Data analysis methods for gravitational waves

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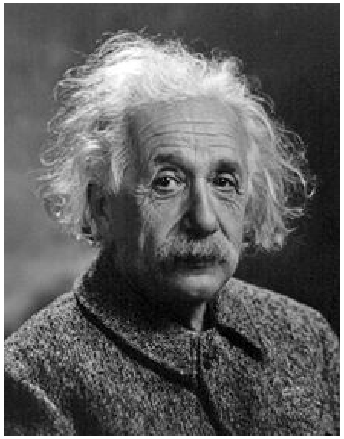
Coalescence of two black holes (credits: SXS)

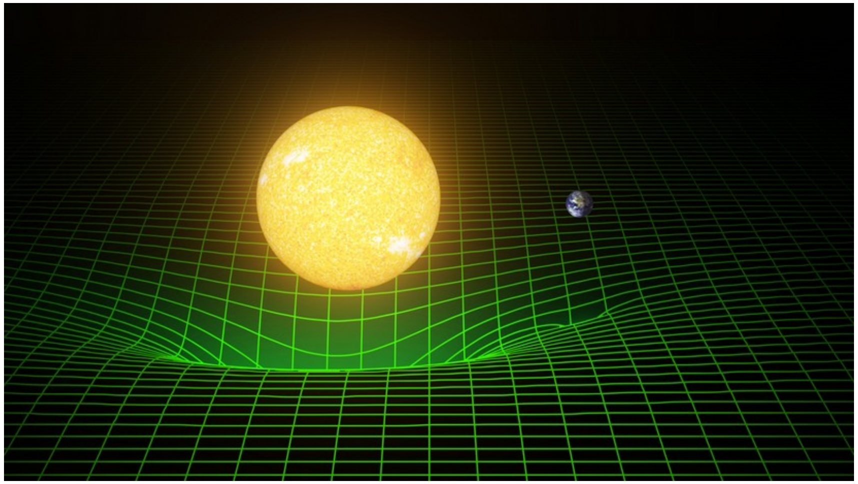
Einstein's General relativity



General relativity – 1915

- Spacetime is a deformable and dynamic object
- Gravity describes as a **geometrical effect coming from spacetime curvature**
- Einstein's fields equations

Space-time geometry $$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$ Energy/ Matter



“spacetime tells matter how to move; matter tells spacetime how to curve”

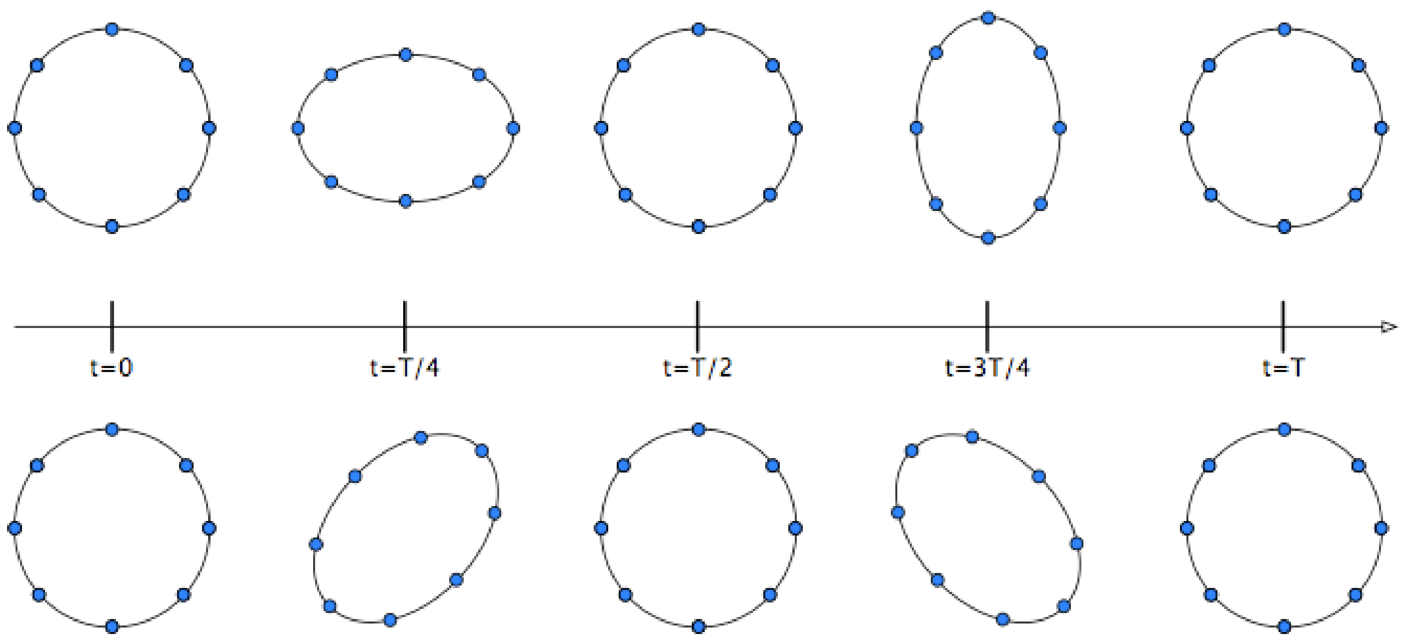
John Archibald Wheeler

Gravitational waves

- Linearization of Einstein equations

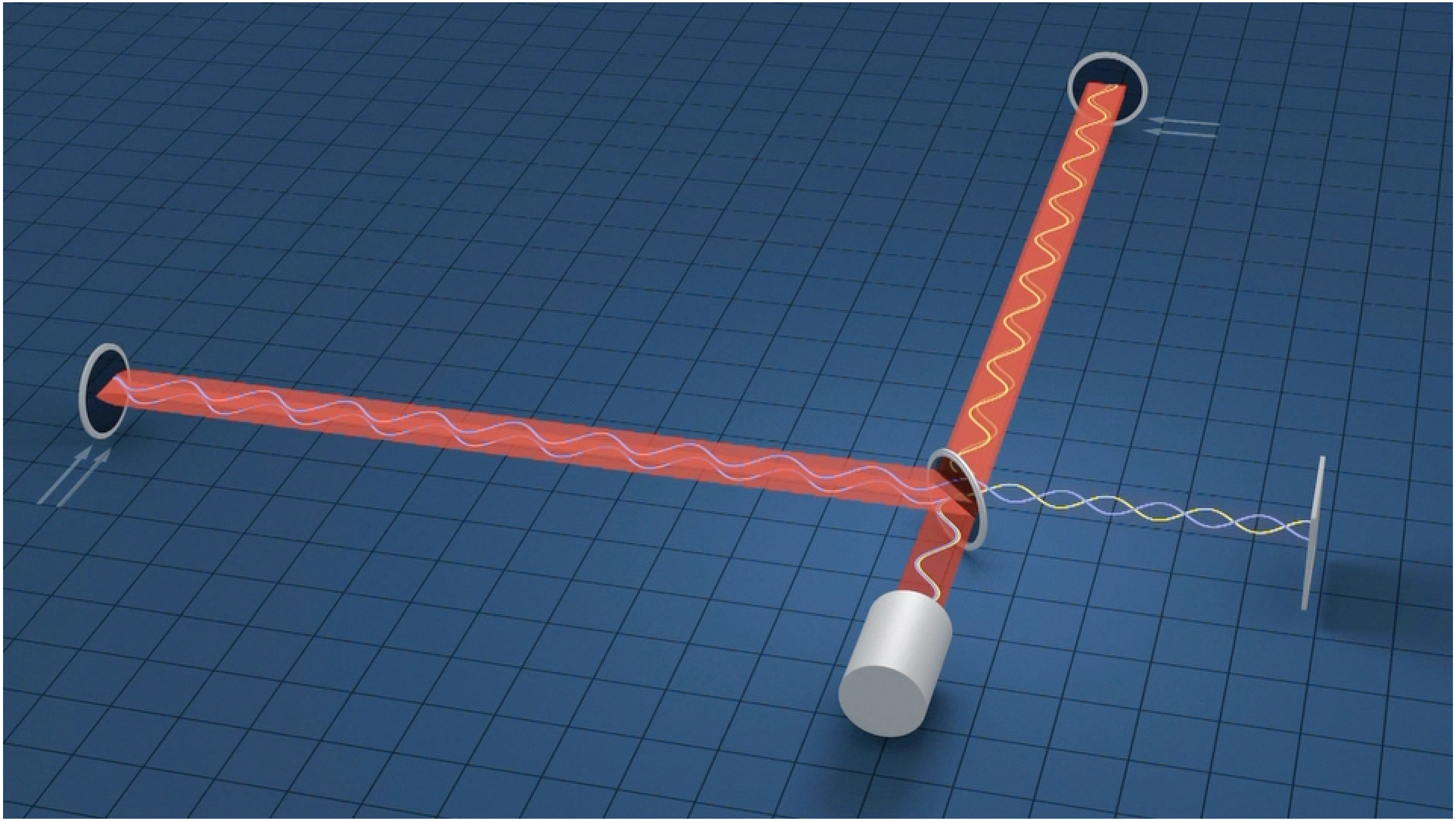
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad |h_{\mu\nu}| \ll 1 \longrightarrow \Box \bar{h}_{\mu\nu} = 0$$

- **Propagating perturbations of space-time metric**
  - Travel at the speed of light
  - Tranverse waves
  - Two polarisations x and +
  - Dimensionless strain amplitude *h*

