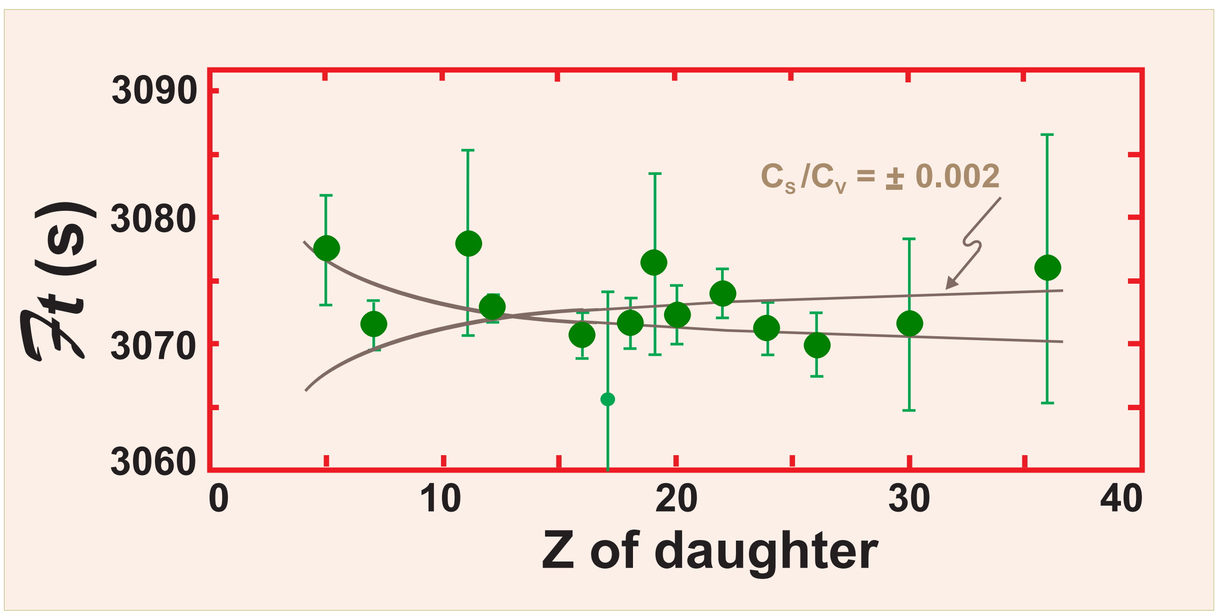
Test Conservation of the Vector current (CVC)

G_V constant to ± 0.011%

Validate correction terms ✔

Test for Scalar current

limit, $C_S/C_V = 0.0012\ (10)$



RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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FROM MANY TRANSITIONS

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Validate correction terms ✔

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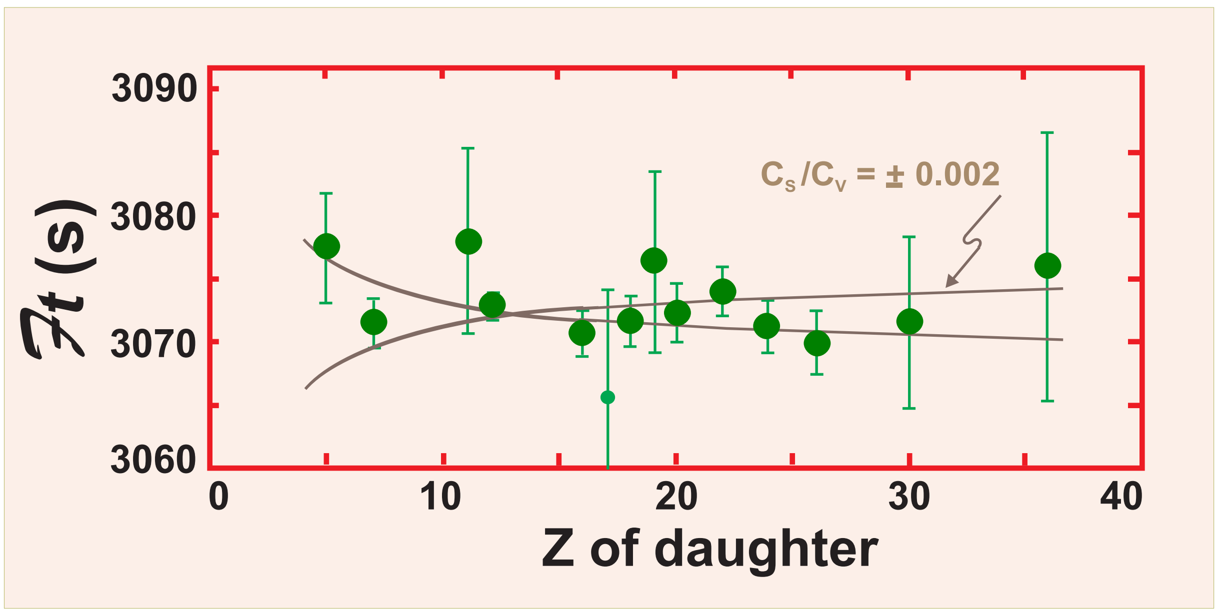

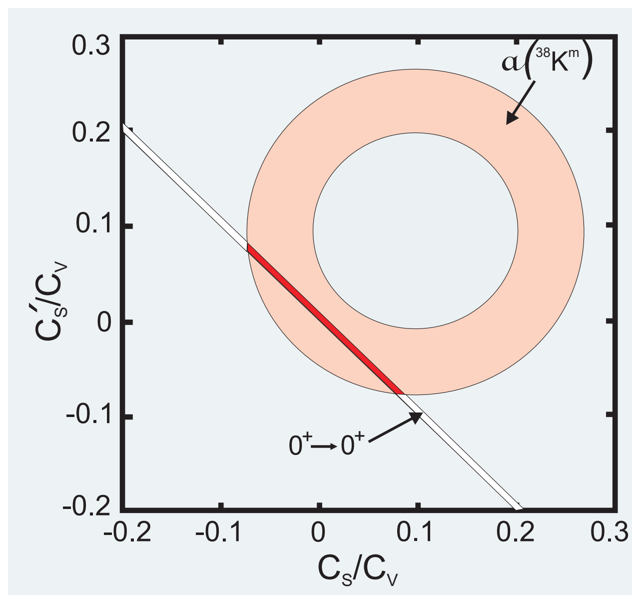

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

G_V constant to ± 0.011%

Test for Scalar current

limit, $C_S/C_V = 0.0012\ (10)$

WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

mass eigenstates

Cabibbo-Kobayashi-Maskawa matrix

Obtain precise value of $G_V^2(1 + \Delta_R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G_\mu^2 = 0.94907 \pm 0.00041$$

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

Test for Scalar current

G_V constant to ± 0.011%

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WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates (left) — mass eigenstates (right)

Cabibbo-Kobayashi-Maskawa matrix

Obtain precise value of $G_V^2(1 + \Delta_R)$

Determine V_{ud}^2

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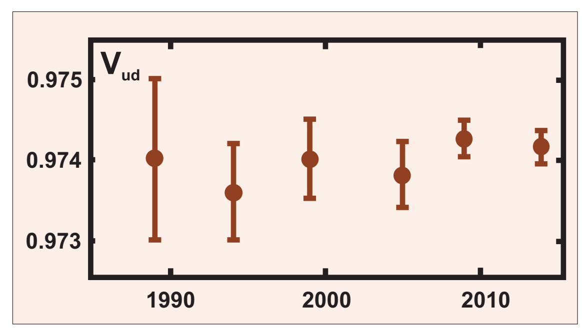

