Non-perturbative analysis of the spectrum of meson resonances in an ultraviolet-complete composite-Higgs model

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▶ New strong dynamics condensates at scale Λ and spontaneously breaks a global symmetry G into H \Rightarrow Higgs is naturally light as a pNGB leaving in G/H

▶ Potential and mass for the Higgs generated from linear coupling between SM fermions and composite spin 1/2 resonances (partial compositeness)

Effective models of composite Higgs

Full gauge theory (hypergluons, hyperfermions as d.o.f) hard to study below Λ because of its non-perturbative nature \Rightarrow Effective models are useful

► Chiral Lagrangians: dictated only by global symmetries ⇒ Little information on the details of the strong dynamics ⇒ Not sure that an UV completion exists

▶ 4-fermion interactions approximate the strong dynamics
 (gauge bosons are froze-out)
 ⇒ Definite UV completion and underlying gauge symmetry respected
 ⇒ Possible to make calculation of non-perturbative quantities with
 Nambu Jona-Lasinio (NJL) techniques [Nambu and Jona-Lasinio '61]

UV completions: EW sector (Higgs as pNGB) + coloured sector (top partners)

Basic requirements for an UV completion

- ▶ Higgs as a composite Nambu-Goldstone boson leaving in coset G/H
- Custodial symmetry: $H \supset SU(2)_L \times SU(2)_R$
- ► No fundamental scalars

All possible minimal UV models classified in [Ferretti, Karateev, '14]

Minimal model: $SU(4)/Sp(4) \cong SO(6)/SO(5)$

- ▶ $SU(4)/Sp(4) \Rightarrow$ only 5 NGBs: Higgs doublet + singlet η
- ▶ 4 Weyl fermions $\psi \Rightarrow SU(4)$ global symmetry

► $Sp(4) \Rightarrow \psi$ belong to a pseudo-real hypercolour representation: the fundamental of Sp(2N) [Barnard et al. '13]

Fermionic bilinears

			Colour	Flavour	
		Lorentz	Sp(2N)	SU(4)	Sp(4)
Hypercolour fermions	ψ^a_i	(1/2, 0)		4^a	4
	$\overline{\psi}_{ai} \equiv \psi^{\dagger}_{aj} \Omega_{ji}$	(0, 1/2)	\Box_i	$\overline{4}_a$	4*
Spin-zero bilinears Spin-one bilinears	$M^{ab} \sim (\psi^a \psi^b)$	(0, 0)	1	6^{ab}	5 + 1
	$\overline{M}_{ab} \sim (\overline{\psi}_a \overline{\psi}_b)$	(0,0)	1	$\overline{6}_{ab}$	5 + 1
	$a^{\mu} \sim (\overline{\psi}_a \overline{\sigma}^{\mu} \psi^a)$	(1/2, 1/2)	1	1	1
	$(V^{\mu},A^{\mu})^b_a\sim (\overline{\psi}_a\overline{\sigma}^{\mu}\psi^b)$	(1/2, 1/2)	1	15^a_b	10 + 5

Hypercolour-invariant fermionic bilinears have the quantum numbers of the meson resonances

Lightest composite meson resonances	
${{Scalars:}\over{Vectors:}}~~\sigma+S^{\hat{A}}\sim1+5$	$rac{ extsf{Pseudo-scalars:}}{ extsf{Axial-vector:}} \eta' + G^{\hat{A}} \sim 1 + 5$

▶ The model is a vector-like gauge theory: all fermions ψ can be made massive $(m_{\psi}\psi\psi)$, while preserving the gauge hypercolour symmetry $G_c = Sp(2N)$

▶ Vafa-Witten theorem: The flavour subgroup H of G preserved by m_{ψ} can not be spontaneously broken \Rightarrow If SU(4) broken, it is broken down to Sp(4)

't Hooft anomaly matching:

Any global UV anomaly (generated by the hyperfermions ψ) must be matched in the IR, either by massless spin-1/2 baryons or Goldstone boson