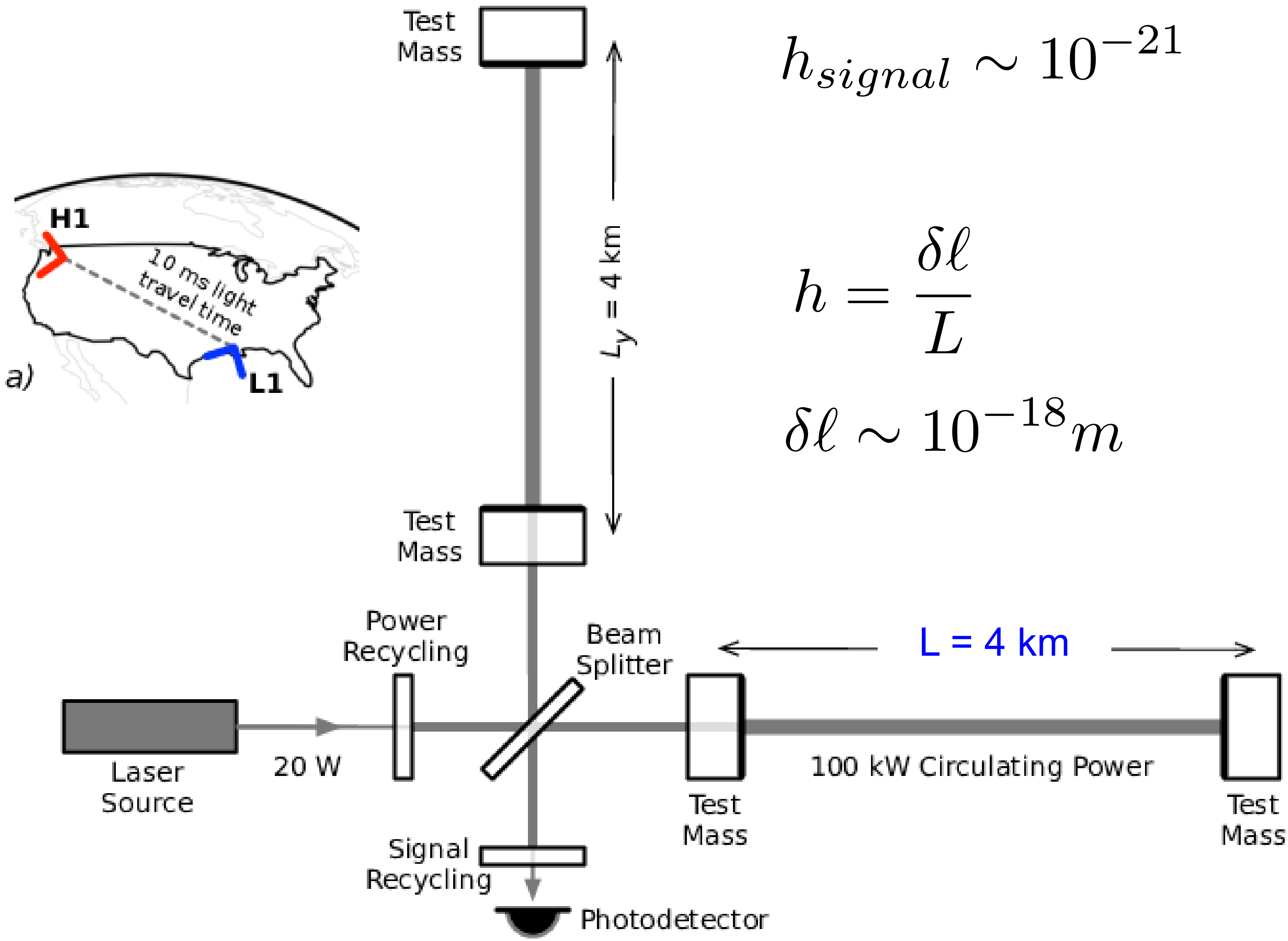


$h \sim 0.5$
in this illustration

Michelson interferometer



Advanced LIGO



$$h_{signal} \sim 10^{-21}$$

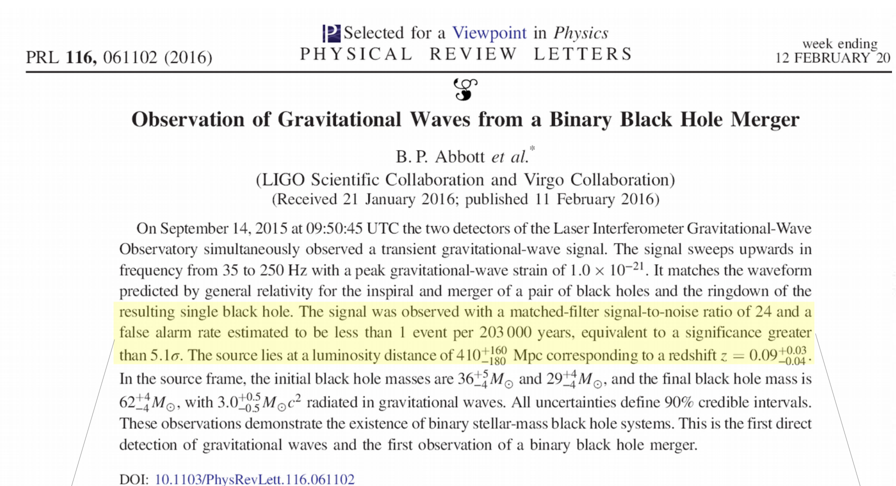
$$h = \frac{\delta\ell}{L}$$

$$\delta\ell \sim 10^{-18} m$$

Detection of gravitational waves is an experimental feat!

Also have to address challenging data analysis problems

GW150914 detected on Sep 14 2015 9:50:45 UTC



PRL 116, 061102 (2016) — Selected for a Viewpoint in *Physics* — PHYSICAL REVIEW LETTERS — week ending 12 FEBRUARY 20

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

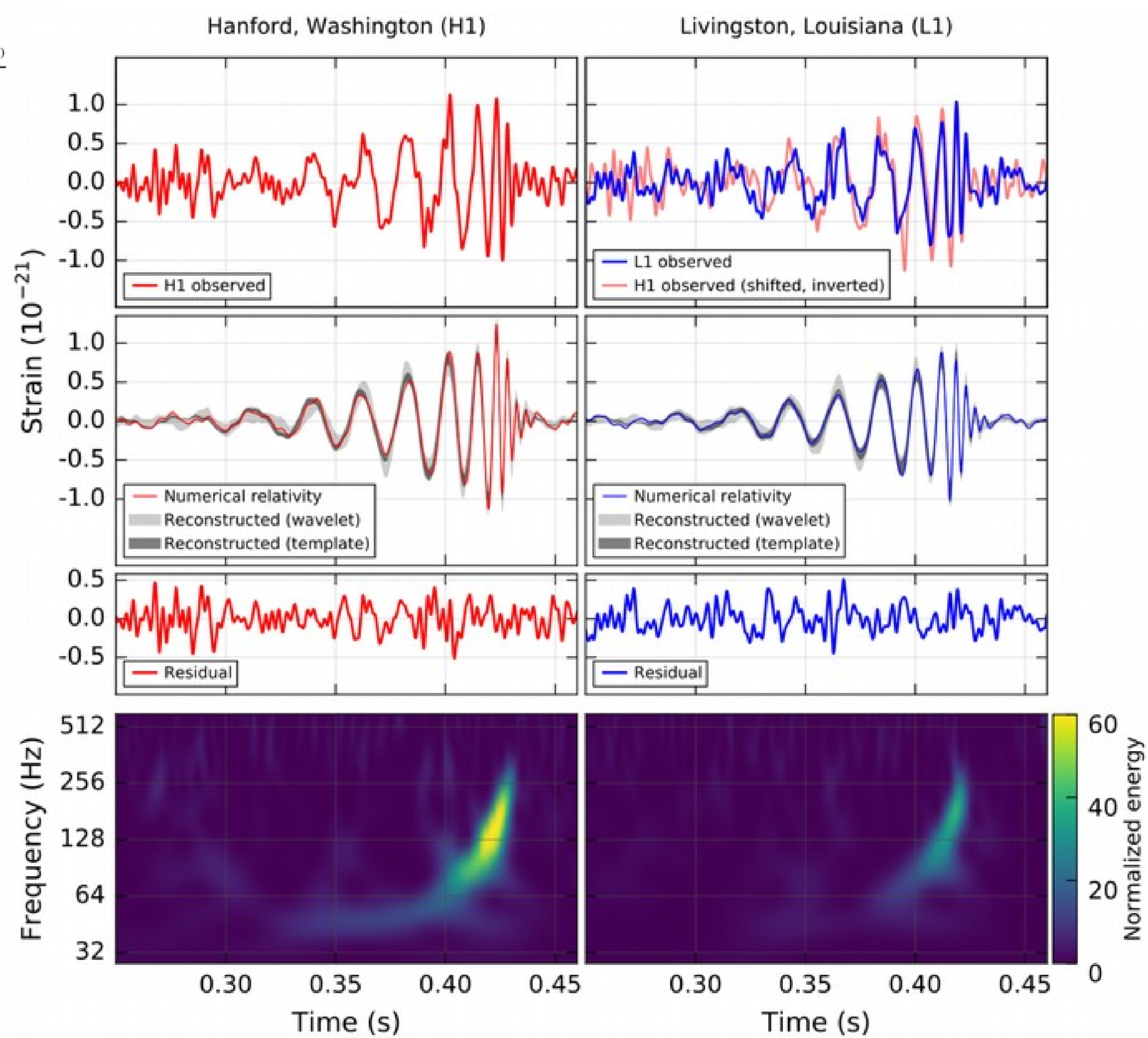
(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102

The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than **1 event per 203 000 years**, equivalent to a **significance greater than 5.1 σ**.



Announced yesterday!

GW151226 detected on Dec 26 2015 03:38:53 UTC

PRL **116,** 241103 (2016) PHYSICAL REVIEW LETTERS

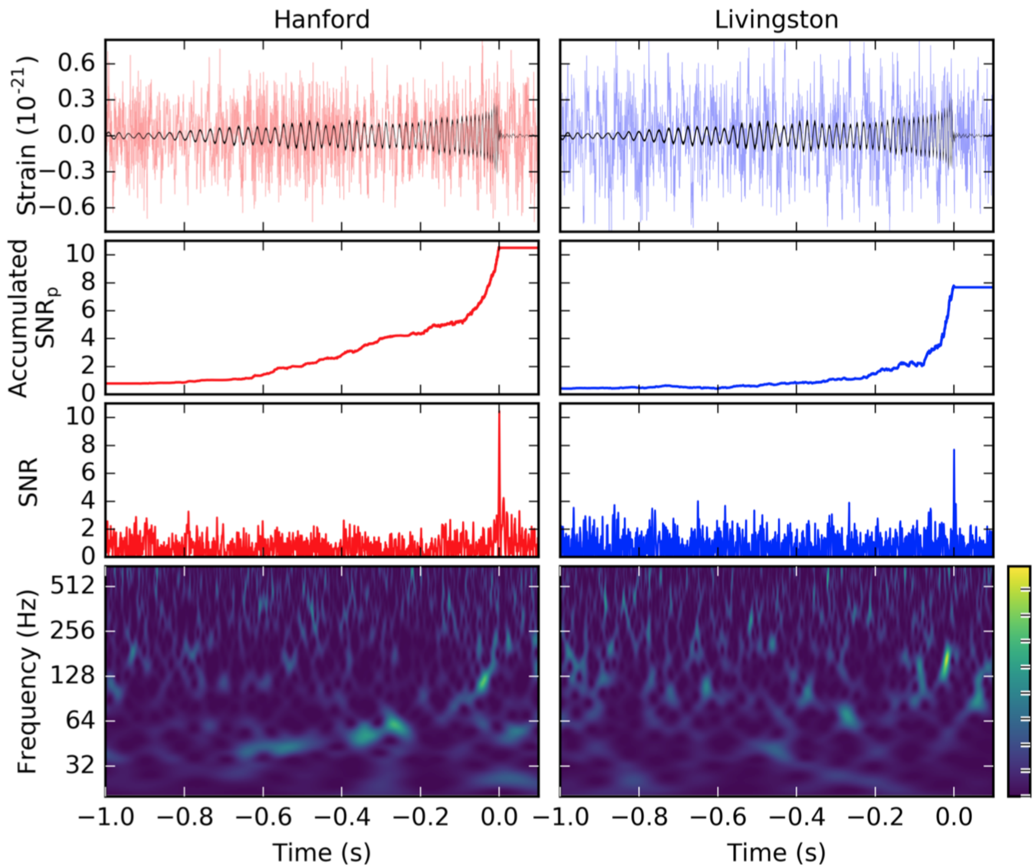
GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 31 May 2016; published 15 June 2016)

We report the observation of a gravitational-wave signal produced by the coalescence of two stellar-mass black holes. The signal, GW151226, was observed by the twin detectors of the Laser Interferometer Gravitational-Wave Observatory (LIGO) on December 26, 2015 at 03:38:53 UTC. The signal was initially identified within 70 s by an online matched-filter search targeting binary coalescences. Subsequent off-line analyses recovered GW151226 with a network signal-to-noise ratio of 13 and a significance greater than 5σ. The signal persisted in the LIGO frequency band for approximately 1 s, increasing in frequency and amplitude over about 55 cycles from 35 to 450 Hz, and reached a peak gravitational strain of $3.4^{+0.7}_{-0.9} \times 10^{-22}$. The inferred source-frame initial black hole masses are $14.2^{+8.3}_{-3.7} M_\odot$ and $7.5^{+2.3}_{-2.3} M_\odot$, and the final black hole mass is $20.8^{+6.1}_{-1.7} M_\odot$. We find that at least one of the component black holes has spin greater than 0.2. This source is located at a luminosity distance of 440^{+180}_{-190} Mpc corresponding to a redshift of $0.09^{+0.03}_{-0.04}$. All uncertainties define a 90% credible interval. This second gravitational-wave observation provides improved constraints on stellar populations and on deviations from general relativity.

DOI: 10.1103/PhysRevLett.116.241103

The signal was initially identified within 70 s by an online **matched filter search** targeting binary coalescences.



