Bayes-Tracking – A new Approach for Gamma-Ray Tracking

P. Napiralla, C. Stahl, H. Egger, M. Reese, N. Pietralla

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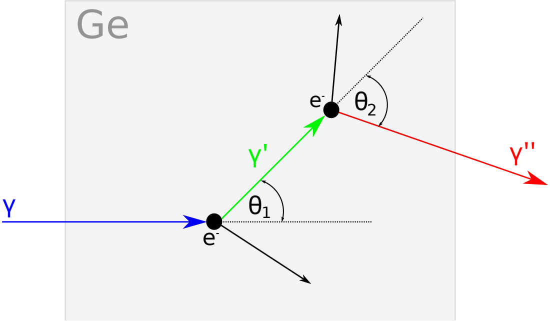

Figure: Compton-Escape Event in a Germanium detector

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05P15RDFN1 - TP9 (Experiment)

Existing Gamma-Ray Tracking Algorithms

Existing algorithms: *Forward-Tracking, Back-Tracking*

- Identification of most probable photon paths
- Summation of measured deposited energies

Problem: Compton-escaped photons “useless”

⇒ New algorithm which also utilises Compton-escaped photons using *Bayesian inference*, called **Bayes-Tracking**

Bayes-Tracking

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Requirements on new *Bayes-Tracking algorithm*

- **Goal**: Identify correct ingoing photon energy E_γ (using Compton-Escape- and/or Photoabsorption-Events; no pair-production yet)
- **Data**: Interactions of photon with detector with deposited energies $\left\{E_{\text{dep}_1}, E_{\text{dep}_2}, \ldots, E_{\text{dep}_N}\right\}$ at measured interaction points $\{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N\}$

$\Rightarrow$ Calculate probability of photon energy e_0 given the interactions with energy depositions $\left\{E_{\text{dep}_1}, \ldots, E_{\text{dep}_N}\right\}$ at $\{\vec{x}_1, \ldots, \vec{x}_N\}$:

$$\Rightarrow P\left(e_0 \middle| \left\{\{E_{\text{dep}_1}, \vec{x}_1\}, \ldots, \{E_{\text{dep}_N}, \vec{x}_N\}\right\}\right)$$

Bayes-Tracking
Bayes' theorem

Let A and B be two events. The conditional probability of B, given A is true

$$P\left(B|A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)}$$

can be calculated by *Bayes' theorem*

$$P\left(hypothesis|data\right) = \frac{P\left(data|hypothesis\right) \cdot P\left(hypothesis\right)}{P\left(data\right)} .$$

Bayes-Tracking
Bayes' theorem

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can be calculated by *Bayes' theorem*

$$P\left(\textit{hypothesis}|\textit{data}\right) = \frac{P\left(\textit{data}|\textit{hypothesis}\right) \cdot P\left(\textit{hypothesis}\right)}{P\left(\textit{data}\right)} .$$

- $P\left(\textit{data}\right)$: *evidence*
  $\rightarrow$ (naive) knowledge about *data*
- $P\left(\textit{hypothesis}\right)$: *prior probability*
  $\rightarrow$ (naive) knowledge about *hypothesis*
- $P\left(\textit{data}|\textit{hypothesis}\right)$: *likelihood function*
  $\rightarrow$ testing plausibility of *data*, given *hypothesis* is true
- $P\left(\textit{hypothesis}|\textit{data}\right)$: *posterior probability*
  $\rightarrow$ probability of *hypothesis* being true, given *data*

Bayes-Tracking

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Probability of photon energy e_0 given a certain set of energy-depositions:

$$P\left(e_0 \middle| \left\{\{E_{\text{dep}_1}, \vec{x}_1\}, \dots, \{E_{\text{dep}_N}, \vec{x}_N\}\right\}\right) \propto \sum_{\pi} P\left(\pi\left(\{E_{\text{dep}_1}, \vec{x}_1\}, \dots, \{E_{\text{dep}_N}, \vec{x}_N\}\right) \middle| e_0\right)$$

Function of permutations $\pi\left(E_{\text{dep}_1}, \dots, E_{\text{dep}_N}\right)$.