 ψ 's can not form baryons because they are in pseudo-real hypercolour irreps \Rightarrow SU(4) necessarily broken

$$d^{AB\hat{C}} = 2Tr[\{T^A, T^B\}T^{\hat{C}}]$$

SU(4) broken $(T^{\hat{A}})$ and unbroken (T^{A}) generators combine in non-zero anomaly coefficients \Rightarrow Global anomalies

$$(\psi^a \psi^b) \equiv \psi^a_i \Omega_{ij} \psi^b_j$$

The unique invariant tensor of Sp(2N)is two-index antisymmetric $\Rightarrow SU(4)$ -flavour contraction also antisymmetric (4 × 4 = 6_A + 10_S)

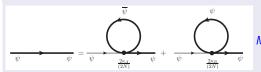
Mass gap from four-fermion interactions

Scalar 4-fermion operators relevant for the spontaneous breaking:

$$\mathcal{L}_{scal}^{\psi} = \frac{\kappa_{\mathbf{A}}}{2N} (\psi^{\mathbf{a}} \psi^{\mathbf{b}}) (\overline{\psi}_{\mathbf{a}} \ \overline{\psi}_{\mathbf{b}}) + \frac{\kappa_{\mathbf{B}}}{8N} \left[\epsilon_{\mathbf{a}\mathbf{b}\mathbf{c}\mathbf{d}} (\psi^{\mathbf{a}} \psi^{\mathbf{b}}) (\psi^{\mathbf{c}} \psi^{\mathbf{d}}) + h.c. \right]$$

▶ κ_A controls spontaneous symmetry breaking $SU(4) \rightarrow Sp(4)$ ▶ κ_B explicitly breaks the anomalous U(1) symmetry ▶ 4-fermion vector operators with couplings $\kappa_{C,D}$

Schwinger Dyson equation determines dynamical fermion mass M_{ψ}



$$M_{\psi} = 4(\kappa_A + \kappa_B)M_{\psi}\tilde{A}_0(M_{\psi}^2)$$

Self-consistence implicitly ressums all diagrams leading in $1/\mathsf{N}$

coupling

$$\xi \equiv \frac{\Lambda^2(\kappa_A + \kappa_B)}{4\pi^2} = \left[1 - \frac{M_{\psi}^2}{\Lambda^2} \ln\left(\frac{\Lambda^2 + M_{\psi}^2}{M_{\psi}^2}\right)\right]^{-1} \rightarrow \text{Non triv}$$
spontaneou
critical et e e e e maximal $\rightarrow \text{Consiste}$

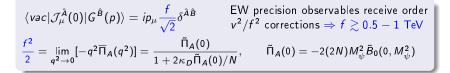
 $1<\xi\lesssim 3.25$

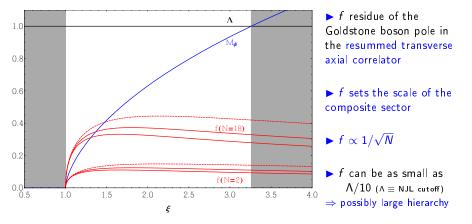
coupling

▶ Non trivial solution $M_{\psi} \neq 0$ (SU(4) spontaneously broken) exists only if $\xi > 1$

▶ Consistent resummation: $0 < M_{\psi}/\Lambda \lesssim 1$

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Bethe-Salpether equation

Resummation (geometrical series) of an infinite number of constituent fermion loops at leading order in $1/N \Rightarrow$ Two-point correlators develop a pole

The pole defines the meson mass M_{ϕ}

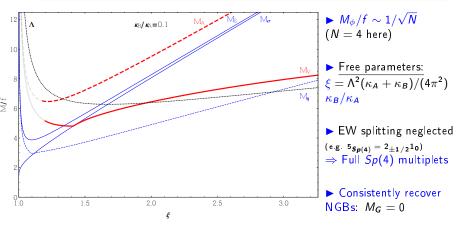
$$\overline{\Pi}_{\phi}(q^2) = \frac{\overline{\Pi}_{\phi}(q^2)}{1 - 2K_{\phi}\overline{\Pi}_{\phi}(q^2)} \longrightarrow 1 - 2K_{\phi}\overline{\Pi}_{\phi}(q^2 = M_{\phi}^2) = 0$$