Will concentrate on first event to introduce GW data analysis

Outline

Primer on gravitational waves

Forewords: Filtering for visualization

Searches: principles and methods

Time-frequency excess power / coherent
Matched filtering / coincidence

Non-Gaussian noise and “glitches”

Statistical significance

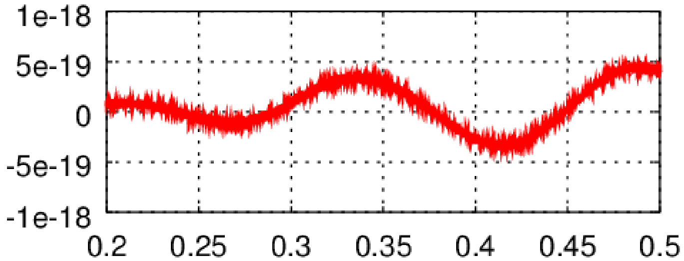
Parameter estimation and
waveform reconstruction

Bayesian inference

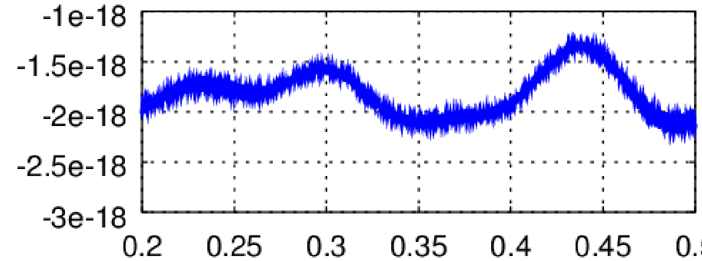
Filtering out signal from noise (1)

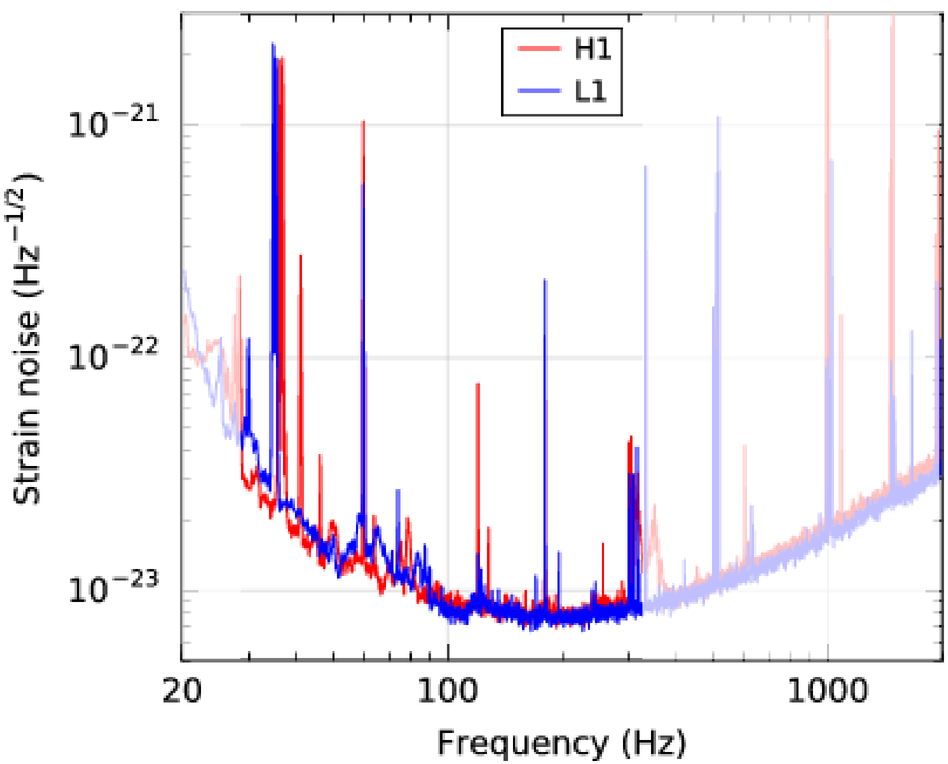
$$h(t)$$

Hanford H1: raw data

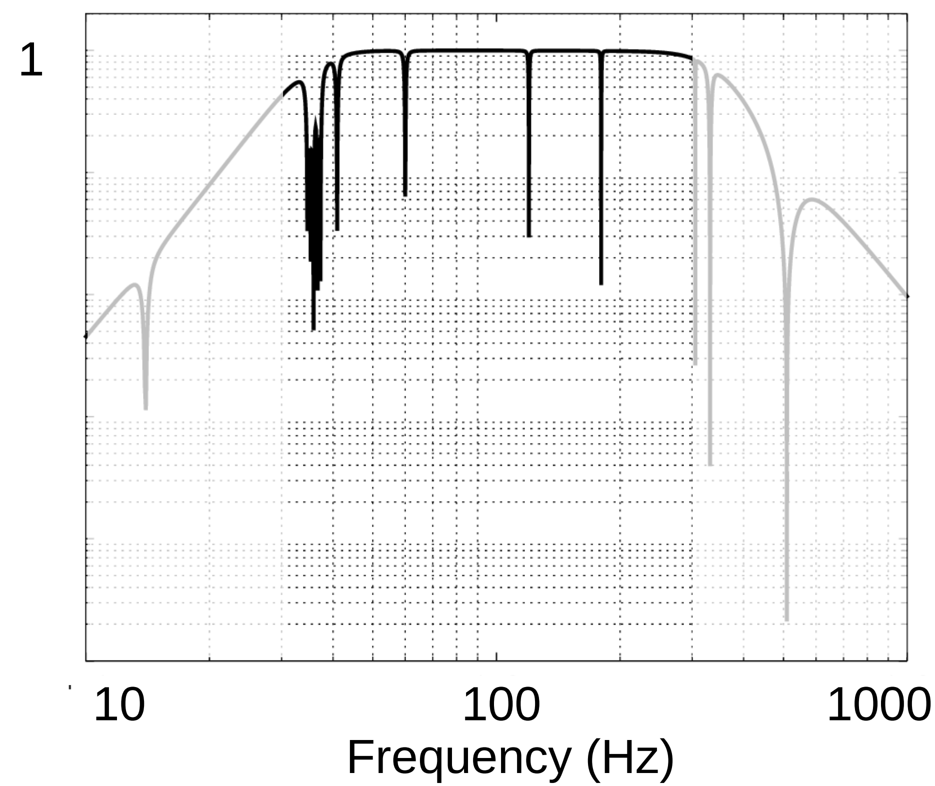


Livingston L1: raw data



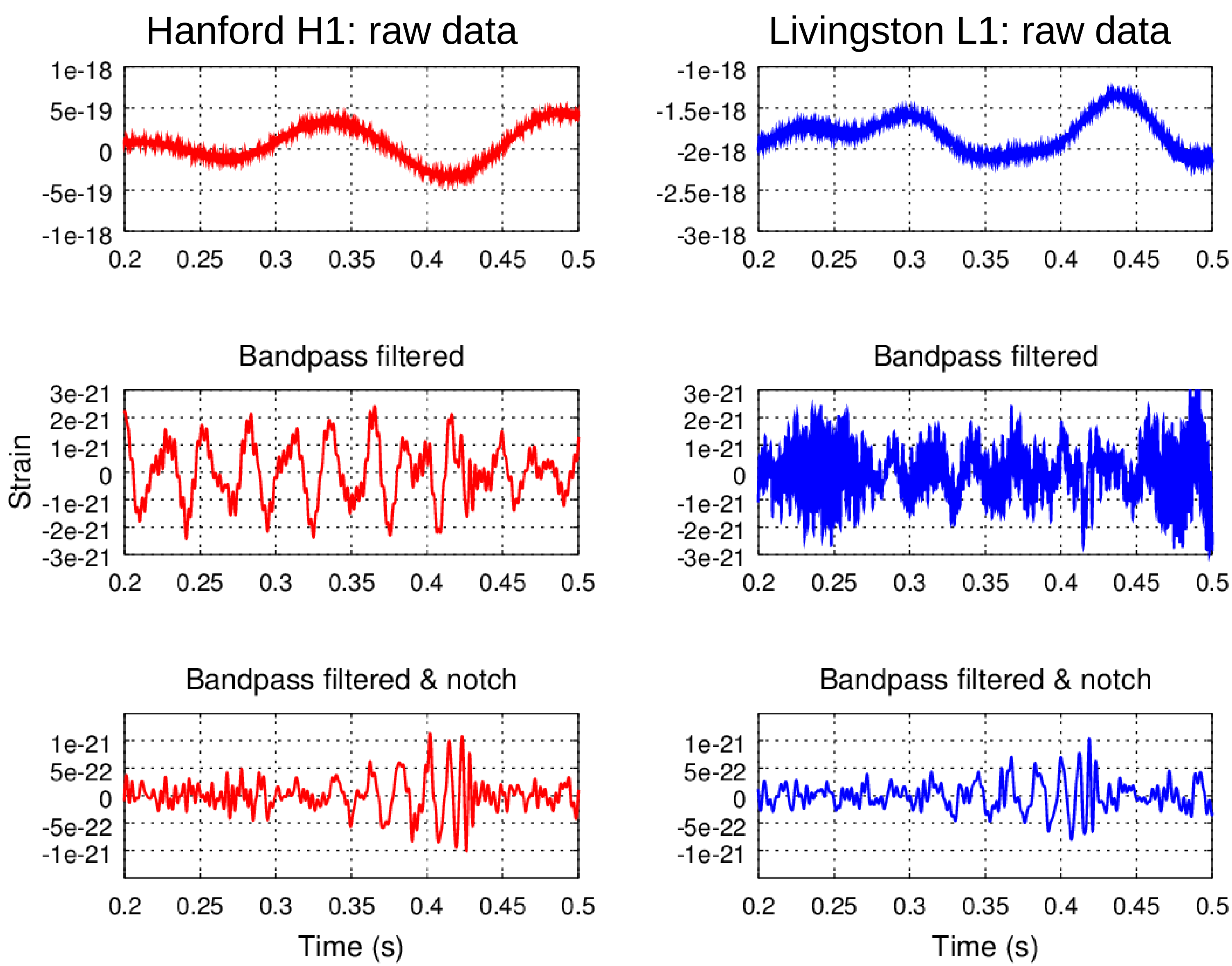


Bandpass filter + frequency notch



Filtering out signal from noise (2)

$h(t)$



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Searches for transient gravitational waves

Search for rare transients
with low signal to noise ratio

Expected signal is **unknown**

Search transients appearing in phase
in all detectors with no waveform prior

Time-frequency excess power

Expected signal is **known**

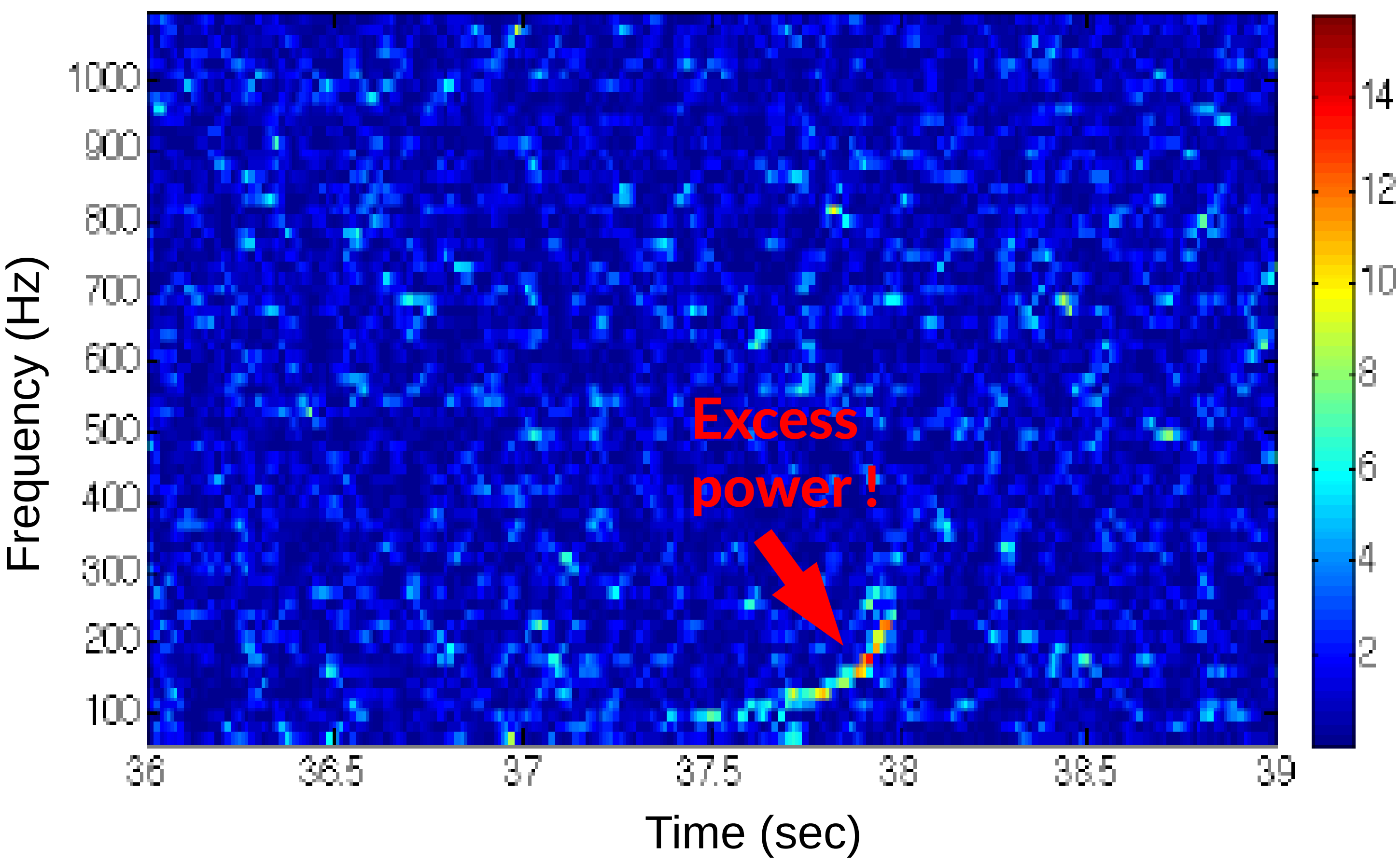
Targeted search
signature of binary black-hole merger
as predicted by general relativity

