Assessing lepton-flavour non-universality with $B \to K^* \mu^+ \mu^-$

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In collaboration with: **B. Capdevila, S. Descotes-Genon, L. Hofer and J. Virto**

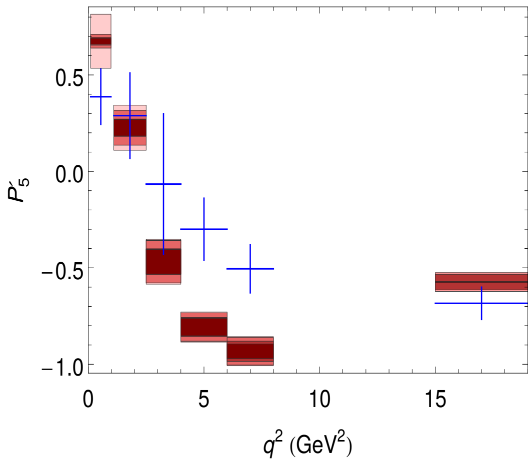
Based on: CDVM'16 and CDHM'16.

Outline

This talk will focus on the following questions:

- Setting the stage.
- A new generation of ULFV observables:
  1. Q_i (and $\hat{Q}_i$ observables)
  2. B_j (an $\widetilde{B}_j$ observables) with j=5,6s.
  3. $\widetilde{M}$ a transversity independent charm free observable at low-q^2 even with $\delta C_9^{\rm NP} \neq 0$.
- Definitions, properties and ability to disentangle NP scenarios.

Present situation

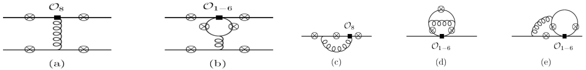


P_5' was proposed in **DMRV, JHEP 1301(2013)048**

$$P_5' = \sqrt{2}\frac{\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*})}{\sqrt{|A_0|^2(|A_\perp|^2 + |A_\parallel|^2)}} = \sqrt{2}\frac{\mathrm{Re}[n_0 n_\perp^\dagger]}{\sqrt{|n_0|^2(|n_\perp|^2 + |n_\parallel|^2)}}.$$

- 2013: 1fb^{-1} dataset LHCb found 3.7σ
- 2015: 3fb^{-1} dataset LHCb found 3σ in 2 bins.
- Belle confirmed it in a bin [4,8] few months ago.

1 Computed in i-QCDF + KMPW+ 4-types of correct. $\mathbf{F^{full}(q^2)} = F^{soft}(\xi_\perp, \xi_\parallel) + \triangle F^{\alpha_s}(q^2) + \triangle F^{p.c.}(q^2)$

type of correction	Factorizable	Non-Factorizable
α_s-QCDF	$\triangle F^{\alpha_s}(q^2)$	
power-corrections	$\triangle F^{p.c.}(q^2)$	LCSR with single soft gluon contribution

2 Another group [BSZ] found using full-FF approach and BSZ-FF very similar result ($\lesssim$ errors).

Other tensions beyond P_5'...

Systematic low-recoil small tensions:

$b \to s\mu^+\mu^-$ (low-recoil)	bin	SM	EXP	Pull
$10^7 \times \mathrm{BR}(B^0 \to K^0\mu^+\mu^-)$	[15,19]	0.91 ± 0.12	0.67 ± 0.12	+1.4
$10^7 \times \mathrm{BR}(B^0 \to K^{*0}\mu^+\mu^-)$	[16,19]	1.66 ± 0.15	1.23 ± 0.20	+1.7
$10^7 \times \mathrm{BR}(B^+ \to K^{*+}\mu^+\mu^-)$	[15,19]	2.59 ± 0.25	1.60 ± 0.32	**+2.5**
$10^7 \times \mathrm{BR}(B_s \to \phi\mu^+\mu^-)$	[15,18.8]	2.20 ± 0.17	1.62 ± 0.20	**+2.2**

After including the BSZ DA correction that affected the error of twist-4:

$10^7 \times \mathrm{BR}(B_s \to \phi\mu^+\mu^-)$	SM	EXP	Pull
[0.1,2]	1.56 ± 0.35	1.11 ± 0.16	+1.1
[2,5]	1.55 ± 0.33	0.77 ± 0.14	**+2.2**
[5,8]	1.89 ± 0.40	0.96 ± 0.15	**+2.2**

Global fit to $\sim$ 90 obs. (radiative+$b \to s\mu^+\mu^-$)

All deviations add up constructively

- A new physics contribution to $C_{9,\mu}$=-1.1 with a pull-SM of 4.5σ alleviates all anomalies and tensions.
  - 3.8σ if only $b \to s\mu^+\mu^-$ excluding [6,8]
  - 2.8σ if only low-recoil considered.
- NP contributions to the rest of Wilson coefficient are not (**for the moment**) yet significantly different from zero.

No $b \to se^+e^-$ data included at this point.

NATURE shows us two very different faces.....

The strongest signal of NP in C_9

- This coefficient is affected by long-distance charm contributions.

$$C_9^{\text{eff},i} = C_{9\ \text{SM pert}}^{\text{eff}}(q^2) + C_9^{\text{NP}} + C_9^{l.d.\ c\bar{c}(i)}(q^2)$$

Hints of lepton-flavour non-universal NP

- Observables probing ULFV are free from long-distance charm pollution in the SM, i.e., free from $C_9^{l.d.\ c\bar{c}(i)}(q^2)$.
- Only NP can explain tensions w.r.t SM in these observables.

A brief (or not so) parenthesis on hadronic uncertainties

There are two ways to discard attempts of explanation (factorizable p.c, charm) of the anomaly in P_5' within the SM:

1. Direct deconstruction of arguments ($\rightarrow$ the case of factorizable power corrections) or by comparison with data of explicit computations (not fits) of long-distance charm contributions (KMPW).

2. With the help of ULFV observables: if P_5' and ULFV observables share the same new physics explanation, no space for long-distance charm or other unknown hadronic uncertainties is left in P_5'.

let's play a bit first with 1 ....

$$\Big($$

Factorizable power corrections

$$\mathbf{F^{full}(q^2)} = F^{soft}(\xi_\perp, \xi_\|) + \triangle F^{\alpha_s}(q^2) + \triangle F^{p.c.}(q^2) \quad \text{with} \quad \triangle F^{p.c.}(q^2) = a_F + b_F(\frac{q^2}{m_B^2}) + \ldots$$

1) **Power correction error size**: In JC'14 (and DHMV'14) they take uncorrelated errors among p.c. missing that this choice introduces scheme (definition of SFF) dependence. *Numerically:*

ONLY power correction error of $\langle P_5' \rangle_{[4,6]}$	error of f.f.+p.c. scheme-1 in transversity basis **DHMV'14**	error of f.f.+p.c. scheme-2 in helicity basis **JC'14**
NO correlations among errors of p.c. (hyp. 10%)	± 0.05	$\pm$**0.15**
WITH correlations among errors of p.c.	± 0.03	± 0.03

Their scheme's choice inflates error **artificially**.