φ	K_{ϕ}	$ ilde{\Pi}_{\phi}(q^2)$	
$G^{\hat{A}}$	$2(\kappa_A + \kappa_B)/(2N)$	$\tilde{\Pi}_P(q^2) = (2N) \left[\tilde{A}_0(M_{\psi}^2) - \frac{q^2}{2} \tilde{B}_0(q^2, M_{\psi}^2) \right]$	
η'	$2(\kappa_A - \kappa_B)/(2N)$		
$S^{\hat{A}}$	$2(\kappa_A - \kappa_B)/(2N)$	$\tilde{\Pi}_{S}(q^{2}) = (2N) \left[\tilde{A}_{0}(M_{\psi}^{2}) - \frac{1}{2}(q^{2} - 4M_{\psi}^{2})\tilde{B}_{0}(q^{2}, M_{\psi}^{2}) \right]$	
σ	$2(\kappa_A + \kappa_B)/(2N)$	$\Pi_{S}(q) = (2\pi) [\pi_{0}(m_{\psi}) - \frac{1}{2}(q - 4m_{\psi})D_{0}(q, m_{\psi})]$	

and similarly for the spin one channels V and A

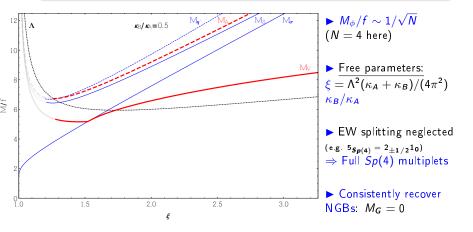
Current-current hypothesis

► Large-N relation among 4-fermion operators dominated by single hypergluon exchange $\rightarrow \kappa_A = \kappa_C = \kappa_D$ $(M_a = M_A)$



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Coloured fundamental fermions

Introduce constituent coloured fermions X^f that can form spin-1/2 baryons mixing with SM top quark

▶ VL embedding of $SU(3)_c$ inside coloured sector implies 6 Weyl fermions X^f ⇒ $SU(6) \rightarrow SO(6) \supset SU(3)_c$

► Need to go beyond Sp(2N) fundamental representation (real rep.) ⇒ 2-index traceless antisymmetric $(N \ge 2)$: $X_{ij}^f = -X_{ji}^f \sim \square \quad X_{ij}^f \Omega_{ji} = 0$

Main changes with in presence of the two sectors

▶ <u>'t Hooft anomaly matching</u>: SU(4) must be broken to Sp(4) (except for N = 8n) No such argument for SU(6)/SO(6)

▶ New conserved U(1) current \mathcal{J}_0^{μ} : Generalise κ_B -term, Extra NGB η_0

Coupling between EW and coloured sectors:

 \Rightarrow Mass gap M_{ψ} and M_X are coupled (novel feature w.r.t QCD)

 \Rightarrow Mixing in scalar and pseudo-scalar singlet sectors

► Coloured pNGBs receive mass from gluon loops: Enough to comply with direct searches even for f = 1 TeV

UV completion of composite Higgs model in term of 4-fermion interactions First application of NJL techniques in composite Higgs models

► <u>NJL well describes SSB</u>: non-perturbative computation of $M_{\psi,X}$ and $f \Rightarrow f$ can be as small as $\Lambda/10 \rightarrow$ large hierarchy could explain that no new states have been observed so far at LHC

► Computation of the composite masses (consistent with lattice results) ⇒ spectrum belong to multi-TeV range but few states can be relatively light (e.g. EW and coloured pNGBs including η_0 , η' for small κ_B/κ_A , vectors for intermediate ξ , σ for small ξ)

► Only few parameters (ξ , κ_{A6}/κ_A , κ_B/κ_A , N, m_X) if current-current hypothesis is assumed \Rightarrow Phenomenologically simple

Outlooks

- Calculation of top partners masses within NJL framework
- Generate Higgs potential by realizing partial compositeness
- Consider other UV completions (other cosets and/or hyperfermions)
- \Rightarrow Completions with $f \sim N^2$ imply lighter composite resonances in EW sector

Thanks for your attention!

