Three loop QCD-threshold results for the Pseudo-scalar Higgs boson production at the LHC.

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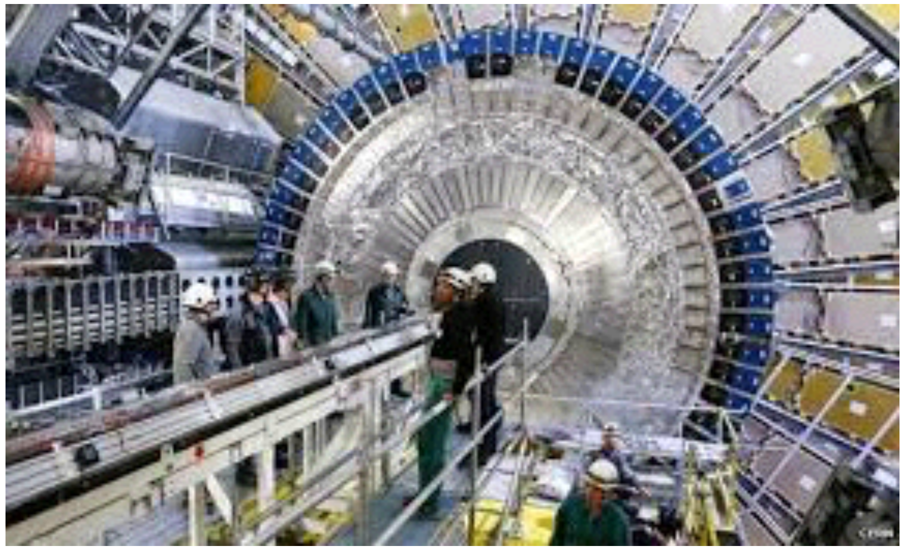
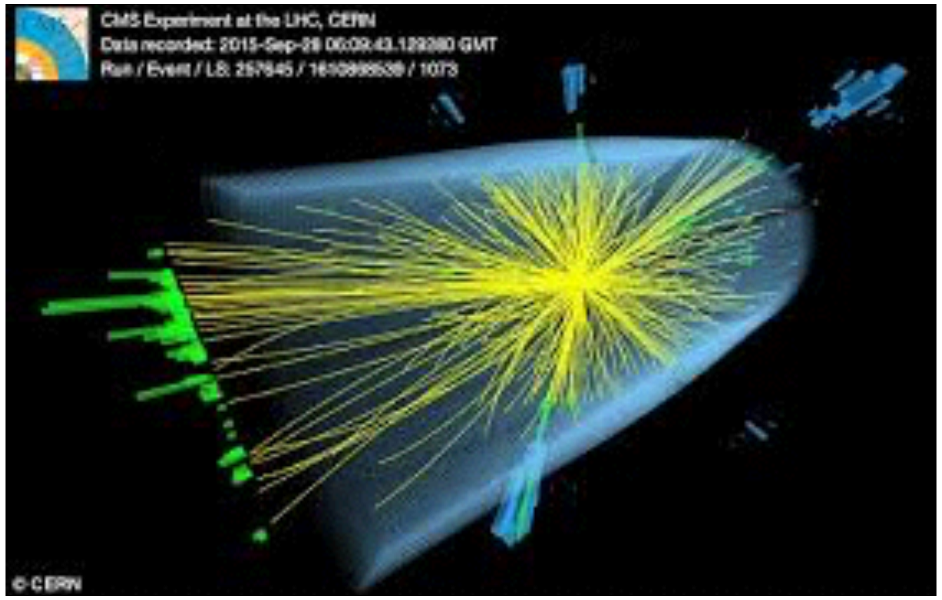
LAPTH, Annecy,19 May 2016

Plan

- Computation of Pseudo-scalar Higgs boson form factor at three loops in QCD
- Discuss UV and IR poles structure using K+G equation
- How IR can be used to obtain UV renormalisation constant
- Applications to
  - N3LO threshold corrections and
  - N-independent part of N3LL and
  - Matching coefficient in SCET
- Leading Transcendentality Principle
- Scale dependence has been studied.

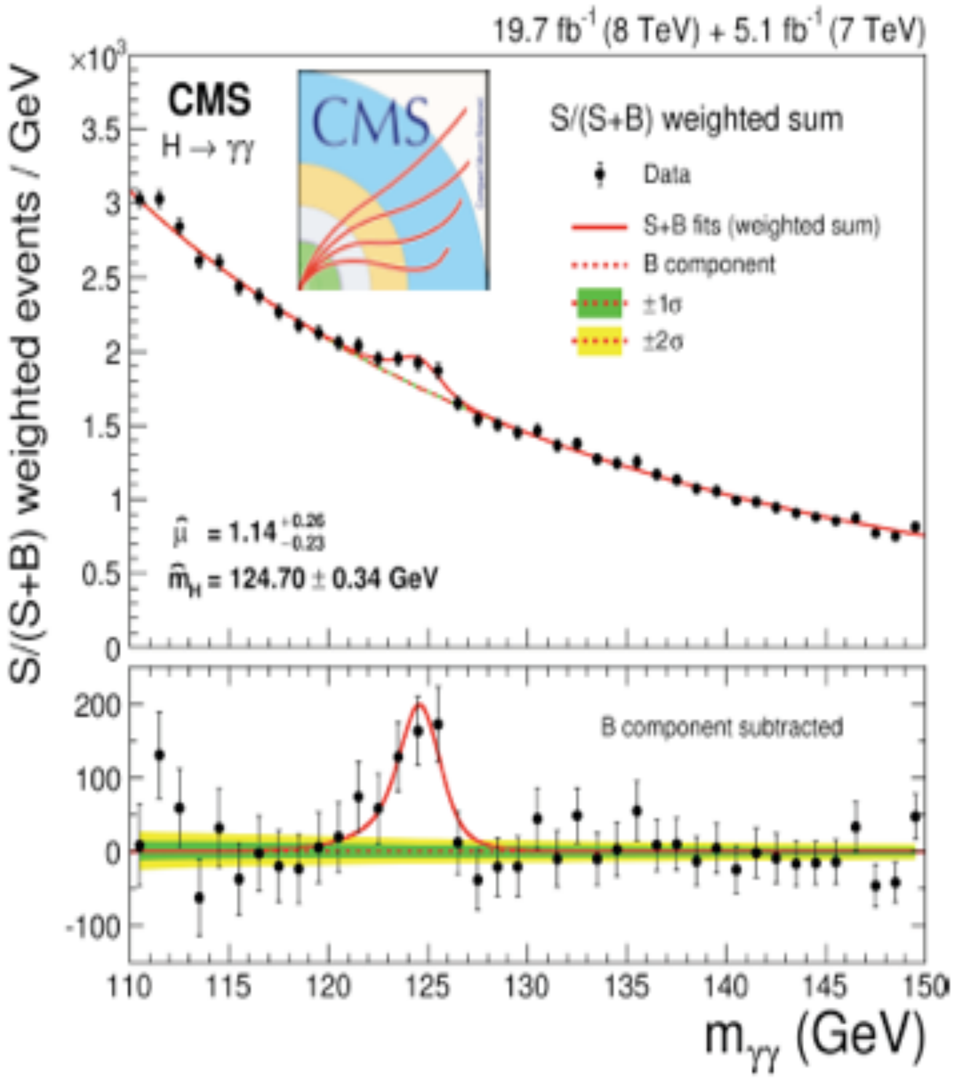
What experimentalists see…

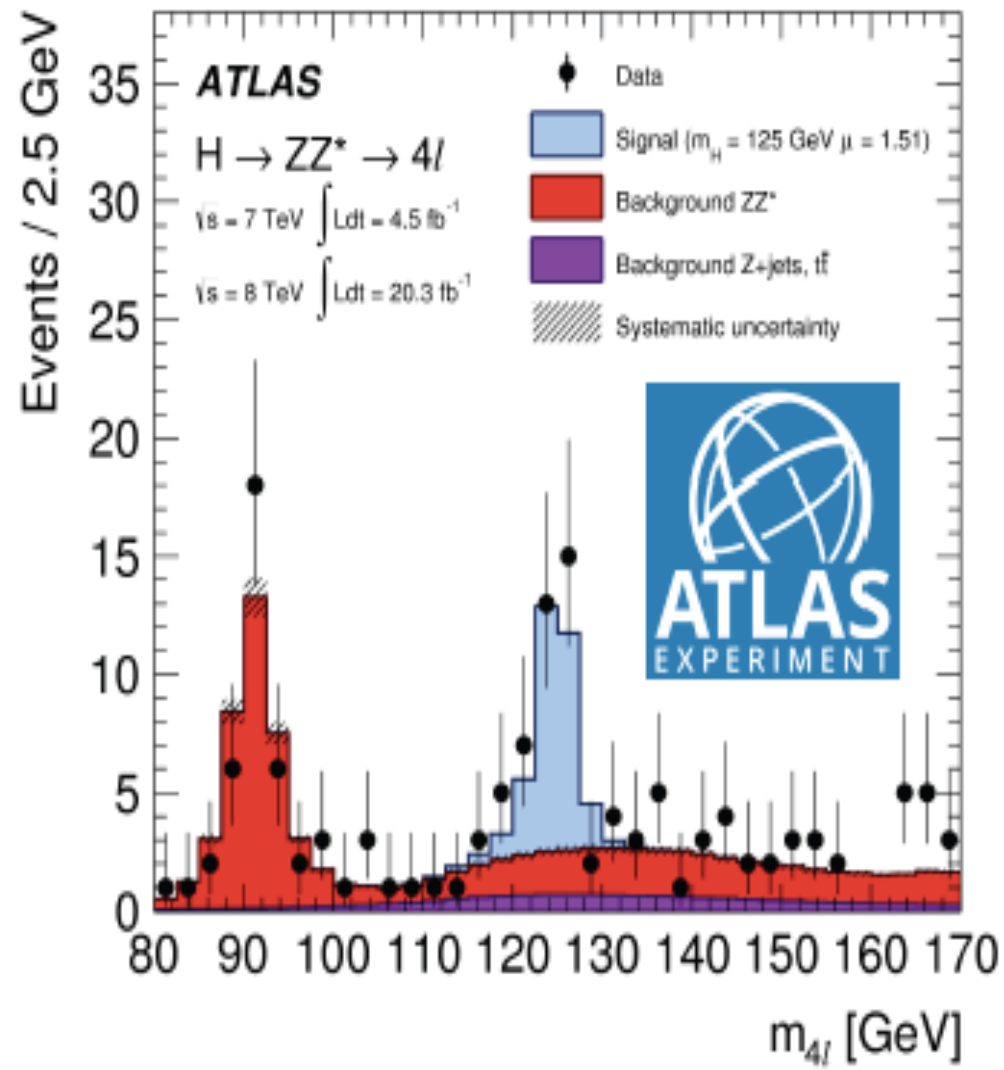




What experimentalist see

[EPJ C74 (2015)]

