GDR Terascale
Avoiding the Goldstone Boson Catastrophe in general renormalisable field theories at two loops

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The context

Going Beyond the Standard Model

- 2012: discovery of a SM-Higgs-like particle by ATLAS and CMS
- No Physics beyond the SM found yet

$\Rightarrow$ properties of the Higgs as a probe for new Physics $\rightarrow$ Higgs mass m_h^2

- A tool to compute the Higgs mass $\rightarrow$ **effective potential** V_{eff}

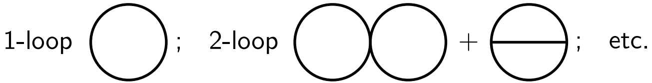
State of the art

- **SM**: V_{eff} (relates $m_h^2 \leftrightarrow \lambda$) is known to full 2-loop (*Ford, Jack and Jones '92*) + leading – QCD – 3-loop and 4-loop (*Martin '13, Martin '15*)
- Some results for m_h^2 in specific SUSY theories: **MSSM** (leading – SQCD – 3-loop order); **NMSSM** (2-loop); **Dirac Gaugino models** (leading – SQCD – 2-loop: *J.B., Goodsell, Slavich '16*)
- **Generic theories**: V_{eff} computed to 2-loop (Martin '01), tadpoles and scalar masses (in gaugeless limit) implemented in `SARAH` (*Goodsell, Nickel, Staub '15*)

The effective potential

$$V_{\text{eff}} = V^{(0)} + \text{quantum corrections}$$

- Quantum corrections = 1PI vacuum graphs computed loop by loop



- Expressed as a function of **running tree-level masses** of particles, in some **minimal substraction scheme** ($\overline{\text{MS}}$, $\overline{\text{DR}}'$, etc.)
- First derivative of V_{eff}: **tadpole equation** ($\leftrightarrow$ minimum condition), relates vev and mass-squared parameters
- Second derivative: same as self-energy diagrams, but with **zero external momentum** $\rightarrow$ **approximate scalar masses**

The Goldstone Boson Catastrophe

- Beyond one loop, V_{eff} only computed in Landau gauge $\Rightarrow$ Goldstones are treated as actual massless bosons *i.e.* $(m_G^2)^{\text{OS}} = 0$

- By **choice** (simplicity) V_{eff} is computed with running masses:

$$(m_G^2)^{\text{run.}} = (m_G^2)^{\text{OS}} - \Pi_G((m_G^2)^{\text{OS}}) = -\Pi_G(0),$$

where Π_G is the Goldstone self-energy

- Under RG flow, $(m_G^2)^{\text{run.}}$ may
  - $\rightarrow$ become 0 $\Rightarrow$ infrared divergence in V_{eff}
  - $\rightarrow$ change sign $\Rightarrow$ imaginary part in V_{eff}

$\equiv$ **Goldstone boson catastrophe**

Illustration: the abelian Goldstone model

- 1 complex scalar $\phi = \frac{1}{\sqrt{2}}(v + h + iG)$, no gauge group and only a potential

$$V^{(0)} = \mu^2|\phi|^2 + \lambda|\phi|^4$$

v: true vev, to all orders in perturbation theory (PT)

- SM: G^+, G^0 Goldstones do not mix, and can be treated separetely
  $\rightarrow$ this model captures the behaviour of the GBC in the SM
- V_{eff} at 2-loop order:

$$V_{\text{eff}} = V^{(0)} + \underbrace{\frac{1}{16\pi^2}\Big[f(m_h^2) + f(m_G^2)\Big]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{1}{(16^2)^2}\Big[\lambda\Big(\frac{3}{4}A(m_G^2)^2 + \frac{1}{2}A(m_G^2)A(m_h^2)\Big) - \lambda^2 v^2 I(m_h^2, m_G^2, m_G^2) + \overbrace{\cdots}^{\text{no Goldstone}}\Big]}_{\text{2-loop}} + \mathcal{O}(\text{3-loop})$$

where $f(x) = \frac{x^2}{4}(\log x/Q^2 - 3/2)$, $A(x) = x(\log x/Q^2 - 1)$ and $I \propto$ 