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Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

G_V constant to $\pm$ 0.011%

Test for Scalar current

limit, $C_S/C_V = 0.0012\ (10)$

WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates (left vector); mass eigenstates (right vector)

Cabibbo-Kobayashi-Maskawa matrix

Obtain precise value of $G_V^2(1 + \Delta_R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G_\mu^2 = 0.94907 \pm 0.00041$$

Test CKM unitarity

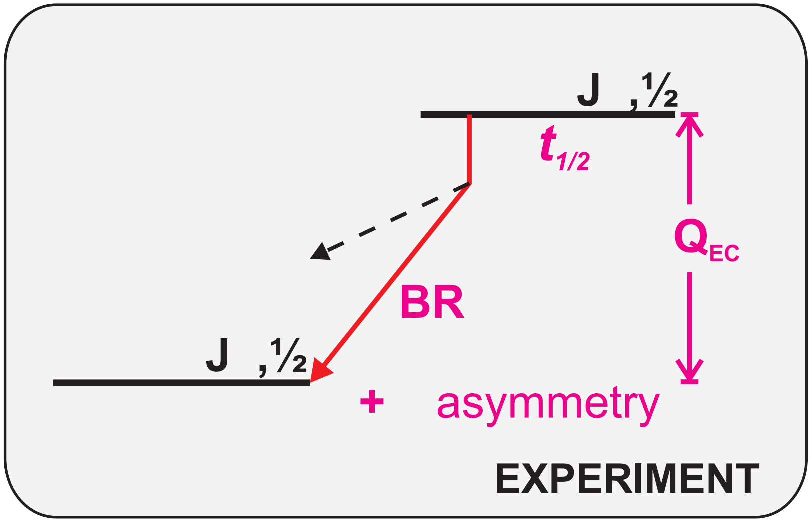
$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99985 \pm 0.00055$$

T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

$$ft = \frac{K}{G_V^2 \langle \quad \rangle^2 + G_A^2 \langle \quad \rangle^2}$$

f = statistical rate function: f(Z, Q_{EC})
t = partial half-life: f($t_{1/2}$, BR)
$G_{V,A}$ = coupling constants
$\langle \ \rangle$ = Fermi, Gamow-Teller matrix elements



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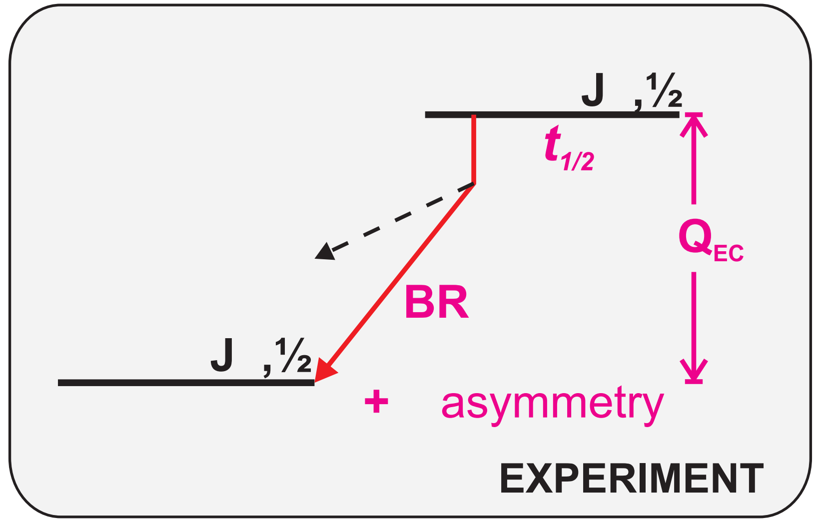

INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft\,(1 + \ '_R)[1 - (\ _C - \ _{NS})] = \frac{K}{G_V^2\,(1 + \ _R)(1 + \ ^2\langle\qquad\rangle^2)}$$

$= G_A/G_V$

T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

$$ft = \frac{K}{G_V^2 < \; >^2 + G_A^2 < \quad >^2}$$

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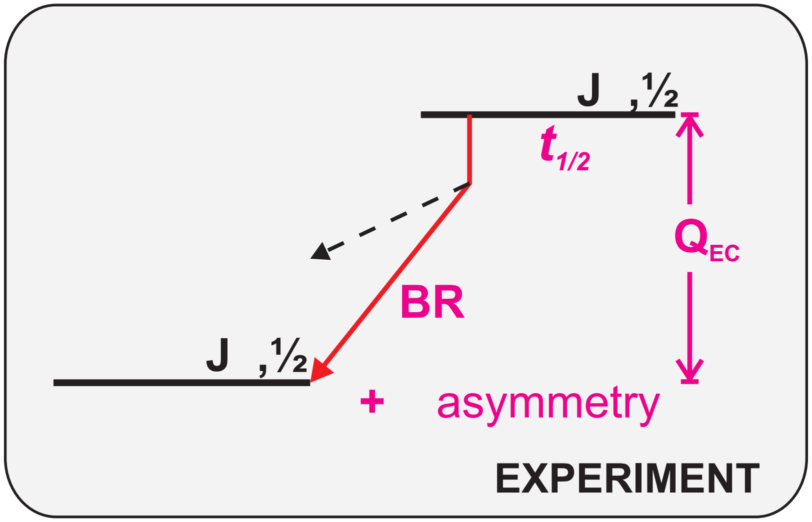

INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft\,(1 + \;'_R)[1 - (\;_C - \;_{NS})] = \frac{K}{G_V^2\,(1 + \;_R)(1 + \;^2 < \quad >^2)}$$

$= G_A/G_V$

Requires additional experiment:
for example, asymmetry (*A*)

T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

$$ft = \frac{K}{G_V^2 \langle \quad \rangle^2 + G_A^2 \langle \quad \rangle^2}$$

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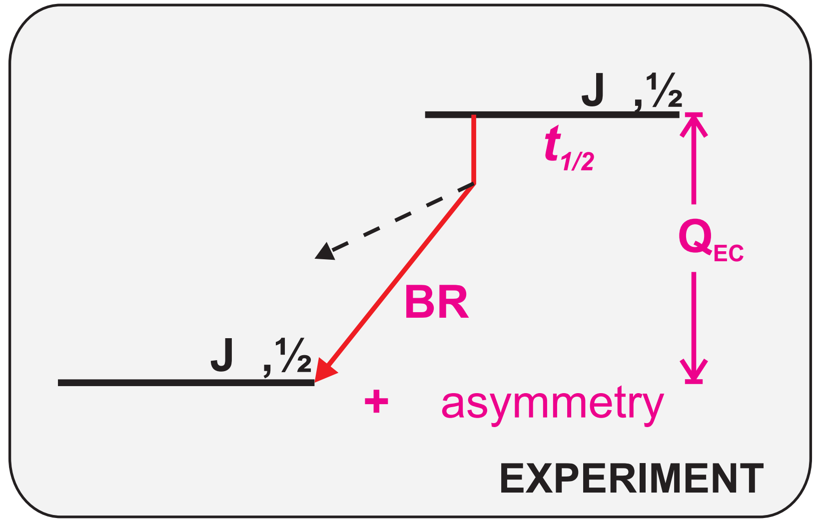


INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft\,(1 + \,'_R)[1 - (\,_C - \,_{NS})] = \frac{K}{G_V^2\,(1 + \,_R)(1 + \,^2 \langle \quad \rangle^2)}$$

$= G_A/G_V$

NEUTRON DECAY

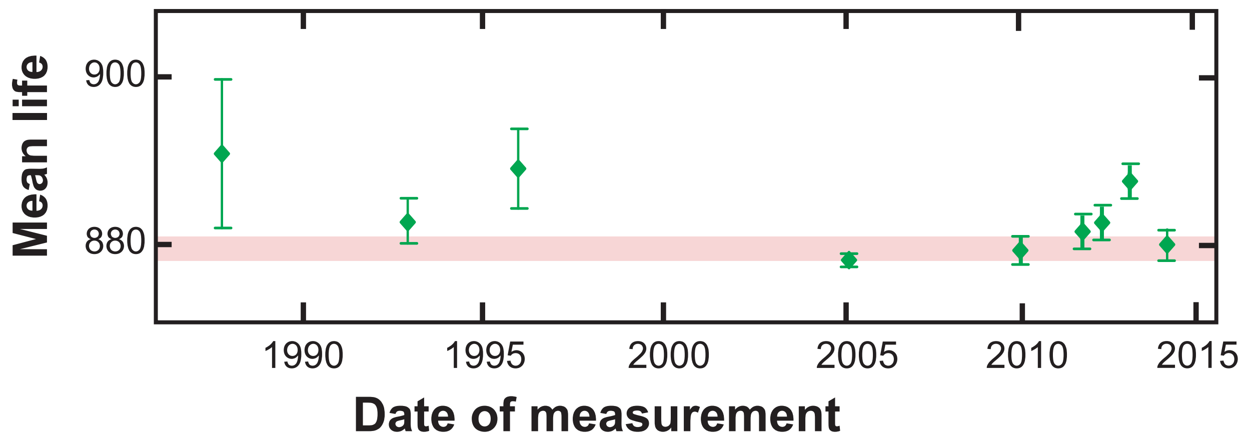
Requires additional experiment:
for example, asymmetry (*A*)

NEUTRON DECAY DATA 2015

Mean life:

τ = **880.2 ± 1.0 s**

χ^2/N = **3.4**

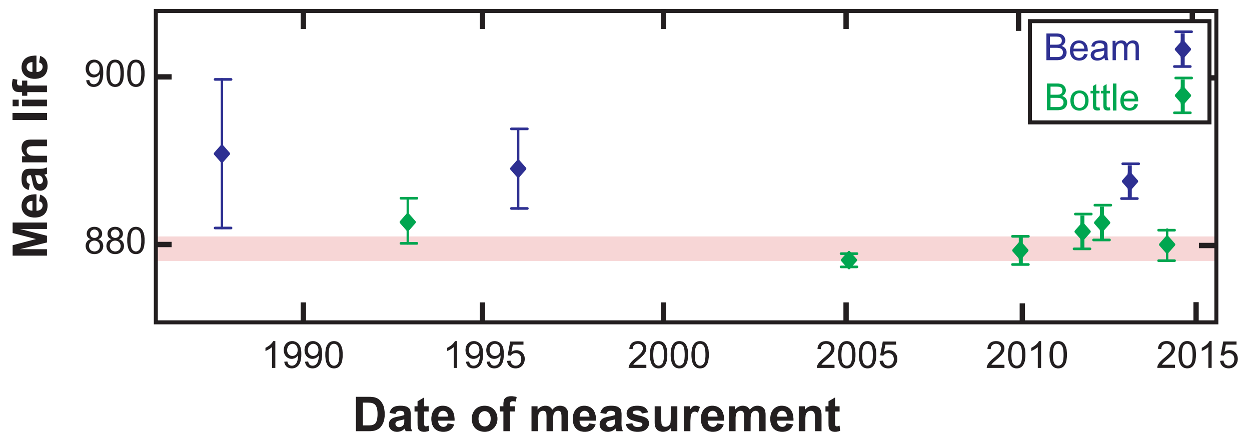


NEUTRON DECAY DATA 2015

Mean life:

τ **= 880.2 ± 1.0 s**

χ^2/N **= 3.4**



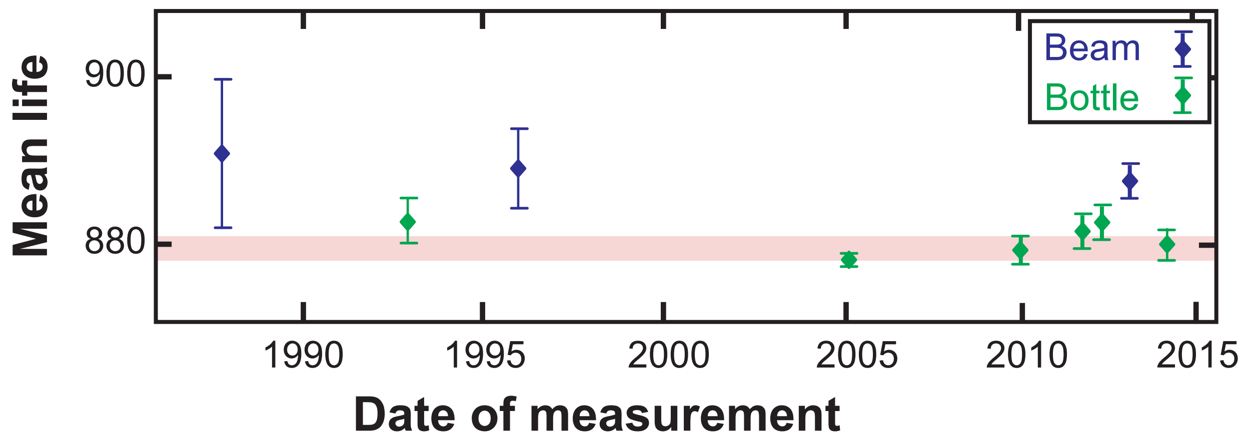
NEUTRON DECAY DATA 2015

Mean life:

τ = 880.2 ± 1.0 s

$\chi^2/N = 3.4$

Beam: 888.1 ± 2.0 s
Bottle: 879.5 ± 0.7 s



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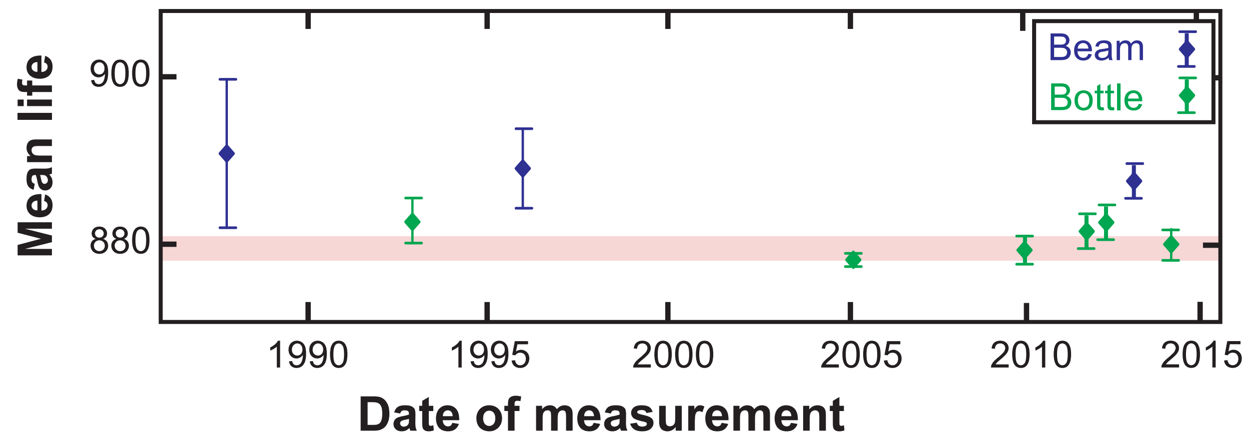

β **asymmetry:**

λ **= -1.2725 ± 0.0020**

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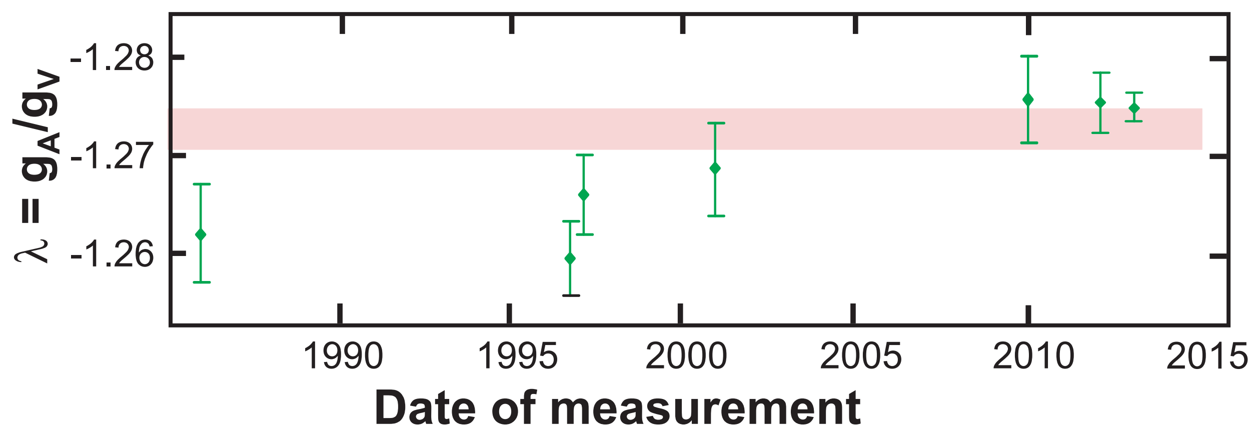

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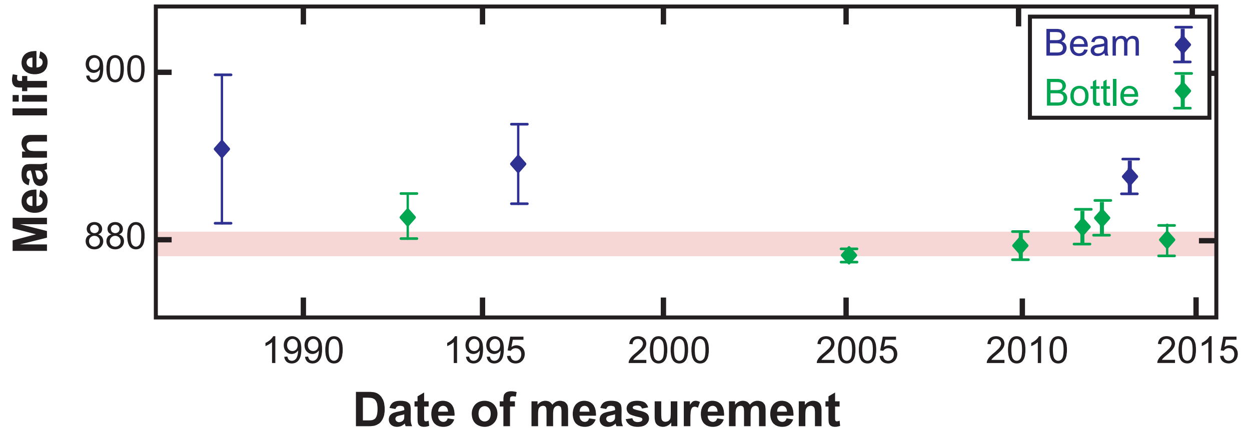

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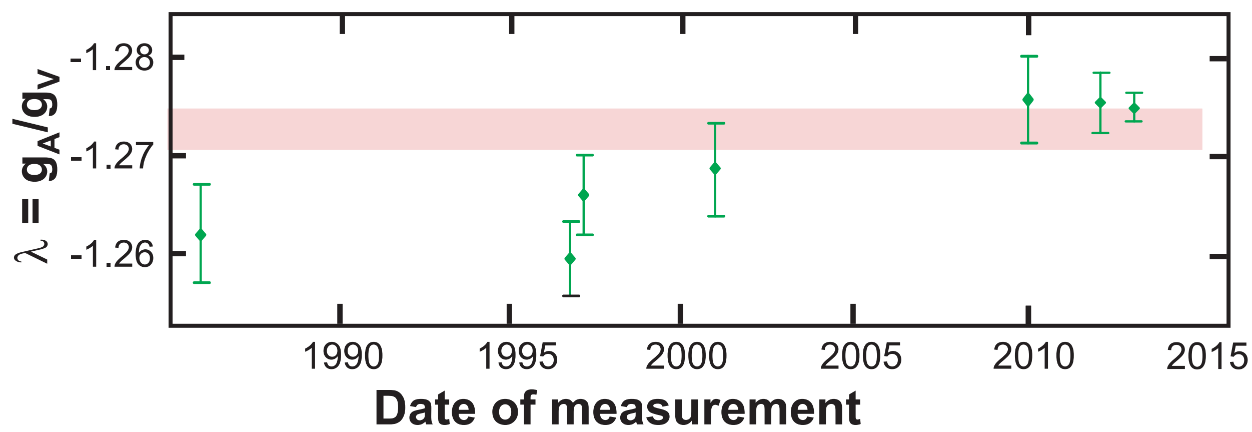

