Data analysis methods for gravitational waves

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Coalescence of two black holes (credits: SXS)
Einstein's General relativity

General relativity – 1915

- Spacetime is a deformable and dynamic object
- Gravity describes as a geometrical effect coming from spacetime curvature
- Einstein's fields equations

\[ G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \]

Energy/Matter

“spacetime tells matter how to move; matter tells spacetime how to curve”

John Archibald Wheeler
Gravitational waves

- Linearization of Einstein equations
\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad |h_{\mu\nu}| \ll 1 \quad \rightarrow \quad \Box \bar{h}_{\mu\nu} = 0 \]

- Propagating perturbations of space-time metric
  - Travel at the speed of light
  - Tranverse waves
  - Two polarisations x and +
  - Dimensionless strain amplitude \( h \)

\[ h \sim 0.5 \]

in this illustration
Michelson interferometer
Advanced LIGO

\[ h_{\text{signal}} \approx 10^{-21} \]

\[ h = \frac{\delta l}{L} \]

\[ \delta l \approx 10^{-18} m \]

L = 4 km
Detection of gravitational waves is an experimental feat! Also have to address challenging data analysis problems.

GW150914 detected on Sep 14 2015 9:50:45 UTC

The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203,000 years, equivalent to a significance greater than 5.1σ.
Announced yesterday!

GW151226 detected on Dec 26 2015 03:38:53 UTC

The signal was initially identified within 70 s by an online matched filter search targeting binary coalescences.

Will concentrate on first event to introduce GW data analysis
Outline

Primer on gravitational waves

Forewords: Filtering for visualization

Searches: principles and methods
  Time-frequency excess power / coherent
  Matched filtering / coincidence

Non-Gaussian noise and “glitches”

Statistical significance

Parameter estimation and waveform reconstruction
  Bayesian inference
Filtering out signal from noise (1)

\[ h(t) \]

Hanford H1: raw data

Livingston L1: raw data

Bandpass filter + frequency notch
Filtering out signal from noise (2)

\[ h(t) \div 500 \]
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Searches for transient gravitational waves

Search for rare transients with low signal to noise ratio

Expected signal is unknown
Search transients appearing in phase in all detectors with no waveform prior

Time-frequency excess power

Expected signal is known
Targeted search signature of binary black-hole merger as predicted by general relativity

Matched filtering
Time-frequency excess power: basic ideas

Transients appears as a local excess in a time-frequency representation of the observations.
Coherent WaveBurst (1)

Time frequency representation(s)

- Projection on bases of functions reasonably localized in time and frequency
- Wilson transforms (modification of Gabor transforms – orthonormal bases)
- Several decompositions at different time scales

S. Klimenko et al., arXiv:1511.05999
Coherent WaveBurst (2)

Combine multi detector data

- **Coherent** analysis: compensate for time delay and phase shift and sum
- Similar to beamforming in phased array radiotelescopes
- Retain time-frequency pixels that are “phase coherent”

- Fast and robust algorithm
  - Event detected **with 3 minutes latency**
  - **General relativity not needed!**
    Can detect unexpected sources

S. Klimenko et al., arXiv:1511.05999
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Matched filtering: basic ideas (1)

Correlate data with expected signal

Expected signal (template)
Here toy model

\[ \rho \]
Matched filtering: basic ideas (2)

Template from astrophysical model

- Characteristic chirp waveform
- Encodes system dynamics
  - Inspiral
    - Leading order: *chirp mass*
    - Next to leading order: mass ratio, spin (assumed aligned with orbital angular momentum)
  - Merger and ringdown
    - Governed by final black-hole mass and spin
- 11 parameters total
  - 4 mass and (aligned) spins, and geometrical params (no excentricity)
Matched filtering: basic ideas (3)

Detect any signal in a space of possible signals ... all with different phase evolution

Do it with a finite set of templates!