Matched filtering

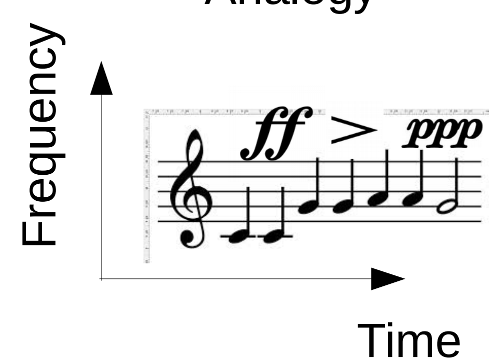
Time-frequency excess power: basic ideas

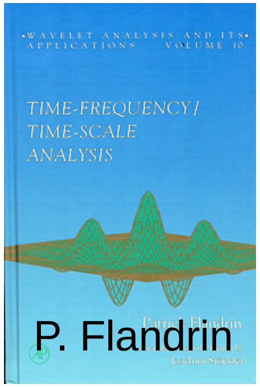
Transients appears as a **local excess in a time-frequency representation** of the observations

Time-frequency representation

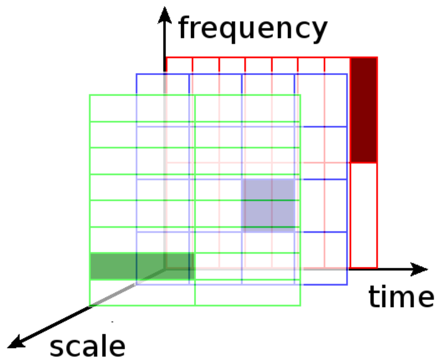


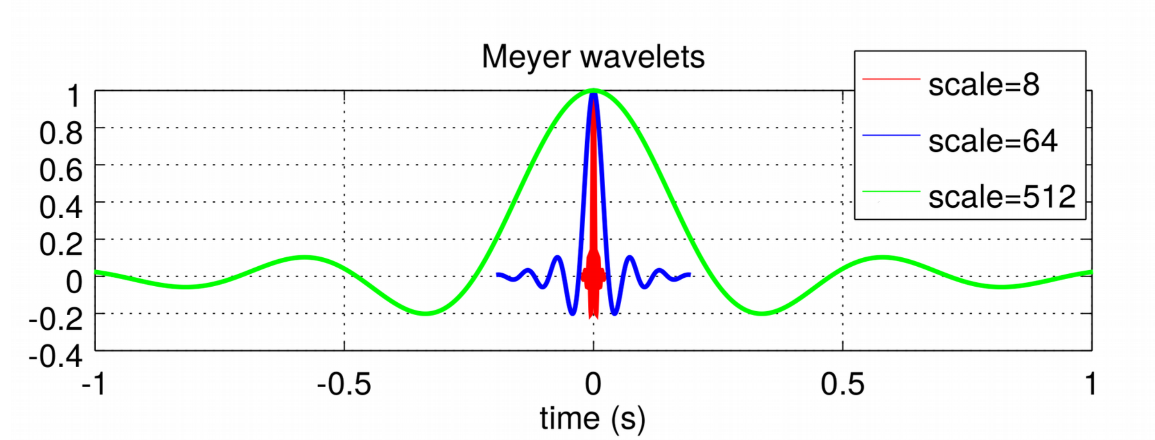
Analogy

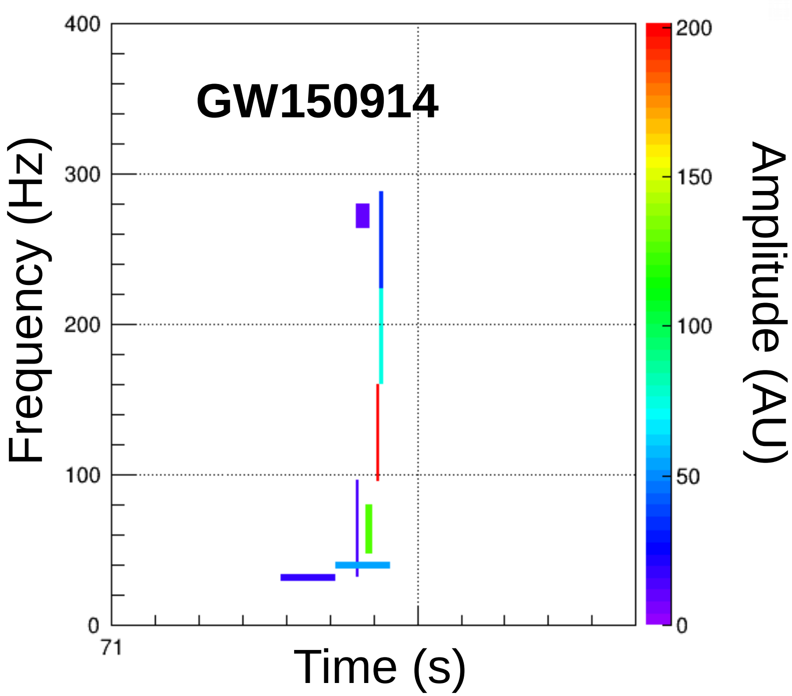




Coherent WaveBurst (1)





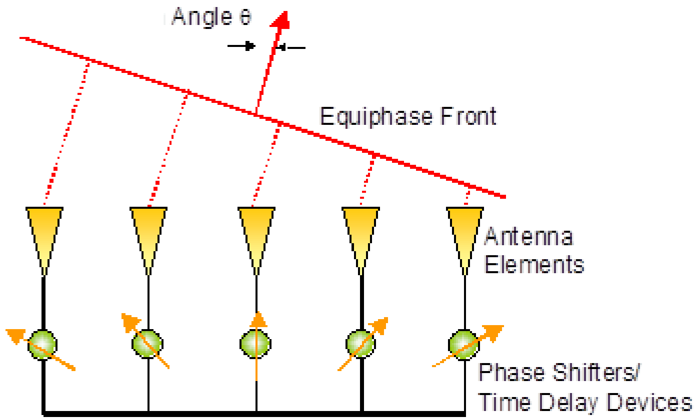


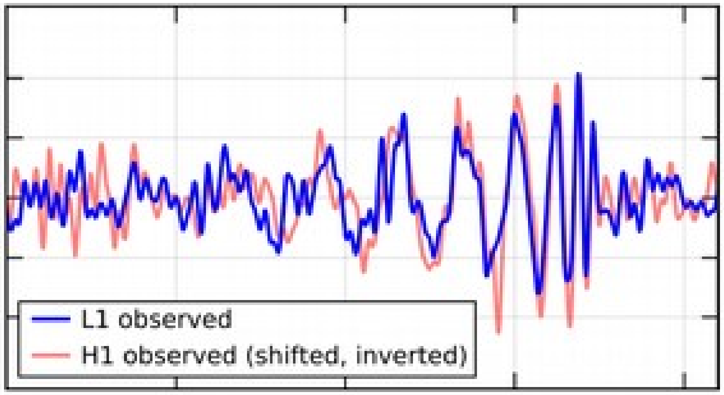
Time frequency representation(s)

- Projection on bases of functions reasonably localized in time and frequency
- Wilson transforms (modification of Gabor transforms – orthonormal bases)
- Several decompositions at different time scales

S. Klimenko et al., arXiv:1511.05999

Coherent WaveBurst (2)





Combine multi detector data

- **Coherent** analysis: compensate for time delay and phase shift and sum
- Similar to beamforming in phased array radiotelescopes
- Retain time-frequency pixels that are "phase coherent"

- Fast and robust algorithm
  - Event detected **with 3 minutes latency**
  - **General relativity not needed!**
    Can detect unexpected sources