$V_{ud} = 0.9754 \pm 0.0014$

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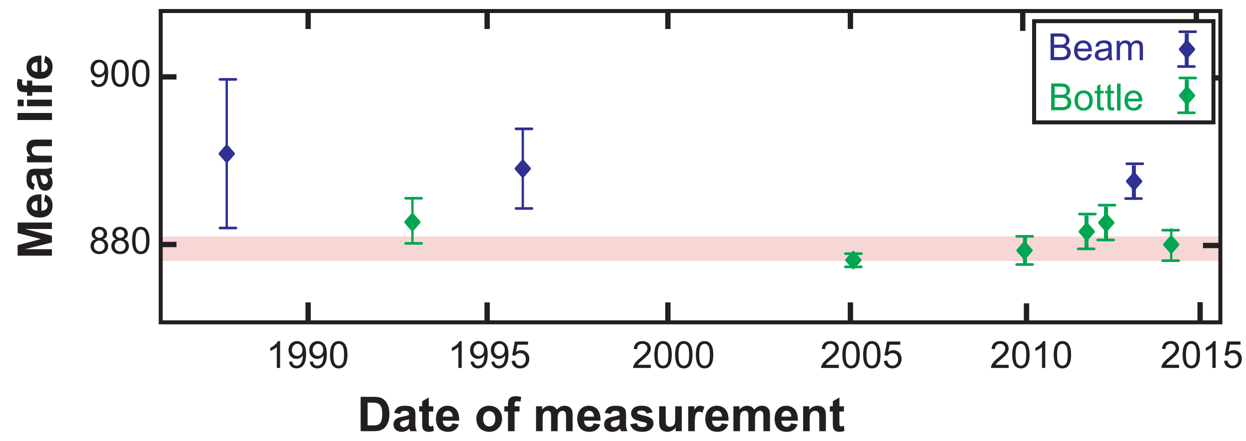

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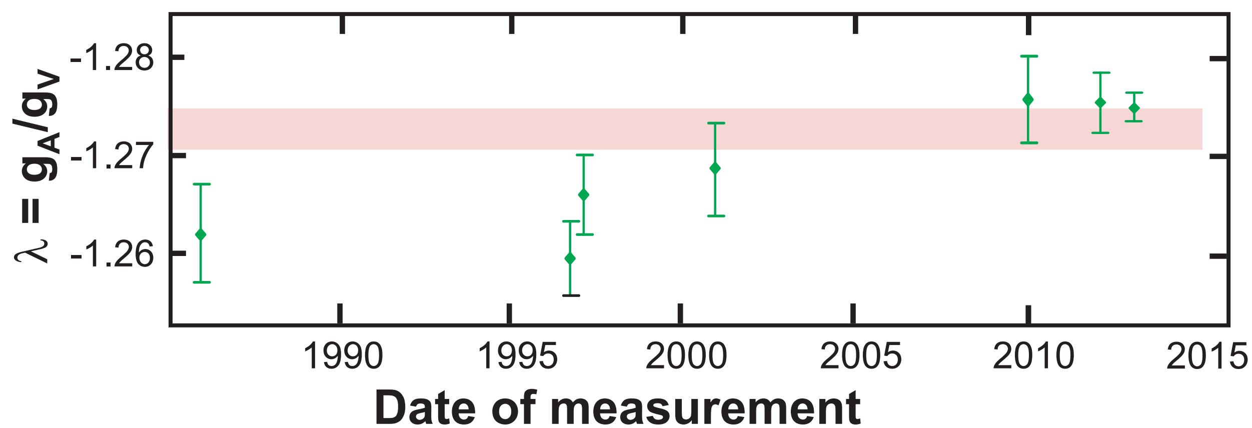

V_{ud} **= 0.9754 ± 0.0014**

Beam-bottle span
$0.9707 \leq V_{ud} \leq 0.9761$

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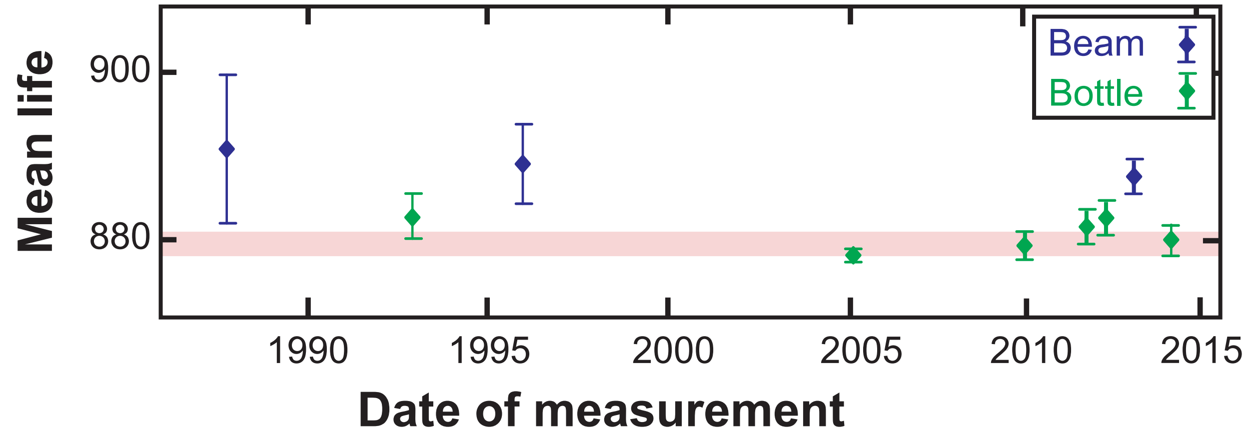

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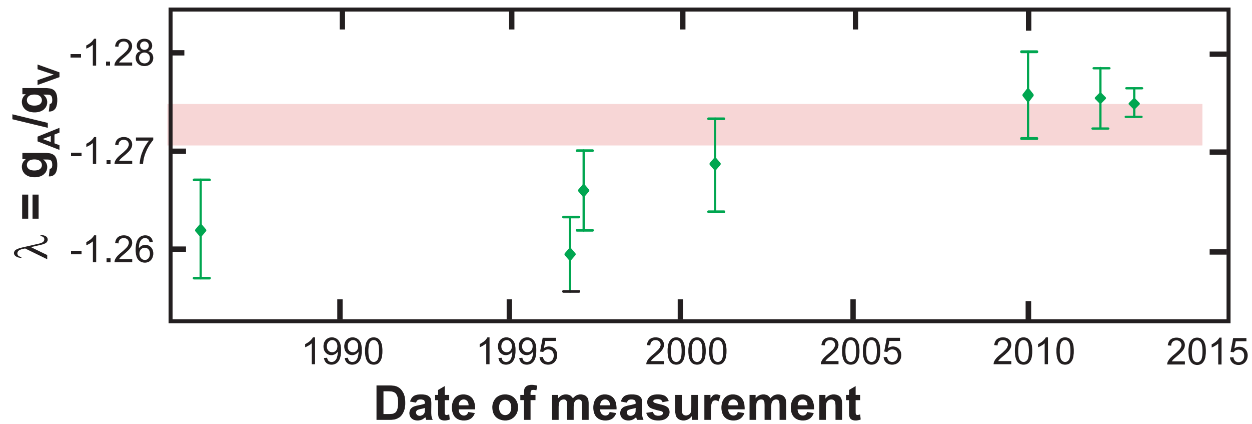