Vector-like gauge theories

Vector-like \equiv An even number of Weyl fermions can be made all massive in a gauge invariant way

Vafa-Witten theorem

In any vector-like gauge theory with massless fermions and vanishing vacuum angles, the subgroup H_m of the flavour group G that corresponds to the remaining global symmetry when all fermion flavours are given identical gauge invariant masses, cannot be spontaneously broken

 \Rightarrow If H_m corresponds to a maximal subgroup of G: either G is not spontaneously broken at all, or G is spontaneously broken towards H_m

Three cases in vector-like theories: [Peskin, '80]

• $G = SU(N_f)_L \times SU(N_f)_R$ and $H_m = SU(N_f)_V$ (complex rep. of \mathcal{G})

•
$$G = SU(2N_f)$$
 and $H_m = SO(2N_f)$ (real rep.)
 $H_m = Sp(2N_f)$ (pseudo-real rep.)

The spectrum of mesons

No confinement in the NJL \Rightarrow Prescription for the unphysical imaginary parts $1 - 2K_{\phi}\tilde{\Pi}_{\phi}(q^2) = c_0^{\phi}(q^2) + c_1^{\phi}(q^2)q^2 \longrightarrow M_{\phi}^2 = Re\left[-\frac{c_0^{\phi}(M_{\phi}^2)}{c_1^{\phi}(M_{\phi}^2)}\right]$ $K_{\phi} \equiv \text{four-fermion couplings} \qquad \tilde{\Pi}_{\phi}(q^2) \equiv \text{Polarisation amplitudes}$

▶ Inserting the gap-equation, one recovers consistently the Goldstone pole: $M_G = 0$

► Singlet pseudo-scalar proportional to U(1)anomaly and mixes with axial vector: $M_{\eta'}^2 = -\frac{\kappa_B}{\kappa_A^2 - \kappa_B^2} \frac{[1 - 2\kappa_a \tilde{\Pi}_A^L(M_{\eta'}^2)]}{\tilde{B}_0(M_{\eta'}^2, M_{\psi}^2)}$

► Scalars proportional to
the mass gap
$$M_{\psi}$$
: $M_{\sigma}^2 = 4M_{\psi}^2$, $M_{\mathbf{S}}^2 = 4M_{\psi}^2 + M_{\eta'}^2 \frac{\tilde{B}_0(M_{\eta'}^2, M_{\psi}^2)}{\tilde{B}_0(M_{\mathbf{S}}^2, M_{\psi}^2)} \simeq M_{\sigma}^2 + M_{\eta'}^2$

 Vector heavy even for vanishing mass gap:

$$M_{V}^{2} = \frac{-3}{4\kappa_{D}\tilde{B}_{0}(M_{V}^{2}, M_{\psi}^{2})} + 2M_{\psi}^{2}\frac{\tilde{B}_{0}(0, M_{\psi}^{2})}{\tilde{B}_{0}(M_{V}^{2}, M_{\psi}^{2})} - 2M_{\psi}^{2}$$

Axial-vector generally the heaviest:

$$M_{A}^{2} = \frac{-3}{4\kappa_{D}\tilde{B}_{0}(M_{A}^{2}, M_{\psi}^{2})} + 2M_{\psi}^{2}\frac{\tilde{B}_{0}(0, M_{\psi}^{2})}{\tilde{B}_{0}(M_{V}A2, M_{\psi}^{2})} + 4M_{\psi}^{2} \simeq M_{V}^{2} + 6M_{\psi}^{2}$$

Adding the coloured sector

► SM fermions may mix with composite resonances (<u>partial compositeness</u>: Linear couplings between the SM fermions and coloured spin 1/2 composite resonances)

 \Rightarrow Generates couplings between SM and composite Higgs and SM fermion masses

 \Rightarrow Source of explicit breaking: generates a potential and a mass for the Higgs (potential generated mainly by the heavy top quark)

▶ Introduce constituent coloured fermions X^f that can form spin-1/2 baryons ⇒ Need to go beyond Sp(2N) fundamental representation (real rep.) → $N \ge 2$

	Lorentz	Sp(2N)	SU(6)	SO(6)	
X_{ij}^f	(1/2, 0)	\square_{ij}	6^{f}	6	
$\overline{X}_{fij} \equiv \Omega_{ik} X_{fkl}^{\dagger} \Omega_{lj}$	(0, 1/2)	\square_{ij}	$\overline{6}_{f}$	6	
$M_c^{fg} \sim (X^f X^g)$	(0, 0)	1	21^{fg}	20' + 1	
$\overline{M}_{cfg} \sim (\overline{X}_f \overline{X}_g)$	(0, 0)	1	$\overline{21}_{fg}$	20' + 1	
$a_X^\mu \sim (\overline{X}^f \overline{\sigma}^\mu X_f)$	(1/2, 1/2)	1	1	1	
$(V_c^{\mu}, A_c^{\mu})_f^g \sim (\overline{X}_f \overline{\sigma}^{\mu} X^g)$	(1/2, 1/2)	1	35_g^f	15 + 20'	