Sources of information:

- Compton-scattering for $i = 1, \dots, N-1$ (Number of interactions N) (P_{int})
- Distances between the interaction points (P_{λ})
- Last interaction: Compton- or photoelectric effect possible (P_{last})

Important: Measurement uncertainties for $\vec{x}_i$ and E_{dep_i} present ($\mathcal{G}_{3D}$ and $\mathcal{G}$)

Bayes-Tracking

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Likelihood function can be calculated via:

$$P\left(\{\vec{x}_1, E_{\text{dep}_1}\}, \ldots, \{\vec{x}_N, E_{\text{dep}_N}\} | e_0\right) = \int \prod_{i=1}^{N-1} \left[P_{\text{int}}\left(\{\vec{\mu}_i\}, e_{i-1}\right) \cdot P_\lambda\left(\vec{\mu}_{i-1}, \vec{\mu}_i, e_{i-1}\right) \right.$$

$$\left. \cdot\, \mathcal{G}_{3D}\left(\vec{x}_i, \vec{\mu}_i, \hat{\sigma}_x\right) \cdot \mathcal{G}\left(E_{\text{dep}_i}, \mathcal{E}(\{\vec{\mu}_i\}, e_{i-1}), \sigma_E\right) \right]$$

$$\cdot\, P_{\text{last}}\left(\vec{\mu}_{N-1}, \vec{\mu}_N, e_{N-1}, E_{\text{dep}_N}\right) \mathrm{d}\vec{\mu}_0 \cdots \mathrm{d}\vec{\mu}_N \,.$$

- Likeliness for scattering of photon with energy e_{i-1} by angle $\theta(\{\vec{\mu}_i\})$)
- Probability for no interaction between $\vec{\mu}_{i-1}$ and $\vec{\mu}_i$
- Probability to measure $\vec{\mu}_i$ as $\vec{x}_i$ ($\mathcal{G}_{3D}$) and $\mathcal{E}\left(\{\vec{\mu}_i\}, e_{i-1}\right)$ as E_{dep_i} ($\mathcal{G}$)
- Last interaction: Possible Photoelectric- or Compton effect

$$\mathcal{E}\left(\{\vec{\mu}_i\}, e_{i-1}\right) = e_{i-1} \cdot \left[1 - \left(1 + \frac{e_{i-1}}{m_{e_0} c^2} \left[1 - \cos\left(\theta(\{\vec{\mu}_i\})\right) \right] \right)^{-1} \right]$$

Bayes-Tracking – Generating “Test-data”

Ge Detector Simulation (using *Geant4*)

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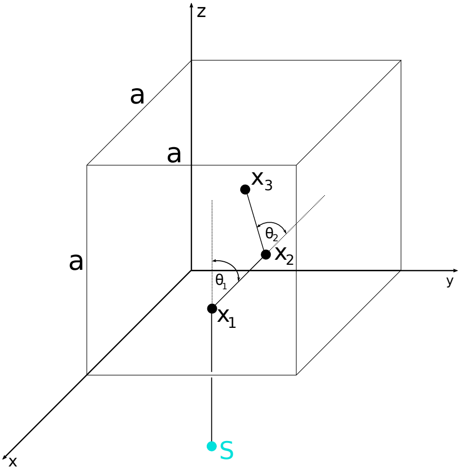


Figure: Geometry of the simulated Ge detector (edge length $a = 8\,\mathrm{cm}$) with photon source S.

Bayes-Tracking – Single Photons

Compton- and Photo-Events

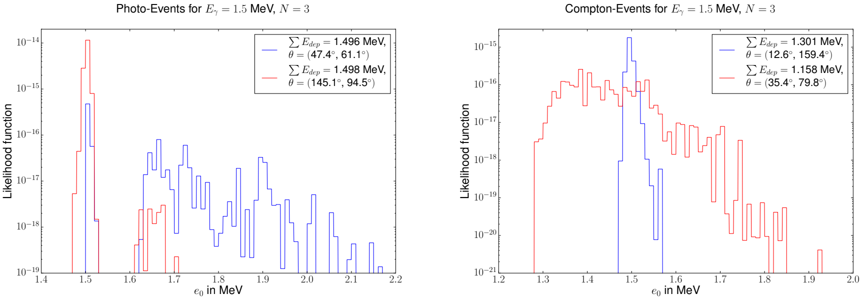
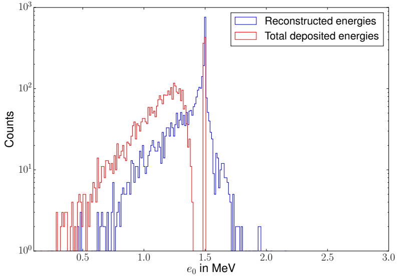