Analytically:

$$P_5' = P_5'|_\infty \Big[1 + \frac{\mathbf{aV_-} - \mathbf{aT_-}}{\xi_\perp} \frac{m_B}{|k|} \frac{m_B^2}{q^2} C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})} - \frac{\mathbf{aV_+}}{\xi_\perp} \frac{\mathbf{2C_{9,\|}}}{\mathbf{C_{9,\perp} + C_{9,\|}}}$$

$$+ \frac{aV_0 - aT_0}{\xi_\|} 2 C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\|}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})} + \mathcal{O}\left(\frac{m_{K^*}^2}{m_B^2}, \frac{q^2}{m_B^2}\right) \Big]$$

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Long-distance contributions from $c\bar{c}$ loops where the lepton pair is created by an electromagnetic current.

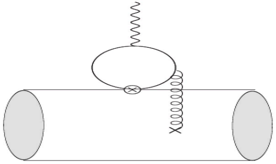
1. The γ couples universally to $\mu^\pm$ and $e^\pm$: R_K nor any LFVU cannot be explained by charm-loops.
2. KMPW is the only real computation of long-distance charm.

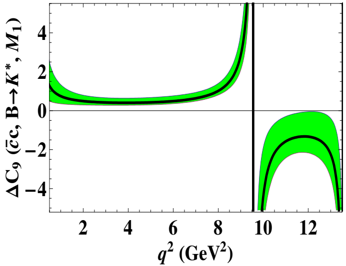
$$C_9^{\rm eff\, i} = C_9^{\rm eff}{}_{\rm SM\, pert}(q^2) + C_9^{\rm NP} + s_i \delta C_9^{c\bar{c}(i)}{}_{\rm KMPW}(q^2)$$

KMPW implies $s_i = 1$, but we vary $s_i = 0 \pm 1$, $i = 0, \perp, \|$.

$$\delta C_9^{{\rm LD},(\perp,\|)}(q^2) = \frac{a^{(\perp,\|)} + b^{(\perp,\|)} q^2 [c^{(\perp,\|)} - q^2]}{b^{(\perp,\|)} q^2 [c^{(\perp,\|)} - q^2]}$$

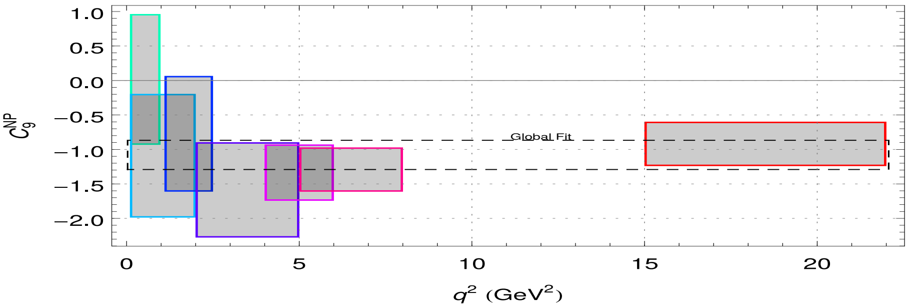
$$\delta C_9^{{\rm LD},0}(q^2) = \frac{a^0 + b^0 [q^2 + s_0][c^0 - q^2]}{b^0 [q^2 + s_0][c^0 - q^2]}$$





$\boxed{3}$ Bin-by-bin global fit analysis of C_9 tells you if a residual q^2 dependence is present.

$\Rightarrow$ if the values obtained are flat, charm is well estimated.



- We use KMPW. Notice the excellent agreement of bins [2,5], [4,6], [5,8].
  $C_9^{NP\,[2,5]} = -1.6 \pm 0.7$, $C_9^{NP\,[4,6]} = -1.3 \pm 0.4$, $C_9^{NP\,[5,8]} = -1.3 \pm 0.3$

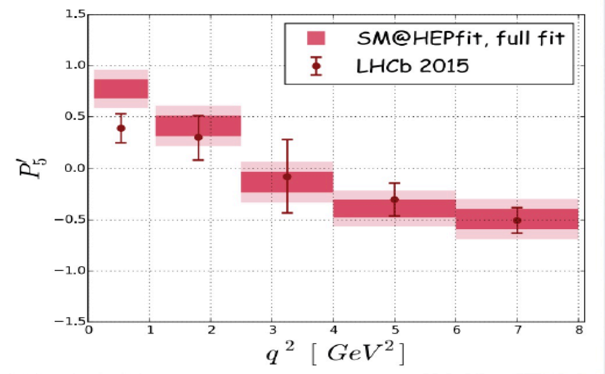
- **We do not find any indication for a q^2-dependence in C_9 neither in the plots nor in a 6D fit adding $a^i + b^i s$ to $C_9^{\rm eff}$ for $i = K^*, K, \phi$.** $\rightarrow$ disfavours again charm explanation.

Another group [Silvestrini et al.] argue that maybe there is an unknown and very hard to compute charm contribution (that they do not even try to compute or estimate) that explain only one anomaly.

An anatomy/deconstruction of (Ciuchini et al'15)

There is certain confusion in the literature related to the correct interpretation of [Ciuchini et al.'15].

1) **Arbitrary** parametrization $h_\lambda = h_\lambda^{(0)} + h_\lambda^{(1)} q^2 + h_\lambda^{(2)} q^4$ and fit ONLY LHCb data @low-q^2.



THIS IS JUST A FIT TO DATA: No dynamics is involved. If one adds 18 free parameters one can fit easily anything.

Can one get a solid conclusion out of this result?

In v1 of that work we found an internal inconsistency of more than 4σ between their predictions.

$\rightarrow$ Reason error in S_4^{theory}. Example in bin [4,6]:

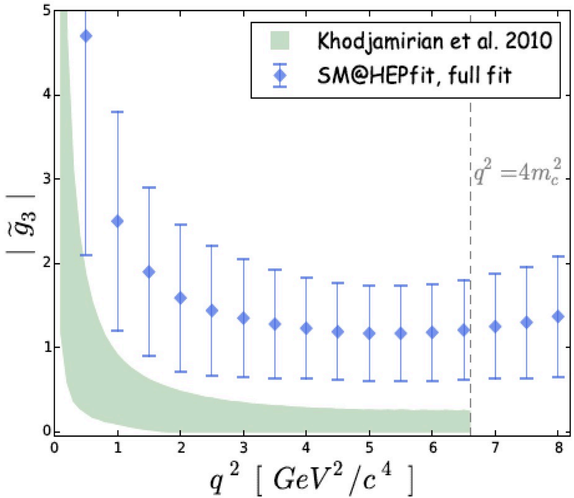
$$S_4^{v1} = -0.120 \pm 0.008 \text{ versus } S_4^{v2} = -0.241 \pm 0.014 \text{ they differ by } 7.5\sigma\text{!!!!!}$$

Surprisingly in abstract v1: " *good description* of current experimental data within SM" (also in v2...)

$\rightarrow$ Difficult to get a robust conclusion. So many parameters can swallow anything (real or spurious).

The paper has basically two parts:

I) Part-I Unconstrained fit: They simply confirm our results of the global fit (we obviously agree).