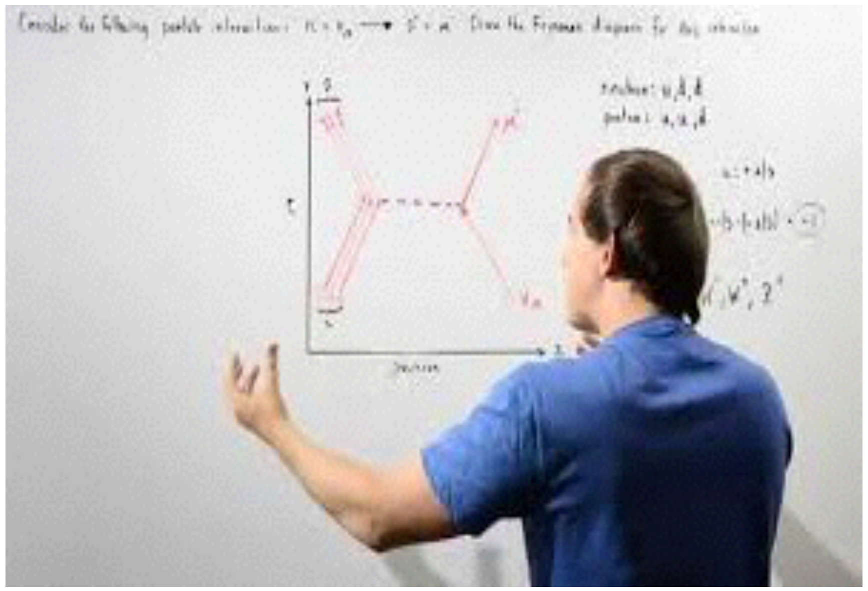


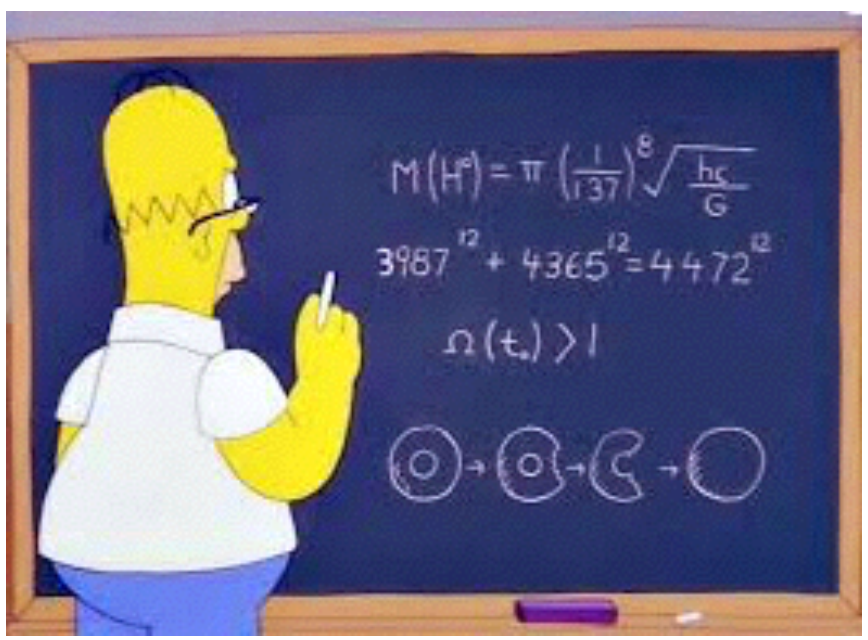


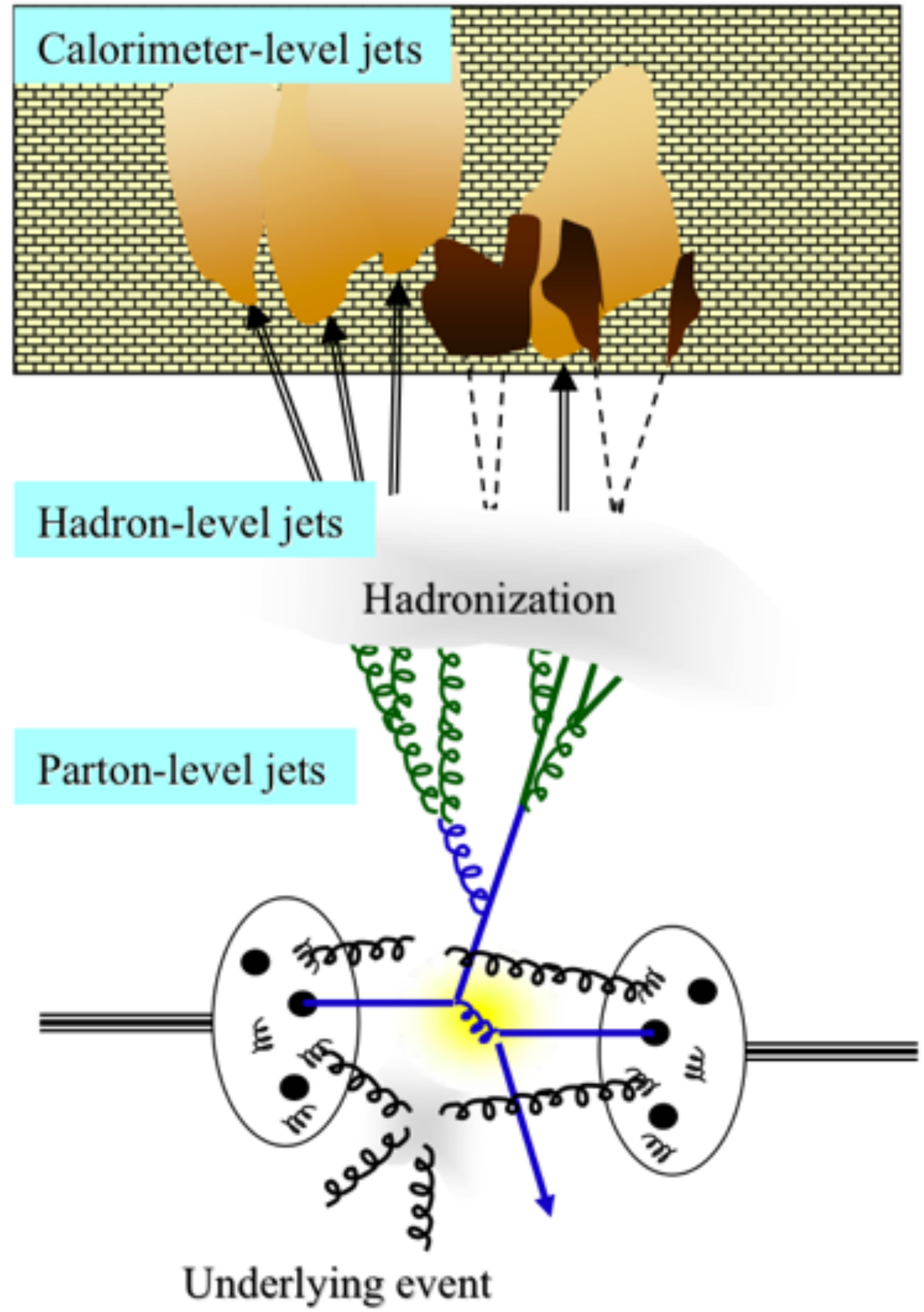
[PRD 91 (2015)]

Excess at 125 GeV

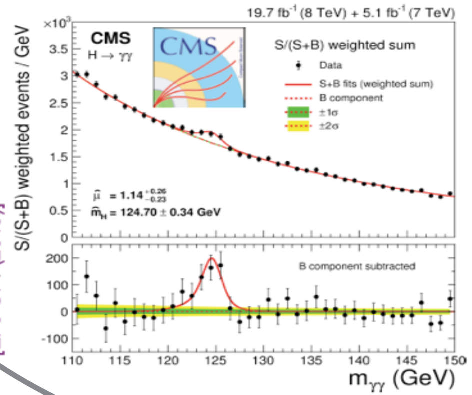
What theorists interpret…



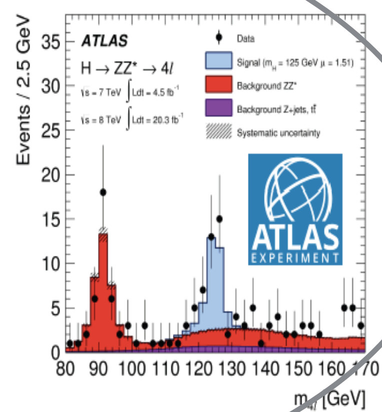




Measurement Vs Theory



[EPJ C74 (2015)]



[PRD 91 (2015)]

Significance of excess:

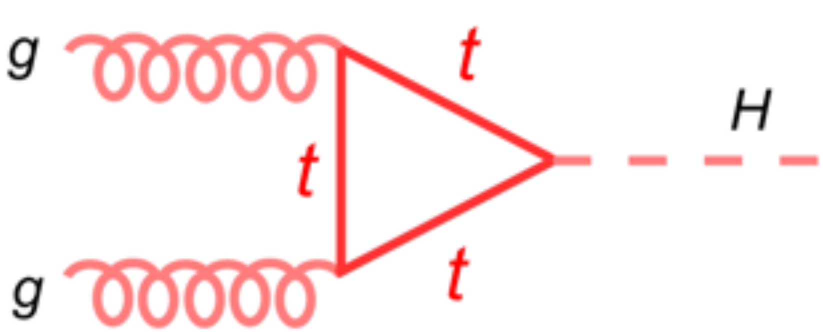
$\gamma\gamma$: 5.6 σ (5.1 exp.)

ZZ: 6.6 σ (5.5 exp.)

Signal strength $\mu = \sigma_{obs} / \sigma_{SM}$

$$\mu = 1.12^{+0.25}_{-0.23}$$

$$\mu = 1.51^{+0.39}_{-0.34}$$



Agreement with SM Higgs Boson

Inputs that can affect

- UV Renormalisation Scale

$$\alpha_s(\mu_R)$$

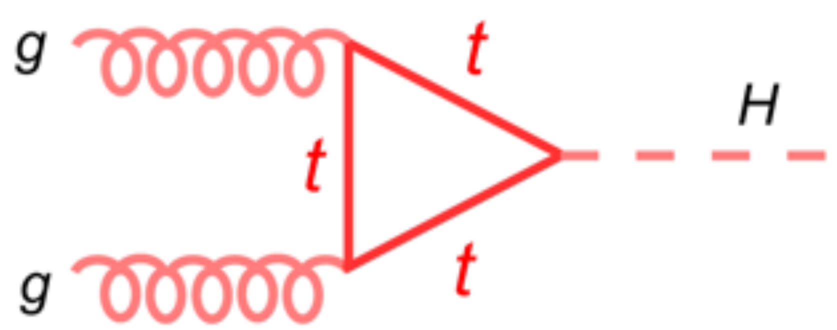
- Factorisation Scale through parton distribution functions

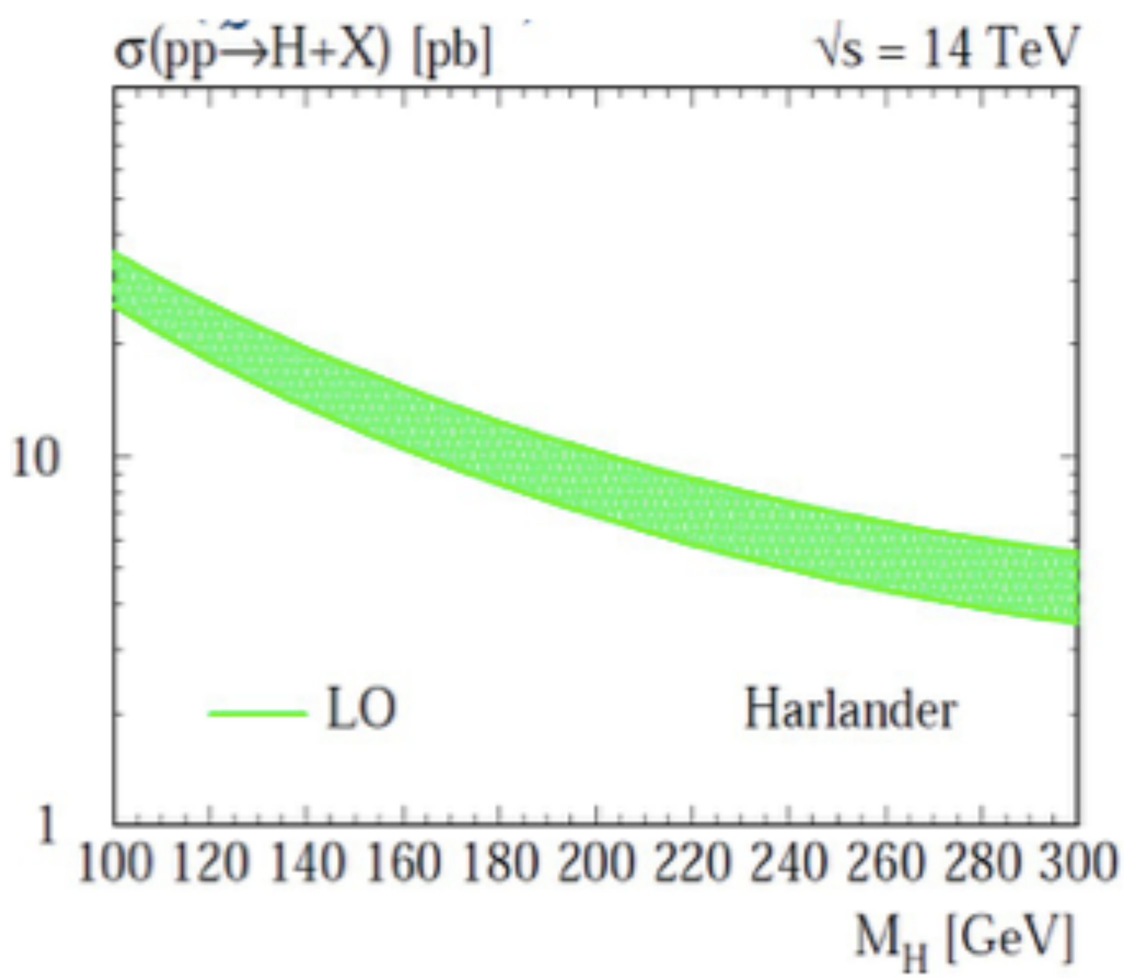
$$f_a(x, \mu_F)$$

- Missing Higher Order corrections
- Stability of the perturbation theory
- Resummation Methods
- Hadronisation models

LO is crude estimate

$$2S\sigma^H(x, m_H) = \int_x^1 \frac{dz}{z}\Phi_{gg}^{(0)}(z, \mu_F) 2\hat{s}\hat{\sigma}_{gg}^{(0)}\left(\frac{x}{z}, m_H^2, \mu_R\right) + \cdots$$



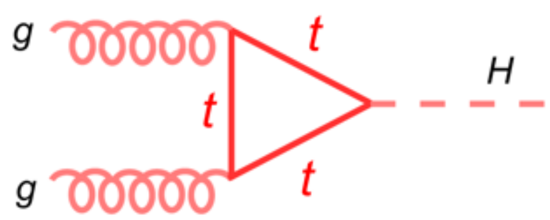


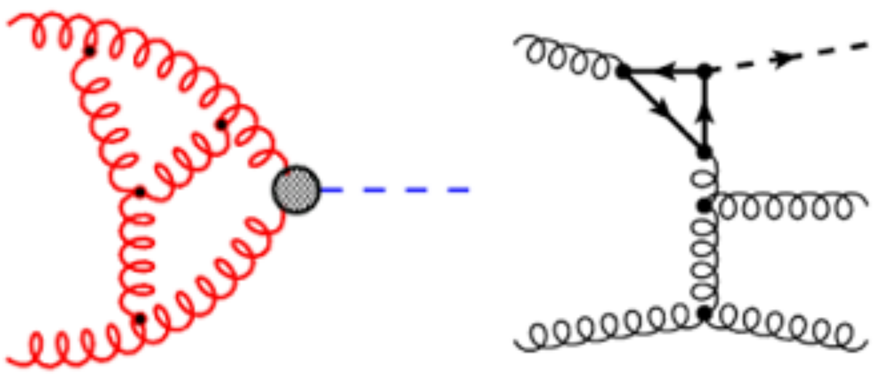
$$2\hat{s}\hat{\sigma}_{gg}^{(0)}\left(\frac{x}{z}, m_H^2, \mu_R\right) = \alpha_s^2(\mu_R) G_F F(m_t, m_H)$$

