CNRS/MASTODONS Interdisciplinary workshop on Time Series Analysis

Astrophysics-Photon

Dimitrios Emmanoulopoulos

17th June 2016, Campus Paris Diderot, Paris, France

UNIVERSITY OF Southampton

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Astrophysical time series and photon counting statistics: Problems/Progress

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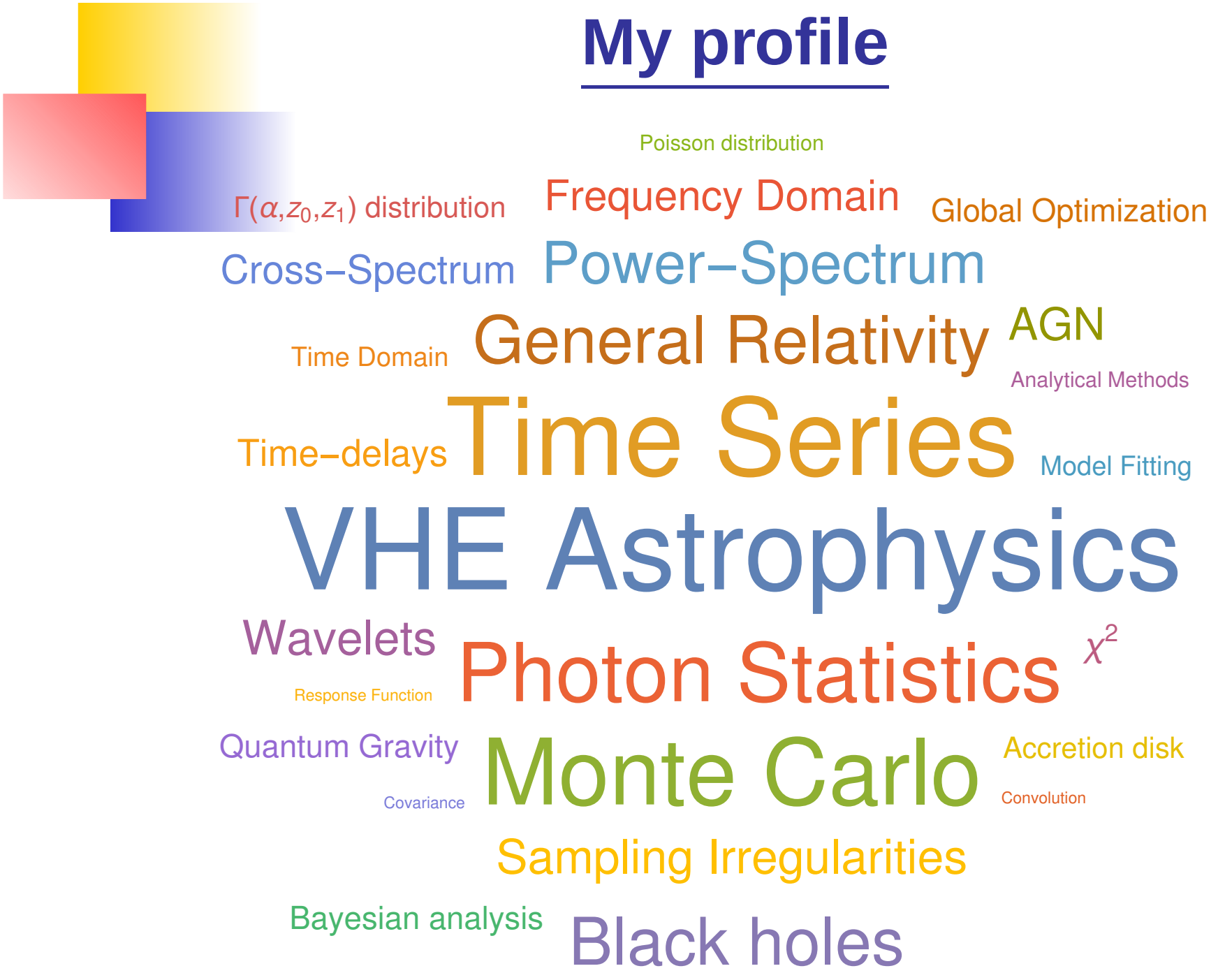
My profile


Poisson distribution
$\Gamma(\alpha,z_0,z_1)$ distribution
Frequency Domain
Global Optimization
Cross–Spectrum
Power–Spectrum
Time Domain
General Relativity
AGN
Analytical Methods
Time–delays
Time Series
Model Fitting
VHE Astrophysics
Wavelets
Photon Statistics
χ^2
Response Function
Quantum Gravity
Monte Carlo
Accretion disk
Covariance
Convolution
Sampling Irregularities
Bayesian analysis
Black holes

Outline

- What is a Black Hole?
- Problems in astrophysical time series.
- Development of new methods.
- Extraction of astrophysical information.

Time Series (in general)

- The only model-independent tool that provides information about a system.

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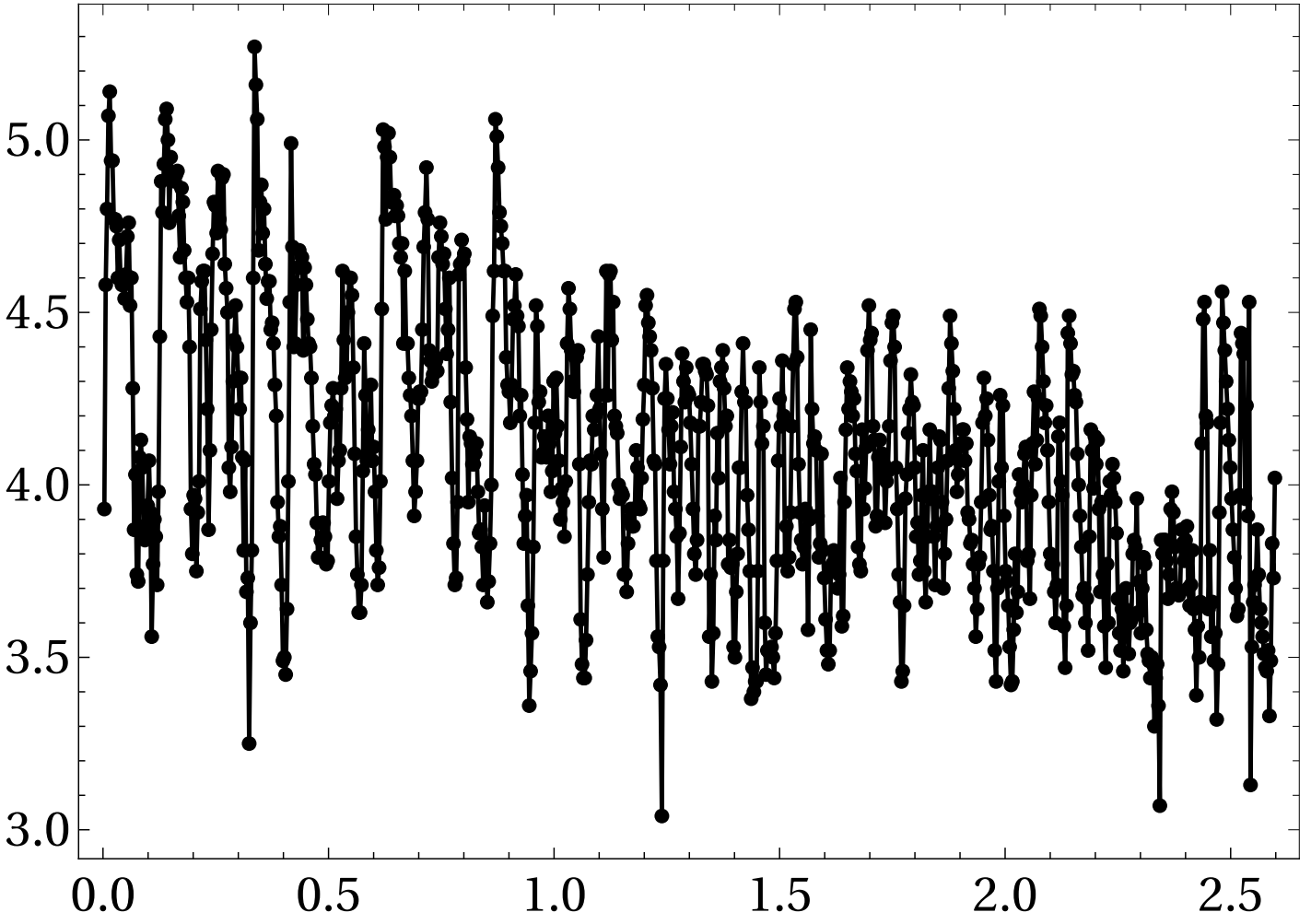
- Through modelling, this information is mapped into a system's property.

Time Series (in general)



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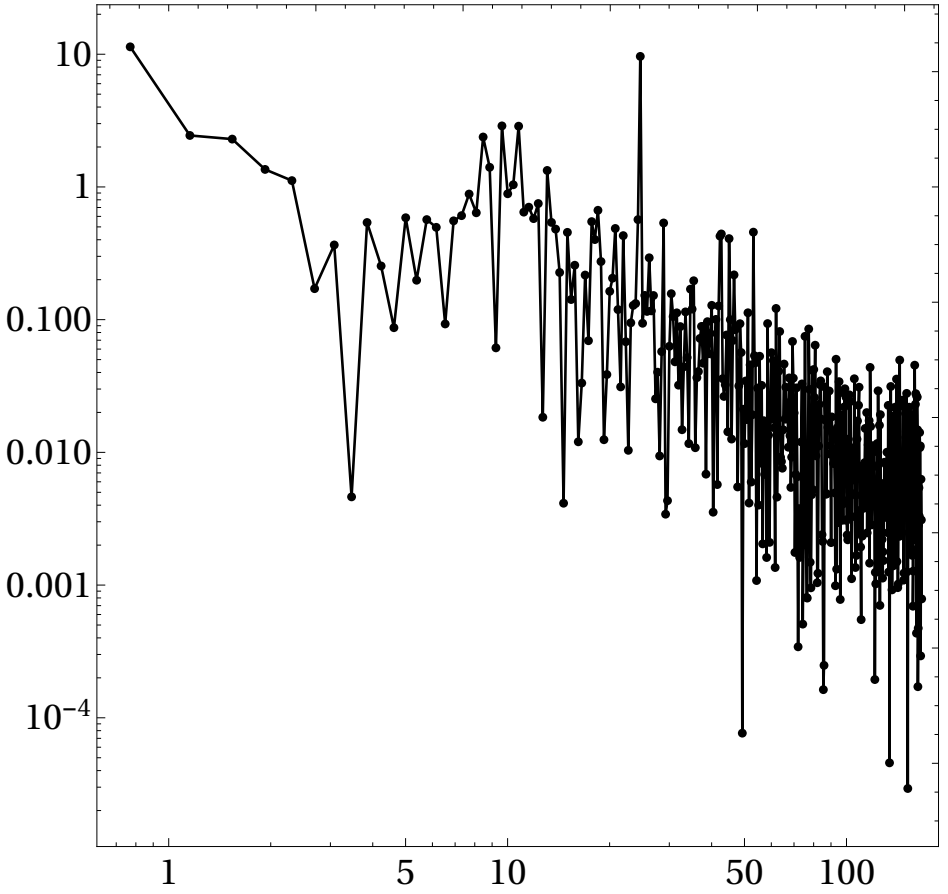

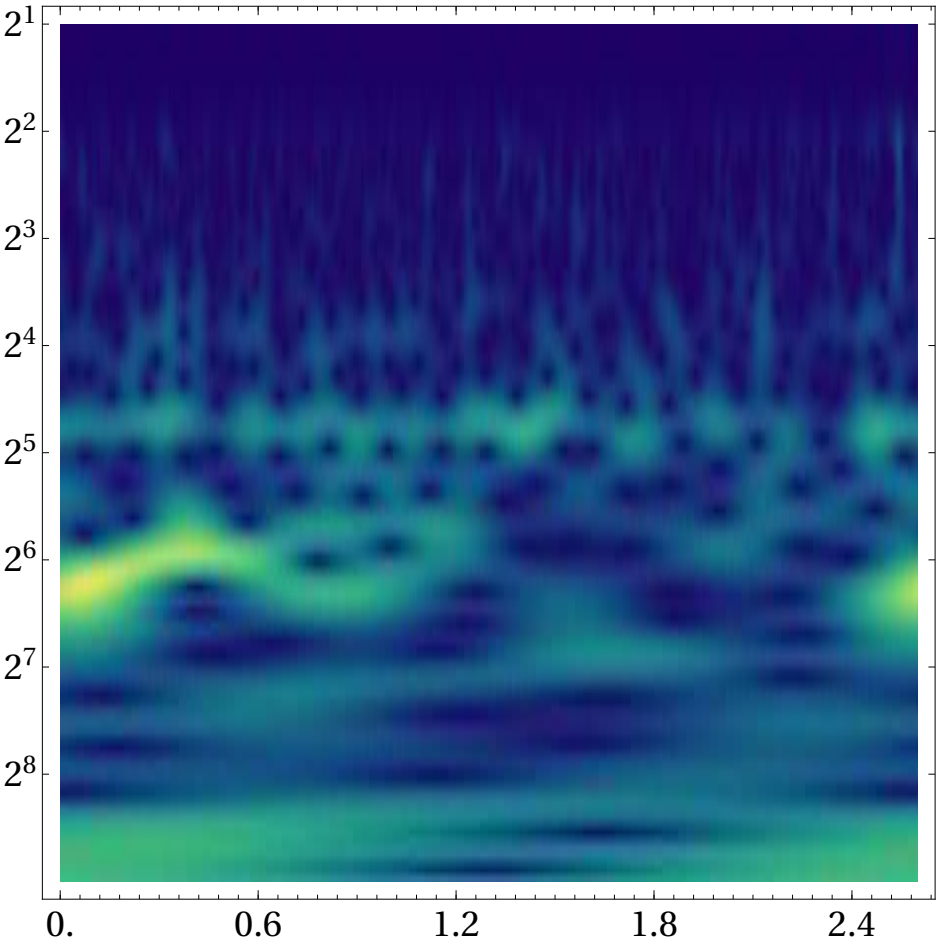

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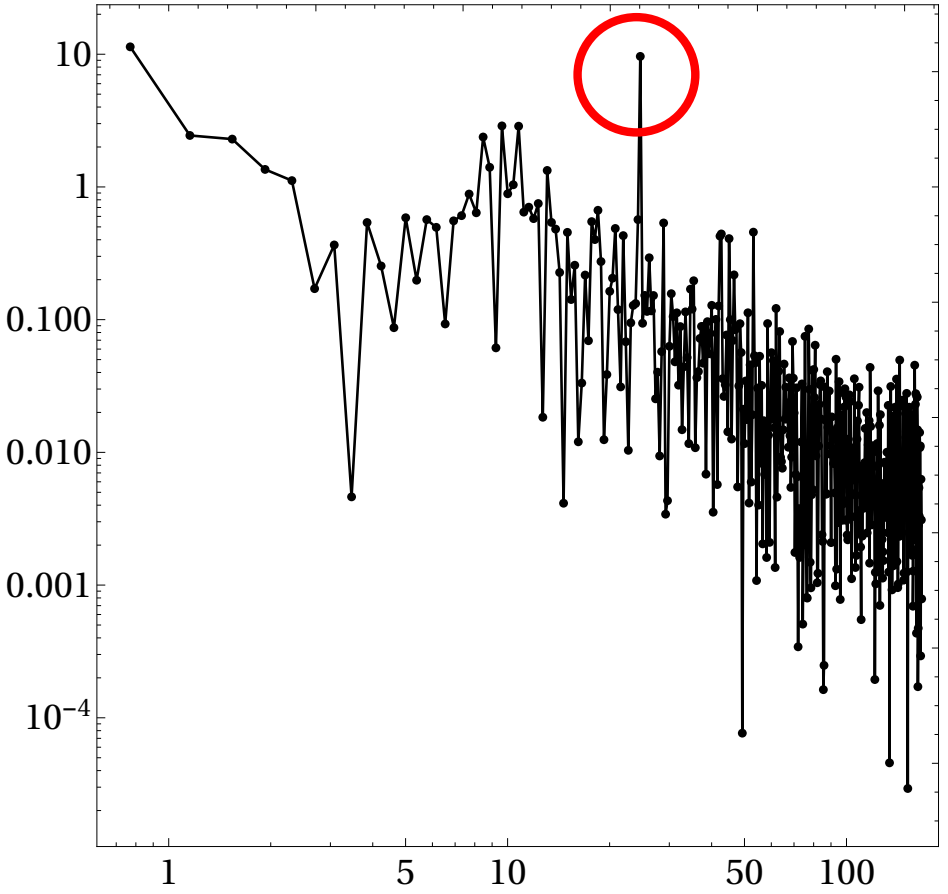

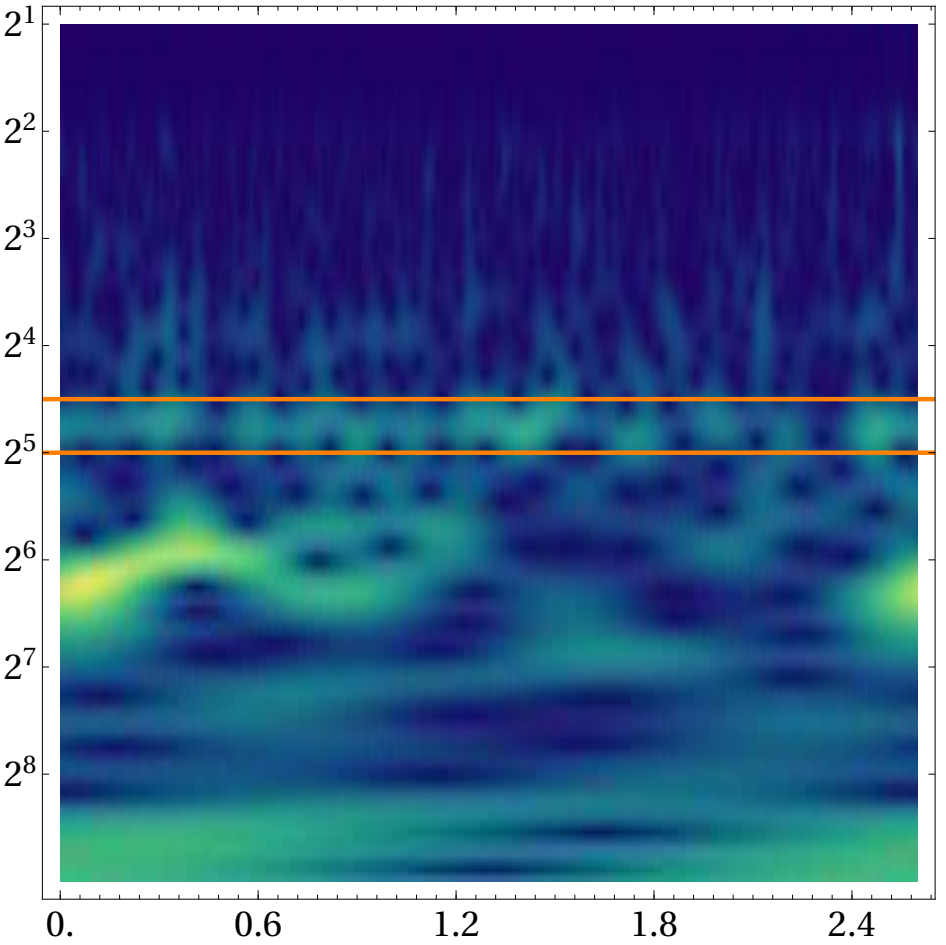

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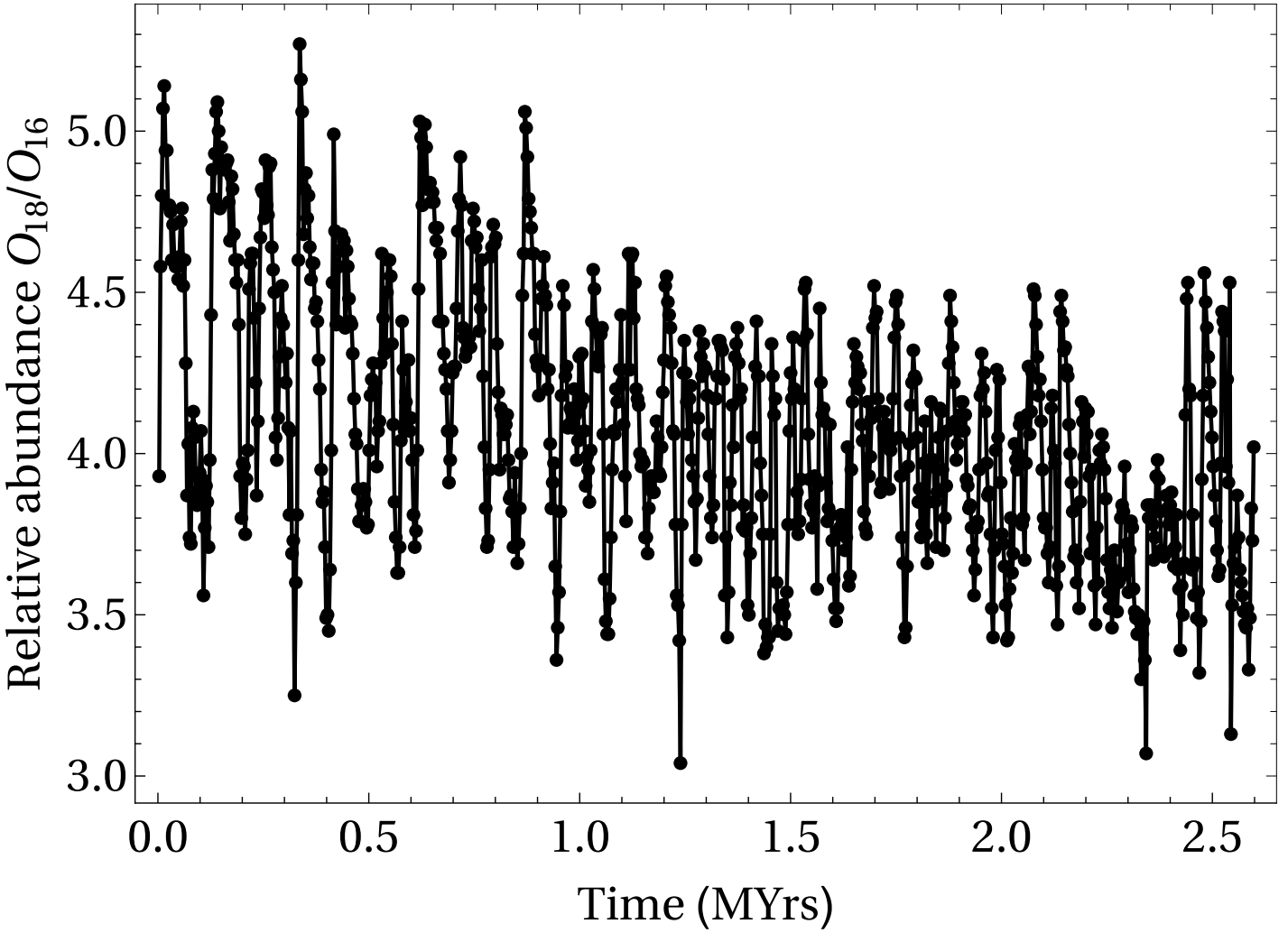

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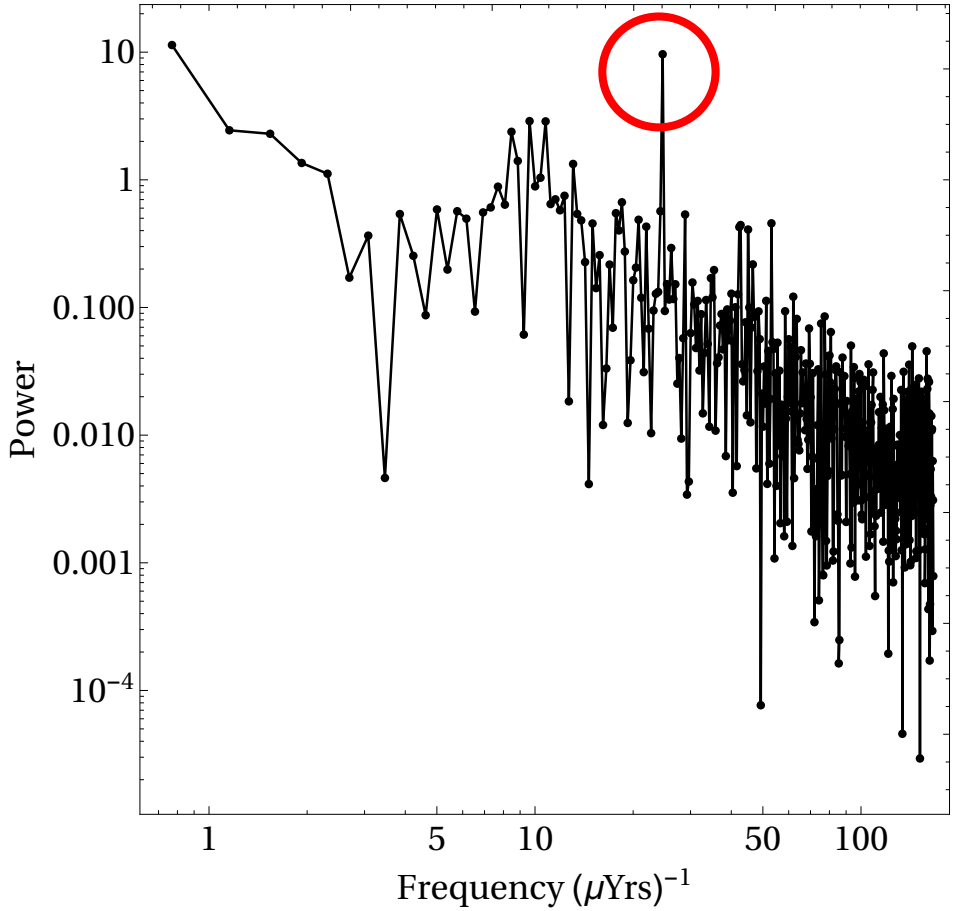

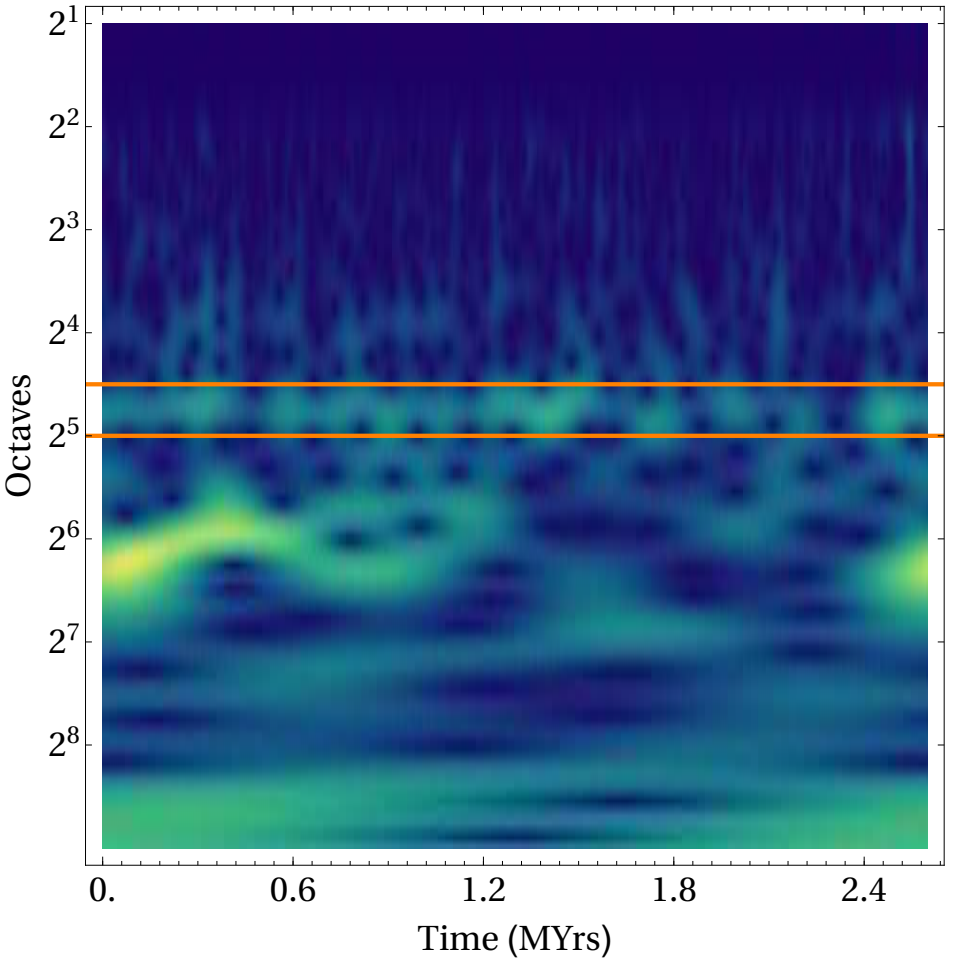

41 kYrs periodicity (Obliquity)

What is a Black Hole?

Luminosity$= (2 \times 10^{10}) \cdot (4 \times 10^{26}\ \mathrm{W}) = 8 \times 10^{36}$ W

What is a Black Hole?

Luminosity$= 10^{44} - 10^{47}$ W

What is a Black Hole?

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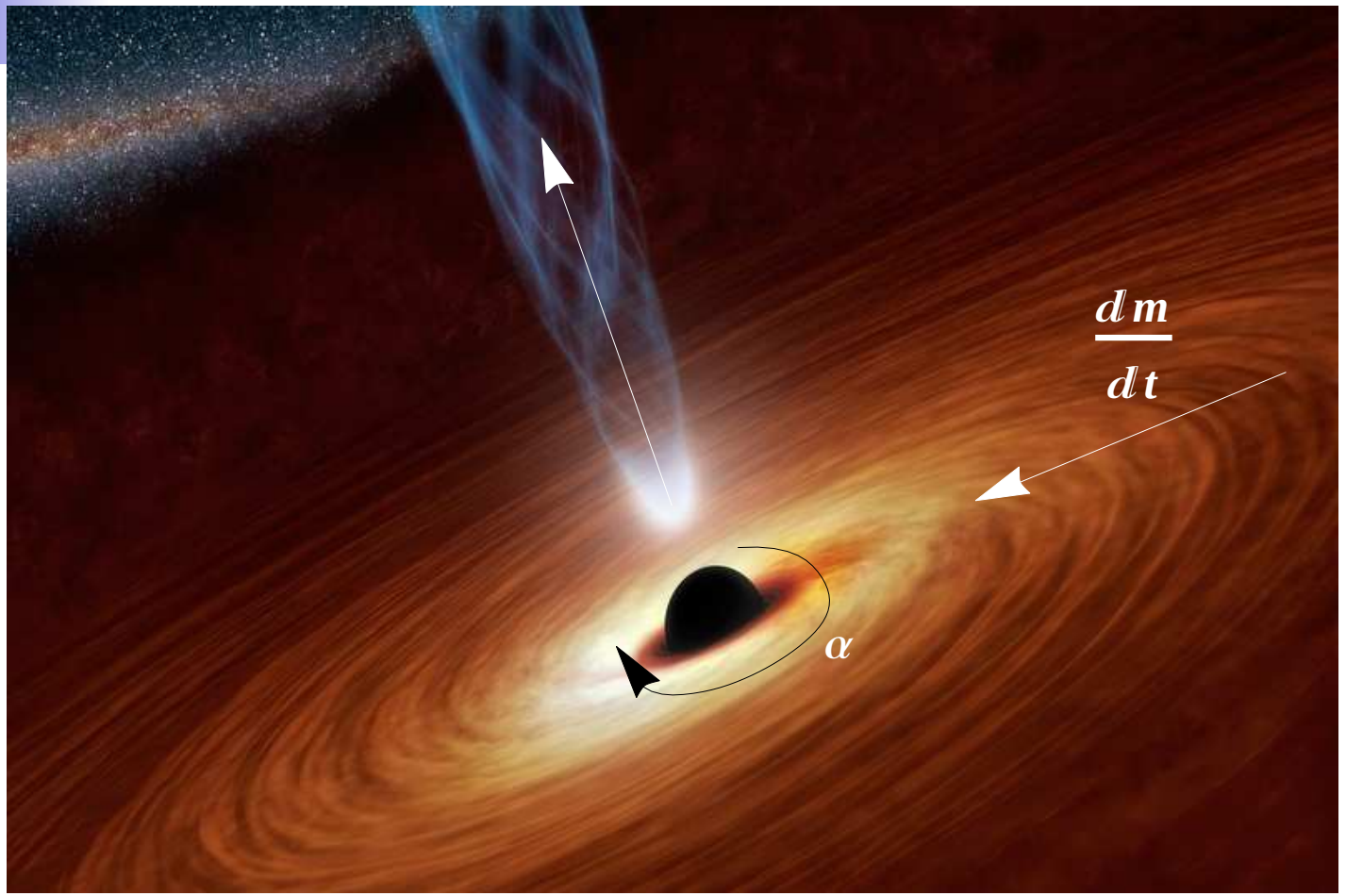
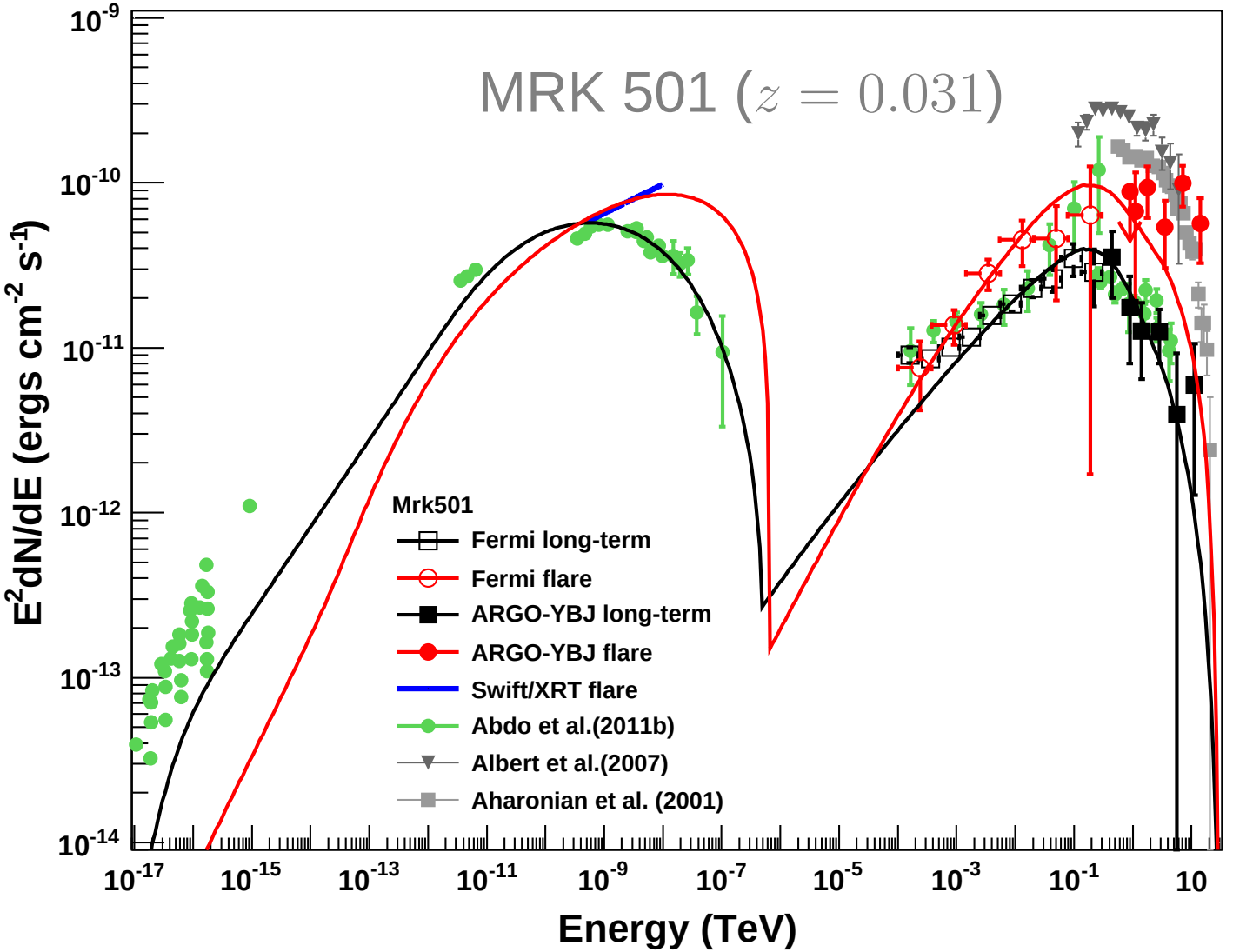


What is a Black Hole?

Radiation in the entire Elec.Magn. spectrum.



SongZhan Chen (2013), Science China Physics, Mechanics & Astronomy, **56**, 1

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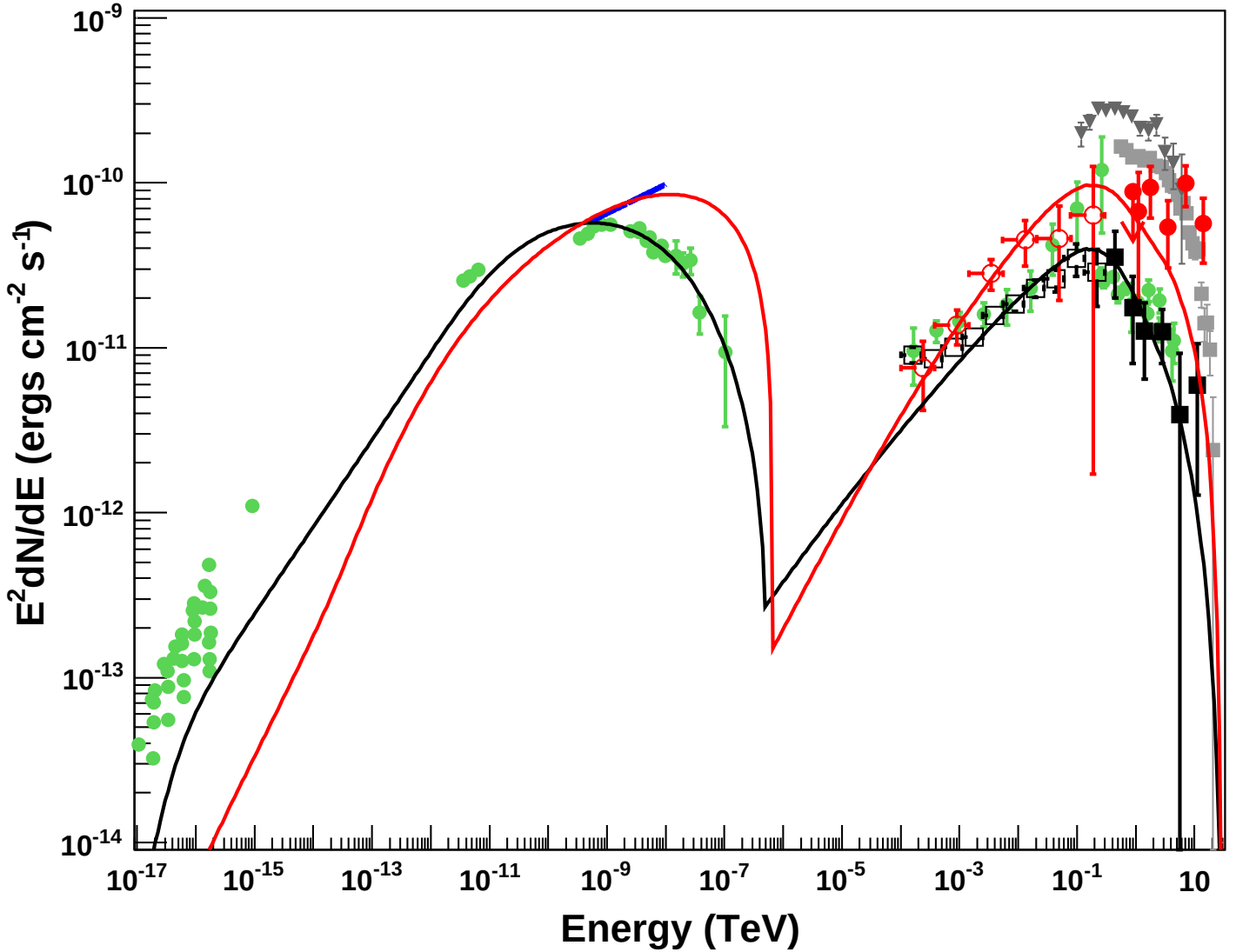
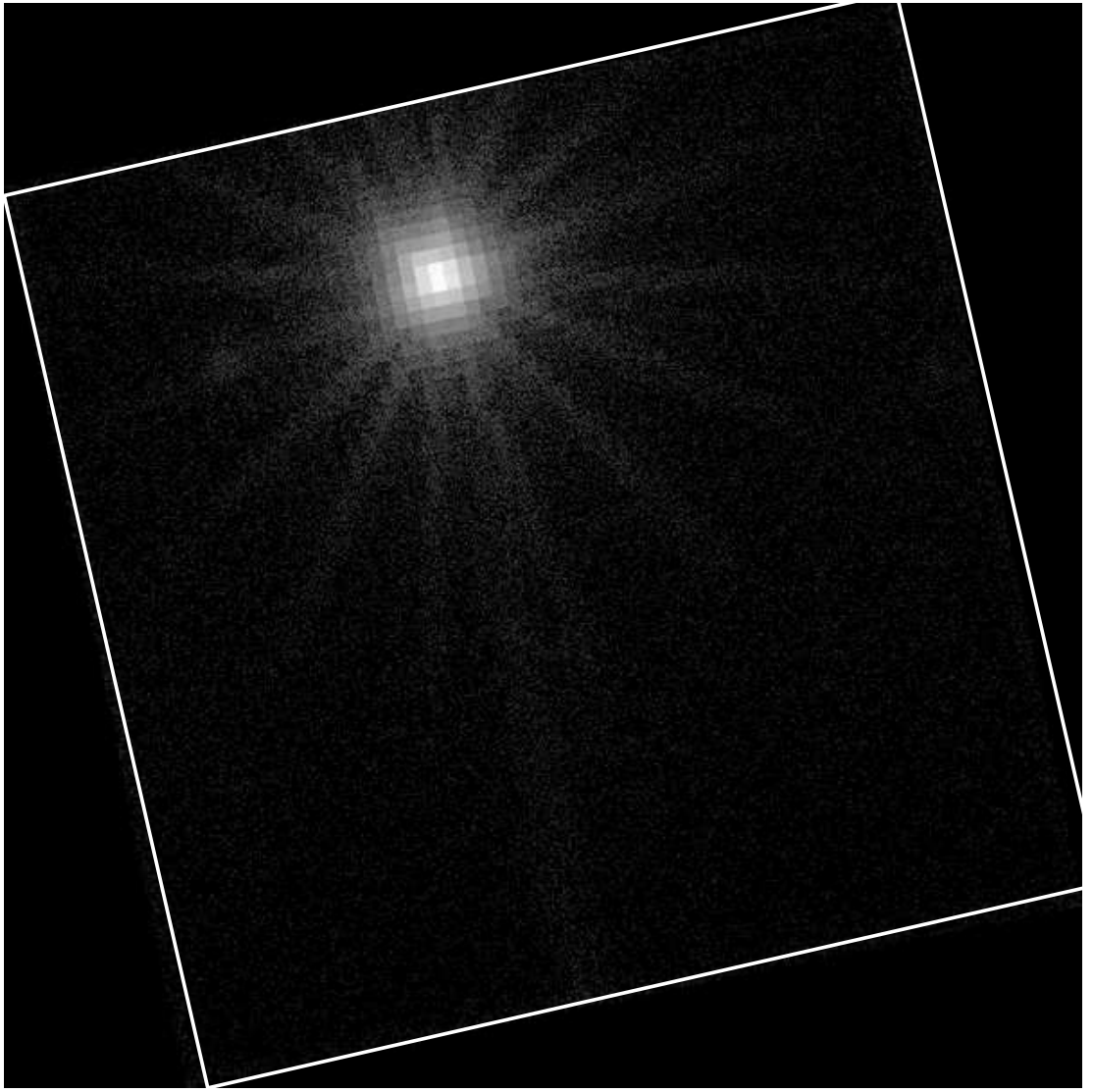


What do we really observe?