Consider again:

$$C_9^{\mathrm{eff\,i}} = C_9^{\mathrm{eff}}{}_{\mathrm{SM\,pert}}(q^2) + C_9^{\mathrm{NP}} + \delta C_9^{c\bar{c}(i)}{}_{\mathrm{KMPW}}(q^2)$$

where

$$\delta C_9^{c\bar{c}(i)}{}_{\mathrm{KMPW}}(q^2) \to |2C_1\tilde{g}_i^{CFFMPSV}| \to h_\lambda$$

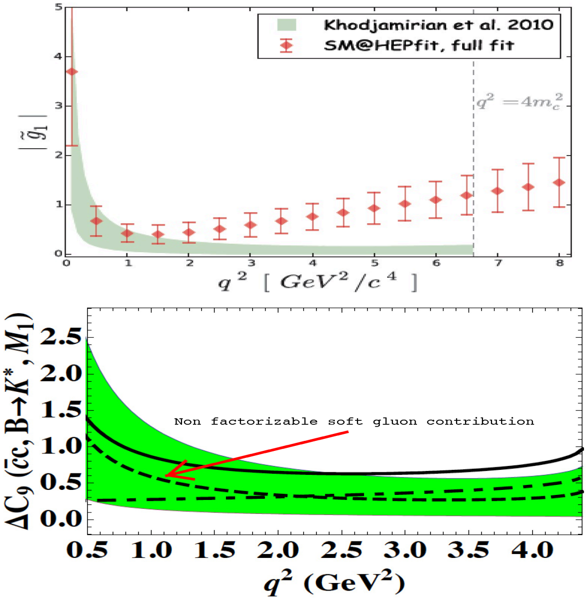
Blue: Their **fit** to $\delta C_9^{c\bar{c}(i)}{}_{\mathrm{KMPW}}(q^2)$
Green: The **computation** of Khodjamirian et al.

They show a constant shift everywhere. Two options:

...this universal shift is C_9^{NP} (same as R_K).

...or a universal charm q^2-**independent** coming from?? unable to explain nor R_K neither any LFVU. (weird)

II) Part-II Constrained fit: This part of the paper is highly 'controversial'.



They consider the result of KMPW at $q^2 \lesssim 1$ GeV^2 as an estimate of the charm loop effect.

- **Problem 1**: They tilt the fit at very-low q^2 inducing artificially a high-q^2 effect.
- **Problem 2**: Precisely below 1 GeV^2 there are well known lepton mass effects not considered here.
- **Problem 3**: KMPW computed the soft gluon effect with respect to LO factorizable (no imaginary part included) but CFFMPSV imposes

$$|g_i|^{LHCb} \simeq g_i^{KMPW} \quad \text{at} \quad q^2 \lesssim 1\text{GeV}^2$$

This makes no sense since on the RHS the imaginary part is not computed.

KMPW (left): Dashed is $2C_1\tilde{g}_1$ indistinguishable from $2C_1\tilde{g}_2$.

$$\Big)$$

let's now explore $\boxed{2}$....

Universal Lepton-Flavour Violating Observables

$$R_K = \frac{\mathrm{Br}(B^+ \to K^+\mu^+\mu^-)}{\mathrm{Br}(B^+ \to K^+e^+e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

⇒ R_K shows a 2.6σ tension with its SM prediction.

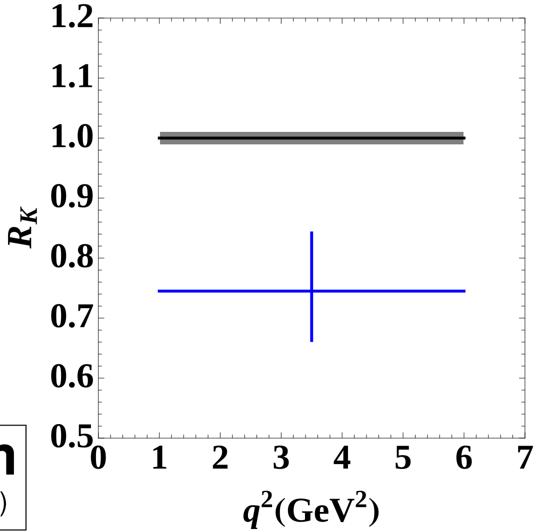
⇒ R_K (but also future measurements of R_{K^*}, R_ϕ, ...) represents the next step:

- New ingredient of the puzzle: Is Nature Universal LFV?
- This tension cannot be resolved within the SM, in particular **long-distance charm cannot explain it**.

If answer is YES:

NP or Charm? $\longmapsto$ **NP $\times$ Charm**
(obsolete question) (disentangling type of NP)

New Physics only possible explanation and charm only enters into game when discussing type of New Physics



The gray box is the SM prediction and blue cross is data.

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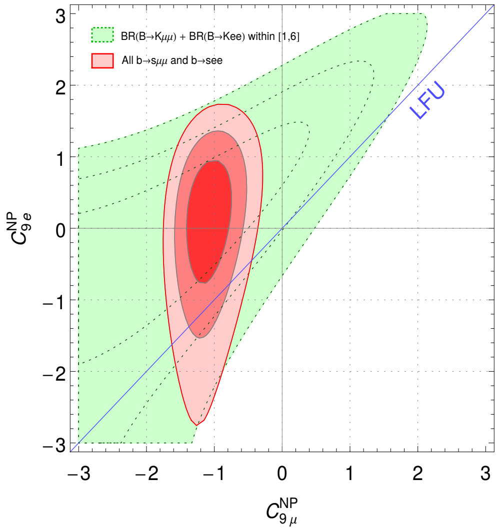
1 A separated fit to $C_{9,\mu}^{\rm NP}$ and $C_{9,e}^{\rm NP}$ shows a preference for $C_{9,\mu}^{\rm NP} \sim -1$ and $C_{9,e}^{\rm NP}$ compatible with zero .

$(C_{9,\mu}^{\rm NP}, C_{9,e}^{\rm NP})$



Universal LFV observables: R_K's hints

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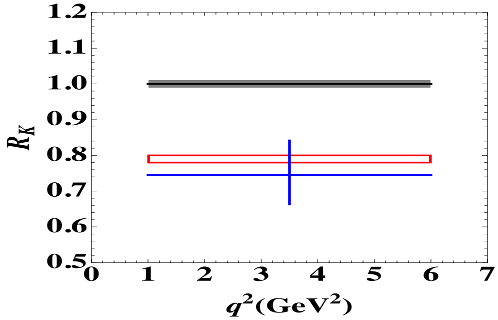
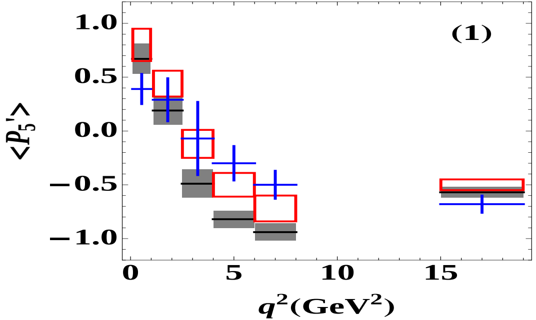
3 Same $C_{9,\mu}^{\mathrm{NP}} = -1.1$ alleviates **both** R_K and P_5' anomalies (with $C_{9,e}$ SM-like). R_K adds coherently in the global fit $+0.4\sigma$ to this NP solution.