- Tree-level masses: $m_h^2 = \mu^2 + 3\lambda v^2$, $m_G^2 = \mu^2 + \lambda v^2$

Illustration: the abelian Goldstone model

Tree-level tadpole

$$\left.\frac{\partial V^{(0)}}{\partial h}\right|_{h=0,G=0} = 0 = \mu^2 v + \lambda v^3 = m_G^2 v$$

Loop-corrected tadpole

$$\left.\frac{\partial V_{\text{eff}}}{\partial h}\right|_{h=0,G=0} = 0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\left[3A(m_h^2) + A(m_G^2)\right]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\log\frac{m_G^2}{Q^2}}{(16^2)^2}\left[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{m_h^2} A(m_h^2)\right] + \overbrace{\cdots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}} + \mathcal{O}(\text{3-loop})$$

Illustration: the abelian Goldstone model

Tree-level tadpole equation

$$\frac{\partial V^{(0)}}{\partial h}\bigg|_{h=0,G=0} = 0 = \mu^2 v + \lambda v^3 = m_G^2 v$$

Loop-corrected tadpole equation

$$\frac{\partial V_{\text{eff}}}{\partial h}\bigg|_{h=0,G=0} = 0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\left[3A(m_h^2) + A(m_G^2)\right]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\overbrace{\log\frac{m_G^2}{Q^2}}^{\text{GBC!}}}{(16^2)^2}\left[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{m_h^2} A(m_h^2)\right] + \overbrace{\cdots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}} + \mathcal{O}(\text{3-loop})$$

First approaches to the GBC

By hand

- ▷ if $m_G^2 < 0$, drop the imaginary part of V_{eff}
- ▷ tune the renormalisation scale Q to ensure $m_G^2 > 0$ (and even m_G^2 not too small)

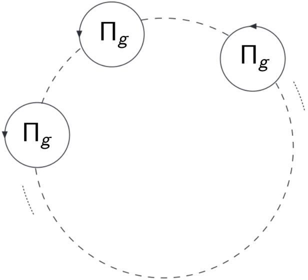
  $\Rightarrow$ may be impossible to achieve and is completely ad hoc

In automated codes (SARAH)

- For SUSY theories **only**
- Rely on the gauge-coupling dependent part of $V^{(0)}$
  - $\rightarrow$ minimize full $V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2} V^{(1)} + \frac{1}{(16\pi^2)^2} V^{(2)}|_{\text{gaugeless}}$
  - $\rightarrow$ compute tree-level masses with $V^{(0)}|_{\text{gaugeless}}$ (= turn off the D-term potential)
  - $\rightarrow$ yields a fake Goldstone mass of order $\mathcal{O}(m_{EW}^2) \Rightarrow$ no GBC

Resummation of the Goldstone contribution

SM: Martin 1406.2355; Ellias-Miro, Espinosa, Konstandin 1406.2652.
MSSM: Kumar, Martin 1605.02059.



[Adapted from arXiv:1406.2652]

- Power counting $\rightarrow$ most divergent contribution to V_{eff} at ℓ-loop $=$ ring of $\ell-1$ Goldstone propagators and $\ell-1$ insertions of 1PI subdiagrams Π_g involving **only** heavy particles
- Π_g obtained from Π_G, Goldstone self-energy, by removing "soft" Goldstone terms
- Resumming Goldstone rings $\Leftrightarrow$ shifting the Goldstone tree-level mass by Π_g in the 1-loop Goldstone term

$$\hat{V}_{\text{eff}} = V_{\text{eff}} + \frac{1}{16\pi^2}\left[f(m_G^2+\Pi_g) - \sum_{n=0}^{\ell-1}\frac{(\Pi_g)^n}{n!}\left(\frac{d}{dm_G^2}\right)^n f(m_G^2)\right]$$

$\rightarrow$ ℓ-loop resummed V_{eff}, free of leading Goldstone boson catastrophe