▶ 6 Weyl fermions

$$X^{f} \sim 6_{SU(6)} = (3 + \overline{3})_{SU(3)_{c}}$$

▶ $SU(6) \supset SU(3)_{c} \rightarrow SO(6)$
▶ $X_{ij} \sim \square$
 $X_{ij}^{f} = -X_{ji}^{f}, X_{ij}^{f}\Omega_{ji} = 0$

Spin-zero coloured mesons

Spin-one coloured mesons

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U(1) (anomalous) symmetries

Lot of changes appears when theory includes both EW and coloured sectors

- lmportant to consider global fermion numbers $U(1)_{\psi}$ and $U(1)_{X}$
- Currents $\mathcal{J}^{0}_{\mu\psi,X}$ both anomalous w.r.t Sp(2N) (like $U(1)_{A}$ in QCD)
- ► However, one linear combination is anomaly free and thus conserved: $\mathcal{J}^0_\mu = \mathcal{J}^0_{\mu X} - 3(N-1)\mathcal{J}^0_{\mu \psi}$

 \Rightarrow New Goldstone boson η_0 appears while η' receive a mass from the anomaly

Construct the minimal operator that preserves all exact symmetries but explicitly breaks the anomalous U(1)

• <u>EW sector</u>: Sp(2N) anomaly breaks $U(1)_{\psi} \rightarrow \mathcal{O}_{\psi} = -\frac{1}{4} \epsilon_{abcd}(\psi^{a}\psi^{b})(\psi^{c}\psi^{d})$

- <u>Colour sector</u>: anomaly breaks $U(1)_X \to \mathcal{O}_X = -\frac{1}{6!} \epsilon_{f_1 \cdots f_6} \epsilon_{g_1 \cdots g_6}(X^{f_1} X^{g_1}) \cdots (X^{f_6} X^{g_6})$
- $\blacktriangleright \text{ Full theory preserves } U(1)_{X-3(N-1)\psi} : \rightarrow \mathcal{L}_{\psi X} = A_{\psi X} \frac{\mathcal{O}_{\psi}}{(2N)^2} \left[\frac{\mathcal{O}_{X}}{[(2N+1)(N-1)]^6} \right]^{(N-1)}$

After spontaneous breaking $\mathcal{L}_{\psi X}$ generates effective 4-fermion operators ψ^4 , X^4 and $\psi^2 X^2$

Possible trilinear baryons:

Anomaly matching condition:

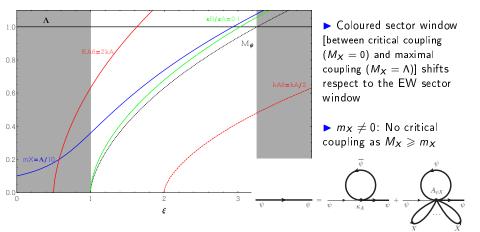
$$\sum_{i=\psi,X} n_i A(r_i) = \sum_{i=baryon} n_{i'} A(r_i), \qquad 2 \operatorname{Tr}[T_r^A \{T_r^B, T_r^C\}] = A(r) d^{ABC}$$

► <u>SU(4)</u>³: Matching impossible for $N \neq 8n \Rightarrow SU(4)$ breaks to Sp(4) and one expects non-zero condensate $\langle \psi \psi \rangle \neq 0$

► $SU(6)^3$: Matching always possible \Rightarrow SU(6) may not break to SO(6) and the condensate $\langle XX \rangle$ may vanish or not

► $SU(4)^2 \times U(1), SU(6)^2 \times U(1), U(1)^3$: U(1) most likely broken by $\langle \psi \psi \rangle$