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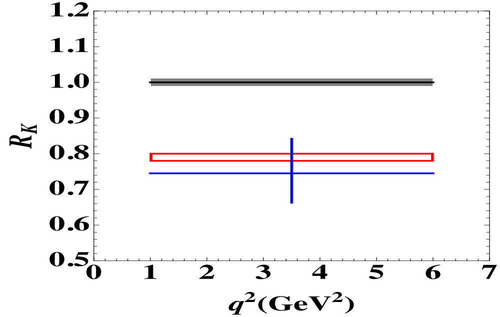
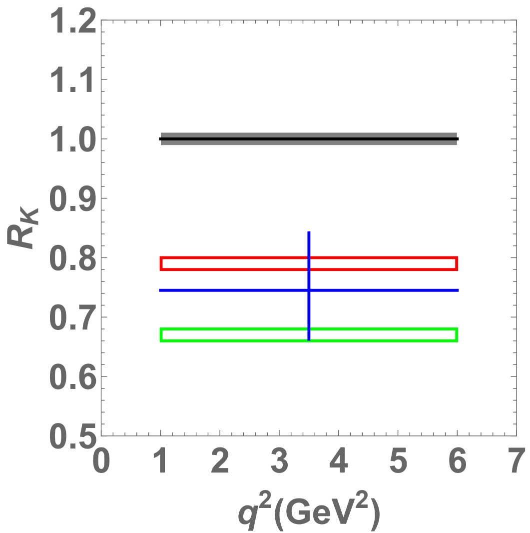
4 **BUT ALSO** low-recoil tensions and $B_s \to \phi\mu\mu$.



$b \to s\mu^+\mu^-$	bin	$SM \to NP$
$\text{BR}(B^0 \to K^0 \mu^+\mu^-)$	[15,19]	$+1.4\sigma \to +0.3\sigma$
$\text{BR}(B^0 \to K^{*0} \mu^+\mu^-)$	[16,19]	$+1.7\sigma \to +0.4\sigma$
$\text{BR}(B^+ \to K^{*+} \mu^+\mu^-)$	[15,19]	$+2.5\sigma \to +1.2\sigma$
$\text{BR}(B_s \to \phi \mu^+\mu^-)$	[15,18.8]	$+2.2\sigma \to +0.5\sigma$

Is it enough R_K to disentangle different New Physics scenarios?

But, with current data, more information than R_K alone is needed to distinguish between NP scenarios.
E.g. $C_{9,\mu}^{NP} = -1.1$ (scenario 1) vs $C_{9,\mu}^{NP} = -C_{10,\mu}^{NP} = -0.65$ (scenario 2).



Blue cross is data and gray band is SM prediction

THE (near) FUTURE:

A new generation of ULFV charm-insensitive observables (in SM).

A New Generation of Observables

$\Rightarrow$ Assume Nature violates universal lepton flavour (muons vs electrons).

Goal: To probe the different NP scenarios suggested by global fits with the highest possible precision.

How? New observables matching the following criteria:

- Sensitivity only to the short distance part of C_9 (**charm free** in the SM).
- Capacity to test for lepton flavour universality violation between the electronic and muonic modes.
- Sensitivity to Wilson coefficients other than C_9.
- In presence of New Physics reduced hadronic uncertainties.

Exploiting the angular analyses of both $B \to K^*\mu\mu$ and $B \to K^*ee$ decays, certain combinations of the angular observables fulfill the requirements

$$\langle Q_i \rangle = \langle P_i^\mu \rangle - \langle P_i^e \rangle \quad \langle \hat{Q}_i \rangle = \langle \hat{P}_i^\mu \rangle - \langle \hat{P}_i^e \rangle \quad \langle B_k \rangle = \frac{\langle J_k^\mu \rangle}{\langle J_k^e \rangle} - 1 \quad \langle \tilde{B}_k \rangle = \frac{\langle J_k^\mu / \beta_\mu^2 \rangle}{\langle J_k^e / \beta_e^2 \rangle} - 1$$

$i = 1, \ldots, 9$ & $k = 5, 6s$

where ˆ means correcting for lepton-mass effects in the first bin (backup slides).

How LFUV NP enter in Wilson coefficients?:

$$C_{i,\mu} = \begin{cases} C_i + \delta C_i, \; i = 10, 9', 10' \\ C_9 + \delta C_9 + \triangle C_9^{(j)} \end{cases} \qquad C_{i,e} = \begin{cases} C_i, \; i = 10, 9', 10' \\ C_9 + \triangle C_9^{(j)} \end{cases}$$

$$j = \perp, \|, 0$$

Notice $C_{7,7'}$ is obviously lepton-mass independent.

$\Rightarrow$ $\delta C_i = C_{i,\mu} - C_{i,e} \equiv$ amount of LFU violation.

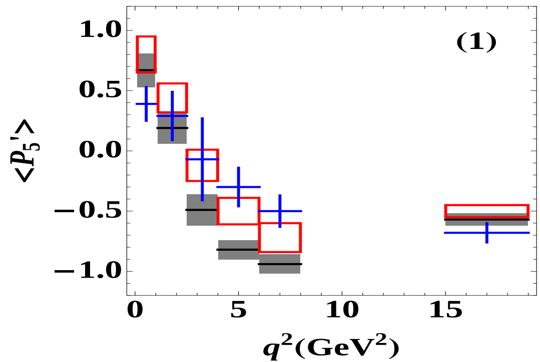
$\Rightarrow$ $C_i \equiv$ SM + LFU NP.

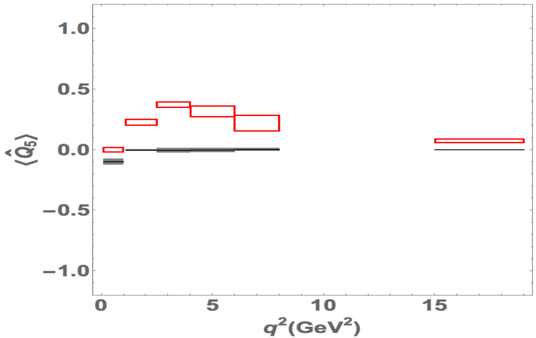
$\Rightarrow$ $\triangle C_9^{(j)} \equiv$ long-distance charm.Two types:

- **Transversity Dependent**: $\triangle C_9^{\perp,\|,0}$ different.
- **Transversity Independent**: $\triangle C_9^{\perp} = \triangle C_9^{\|} = \triangle C_9^{0}$.

Q_i observables. The example: P_5' versus $Q_5 = P_5'^{\mu} - P_5'^{e}$

Gray-SM, Red-NP $C_{9,\mu}^{\rm NP} = -1.11$, $C_{9,e}^{\rm NP} = 0$ and data





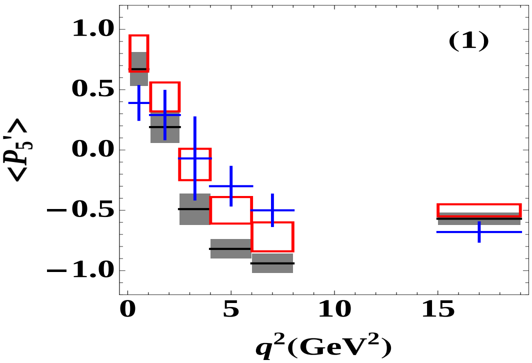
- Soft FF independent at LO exactly in SM
  Soft FF independent at LO exactly in NP.
- Large sensitivity to $C_{9,\mu}$. SM (DHMV'15):