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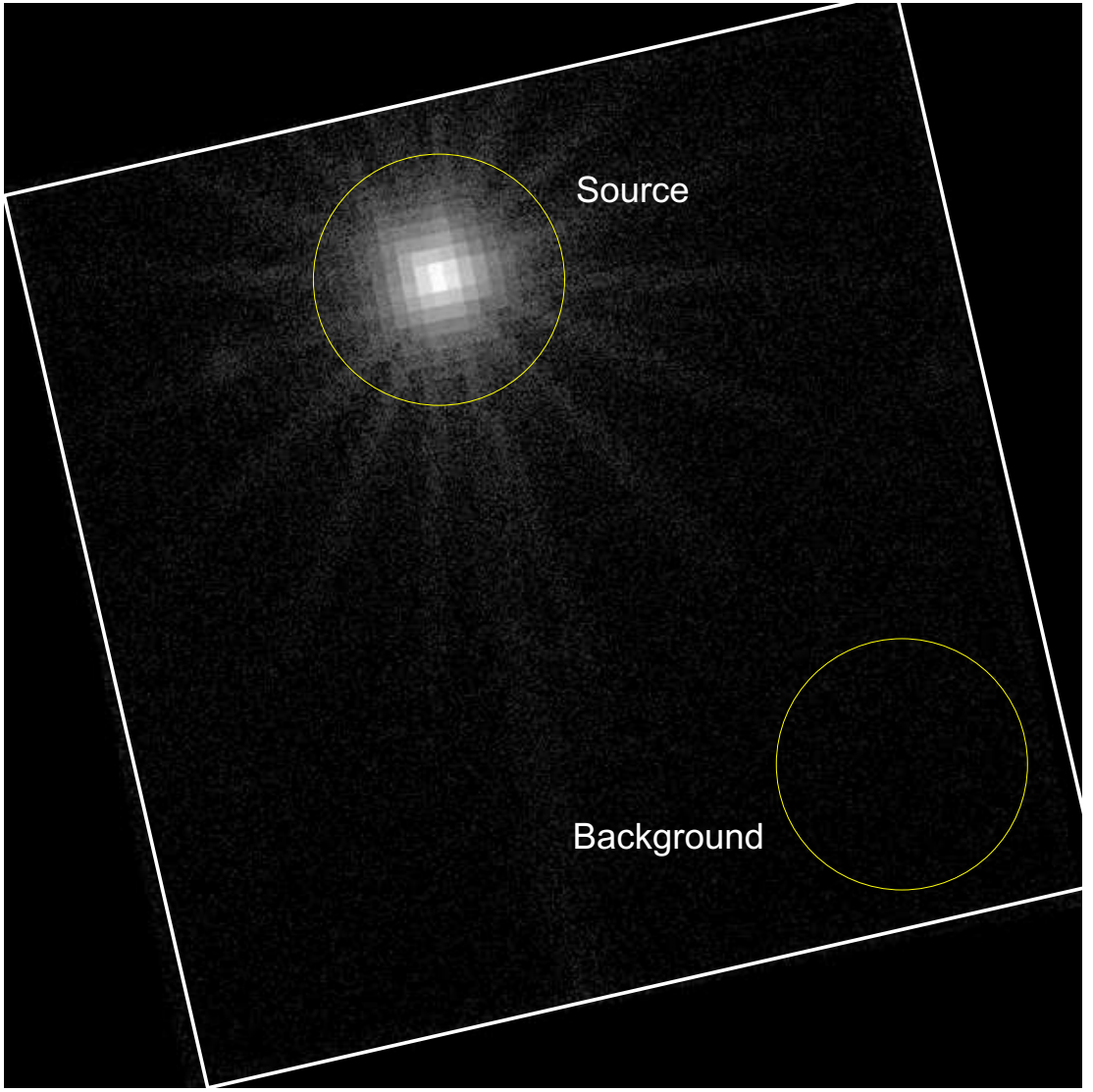
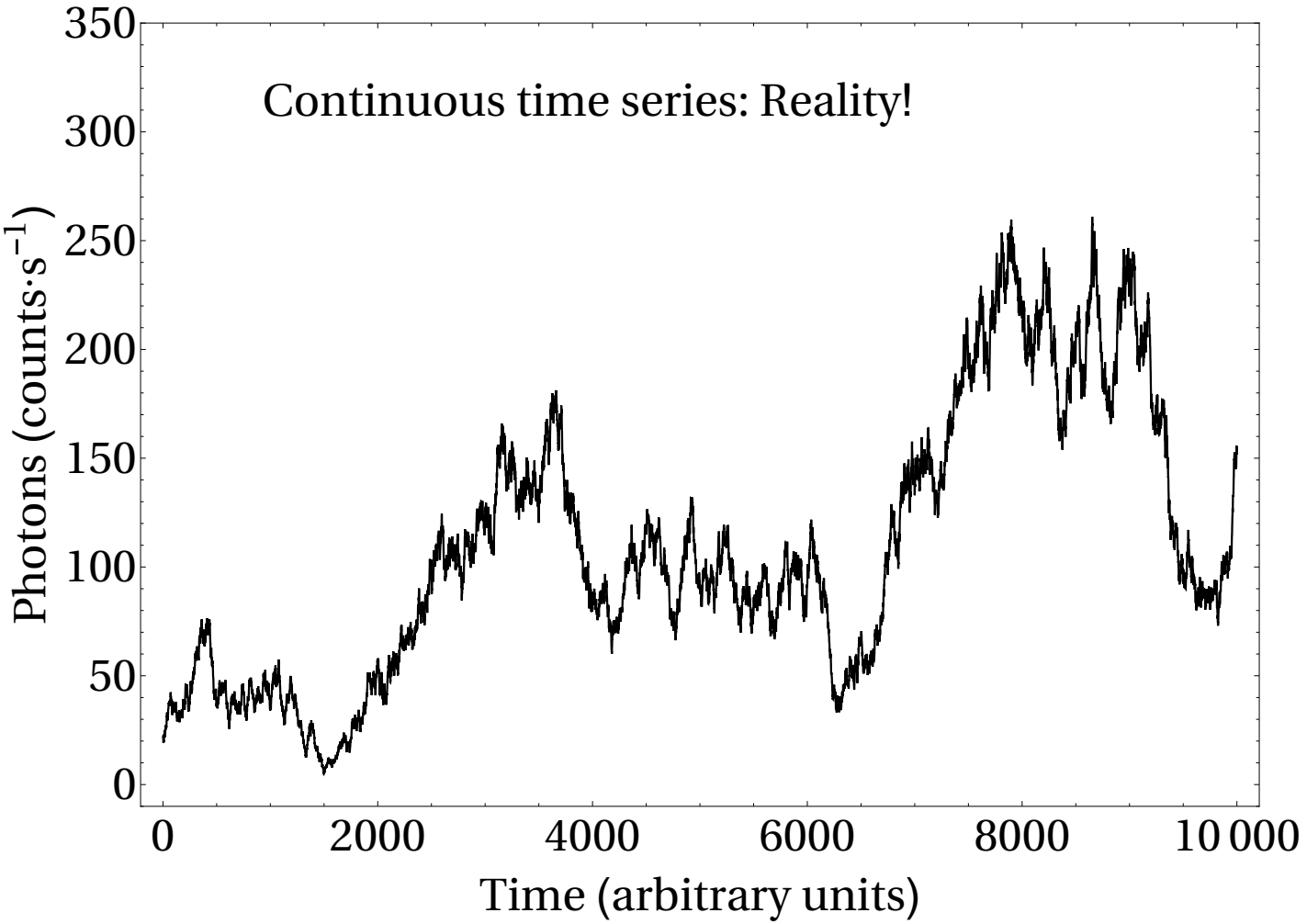


Data-set problems



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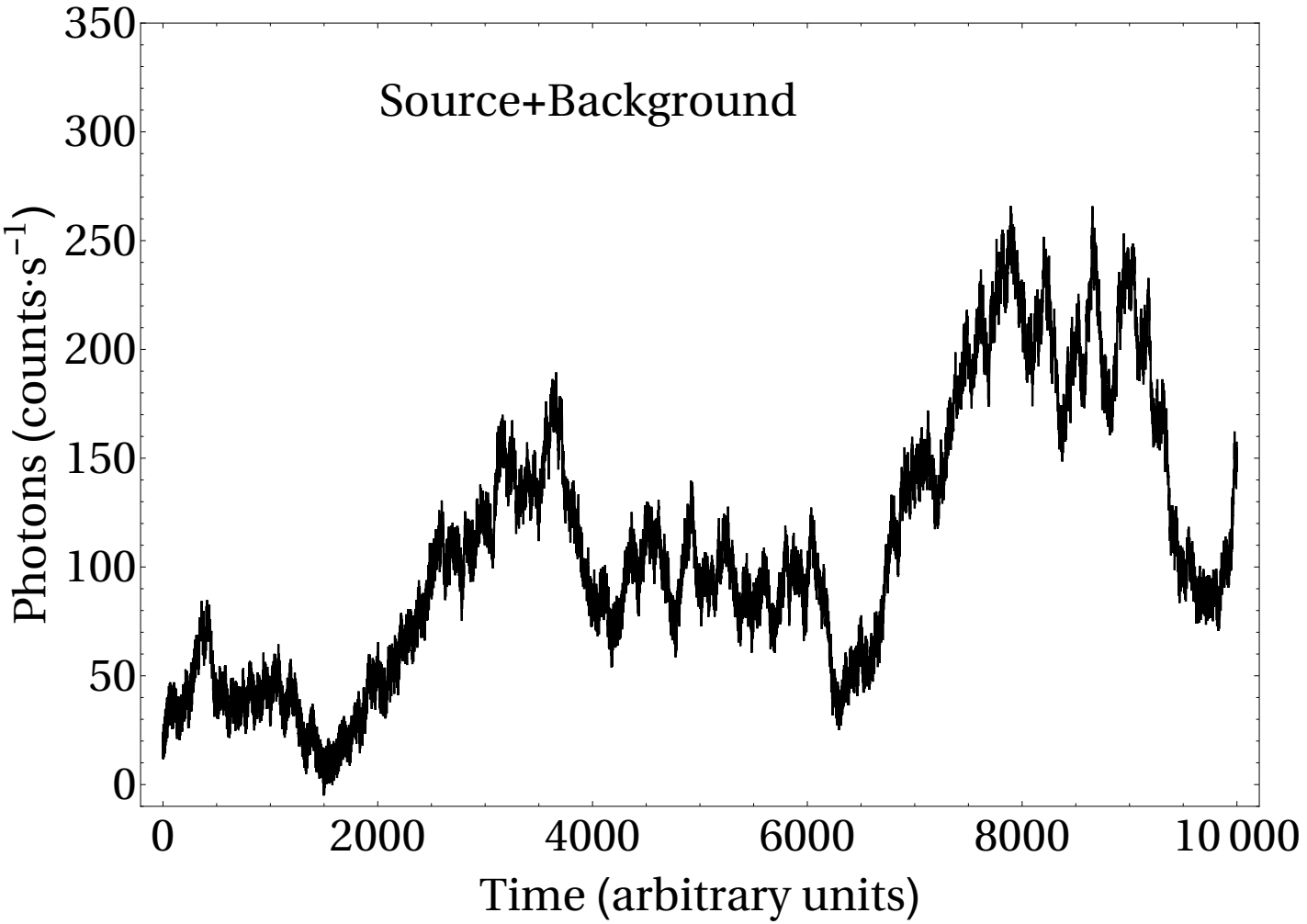

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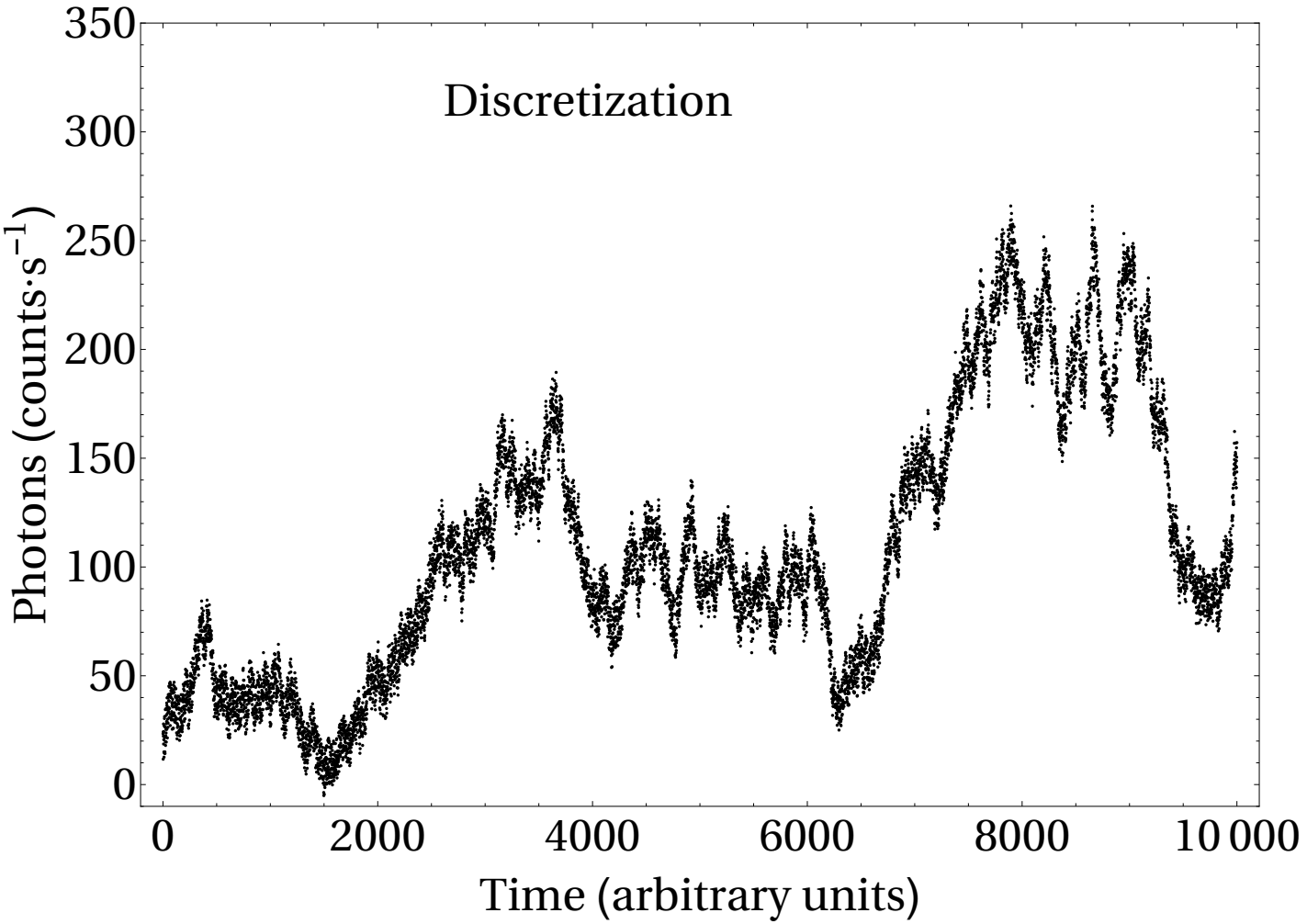

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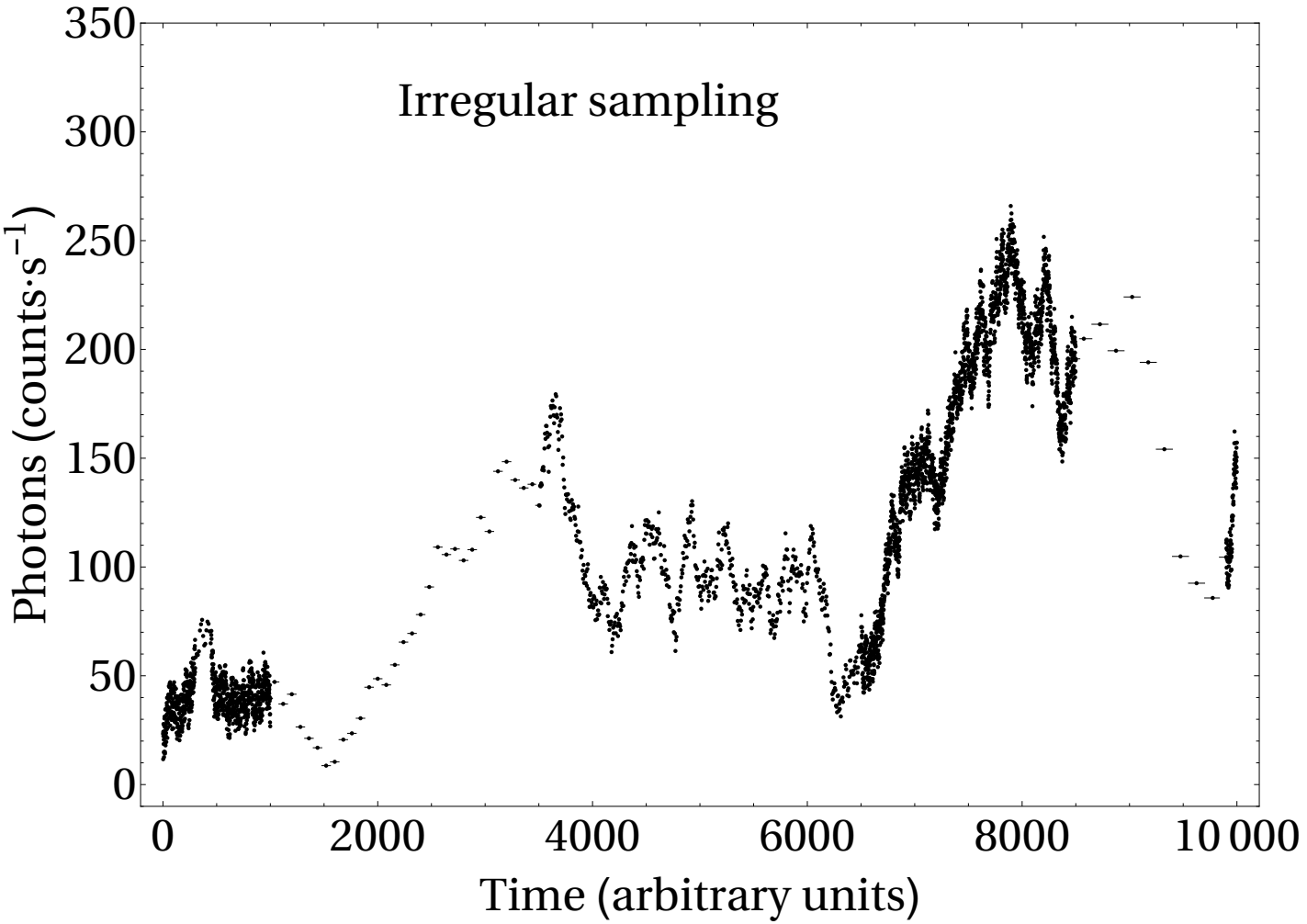

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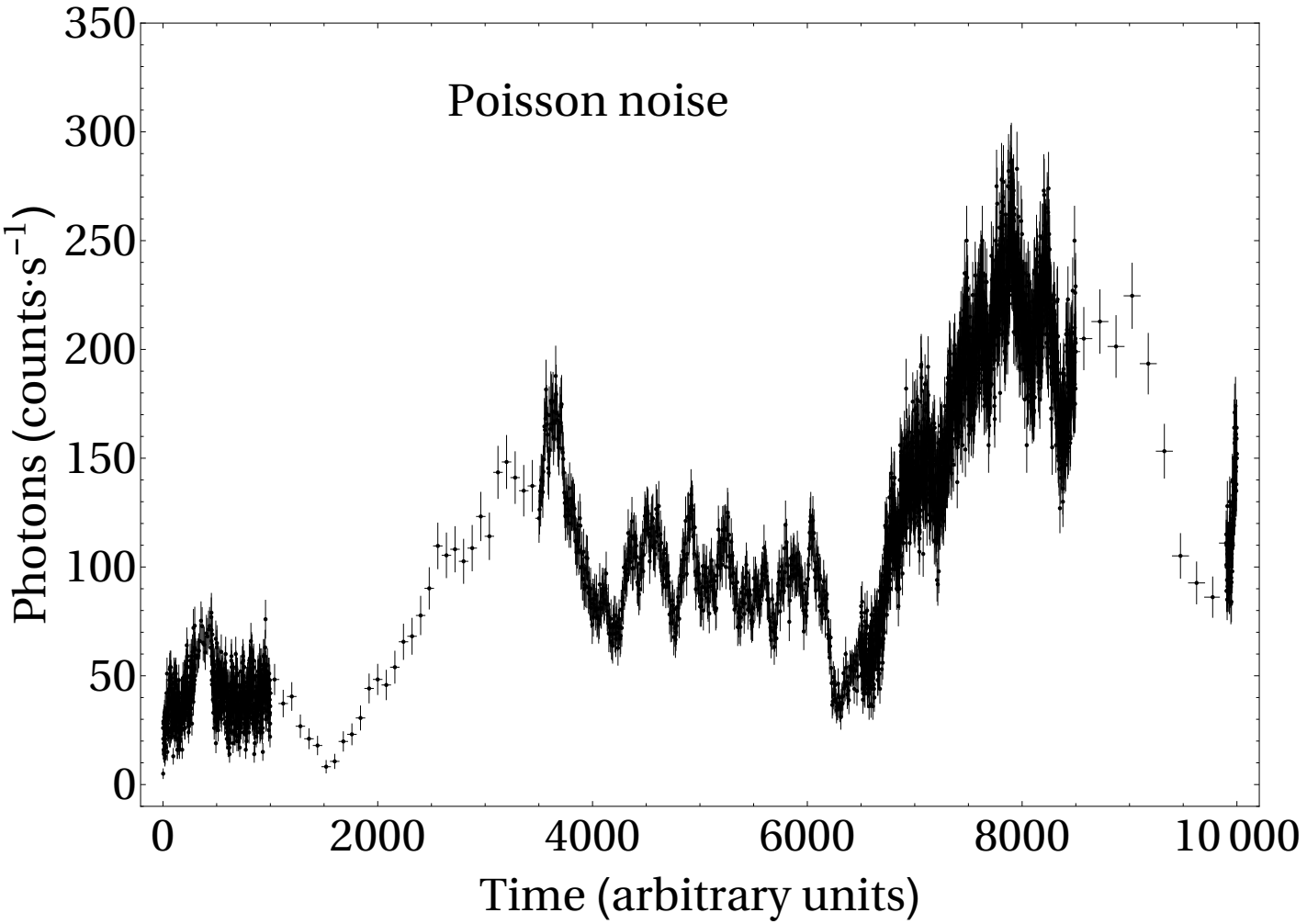

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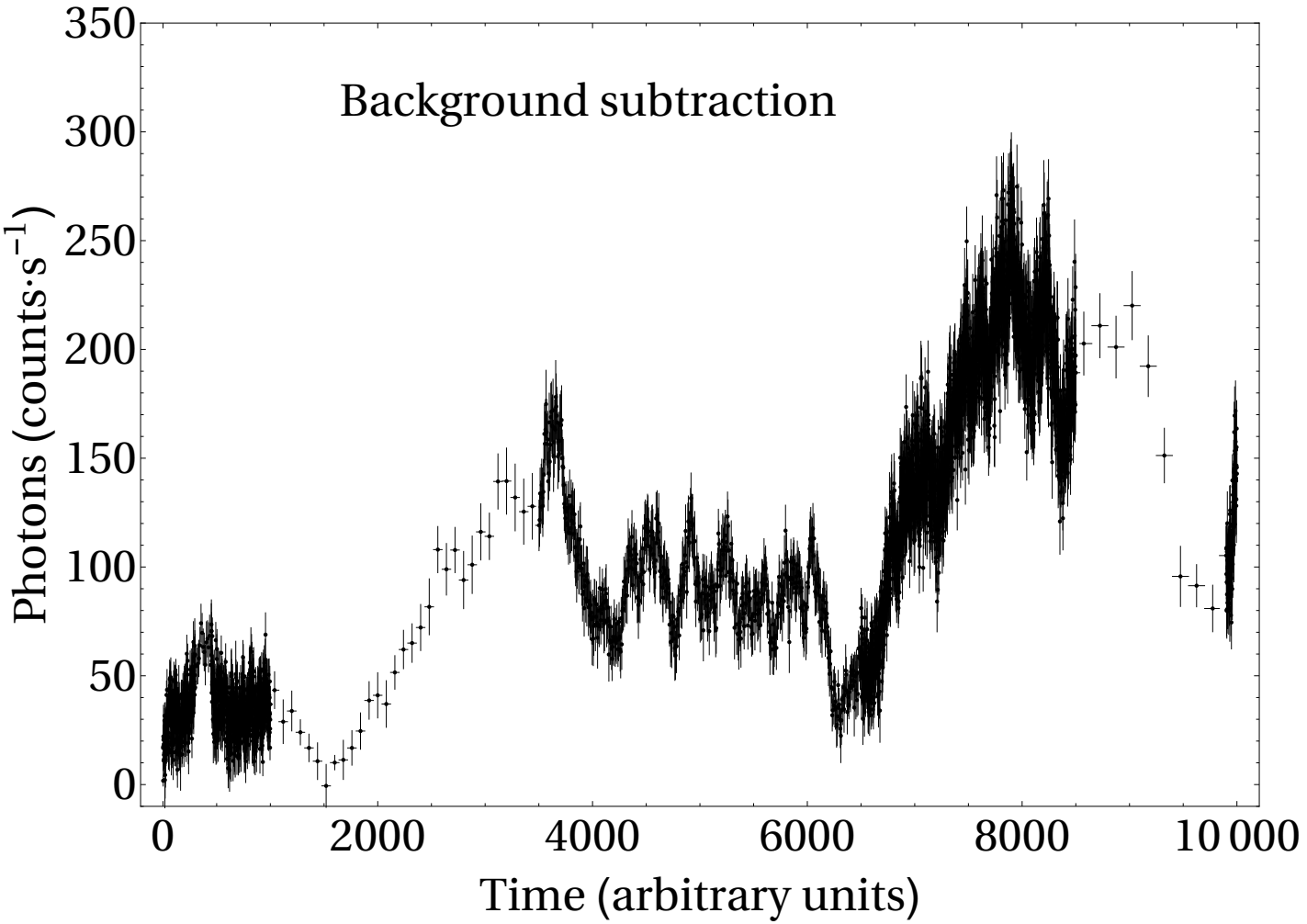

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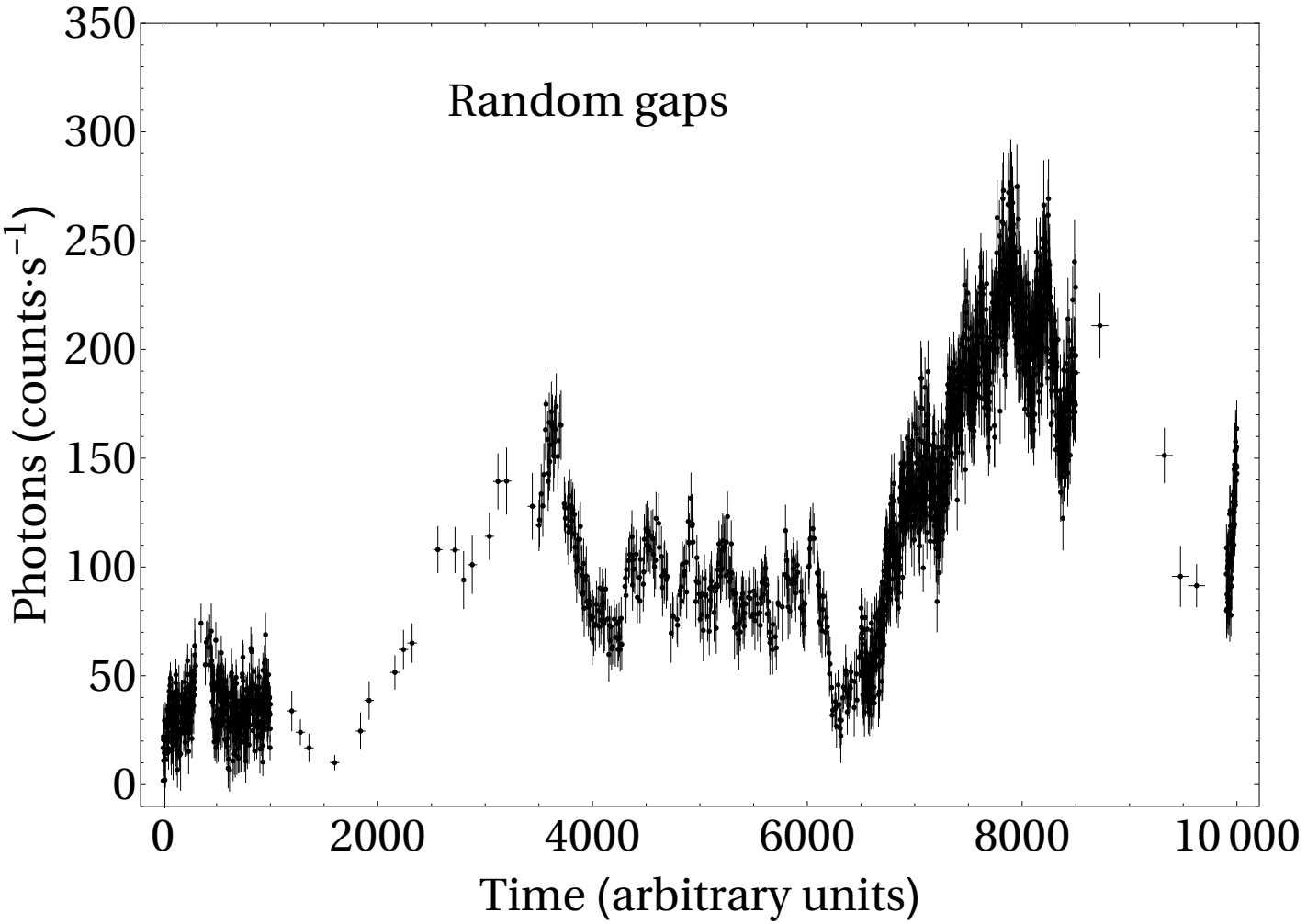

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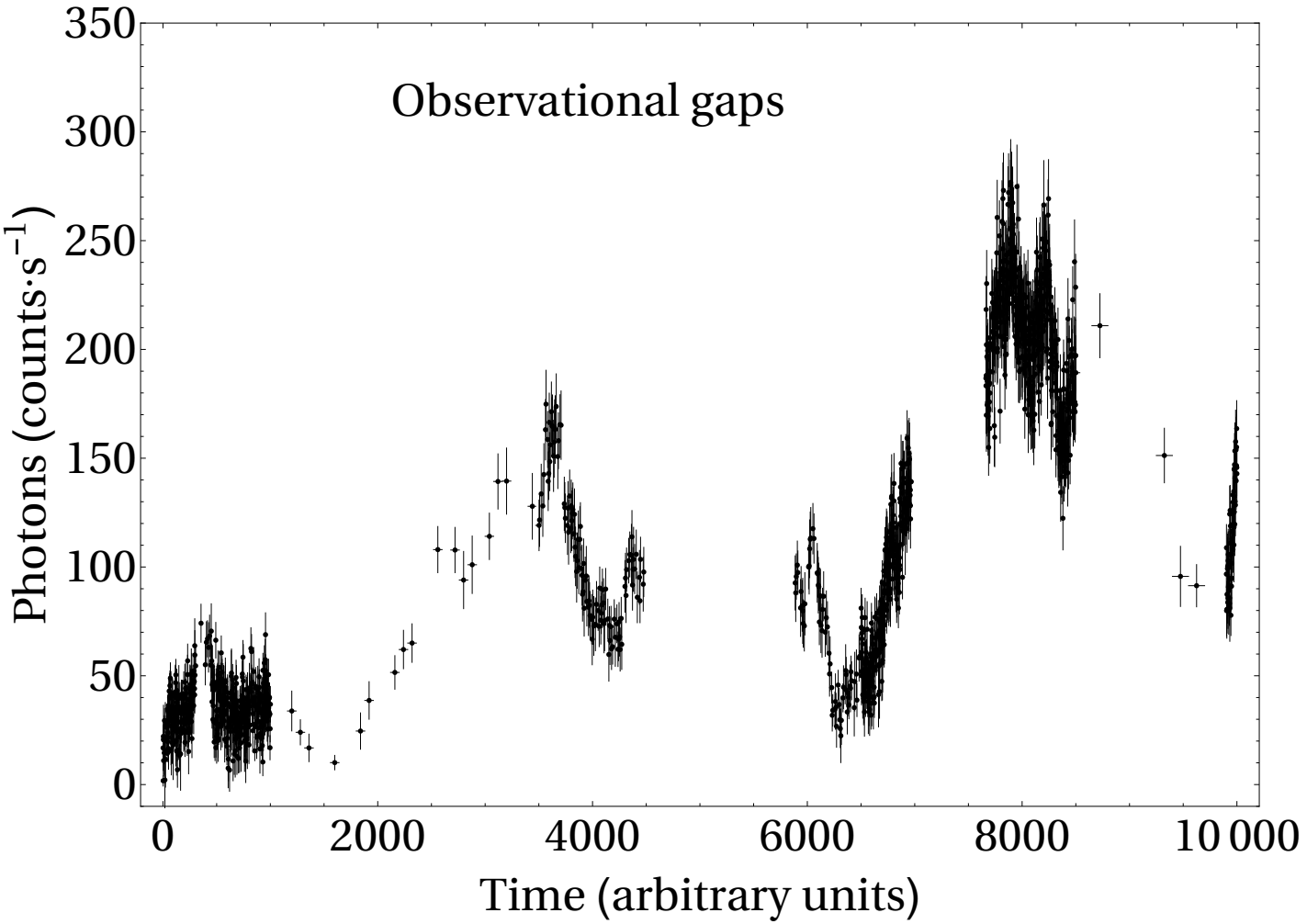

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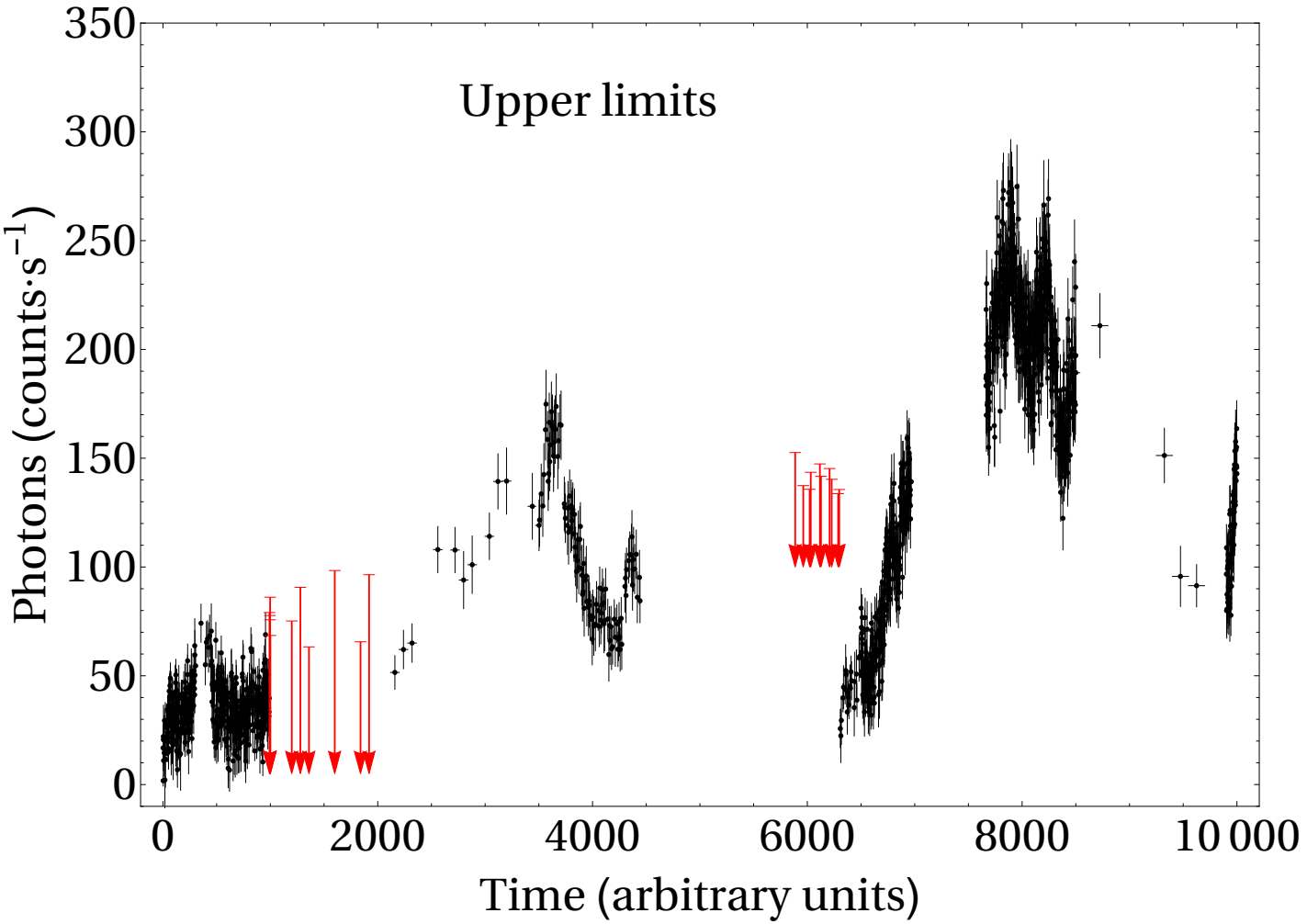

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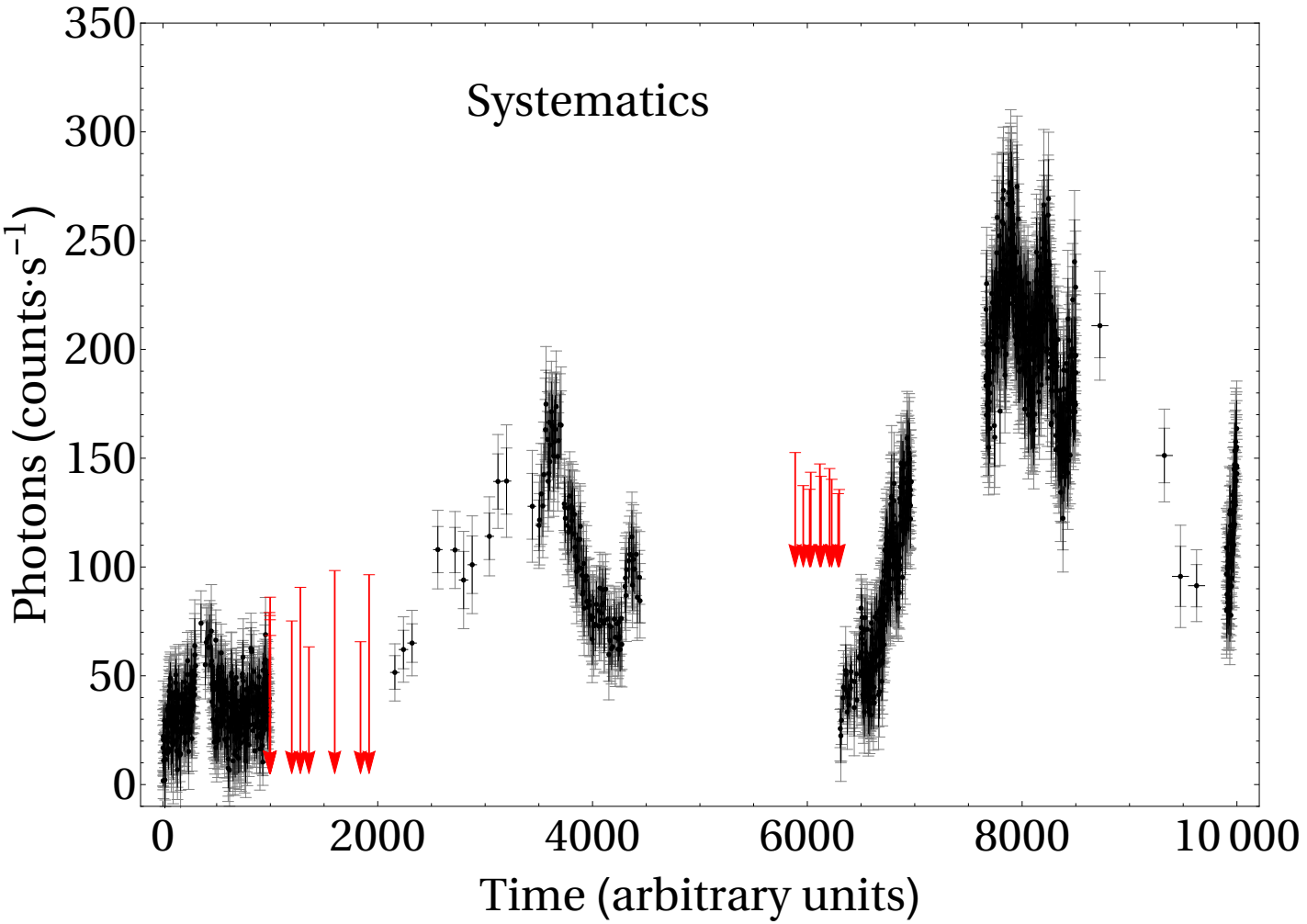