S. Klimenko et al., arXiv:1511.05999

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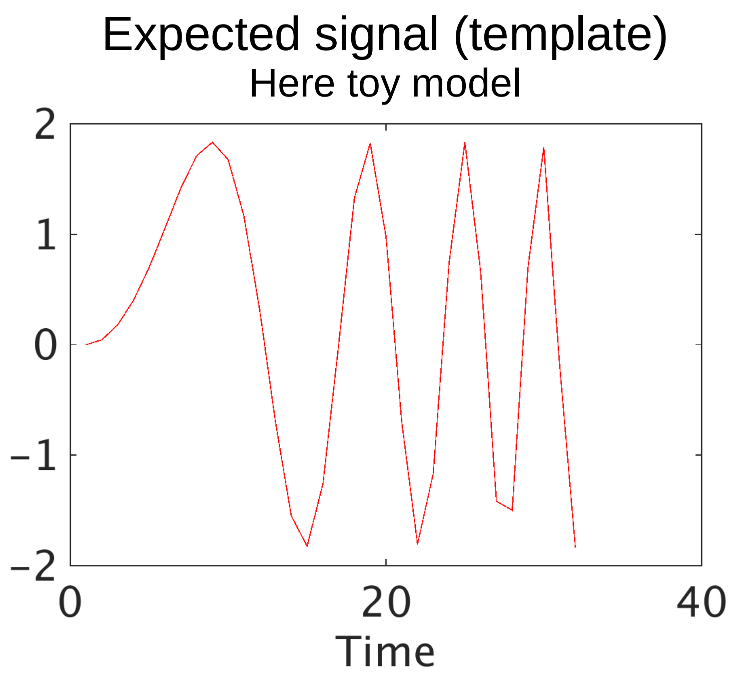
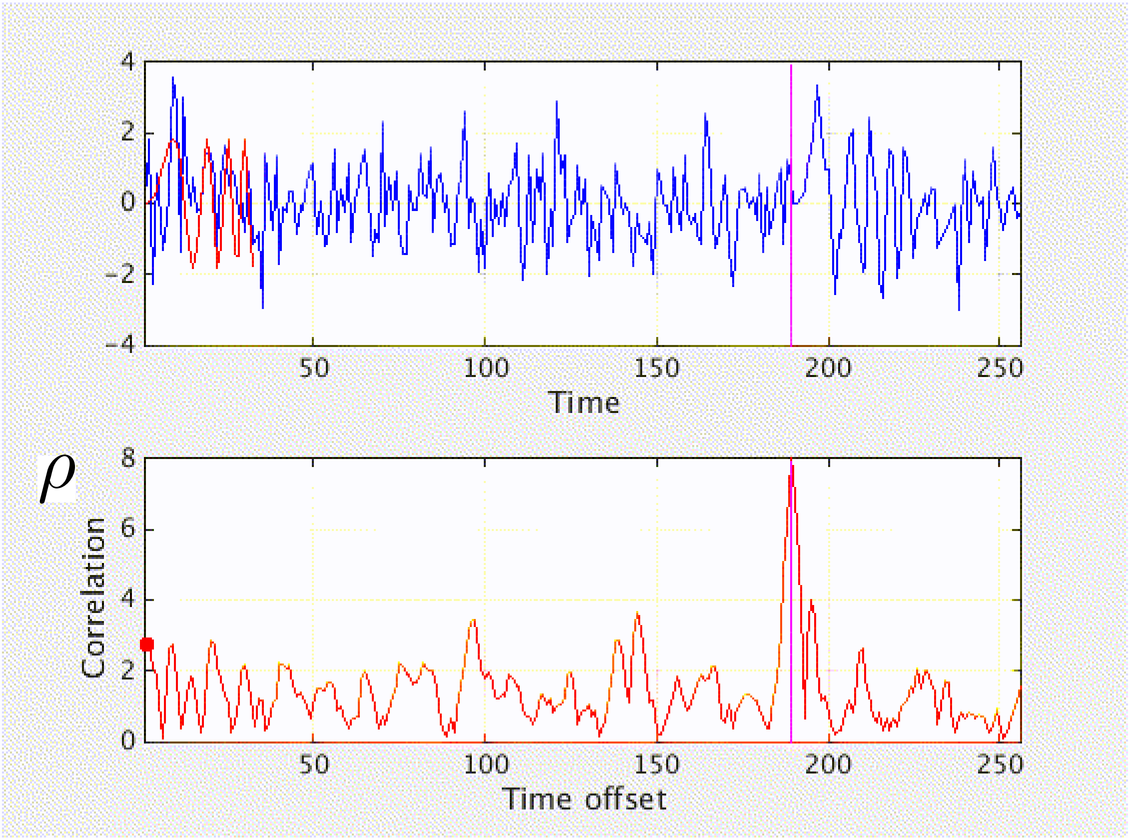
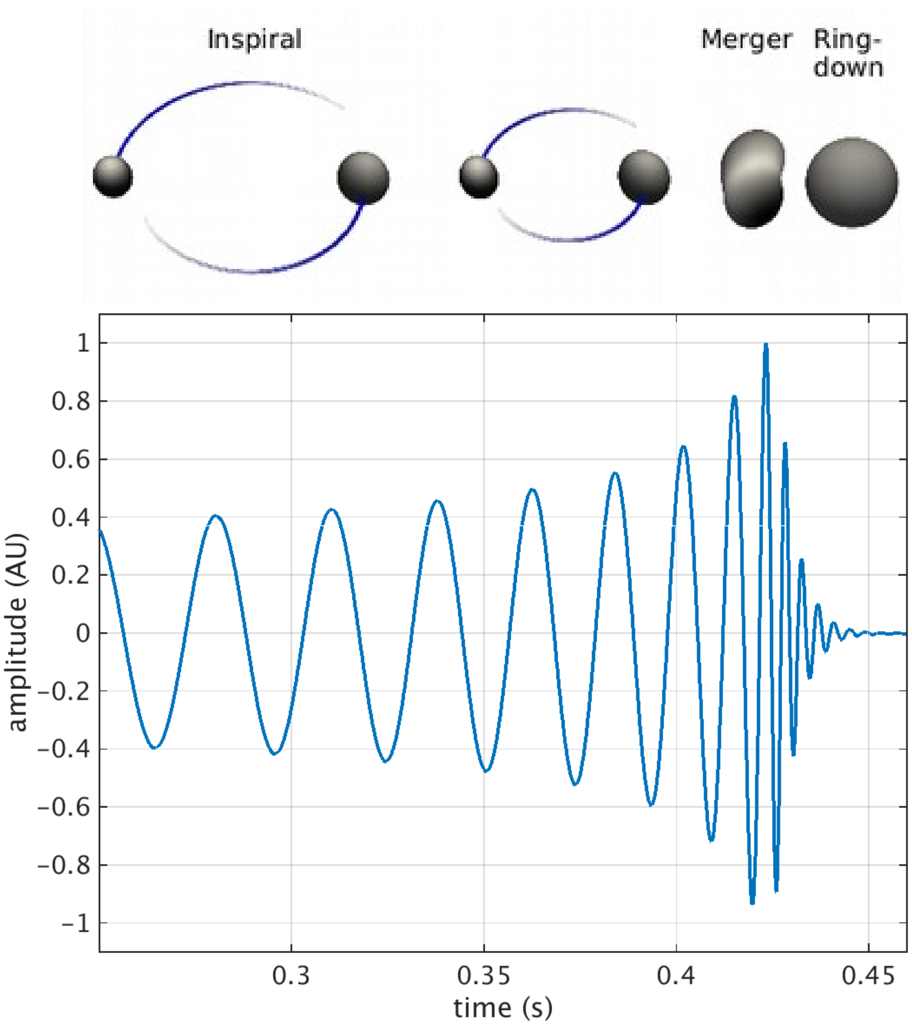
Matched filtering: basic ideas (1)

Correlate data with expected signal





Matched filtering: basic ideas (2)

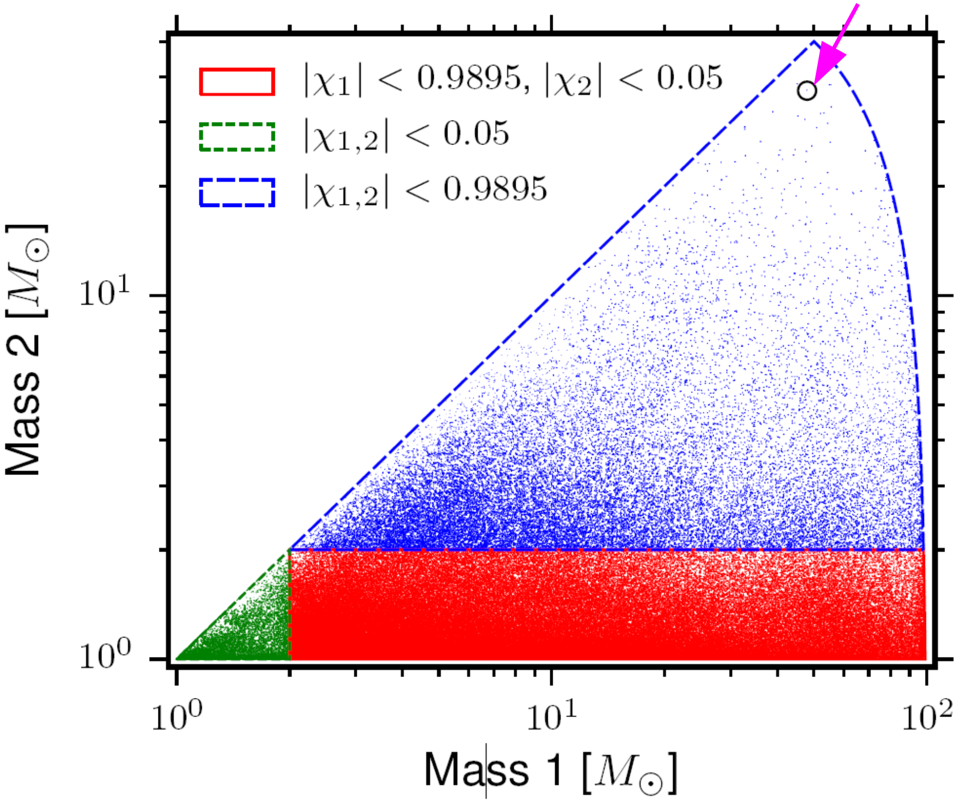


Template from astrophysical model

- Characteristic chirp waveform
- Encodes system dynamics
  - Inspiral
    - Leading order: *chirp mass*
    - Next to leading order: mass ratio, spin (assumed aligned with orbital angular momentum)
  - Merger and ringdown
    - Governed by final black-hole mass and spin
- 11 parameters total
  - 4 mass and (aligned) spins, and geometrical params (no excentricity)

Matched filtering: basic ideas (3)

Template bank



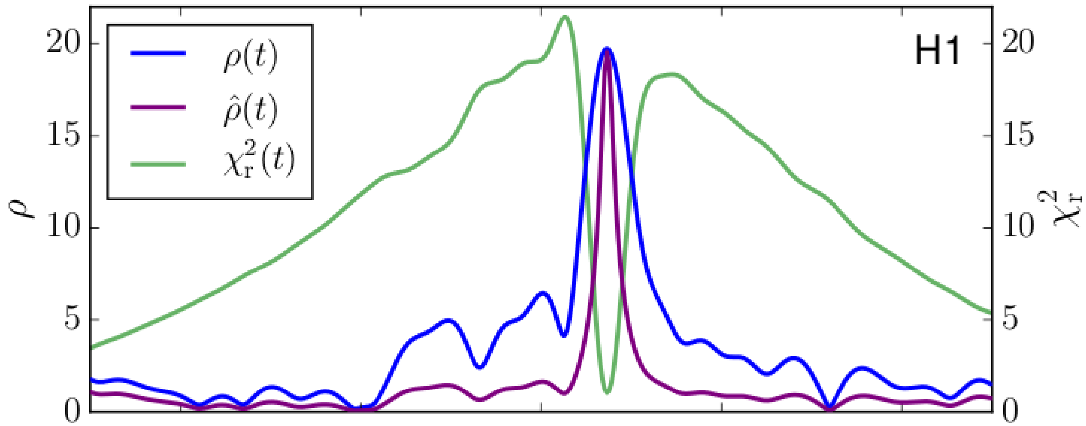
250 000 templates
covers BNS, NS-BH, BBH

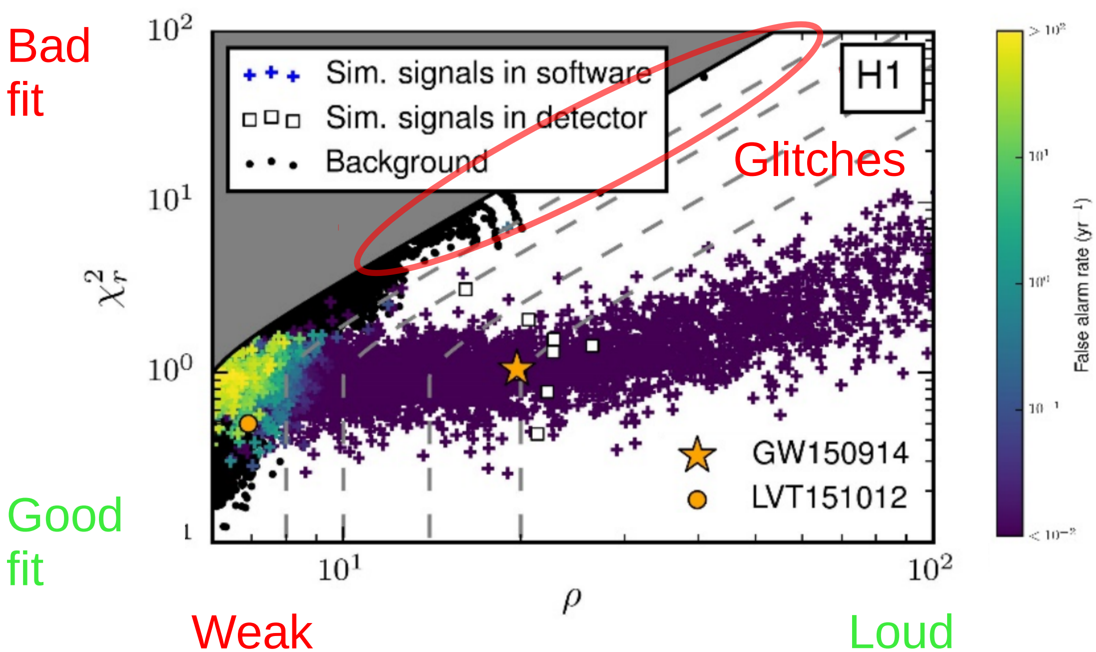
Detect **any signal**
in a ***space of possible signals*** ...
… all with different phase evolution

Do it with a **finite set of templates**!