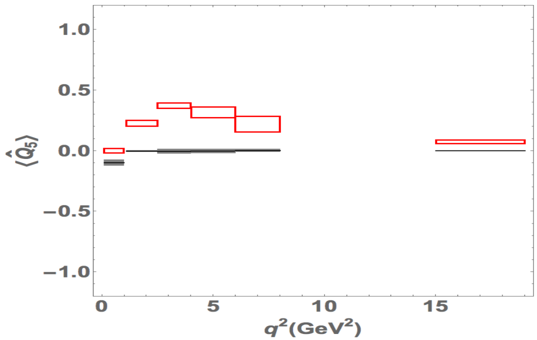
$$\langle P_5' \rangle_{[4,6]} = -0.82 \pm 0.08$$
$$\langle P_5' \rangle_{[6,8]} = -0.94 \pm 0.08$$

- FF independent at all orders in SM (up to $\triangle m_\ell^2$).
  Soft FF independent at LO exactly in NP.
- Long-distance charm insensitive in the SM.
  Large sensitivity to $\delta C_9 = C_{9,\mu} - C_{9,e}$.
  (CDMV'16): ($< 10^{-3}$ without lepton mass)

$$\left\langle \hat{Q}_5 \right\rangle_{[4,6]} = -0.002 \pm 0.017$$
$$\left\langle \hat{Q}_5 \right\rangle_{[6,8]} = +0.002 \pm 0.010$$

Q_i observables. The example: P_5' versus $Q_5 = P_5'^{\mu} - P_5'^{e}$ for $C_{9,\mu}^{\rm NP} = -1.1$





Remark: In presence of NP hadronic uncertainties reemerge in Q_5 (& $\hat{Q}_5$)...

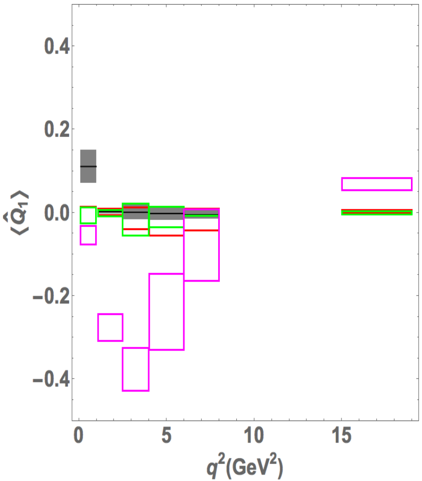
P_5'	Prediction $C_{9,\mu}^{\rm NP} = -1.1$	Q_5	Prediction $\delta C_9 = -1.1$	$\hat{Q}_5$	Prediction $\delta C_9 = -1.1$
$[0.1, 0.98]$	0.80 ± 0.14	$[0.1, 0.98]$	0.172 ± 0.016	$[0.1, 0.98]$	-0.000 ± 0.018
$[1.1, 2.5]$	0.43 ± 0.12	$[1.1, 2.5]$	0.241 ± 0.025	$[1.1, 2.5]$	0.227 ± 0.023
$[2.5, 4]$	-0.12 ± 0.13	$[2.5, 4]$	0.370 ± 0.022	$[2.5, 4]$	0.370 ± 0.021
$[4, 6]$	-0.50 ± 0.11	$[4, 6]$	0.312 ± 0.047	$[4, 6]$	0.314 ± 0.046
$[6, 8]$	-0.73 ± 0.12	$[6, 8]$	0.212 ± 0.063	$[6, 8]$	0.216 ± 0.061

BUT, it only matters when discussing the **type** of NP we can see.

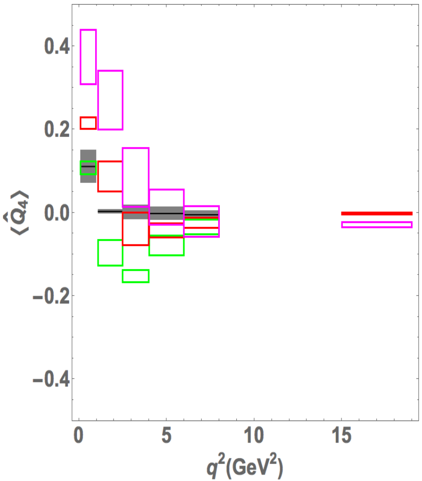
Probing right-handed currents (RHC) with Q_i

SM predictions (grey boxes),
NP: $C_{9,\mu}^{\rm NP} = -1.11$ & $C_{9,\mu}^{\rm NP} = -C_{10,\mu}^{\rm NP} = -0.65$ & $C_{9,\mu}^{\rm NP} = -C_{9,\mu}^{\prime\rm NP} = -1.18$ & $C_{10,\mu}^{\rm NP} = C_{10,\mu}^{\prime\rm NP} = 0.38$.

with $\delta C_i = C_{i,\mu} - C_{i,e}$ (and $C_{i,e}$ SM)



$Q_1 = P_1^{\mu} - P_1^{e}$



$Q_4 = P_4^{\prime\mu} - P_4^{\prime e}$

⇒ $Q_{1,4}$ provide excellent opportunities to probe RHC in $C'_{9,\mu}$ & $C'_{10,\mu}$.

- Q_1 shows significant deviations in presence of RHC. If $C'_7 = 0$ at LO

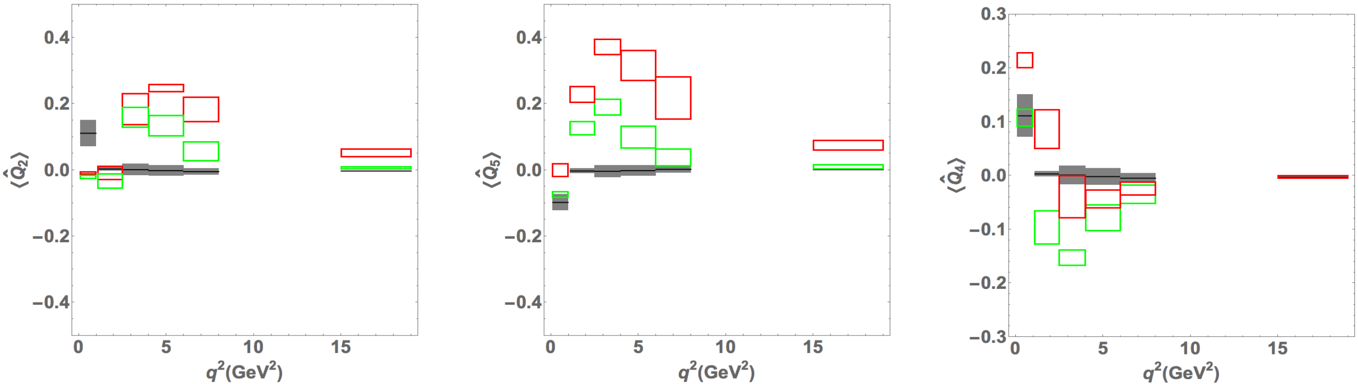
$$s_0^{LO} = -2\frac{C_7 \delta C'_9 m_b M_B}{C_{10,\mu}\delta C'_{10} + C_{9,\mu}\delta C'_9}$$

no zero (except $s = 0$) if $\delta C'_9 = 0$.
no sensitivity to C_i if $C'_i = 0$.

- Q_4 at low-q^2 exhibits deviations for $C'_{9,10,\mu}$ when accurate precision in measurements is achieved.