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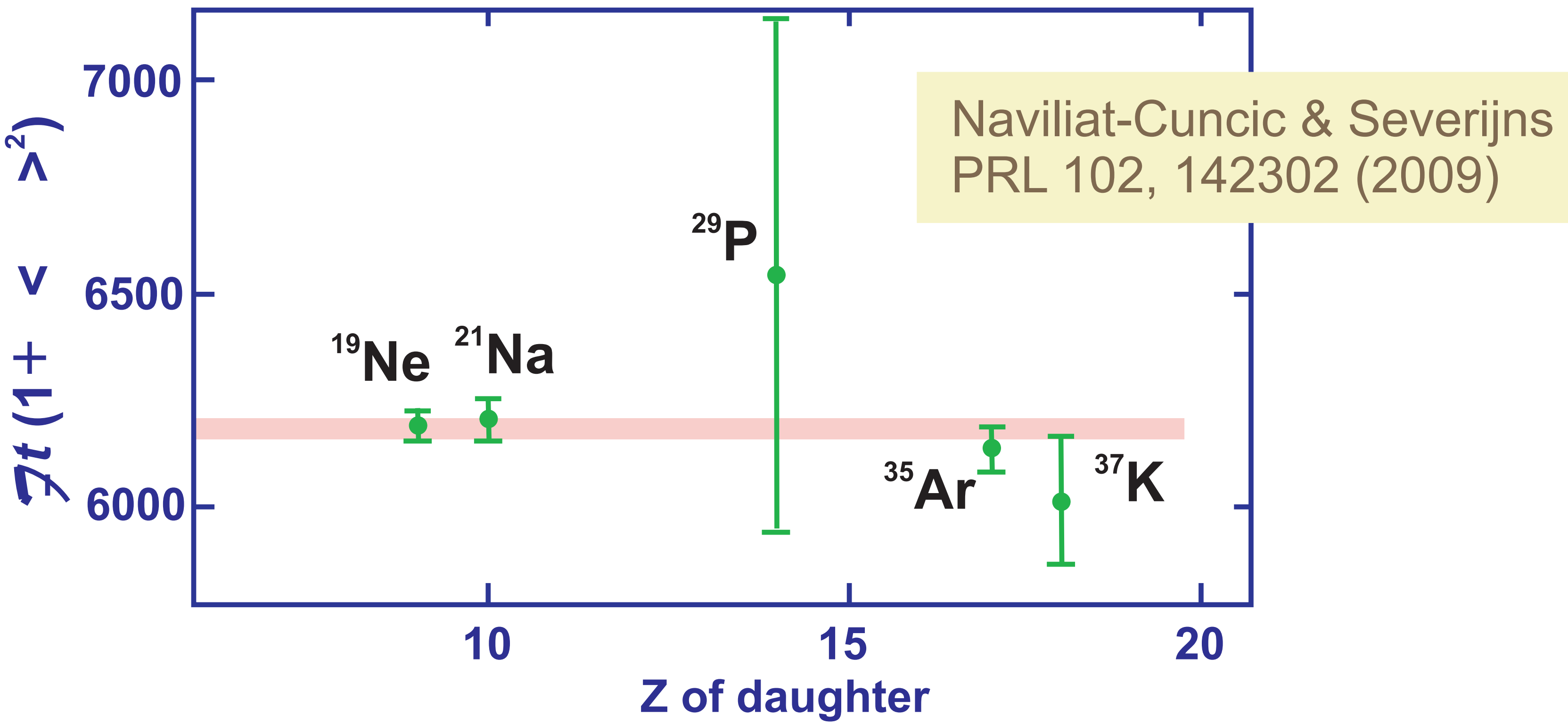
nuclear $0^+ \rightarrow 0^+$
V_{ud} **= 0.9742 ± 0.0002**

NUCLEAR T=1/2 MIRROR DECAY DATA 2009

$$\mathcal{F}t = ft\,(1 + {}'_{R})[1 - ({}_{C} - {}_{NS})] = \frac{K}{G_V^2\,(1 + {}_{R})(1 + < \quad >^2)}$$