Make sure there is a “close” template
for every part of the signal space

Matched filtering: basic ideas (4)





Robust to non-Gaussian noise

- χ_r^2 test that **checks consistency of spectral power distribution**
- Detection statistic

$$\hat{\rho} = \rho\left\{[1 + (\chi_r^2)^3]/2\right\}^{-1/6}$$

Combine multi detector data

- Triggers generated from each detector independently
- **Coincident** in time and mass/spins

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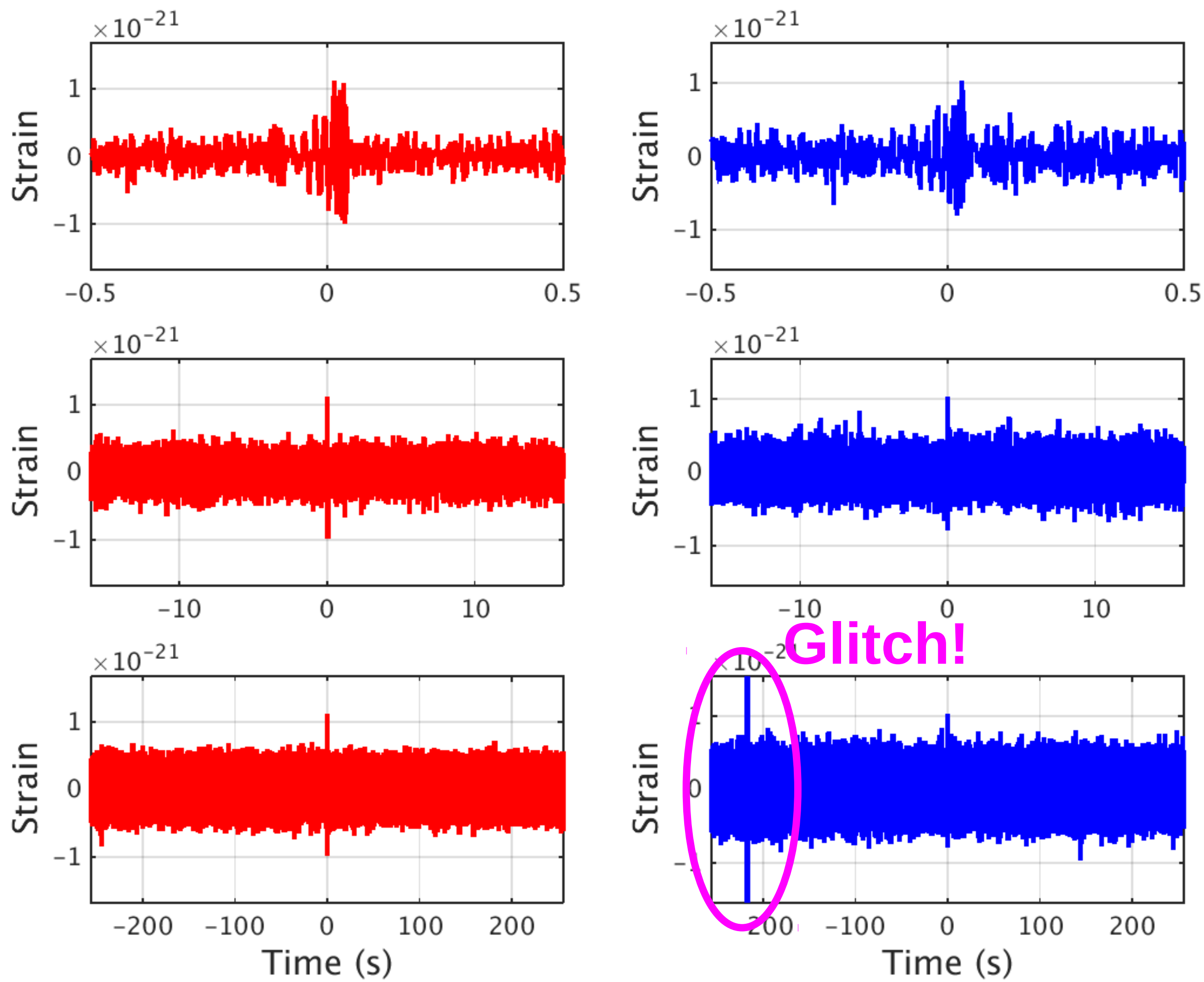
Glitches in the vicinity of GW150914

second

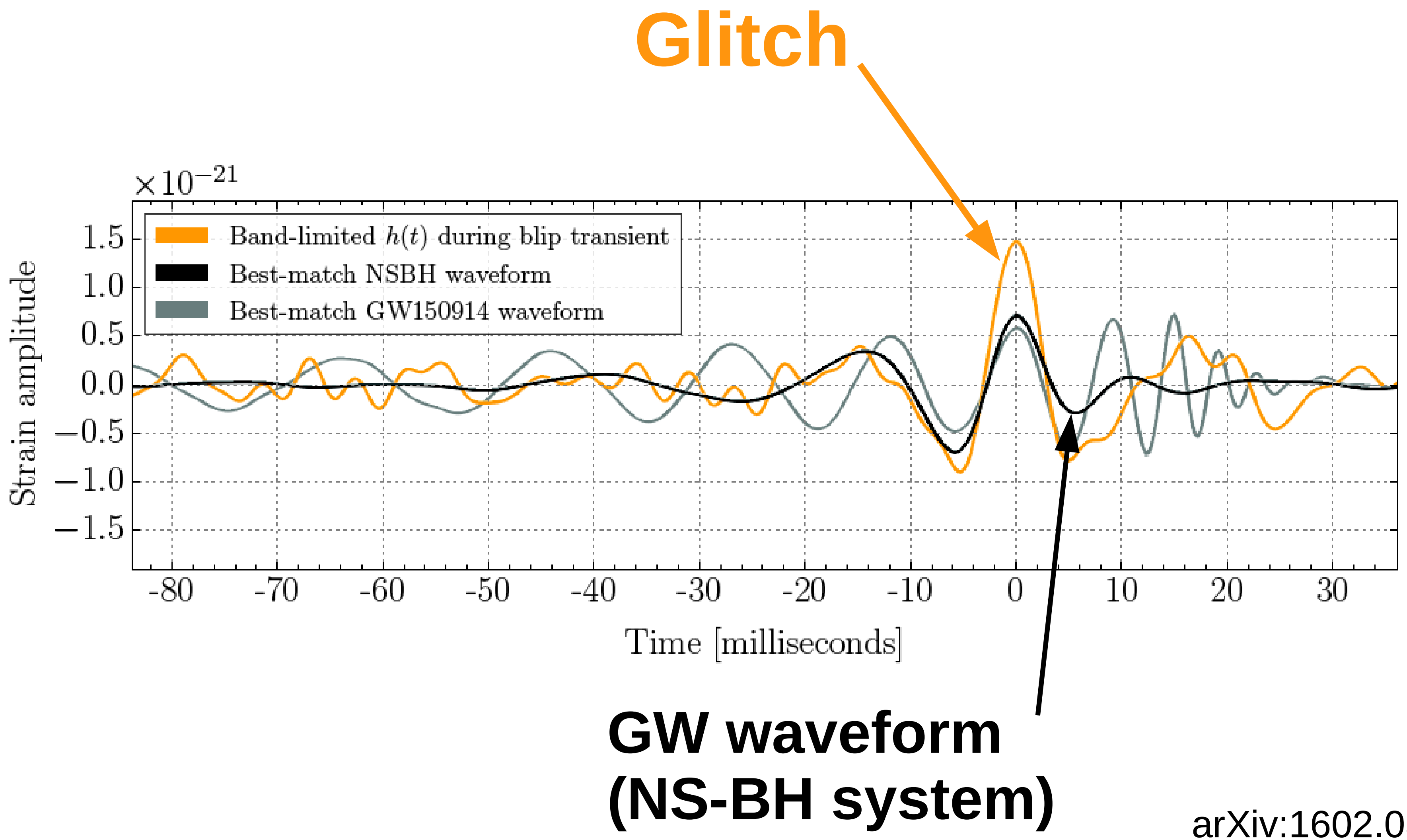
~minutes

~ 10 minutes



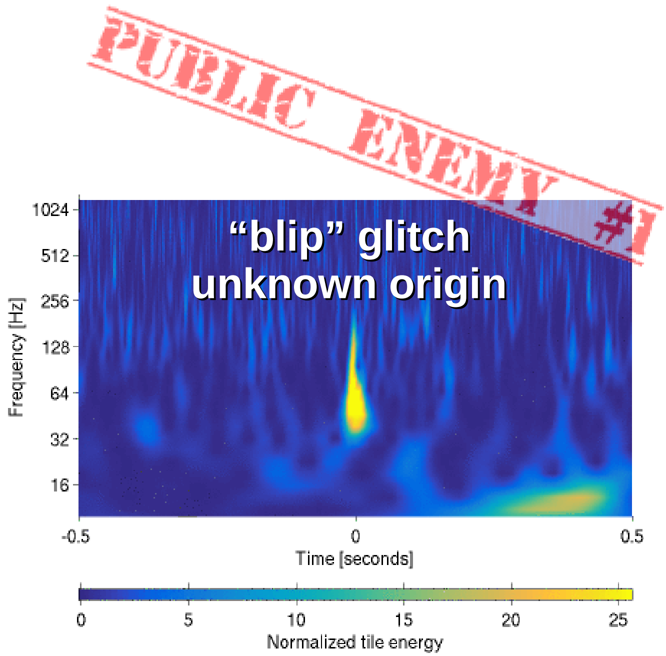
Glitch rate ~ 1 per few seconds to 1 per 20 min