Extending the resummation to generic theories arXiv:1609.06977

Generic theories: J.B., Goodsell arXiv:1609.06977

Real scalar fields $\varphi_i^0 = v_i + \phi_i^0$, where v_i are the vevs **to all order in PT**

$$V^{(0)}(\{\varphi_i^0\}) = V^{(0)}(v_i) + \frac{1}{2} m^2_{0,ij}\phi_i^0\phi_j^0 + \frac{1}{6}\hat{\lambda}_0^{ijk}\phi_i^0\phi_j^0\phi_k^0 + \frac{1}{24}\hat{\lambda}_0^{ijkl}\phi_i^0\phi_j^0\phi_k^0\phi_l^0$$

$m^2_{0,ij}$ solution of the tree-level tadpole equation

To work in minimum of loop-corrected $V_{\text{eff}} \to$ new couplings m^2_{ij}

$\Downarrow$

Diagonalise to work with mass eigenstates in both bases

$$(\phi_i^0, m^2_{0,ij}) \xrightarrow{\phi_i^0 = \tilde{R}_{ij}\tilde{\phi}_j} (\tilde{\phi}_i, \tilde{m}_i) \textit{ (no loop corrections)}$$

$$(\phi_i^0, m^2_{ij}) \xrightarrow{\phi_i^0 = R_{ij}\phi_j} (\phi_i, m_i) \textit{ (with loop corrections)}$$

$\Downarrow$

Single out the Goldstone boson(s), index $G, G', \ldots$ and its/their mass(es)

$$m_G^2 = -\sum_i \frac{1}{v_i}(\tilde{R}_{iG})^2 \left.\frac{\partial(V_{\text{eff}} - V^{(0)})}{\partial\phi_i^0}\right|_{\phi_i^0=0} = \mathcal{O}(1\text{-loop})$$

Our solution: setting the Goldstone boson on-shell arXiv:1609.06977

Issues with the resummation

- taking derivatives of $\hat{V}_{\text{eff}}$ can be very difficult (involves derivatives of the rotation matrices, etc.) $\rightarrow$ in practice resummation was **only** used to find the **tadpole equations**.
- the choice of "soft" Goldstone terms to remove from Π_G to find Π_g may be ambiguous and it is difficult to justify which terms to keep

Setting the Goldstone boson on-shell

- Adopt an on-shell scheme for the Goldstone(s): replace $(m_G^2)^{\text{run.}}$ by $(m_G^2)^{\text{OS}}(=0)$ and $\Pi_G(0)$

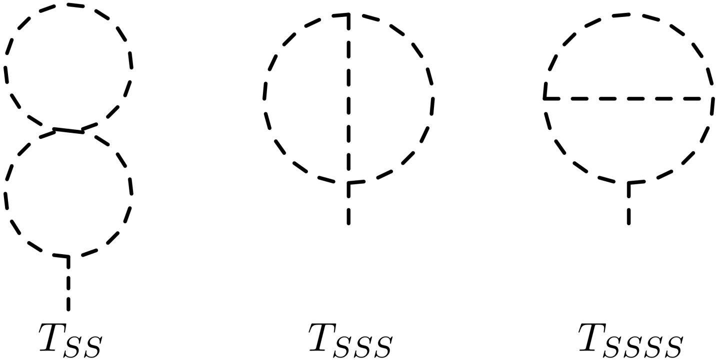
$$(m_G^2)^{\text{run.}} = (m_G^2)^{\text{OS}} - \Pi_G((m_G^2)^{\text{OS}}) = -\Pi_G(0)$$

- This can be done **directly** in the tadpole equations or mass diagrams!

Canceling the IR divergences in the tadpole equations arXiv:1609.06977

2-loop tadpole diagrams involving scalars only:

The GBC also appears in diagrams with scalars and fermions or gauge bosons, and is cured with the same procedure → we present the purely scalar case.