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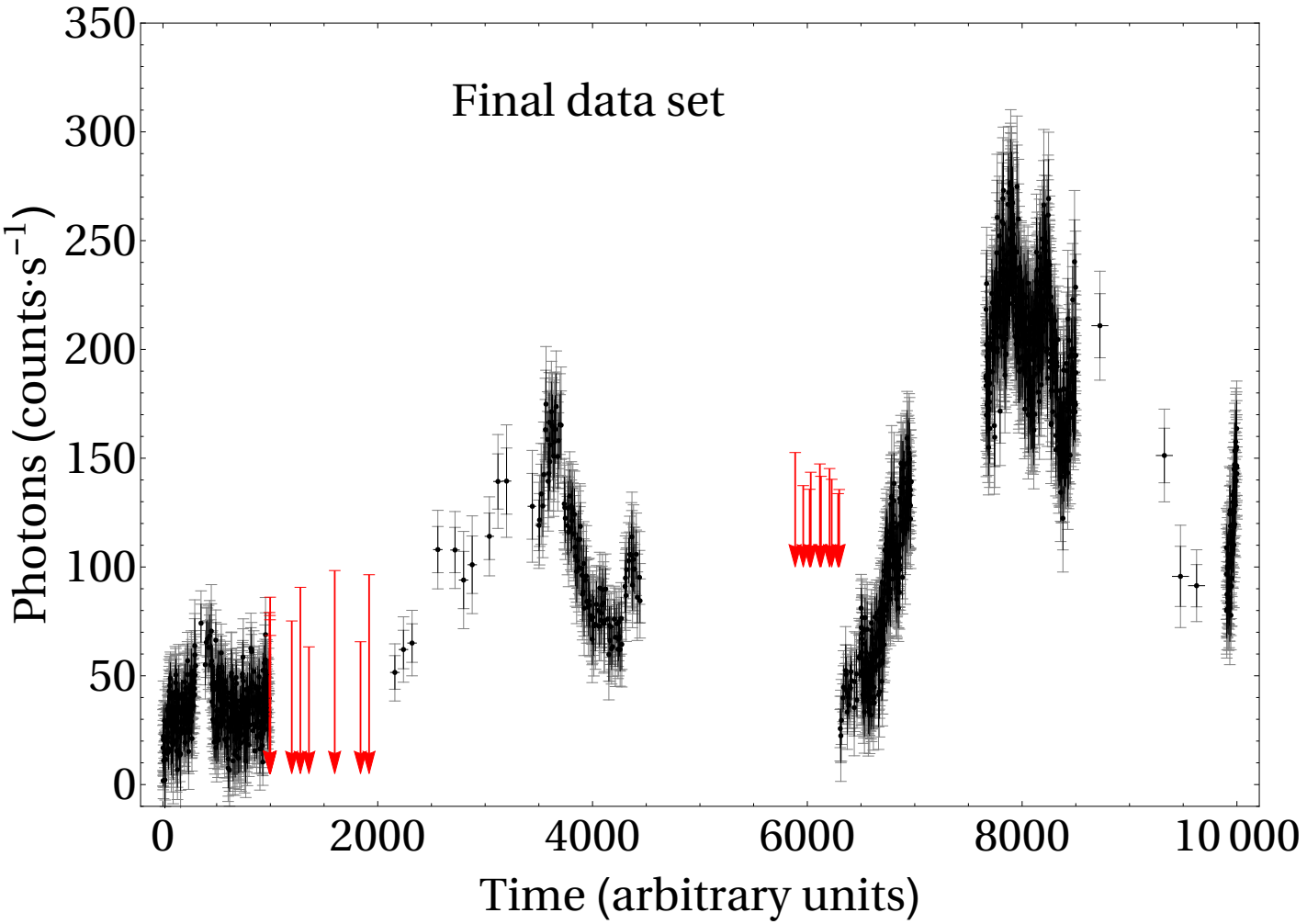

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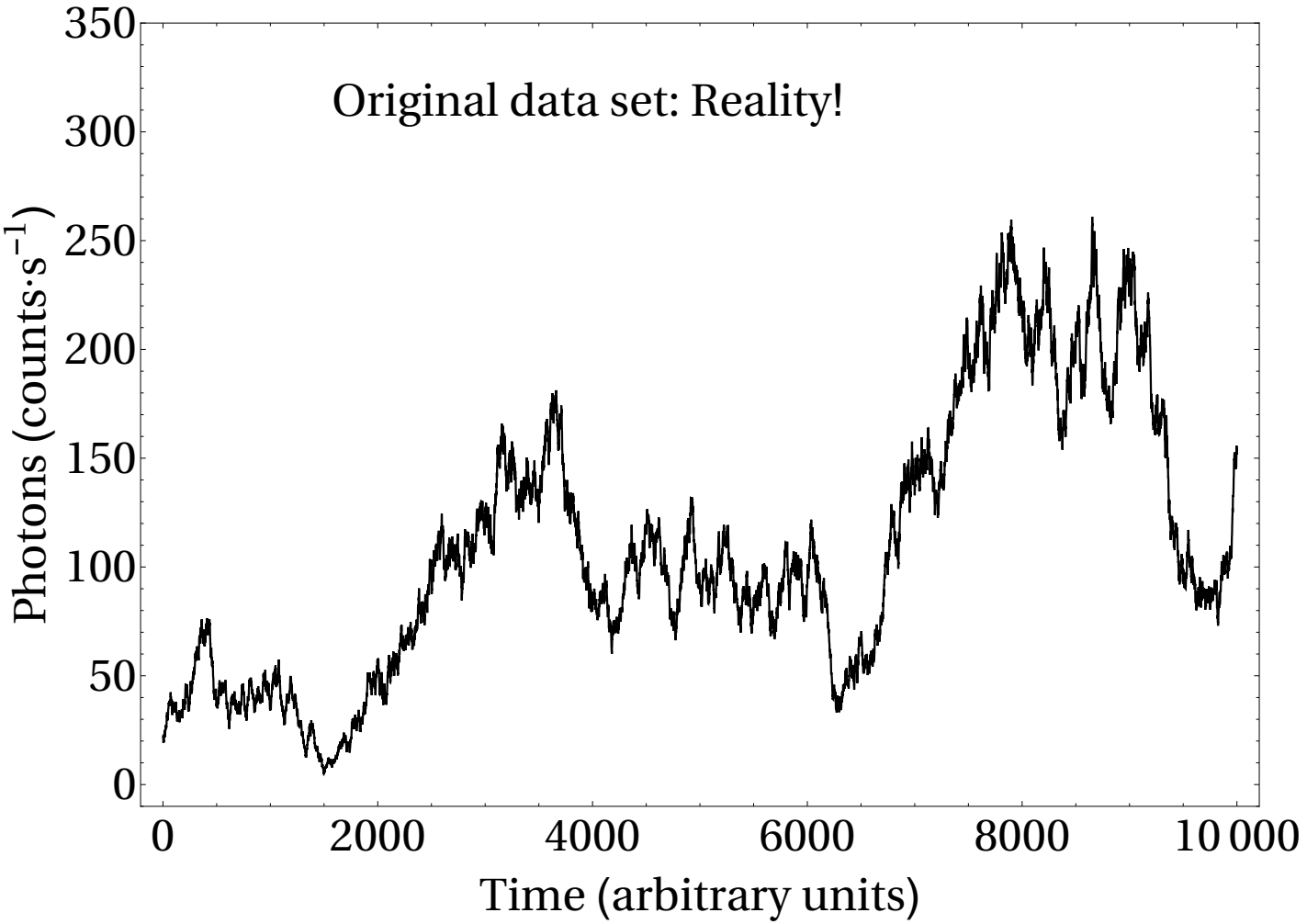

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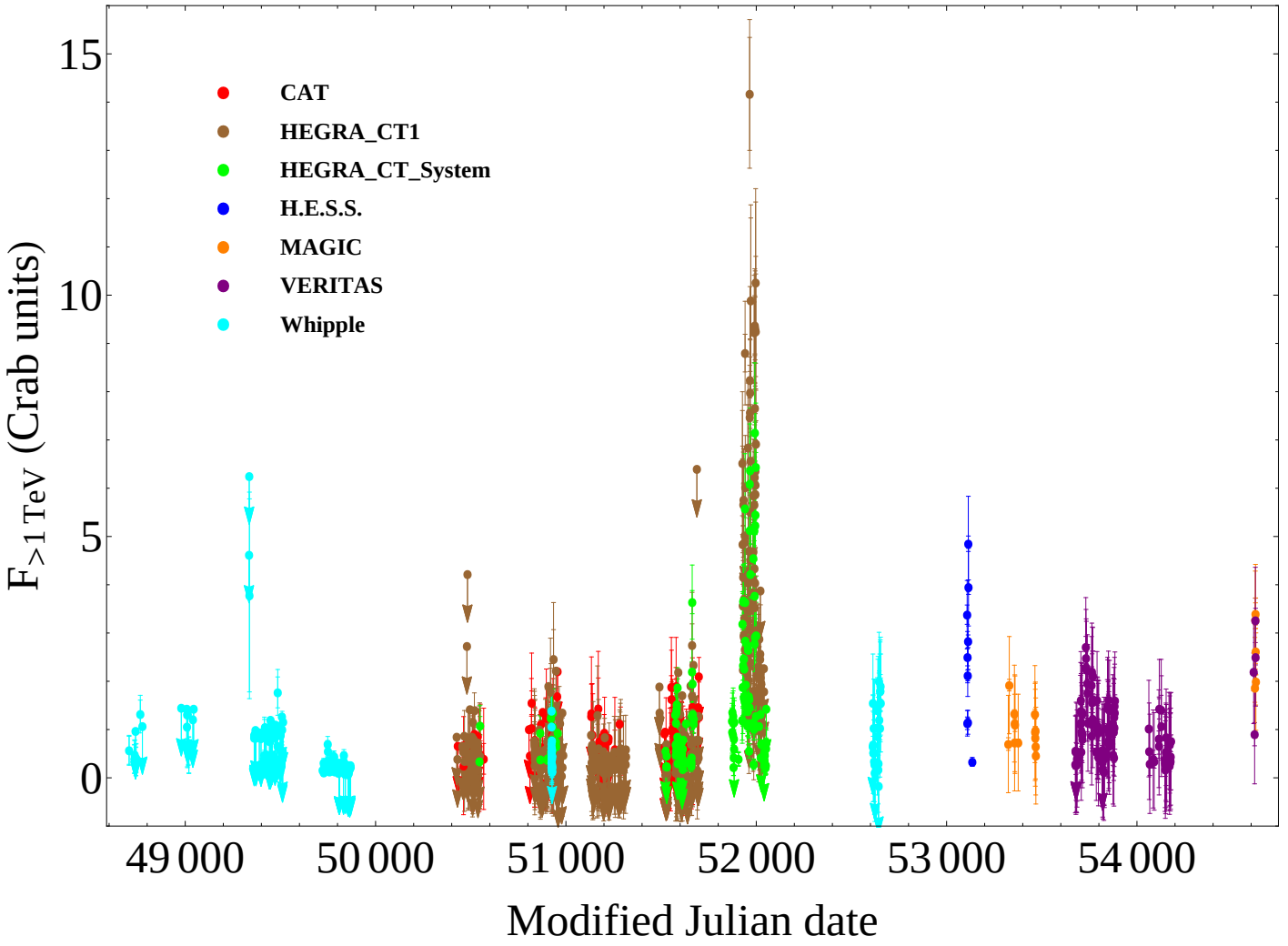

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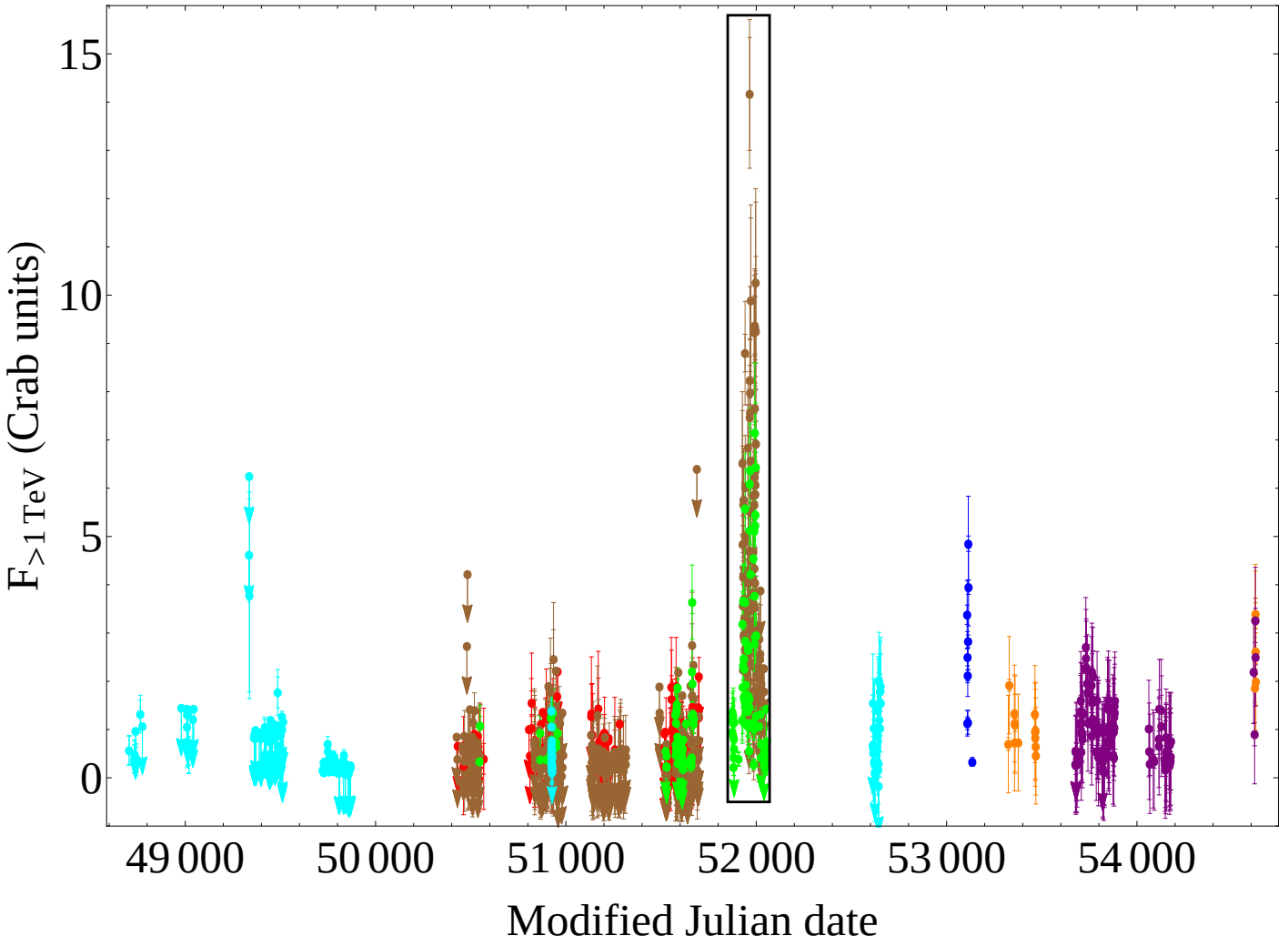

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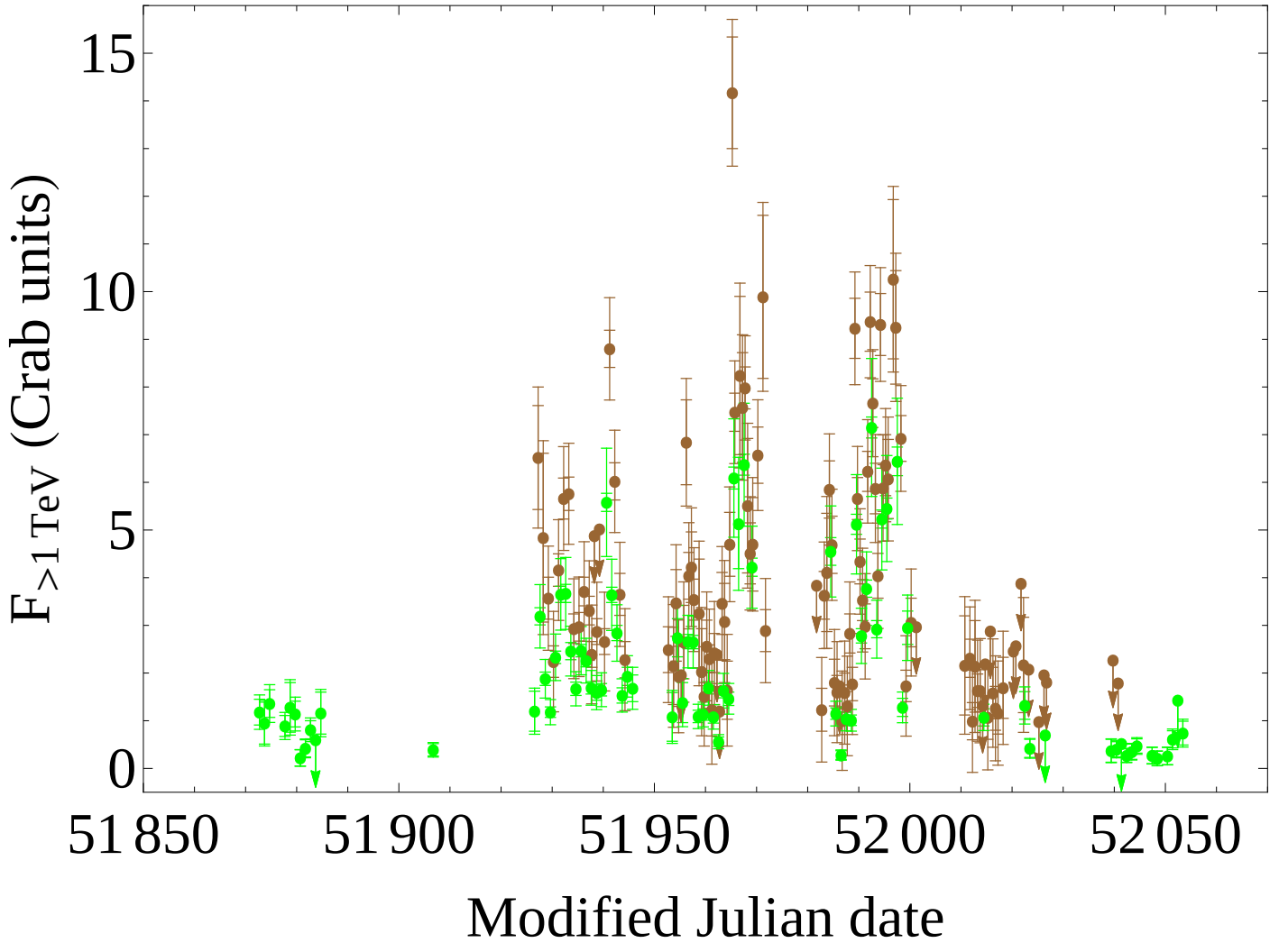

Methodological problems

For a set of observations x_i measured at t_i ($i = 1, \ldots, N$)

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$$\text{Periodogram} = |DFT(f_j)|^2 = \left| \sum_{i=1}^{N} x_i e^{2\pi i f_j t_i} \right|^2$$

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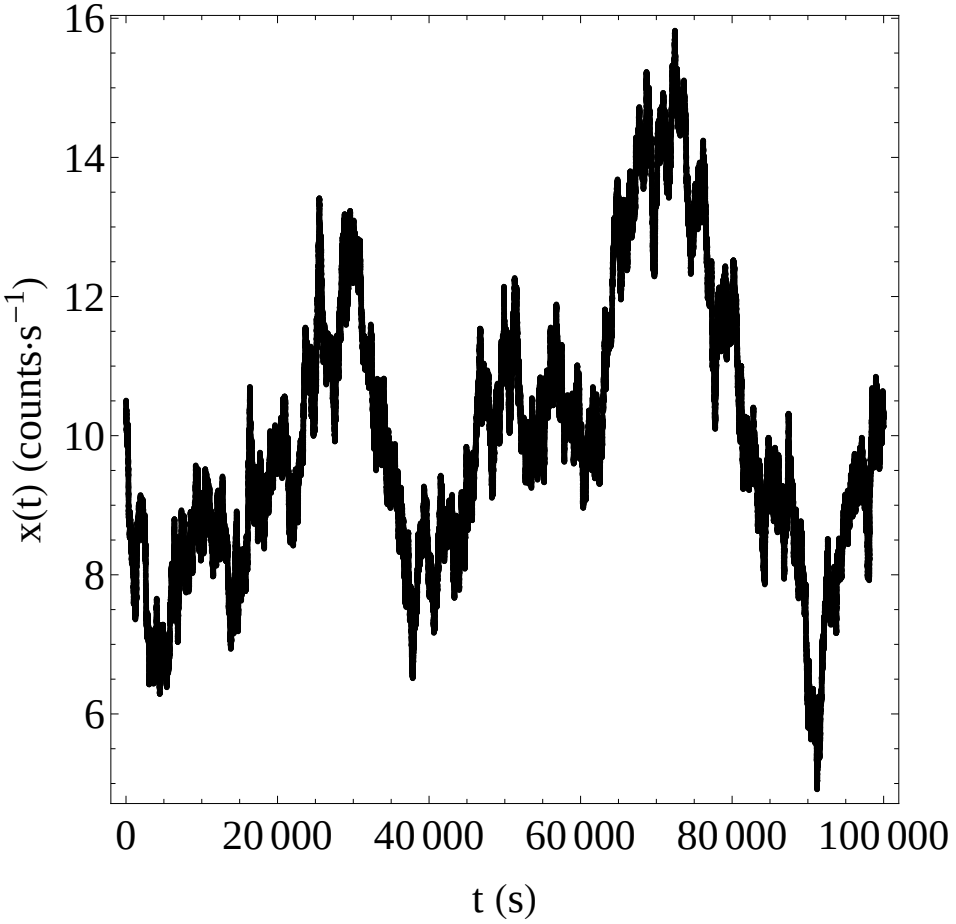
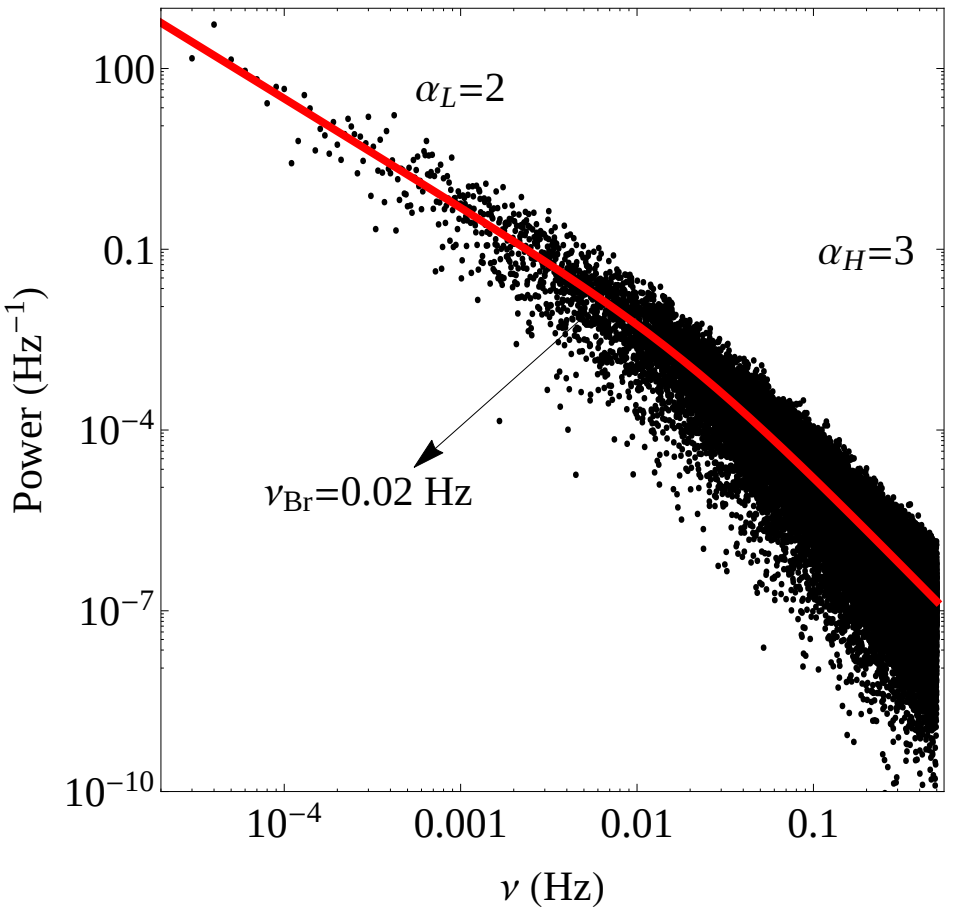
Fitting a model to the Periodogram $\longrightarrow$ PSD, $P(\nu)$

Methodological problems

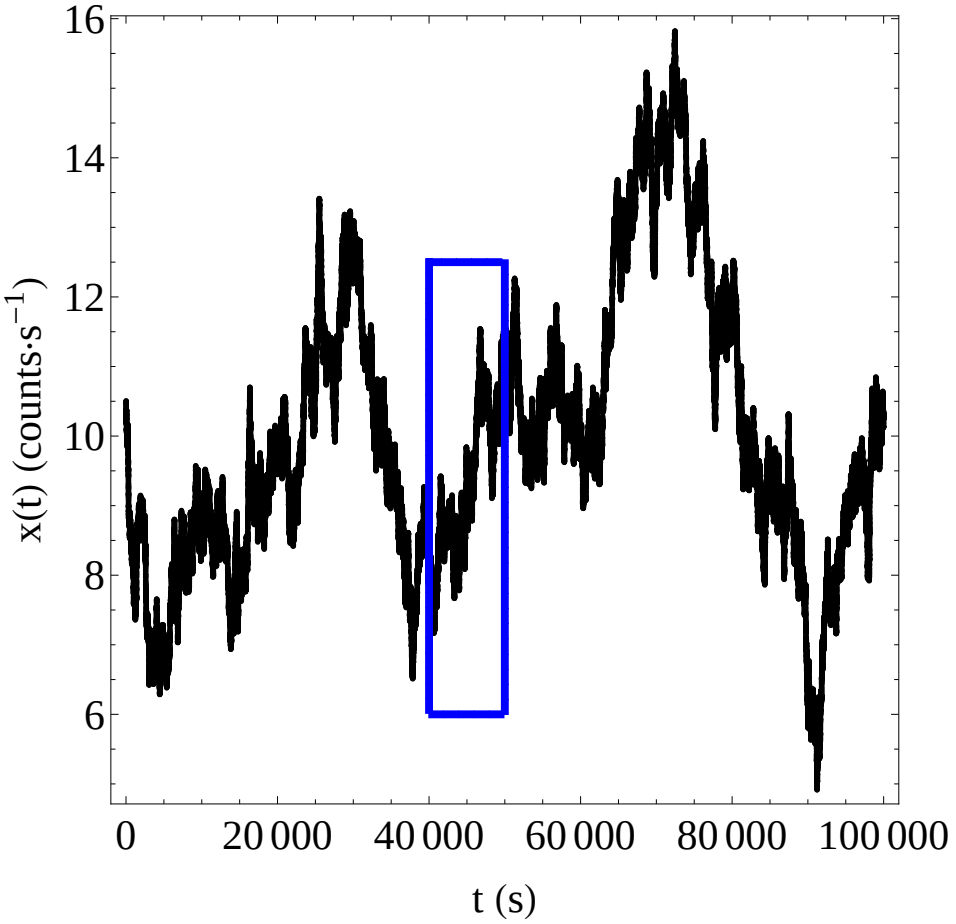
Original light curve

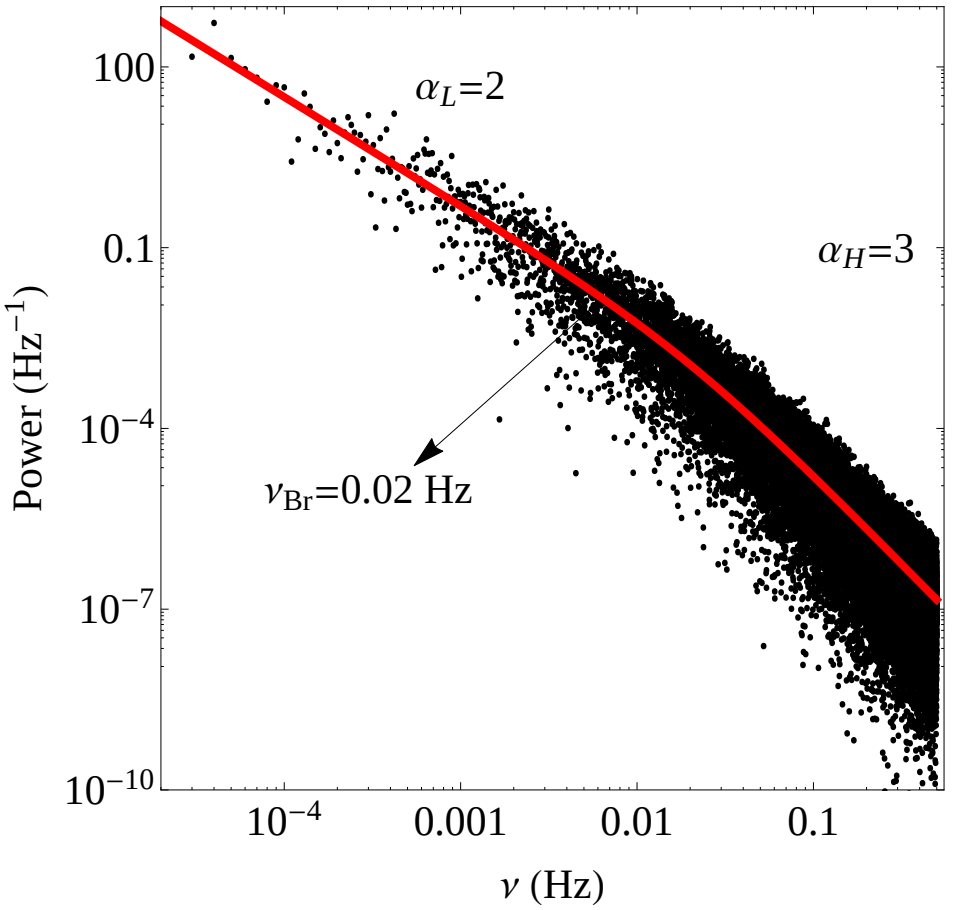




Methodological problems

Light curve segment





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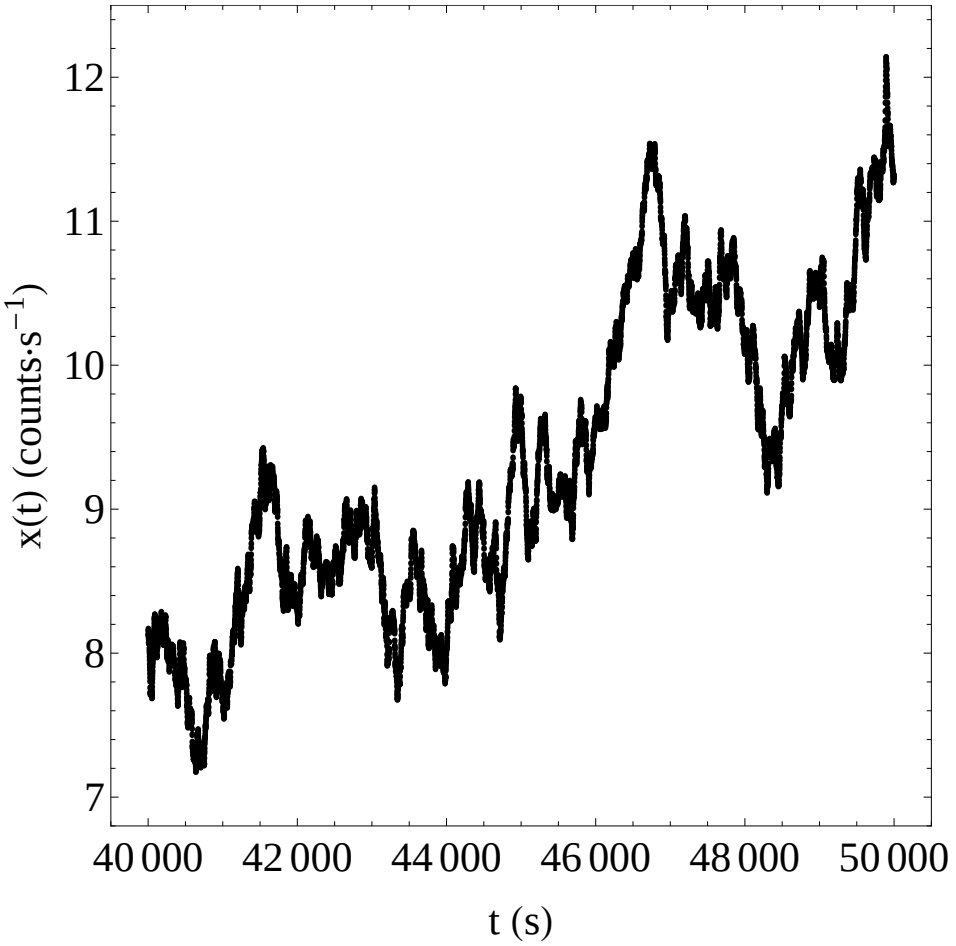
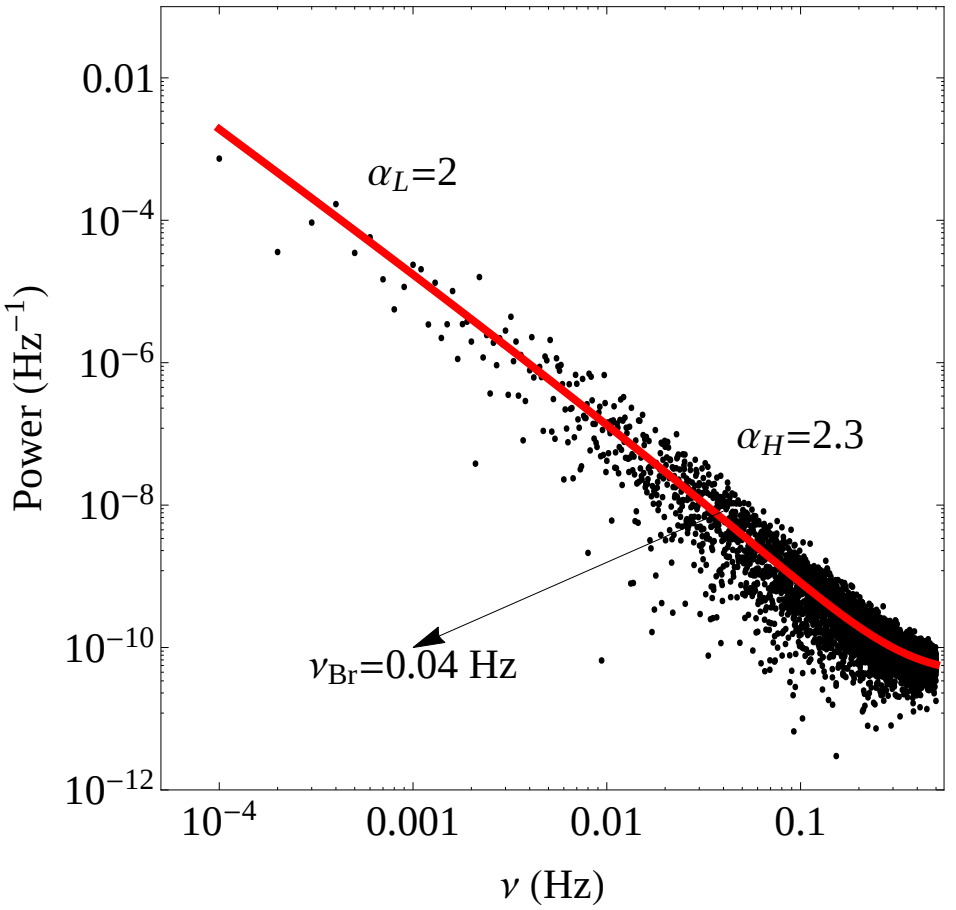
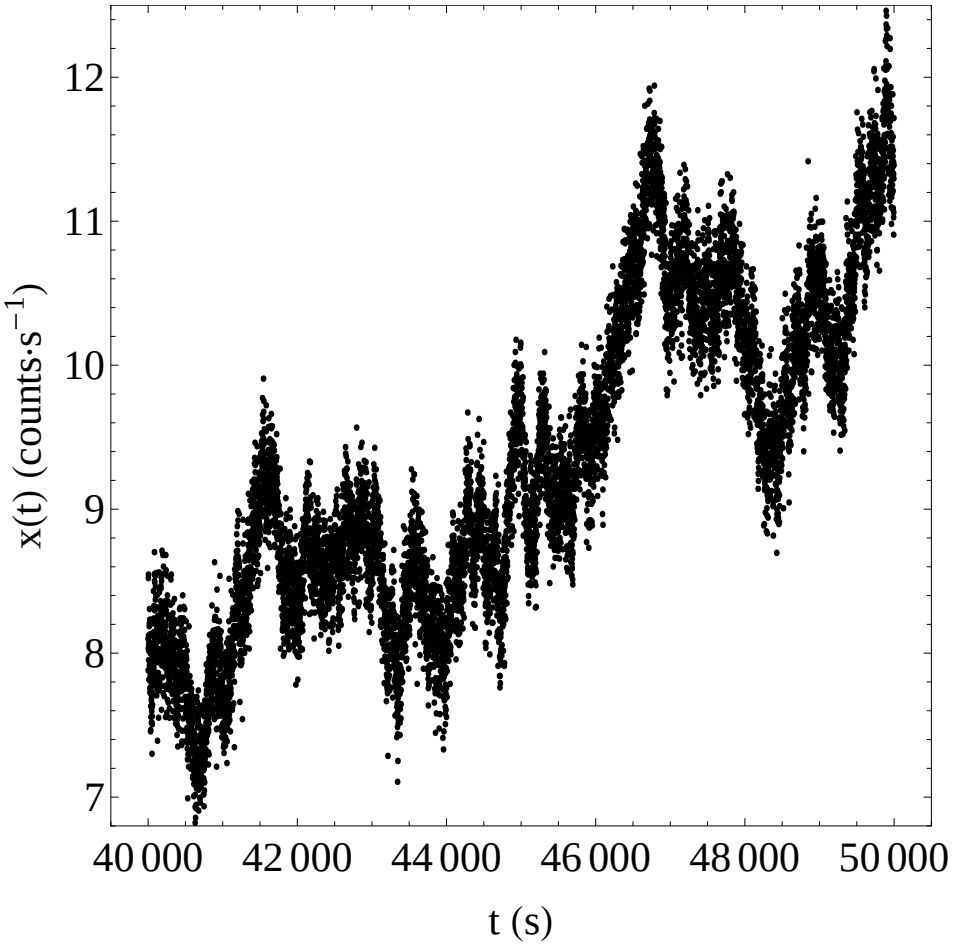
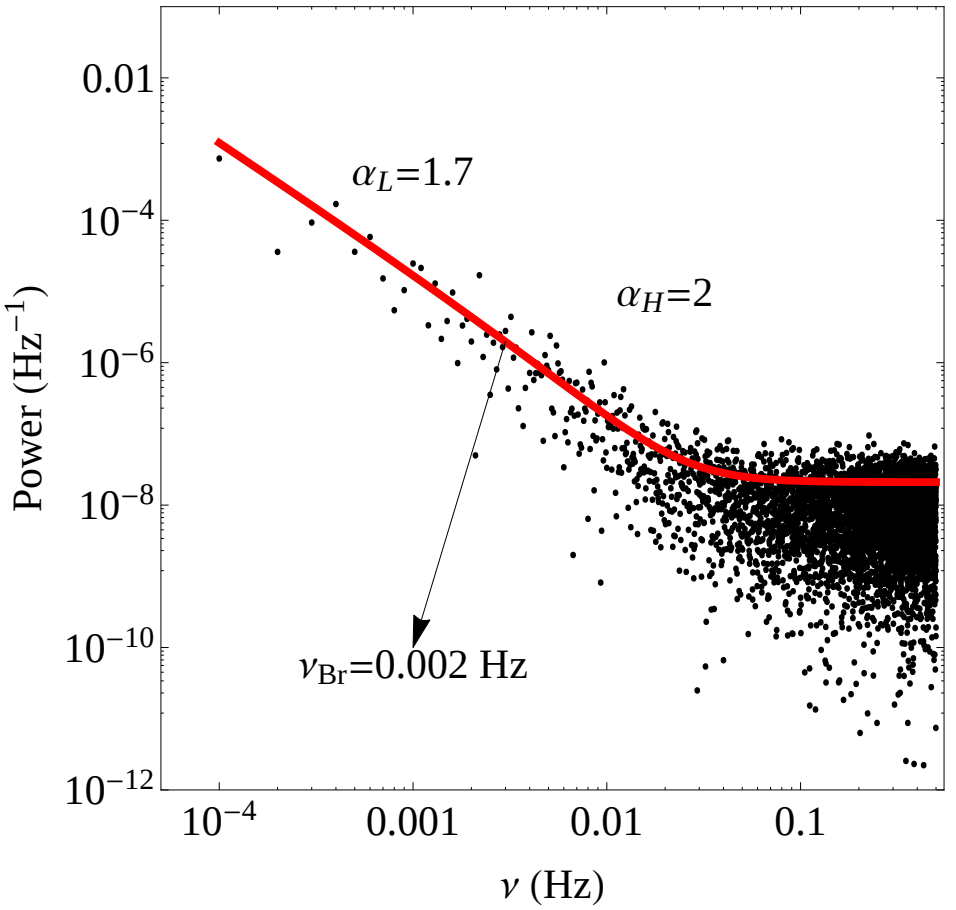