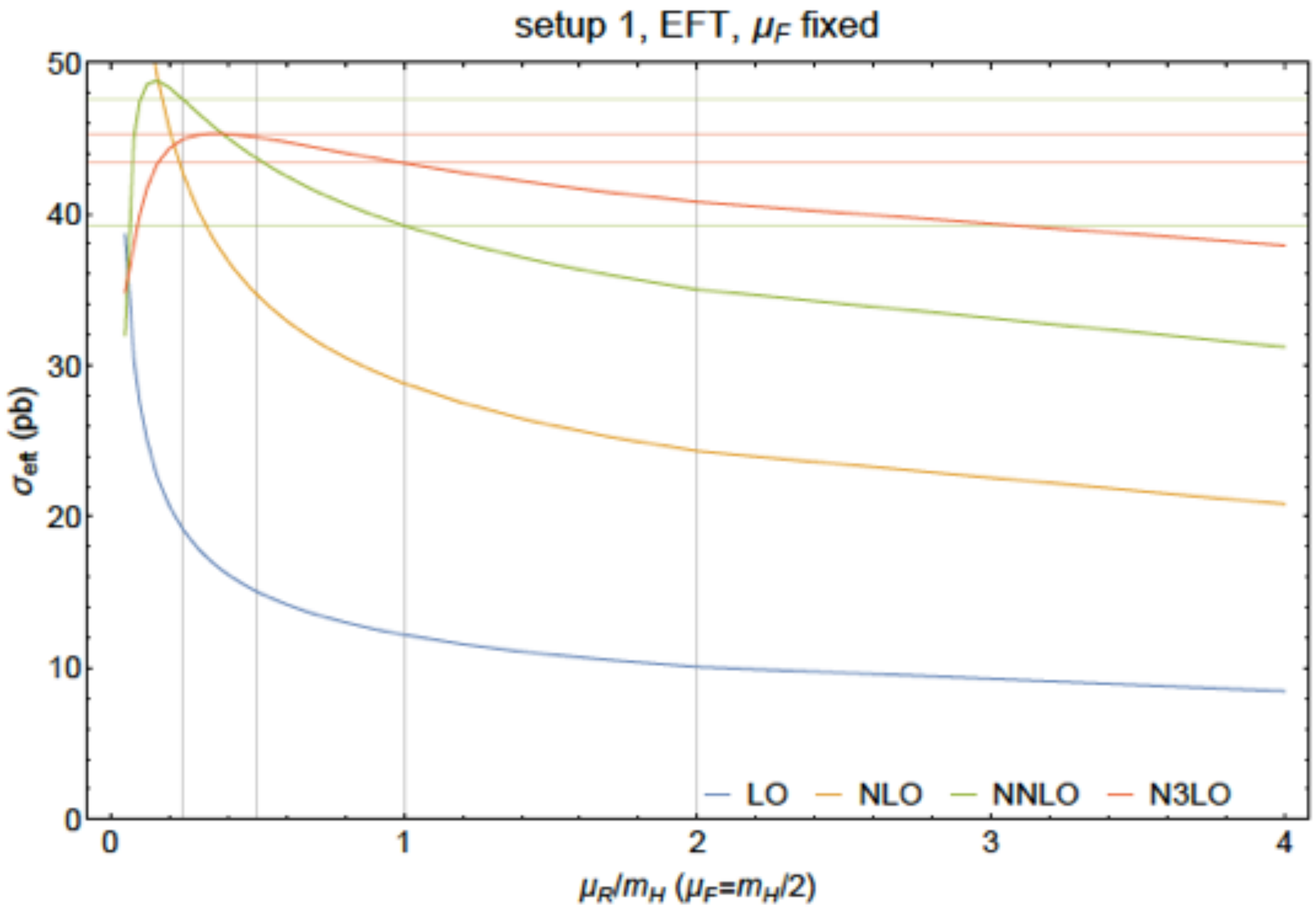
LO prediction is unreliable due to $100 - 200\%$ scale uncertainty

True Result

N3LO result gg fusion to Higgs







$\Delta^{\rm scale}_{EFT,k}$ $(\mu_F = m_H/2)$		
LO	$(k=0)$	$\pm 22.0\%$
NLO	$(k=1)$	$\pm 19.2\%$
NNLO	$(k=2)$	$\pm 9.5\%$
$\mathrm{N^3LO}$	$(k=3)$	$\pm 2.2\%$

$\Delta^{\rm scale}_{EFT,k}$		
LO	$(k=0)$	$\pm 14.8\%$
NLO	$(k=1)$	$\pm 16.6\%$
NNLO	$(k=2)$	$\pm 8.8\%$
$\mathrm{N^3LO}$	$(k=3)$	$\pm 1.9\%$

What is next?

Precise results in other Higgs production channels

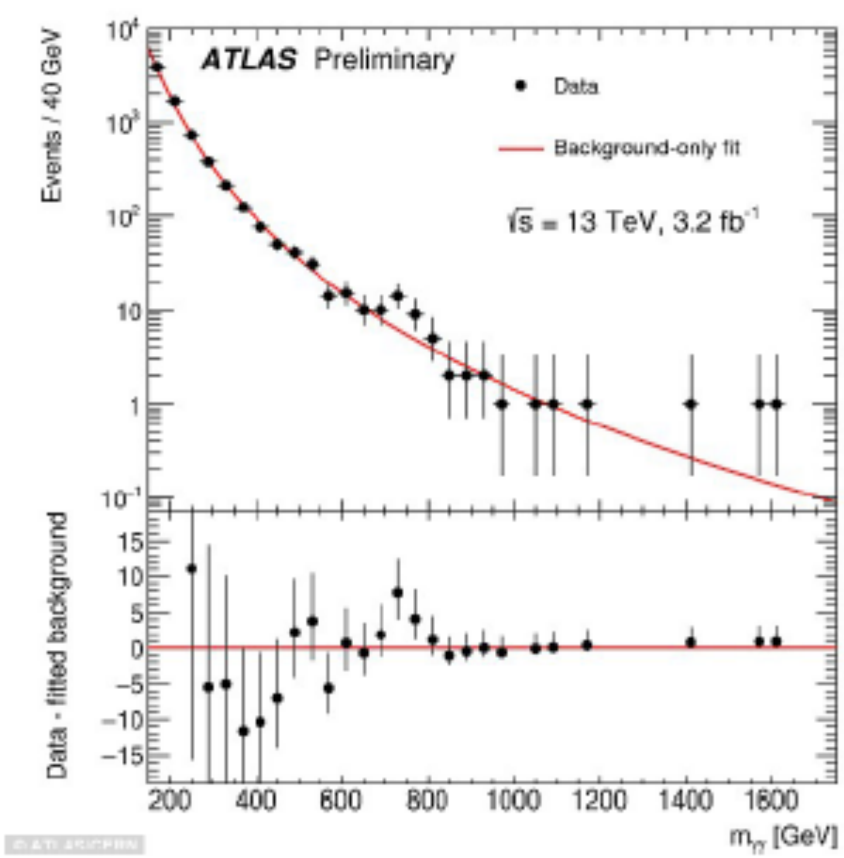
Measure the Higgs couplings more precisely

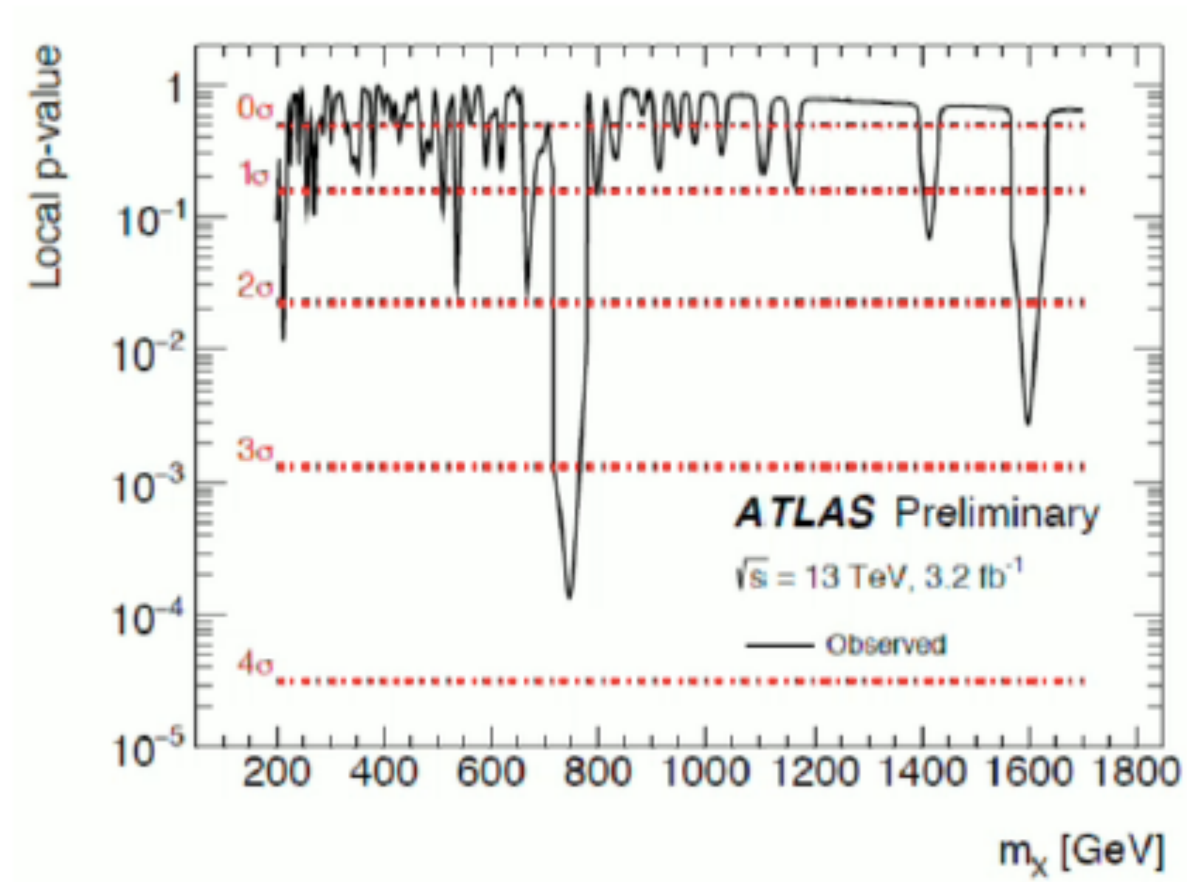
Study the fiducial cross sections with Higgs boson

Look for more more bosons

New particle at the LHC?

ATLAS and CMS have seen the Excess in di-photon channel





New resonance at 750 GeV

New scalar boson or Spin-2 Graviton or Pseudo-Scalar

Pseudo-Scalar Higgs Boson

Pseudoscalar Decaying Only via Loops as an Explanation for the 750 GeV Diphoton Excess

Gang Li, Ying-nan Mao, Yi-Lei Tang, Chen Zhang, Yang Zhou, and Shou-hua Zhu

Phys. Rev. Lett. **116**, 151803 – Published 12 April 2016

Physics See Synopsis: Explaining a 750 GeV Bump

arXiv.org > hep-ph > arXiv:1603.07263

High Energy Physics – Phenomenology

Large loop-coupling enhancement of a 750 GeV pseudoscalar dark sector

Stefano Di Chiara, Andi Hektor, Kristjan Kannike, Luca Marzola, Martti Raidal

arXiv.org > hep-ph > arXiv:1601.00602

High Energy Physics – Phenomenology

The 750 GeV Diphoton Excess from a Pseudoscalar in Matter Scenario

Karim Ghorbani (Arak U.), Hossein Ghorbani (IPM, Teh…

arXiv.org > hep-ph > arXiv:1602.03344

High Energy Physics – Phenomenology

A 750 GeV Diphoton Signal from a Very Light Pseudoscalar in the NMSSM

Ulrich Ellwanger, Cyril Hugonie

CP Odd Higgs boson

CP even Higgs boson Production cross section at N^3LO

[Anastasiou, Duhr, Dulat, Furlan, Herzog, Mistlberger]

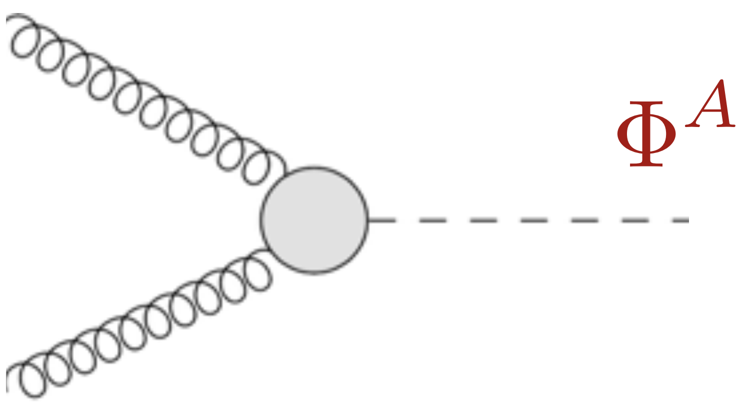
CP odd Higgs boson Production cross section at N^2LO

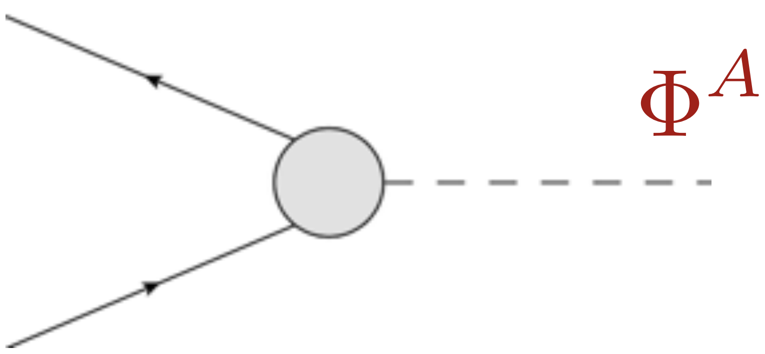
[Harlander, Kilgore; Anastasiou, Melnikov; VR, Smith, Neerven]