Canceling the IR divergences in the tadpole equations arXiv:1609.06977

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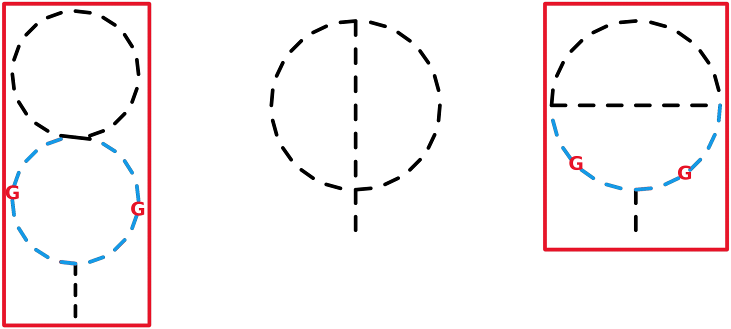
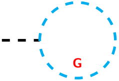


T_{SS} T_{SSS} T_{SSSS}

Some diagrams of T_{SS} and T_{SSSS} topologies diverge for $m_G^2 \to 0$

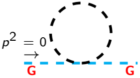
What happens when setting the Goldstone on-shell?

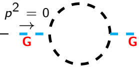
- Contribution of the Goldstone(s) to the 1-loop tadpole:

$$T_S \supset$$ 

$$\propto A(m_G^2) = m_G^2\left(\log \frac{m_G^2}{Q^2} - 1\right)$$

- At 1-loop order the scalar-only diagrams in $\Pi_G(0)$ are

$$(m_G^2)^{\text{run.}} = \underbrace{(m_G^2)^{\text{OS}}}_{=0} -$$ 

$$-$$ 

$$+ \cdots$$

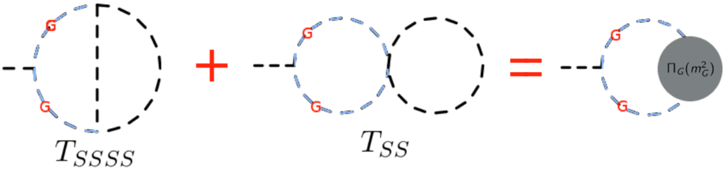
- Shifting m_G^2 by a 1-loop quantity, $\Pi_G(0)$, in the 1-loop tadpole

$\Rightarrow$ **2-loop shift !**

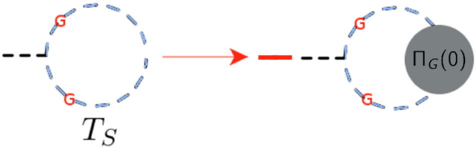
$$A((m_G^2)^{\text{run.}}) = \underbrace{A(0)}_{=0} - \underbrace{\log \frac{m_G^2}{Q^2}}_{\text{1-loop}} \underbrace{\Pi_G(0)}_{\text{1-loop}}$$

Canceling the IR divergences in the tadpole equations arXiv:1609.06977

- 2-loop divergent tadpole diagrams



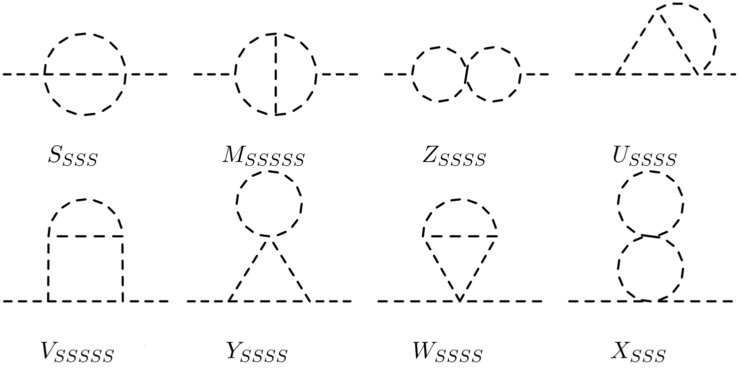
- shifting the Goldstone term in the 1-loop tadpole T_S



$\Rightarrow$ the divergent parts from the diagrams and the shift will cancel out!