Methodological problems

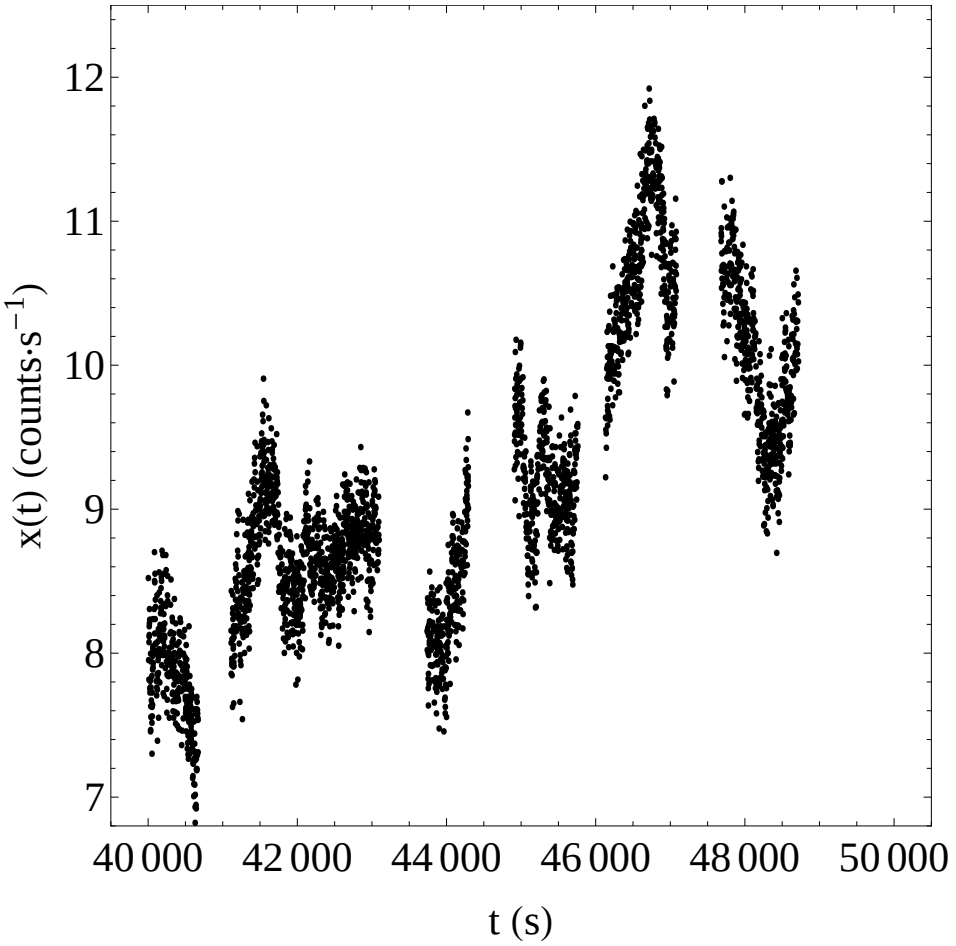
Light curve segment + Poisson noise

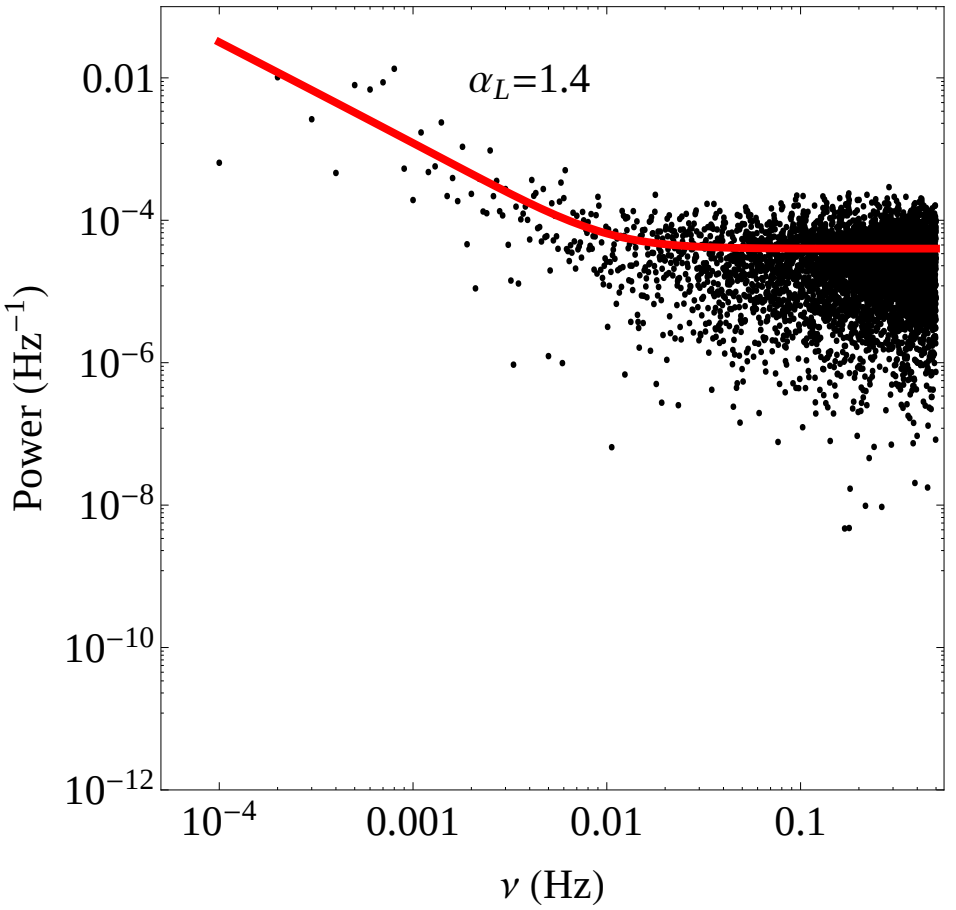




Methodological problems

Light curve segment + Poisson noise + Missing data





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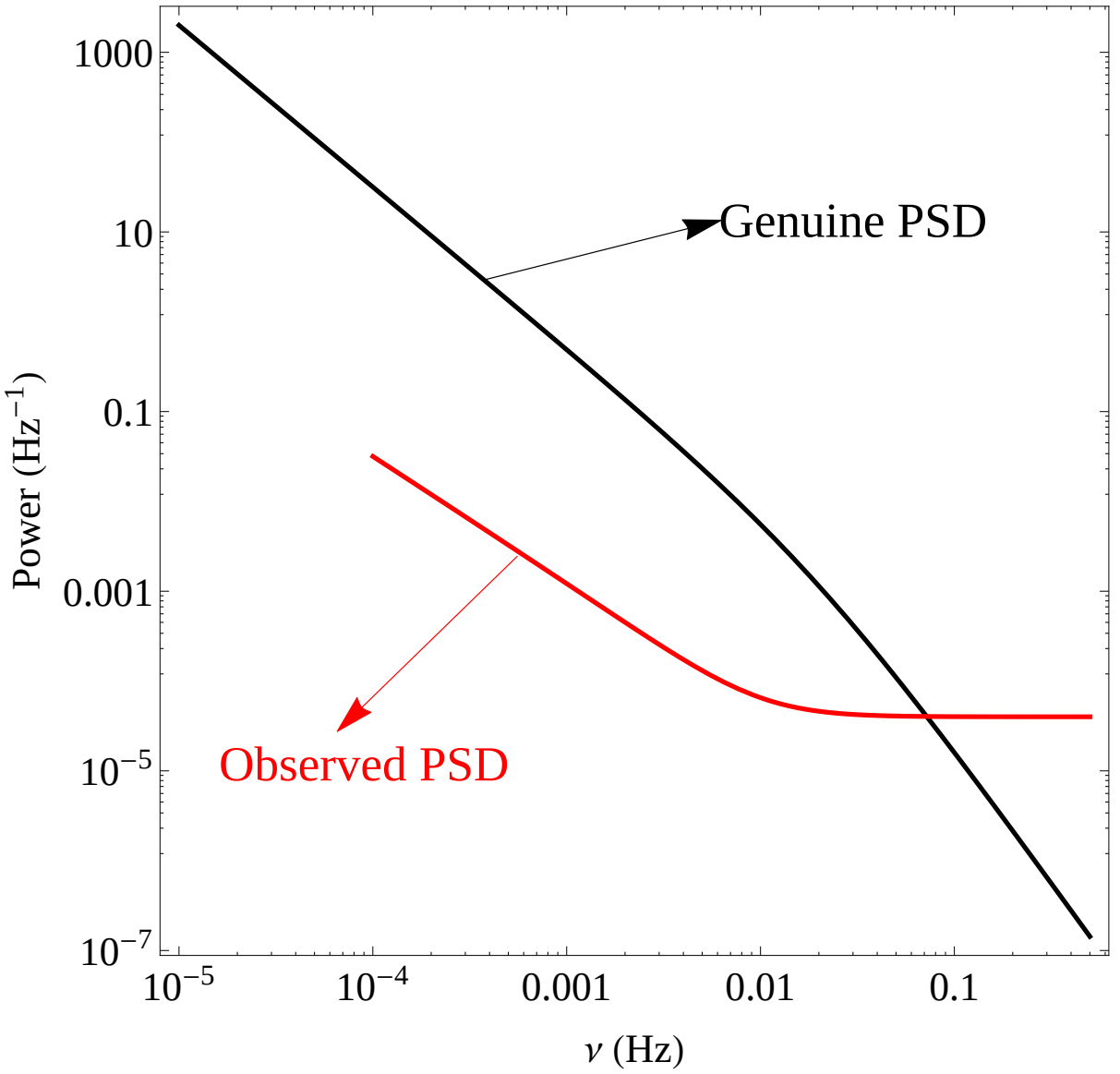

Monte Carlo approach

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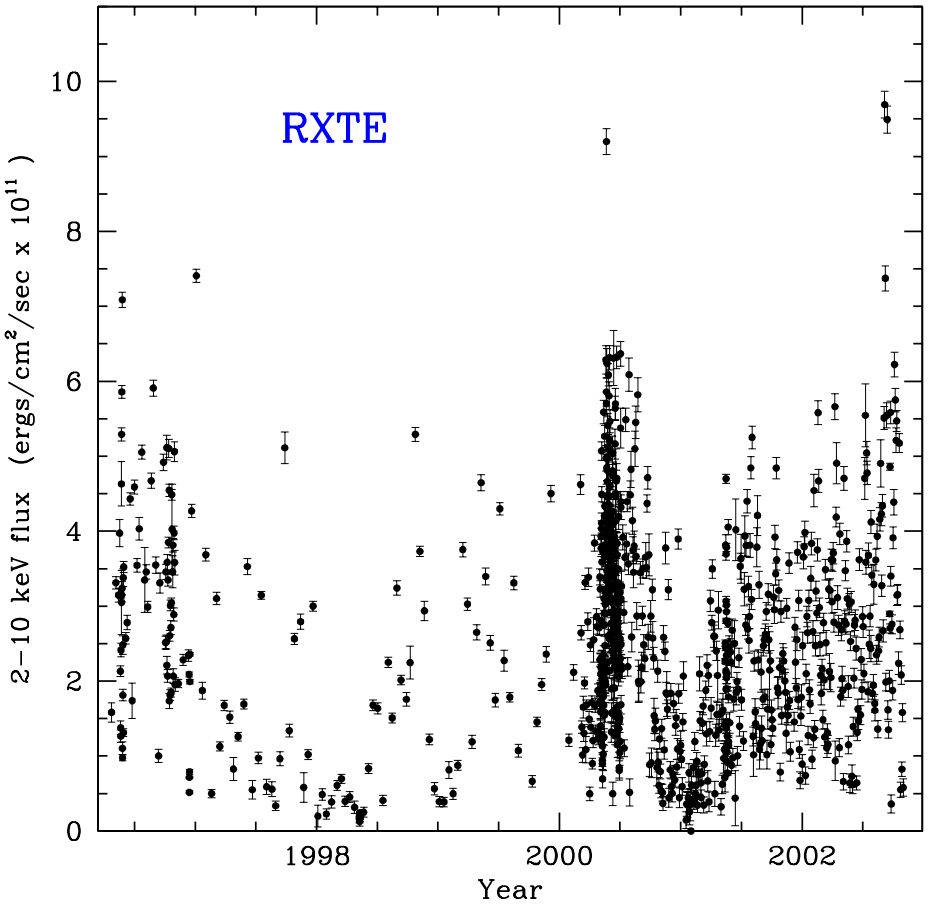
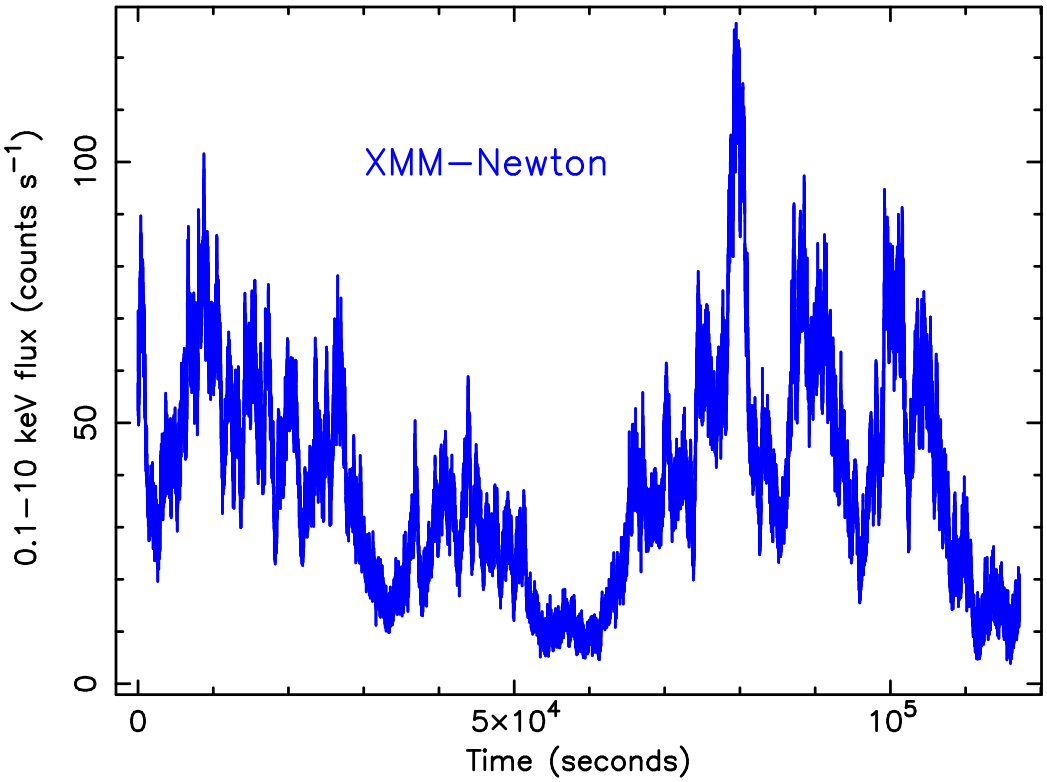
- Fit to the observed 'dirty' periodogram.
- Repeat the process for another PSD model.
- Establish a 'goodness-of-fit' criterion.

Application

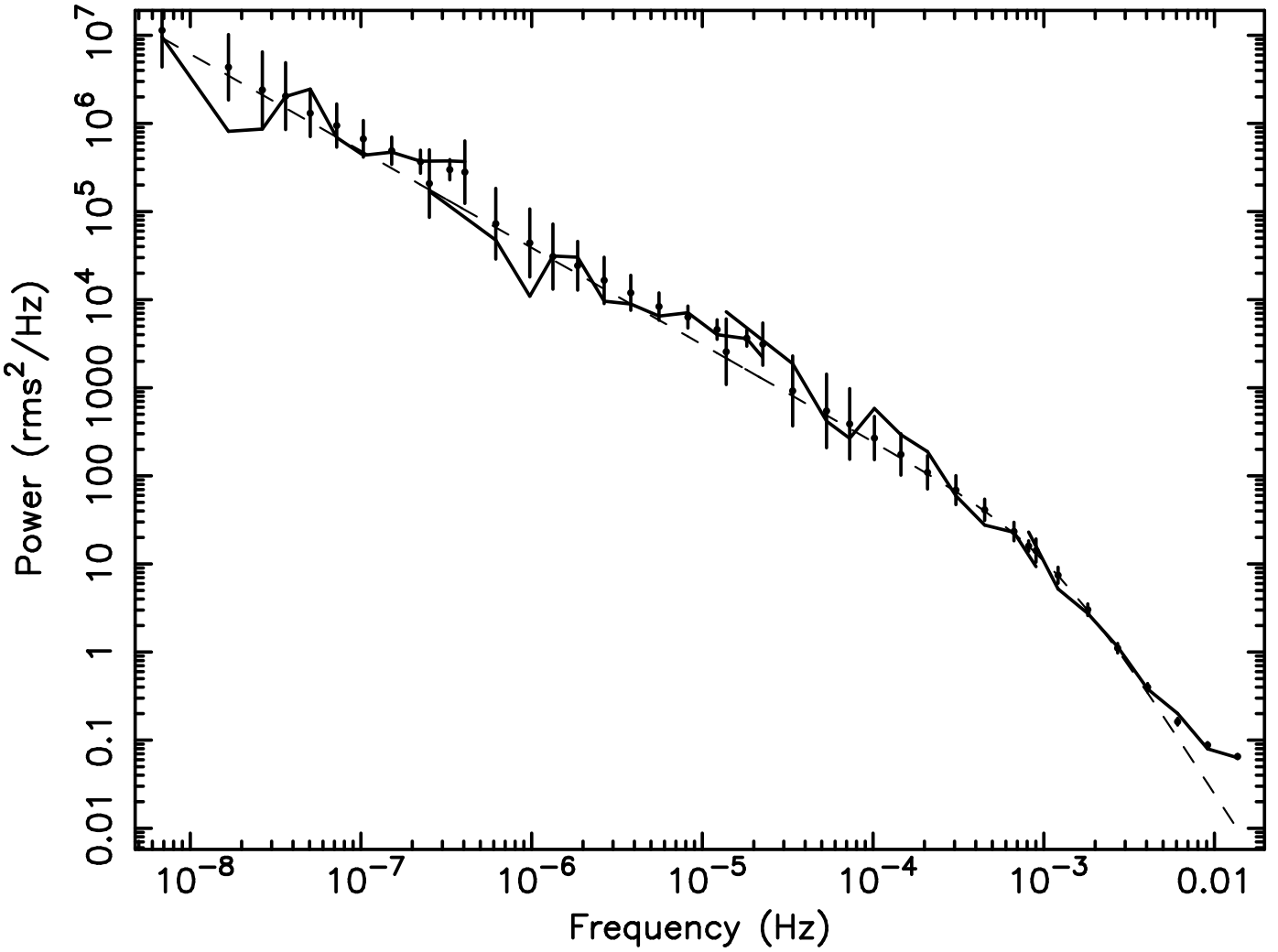
NGC 4051





McHardy et al. 2004, MNRAS, **348**, 783

Application



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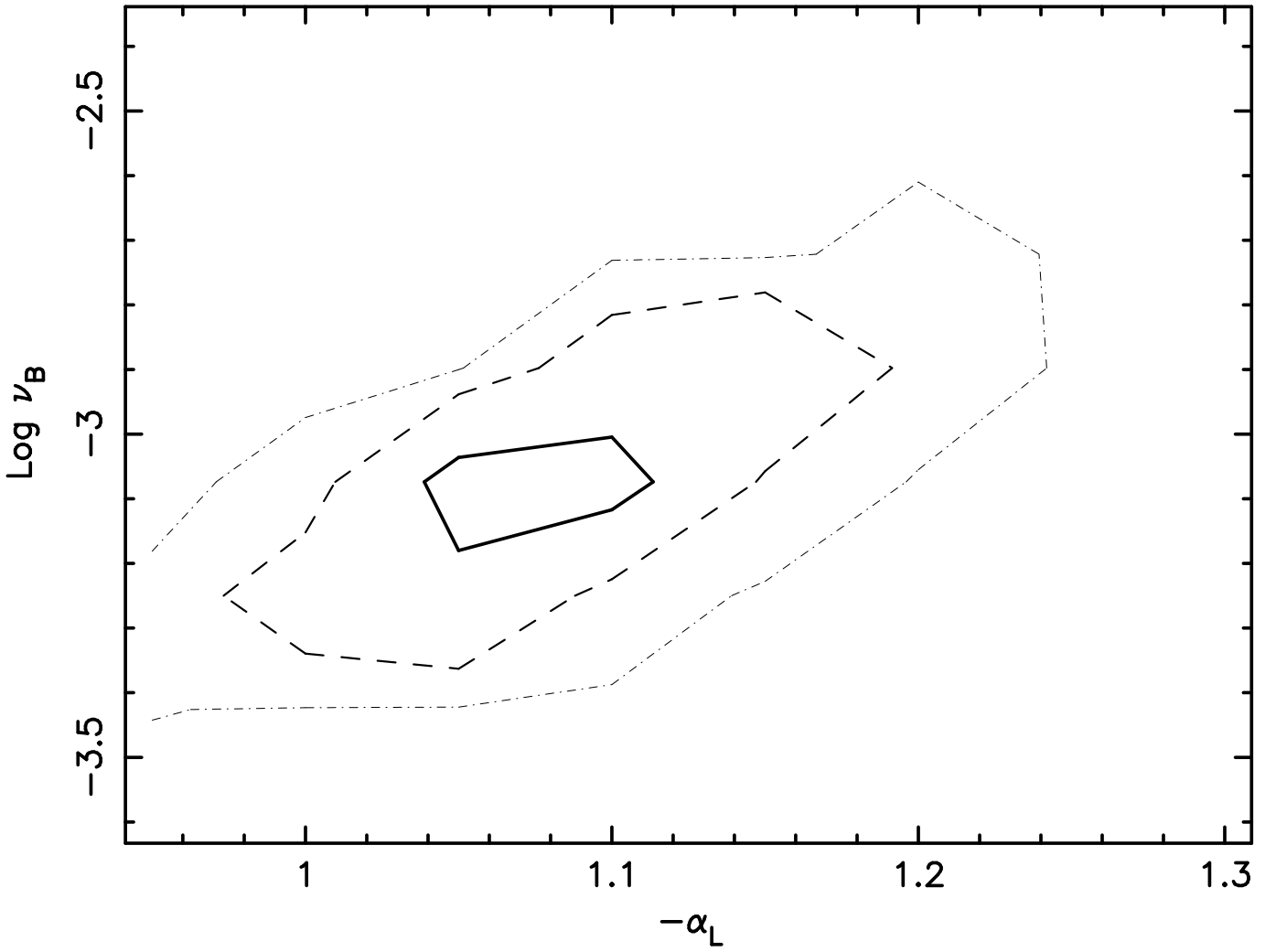

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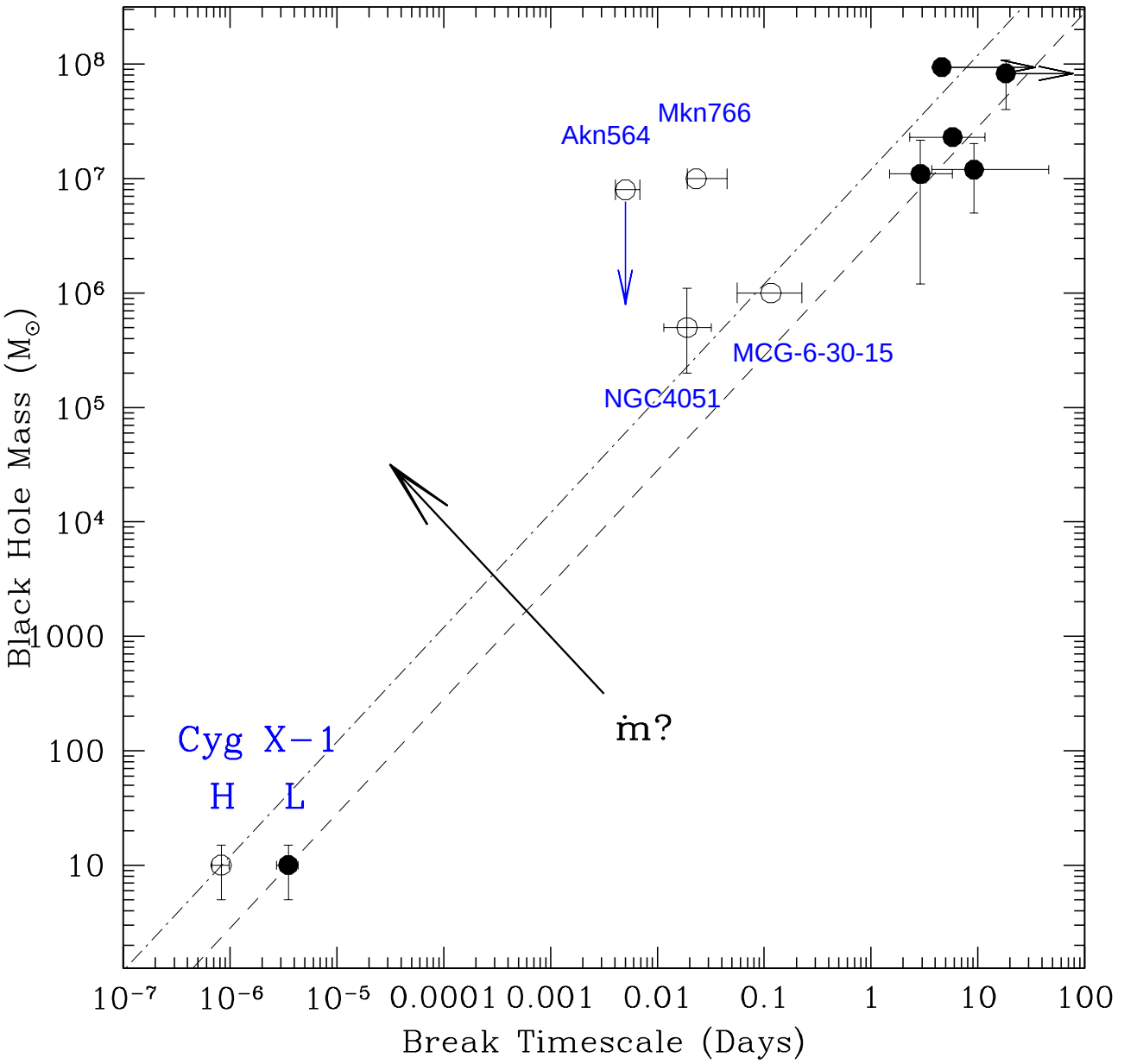

Monte Carlo problems

1) White noise: Random number generator

PSD always flat

Monte Carlo problems

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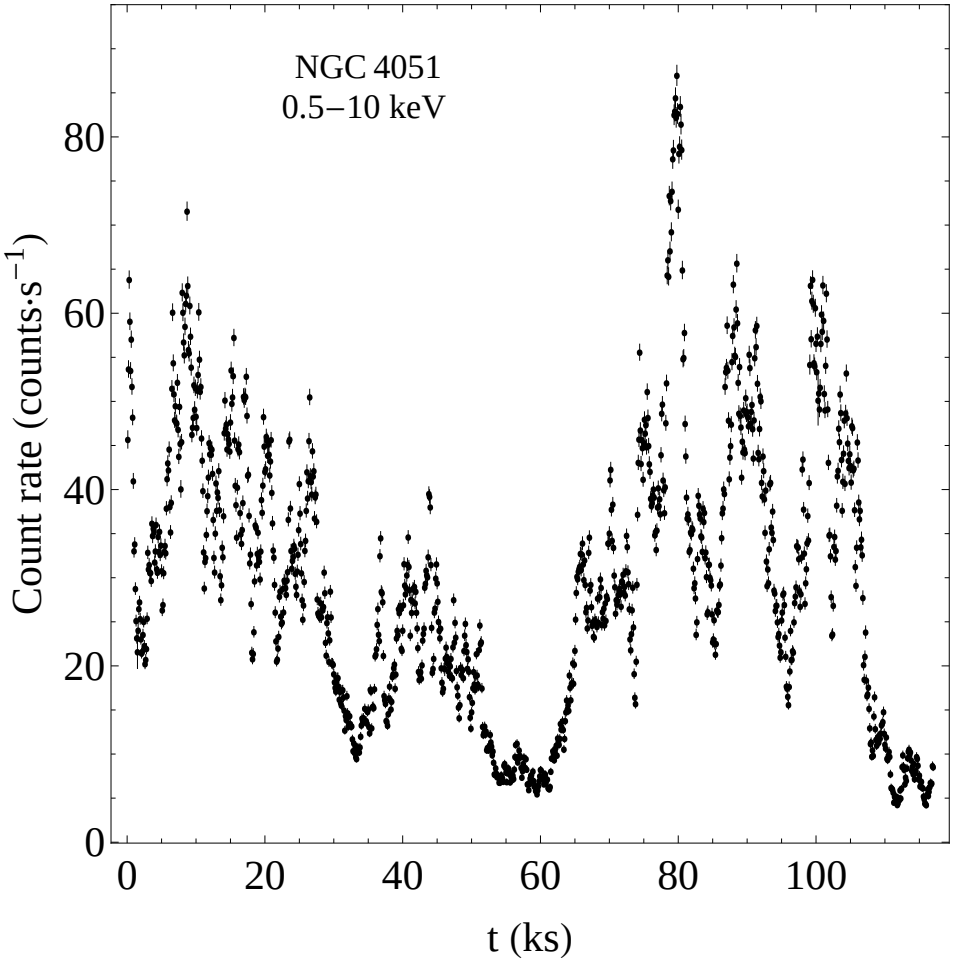

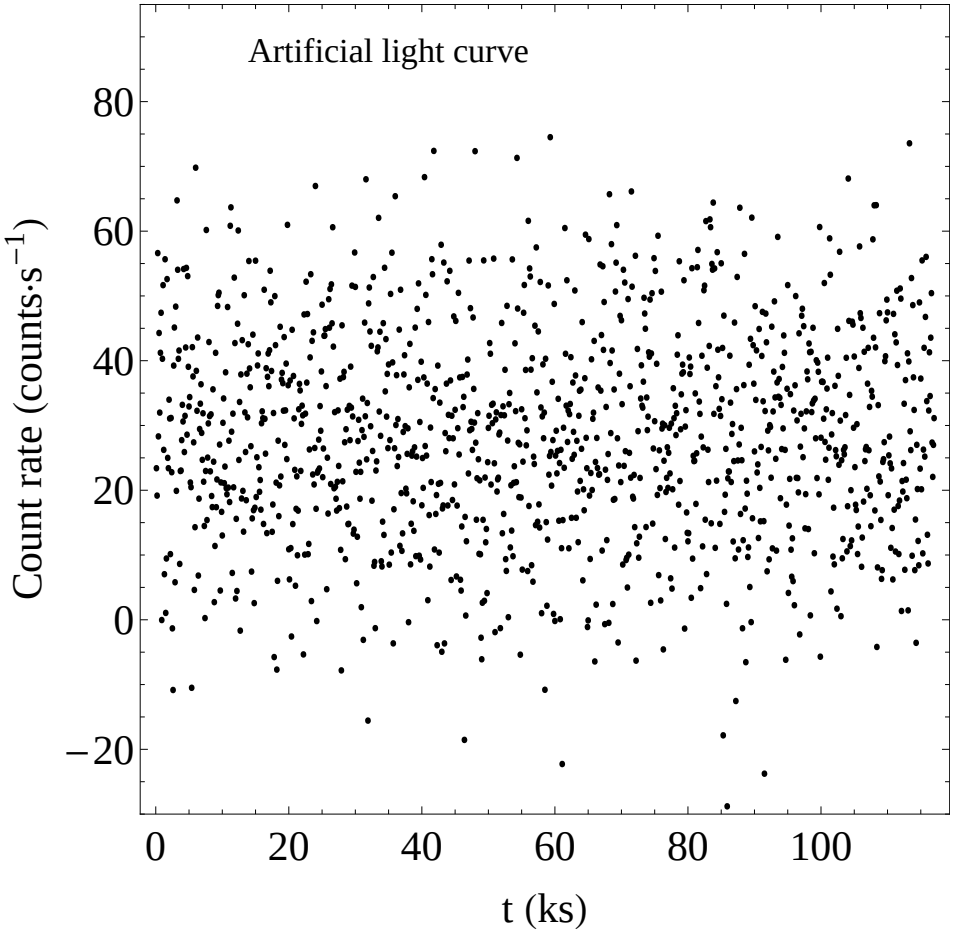

Monte Carlo problems

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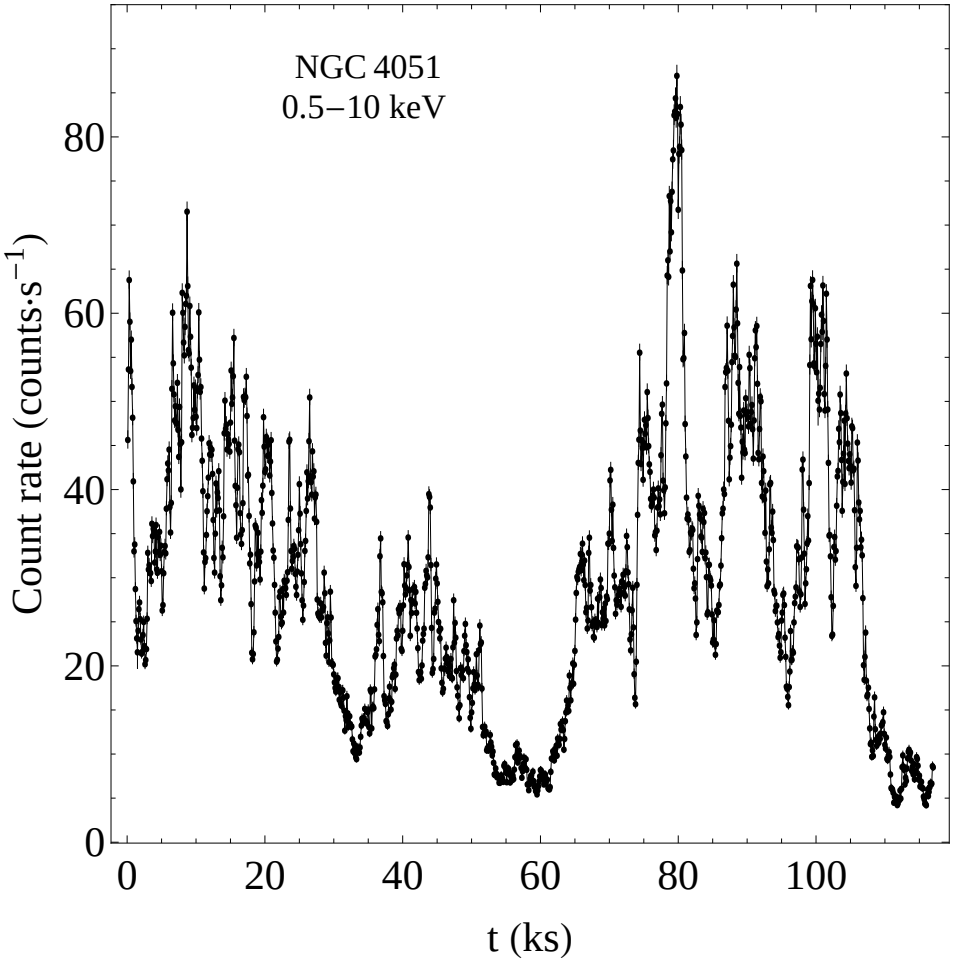
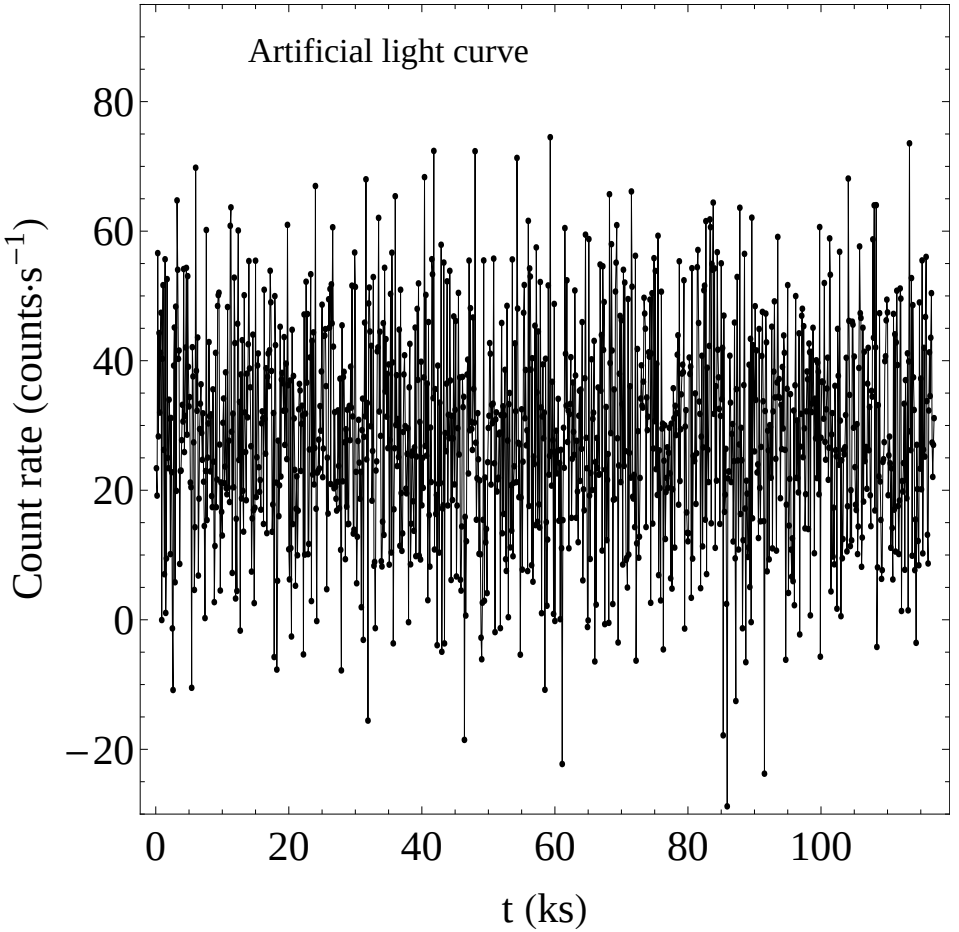