CP odd Higgs boson production at N^3LO

1. Virtual Corrections
2. Real Emission Corrections

Three Loop Virtual Contributions





Yukawa Interaction

$$\mathcal{L}^A = -i\frac{g_c}{v}\Phi^A\left(m_t\bar{\psi}_t\gamma_5\psi_t + \sum_{i=1}^{n_l} m_i\bar{\psi}_i\gamma_5\psi_i\right)$$

$m_t =$ top quark mass

$g_c =$ coupling constant

$v =$ vev = $2^{-\frac{1}{4}}G_F^{-\frac{1}{2}}$

$g_c = \cot\beta$ in MSSM

$\Phi^A =$ pseudo scalar field

$\psi_t =$ top quark field

$n_l =$ no of light quarks = 5

Effective Theory

[Chetyrkin, Kniehl, Steinhauser and Bardeen]

Integrating top quark fields:

$$\mathcal{L}^A_{\text{eff}} = \Phi^A\Big[-\frac{1}{8}C_G O_G - \frac{1}{2}C_J O_J\Big]$$

Effective Operators $\mathcal{O}_G$ & $\mathcal{O}_J$

$$O_G(x) = G_a^{\mu\nu}\tilde{G}_{a,\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma}G_a^{\mu\nu}G_a^{\rho\sigma}$$

$$O_J(x) = \partial_\mu\left(\bar{\psi}\gamma^\mu\gamma_5\psi\right)$$

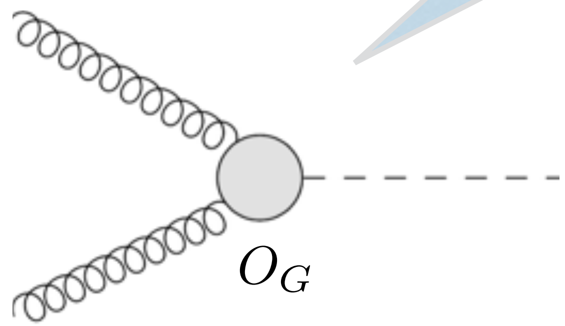
$$C_G = -a_s 2^{\frac{5}{4}} G_F^{\frac{1}{2}} \cot\beta$$

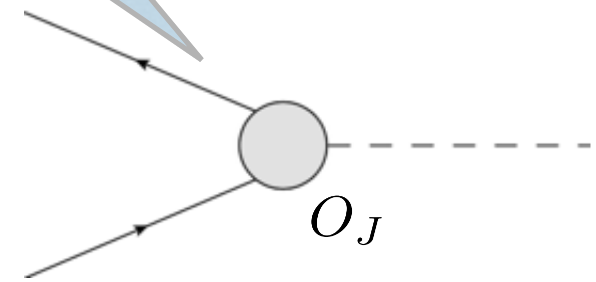
$$C_J = -\left[a_s C_F\left(\frac{3}{2} - 3\ln\frac{\mu_R^2}{m_t^2}\right) + a_s^2 C_J^{(2)} + \cdots\right] C_G$$

- C_G and C_J are Wilson Coefficients
- C_G is exact to all orders due to Adler-Bardeen theorem
- C_J is known to a_s only

Effective Couplings

$$\mathcal{L}_{\text{eff}}^{A} = \Phi^{A}\Big[-\frac{1}{8}C_G O_G - \frac{1}{2}C_J O_J\Big]$$

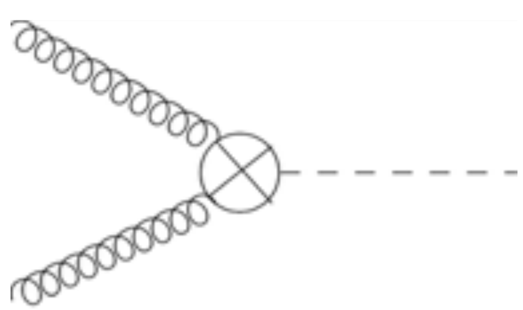


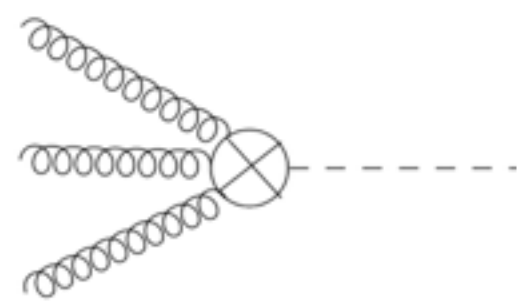


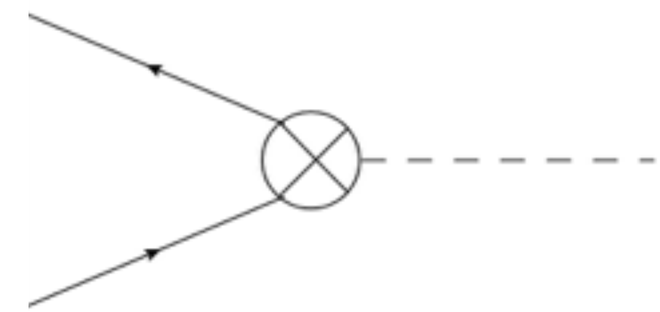
$$O_G(x) = G_a^{\mu\nu}\tilde{G}_{a,\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma}G_a^{\mu\nu}G_a^{\rho\sigma}$$

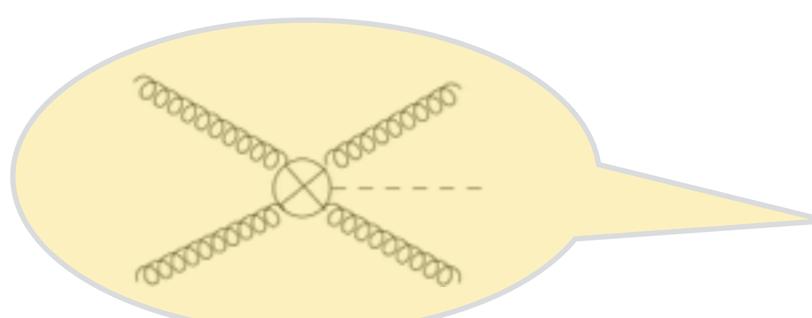
$$O_J(x) = \partial_\mu\left(\bar{\psi}\gamma^\mu\gamma_5\psi\right)$$

Feynman Rules:







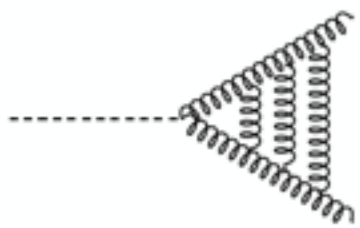
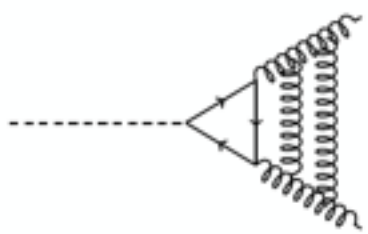
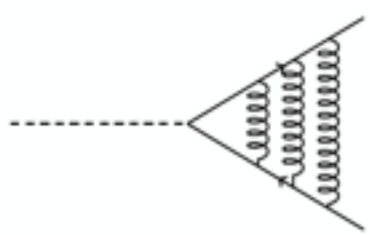
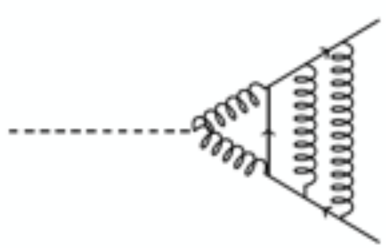


Vanishes

Qgraf and FORM …

[Nogueira,Vermaseren]

Form Factors at Three loops in QCD

1. 
2. 
3. 
4. 

# of diagrams
1586
447
244
400

Qgraf

Diagram generation

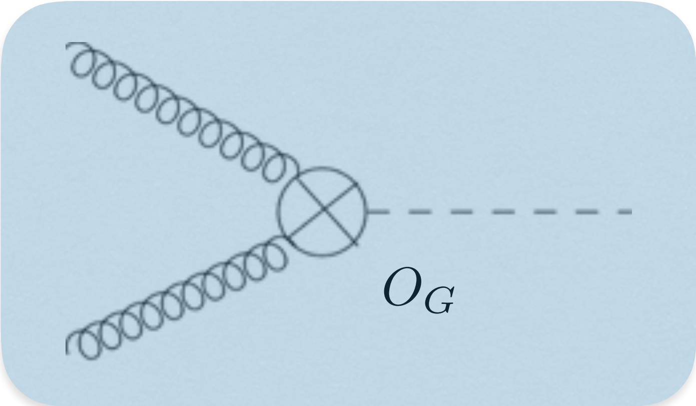
FORM

Feynman rules
SU(N) color algebra
Lorentz contraction

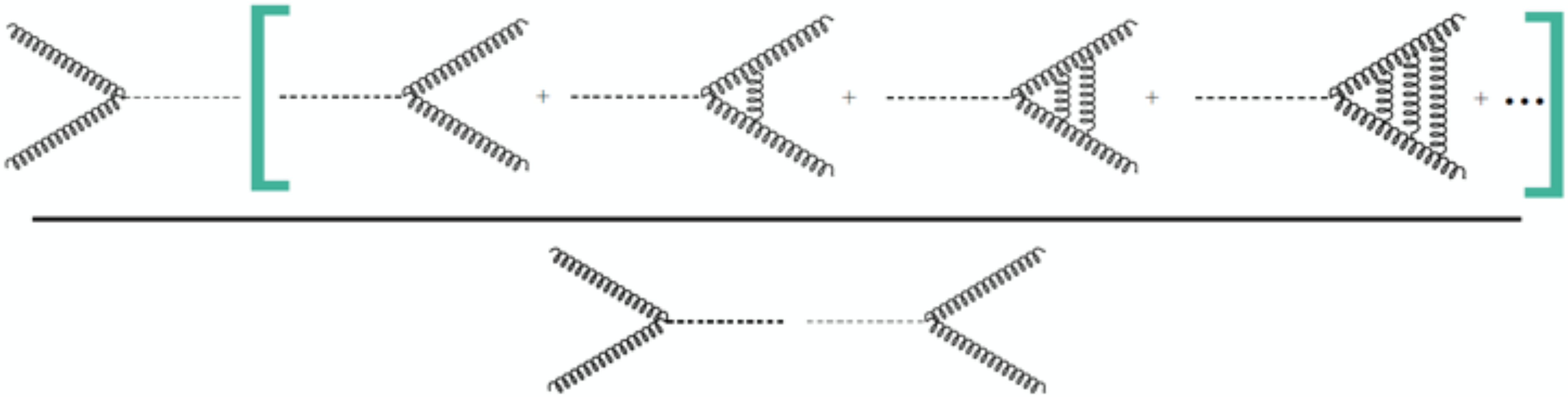
Mathematica

IBP reductions

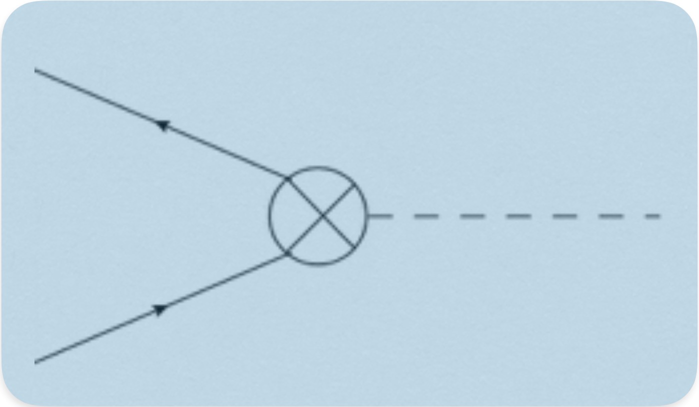
$\mathcal{F}_g^G$ at Three loops



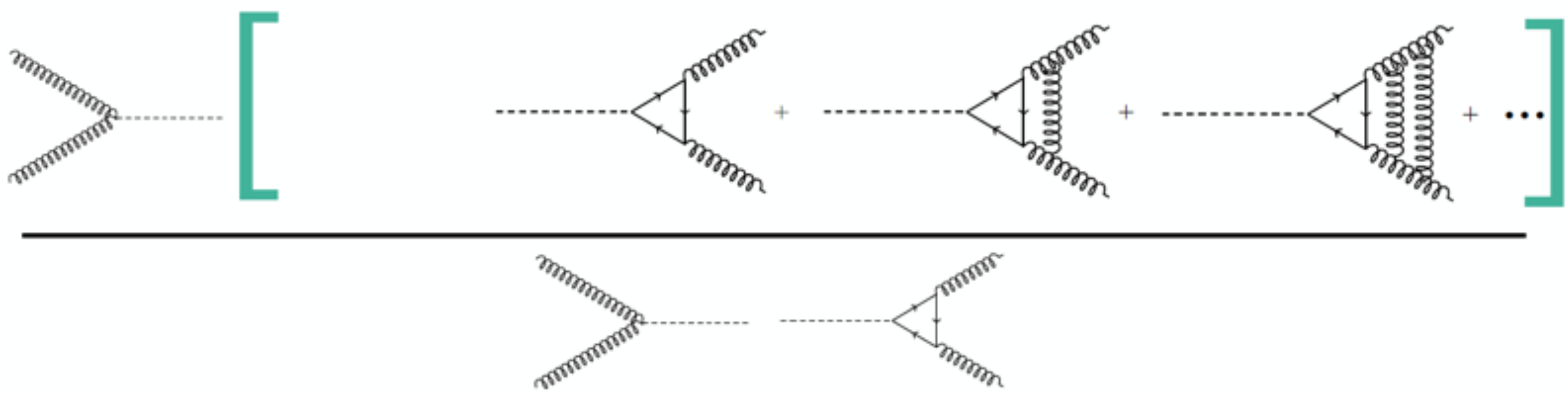
$$\mathcal{F}_g^G = 1 + \hat{a}_s \left(\frac{Q^2}{\mu^2}\right)^{\frac{\epsilon}{2}} S_\epsilon \hat{\mathcal{F}}_g^{G,(1)} + \hat{a}_s^2 \left(\frac{Q^2}{\mu^2}\right)^{2\frac{\epsilon}{2}} S_\epsilon^2 \hat{\mathcal{F}}_g^{G,(2)} + \hat{a}_s^3 \left(\frac{Q^2}{\mu^2}\right)^{3\frac{\epsilon}{2}} S_\epsilon^3 \hat{\mathcal{F}}_g^{G,(3)} + \cdots$$