Figure: Likelihood functions for Photo- and Compton-Events with $N = 3$ and correct photon energy $E_\gamma = 1.5$ MeV

- For large scattering angles θ: "Reconstruction" of E_γ accurately
  $\Rightarrow$ **even for Compton-Escape Events!**
- For small scattering angles θ: Almost no ability to reconstruct E_γ correctly

Bayes-Tracking – Tracking Performance

Compton- and Photo-Events



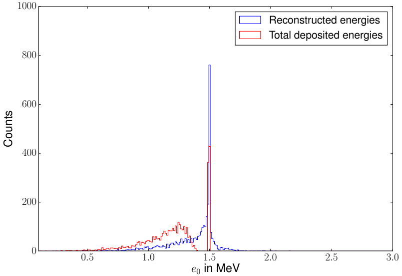


Figure: Histogram (logarithmic and linear) for most likely reconstructed photon energies (3000 Compton-Events, 792 Photo-Events) with $E_\gamma = 1.5\,\text{MeV}$ compared to total deposited energy spectrum ($\hat{=}$ measured spectrum of a plain Ge detector).

Photo-Efficiencies $\epsilon = N_{\text{Photo-Peak}} / N_{\text{total}}$:

- Total deposited energies: $\epsilon = 20.9\,\%$
- Reconstructed energies: $\epsilon = \epsilon_{\text{Photo-Ev.}} + \epsilon_{\text{Compton-Ev.}} = 17\% + 3.4\% = 20.4\,\%$

Conclusion & Outlook

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Conclusion:

- Bayes-Tracking as a new method using Bayesian inference
- Good energy reconstruction using Photo- and Compton-Events

Outlook:

-
-
-

Conclusion & Outlook

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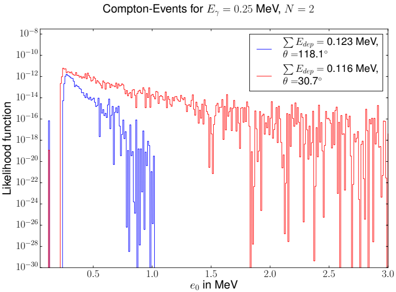
Conclusion:

- Bayes-Tracking as a new method using Bayesian inference
- Good energy reconstruction using Photo- and Compton-Events

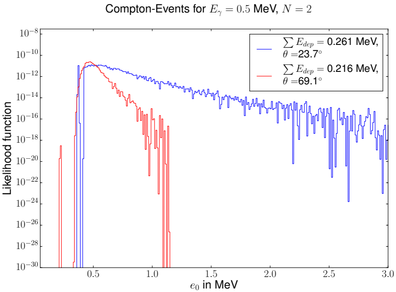
Outlook:

- Faster integration method (e.g. using sparse grids)
- Incorporation of pair production and photon polarization
- Implement AGATA geometry and embed Bayes-Tracking into AGATA framework NARVAL/FEMUL

Additional plots



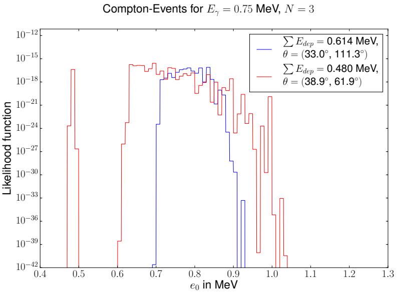
(a) E_γ = 0.25 MeV



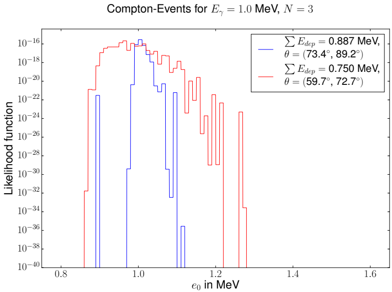
(b) E_γ = 0.5 MeV

Figure: Results of the Bayes-Tracking for Compton-Events with ingoing photon energies E_γ = 0.25 MeV (a) and 0.5 MeV (b) with N = 2.