Monte Carlo problems

2)Done, C. et al. 1992, ApJ, **400**, 138

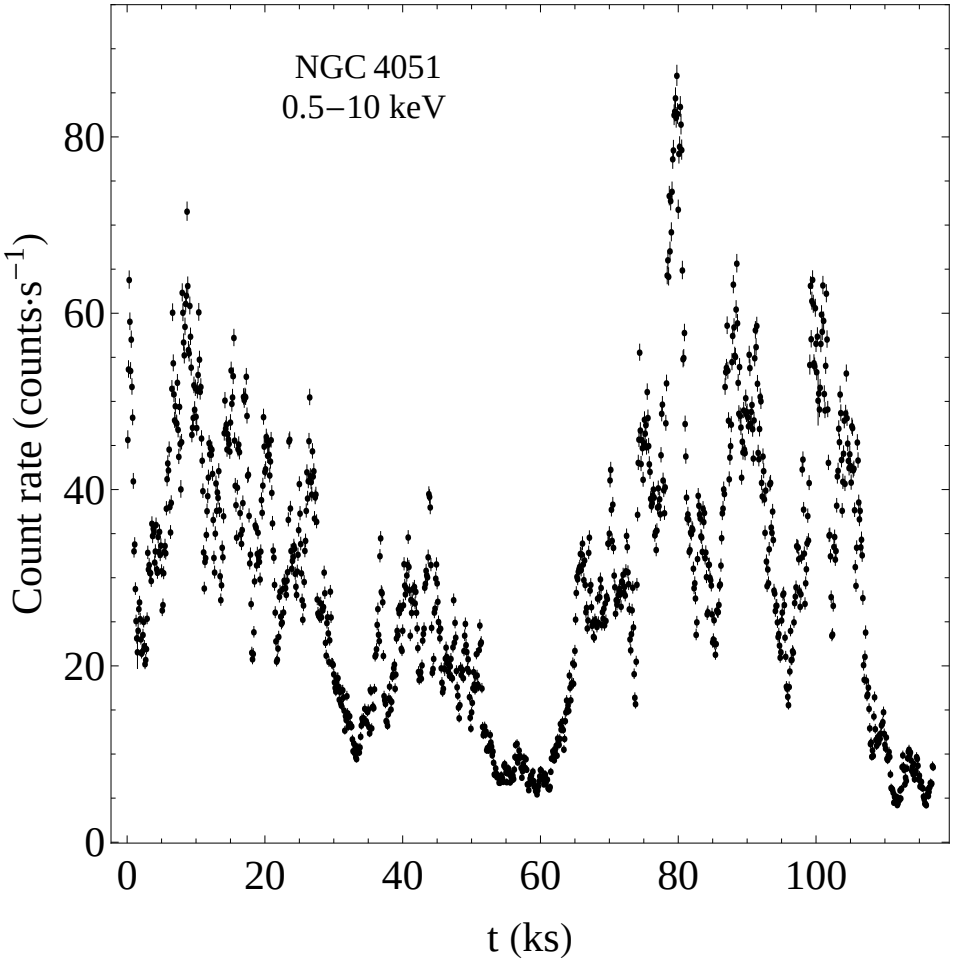
Phase randomisation BUT fixed amplitude

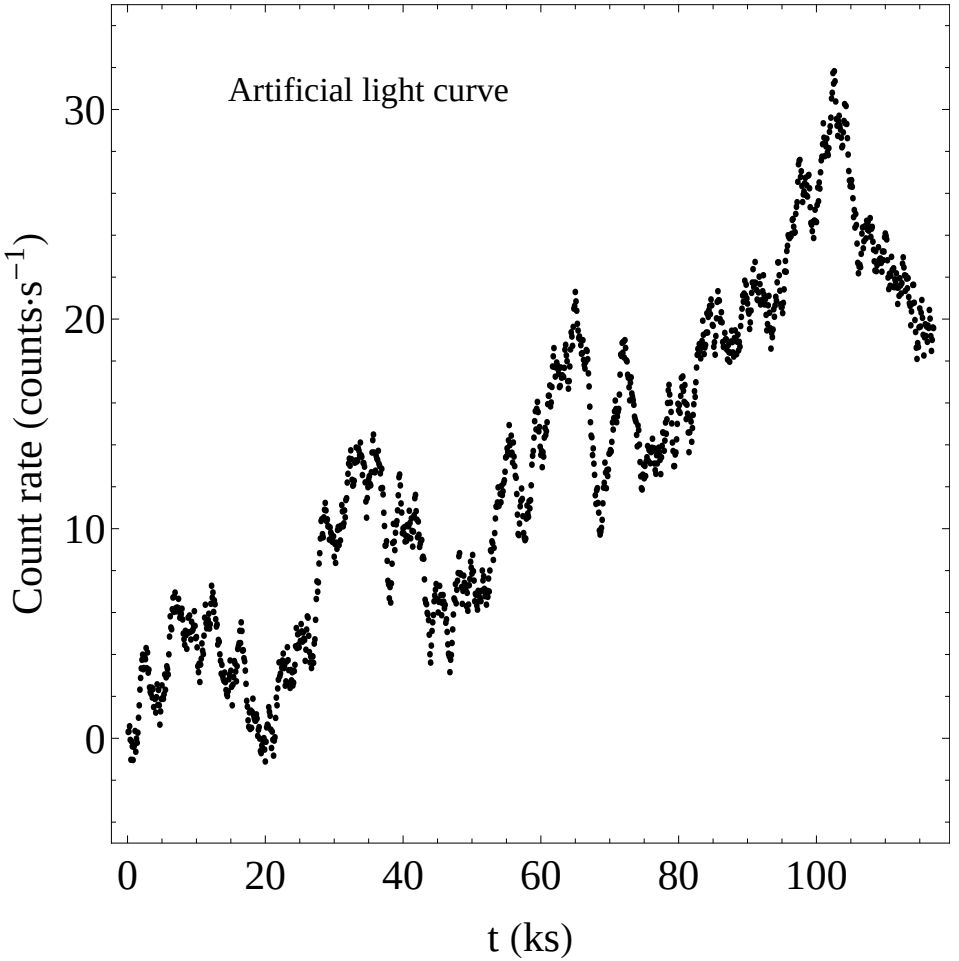
$$x(t) \sim \sum \sqrt{P(\nu)} \cos(2\pi\nu t + \phi)$$

with $\phi \in [0, 2\pi]$

Monte Carlo problems

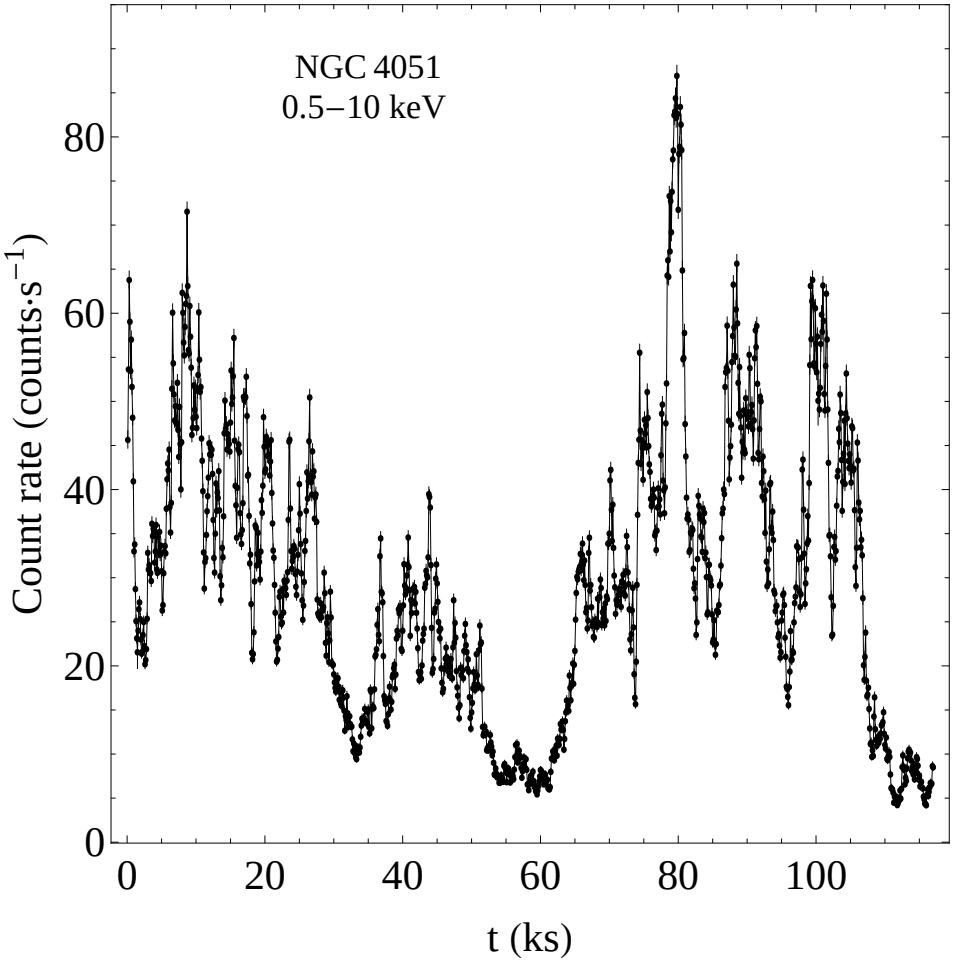
2)Done, C. et al. 1992, ApJ, **400**, 138

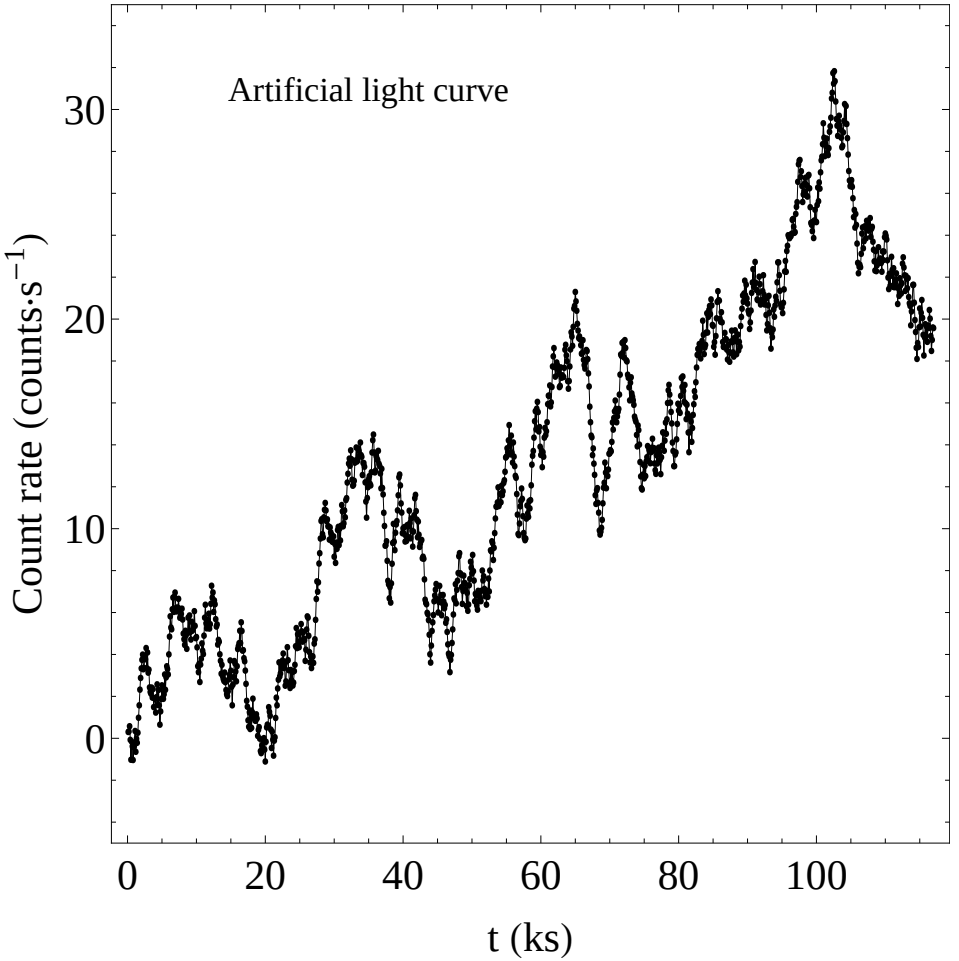




Monte Carlo problems

2)Done, C. et al. 1992, ApJ, **400**, 138





Monte Carlo problems

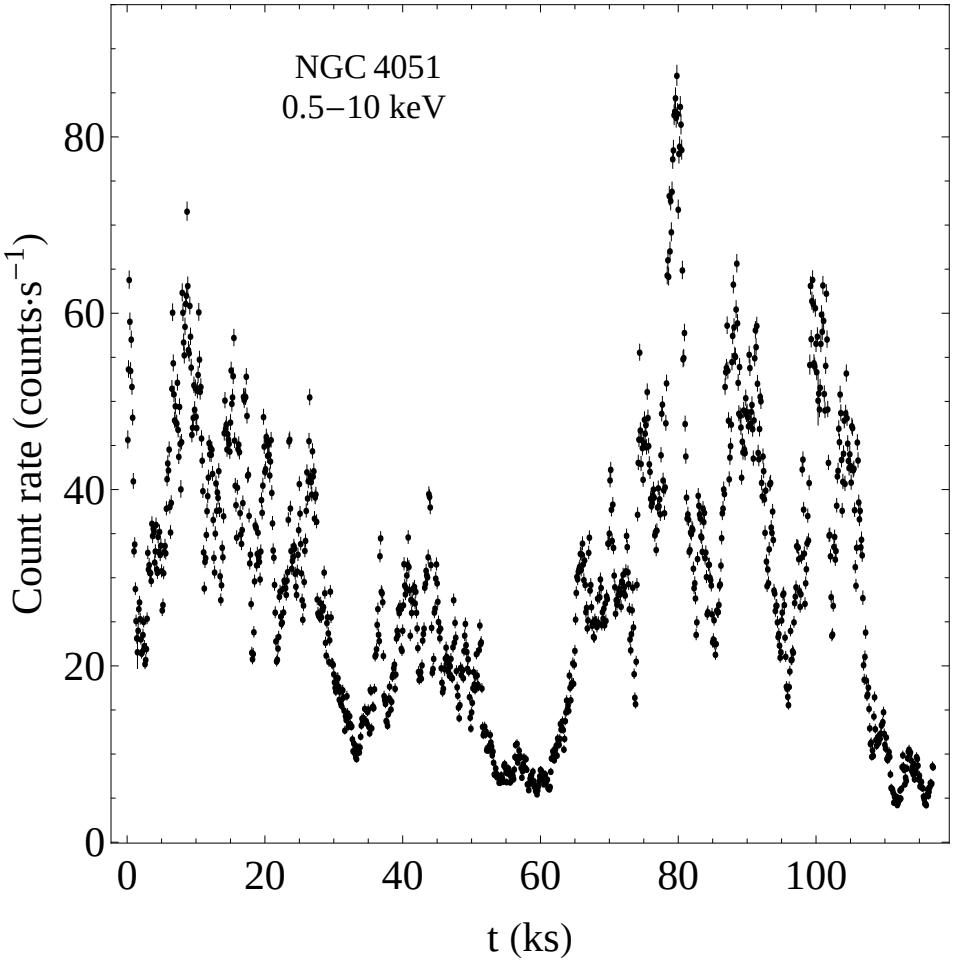
3)Timmer & König 1995, A&A, **300**, 707

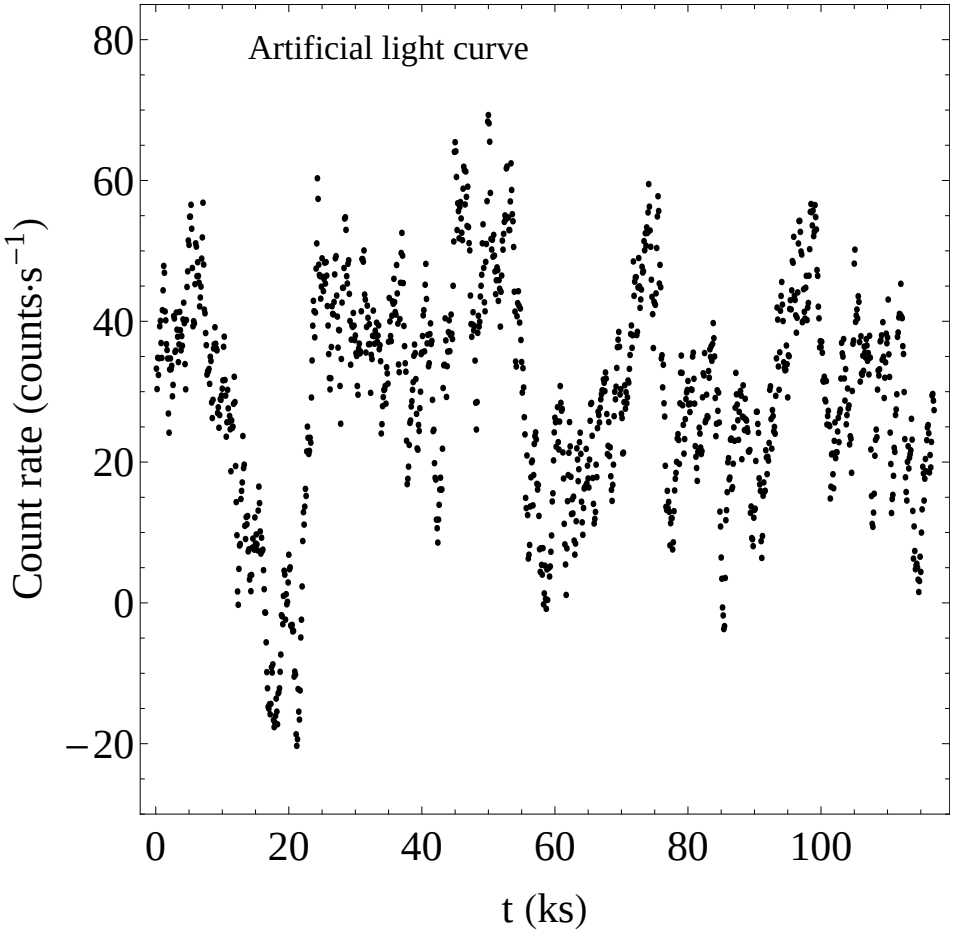
Phase randomisation AND Amplitude randomisation

$$FT_{x(t)}(\nu) = \mathcal{N}(0, \frac{1}{2}P(\nu)) + i\mathcal{N}(0, \frac{1}{2}P(\nu))$$

Monte Carlo problems

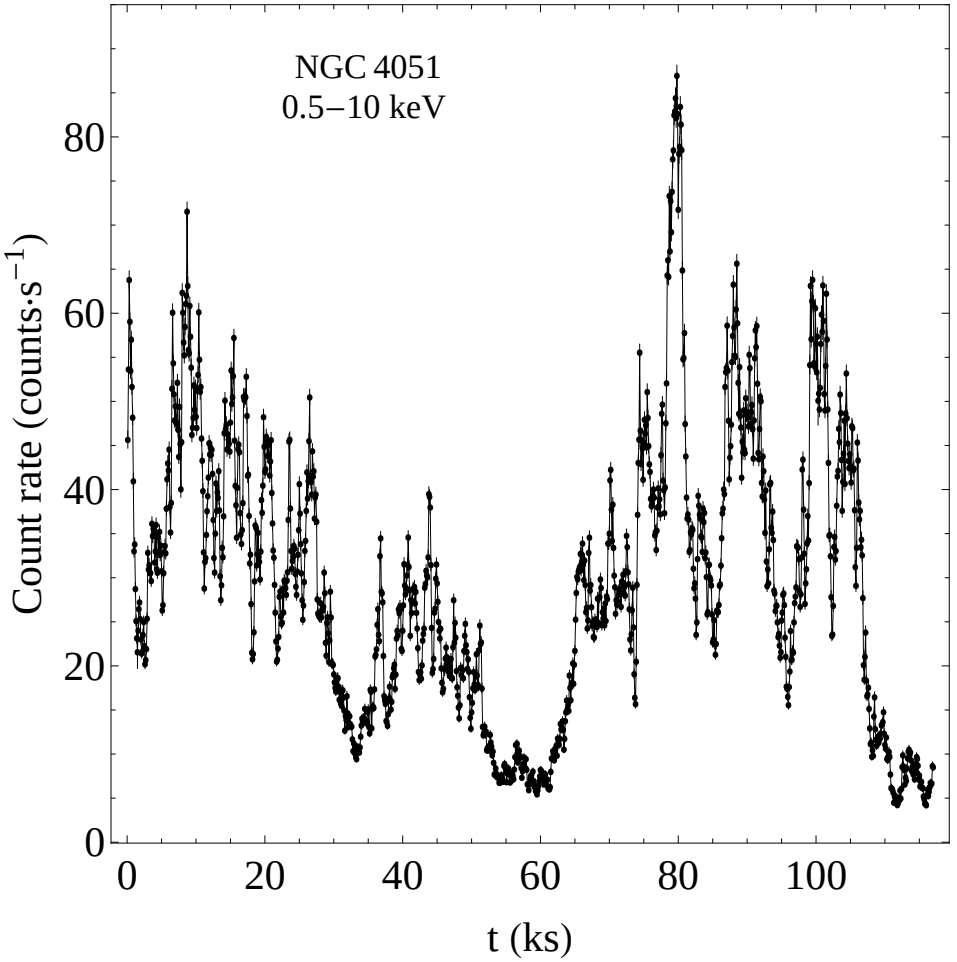
3) Timmer & König 1995, A&A, **300**, 707

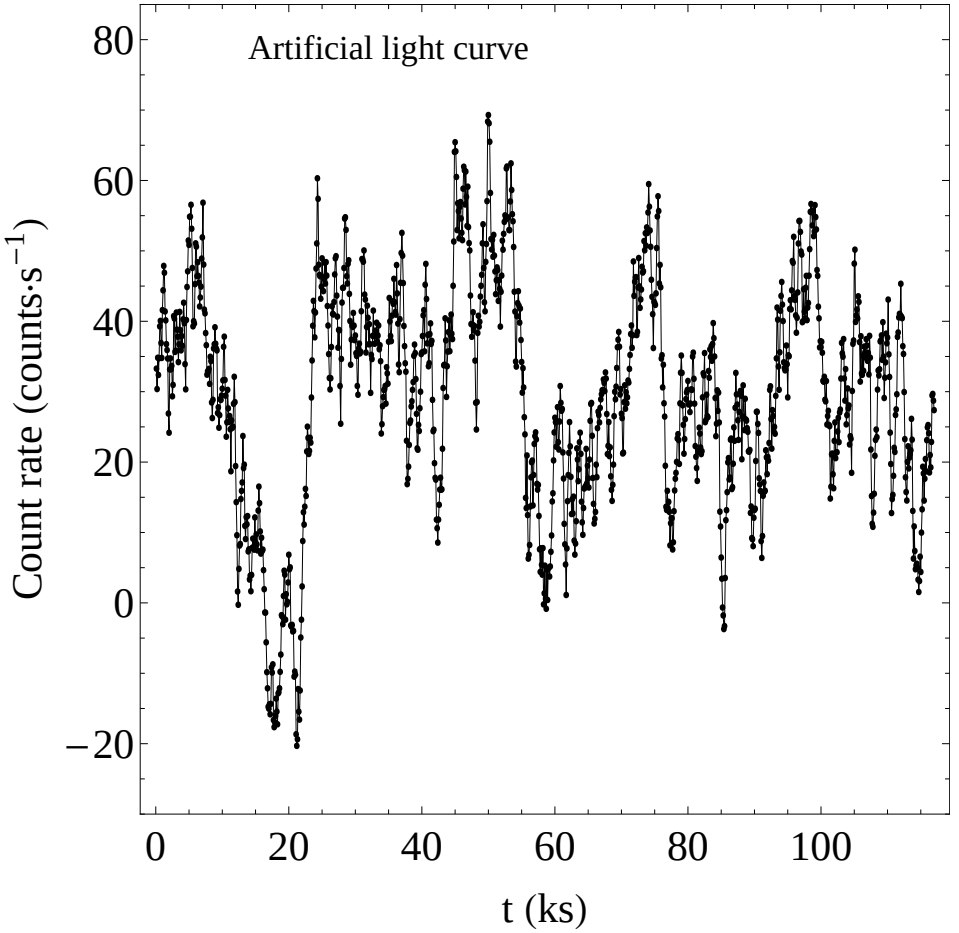




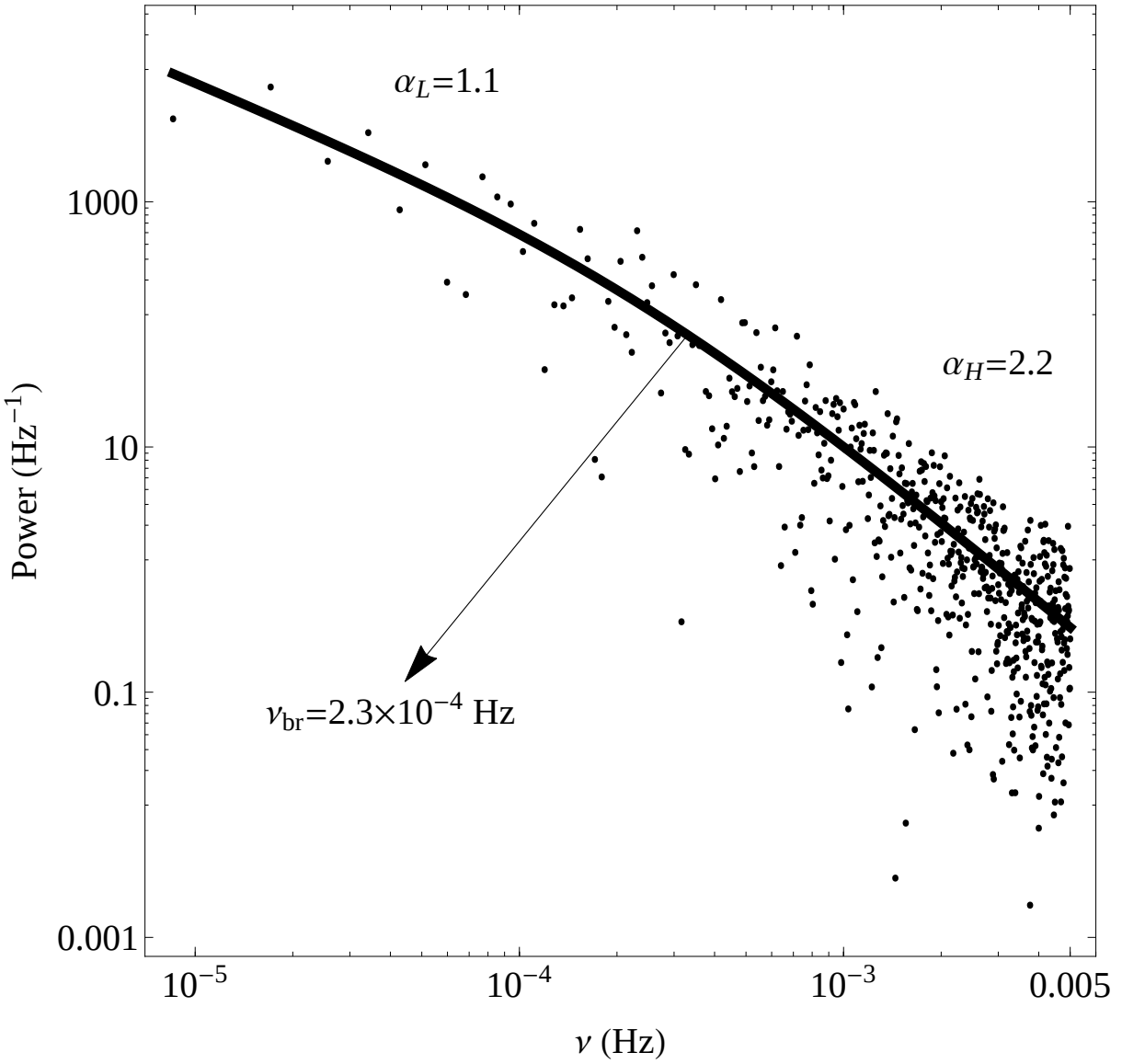
Monte Carlo problems

3)Timmer & König 1995, A&A, **300**, 707

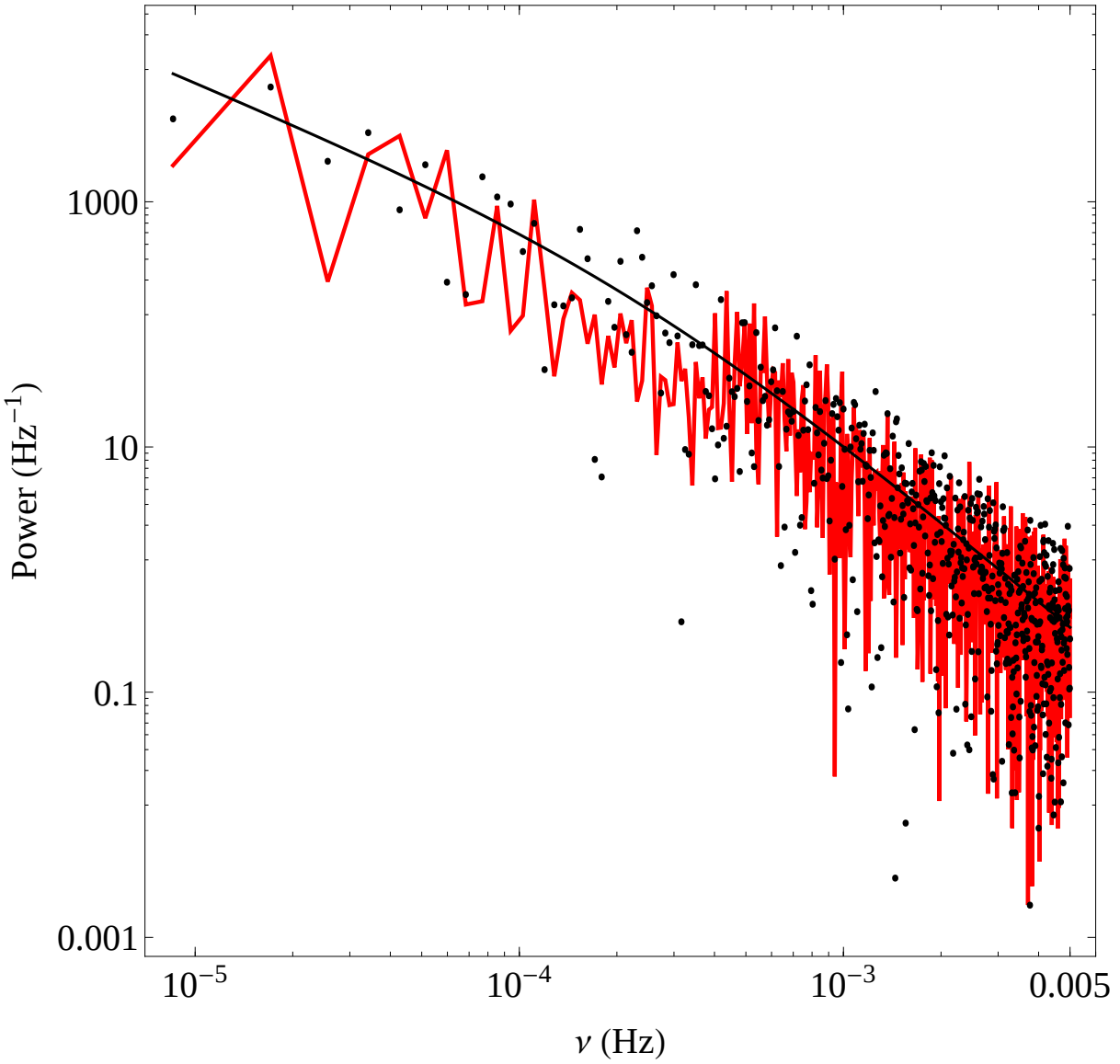




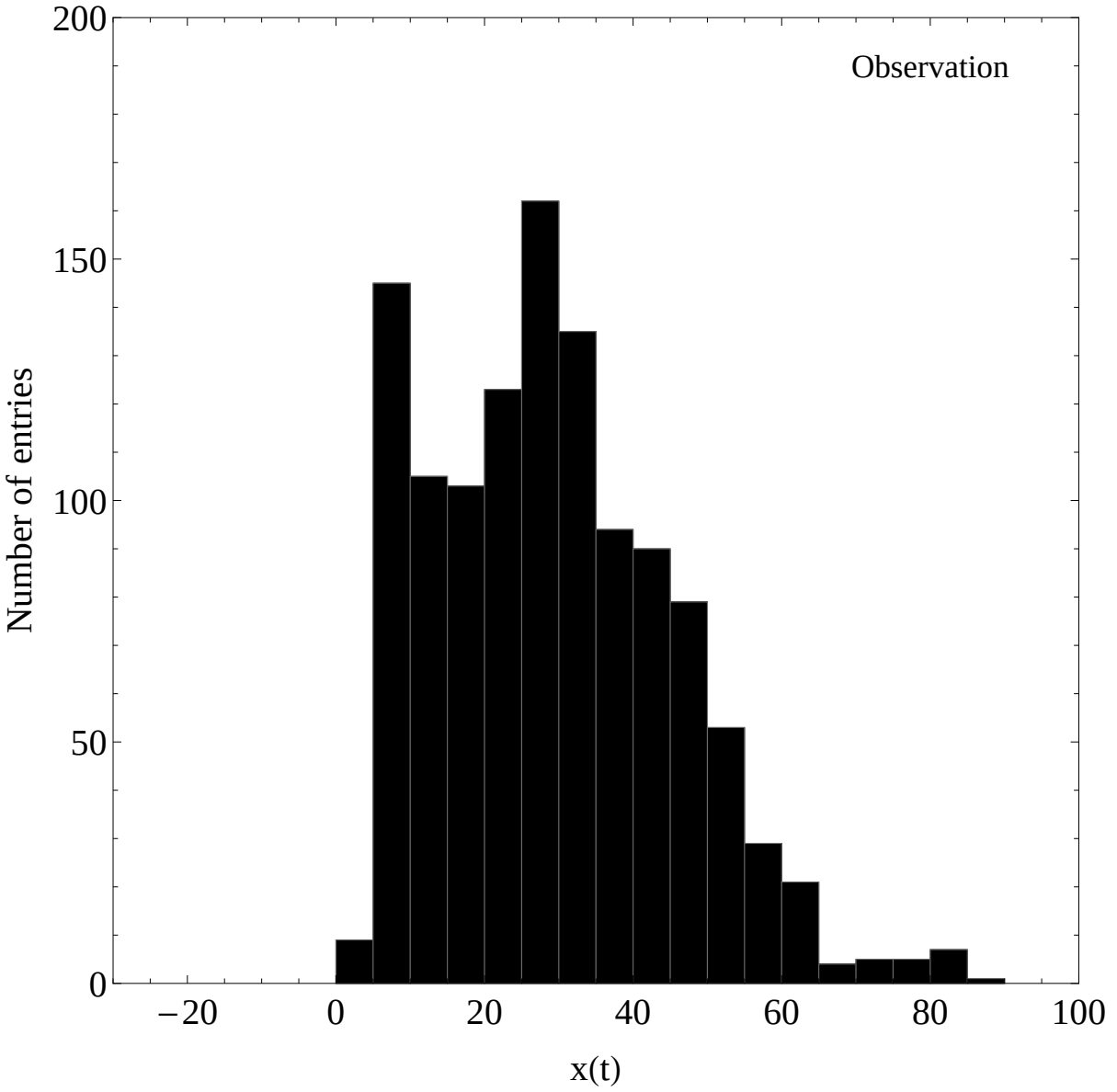
Monte Carlo problems



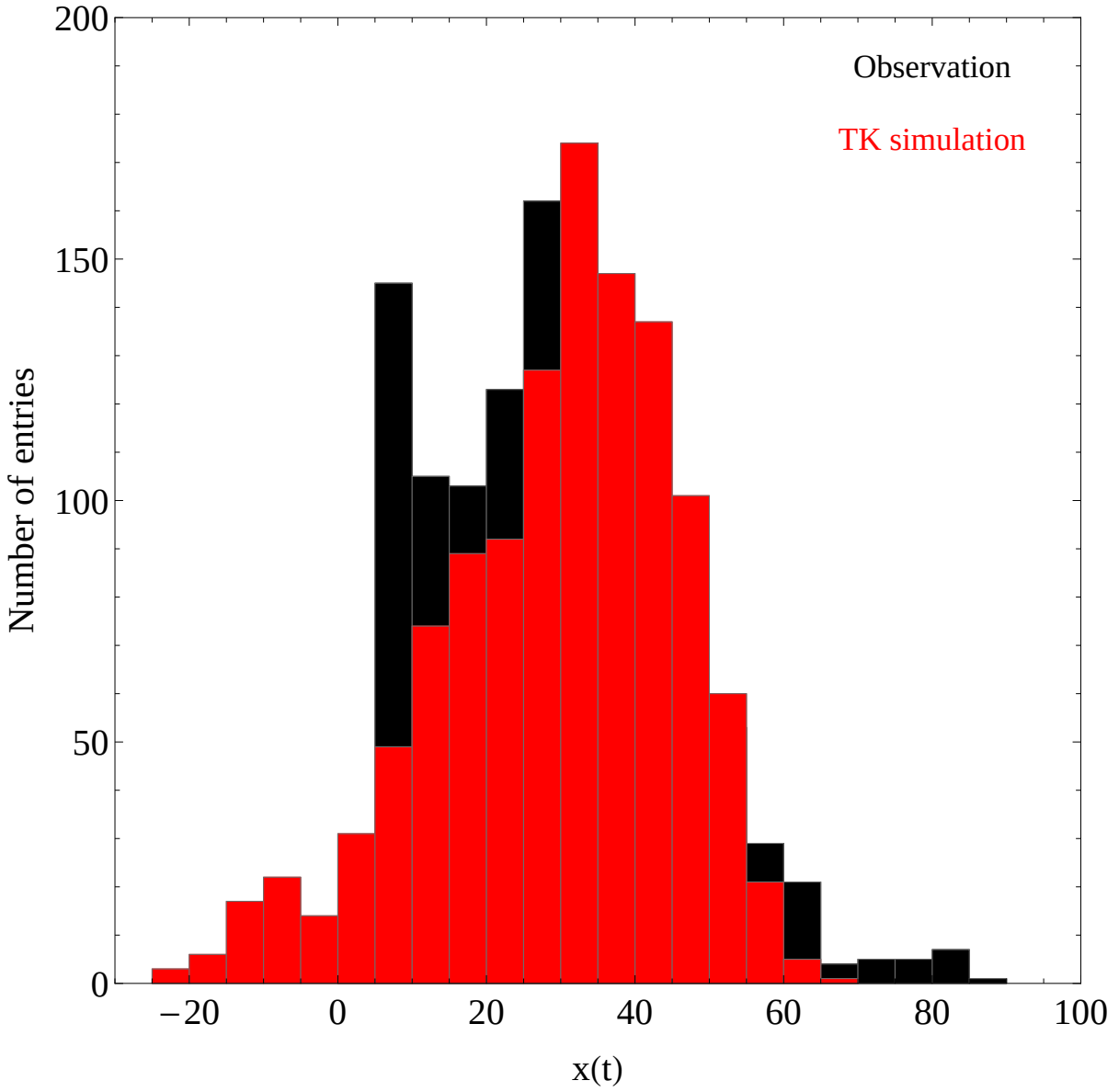
Monte Carlo problems



Monte Carlo problems



Monte Carlo problems



Monte Carlo problems

Few times that **Normality** is a problem!

Monte Carlo problems

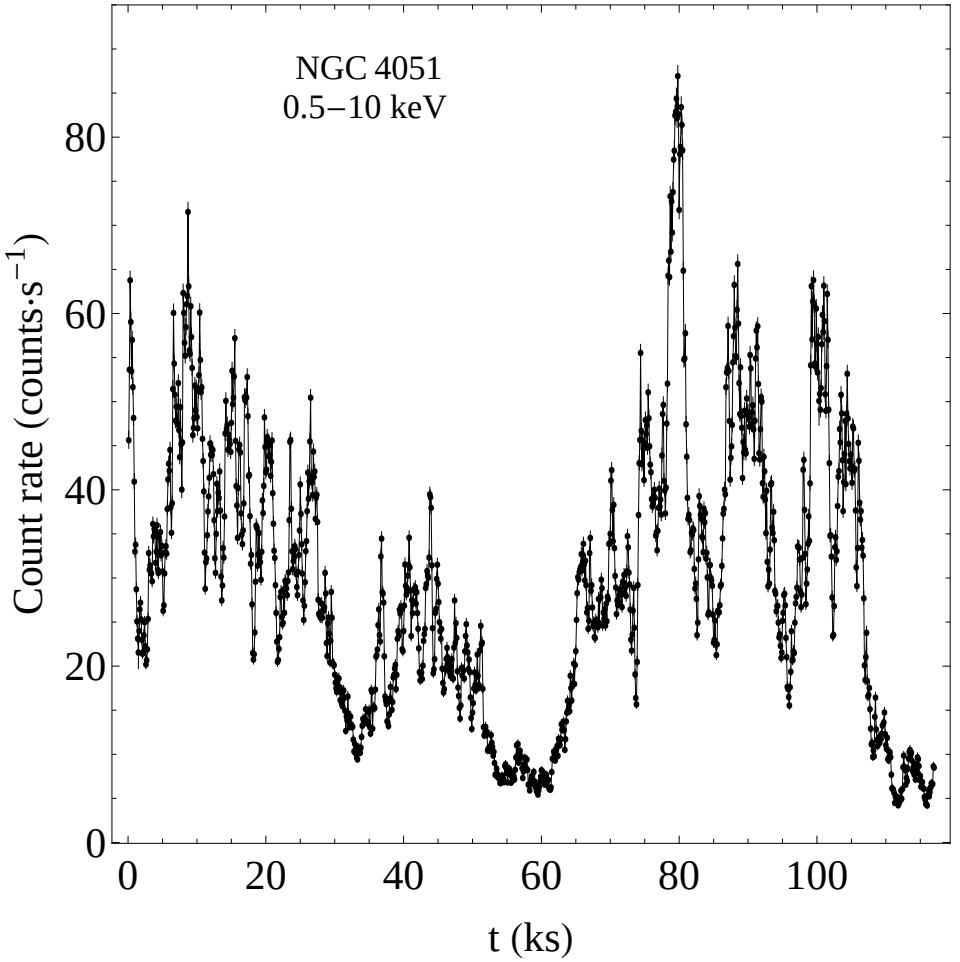
Few times that **Normality** is a problem!

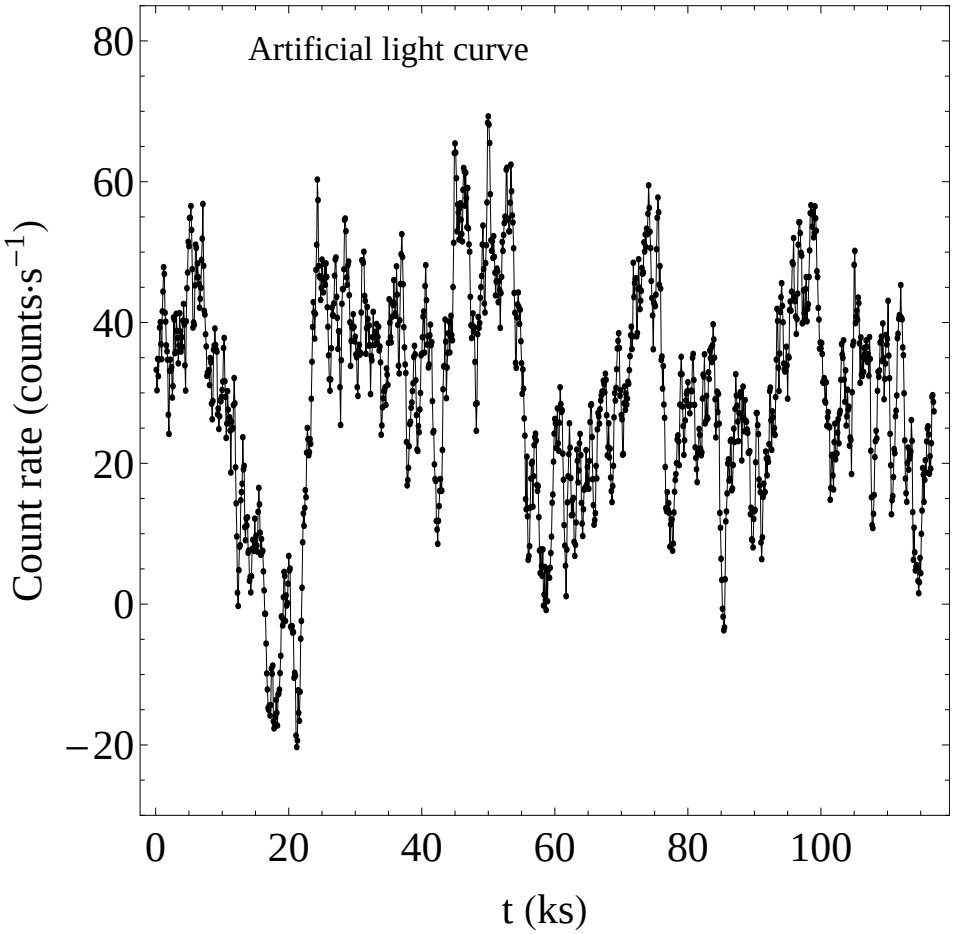
- Wrong underlying probability distribution.
- Physically unrealistic artificial light curves.