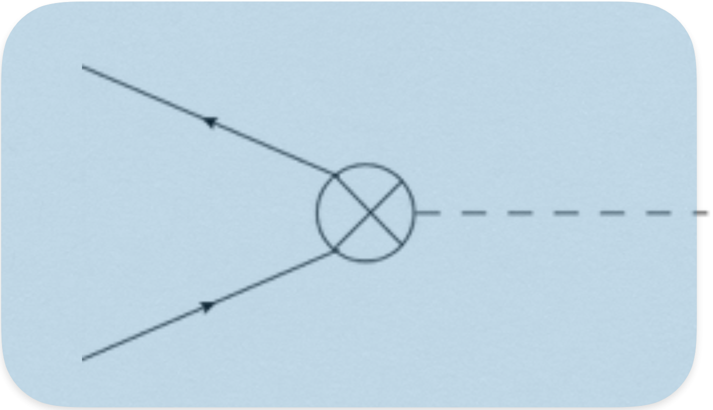
$\mathcal{F}_g^J$ at Three loops



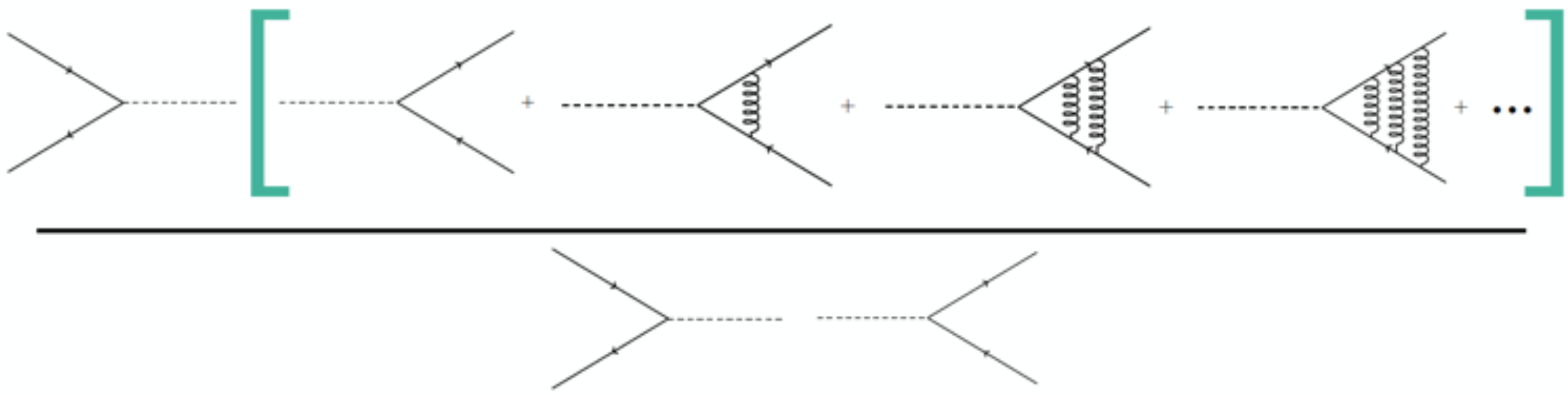
$$\mathcal{F}_g^J = 1 + \hat{a}_s \left(\frac{Q^2}{\mu^2}\right)^{\frac{\epsilon}{2}} S_\epsilon \hat{\mathcal{F}}_g^{J,(1)} + \hat{a}_s^2 \left(\frac{Q^2}{\mu^2}\right)^{2\frac{\epsilon}{2}} S_\epsilon^2 \hat{\mathcal{F}}_g^{J,(2)} + \cdots$$



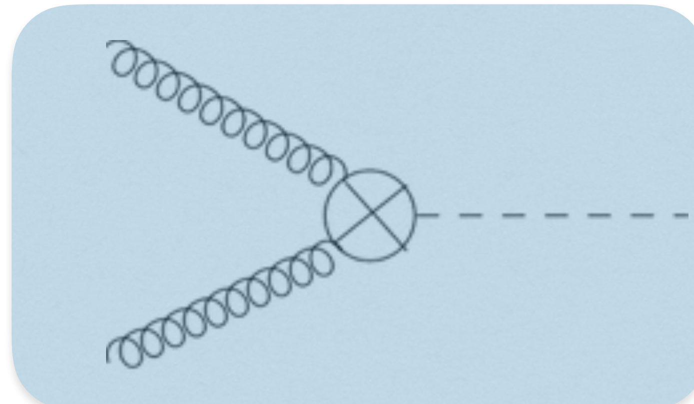
$\mathcal{F}_q^J$ at Three loops



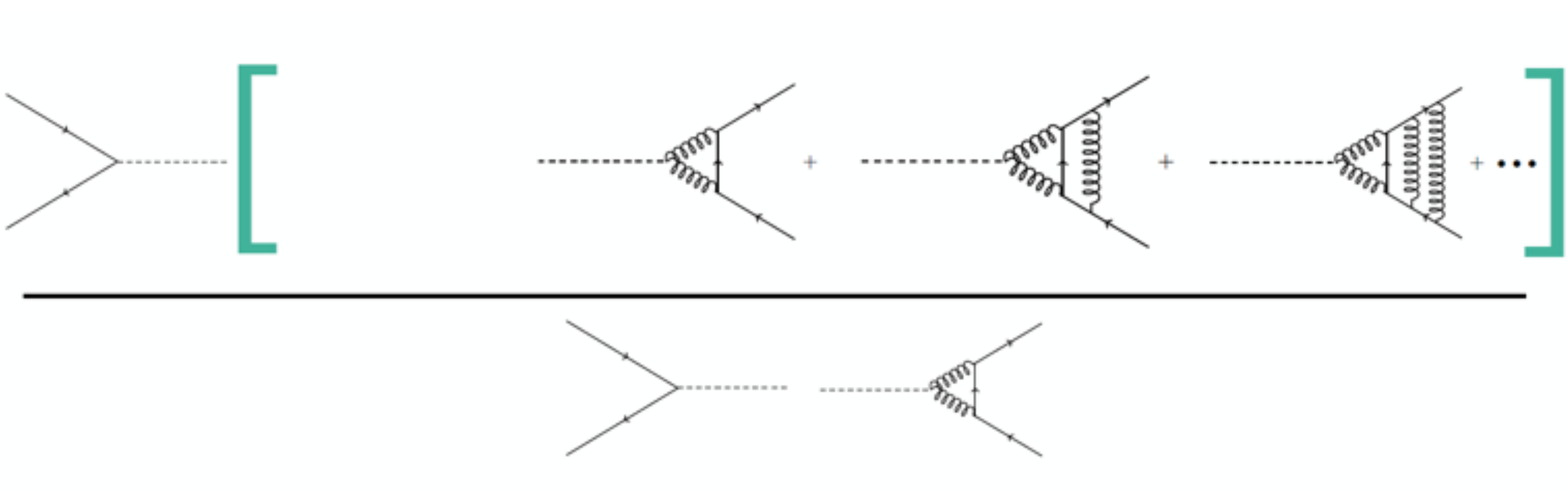
$$\mathcal{F}_q^J = 1 + \hat{a}_s \left(\frac{Q^2}{\mu^2}\right)^{\frac{\epsilon}{2}} S_\epsilon \hat{\mathcal{F}}_q^{J,(1)} + \hat{a}_s^2 \left(\frac{Q^2}{\mu^2}\right)^{2\frac{\epsilon}{2}} S_\epsilon^2 \hat{\mathcal{F}}_q^{J,(2)} + \hat{a}_s^3 \left(\frac{Q^2}{\mu^2}\right)^{3\frac{\epsilon}{2}} S_\epsilon^3 \hat{\mathcal{F}}_q^{J,(3)} + \cdots$$



$\mathcal{F}_q^G$ at Two loops



$$\mathcal{F}_q^G = 1 + \hat{a}_s \left(\frac{Q^2}{\mu^2}\right)^{\frac{\epsilon}{2}} S_\epsilon \hat{\mathcal{F}}_q^{G,(1)} + \hat{a}_s^2 \left(\frac{Q^2}{\mu^2}\right)^{2\,\frac{\epsilon}{2}} S_\epsilon^2 \hat{\mathcal{F}}_q^{G,(2)} + \cdots$$



γ_5 & $\epsilon_{\mu\nu\lambda\sigma}$ in n-dimensions

Defining γ_5 & $\epsilon_{\mu\nu\lambda\sigma}$ in $n \neq 4$ dimension ?

Many Prescriptions exist $\{\gamma_5, \gamma^\mu\} \neq 0 \quad n \neq 4$

['t Hooft and Veltman]

We follow:

$$\gamma_5 = \frac{i}{4!}\epsilon_{\mu_1\mu_2\mu_3\mu_4}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}$$

$$\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4} = 4!\ \delta^{\nu_1\cdots\nu_4}_{[\mu_1\cdots\mu_4]}$$

n-dim

[Larin]

Breaks Chiral Ward identity !

Remedy: Finite renormalisation

Method

Unphysical degrees of Freedom of gluons:

1. Feynman Gauge for internal gluons
2. Physical Polarisation for external gluons

Large number of 3- loop Integrals:

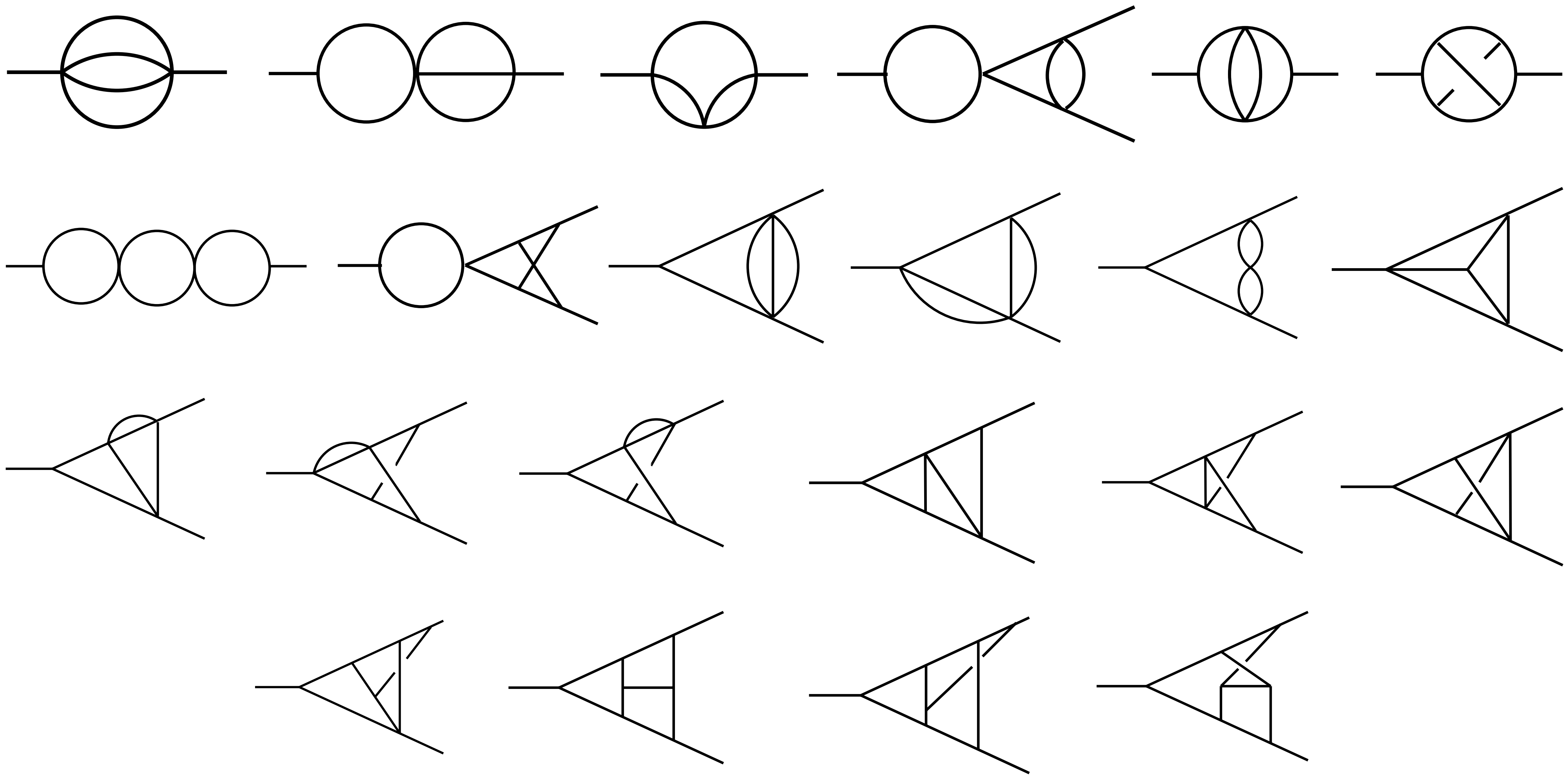
[Chetyrkin, Tkachov; Gehrmann, Remiddi]