Canceling the IR divergences in the mass diagrams arXiv:1609.06977

- ▷ Earlier literature: inclusion of momentum cures all the IR divergences
- ▷ We found
  - ⇒ true at 1-loop order
  - ⇒ at 2-loop, ∃ diagrams that still diverge for $m_G^2 \to 0$ **even with external momentum included**



S_{SSS} M_{SSSSS} Z_{SSSS} U_{SSSS}

V_{SSSSS} Y_{SSSS} W_{SSSS} X_{SSS}

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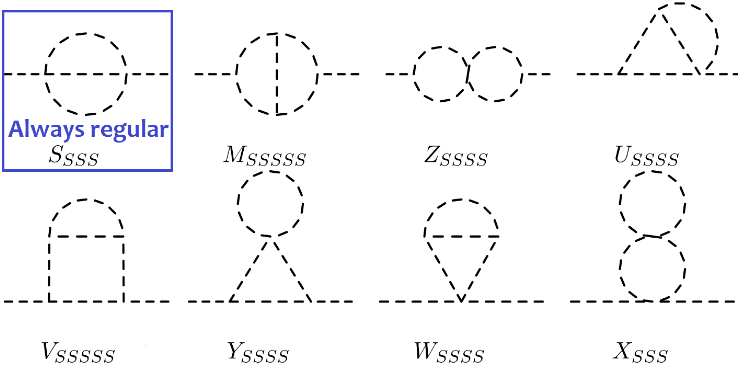

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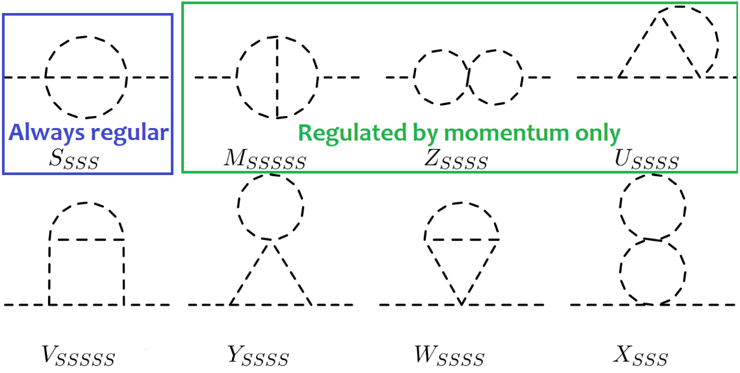

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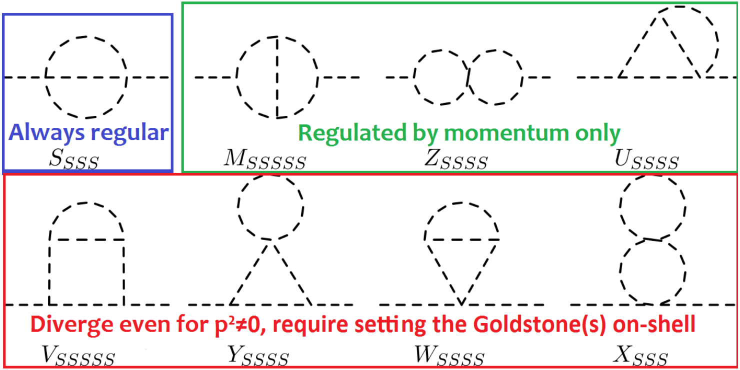
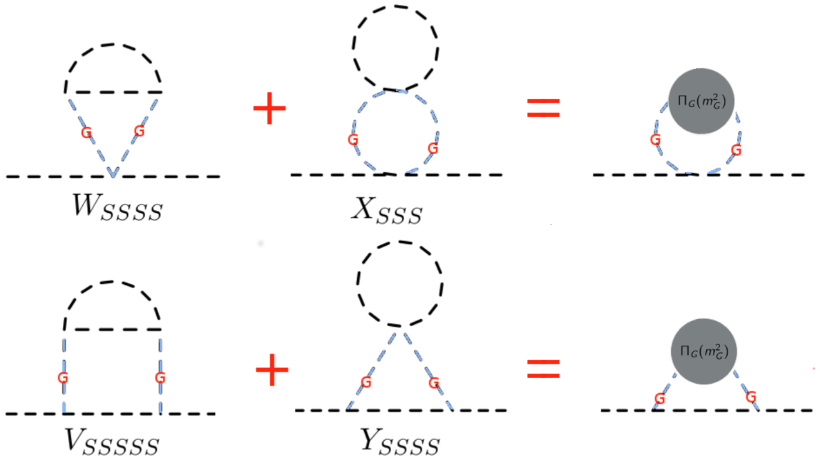