Additional plots



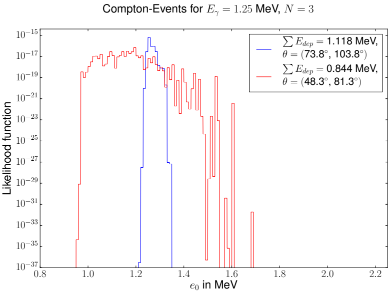
(a) E_γ = 0.75 MeV



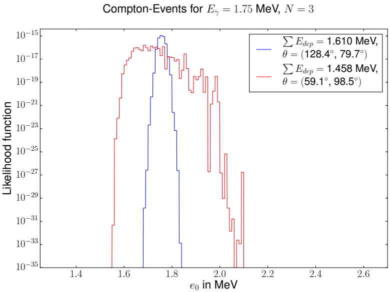
(b) E_γ = 1.0 MeV

Figure: Results of the Bayes-Tracking for Compton-Events with ingoing photon energies E_γ = 0.75 MeV (a) and 1.0 MeV (b) with N = 3.

Additional plots



(a) $E_\gamma = 1.25$ MeV



(b) $E_\gamma = 1.75$ MeV

Figure: Results of the Bayes-Tracking for Compton-Events with ingoing photon energies $E_\gamma = 1.25$ MeV (a) and 1.75 MeV (b) with $N = 3$.

Additional plots

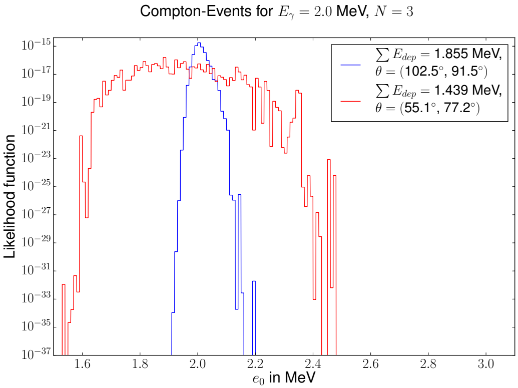


Figure: Results of the Bayes-Tracking for Compton-Events with ingoing photon energies $E_\gamma = 2.0\,\text{MeV}$ with $N = 3$.

Additional plots

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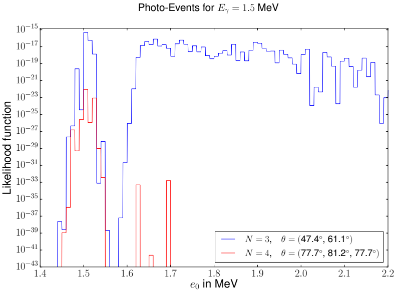
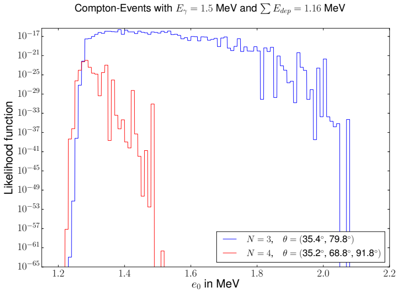


(a) $\sum E_{\text{dep}} = 1.5\,\text{MeV}$



(b) $\sum E_{\text{dep}} = 1.16\,\text{MeV}$

Figure: Comparison of photons with three and four interactions inside the detector that either deposited their whole energy (a), or 1.16 MeV (b).

Additional plots

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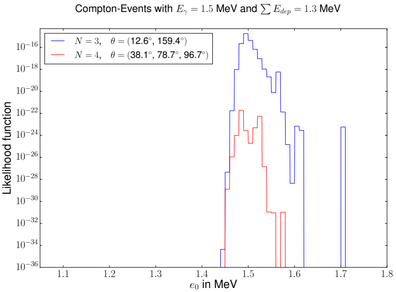
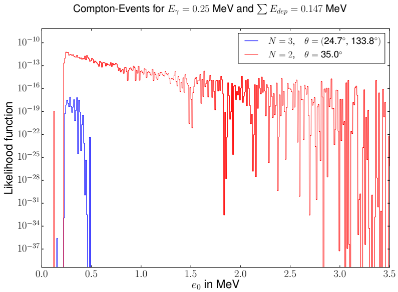


(a) $\sum E_{\text{dep}} = 1.3\,\text{MeV}$, $N = 3$ and $N = 4$



(b) $\sum E_{\text{dep}} = 0.147\,\text{MeV}$, $N = 2$ and $N = 3$

Figure: Comparison of photons that deposited 1.3 MeV inside the detector in three and four interactions (a). In addtion, the influence of a smaller amount of interactions is shown in (b) for $E_\gamma = 0.25\,\text{MeV}$ and $\sum E_{\text{dep}} = 0.147\,\text{MeV}$ for three and two interactions.