1. IBP reduction
2. Lorentz Invariant (LI) Identities

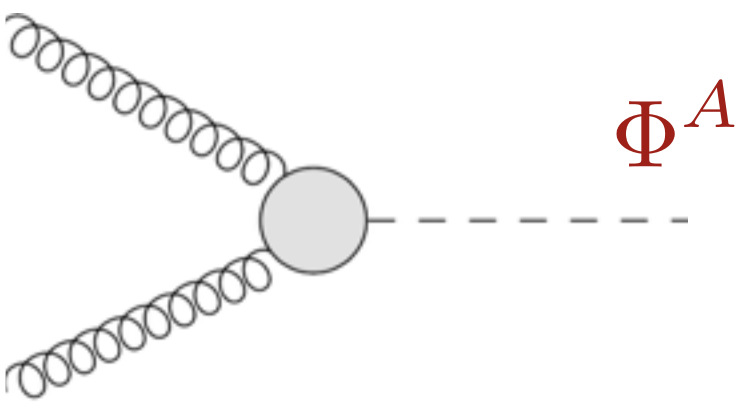
Reduze2 and ***LiteRed*** → *22 Master Integrals*

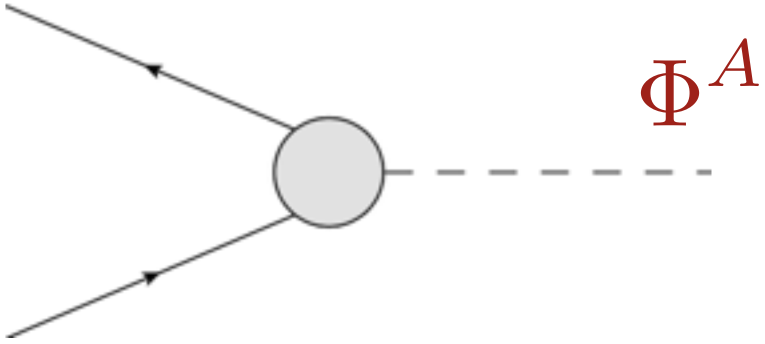
Master Integrals

[T.Gehrmann,T.Huber,D.Maitre,G.Heinrich,C.Studerus,D.A.Kosower,V.A.Smirnov,A.V.Smirnov, R.N.Lee]



UV Renormalisation Z_{IJ}





UV renormalisation

[Larin]

- O_G mixes under renorm with O_J

Renormalised

Bare

$$[O_G]_R = Z_{GG}\,[O_G]_B + Z_{GJ}\,[O_J]_B$$

O_J needs finite renormalisation Z_5^s

$$[O_J]_R = Z_5^s Z^s_{\overline{MS}}\,[O_J]_B$$

needs finite renormalisation

On-shell matrix elements

$$[O_G]_R = Z_{GG}\,[O_G]_B + Z_{GJ}\,[O_J]_B$$

Matrix elements between quark and gluon fields :

$$\mathcal{S}_g^G \equiv Z_{GG}\langle\hat{\mathcal{M}}_g^{G,(0)}|\mathcal{M}_g^G\rangle + Z_{GJ}\langle\hat{\mathcal{M}}_g^{G,(0)}|\mathcal{M}_g^J\rangle$$

$$\mathcal{S}_q^G \equiv Z_{GG}\langle\hat{\mathcal{M}}_q^{J,(0)}|\mathcal{M}_q^G\rangle + Z_{GJ}\langle\hat{\mathcal{M}}_q^{J,(0)}|\mathcal{M}_q^J\rangle$$

Form Factors :

$$\left[\mathcal{F}_g^G\right]_R \equiv \frac{\mathcal{S}_g^G}{\mathcal{S}_g^{G,(0)}} \equiv 1 + \sum_{n=1}^{\infty} a_s^n \left[\mathcal{F}_g^{G,(n)}\right]_R \qquad n = 3$$

$$\left[\mathcal{F}_q^G\right]_R \equiv \frac{\mathcal{S}_q^G}{a_s\mathcal{S}_q^{G,(1)}} \equiv 1 + \sum_{n=1}^{\infty} a_s^n \left[\mathcal{F}_q^{G,(n)}\right]_R \qquad n = 2$$

On-shell matrix elements

$$[O_J]_R = Z_5^s Z_{\overline{MS}}^s [O_J]_B$$

Matrix elements between quark and gluon fields :

$$\mathcal{S}_g^J \equiv Z_5^s Z_{\overline{MS}}^s \langle \hat{\mathcal{M}}_g^{G,(0)} | \mathcal{M}_g^J \rangle$$

Form Factors :

$$[\mathcal{F}_g^J]_R \equiv \frac{\mathcal{S}_g^J}{a_s \mathcal{S}_g^{J,(1)}} \equiv 1 + \sum_{n=1}^{\infty} a_s^n \left[\mathcal{F}_g^{J,(n)}\right]_R \qquad n = 2$$

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UV and IR poles mix

On-shell matrix elements between quark and gluon fields :

$$< \hat{\mathcal{M}}_i^{I,(0)} | \mathcal{M}_i^{K,(n)} >$$

$$I, K = G, J$$
$$i = g, q$$

UV and IR poles mix in n-dimensions

Trick!

Exploit Univerality of IR poles → UV poles

Sudakov Equation (K+G Eqn.)

[Moch, Vogt, Vermaseren; Ravindran; Magnea]

$$Q^2 \frac{d}{dQ^2} \ln \mathcal{F}^\lambda_\beta(\hat{a}_s, Q^2, \mu^2, \epsilon) = \frac{1}{2}\left[K^\lambda_\beta(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon) + G^\lambda_\beta(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon)\right]$$

poles

No poles

RG invariance

$$\mu_R^2 \frac{d}{d\mu_R^2} K^\lambda_\beta(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon) = -\mu_R^2 \frac{d}{d\mu_R^2} G^\lambda_\beta(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon) = -A^\lambda_\beta(a_s(\mu_R^2))$$

Cusp Anomalous dim.

$$A^g_\beta = \frac{C_A}{C_F} A^q_\beta$$

UV + IR Anomalous dim.

$$G^\lambda_{\beta,i}(\epsilon) = 2\left(B^\lambda_{\beta,i} - \gamma^\lambda_{\beta,i}\right) + f^\lambda_{\beta,i} + C^\lambda_{\beta,i} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,i}$$

Solution in n-dimensions

[Moch, Vogt, Vermaseren; Ravindran; Magnea]

Solution in $4+\epsilon$ dim:

$$\ln \mathcal{F}^{\lambda}_{\beta}(\hat{a}_s, Q^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \hat{\mathcal{L}}^{\lambda}_{\beta,i}(\epsilon)$$

with

$$\hat{\mathcal{L}}^{\lambda}_{\beta,1}(\epsilon) = \frac{1}{\epsilon^2}\Big\{-2A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon}\Big\{G^{\lambda}_{\beta,1}(\epsilon)\Big\}$$

$$\hat{\mathcal{L}}^{\lambda}_{\beta,2}(\epsilon) = \frac{1}{\epsilon^3}\Big\{\beta_0 A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon^2}\Big\{-\frac{1}{2}A^{\lambda}_{\beta,2} - \beta_0 G^{\lambda}_{\beta,1}(\epsilon)\Big\} + \frac{1}{\epsilon}\Big\{\frac{1}{2}G^{\lambda}_{\beta,2}(\epsilon)\Big\}$$

$$\hat{\mathcal{L}}^{\lambda}_{\beta,3}(\epsilon) = \frac{1}{\epsilon^4}\Big\{-\frac{8}{9}\beta_0^2 A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon^3}\Big\{\frac{2}{9}\beta_1 A^{\lambda}_{\beta,1} + \frac{8}{9}\beta_0 A^{\lambda}_{\beta,2} + \frac{4}{3}\beta_0^2 G^{\lambda}_{\beta,1}(\epsilon)\Big\}$$
$$+\frac{1}{\epsilon^2}\Big\{-\frac{2}{9}A^{\lambda}_{\beta,3} - \frac{1}{3}\beta_1 G^{\lambda}_{\beta,1}(\epsilon) - \frac{4}{3}\beta_0 G^{\lambda}_{\beta,2}(\epsilon)\Big\} + \frac{1}{\epsilon}\Big\{\frac{1}{3}G^{\lambda}_{\beta,3}(\epsilon)\Big\}$$

Cusp Anomalous dim.

$$G^{\lambda}_{\beta,i}(\epsilon) = 2\left(B^{\lambda}_{\beta,i} - \gamma^{\lambda}_{\beta,i}\right) + f^{\lambda}_{\beta,i} + C^{\lambda}_{\beta,i} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,i}$$

Single Pole term & UV from IR

[Ravindran, Smith, van Neerven; Moch et. al.]

UV Anomalous dim.

$$C^{\lambda}_{\beta,i} = \sum_j s_j C^{\lambda}_{\beta,j},\ j < i$$

$$G^{\lambda}_{\beta,i}(\epsilon) = 2\left(B^{\lambda}_{\beta,i} - \gamma^{\lambda}_{\beta,i}\right) + f^{\lambda}_{\beta,i} + C^{\lambda}_{\beta,i} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,i}$$

Collinear Anomalous dim.

Soft Anomalous dim.

$$f^g_\beta = \frac{C_A}{C_F} f^q_\beta$$

- Computation of $G^{\lambda}_{\beta,i}$, $C^{\lambda}_{\beta,i} = C^{\lambda}_{\beta,j}, j < i$
- Knowledge of $B^{q,g}_{\beta,i}, f^{q,g}_{\beta,i}$

$\gamma^{\lambda}_{\beta,i}$ → $Z^{\lambda}_{\beta,i}$

UV Renormalisation Constant

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z^\lambda(a_s, \mu_R^2, \epsilon) = \sum_{i=1}^{\infty} a_s^i \gamma_i^\lambda$$

Solution to third order

$$Z^\lambda = 1 + a_s\left[\frac{1}{\epsilon}2\gamma_1^\lambda\right] + a_s^2\left[\frac{1}{\epsilon^2}\left\{2\beta_0\gamma_1^\lambda + 2(\gamma_1^\lambda)^2\right\} + \frac{1}{\epsilon}\gamma_2^\lambda\right] + a_s^3\left[\frac{1}{\epsilon^3}\left\{8\beta_0^2\gamma_1^\lambda + 4\beta_0(\gamma_1^\lambda)^2 + \frac{4(\gamma_1^\lambda)^3}{3}\right\} + \frac{1}{\epsilon^2}\left\{\frac{4\beta_1\gamma_1^\lambda}{3} + \frac{4\beta_0\gamma_2^\lambda}{3} + 2\gamma_1^\lambda\gamma_2^\lambda\right\} + \frac{1}{\epsilon}\left\{\frac{2\gamma_3^\lambda}{3}\right\}\right].$$

$$[O_J]_R = Z_5^s Z_{\overline{MS}}^s [O_J]_B$$

[Larin,Zoller]

$$Z_5^s = 1 + a_s\{-4C_F\} + a_s^2\left\{22C_F^2 - \frac{107}{9}C_A C_F + \frac{31}{18}C_F n_f\right\}$$

All UV Z_{IK} agree with those in the literature

Adler-Bell-Jackie Anomaly

CP odd operators

$O_J(x) = \partial_\mu \left(\bar\psi \gamma^\mu \gamma_5 \psi\right)$ and $O_G(x) = G_a^{\mu\nu} \tilde G_{a,\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma} G_a^{\mu\nu} G_a^{\rho\sigma}$

related by

ABJ Anomaly

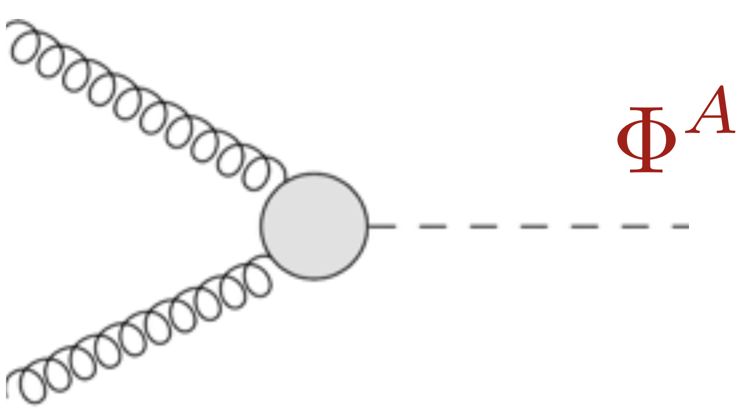
$$[O_J]_R = a_s \frac{n_f}{2} [O_G]_R$$

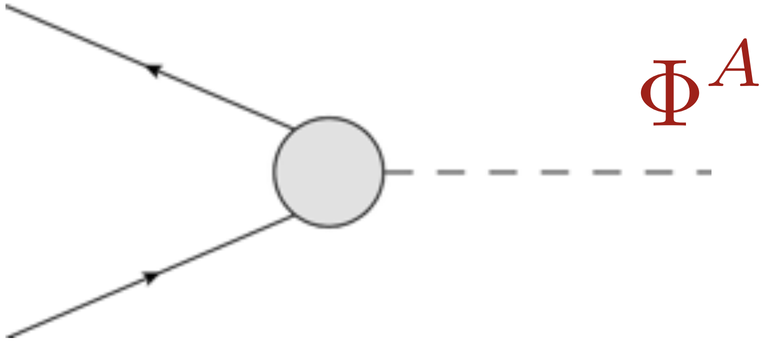
Renormalisation Group Invariance

$$\gamma_{JJ} = \frac{\beta}{a_s} + \gamma_{GG} + a_s \frac{n_f}{2} \gamma_{GJ}$$

Check !