Canceling the IR divergences in the mass diagrams arXiv:1609.06977

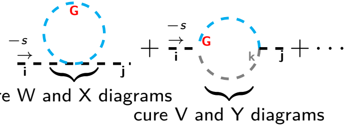


Canceling the IR divergences in the mass diagrams arXiv:1609.06977

Setting the Goldstone(s) on-shell in mass diagrams

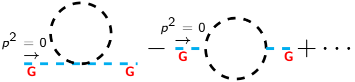
- Goldstone contributions to the 1-loop scalar self-energy

$$\Pi^{(1)}_{ij}(s = \text{-}p^2) =$$



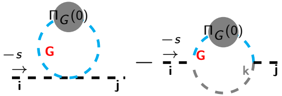
- Again, shifting the Goldstone mass to on-shell scheme gives

$$(m_G^2)^{\text{run.}} = -$$



$\rightarrow$ 2-loop shift to the mass diagrams

$$\delta\Pi^{(1)}_{ij}(s) = -$$



$\longrightarrow$ cancels the divergence in the V, X, Y, W diagrams !

Our results

- Results for generic theories (scalars, fermions, gauge bosons), *avoiding the Goldstone boson catastrophe*
  - → full **two-loop tadpole equations**
  - → **two-loop mass diagrams** for neutral scalars in *gaugeless limit*, in a *generalised effective potential approach* (*i.e.* neglect terms of order $\mathcal{O}(s)$ and higher)

- Numerical implementation *(soon)*: `SARAH` and/or stand-alone code
  - → no more numerical instability associated with the GBC
  - → in particular useful for automated study of non-SUSY theories (for which there was previously no way of evading the GBC)

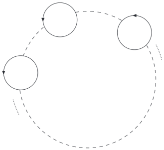
Outlook

- Further work on the GBC
  - investigate further the link between resummation and on-shell method
  - extend the solution of GBC to higher loop order
    - → on-shell method still working?
    - → how to formalise/prove the resummation prescription? ($i.e.$ how to find Π_g)
  - extend mass-diagram calculations to quartic order in the gauge couplings (go beyond the gaugeless limit)
- Apply similar techniques to address other IR divergences

Thank you for your attention !

Backup

More details on the resummation of Goldstone contributions

$R_\ell \equiv$ 

$$\propto \int \frac{d^d k}{i(2\pi)^d} \left(\frac{\Pi_g}{k^2 - m_G^2} \right)^{\ell-1}$$

$$\propto \frac{(\Pi_g)^{\ell-1}}{(\ell-1)!} \left(\frac{d}{dm_G^2} \right)^{\ell-1} \int \frac{d^d k}{i(2\pi)^d} \log(k^2 - m_G^2)$$

$$= \frac{1}{16\pi^2} \frac{(\Pi_g)^{\ell-1}}{(\ell-1)!} \left(\frac{d}{dm_G^2} \right)^{\ell-1} f(m_G^2)$$

$$\text{so} \quad \sum_\ell R_\ell = \frac{1}{16\pi^2} f(m_G^2 + \Pi_g)$$

where $f(x) = \frac{x^2}{4}(\overline{\log}\, x - \frac{3}{2})$

More details about the calculations for the scalar-only tadpole

Divergent terms

- From T_{SS}:

$$\left.\frac{\partial V_S^{(2)}}{\partial \phi_r^0}\right|_{\varphi=v} \supset \frac{1}{4} R_{rp} \sum_{l \neq G} \lambda^{GGll} \lambda^{GGp} \,\overline{\log}\, m_G^2 A(m_l^2)$$