The underlying physical flux distribution is always defined for positive flux values.

Monte Carlo problems

Few times that **Normality** is a problem!





Simulating realistic light curves

Emmanoulopoulos et al. 2013, MNRAS, **433**, 907

+

Out[23]=

Generating artificial light curves:Revisited and updated

The numerical code

Dimitrios Emmanoulopoulos†

†Physics and Astronomy, University of Southampton, SO17 1BJ, Southampton, United Kingdom

D.Emmanoulopoulos@soton.ac.uk

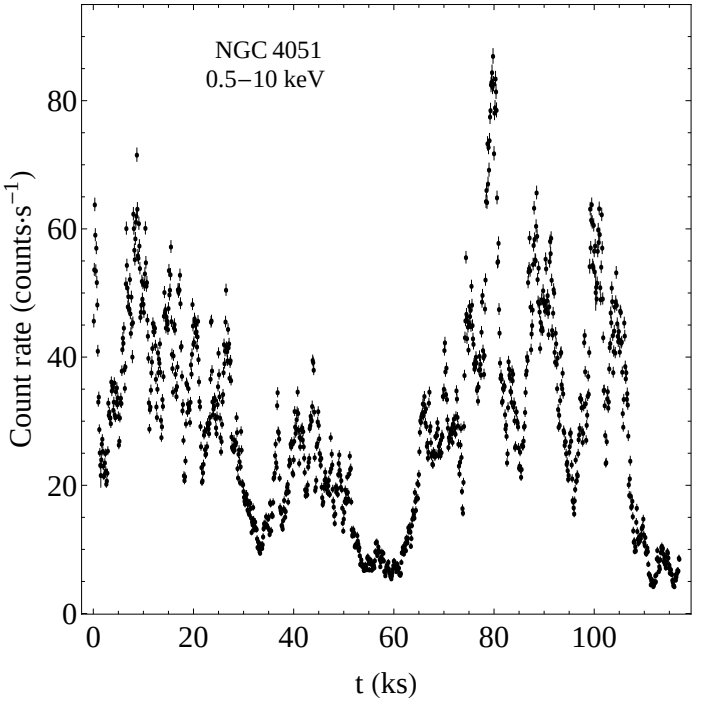
Abstract

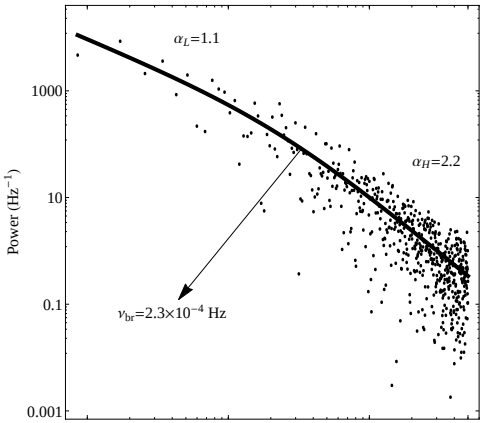
This notebook presents the entire methodology for the production of artificial light curves as presented in § 3.1 in the corresponding MNRAS paper. As it is done in the paper (
Newton (obs ID: 0109141401), for the production of a single artificial light curve having the same statistical and variability properties as the observed light curves. The method
described in the MNRAS paper. As in any time series methodology we caution the readers that the outputs of the method depends ONLY in the input parameters and thus the

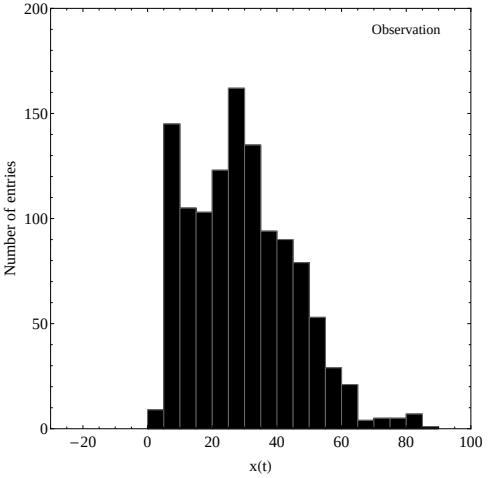
For all the astrophysical fields!

Simulating realistic light curves

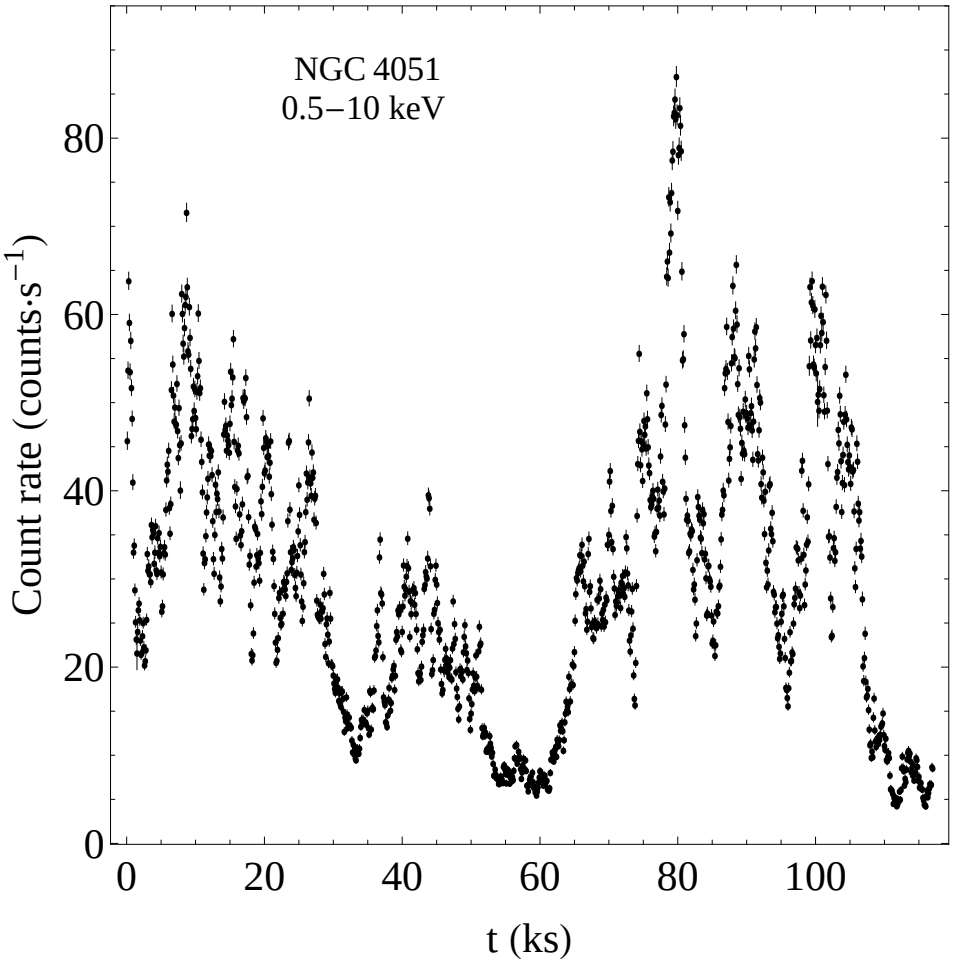
We randomize amplitudes and phases in an iterated fashion.