Additional plots

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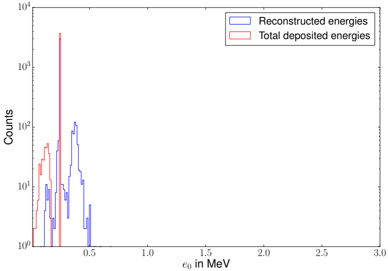
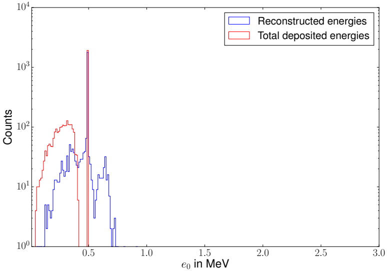


(a) $E_\mu = (0.25 \pm 0.005)$ MeV, $N_P/N_C = 1$



(b) $E_\mu = (0.5 \pm 0.005)$ MeV, $N_P/N_C = 0.96$

Figure: Energy reconstruction for $N = 3$ with $E_\gamma = 0.25$ MeV and 0.5 MeV compared to the respective total deposited energy.

Additional plots

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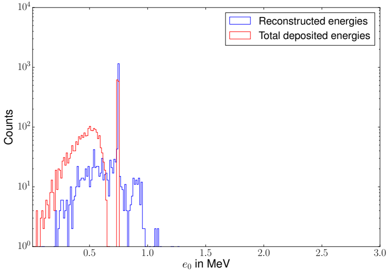
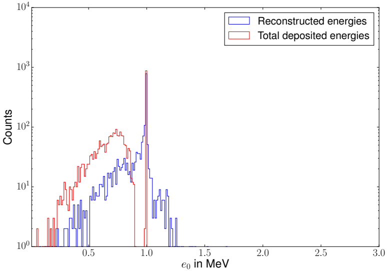


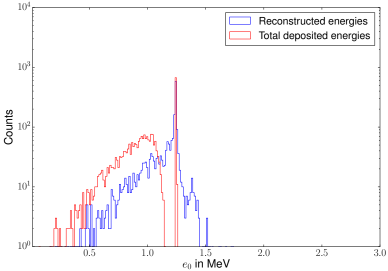
(a) $E_\mu = (0.75 \pm 0.005)$ MeV, $N_P/N_C = 0.6$



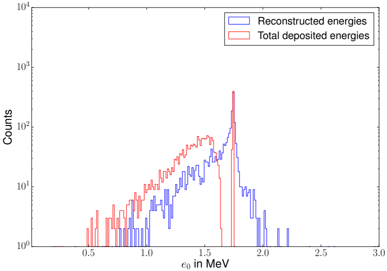
(b) $E_\mu = (1.0 \pm 0.005)$ MeV, $N_P/N_C = 0.44$

Figure: Energy reconstruction for $N = 3$ with $E_\gamma = 0.75$ MeV and 1.0 MeV compared to the respective total deposited energy.

Additional plots



(a) $E_\mu = (1.25 \pm 0.005)$ MeV, $N_P/N_C = 0.34$



(b) $E_\mu = (1.75 \pm 0.01)$ MeV, $N_P/N_C = 0.22$

Figure: Energy reconstruction for $N = 3$ and $E_\gamma = 1.25$ MeV and 1.75 MeV compared to the respective total deposited energy.