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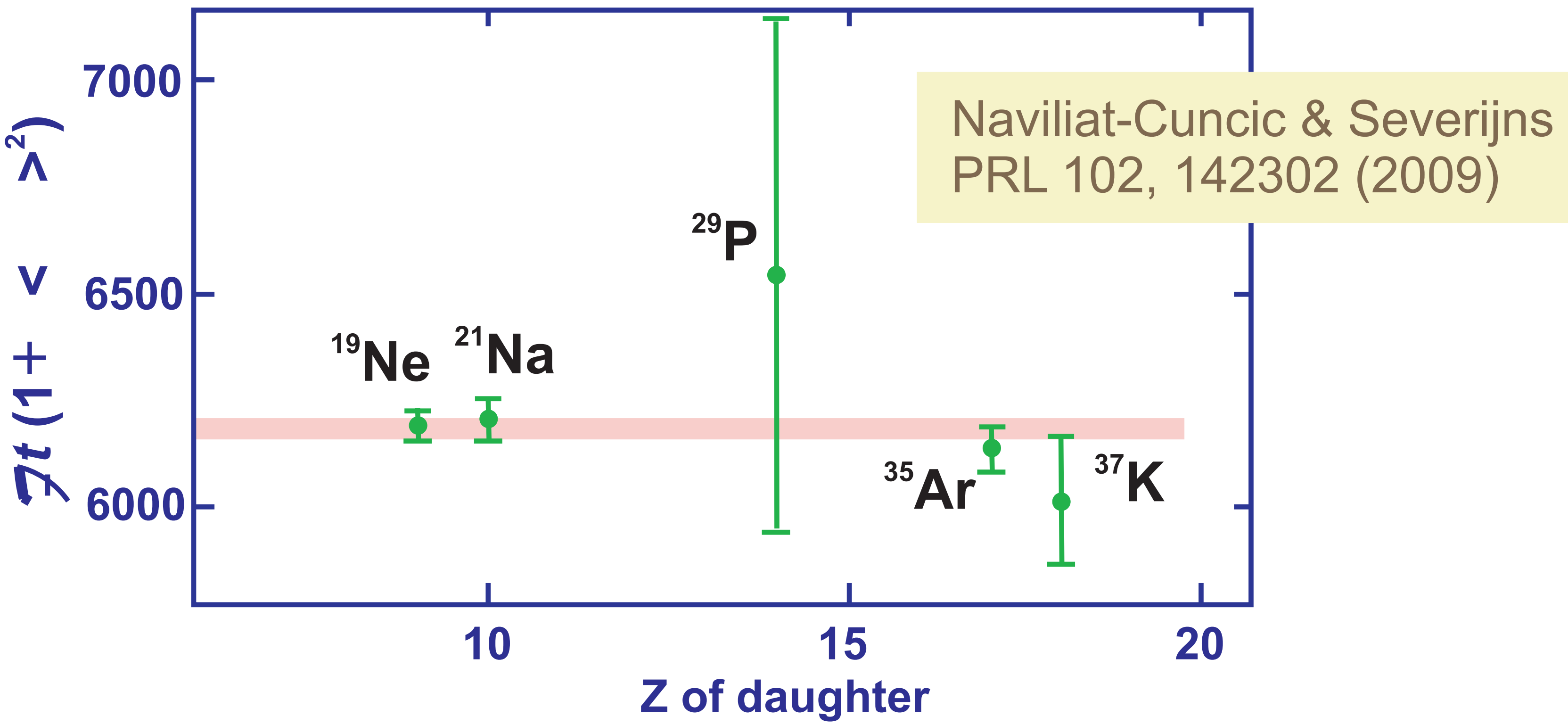
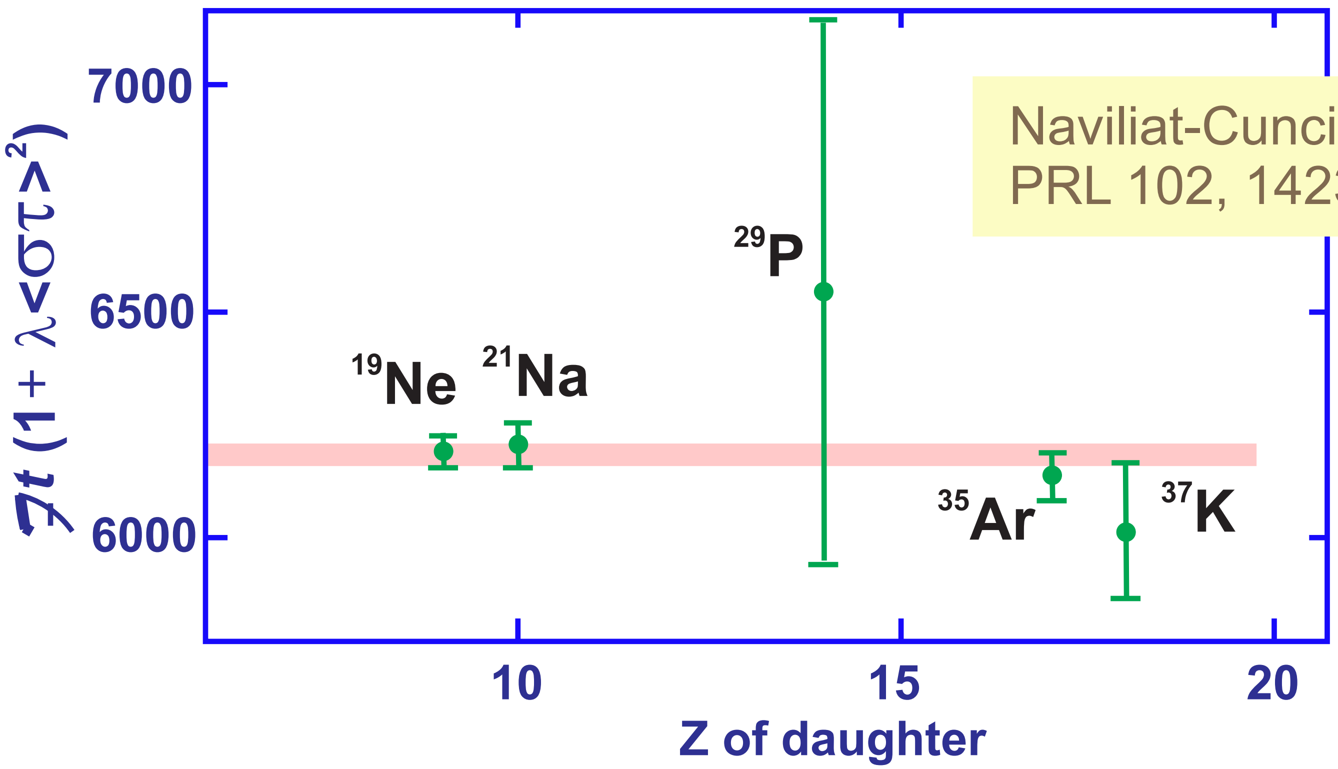


$V_{ud} = 0.9719 \pm 0.0017$

NUCLEAR T=1/2 MIRROR DECAY DATA 2009

$$\mathcal{F}t = ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{G_V^2\,(1 + \Delta_R)(1 + \lambda^2 \langle\sigma\tau\rangle^2)}$$



Naviliat-Cuncic & Severijns
PRL 102, 142302 (2009)

$V_{ud} = 0.9719 \pm 0.0017$

nuclear $0^+ \rightarrow 0^+$

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PION BETA DECAY

Decay process:

$$^{+} \longrightarrow {}^{0}\, e^{+} \quad {}_{e}$$

$$0^{-},1 \longrightarrow 0^{-},1$$

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Experimental data:

$= 2.6033 \pm 0.0005 \times 10^{-8}$ s (PDG 2009)

BR $= 1.036 \pm 0.007 \times 10^{-8}$

Pocanic *et al*,
PRL 93, 181803 (2004)

Result:

$$V_{ud} = 0.9749 \pm 0.0026$$

PION BETA DECAY

Decay process:

$$\pi^+ \longrightarrow \pi^0 e^+ \nu_e$$

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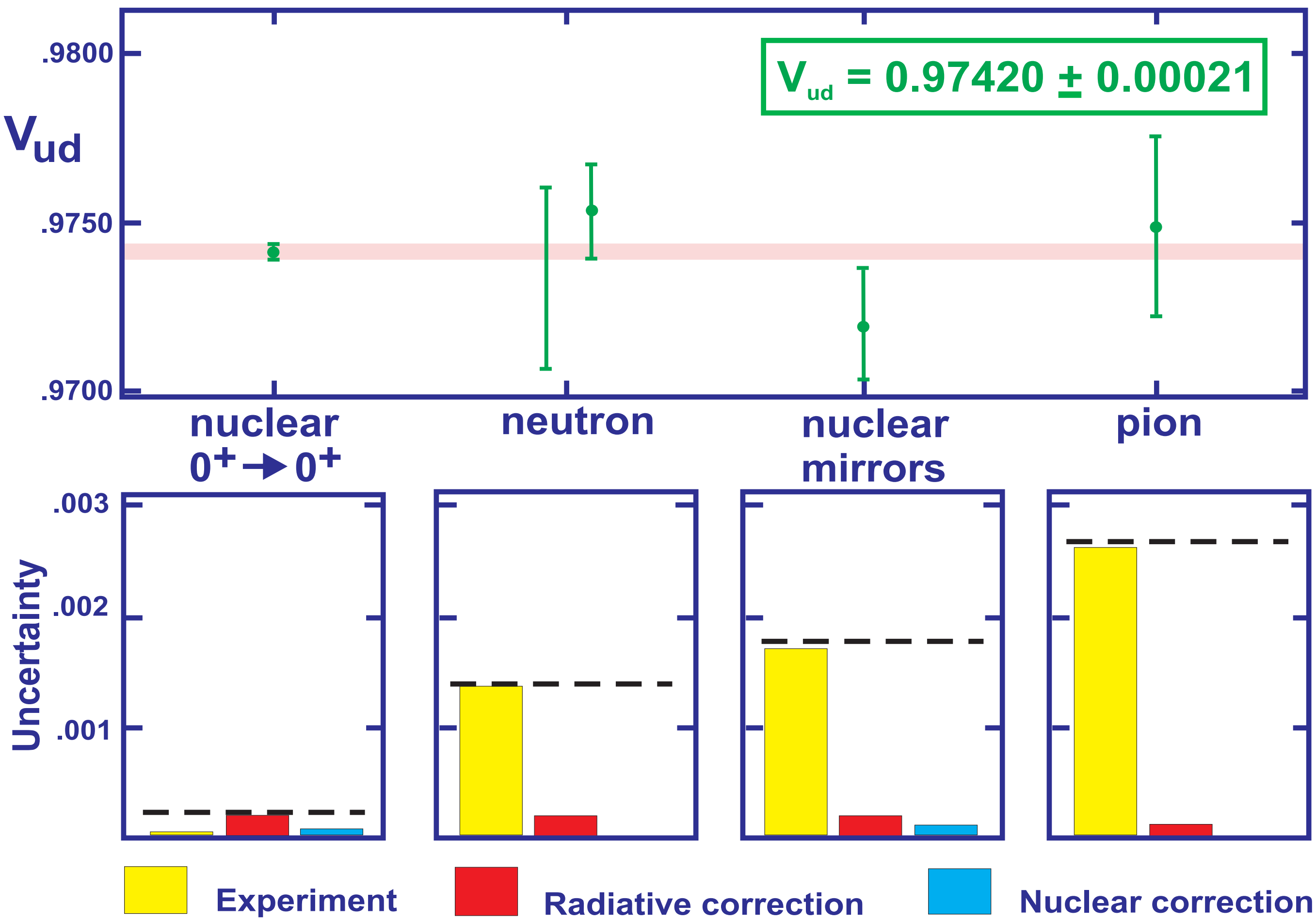
CURRENT STATUS OF V_{ud} AND CKM UNITARITY