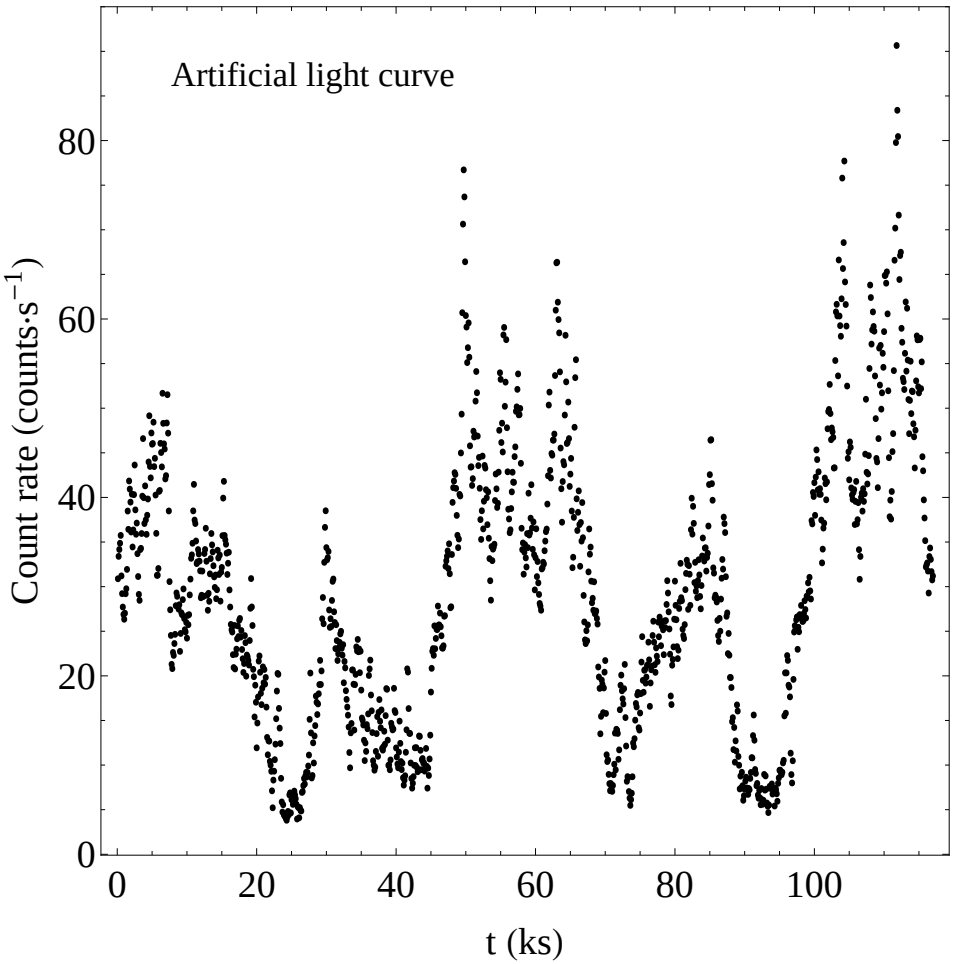




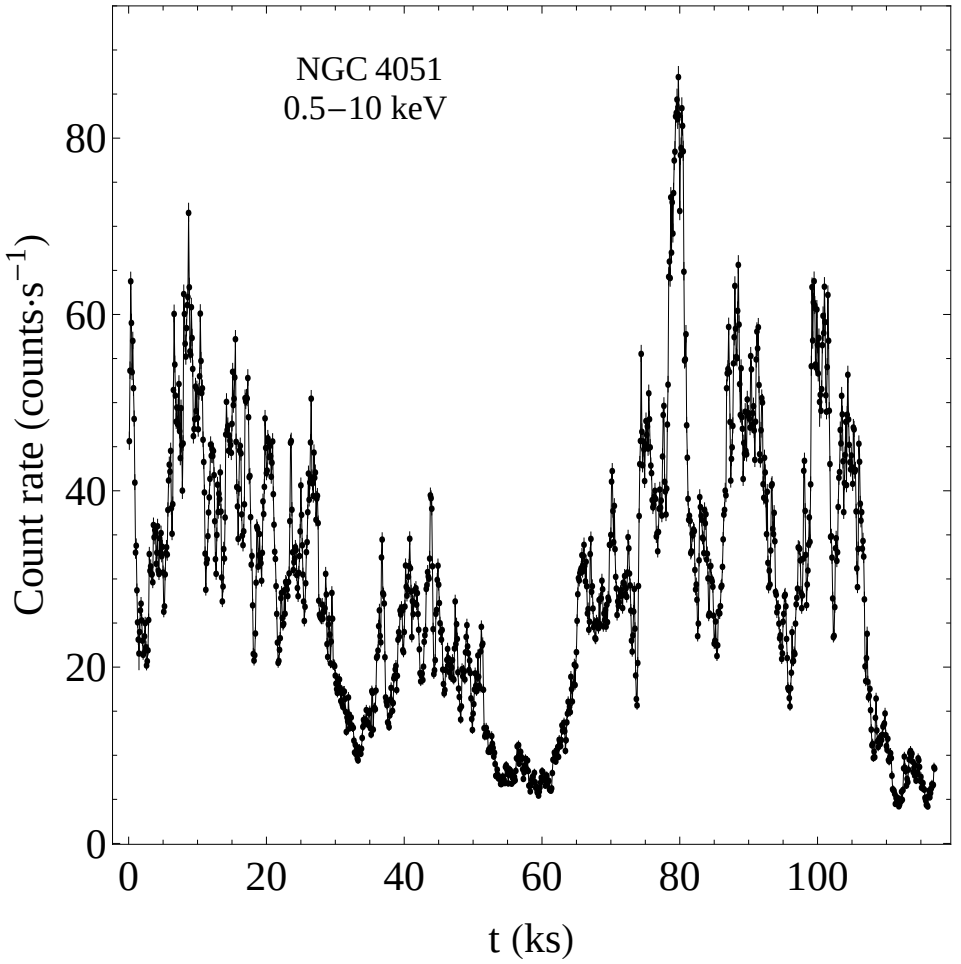


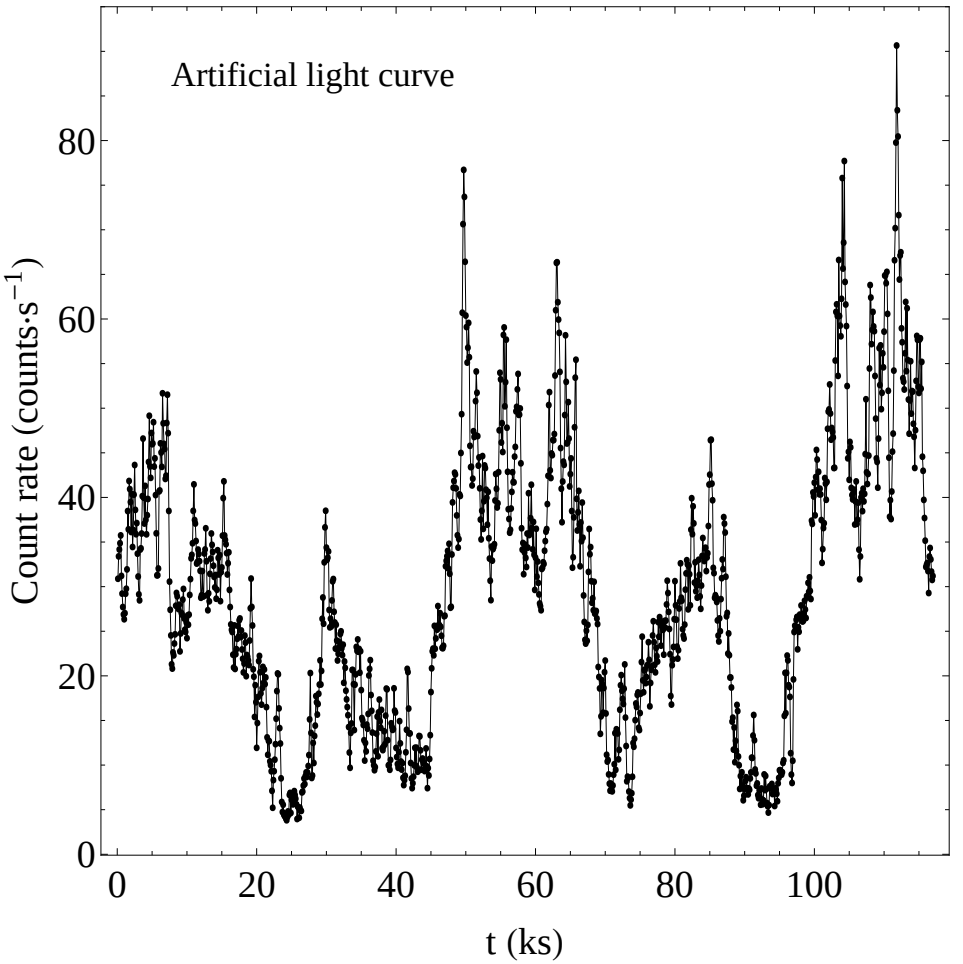
Simulating realistic light curves



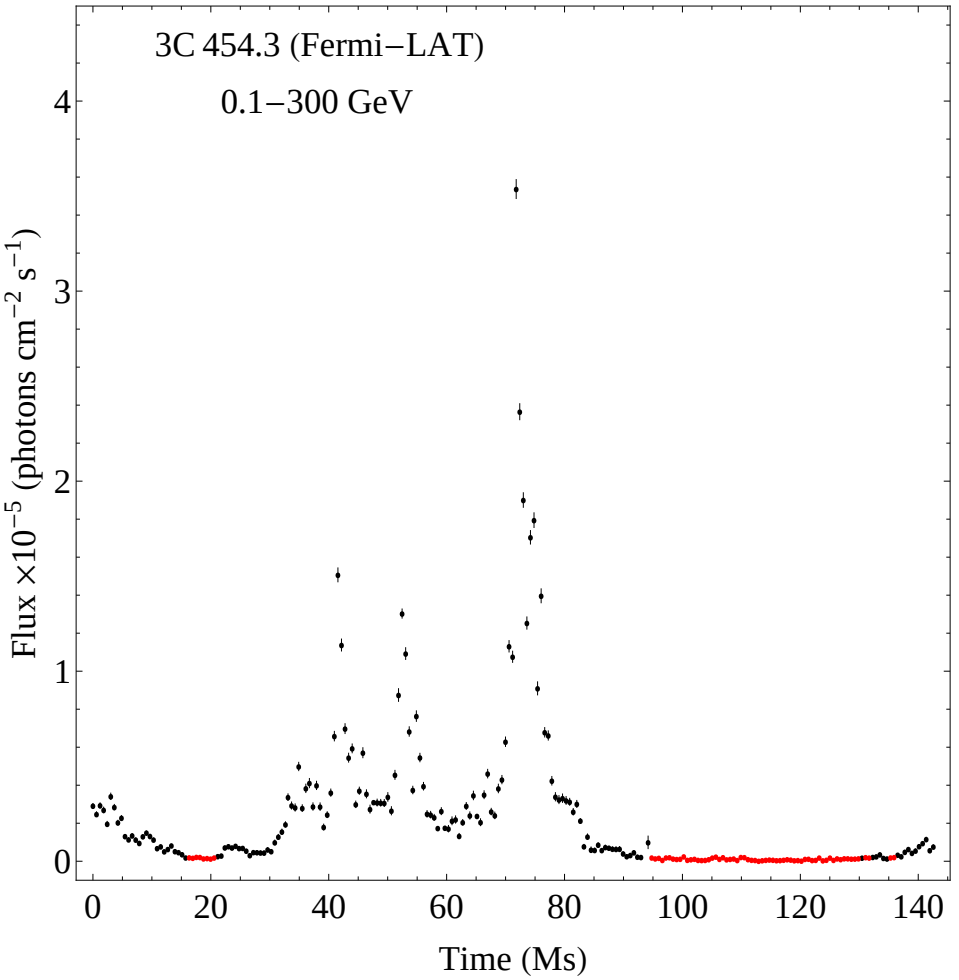


Simulating realistic light curves

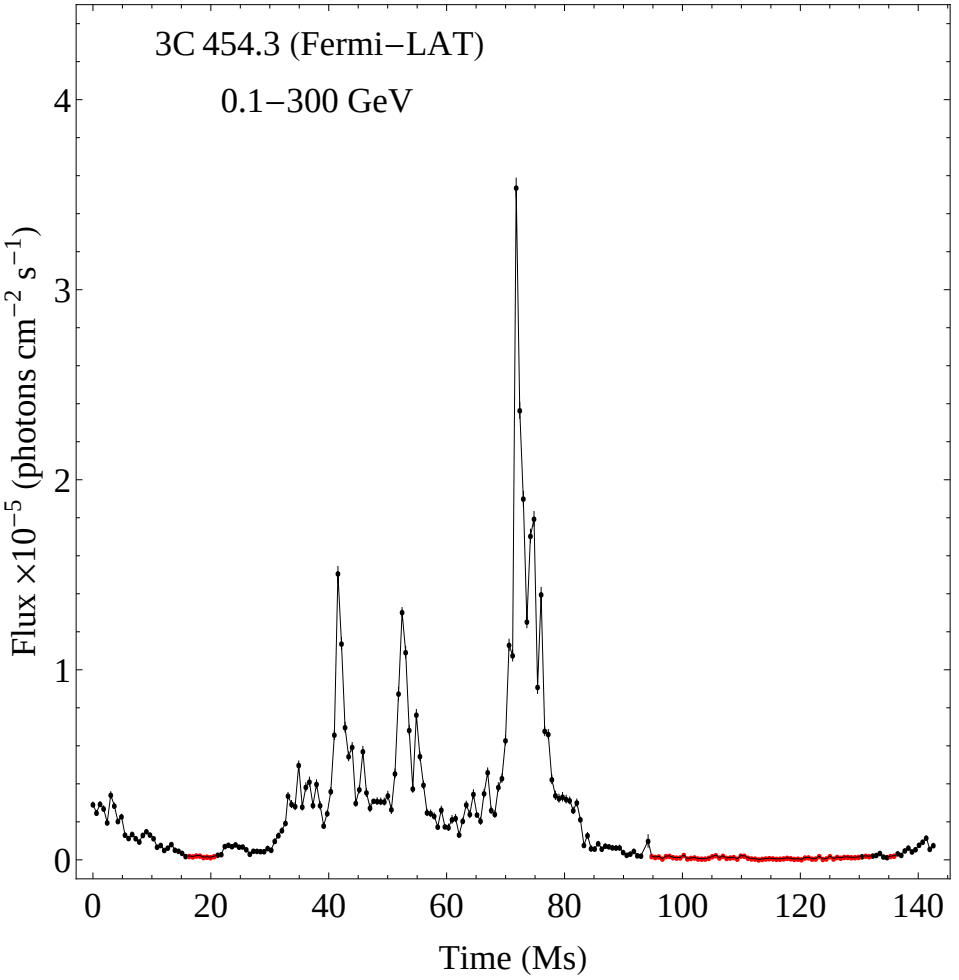


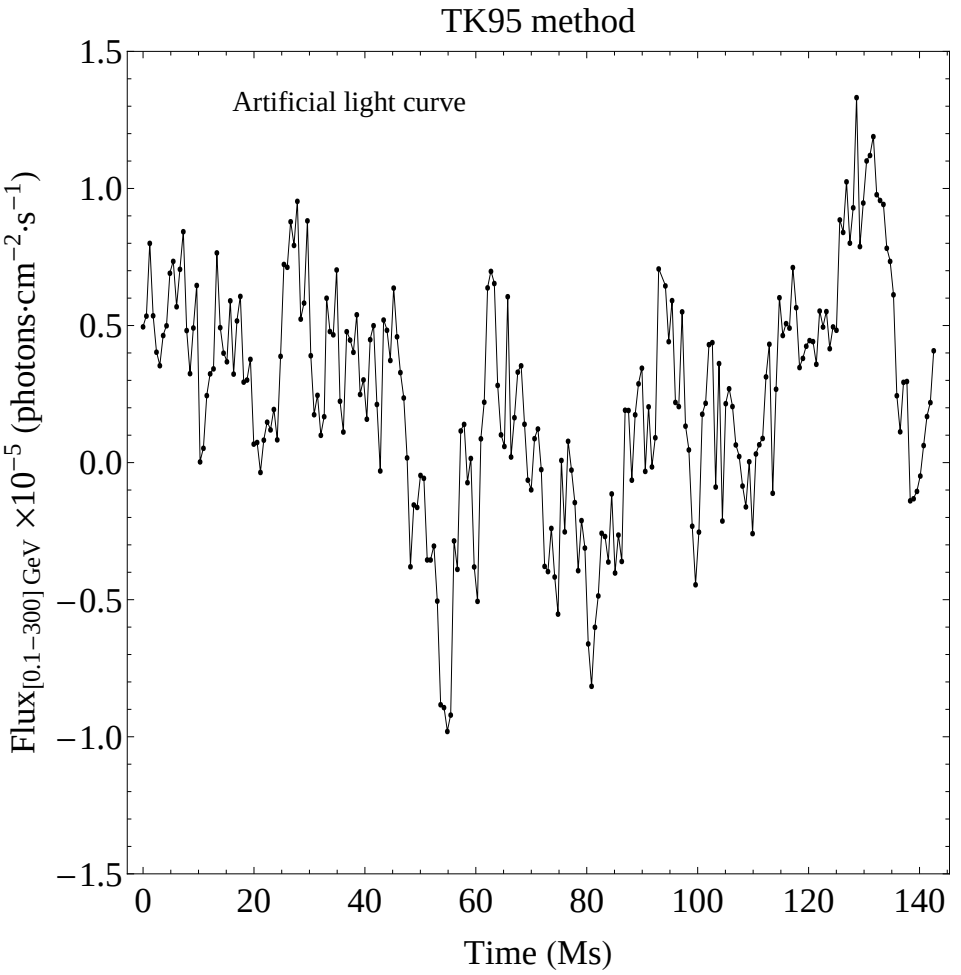


Simulating realistic light curves

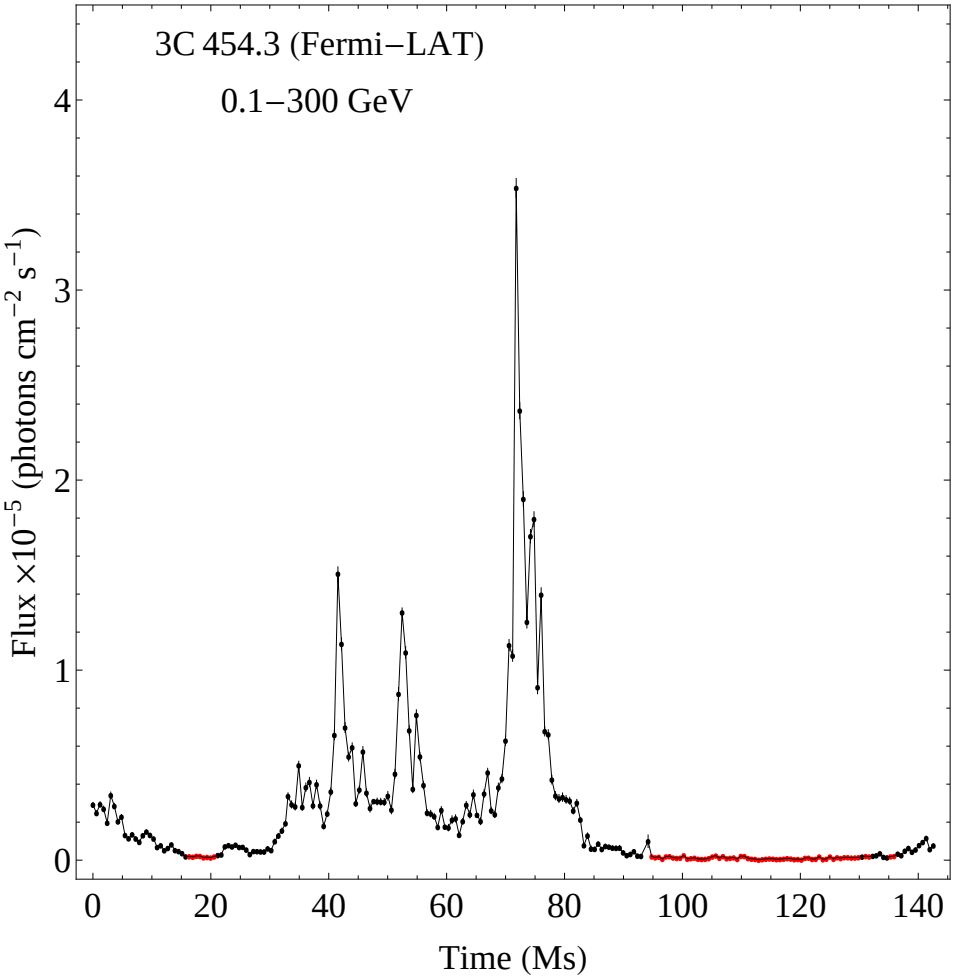


Simulating realistic light curves

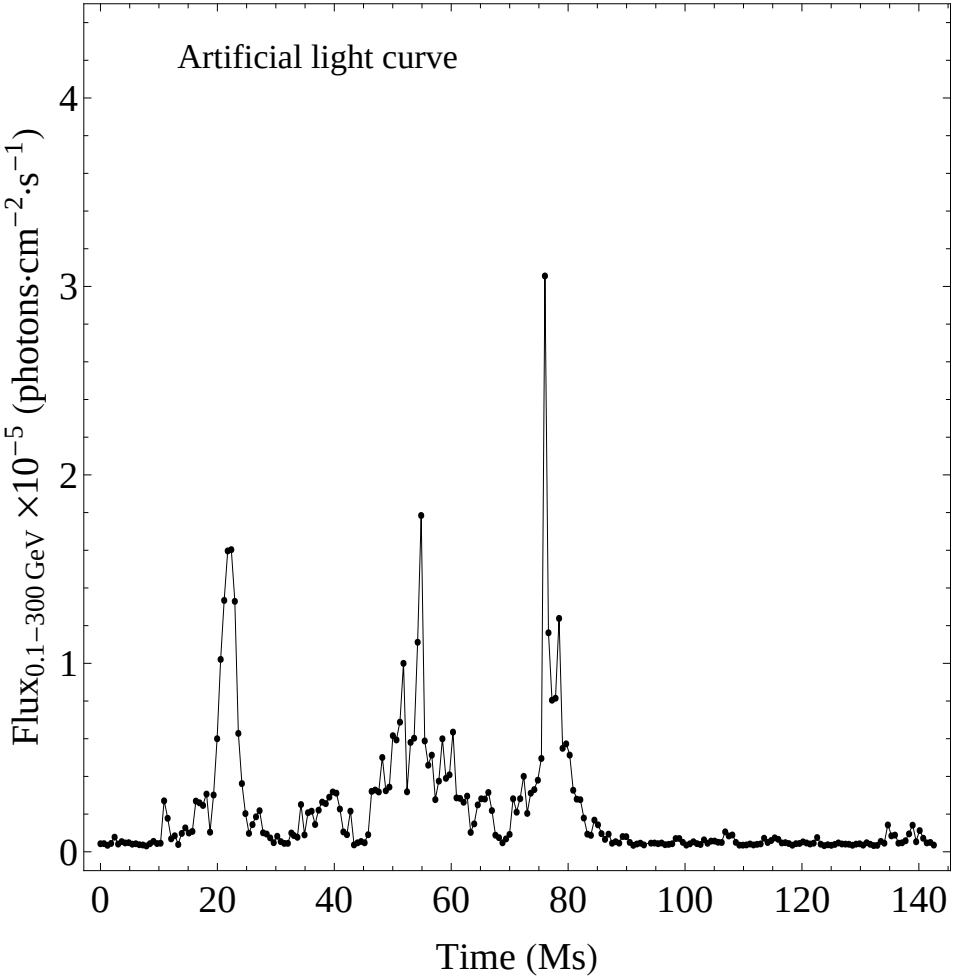




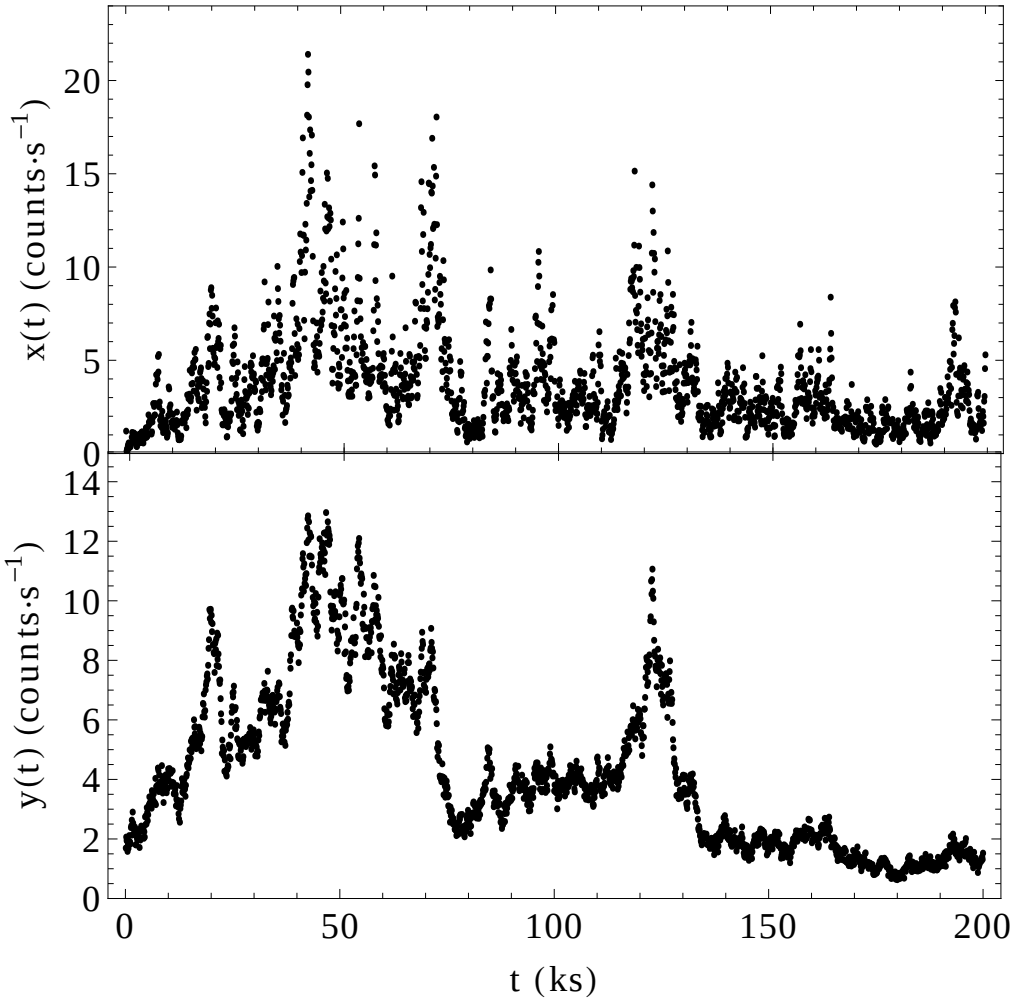
Simulating realistic light curves



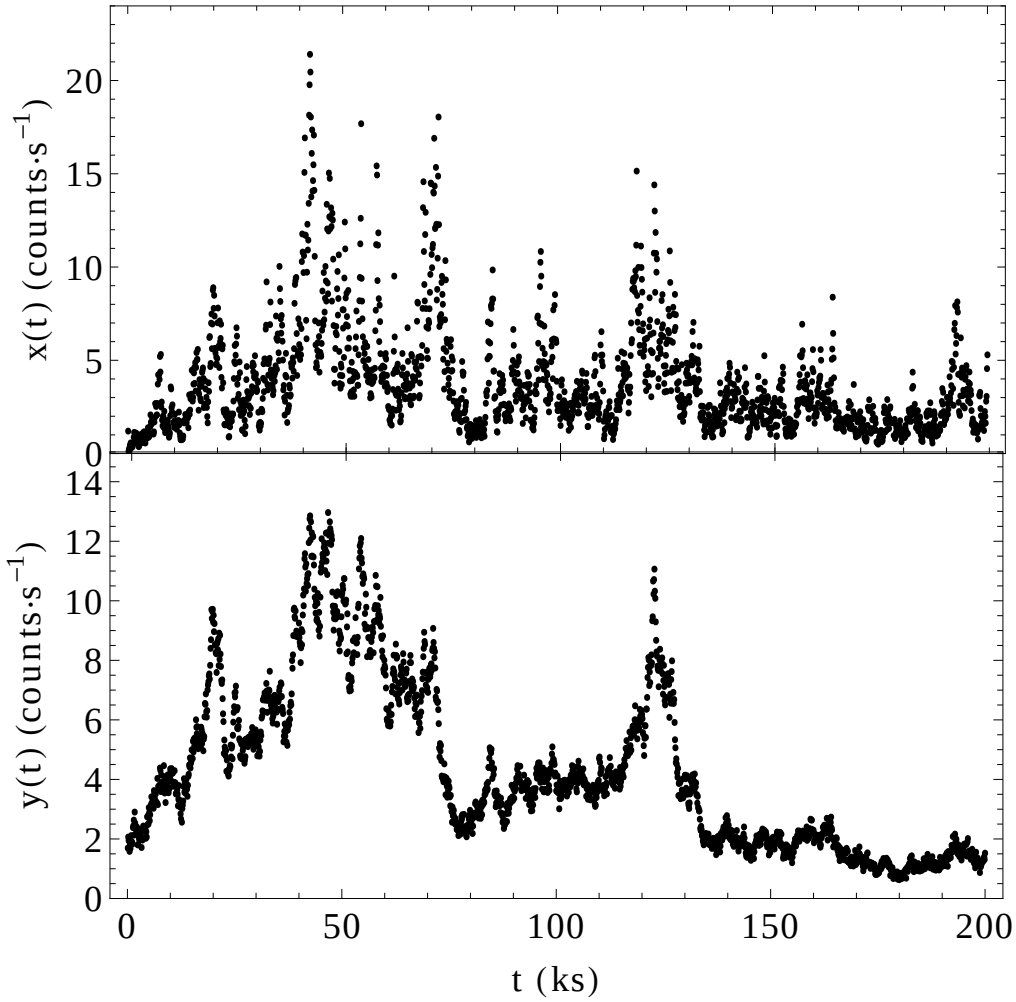
New method

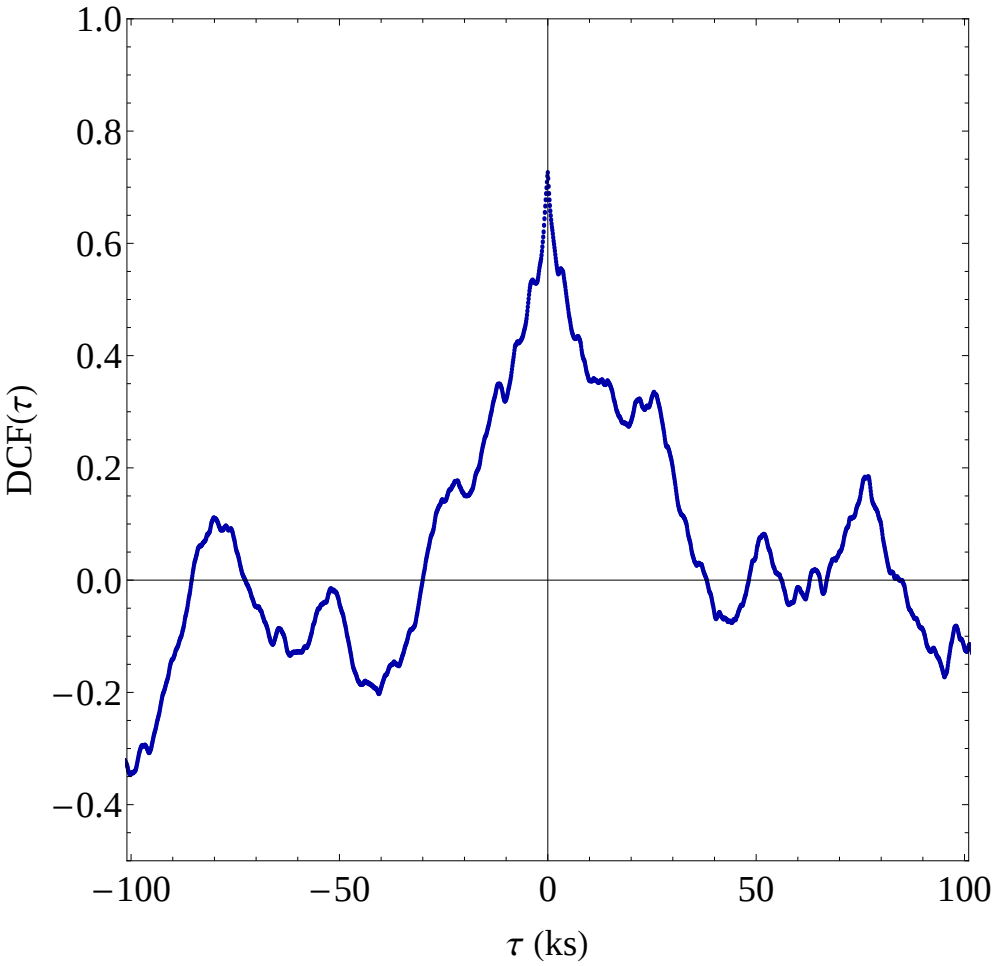


Simulating realistic light curves

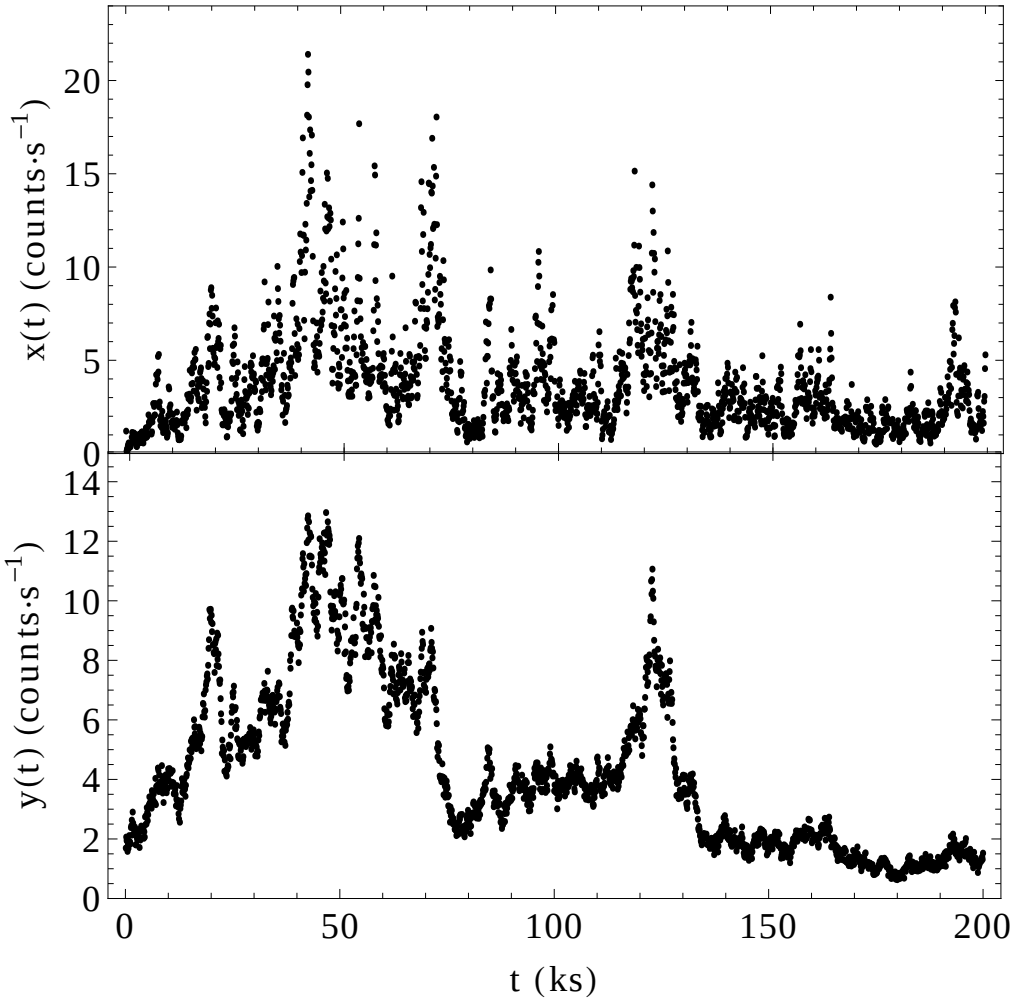


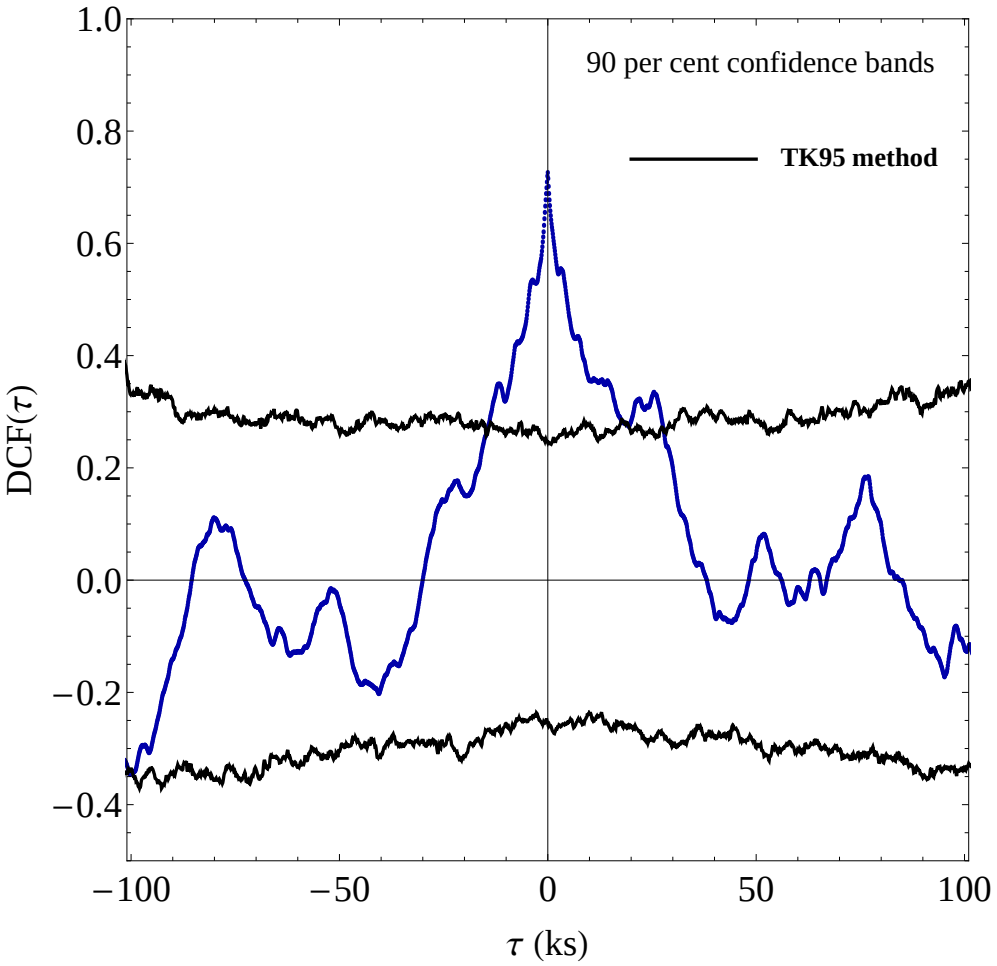
Simulating realistic light curves



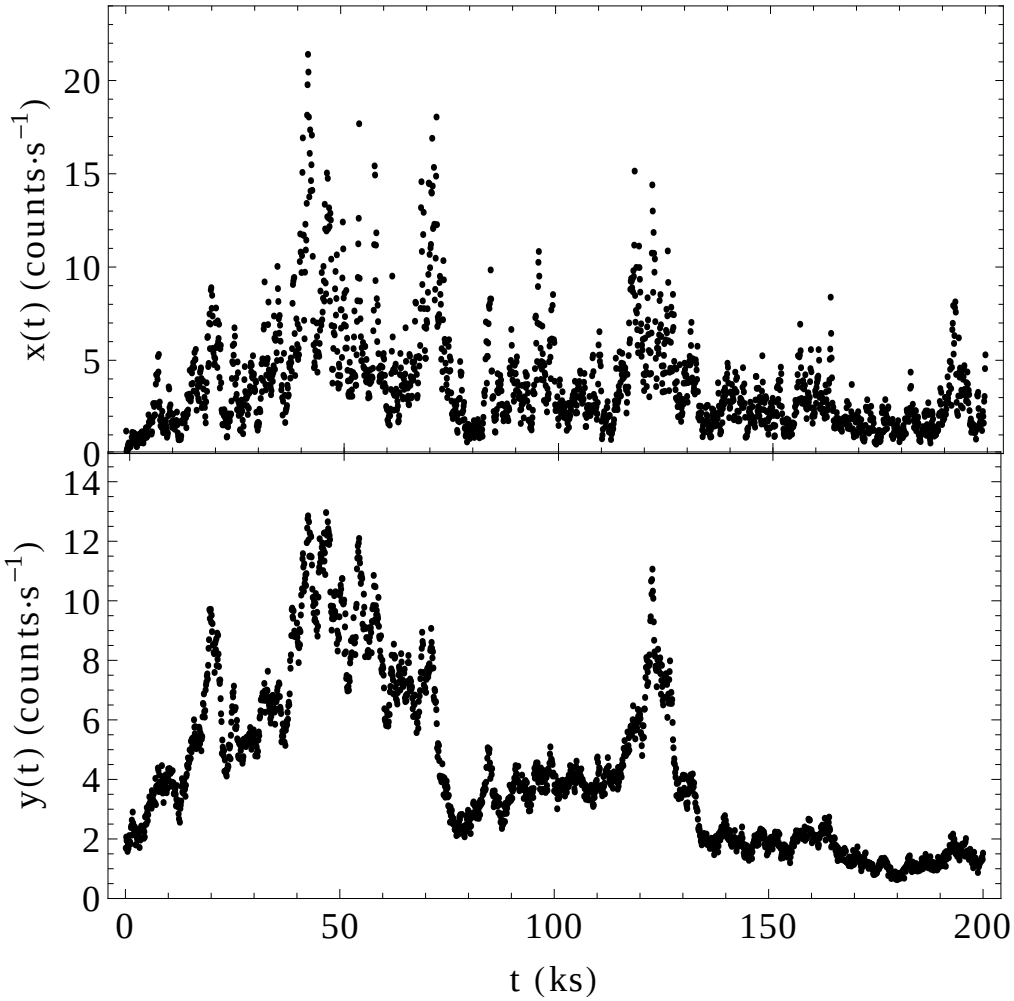


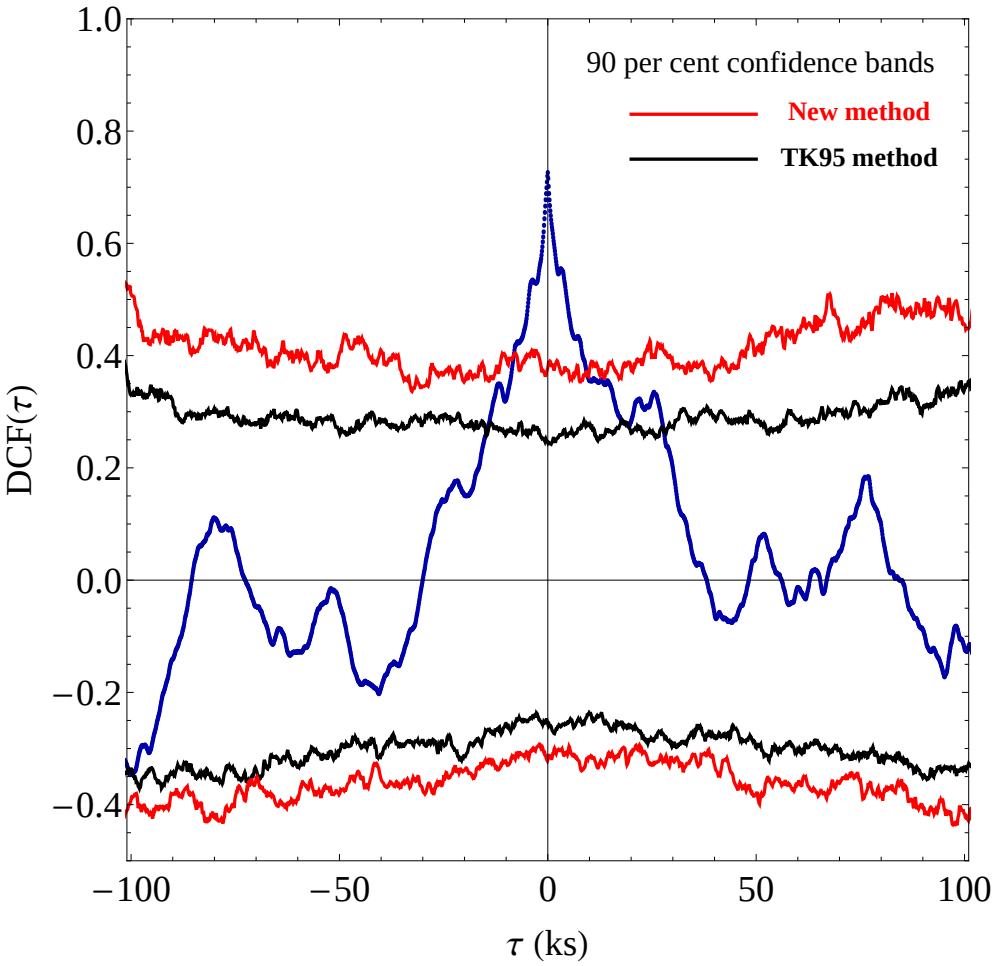
Simulating realistic light curves



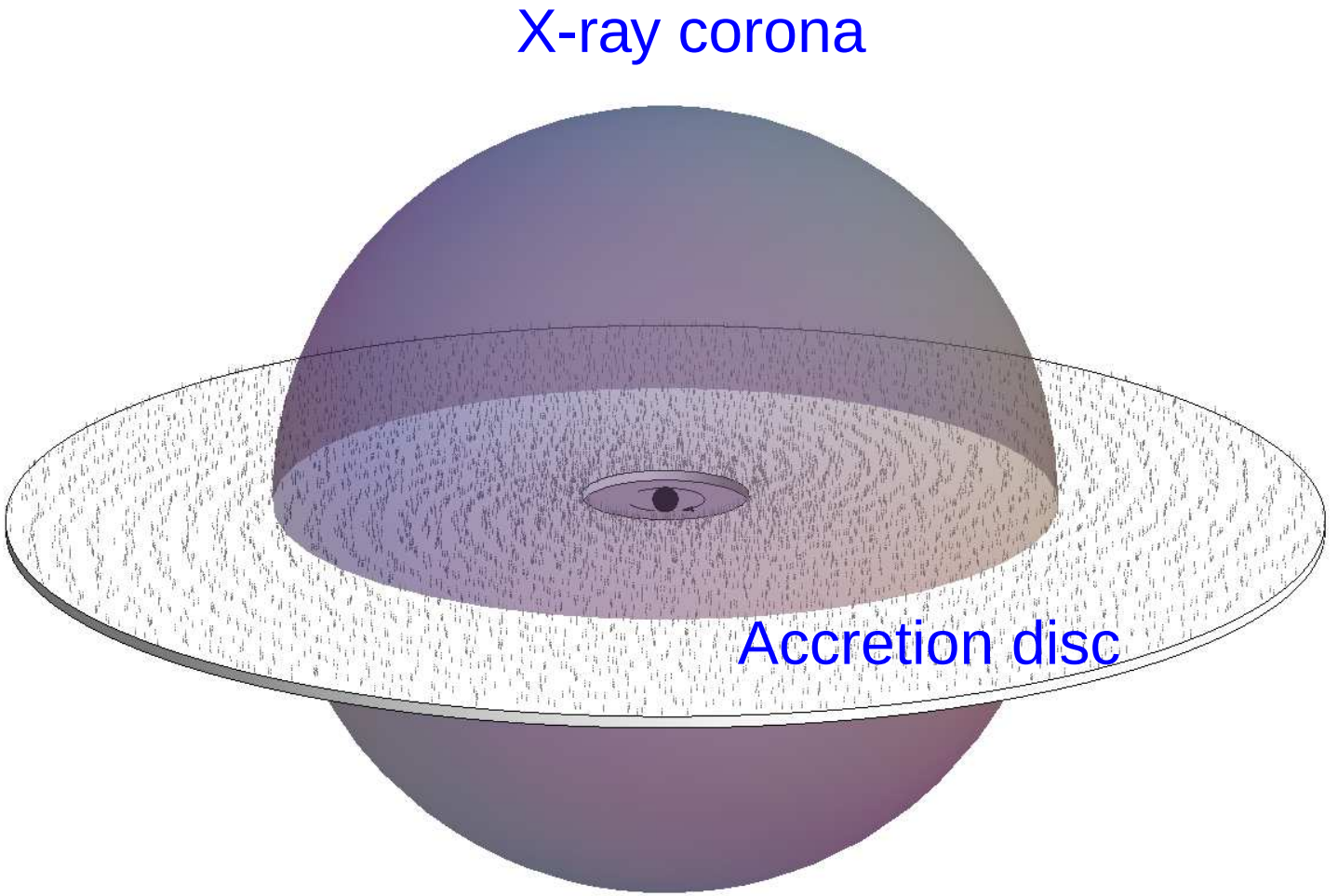


Simulating realistic light curves



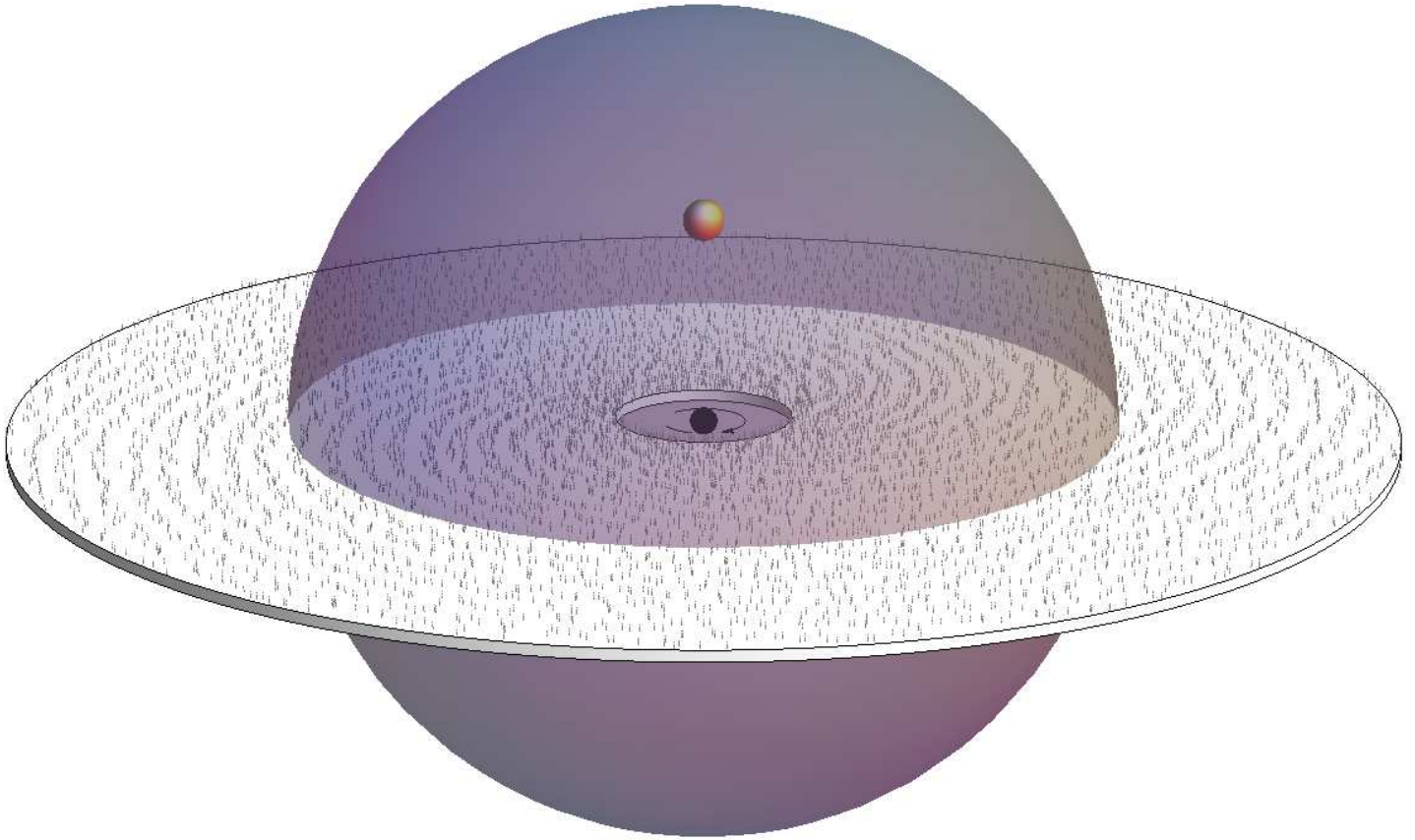


Current paradigm

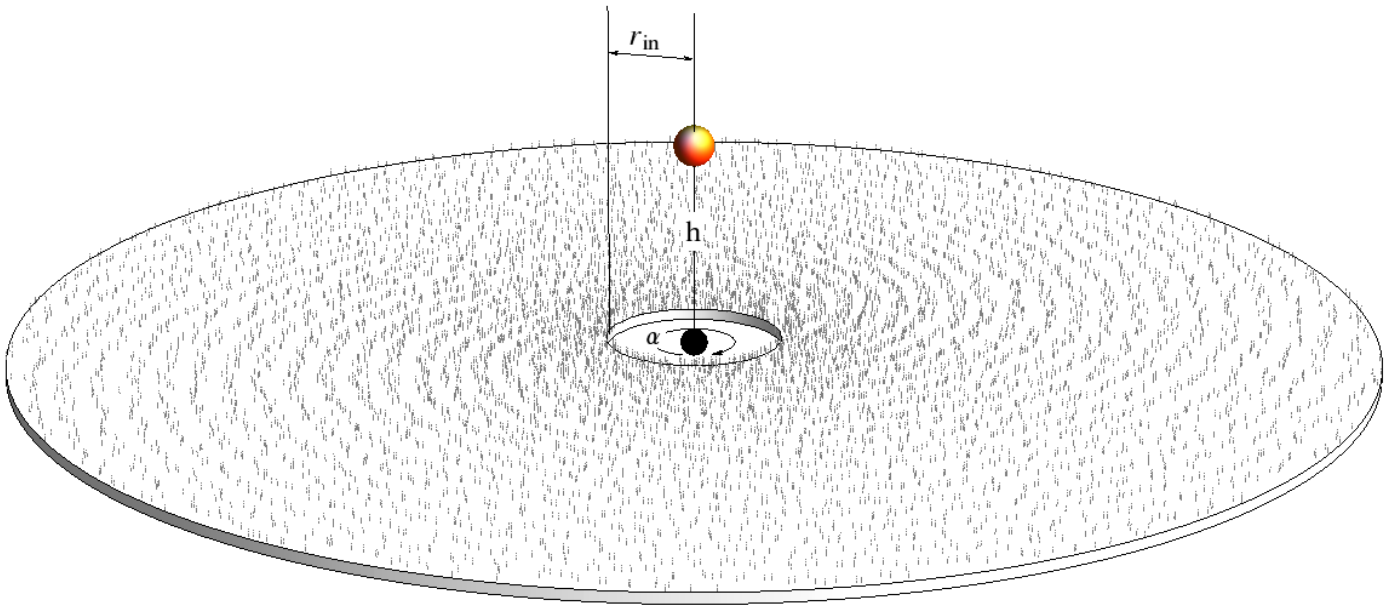


Current paradigm

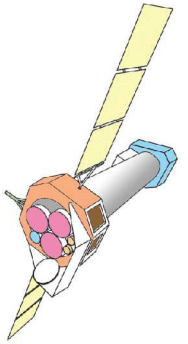
Approximation of spherical corona by point source

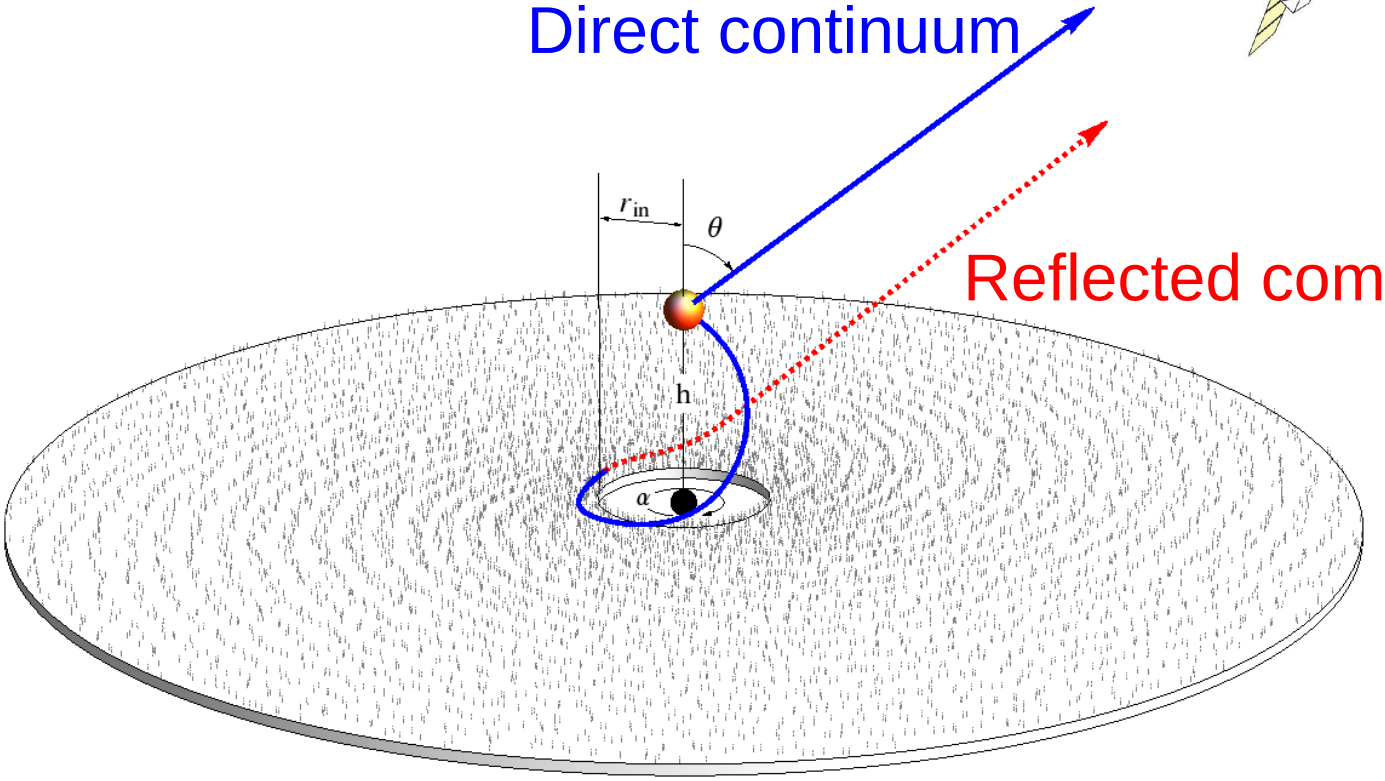


Current paradigm

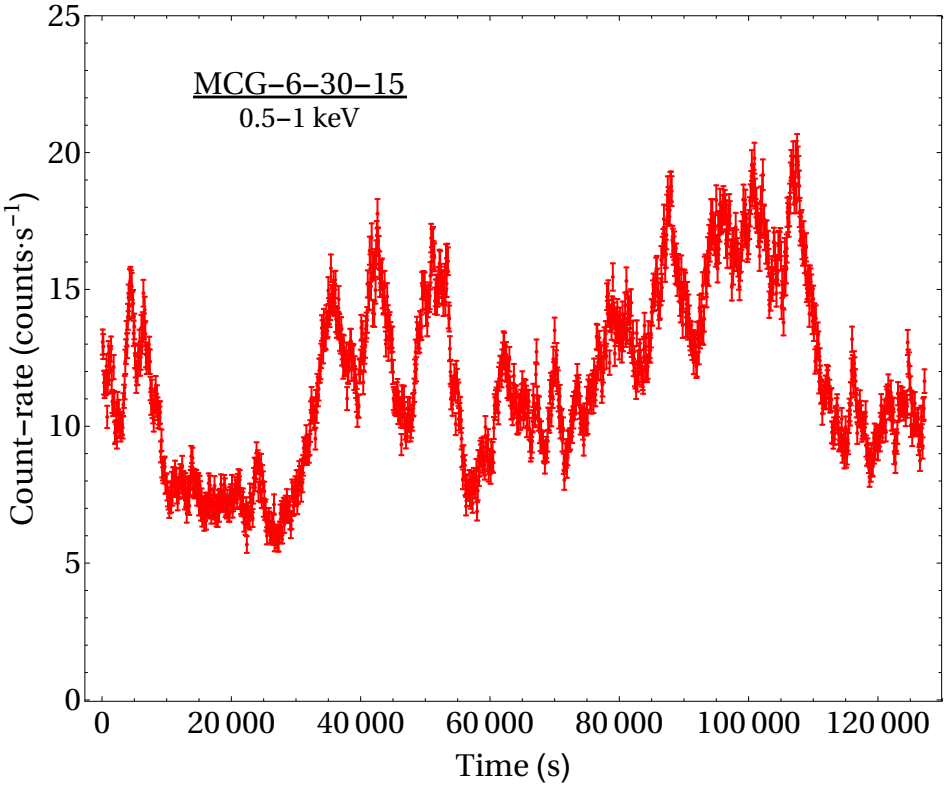


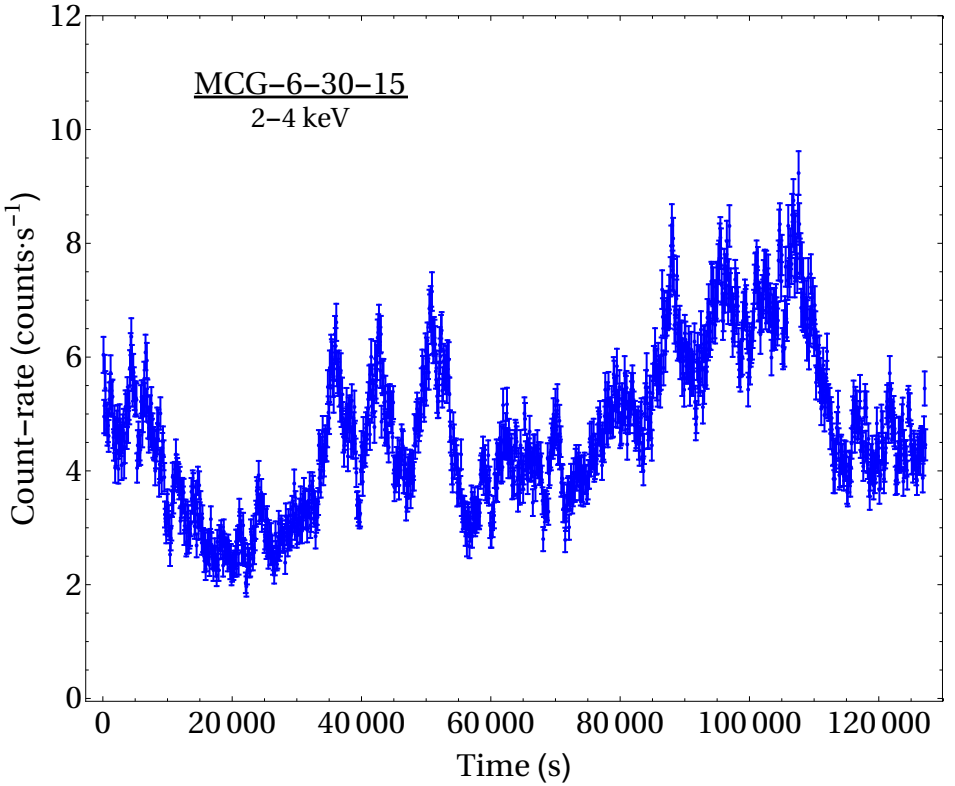
Current paradigm





Time Series Approach





Time Series Approach

Cross spectrum

$$\mathcal{C}_{s,h}(f_j) = S^*(f_j)H(f_j) = |S(f_j)||H(f_j)|e^{i(\phi_H(f_j)-\phi_S(f_j))}$$

Time Series Approach

Cross spectrum

$$\mathcal{C}_{s,h}(f_j) = S^*(f_j)H(f_j) = |S(f_j)||H(f_j)|e^{i(\phi_H(f_j)-\phi_S(f_j))}$$

Phase spectrum

$$\phi(f_{\mathrm{bin},i}) = \arg\left[\langle \mathcal{C}_{s,h}(f_{\mathrm{bin},i})\rangle\right] = \arctan\left[\frac{\mathrm{Im}\left[\langle \mathcal{C}_{s,h}(f_{\mathrm{bin},i})\rangle\right]}{\mathrm{Re}\left[\langle \mathcal{C}_{s,h}(f_{\mathrm{bin},i})\rangle\right]}\right]$$

Time Series Approach

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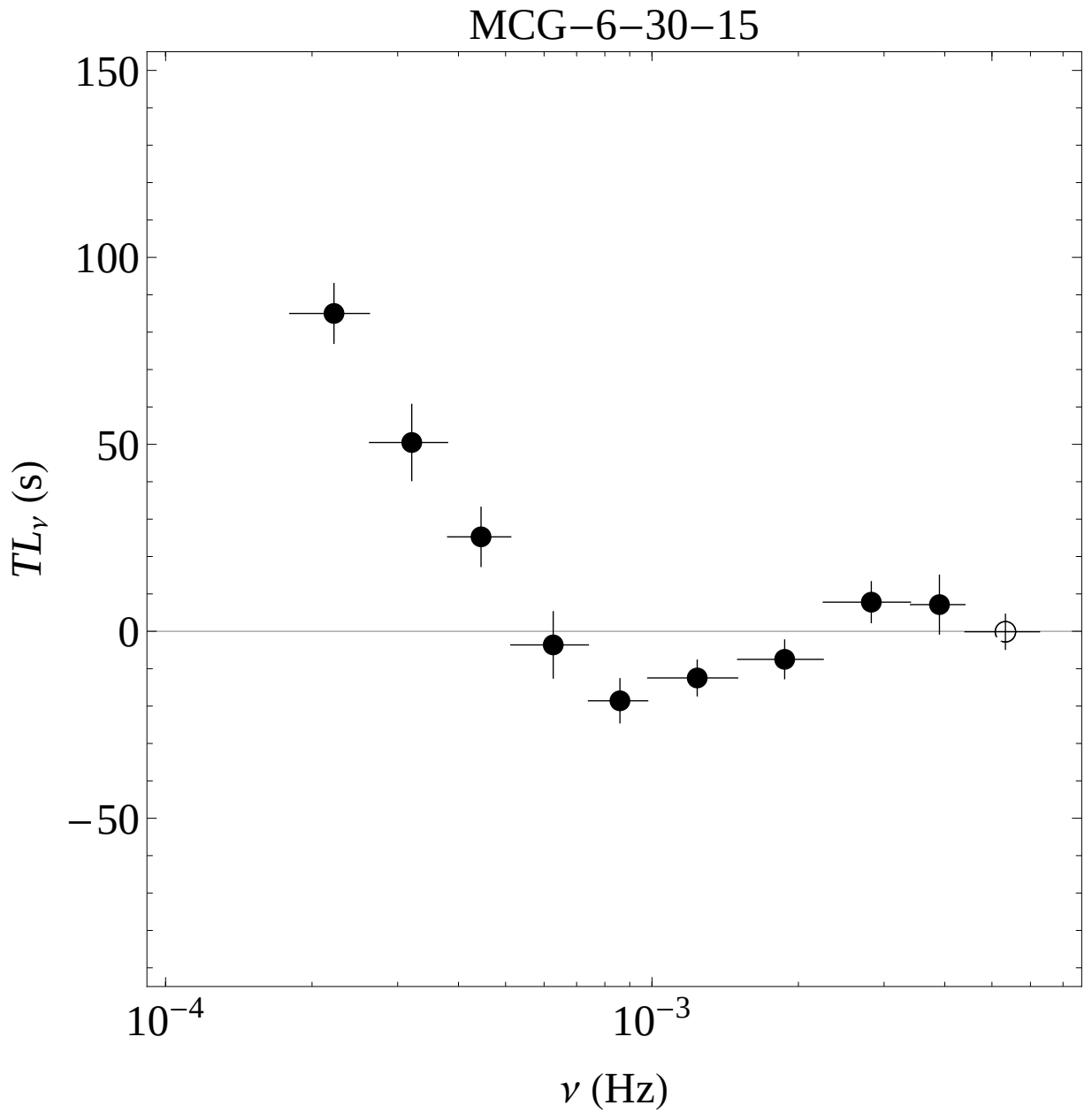
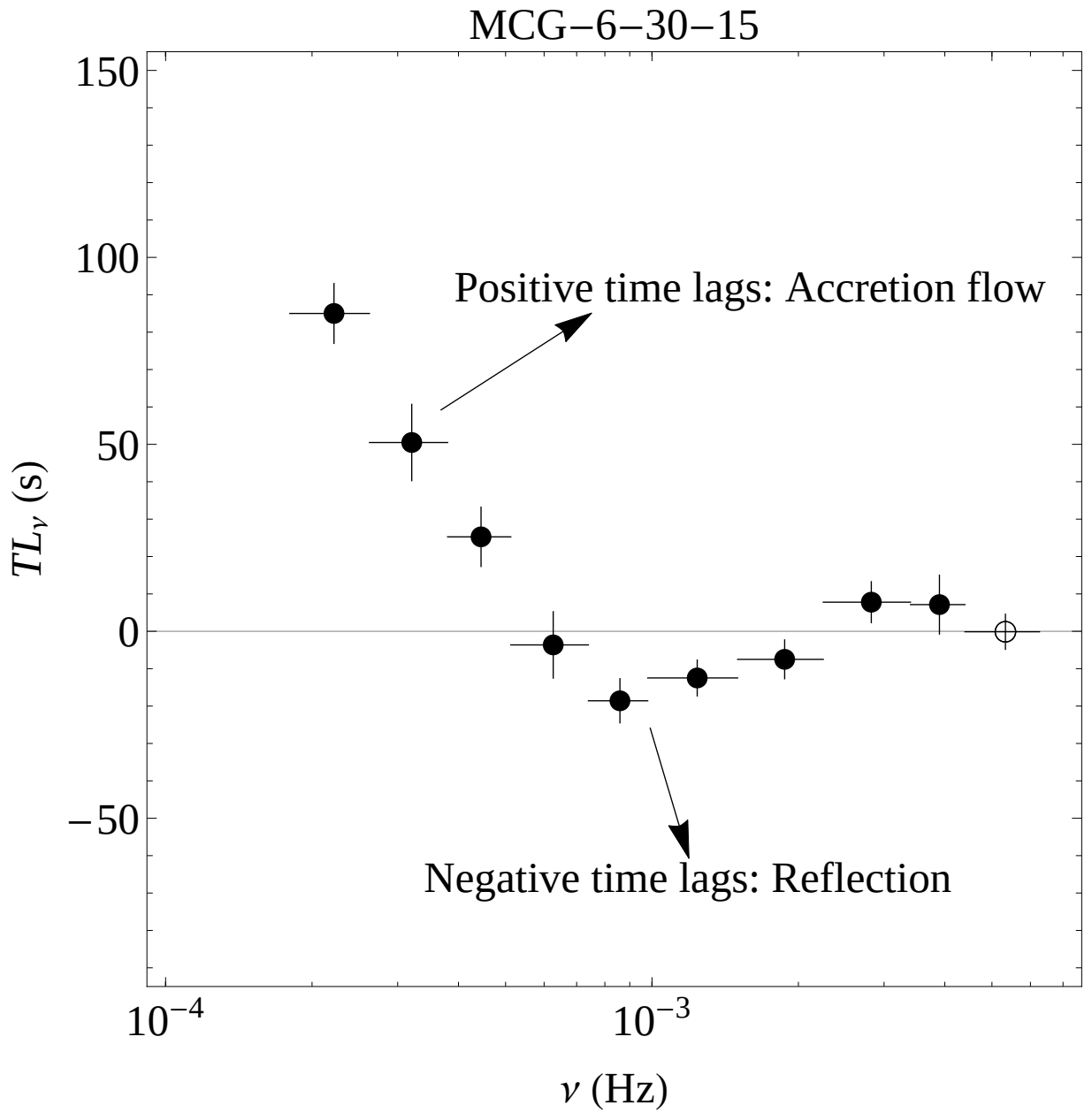
Time-lag spectrum

$$TL(f_{\mathrm{bin},i}) = \frac{\phi(f_{\mathrm{bin},i})}{2\pi f_{\mathrm{bin},i}}$$

Time Series Approach



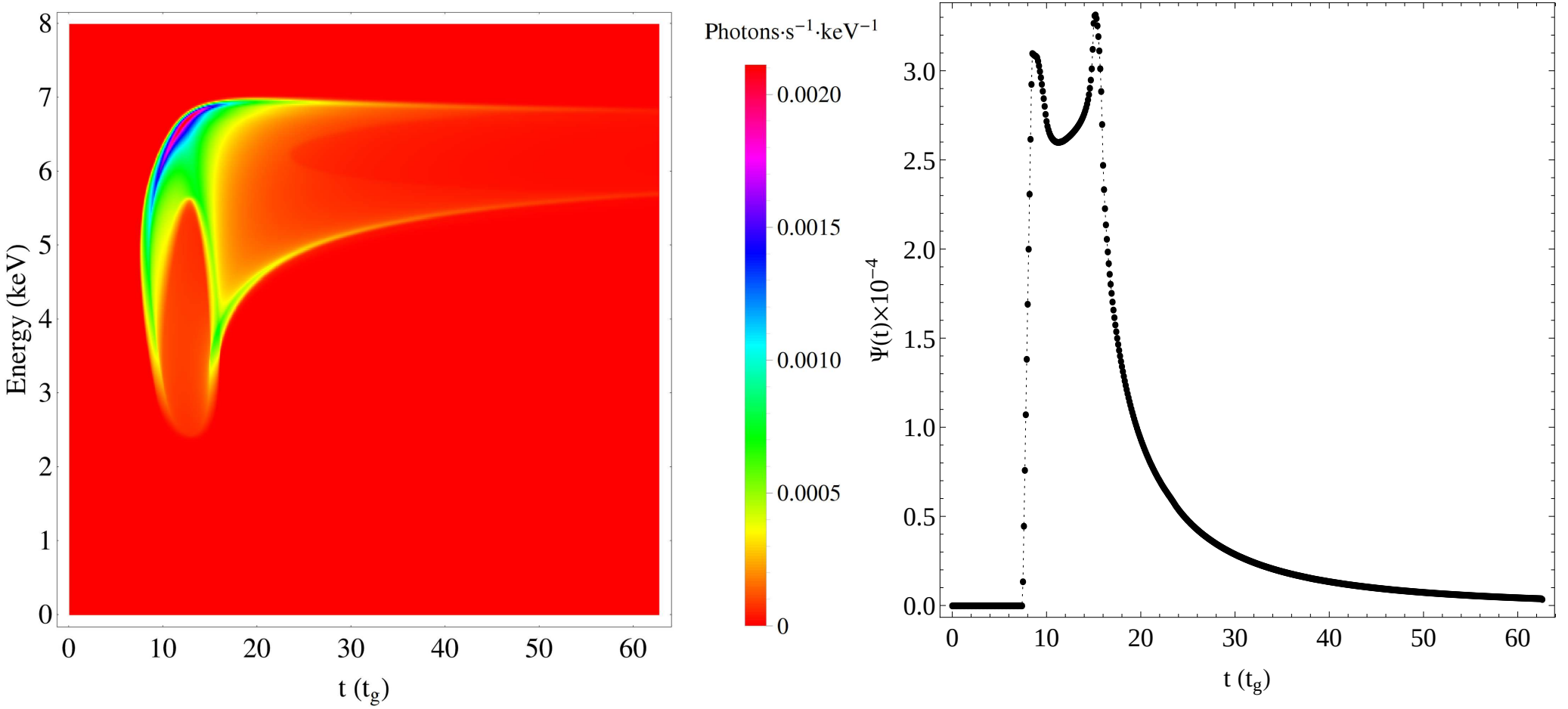
Time Series Approach



X-ray reverberation: Modelling

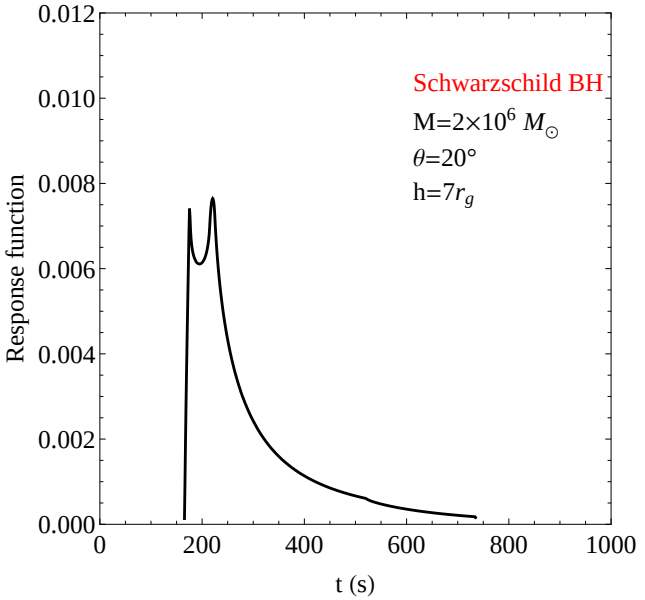
Response profiles: Dovčiak et al. 2011, ApJ, **731**, 75