Make sure there is a “close” template for every part of the signal space

250 000 templates covers BNS, NS-BH, BBH
Matched filtering: basic ideas (4)

Robust to non-Gaussian noise

- $\chi_r^2$ test that checks consistency of spectral power distribution
- Detection statistic
  \[ \hat{\rho} = \rho \left\{ \left[1 + \left(\chi_r^2\right)^3/2 \right] \right\}^{-1/6} \]

Combine multi detector data

- Triggers generated from each detector independently
- **Coincident** in time and mass/spins
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Glitches in the vicinity of GW150914

second

~minutes

~ 10 minutes

Glitch rate ~ 1 per few seconds to 1 per 20 min
Glitches can mimic gravitational waves

![Graph showing GW waveform and Glitch](arXiv:1602.03844)
Glitch zoo

“blip” glitch
unknown origin

Malfunction of 45 MHz modulation signal

Power line (60 Hz)

Violin mode (500 Hz and harmonics)

Scattered light

Credits: Coughlin, Smith et al, Gravity-spy zooniverse.org
Mitigating glitches

- Monitor instrument and environment
  - 200,000 aux. channels
  - Seismometers, microphones, magnetometers, …
  - Coupling between the environment and h(t)

- Data quality flags
  - Vetoed: 22 h [~5 %]
  - Vetoed: 20 min [< 0.1 %]
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Estimate noise background

- What is the chance that this event is noise? (i.e., the event **statistical significance**)
  - Probability that glitches occur in coincidence at both detectors
  - Challenging to **measure the experimental background**
    - Non-Gaussian noise (glitches) is **impossible to model**
    - **Can't shield the detector** from gravitational wave!
    - Estimate background to **high-significance (p-value < 10^{-6})**
      For comparison: glitch occurrence ~1–10% of observation time

- **Empirical estimate from the data – resampling**
  - Data time-stamps are **artificially shifted by an offset** much larger than the inter-site propagation time
  - Repeat this operation million times with different offsets
Analysis results for GW150914

Binary coalescence search
Matched filtering

Matched filtering results:
- $> 5.1\sigma$

Generic transient search
Time-frequency excess power

Transient search results:
- $> 4.6\sigma$

Probability that this event is due to background alone is $< 1/5\,000\,000$

16 days of observation $\rightarrow$ less than 1 noise event per 203,000 years
Analysis results for GW151226

Binary coalescence search
Matched filtering

Generic transient search
Time-frequency excess power

Not reliably detected by generic transient searches

- Lower SNR
- Longer signal – power distributed over a larger time-frequency region

\[ > 5.0 \sigma \]
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Beyond detection:

Parameter estimation

• Recall: 11 params; 4 for mass and spins & geometry

• Matched filtering search offers **crude** parameter estimates
  • From best matching template: point estimates for mass and spins obtained separately at each detector
Beyond detection: Parameter estimation

- Joint and **coherent** analysis of multi-detector data
- Parameter estimation using **Bayesian inference**

\[
\text{posterior } p(\text{param}|\text{data}) = \frac{p(\text{param}) \, p(\text{data}|\text{param})}{p(\text{data})}
\]

- Marginalization requires computing high-dimensional integrals
  \[
p(\theta_1|\text{data}) = \int p(\{\theta_k\}|\text{data}) \, d\theta_2 \ldots d\theta_n
\]

- Use Monte-Carlo! (stochastic sampling)
- Samplers e.g., Markov Chain Monte Carlo
  series of \{\theta_k\} representative of posterior distribution
Beyond detection: Parameter estimation

Uniform prior in mass

\[ m_{1,2} \in [10, 80] M_\odot \]

with \( m_1 > m_2 \)
Beyond detection: Waveform reconstruction

Estimate $t$, $f$, duration coordinates of wavelets that fit the observations coherently in both detectors

Very good agreement with best-fit search template and prediction from numerical relativity – No excess detected in residual

Observe amplitude decay consistent with damped oscillations of relaxing black hole – but low SNR $\rightarrow$ no reliable reconstruction
Conclusions

Gravitational-wave data analysis reached maturity

- More than 20 years of research
- Succeed in reliably detecting GW150914 and GW151226!
- Interdisciplinary endeavor: Physics (from signal modelling to detector physics) ; Maths, stats, signal processing ; computer science

Beginning of a long story!

- More/other sources to discover (NS!)

Opens new challenges!

- **Digging deeper**: tractable, fully coherent all-sky searches for binary coalescences
- Cover a **larger signal space**: long (> 10 s) transients
- **Rapid event reconstruction** for multimessenger astronomy: fast Bayesian inference
- **Better noise/glitch rejection**: deep learning?
- **Beyond gravitational-wave physics**
  - Cosmology with BBH as standard candles?