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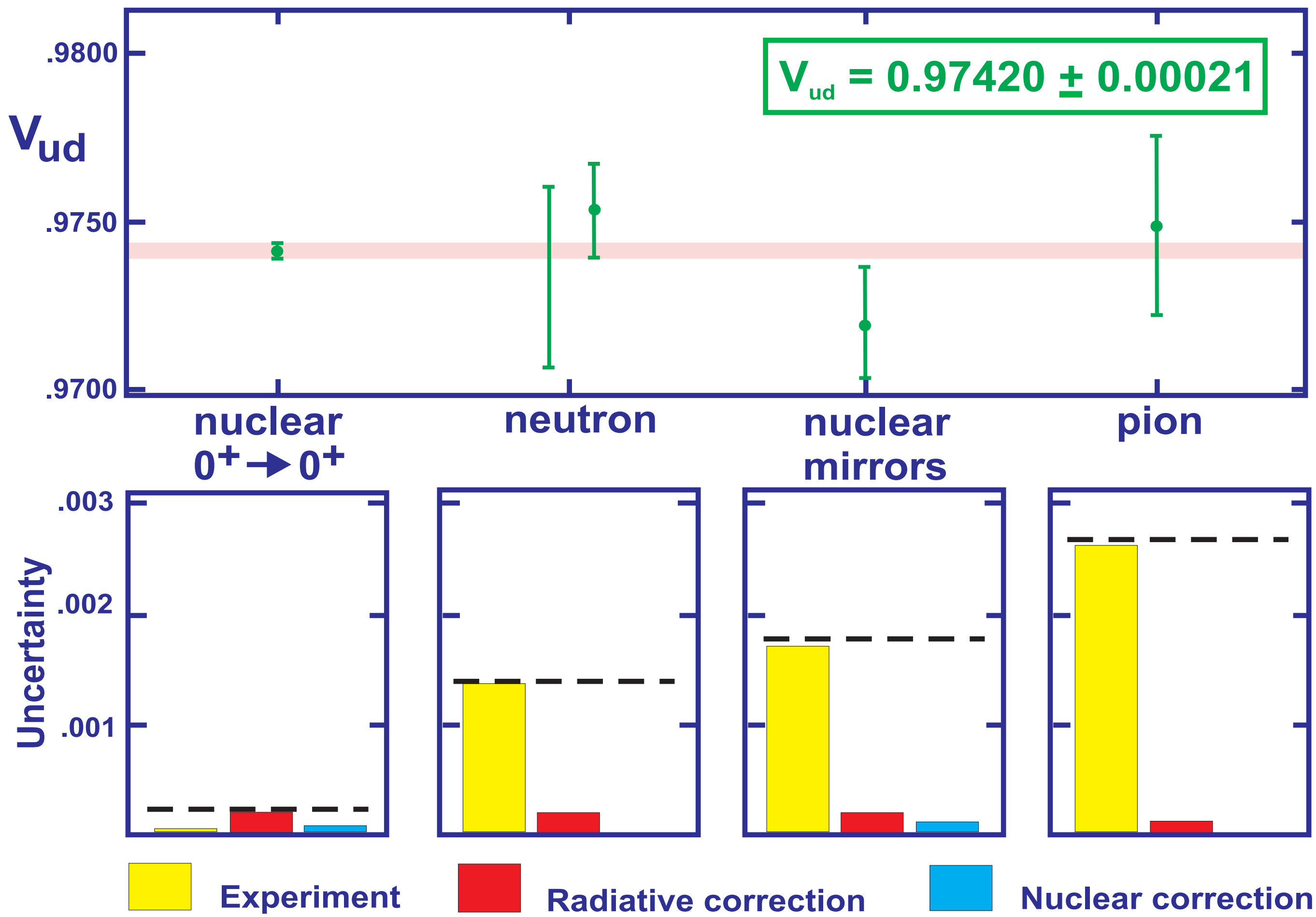

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99985 \pm 0.00055$$

V_{ud}^2 nuclear decays, muon decay: 0.94907 ± 0.00041

V_{us}^2 PDG, kaon decays: 0.05076 ± 0.00036

V_{ub}^2 B decays: 0.00002 ± 0.00001

FINAL REMARK ON V_{us}

Kaon decay yields two independent determinations of V_{us}:

1) Semi-leptonic $K \rightarrow \pi \ell \nu_\ell$ decay ($K_{\ell 3}$) yields $|V_{us}|$.

2) Pure leptonic decays $K^+ \rightarrow \mu^+ \nu_\mu$ and $\pi^+ \rightarrow \mu^+ \nu_\mu$ together yield $|V_{us}| / |V_{ud}|$.

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BUT, Bazavov et al. [PRL 112, 112001 (2014)] produced a new lattice calculation of the form factor used for $K_{\ell 3}$ decays.

Their new result for $|V_{us}|$ is inconsistent with the $|V_{us}|/|V_{ud}|$ result and, when combined with the superallowed result for $|V_{ud}|$, leads to a unitarity sum over two standard deviations below 1.

Stay tuned ...

SUMMARY AND OUTLOOK

1. Analysis of superallowed $0^+ \rightarrow 0^+$ nuclear β decay confirms CVC to ±0.011% and thus yields V_{ud} = 0.97420(21).
2. The three other experimental methods for determining V_{ud} yield consistent results, but are less precise by a factor of 7 or more.
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4. The largest contribution to V_{ud} uncertainty is from the inner radiative correction, Δ_R. Very little reduction in V_{ud} uncertainty is possible without improved calculation of Δ_R.
5. Isospin symmetry-breaking correction, δ_C, has been tested by requiring consistency among the 14 known transitions (CVC), and agreement with mirror-transition pairs. It contributes much less to V_{ud} uncertainty than does Δ_R.
6. Tests on new mirror pairs are in progress. This requires precise half-life and branching-ratio measurements of T_Z = -1 parent decays.