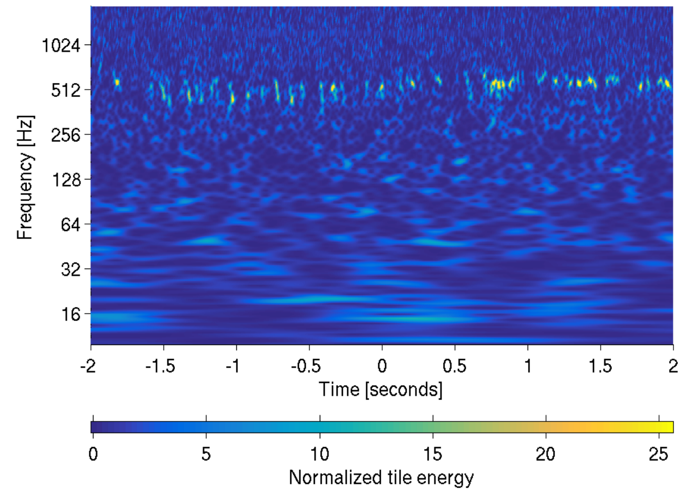
Glitches can mimic gravitational waves

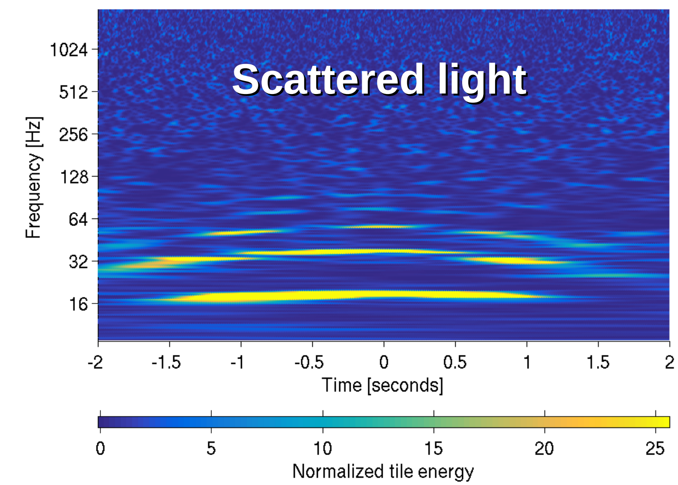


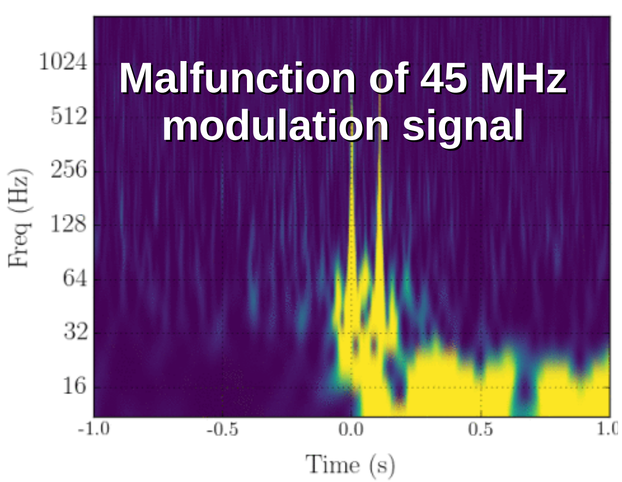
arXiv:1602.03844

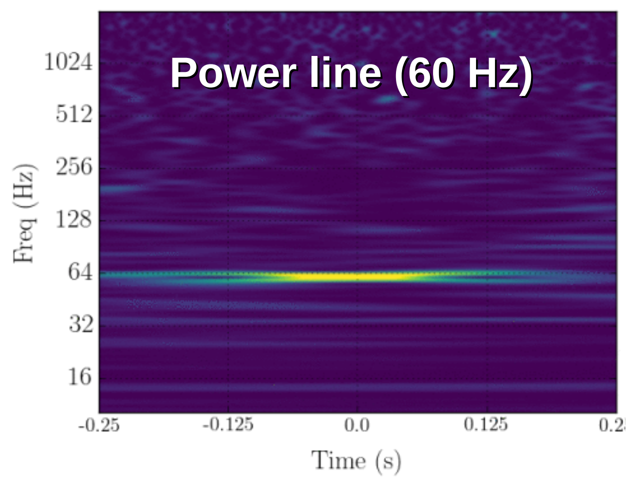
Glitch zoo

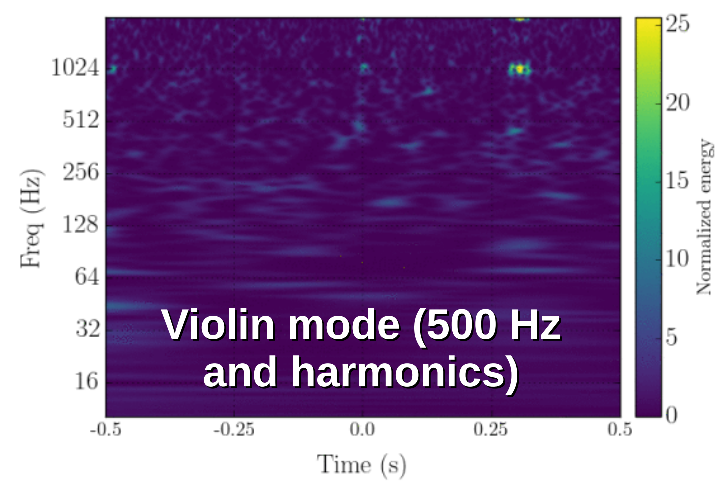








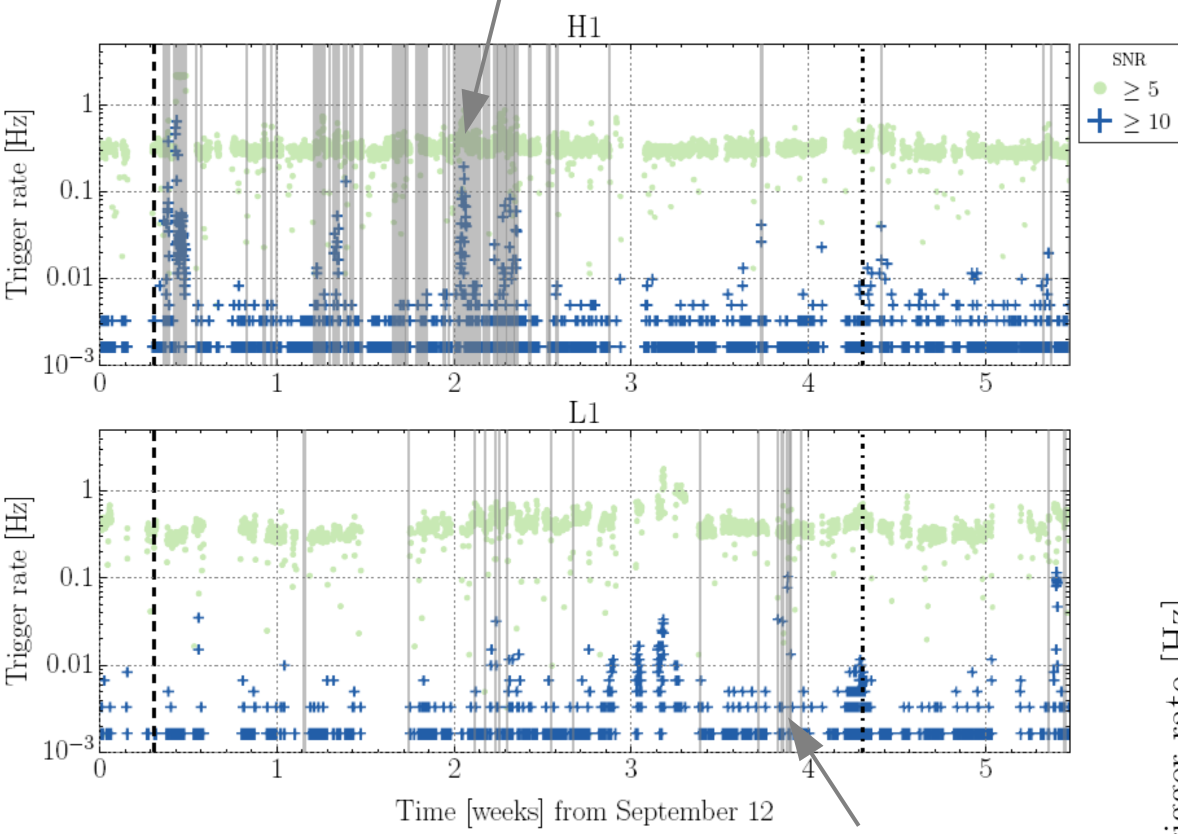




Credits: Coughlin, Smith et al, Gravity-spy zooniverse.org

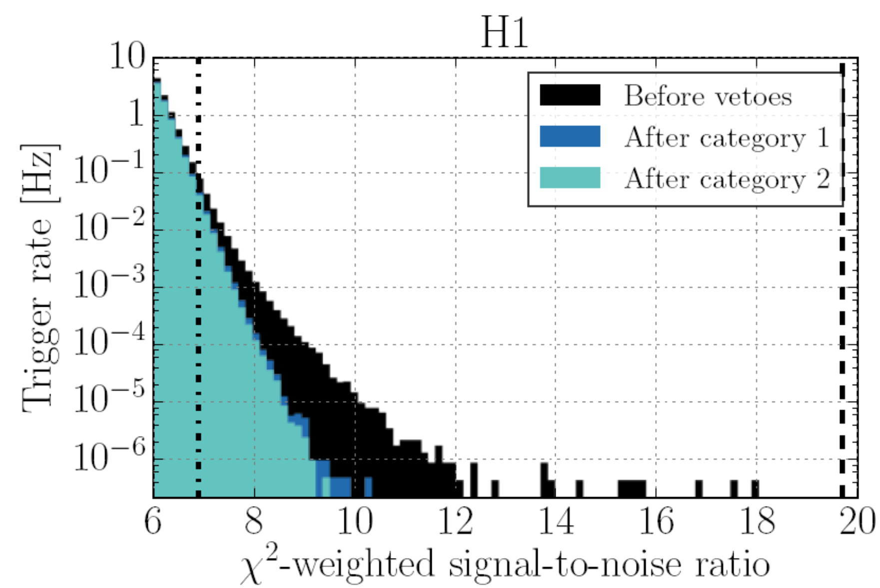
Mitigating glitches

Vetoed: 22 h [~5 %]



Vetoed: 20 min [< 0.1 %]

- Monitor instrument and environment
  - 200 000 aux. channels
  - Seismometers, microphones, magnetometers, …
  - Coupling between the environment and h(t)
- Data quality flags



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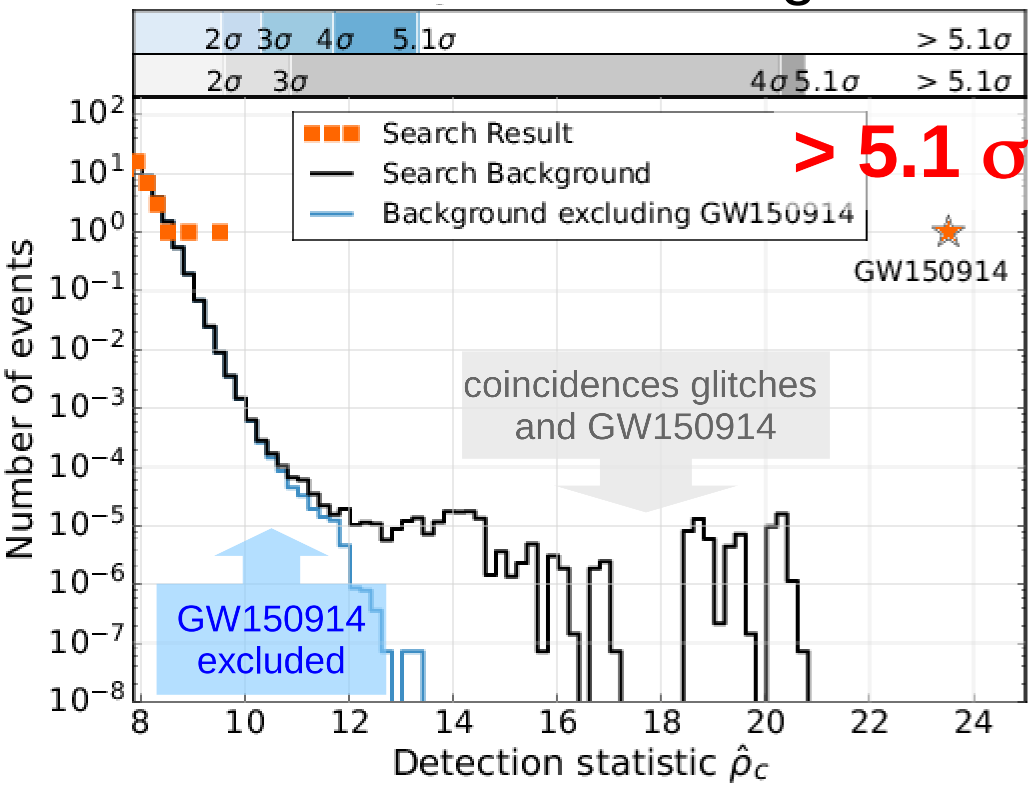
Estimate noise background

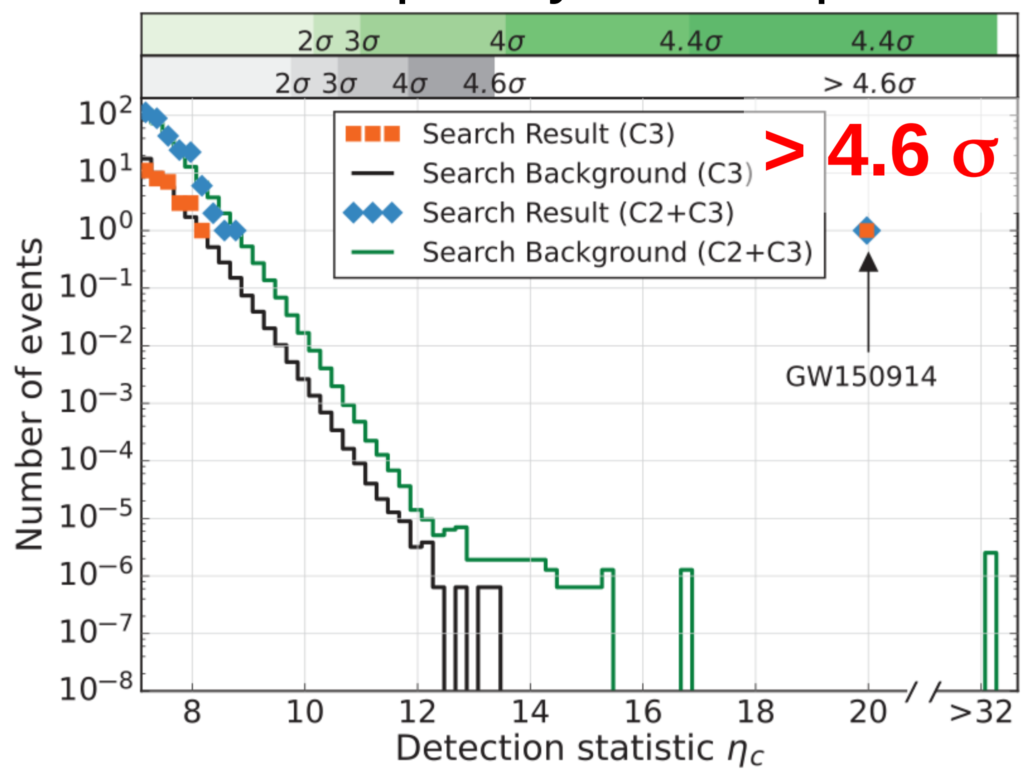
- What is the chance that this event is noise? (i.e., the event **statistical significance**)
  - Probability that glitches occur in coincidence at both detectors
  - Challenging to **measure the experimental background**
    - Non-Gaussian noise (glitches) is **impossible to model**
    - **Can't shield the detector** from gravitational wave!
    - Estimate background to **high-significance (*p-value* $< 10^{-6}$)**
      For comparison: glitch occurrence ~1–10% of observation time
- **Empirical estimate from the data – resampling**
  - Data time-stamps are artificially shifted by an offset much larger than the inter-site propagation time
  - Repeat this operation million times with different offsets