N3LO Soft+Virtual Cross section





Soft+Virtual at N3LO

Inclusive cross section:

$$\sigma^A(\tau, m_A^2) = \sigma^{A,(0)}(\mu_R^2) \sum_{a,b=q,\bar{q},g} \int_\tau^1 dy\, \Phi_{ab}(y, \mu_F^2) \Delta_{ab}^A \left(\frac{\tau}{y}, m_A^2, \mu_R^2, \mu_F^2\right)$$

Born cross section:

$$\sigma^{A,(0)}(\mu_R^2) = \frac{\pi\sqrt{2}G_F}{16} a_s^2 \cot^2\beta \left|\tau_A f(\tau_A)\right|^2.$$

Partonic Flux:

$$\Phi_{ab}(y, \mu_F^2) = \int_y^1 \frac{dx}{x} f_a(x, \mu_F^2) f_b\left(\frac{y}{x}, \mu_F^2\right),$$

Partonic Cross section:

$$\Delta_{ab}^A(z, q^2, \mu_R^2, \mu_F^2) = \Delta_{ab}^{A,\mathrm{SV}}(z, q^2, \mu_R^2, \mu_F^2) + \Delta_{ab}^{A,\mathrm{hard}}(z, q^2, \mu_R^2, \mu_F^2)$$

Soft+Virtual:

$$\Delta_g^{A,\mathrm{SV}}(z, q^2, \mu_R^2, \mu_F^2) = \sum_{i=0}^{\infty} a_s^i \Delta_{g,i}^{A,\mathrm{SV}}(z, q^2, \mu_R^2, \mu_F^2)$$

New!

SV part to N3LO

Hard part to NNLO

Soft+Virtual at N3LO

Soft+Virtual:

$$\Delta_{g,i}^{A,\mathrm{SV}} = \Delta_{g,i}^{A,\mathrm{SV}}|_{\delta}\delta(1-z) + \sum_{j=0}^{2i-1} \Delta_{g,i}^{A,\mathrm{SV}}|_{\mathcal{D}_j}\mathcal{D}_j\,.$$

Plus distributions:

$$\mathcal{D}_i \equiv \left[\frac{\ln^i(1-z)}{1-z}\right]_+$$

z-space exponentiation of SV cross section:

$$\Delta_g^{A,\mathrm{SV}}(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right)\right)\Big|_{\epsilon=0}$$

Mellin Convolution in z-space:

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!}f(z) + \frac{1}{2!}f(z)\otimes f(z) + \cdots\,.$$

z-space Exponent at α_s^3

[V.Ravindran]

RG invariance, K+G equation, Mass factorisation:

$$\Delta_g^{A,\mathrm{SV}}(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right)\right)\Big|_{\epsilon=0}$$

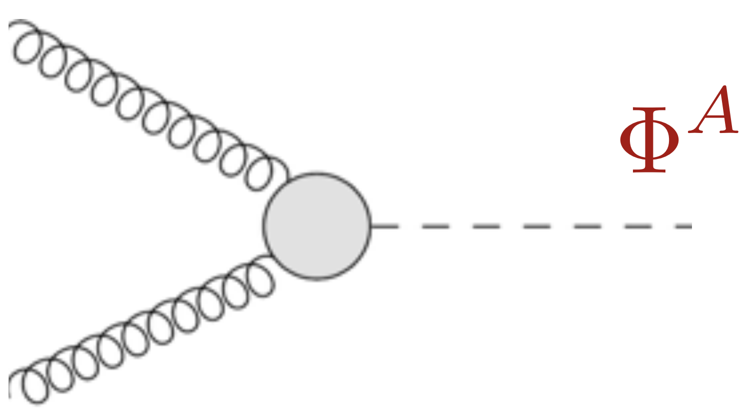
$$\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right) = \left(\ln\left[Z_g^A(\hat{a}_s,\mu_R^2,\mu^2,\epsilon)\right]^2 + \ln\left|\mathcal{F}_g^A(\hat{a}_s,Q^2,\mu^2,\epsilon)\right|^2\right)\delta(1-z) + 2\Phi_g^A(\hat{a}_s,q^2,\mu^2,z,\epsilon) - 2\mathcal{C}\ln\Gamma_{gg}(\hat{a}_s,\mu_F^2,\mu^2,z,\epsilon)\,.$$

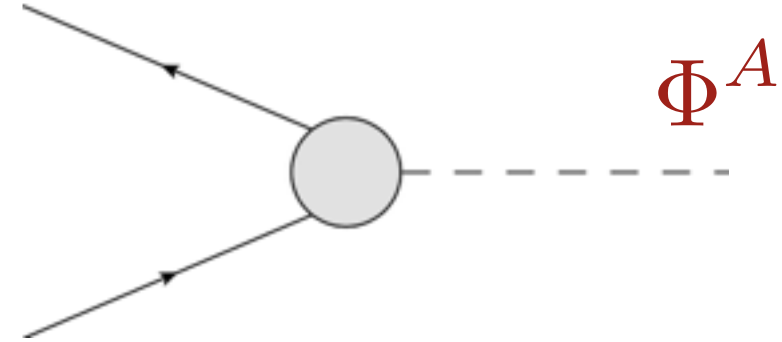
α_s^3

- Z_g^A is operator renormalisation → Checked
- $\mathcal{F}_g^A$ is the Form Factor → New !
- Φ_g^A is the Soft distribution function → Known
- Γ_{gg} is the Altarelli Parisi kernel → Known

N3LO $\Delta_{g,3}^{A,SV}(z,q^2)$ New

N3LL Resumed Cross section





Constant part at N3LL

[S.Catani, L.Trentadue, G.F.Sterman]

Resumed Cross section

$$\Delta_{g,N}^{A,\text{res}}(q^2,\mu_R^2,\mu_F^2) = C_g^{A,\text{th}}(q^2,\mu_R^2,\mu_F^2)\Delta_{g,N}(q^2)\,.$$

$$\Delta_{g,N} = \exp\left[\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\left\{2\int_{q^2}^{q^2(1-z)^2}\frac{d\lambda^2}{\lambda^2}A_g\left(a_s(\lambda^2)\right) + D_g\left(a_s(q^2(1-z)^2)\right)\right\}\right]$$

$$C_g^{A,\text{th}} = 1 + \sum_{j=1}^{\infty} a_s^j\, C_{g,j}^{A,\text{th}}\,,$$

N3LL

$$\begin{aligned}
C_{g,3}^{A,\text{th}} = {} & n_f C_J^{(2)}\Big\{-4\Big\} + C_F n_f^2\Big\{\frac{1498}{9} - \frac{40}{9}\zeta_2 - \frac{32}{45}\zeta_2^2 - \frac{224}{3}\zeta_3\Big\} + C_F^2 n_f\Big\{\frac{457}{3} + 208\zeta_3 \\
& - 320\zeta_5\Big\} + C_A^2 n_f\Big\{-\frac{113366}{81} - \frac{10888}{81}\zeta_2 + \frac{17192}{135}\zeta_2^2 + \frac{584}{3}\zeta_3 - \frac{464}{3}\zeta_2\zeta_3 + \frac{808}{9}\zeta_5\Big\} \\
& + C_A^3\Big\{\frac{114568}{27} + \frac{137756}{81}\zeta_2 - \frac{4468}{27}\zeta_2^2 - \frac{32}{5}\zeta_2^3 - \frac{80308}{27}\zeta_3 - \frac{616}{3}\zeta_2\zeta_3 + 96\zeta_3^2 \\
& + \frac{3476}{9}\zeta_5\Big\} + C_A n_f^2\Big\{\frac{6914}{81} - \frac{1696}{81}\zeta_2 - \frac{608}{45}\zeta_2^2 + \frac{688}{27}\zeta_3\Big\} + C_A C_F n_f\Big\{-1797 \\
& + 96\ln\left(\frac{q^2}{m_t^2}\right) - \frac{4160}{9}\zeta_2 + 96\ln\left(\frac{q^2}{m_t^2}\right)\zeta_2 + \frac{176}{45}\zeta_2^2 + \frac{1856}{3}\zeta_3 + 192\zeta_2\zeta_3 \\
& + 160\zeta_5\Big\}\,.
\end{aligned}$$

New!

N3LO Matching Coefficient in SCET

Matching Coefficient in SCET

IR finite Matching Coefficient

[T.Becher, M.Neubert]

$$C_g^{A,\text{eff}}(Q^2,\mu_h^2) \equiv \lim_{\epsilon\to 0}(Z_g^{A,h})^{-1}(\epsilon,Q^2,\mu_h^2)\,[\mathcal{F}_g^A]_R(\epsilon,Q^2)$$

$$Z_g^{A,h}(\epsilon,Q^2,\mu_h^2) = 1 + \sum_{i=1}^{\infty} a_s^i(\mu_h^2) Z_{g,i}^{A,h}(\epsilon,Q^2,\mu_h^2)\,,$$

$$C_g^{A,\text{eff}}(Q^2,\mu_h^2) = 1 + \sum_{i=1}^{\infty} a_s^i(\mu_h^2) C_{g,i}^{A,\text{eff}}(Q^2,\mu_h^2)$$

N3LL

$$\mu_h^2 \frac{d}{d\mu_h^2} \ln C_{g,i}^{A,\text{eff}} = \frac{1}{2} A_{g,i} L - \left(B_{g,i} + \frac{1}{2} f_{g,i}\right)$$

$$\begin{aligned}
C_{g,3}^{A,\text{eff}} = {} & n_f C_J^{(2)}\left\{-2\right\} + C_F n_f^2\left\{L\left(-\frac{320}{9} + 8\ln\left(\frac{\mu_h^2}{m_t^2}\right) + \frac{32}{3}\zeta_3\right) + \frac{749}{9} - \frac{20}{9}\zeta_2 - \frac{16}{45}\zeta_2^2\right.\\
&\left. - \frac{112}{3}\zeta_3\right\} + C_F^2 n_f\left\{\frac{457}{6} + 104\zeta_3 - 160\zeta_5\right\} + C_A^2 n_f\left\{\frac{2}{9}L^5 - \frac{8}{27}L^4 + L^3\left(-\frac{752}{81}\right.\right.\\
&\left. - \frac{2}{3}\zeta_2\right) + L^2\left(\frac{512}{27} - \frac{103}{9}\zeta_2 + \frac{118}{9}\zeta_3\right) + L\left(\frac{129283}{729} + \frac{4198}{81}\zeta_2 - \frac{48}{5}\zeta_2^2 + \frac{28}{9}\zeta_3\right)\\
&\left. - \frac{7946273}{13122} - \frac{19292}{729}\zeta_2 + \frac{73}{45}\zeta_2^2 - \frac{2764}{81}\zeta_3 - \frac{82}{9}\zeta_2\zeta_3 + \frac{428}{9}\zeta_5\right\} + C_A^3\left\{-\frac{1}{6}L^6 - \frac{11}{9}L^5\right.\\
& + L^4\left(\frac{389}{54} - \frac{3}{2}\zeta_2\right) + L^3\left(\frac{2206}{81} + \frac{11}{3}\zeta_2 + 2\zeta_3\right) + L^2\left(-\frac{20833}{162} + \frac{757}{18}\zeta_2 - \frac{73}{10}\zeta_2^2\right.\\
&\left. + \frac{143}{9}\zeta_3\right) + \frac{2222}{9}\zeta_5 + L\left(-\frac{500011}{1458} - \frac{16066}{81}\zeta_2 + \frac{176}{5}\zeta_2^2 + \frac{1832}{27}\zeta_3 + \frac{34}{3}\zeta_2\zeta_3\right.\\
&\left. + 16\zeta_5\right) + \frac{41091539}{26244} + \frac{316939}{1458}\zeta_2 - \frac{1399}{270}\zeta_2^2 - \frac{24389}{1890}\zeta_2^3 - \frac{176584}{243}\zeta_3 - \frac{605}{9}\zeta_2\zeta_3\\
&\left. - \frac{104}{9}\zeta_3^2\right\} + C_A n_f^2\left\{-\frac{2}{27}L^4 + \frac{40}{81}L^3 + L^2\left(\frac{80}{81} + \frac{8}{9}\zeta_2\right) + L\left(-\frac{12248}{729} - \frac{80}{27}\zeta_2\right.\right.\\
&\left.\left. - \frac{128}{27}\zeta_3\right) + \frac{280145}{6561} + \frac{4}{9}\zeta_2 + \frac{4}{27}\zeta_2^2 + \frac{4576}{243}\zeta_3\right\} + C_A C_F n_f\left\{-\frac{2}{3}L^3 + L^2\left(\frac{215}{6}\right.\right.\\
&\left. - 6\ln\left(\frac{\mu_h^2}{m_t^2}\right) - 16\zeta_3\right) + L\left(\frac{9173}{54} - 44\ln\left(\frac{\mu_h^2}{m_t^2}\right) + 4\zeta_2 + \frac{16}{5}\zeta_2^2 - \frac{376}{9}\zeta_3\right)\\
& + 24\ln\left(\frac{\mu_h^2}{m_t^2}\right) - \frac{726935}{972} - \frac{415}{18}\zeta_2 + 6\ln\left(\frac{\mu_h^2}{m_t^2}\right)\zeta_2 - \frac{64}{45}\zeta_2^2 + \frac{20180}{81}\zeta_3 + \frac{64}{3}\zeta_2\zeta_3\\
&\left. + \frac{608}{9}\zeta_5\right\}.
\end{aligned}$$

New!