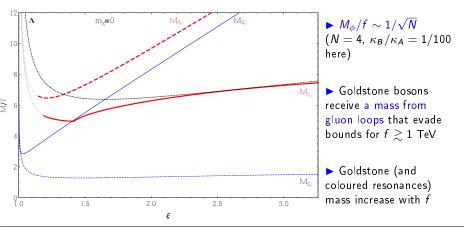
Two coupled mass gap equations: $\begin{cases}
M_{\psi} = 4 \left[\kappa_{A} + \kappa_{B}(M_{X}^{2})\right] M_{\psi} \tilde{A}_{0}(M_{\psi}^{2}) \\
M_{X} = 4 \left[\kappa_{A6} + \kappa_{B6}(M_{\psi}^{2}, M_{X}^{2})\right] M_{X} \tilde{A}_{0}(M_{X}^{2}) + m_{X}
\end{cases}
\begin{cases}
\kappa_{B} = \kappa_{B6} = 0 \\
\kappa_{A} = \kappa_{A6}, m_{X} = 0 \\
\Rightarrow M_{\psi} = M_{X}
\end{cases}$



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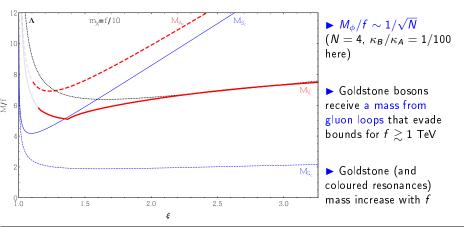
Current-current hypothesis

The ratio EW masses/ coloured masses strongly depends on the ratio κ_{A6}/κ_A Unfortunately the large-N approximation does not determine this ratio uniquely (but still determines $\kappa_{A6} = \kappa_{C6} = \kappa_{D6}$) \Rightarrow Choose $\kappa_A = \kappa_{A6}$



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The Sp(4) singlet mesons σ , η' , a^{μ} may mix with the SO(6) singlet ones σ_c , η'_c , a^{μ}_c (all SM singlets)

If on ignores mixing: $\left\{ \begin{array}{l} M_{\sigma} = 2 \, M_{\psi} \\ M_{\sigma_c} = 2 \, M_X \end{array} \right. \left\{ \begin{array}{l} M_{\eta'}^2 \sim \kappa_B \\ M_{\eta'_c}^2 \sim (\kappa_{B6}, m_X) \end{array} \right. \left\{ \begin{array}{l} M_a = M_A \\ M_{a_c} = M_{A_c} \end{array} \right.$

► Axial-vectors: Sp(2N) current-current operators do not induce singlet-singlet mixing operators

 \Rightarrow Axial singlet mixing is subleading in 1/N

▶ (Pseudo-)scalars: Anomalous operator $A_{\psi X}$ induces a coupling $\psi^2 X^2$ of the same order as the couplings ψ^4 , X^4

 \Rightarrow The mixing is a leading effect for (pseudo-)scalars

 \Rightarrow One linear combination of *eta'*, η'_c is massless for $m_X = 0$: U(1) Goldstone

$$\mathbf{K}_{\eta_{\psi}\eta_{X}} = \begin{pmatrix} K_{\eta_{\psi}} & -K_{\psi X} & 0 & 0 \\ -K_{\psi X} & K_{\eta X} & 0 & 0 \\ 0 & 0 & K_{a} & 0 \\ 0 & 0 & 0 & K_{a_{c}} \end{pmatrix} , \qquad \mathbf{\Pi}_{\eta_{\psi}\eta_{X}} = \begin{pmatrix} \tilde{\Pi}_{P}^{\psi} & 0 & \sqrt{p^{2}}\tilde{\Pi}_{AP}^{\psi} & 0 \\ 0 & \tilde{\Pi}_{P}^{X} & 0 & \sqrt{p^{2}}\tilde{\Pi}_{AP}^{X} \\ \sqrt{p^{2}}\tilde{\Pi}_{AP}^{\psi} & 0 & \tilde{\Pi}_{A}^{L\psi} & 0 \\ 0 & \sqrt{p^{2}}\tilde{\Pi}_{AP}^{X} & 0 & \tilde{\Pi}_{A}^{LX} \end{pmatrix}$$

Mixed states may couple both to EW gauge boson ($\phi \rightarrow \gamma \gamma$) and gluons ($gg \rightarrow \phi$) \Rightarrow Potential discovery channel