Probing NP in $C_{9,10}$ with Q_i

SM predictions (grey boxes),
NP: $C_{9,\mu}^{\rm NP} = -1.11$ (scenario1) & $C_{9,\mu}^{\rm NP} = -C_{10,\mu}^{\rm NP} = -0.65$ (scenario 2) with $\delta C_i = C_{i,\mu} - C_{i,e}$ (and $C_{i,e}$ SM)



$Q_2 = P_2^{\mu} - P_2^{e}$ $Q_5 = P_5^{\prime\mu} - P_5^{\prime e}$ $Q_4 = P_4^{\prime\mu} - P_4^{\prime e}$

⇒ Q_2, Q_4 & Q_5 show distinctive signatures for the two NP scenarios considered.

- Differences in the high-q^2 bins of the large recoil region of Q_2 & Q_5 are quite significant. Lack of difference between scenario 2 and SM same reason why P_5' in scenario 2 is worst than scenario 1.
- Q_4 at very low-q^2 (second bin) is very promising to disentangle scenario 1 from 2.

Idea: Combine J_i^{μ} & J_i^{e} to build combinations sensitive to some C_i, with controlled sensitivitiy to long-distance charm.

$$\beta_\ell J_5 - 2iJ_8 = 8\beta_\ell^2 N^2 m_B^2 (1-\hat{s})^3 \frac{\hat{m}_{K^*}}{\hat{s}\sqrt{\hat{s}}} C_{10}^\ell \left[C_7 \hat{m}_b (1+\hat{s}) + \hat{\mathbf{s}} C_9^\ell\right] \xi_\perp \xi_{||} + \ldots$$

$$\beta_\ell J_{6s} - 2iJ_9 = 16\beta_\ell^2 N^2 m_B^2 \frac{(1-\hat{s})^2}{\hat{s}} C_{10}^\ell \left[2C_7 \hat{m}_b + \hat{\mathbf{s}} C_9^\ell\right] \xi_\perp^2 + \ldots$$

where $\beta_\ell = \sqrt{1-4m_\ell^2/q^2}$.

Assuming real NP & maximal LFUV μ vs e, natural combinations are

$$B_5 = \frac{J_5^\mu}{J_5^e} - 1 \quad B_{6s} = \frac{J_{6s}^\mu}{J_{6s}^e} - 1$$

- Form factor independent at all orders (up to $\triangle$ lepton mass).
- Full charm insensitive in the SM.
- Linear sensitivity to δC_9 **kinematically suppressed.**

B_5 & B_{6s} Observables (unique properties)

In the large-recoil limit and in absence of RHC currents [CDMV'16]:

$$B_5 = \frac{J_5^\mu - J_5^e}{J_5^e} = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2}\frac{\delta C_{10}}{C_{10}} + \frac{\beta_\mu^2}{\beta_e^2}\frac{2(C_{10}+\delta C_{10})\delta C_9 \hat{\mathbf{s}}}{C_{10}\left(2C_7\hat{m}_b(1+\hat{s}) + (2C_9 + \triangle C_{9,0} + \triangle C_{9,\perp})\hat{\mathbf{s}}\right)} + \ldots$$

$$B_{6s} = \frac{J_{6s}^\mu - J_{6s}^e}{J_{6s}^e} = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2}\frac{\delta C_{10}}{C_{10}} + \frac{\beta_\mu^2}{\beta_e^2}\frac{2(C_{10}+\delta C_{10})\delta C_9 \hat{\mathbf{s}}}{C_{10}\left(4C_7\hat{m}_b + (2C_9 + \triangle C_{9,\perp} + \triangle C_{9,\|})\hat{\mathbf{s}}\right)} + \ldots$$

In the limit of $s \to 0$ δC_{10} is cleanly disentangled:

$$B_5(s\to 0) = B_{6s}(s\to 0) = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2}\frac{\delta C_{10}}{C_{10}} + \ldots$$

This shows the IMPORTANCE of the normalization to the electronic mode. IF NOT normalized:

$$J_5^\mu - J_5^e \propto \mathbf{C_7}\delta C_{10}\xi_\perp \xi_\|$$

Several PROBLEMS in extracting δC_{10} if not normalized:

1) $\xi_\perp \xi_\|$: SFF error? KMPW or BSZ
2) Charm contribution possible inside $\mathbf{C_7}$.

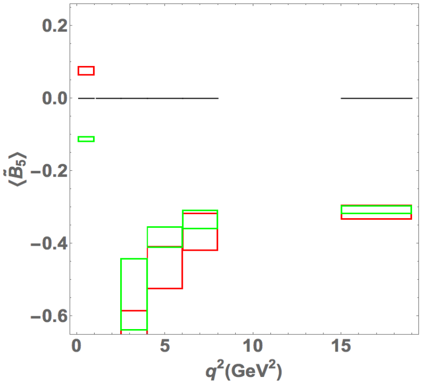
B_5 & B_{6s} are **not identically 0** in the SM.

Lepton mass differences generates a non-zero contribution mainly in the first bin.

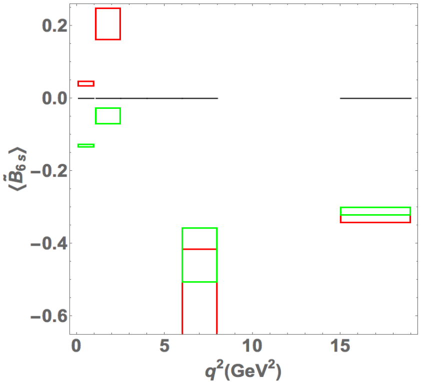
$\Rightarrow$ If on an event-by-event basis experimentalist can measure $\langle J_i^\mu/\beta_\mu^2\rangle$:

$$\langle\widetilde{B}_5\rangle = \frac{\langle J_5^\mu/\beta_\mu^2\rangle}{\langle J_5^e/\beta_e^2\rangle} - 1 \quad \langle\widetilde{B}_{6s}\rangle = \frac{\langle J_{6s}^\mu/\beta_\mu^2\rangle}{\langle J_{6s}^e/\beta_e^2\rangle} - 1$$

- SM Predictions: $\langle\widetilde{B}_i\rangle = 0.00 \pm 0.00$.
- All good properties of $B_{5,6s}$ + simpler structure $\beta_i \to 1$.



$\widetilde{B}_5$



$\widetilde{B}_{6s}$

- When $\hat{s} \to 0$, $\widetilde{B}_5 = \widetilde{B}_{6s} = \delta C_{10}/C_{10}$
  $\Rightarrow$ Sensitivity to δC_{10}!
  Exactly as B_5, B_{6s} but simpler.
- 1st Bins: Capacity to distinguish
  $C_{9,\mu}^{\rm NP} = -1.11$ from
  $C_{9,\mu}^{\rm NP} = -C_{10,\mu}^{\rm NP} = -0.65$.

$\widetilde{M}$: Transversity Independent Charm Free Observables at low q^2

Goals: Can one construct a ULFV observable not only free from hadronic uncertainties in the SM but also free from long-distance charm in presence of New Physics? **Yes** BUT only under two conditions:

- Only if New Physics is dominated by δC_9.
- Only if long-distance charm is transversity independent $\triangle C_9^{\perp} = \triangle C_9^{\parallel} = \triangle C_9^0 = \triangle C_9$.

$$\widetilde{M} = \frac{\widetilde{B_5}\widetilde{B_{6s}}}{\widetilde{B_{6s}} - \widetilde{B_5}} = -\frac{\delta C_9 \hat{s}}{C_7 \hat{m}_b (1 - \hat{s})} + \delta C_{10} \text{ terms} + \delta C_{10} \triangle C_9 \text{ terms} + \ldots$$

- If charm is transversity dependent (as expected) is impossible to remove it in presence of NP.