Additional plots

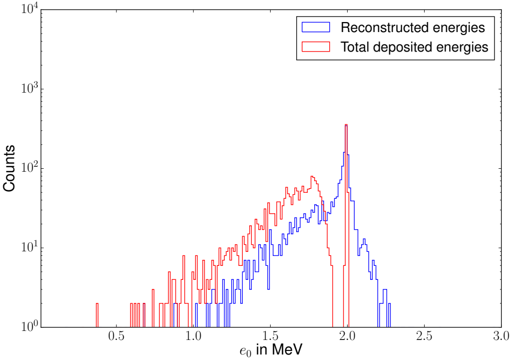
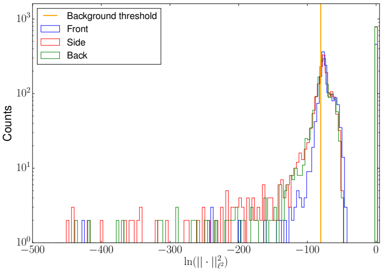
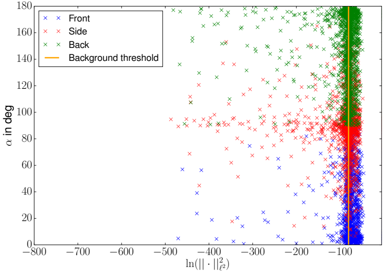


Figure: Energy reconstruction for $N = 3$ with $E_\gamma = (2.0 \pm 0.01)$ MeV ($N_P/N_C = 0.21$) compared to the respective total deposited energy.

Additional plots



(a) $\ln(||(p_n)_n||^2_{\ell^2})$ for general directions



(b) $\ln(||p_n)_n||^2_{\ell^2})$ depending on α

Figure: Histogram of $\ln(||(p_n)_n||^2_{\ell^2})$ for the general ingoing directions of the background photons (front, side, back of detector) (a) and $\ln(||(p_n)_n||^2_{\ell^2})$ depending on the exact angle between the source photon direction and the background photon direction α (b).

Additional plots

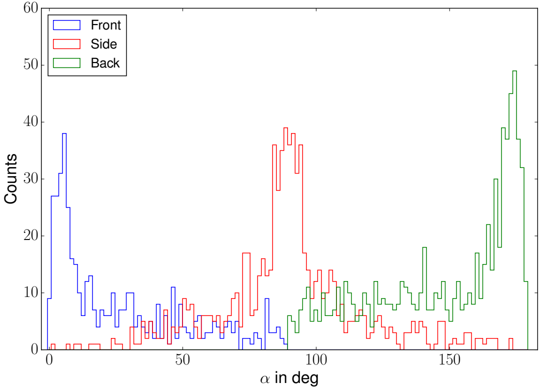


Figure: Histogram for background photons that yielded a likelihood function of zero depending on their angle of incidence α.

Additional plots

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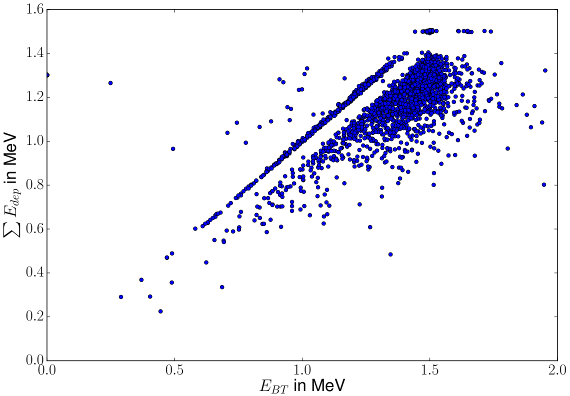

Figure: Influence of the total deposited energy $\sum E_{\text{dep}}$ on the reconstructed energy E_{BT}.

Additional plots

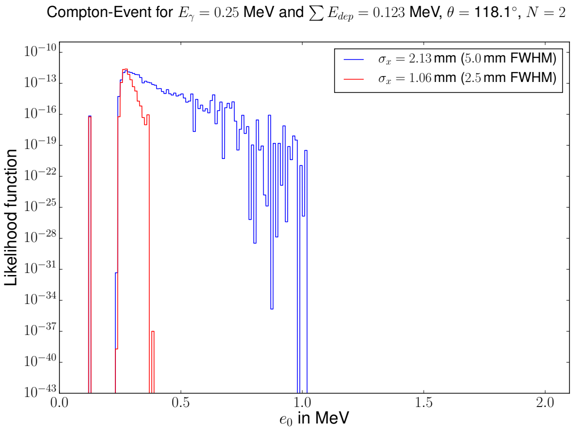

Figure: Influence of the interaction point measurement uncertainty σ_x on the likelihood function5.