- From T_{SSSS}:

$$\left.\frac{\partial V_S^{(2)}}{\partial \phi_r^0}\right|_{\varphi=v} \supset \frac{1}{4} R_{rp} \lambda^{pGG} \lambda^{Gkl} \lambda^{Gkl} \,\overline{\log}\, m_G^2 P_{SS}(m_k^2, m_l^2)$$

Setting the Goldstone mass on-shell

$$\Pi_{GG}^{(1),S}\left(p^2\right) = \frac{1}{2}\lambda^{GGjj} A(m_j^2) - \frac{1}{2}(\lambda^{Gjk})^2 B(p^2, m_j^2, m_k^2)$$

- Hence a 2-loop shift:

$$\frac{\partial V_S^{(2)}}{\partial \phi_r^0}((m_G^2)^{\mathrm{OS}}) = \left.\frac{\partial V_S^{(2)}}{\partial \phi_r^0}\right|_{m_G^2 \to (m_G^2)^{\mathrm{OS}}} - \frac{1}{4} R_{rp} \lambda^{GGp} \,\overline{\log}(m_G^2)^{\mathrm{OS}} \left(\lambda^{GGjj} A(m_j^2) - (\lambda^{Gjk})^2 B(0, m_j^2, m_k^2)\right).$$

The full 2-loop tadpole equation free of GBC

$$\left.\frac{\partial \hat{V}^{(2)}}{\partial \phi_r^0}\right|_{\varphi=v} = R_{rp}\Big[\overline{T}^p_{SS} + \overline{T}^p_{SSS} + \overline{T}^p_{SSSS} + \overline{T}^p_{SSFF} + \overline{T}^p_{FFFS}$$

$$+ \overline{T}^p_{SSV} + \overline{T}^p_{VS} + \overline{T}^p_{VVS} + \overline{T}^p_{FFV} + \overline{T}^p_{\overline{FF}V} + \overline{T}^p_{\text{gauge}}\Big].$$

Notations: see 1609.06977, 1503.03098

The full 2-loop tadpole equation free of GBC

The all-scalar diagrams are

$$
\begin{aligned}
\overline{T}^{p}_{SS} =& \frac{1}{4} \sum_{j,k,l \neq G} \lambda^{jkll} \lambda^{jkp} P_{SS}(m_j^2, m_k^2) A(m_l^2) \\
&+ \frac{1}{2} \sum_{k,l \neq G} \lambda^{Gkll} \lambda^{Gkp} P_{SS}(0, m_k^2) A(m_l^2), \\
\overline{T}^{p}_{SSS} =& \frac{1}{6} \lambda^{pjkl} \lambda^{jkl} f_{SSS}(m_j^2, m_k^2, m_l^2)\big|_{m_G^2 \to 0}, \\
\overline{T}^{p}_{SSSS} =& \frac{1}{4} \sum_{(j,j') \neq (G,G')} \lambda^{pjj'} \lambda^{jkl} \lambda^{j'kl} U_0(m_j^2, m_{j'}^2, m_k^2, m_l^2) \\
&+ \frac{1}{4} \sum_{(k,l) \neq (G,G')} \lambda^{pGG'} \lambda^{Gkl} \lambda^{G'kl} R_{SS}(m_k^2, m_l^2),
\end{aligned}
$$

where by $(j, j') \neq (G, G')$ we mean that j, j' are not both Goldstone indices.