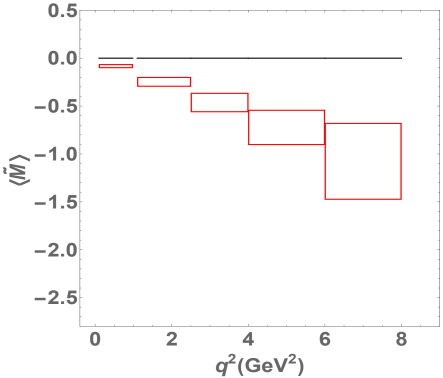
$$\widetilde{M} = \frac{\widetilde{B_5}\widetilde{B_{6s}}}{\widetilde{B_{6s}} - \widetilde{B_5}} = -\frac{\delta C_9 \hat{s}}{C_7 \hat{m}_b (1 - \hat{s}) - (\triangle C_9^0 - \triangle C_9^{\parallel})\hat{s}/2} + \ldots$$

(Leading order expression)

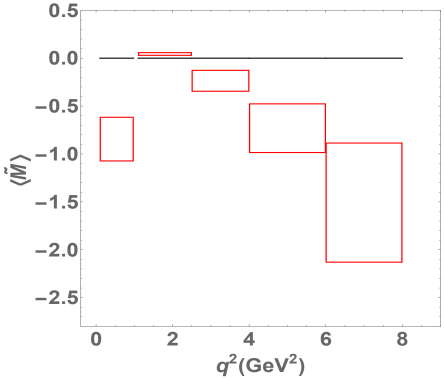
- Maximal sensitivity to NP at very low-q^2.
- **Even if for $\delta C_{10} \neq 0 \Rightarrow$ long-distance charm reemerges, this observable is particularly promising to measure δC_{10}.**
- Singular in the region where $B_5 \simeq B_{6s}$.

Error size comes from TD charm suppressed at low-q^2

Error size comes from all type of charm TD and TI (due to $\delta C_{10} \neq 0$)



Scenario 1:
$\delta C_{9\mu}^{NP} = -1.11$



Scenario 2:
$\delta C_{9\mu}^{NP} = -\delta C_{10\mu}^{NP} = -0.65$

Figure: SM predictions (grey boxes) and NP predictions (red boxes) for ($\tilde{M}$ down) in the 2 scenarios.

Conclusions

- Global view: We have shown that the same NP solution $C_{9,\mu}^{\mathrm{NP}} = -1.1$, $C_{9,e}^{\mathrm{NP}} = 0$ alleviates all tensions: P_5', R_K, low-recoil, $B_s \to \phi\mu^+\mu^-$,...
  $\to$ SM 'alternative explanations' becomes obsolete by construction from a global point of view.

- Local view:
  - Factorizable p.c.: We have proven that an inappropriate scheme's choice if correlations are not considered inflates artificially the errors.
  - Long-distance charm: Explicit computation by KMPW do not explain the anomaly and a bin-by-bin analysis does not find any indication for a q^2-dependence.

- We have proposed different sets of **observables comparing** $B \to K^* ee$ & $B \to K^* \mu\mu$.
  - Q_i Observables: $Q_i \leftrightsquigarrow P_i^\ell$
  - $C_{9\ell}$ linear Observables: $B_{5,6s}$, $\tilde{B}_{5,6s} \leftrightsquigarrow J_{5,6s}$
  - TI charm free Observables: M ($\tilde{M}$)

- $\langle Q_i \rangle$ observables allows us to **distinguish** different **NP** scenarios: RHC or δC_9 versus $\delta C_9 = -\delta C_{10}$.

- $\langle B_5 \rangle$ & $\langle B_{6s} \rangle$ but also $\langle \widetilde{M} \rangle$ can be used to **measure** δC_{10} at very low-q^2.

Backup Slide

“Hats”

LHCb currently determines $F_{L,T}$ using a simplified description of the angular kinematics:

$$\left.\begin{matrix} J_{2s} \\ J_{2c} \end{matrix}\right\} \longmapsto J_{1c} \text{ (equivalent in the massless limit)}$$

Then, to match this convention, the angular observables are redefined in the following way:

$$F_L = \frac{-J_{2c}}{dG/dq^2} \to \hat{F}_L = \frac{J_{1c}}{dG/dq^2} \qquad F_T = \frac{4J_{2s}}{dG/dq^2} \to \hat{F}_T = 1 - \hat{F}_L$$

$$P_1 = \frac{J_3}{2J_{2s}} \to \hat{P}_1 = \frac{J_3}{2\hat{J}_{2s}} \qquad P_2 = \frac{J_{6s}}{8J_{2s}} \to \hat{P}_2 = \frac{J_{6s}}{8\hat{J}_{2s}}$$

$$P_3 = -\frac{J_9}{4J_{2s}} \to \hat{P}_3 = -\frac{J_9}{4\hat{J}_{2s}} \qquad P_4' = \frac{J_4}{\sqrt{-J_{2s}J_{2c}}} \to \hat{P}_4' = \frac{J_4}{\sqrt{\hat{J}_{2s}J_{1c}}}$$

$$P_5' = \frac{J_5}{2\sqrt{-J_{2s}J_{2c}}} \to \hat{P}_5' = \frac{J_5}{2\sqrt{\hat{J}_{2s}J_{1c}}} \qquad P_6' = -\frac{J_7}{2\sqrt{-J_{2s}J_{2c}}} \to \hat{P}_6' = -\frac{J_7}{2\sqrt{\hat{J}_{2s}J_{1c}}}$$

$$P_8' = -\frac{J_8}{\sqrt{-J_{2s}J_{2c}}} \to \hat{P}_8' = -\frac{J_8}{\sqrt{\hat{J}_{2s}J_{1c}}} \qquad \text{with } \hat{J}_{2s} = \frac{1}{16}(6J_{1s} - J_{1c} - 2J_{2s} - J_{2c})$$

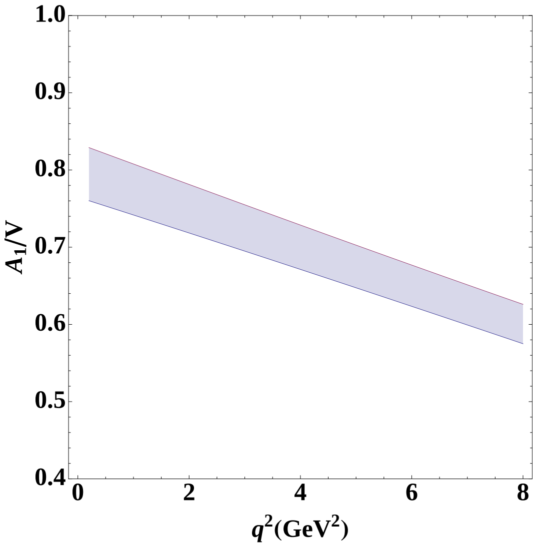
Why is there a need to compute the predictions from $\hat{F}_{L,T}$ instead of $F_{L,T}$? Let's consider the decay distribution

$$\frac{1}{d(G+\bar{G})/dq^2}\frac{d^3(G+\bar{G})}{dO} = \frac{9}{32\pi}\Big[\frac{3}{4}\hat{F}_T \sin^2\theta_K + \hat{F}_L \cos^2\theta_K + \frac{1}{4}F_T \sin^2\theta_K \cos 2\theta_l - F_L \cos^2\theta_K \cos 2\theta_l + ...\Big]$$