Analysis results for GW150914

Binary coalescence search
Matched filtering

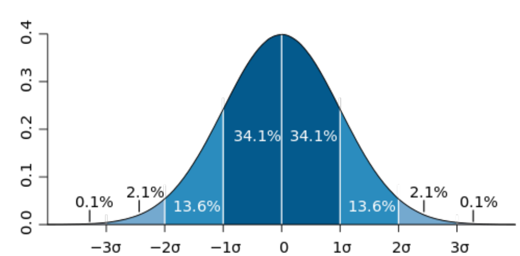


Generic transient search
Time-frequency excess power



Probability that this event is due to background alone is < 1/5 000 000

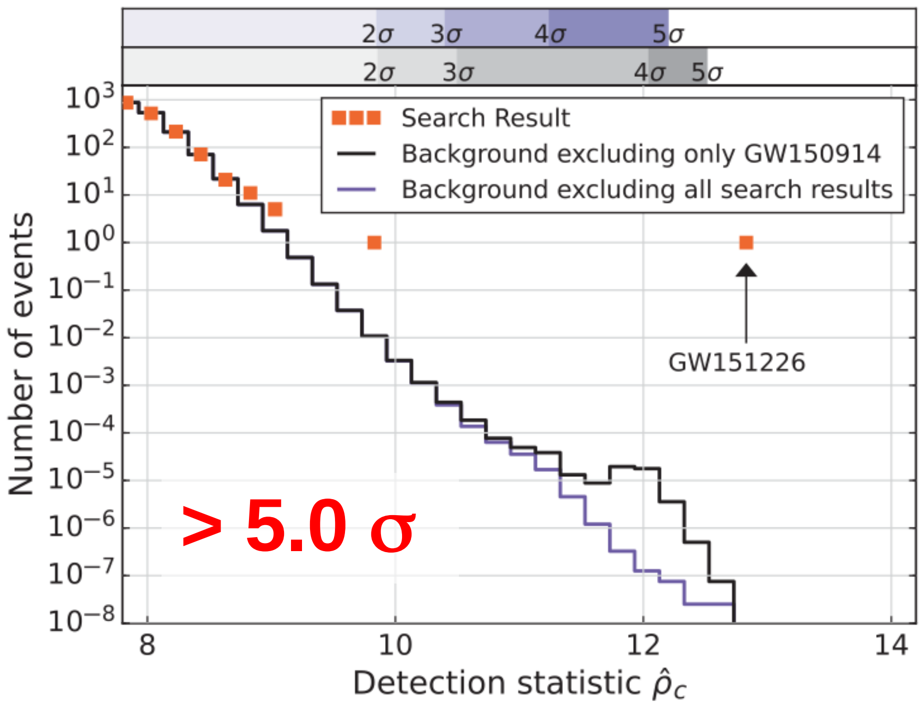
16 days of observation → less than 1 noise event per 203 000 years



5σ

Analysis results for GW151226

Binary coalescence search
Matched filtering



Generic transient search
Time-frequency excess power

Not reliably detected by generic transient searches

- Lower SNR
- Longer signal – power distributed over a larger time-frequency region

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Beyond detection: Parameter estimation

- Recall: 11 params ; 4 for mass and spins & geometry
- Matched filtering search offers **crude** parameter estimates
  - From best matching template: point estimates for mass and spins obtained separately at each detector

Beyond detection: Parameter estimation

- Joint and **coherent** analysis of multi-detector data
- Parameter estimation using **Bayesian inference**

posterior $p(\text{param}|\text{data}) = \frac{p(\text{param})\, p(\text{data}|\text{param})}{p(\text{data})}$ likelihood (obs model)

prior (assumptions)

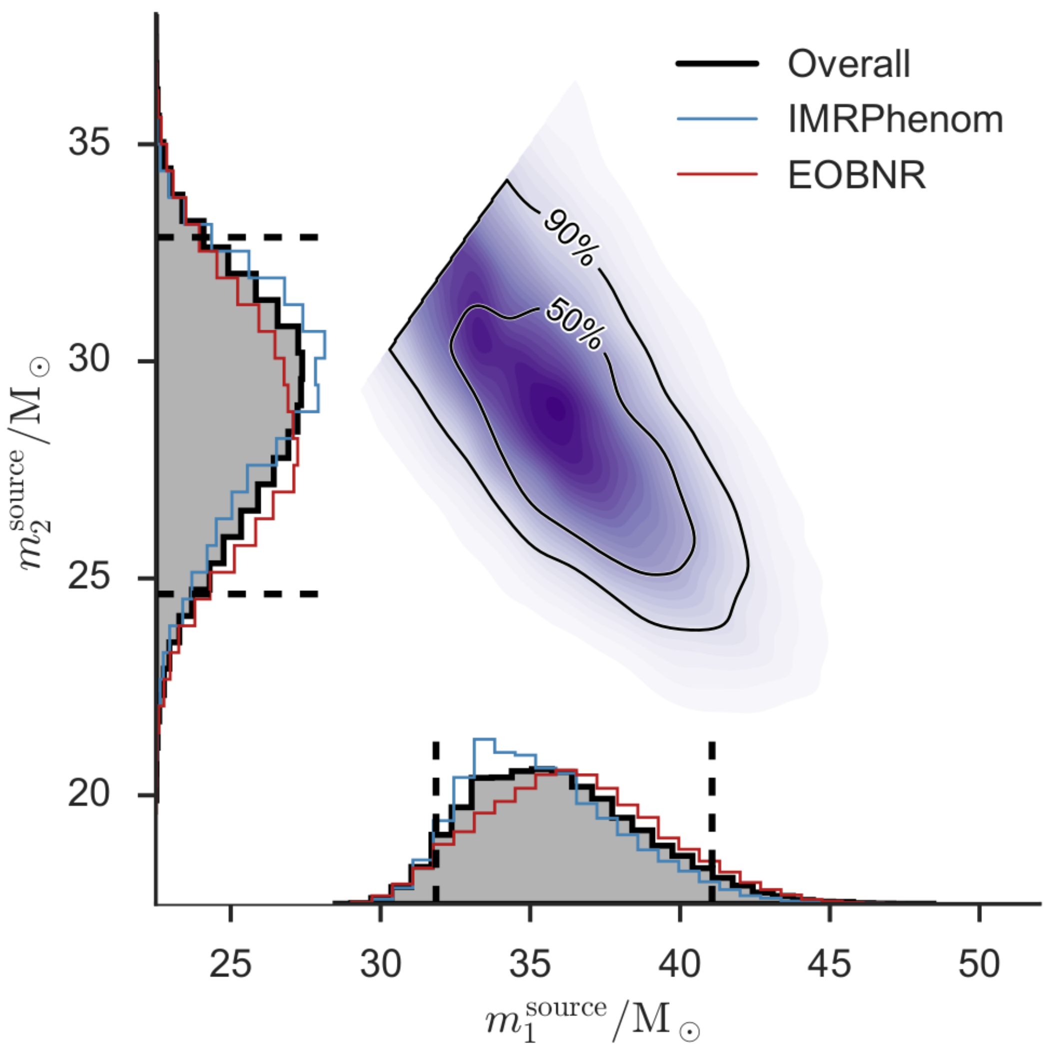
- Marginalization requires computing high-dimensional integrals

$$p(\theta_1|\text{data}) = \int p(\{\theta_k\}|\text{data})\, d\theta_2 \ldots d\theta_n$$

  - Use Monte-Carlo! (stochastic sampling)
  - Samplers e.g., Markov Chain Monte Carlo

series of $\{\theta_k\}$ representative of posterior distribution

Beyond detection: Parameter estimation

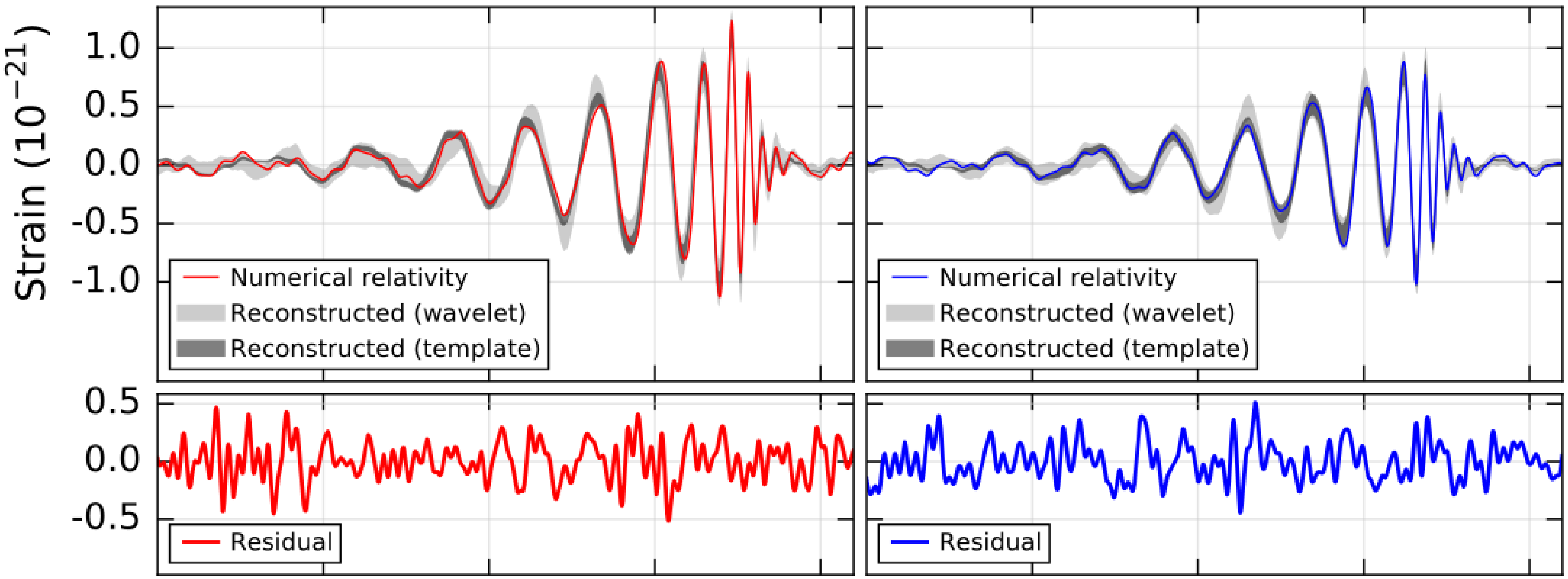


Uniform prior in mass

$m_{1,2} \in [10, 80] M_\odot$

with $m_1 > m_2$

Beyond detection: Waveform reconstruction



Estimate *t, f, duration* coordinates of **wavelets that fit the observations coherently in both detectors**

Very good agreement with best-fit search template and prediction from numerical relativity – **No excess detected in residual**

Observe amplitude decay consistent with damped oscillations of relaxing black hole – but low SNR → no reliable reconstruction

Conclusions

Gravitational-wave data analysis reached maturity

- More than **20 years of research**
- Succeed in reliably detecting GW150914 and GW151226!
- **Interdisciplinary endeavor**: Physics (from signal modelling to detector physics) ; Maths, stats, signal processing ; computer science

Beginning of a long story!

- More/other sources to discover (NS!)

Opens new challenges!

- **Digging deeper**: tractable, fully coherent all-sky searches for binary coalescences
- Cover a **larger signal space**: long (> 10 s) transients
- **Rapid event reconstruction** for multimessenger astronomy: fast Bayesian inference
- **Better noise/glitch rejection**: deep learning?
- **Beyond gravitational-wave physics**
  - Cosmology with BBH as standard candles?