The full 2-loop tadpole equation free of GBC

The fermion-scalar diagrams are

$$\overline{T}^p_{SSFF} = \sum_{(k,l)\neq(G,G')} \Bigg\{ \frac{1}{2} y^{IJk} y_{IJl} \lambda^{klp} f^{(0,0,1)}_{FFS}(m_I^2, m_J^2; m_k^2, m_l^2)$$

$$-\text{Re}\left[y^{IJk} y^{I'J'k} M^*_{II'} M^*_{JJ'} \right] \lambda^{klp} U_0(m_k^2, m_l^2, m_I^2, m_J^2) \Bigg\}$$

$$+ \frac{1}{2} \lambda^{GG'p} y^{IJG} y_{IJG'} \left(-I(m_I^2, m_J^2, 0) - (m_I^2 + m_J^2) R_{SS}(m_I^2, m_J^2) \right)$$

$$- \lambda^{GG'p} \text{Re}\left[y^{IJG} y^{I'J'G'} M^*_{II'} M^*_{JJ'} \right] R_{SS}(m_I^2, m_J^2),$$

$$\overline{T}^p_{FFFS} = T^p_{FFFS}\Big|_{m_G^2 \to 0},$$

The full 2-loop tadpole equation free of GBC

The gauge boson-scalar tadpoles are

$$\overline{T}^p_{SSV} = T^p_{SSV}\big|_{m_G^2\to 0},$$

$$\overline{T}^p_{VS} = \frac{1}{4} g^{abii} g^{abp} f^{(1,0)}_{VS}(m_a^2, m_b^2; m_i^2)\big|_{m_G^2\to 0} + \sum_{(i,k)\neq(G,G')} \frac{1}{4} g^{aaik} \lambda^{ikp} f^{(0,1)}_{VS}(m_a^2; m_i^2, m_k^2),$$

$$\overline{T}^p_{VVS} = \frac{1}{2} g^{abi} g^{cbi} g^{acp} f^{(1,0,0)}_{VVS}(m_a^2, m_c^2; m_b^2, m_i^2)\big|_{m_G^2\to 0} + \sum_{(i,j)\neq(G,G')} \frac{1}{4} g^{abi} g^{abj} \lambda^{ijp} f^{(0,0,1)}_{VVS}(m_a^2, m_b^2; m_i^2, m_j^2) - \frac{1}{4} g^{abG} g^{abG'} \lambda^{GG'p} R_{VV}(m_a^2, m_b^2).$$

The full 2-loop tadpole equation free of GBC

The gauge boson-fermion and gauge diagrams are not affected by the Goldstone boson catastrophe

$$\begin{aligned}
\overline{T}^{p}_{FFV} =& 2 g^{aJ}_{I} \overline{g}^{K}_{bJ} \mathrm{Re}[M_{KI'} y^{I'Ip}] f^{(1,0,0)}_{FFV}(m_I^2, m_K^2; m_J^2, m_a^2) \\
&+ \frac{1}{2} g^{aJ}_{I} \overline{g}^{I}_{bJ} g^{abp} f^{(0,0,1)}_{FFV}(m_I^2, m_J^2; m_a^2, m_b^2), \\
\overline{T}^{p}_{\overline{FF}V} =& g^{aJ}_{I} g^{aJ'}_{I'} \mathrm{Re}[y^{II'p} M^{*}_{JJ'}] \left[f_{\overline{FF}V}(m_I^2, m_J^2, m_a^2) + M_I^2 f^{(1,0,0)}_{\overline{FF}V}(m_I^2, m_{I'}^2; m_J^2, m_a^2) \right] \\
&+ g^{aJ}_{I} g^{aJ'}_{I'} \mathrm{Re}[M^{IK'} M^{KI'} M^{*}_{JJ'} y_{KK'p}] f^{(1,0,0)}_{\overline{FF}V}(m_I^2, m_{I'}^2; m_J^2, m_a^2) \\
&+ \frac{1}{2} g^{aJ}_{I} g^{bJ'}_{I'} g^{abp} M^{II'} M^{*}_{JJ'} f^{(0,0,1)}_{\overline{FF}V}(m_I^2, m_J^2; m_a^2, m_b^2), \\
\overline{T}^{p}_{\text{gauge}} =& \frac{1}{4} g^{abc} g^{dbc} g^{adp} f^{(1,0,0)}_{\text{gauge}}(m_a^2, m_d^2; m_b^2, m_c^2).
\end{aligned}$$