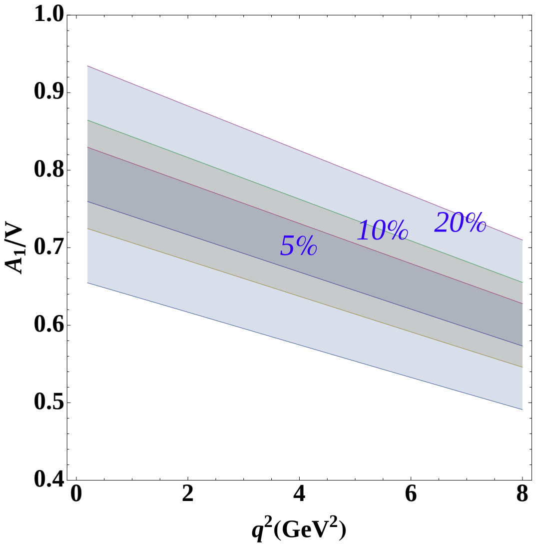
- With the current limited statistics, $\hat{F}_{L,T}$ and $F_{L,T}$ cannot be distinguished by LHCb.
- $\cos\theta_K^2$ is the dominant term, so it is the natural place to extract F_L.

The size of power corrections

The ratio A_1/V is particularly relevant. Let's illustrate that the size of the error associated to power corrections is much below 10%. We use BSZ for this example.



Ratio of FF computed in BSZ including correlations.



Ratio of FF computed in BSZ taking 5%, 10% and 20% for the error associated to p.c.

Notice that already a 5% error of power correction is of the same size of the error of the full-FF.

JC-I: Without leaving any loose ends... Is the procedure to compute P_5' accidentally scheme independent? NO if errors are taken uncorrelated

CDHM'16: *In JC'14 the computation of P_5' is argued to be scheme independent. In helicity basis we find:*

$$P_5' = P_5'|_\infty \Big[1 + \frac{\mathbf{aV_-} - \mathbf{aT_-}}{\xi_\perp} \frac{m_B}{|k|} \frac{m_B^2}{q^2} C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})} - \frac{\mathbf{aV_+}}{\xi_\perp} \frac{\mathbf{2C_{9,\|}}}{\mathbf{C_{9,\perp} + C_{9,\|}}}$$
$$+ \frac{aV_0 - aT_0}{\xi_\|} 2 C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\|}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})} + \mathcal{O}\left(\frac{m_{K^*}^2}{m_B^2}, \frac{q^2}{m_B^2} \right) \Big]$$

OK with JC'14 except for the missing term $\mathbf{aV_+}$. Choosing a scheme with $\mathbf{aV_-}$ or $\mathbf{aT_-}$ is equivalent.

Only apparently a scheme independent computation in helicity basis for a subset of schemes!
The computation should be scheme independent in any basis!!!!

In transversity basis becomes obvious that scheme's choice matters if no correlations are considered:

$$P_5' = P_5'|_\infty \Big[1 + \frac{\mathbf{aV}}{\xi_\perp} \frac{C_{9,\|}}{C_{9,\perp} + C_{9,\|}} + \frac{\mathbf{aV} - 2\mathbf{aT_1}}{\xi_\perp} \frac{m_B^2}{q^2} C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})} - \frac{aA_1}{\xi_\perp} \frac{C_{9,\perp} C_{9,\|} + C_{10}^2}{2(C_{9,\perp}^2 + C_{10}^2)} + \ldots$$

The weights of $\mathbf{aV}$ & $\mathbf{aT_1}$ are MANIFESTLY different: $P_5'^{(q^2=6)} = P_5'|_\infty (1 + [\mathbf{0.78}\,\mathbf{aV} - \mathbf{0.20}\,\mathbf{aT_1}]/\xi_\perp(6) + \ldots$

$\xi_\perp^{(1)}(q^2) \equiv \frac{m_B}{m_B + m_{K^*}} V(q^2) \Rightarrow \mathbf{aV} = 0$ *(our)* *or* $\xi_\perp^{(2)}(q^2) \equiv T_1(q^2) \Rightarrow \mathbf{aT_1 = 0}$ $(JC) > 3$ *times bigger*

Why JC'14 has FFI observables with huge errors and FFD smaller errors?

3) **Soft form factor error** (undervaluated error):

DHMV: $\xi_\perp = \mathbf{0.31}^{+0.20}_{-0.10}$ from KMPW $V = 0.36^{+0.23}_{-0.12} \to err[\langle F_L \rangle^{DHMV'16}_{[0.1,0.98]}] = \pm\mathbf{0.25}$

JC'14: $\xi_\perp = \mathbf{0.31} \pm \mathbf{0.04}$ spread of **only** central values (KMPW,BZ,..) no error! $\to err[\langle F_L \rangle^{JC'14}_{[0.1,0.98]}] = \pm\mathbf{0.18}$.

$\Rightarrow$ **This choice of error in $\xi_\perp$ induces an undervaluation in JC'14 of the errors for FFD observables**

Summary:

Now you have all arguments to analyze misleading statements like:

"Since observables cannot depend on arbitrary scheme definitions, their deviation from the ∞-mass limit cannot be reduced" †

1) It is not the observables, but **the way to compute them** where scheme dependence enter!
2) The goal is not to reduce it but the opposite **NOT TO INFLATE THEM.**

3) **Soft form factor error** (undervaluated error):

DHMV: $\xi_\perp = \mathbf{0.31}^{+0.20}_{-0.10}$ from Full-FF of KMPW $V = 0.36^{+0.23}_{-0.12}$ with error included.

JC'14: $\xi_\perp = \mathbf{0.31} \pm \mathbf{0.04}$ (spread of **only** central values (KMPW,BZ,..) no error taken!).

FF budget:

$$A_1 = A_1^{soft} + \blacksquare A_1^{\alpha_s} + \blacksquare A_1^{\blacksquare}$$

$$A_1 = \mathbf{0.25}^{+0.16}_{-0.10} \text{ (KMPW)}$$

- Our error budget:
  - $A_1^{soft} = \frac{m_B}{m_B+m_K^*}\xi_\perp(0) = \mathbf{0.26}^{+0.17}_{-0.09}$ (KMPW)
  - $\blacksquare A_1^{\alpha_s}$ is $\mathcal{O}(\alpha_s)$ and $\blacksquare A_1^{\blacksquare}$ is $\mathcal{O}(\blacksquare/m_b) \times \mathrm{FF} \simeq 0.1\mathrm{FF}$ of full-FF.
- JC error budget:
  - $A_1^{soft} = \frac{m_B}{m_B+m_K^*}\xi_\perp(0) = \mathbf{0.26} \pm \mathbf{0.03}$
  - $\blacksquare A_1^{\alpha_s}$ is $\mathcal{O}(\alpha_s)$ and $\blacksquare A_1^{\blacksquare}$ is $\mathcal{O}(\blacksquare/m_b) \times \mathrm{FF} \simeq 0.1\mathrm{FF}$ of full-FF.

$\Rightarrow$ **This choice of error in $\xi_\perp$ induces an undervaluation in JC'14 of the errors for FFD observables**: A_{FB}, F_L and S_i.