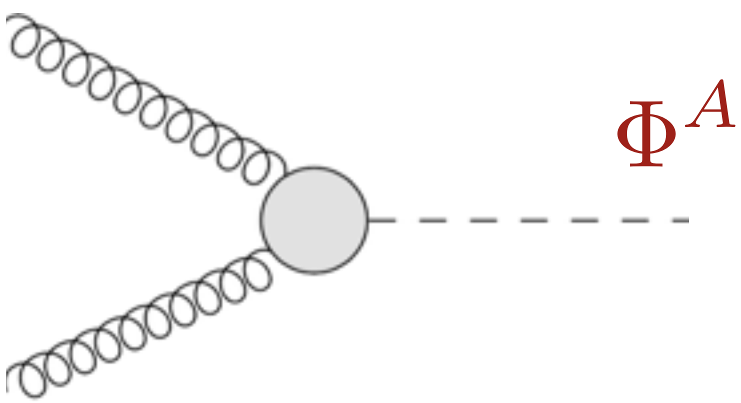
Relations in $\mathcal{N}=4$ SYM

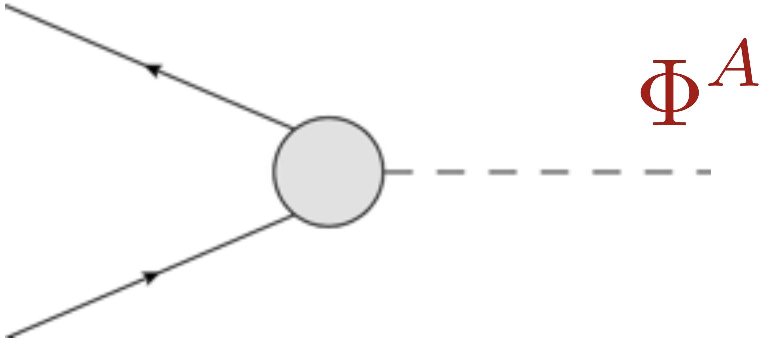
[A.V.Kotikov,L.N.Lipatov,A.I.Onishchenko,V.N.Velizhanin,T. Gehrmann,J. Henn]

Leading Transcendentality Principle

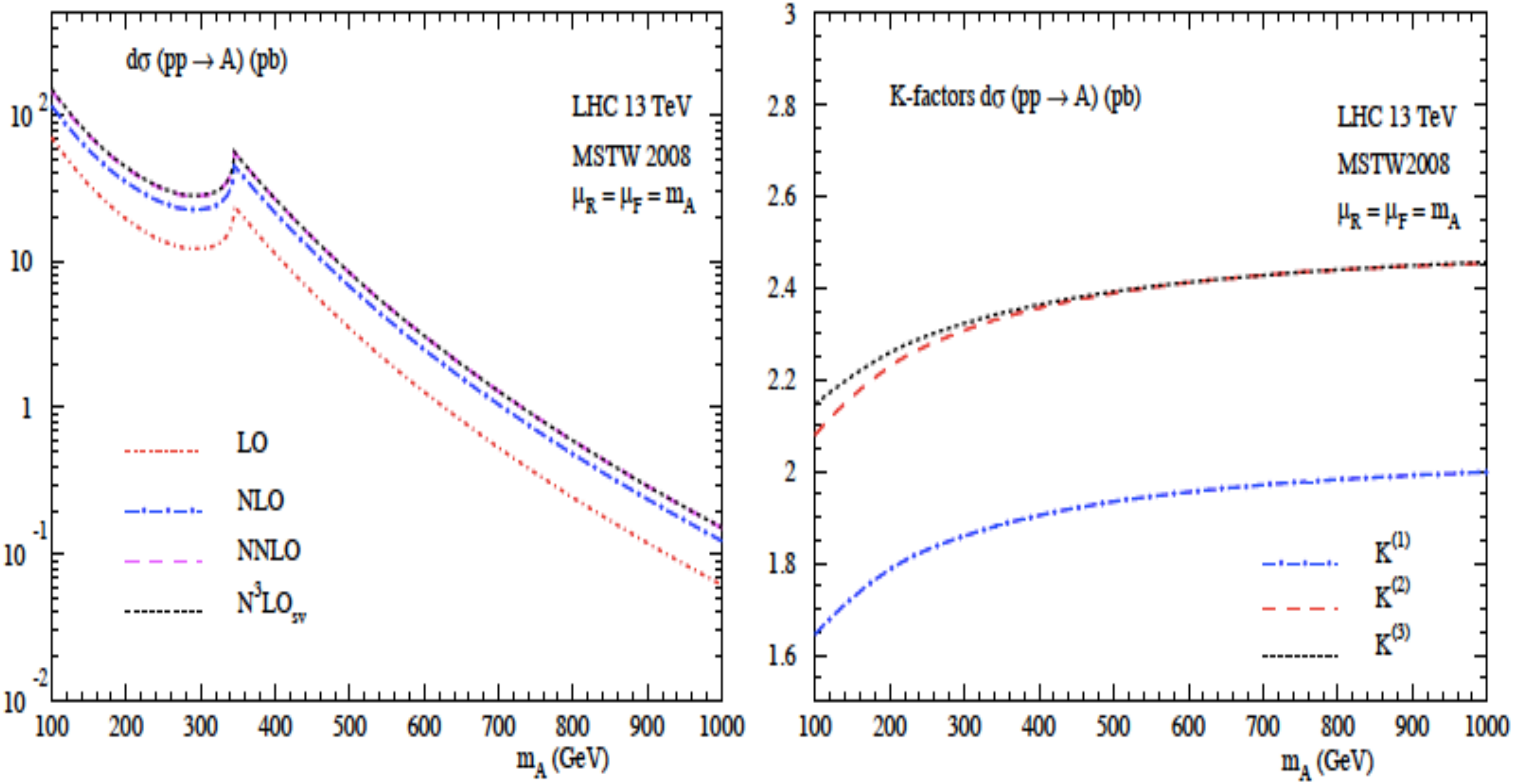
- Set $C_A = C_F = N, T_f n_f = N/2$ for $SU(N)$
- Leading Transcendental (LT) parts of quark and gluon form factors in QCD are equal upto a factor 2^l
- LT part of quark and gluon form factors are identical to the scalar form factor in $\mathcal{N}=4$ SYM
- LT part of pseudo scalar form factor is identical to quark and gluon form factors in QCD upto a factor 2^l also to scalar form factor in $\mathcal{N}=4$ SYM

N3LO Phenomenology

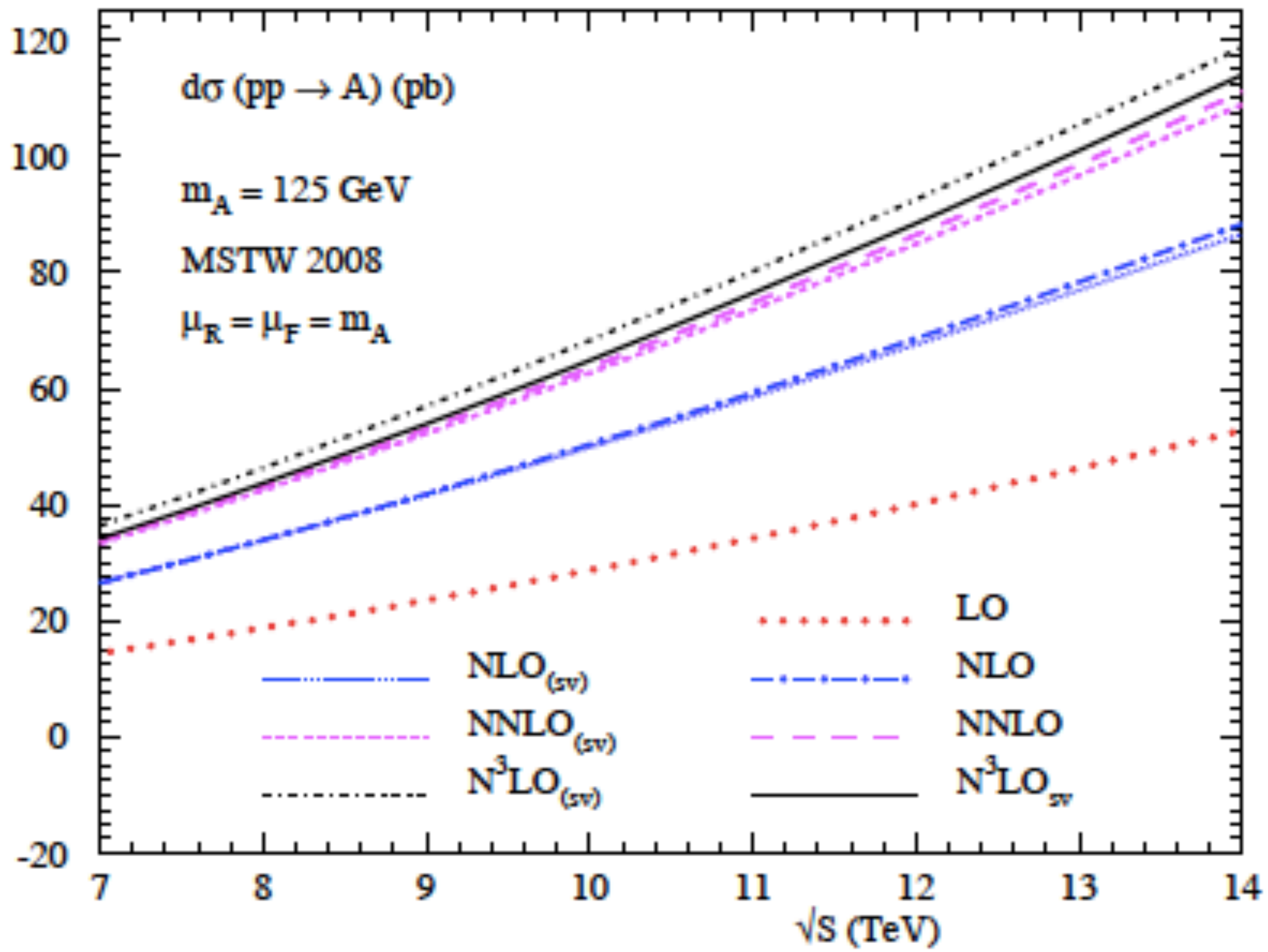




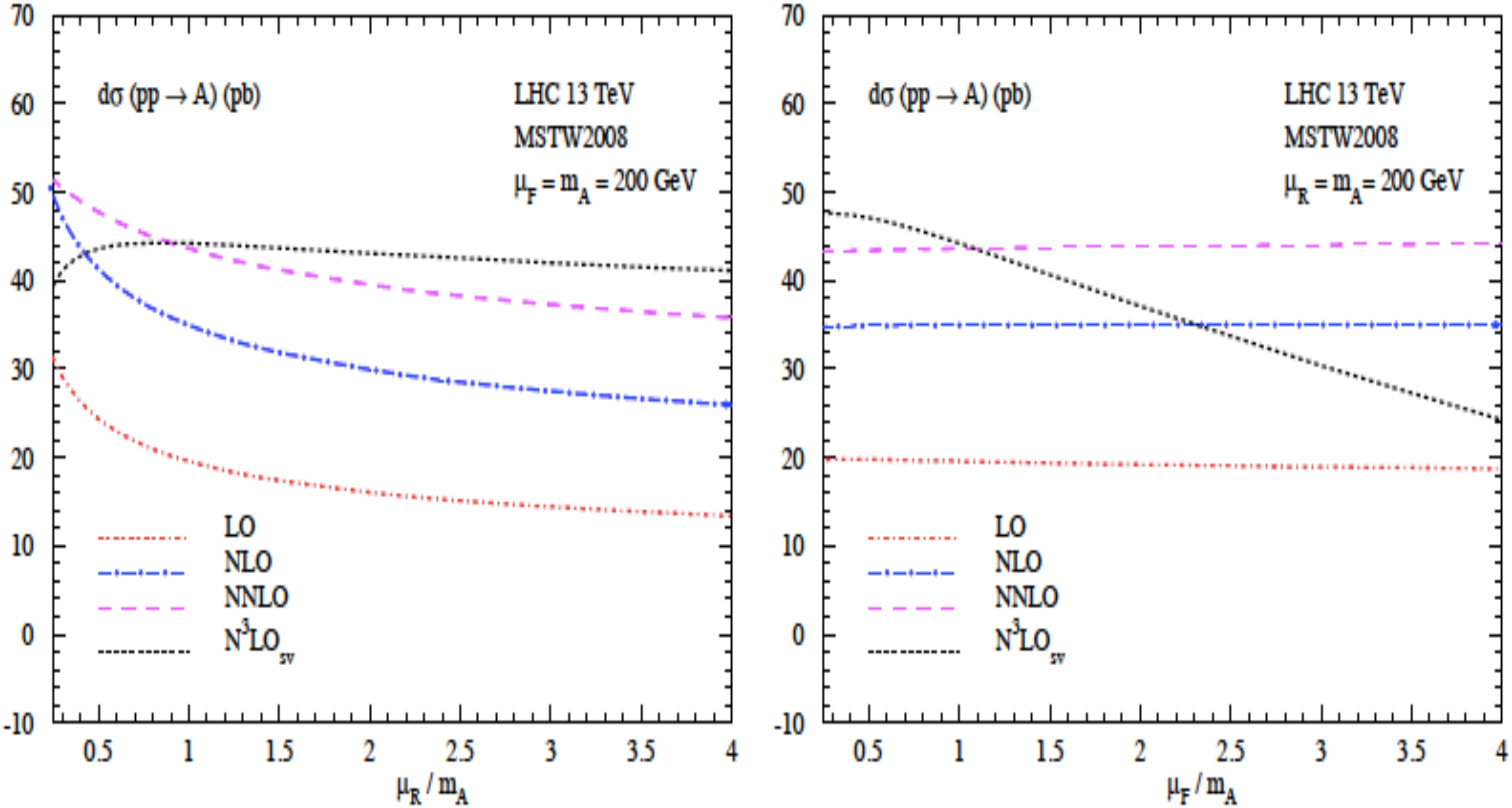
Cross section at N3LO sv



$\sqrt{S}$ dependence at N3LO (sv)



Scale dependence at N3LO sv



PDF dependence at N3LO sv

PDF set	SM Higgs			Pseudo-scalar		
	NLO	NNLO	N^3LO_{SV}	NLO	NNLO	N^3LO_{SV}
ABM11	33.19	39.59	41.99	77.42	92.66	94.64
CT10	31.79	41.84	44.67	74.15	97.94	100.44
MSTW2008	33.59	42.13	44.92	78.35	98.61	101.06
NNPDF 23	33.55	43.01	45.87	78.26	100.70	103.19

Conclusions

- Pseudo-scalar Higgs form factor at three loops in QCD
- UV and IR poles structure using K+G equation
- Subtraction of IR poles results in UV renormalisation constant
- N3LO threshold corrections and N-independent part of resumed cross section at N3LL, Matching coefficients at N3LO in SCET are available
- Scale dependence has been studied.