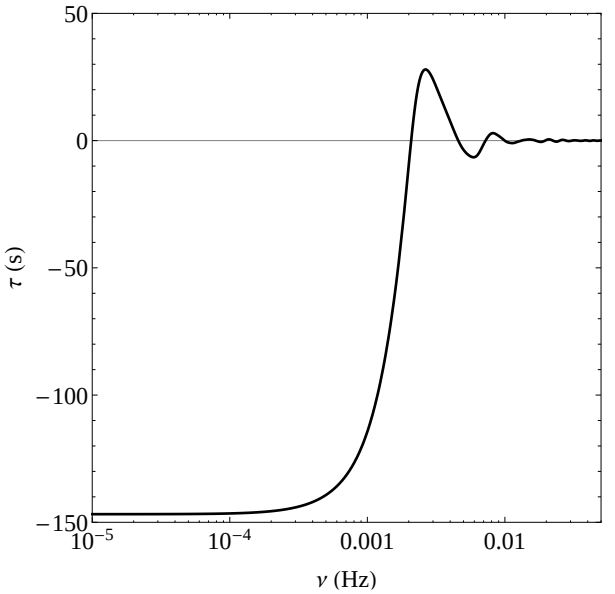
Spin$= 0.676$, Height$= 3.6\ r_{\rm g}$ and Angle=40°

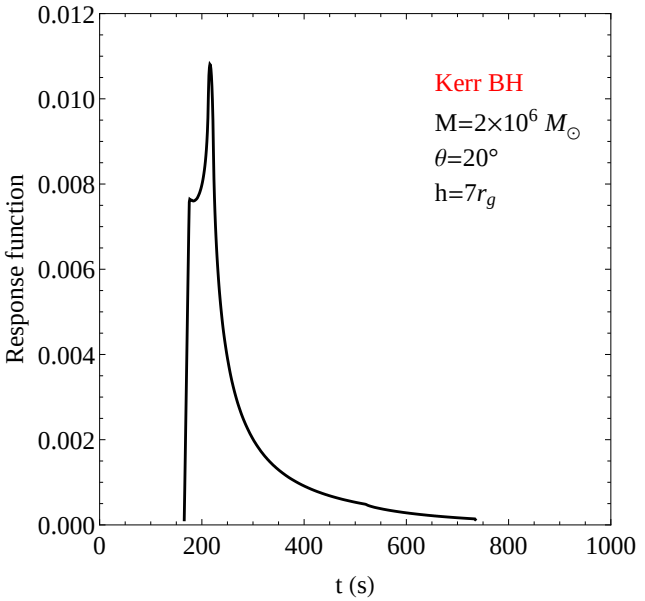


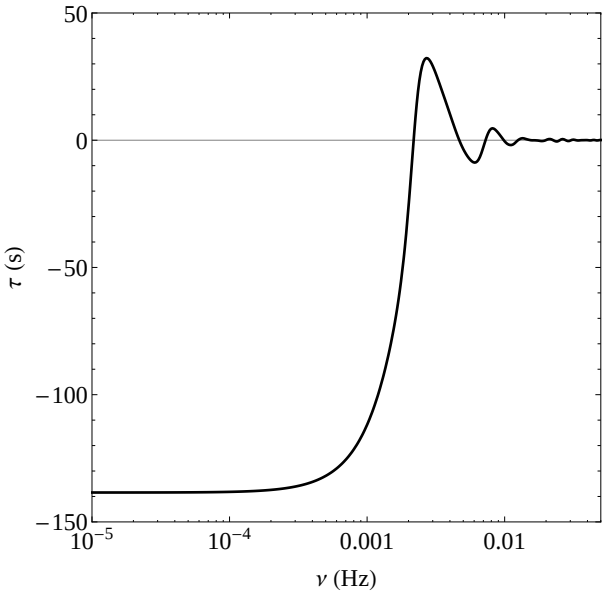
X-ray reverberation: Modelling

GR reflection component: BH Spin

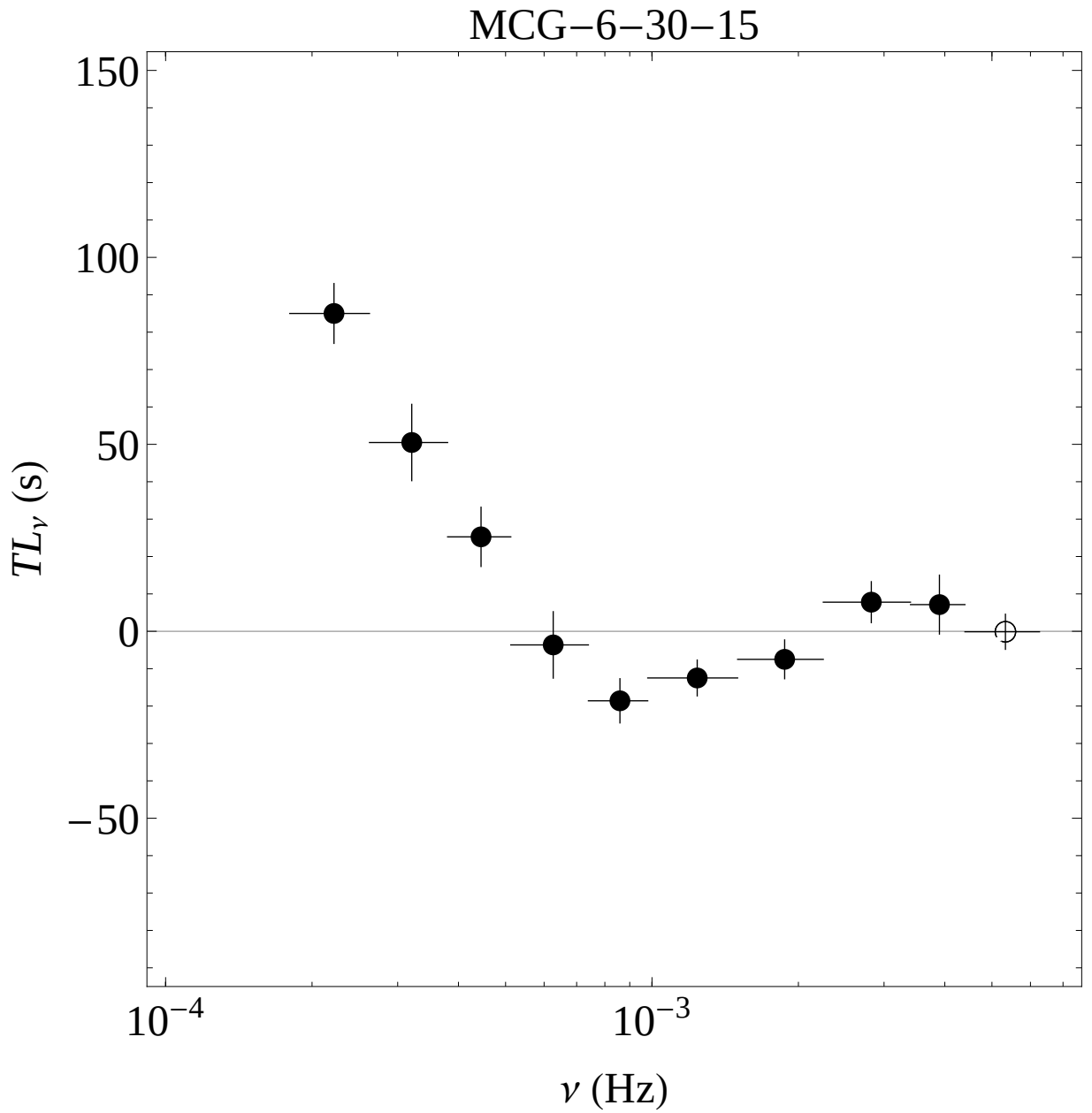




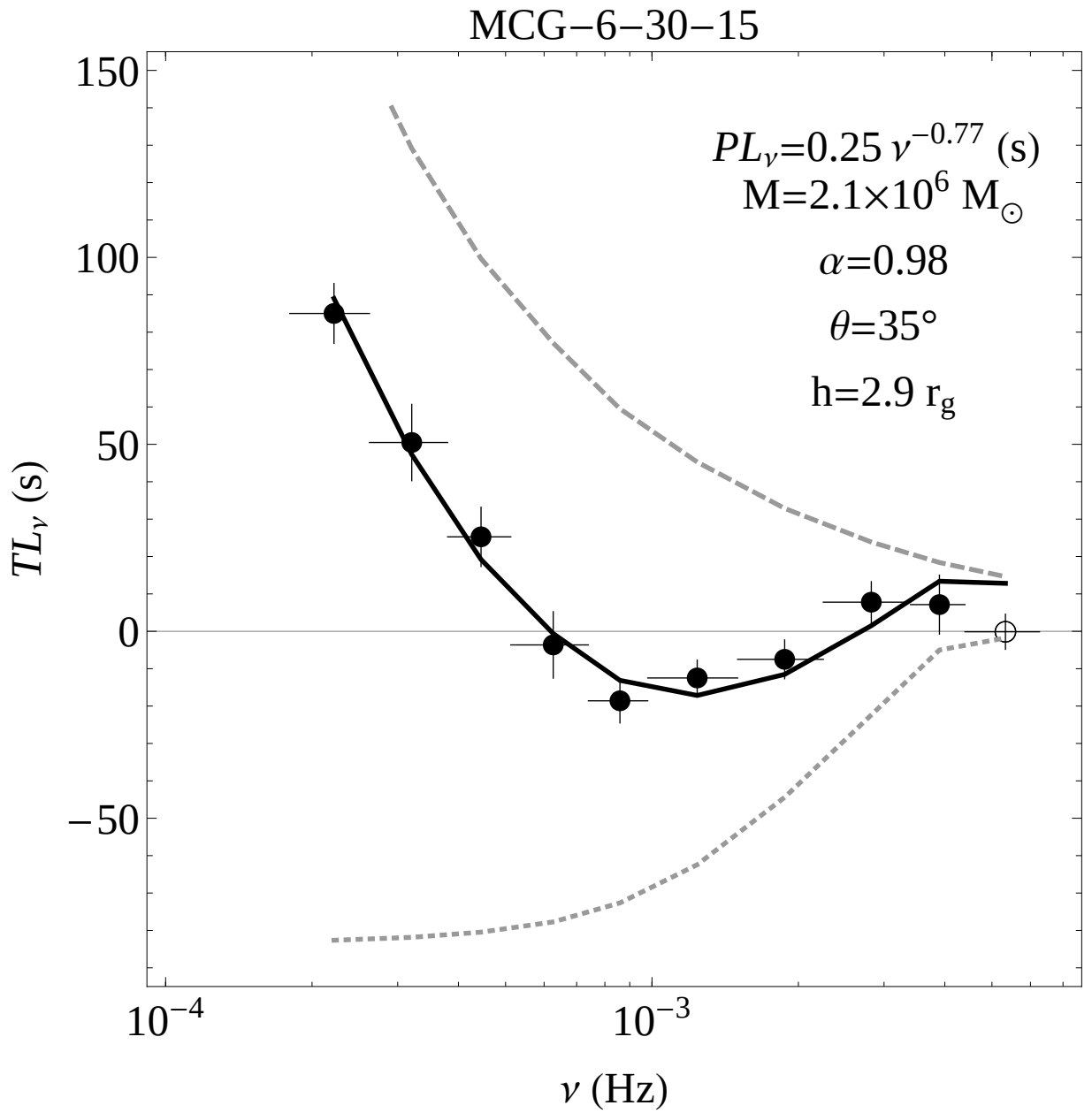




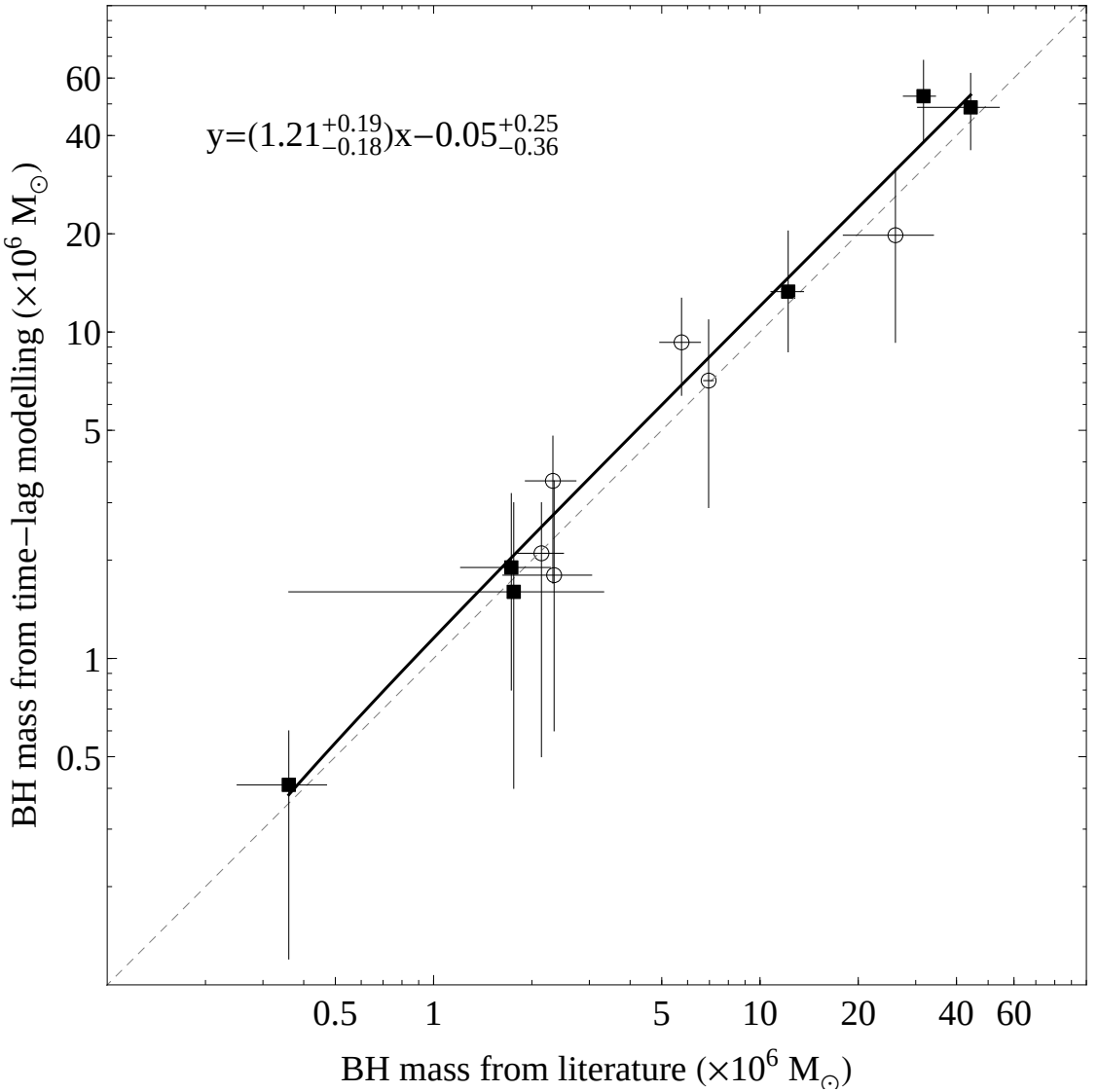
X-ray reverberation: Modelling



X-ray reverberation: Modelling

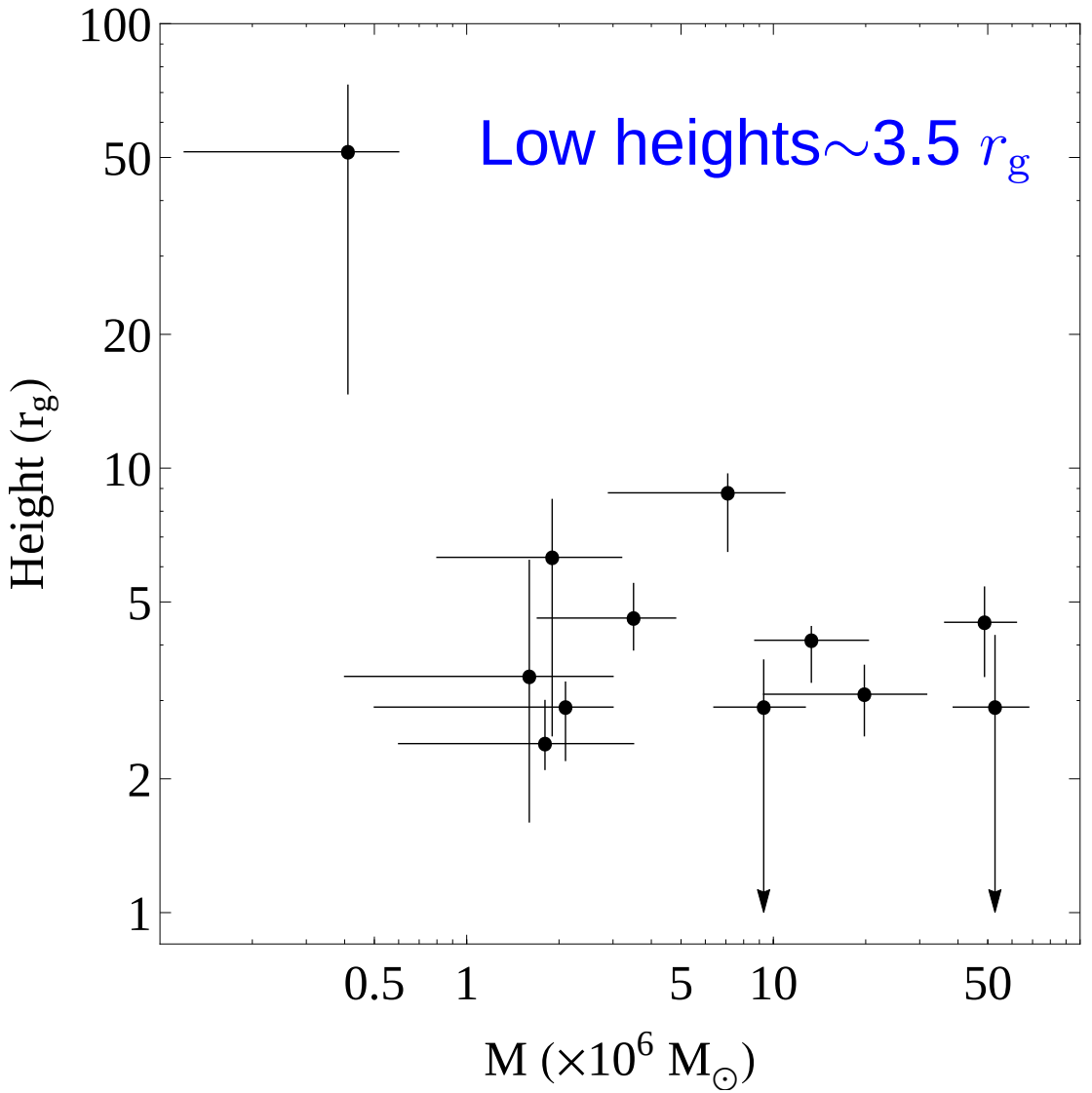


X-ray reverberation: Modelling



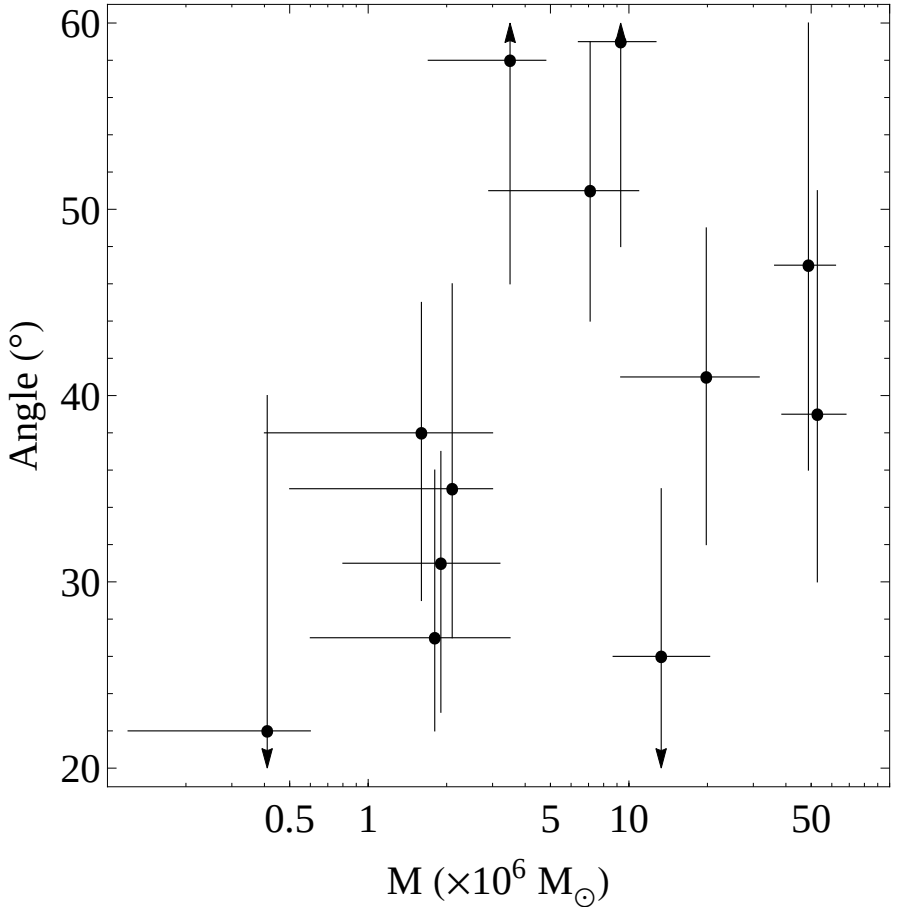
Emmanoulopoulos et al. 2014, MNRAS, **439**, 3931

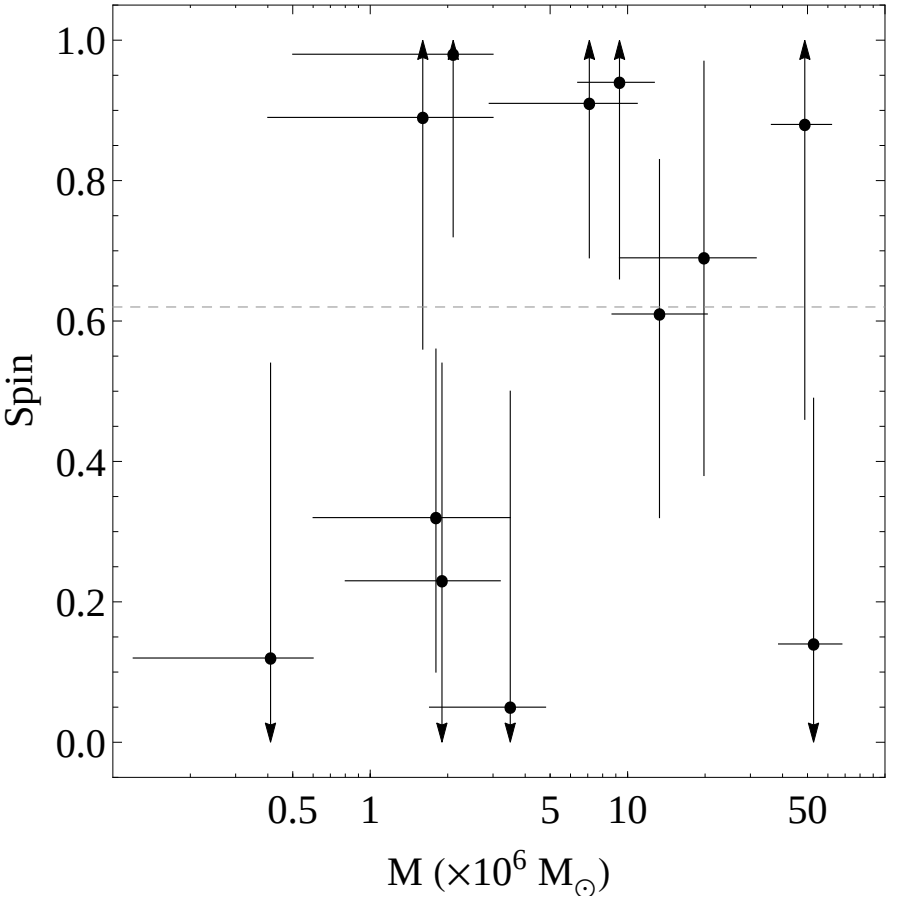
X-ray reverberation: Modelling



Emmanoulopoulos et al. 2014, MNRAS, **439**, 3931

X-ray reverberation: Modelling



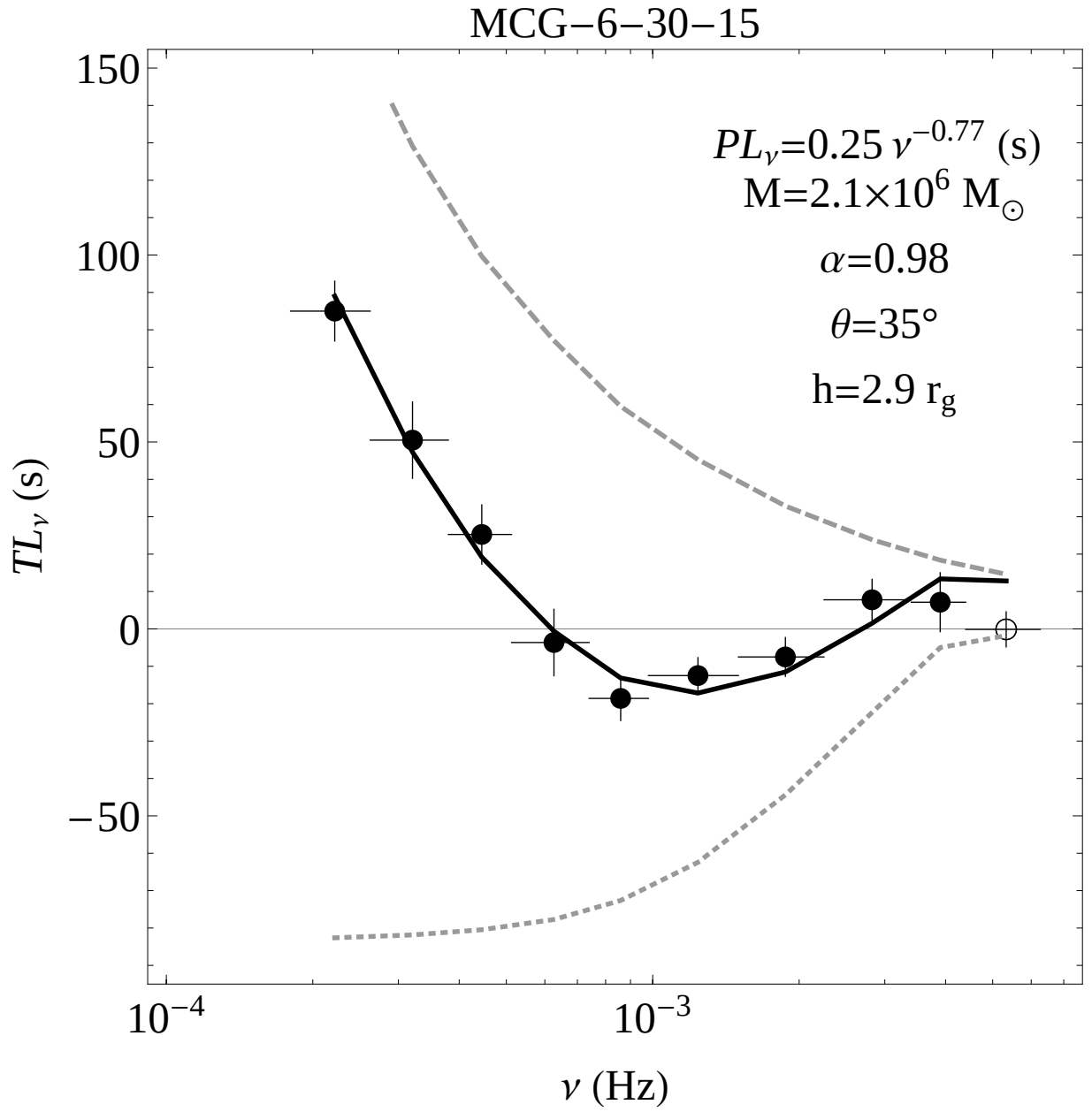


Real range of angles

Spin bimodality

Emmanoulopoulos et al. 2014, MNRAS, **439**, 3931

X-ray reverberation: Problems



<u>9 measurements</u>

6 fitting parameters

1 fixed parameter

X-ray reverberation: Problems

Overfitting $\longrightarrow$ Model degeneracies

X-ray reverberation: Problems

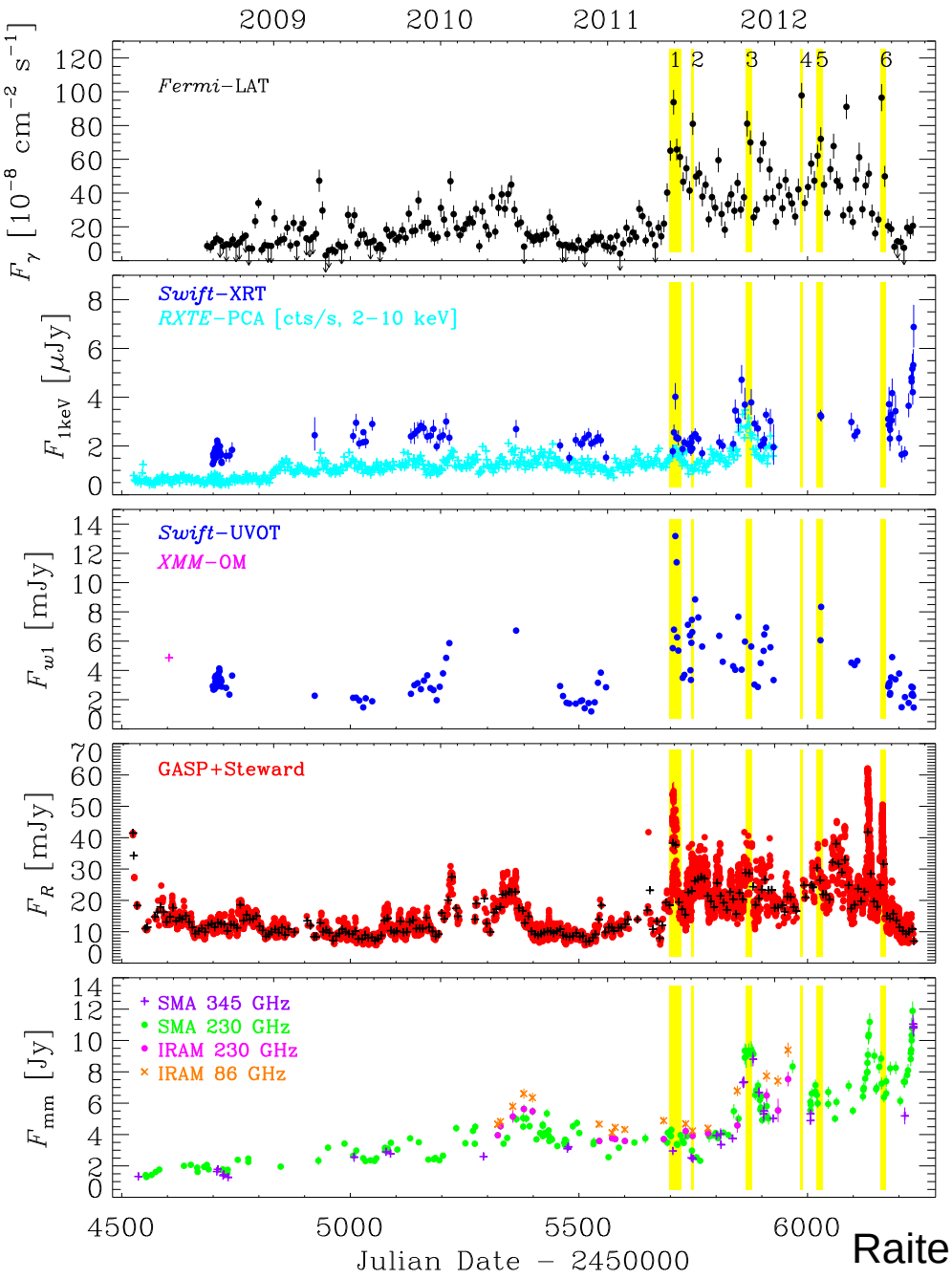
Overfitting $\longrightarrow$ Model degeneracies

Simultaneous multiwavelength observations

$\downarrow$

Causal connection of different emission regions

X-ray reverberation: Problems



Raiteri et al. 2013, MNRAS, **436**, 1530

Overfitting

Quote of John von Neumann

(Freeman, D. 2004, Nature **427**, 297)

Overfitting

Quote of John von Neumann

(Freeman, D. 2004, Nature **427**, 297)

‘With four parameters I can fit an elephant, and with five I can make him wiggle his trunk’

Overfitting

Quote of John von Neumann

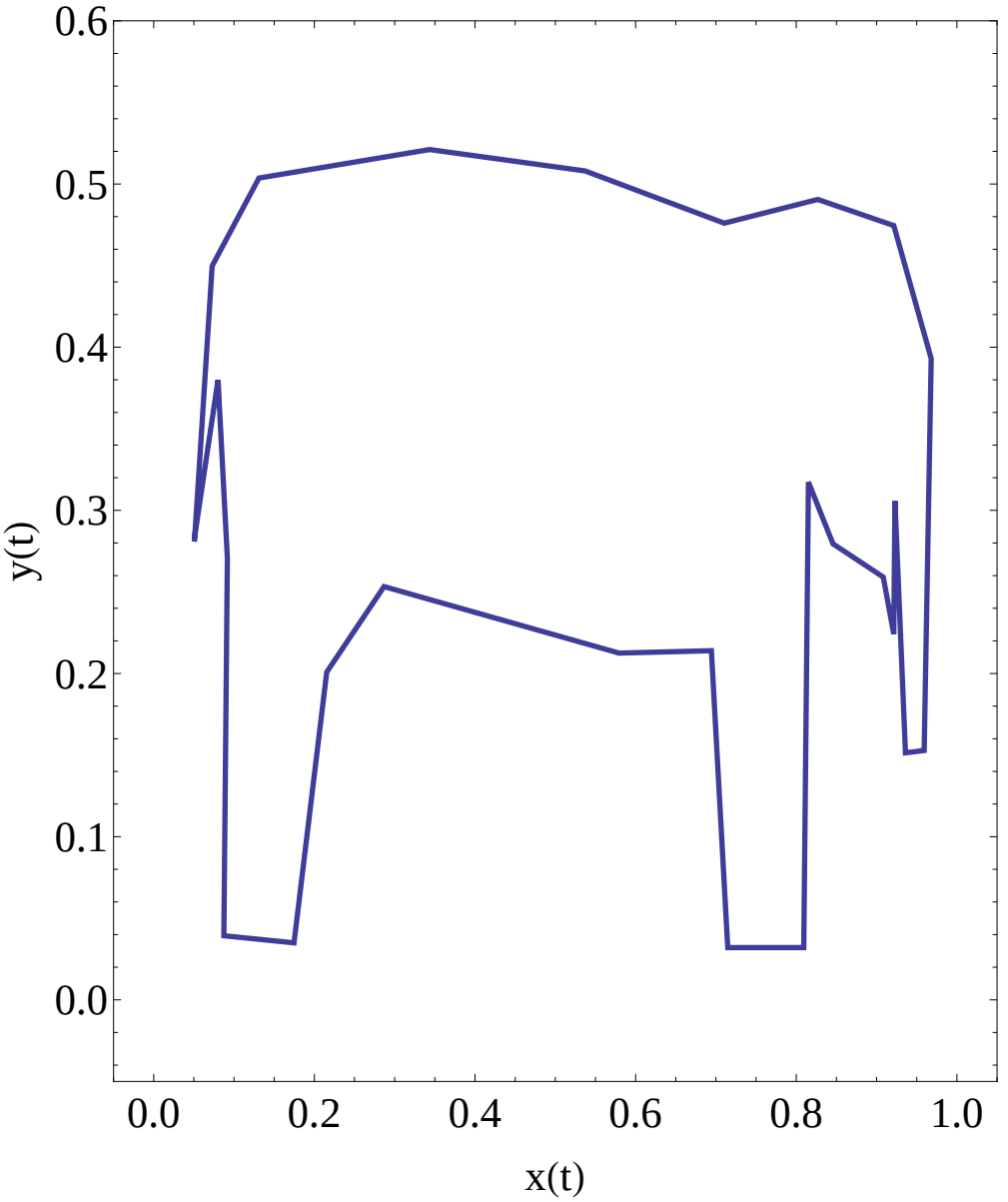
(Freeman, D. 2004, Nature **427**, 297)

‘With four parameters I can fit an elephant, and with five I can make him wiggle his trunk’

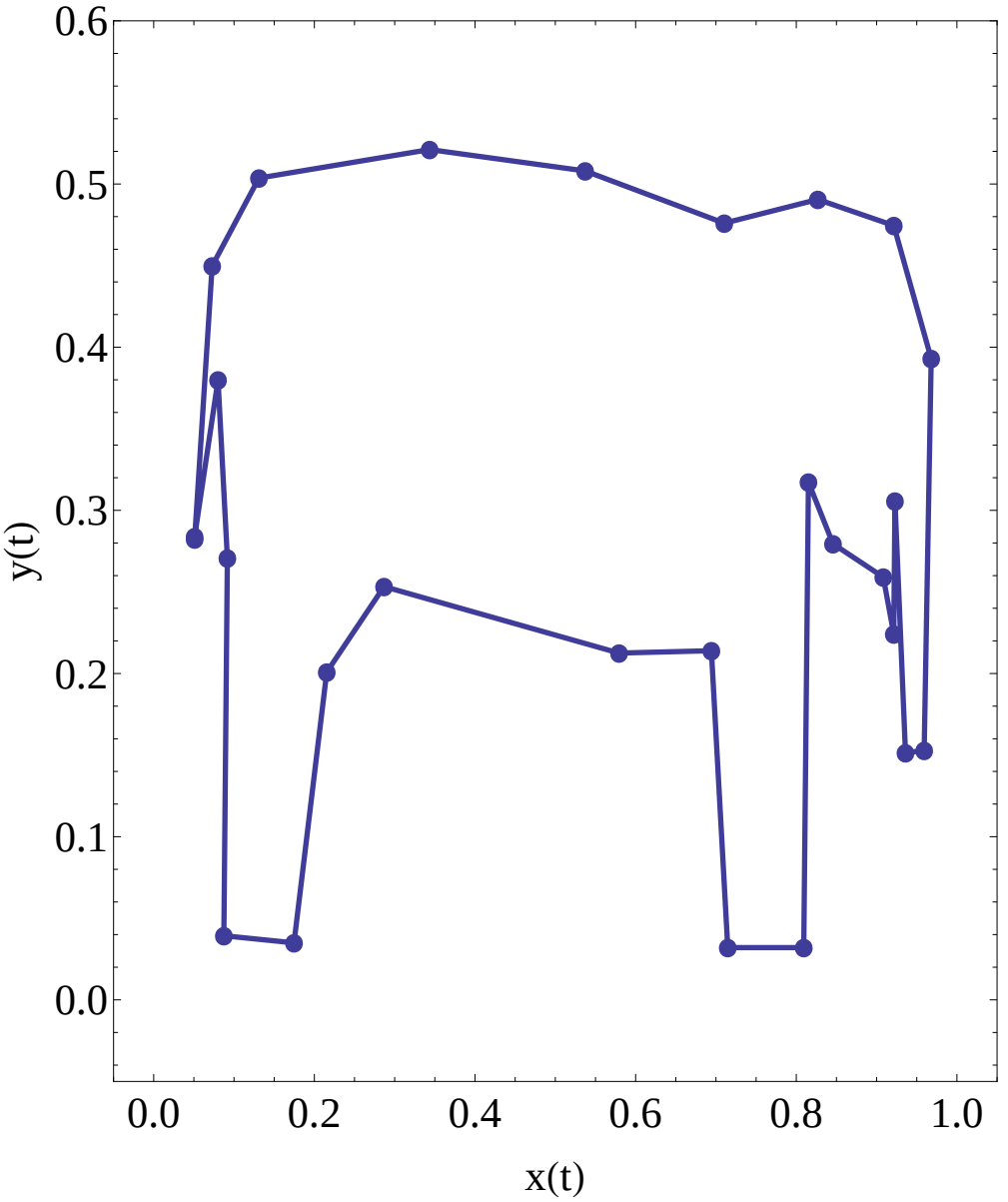
Using a least squares Fourier sine series

J.Wei, 1975, ‘Least Square Fitting of an Elephant’, CHEMTECH, **5**, 128

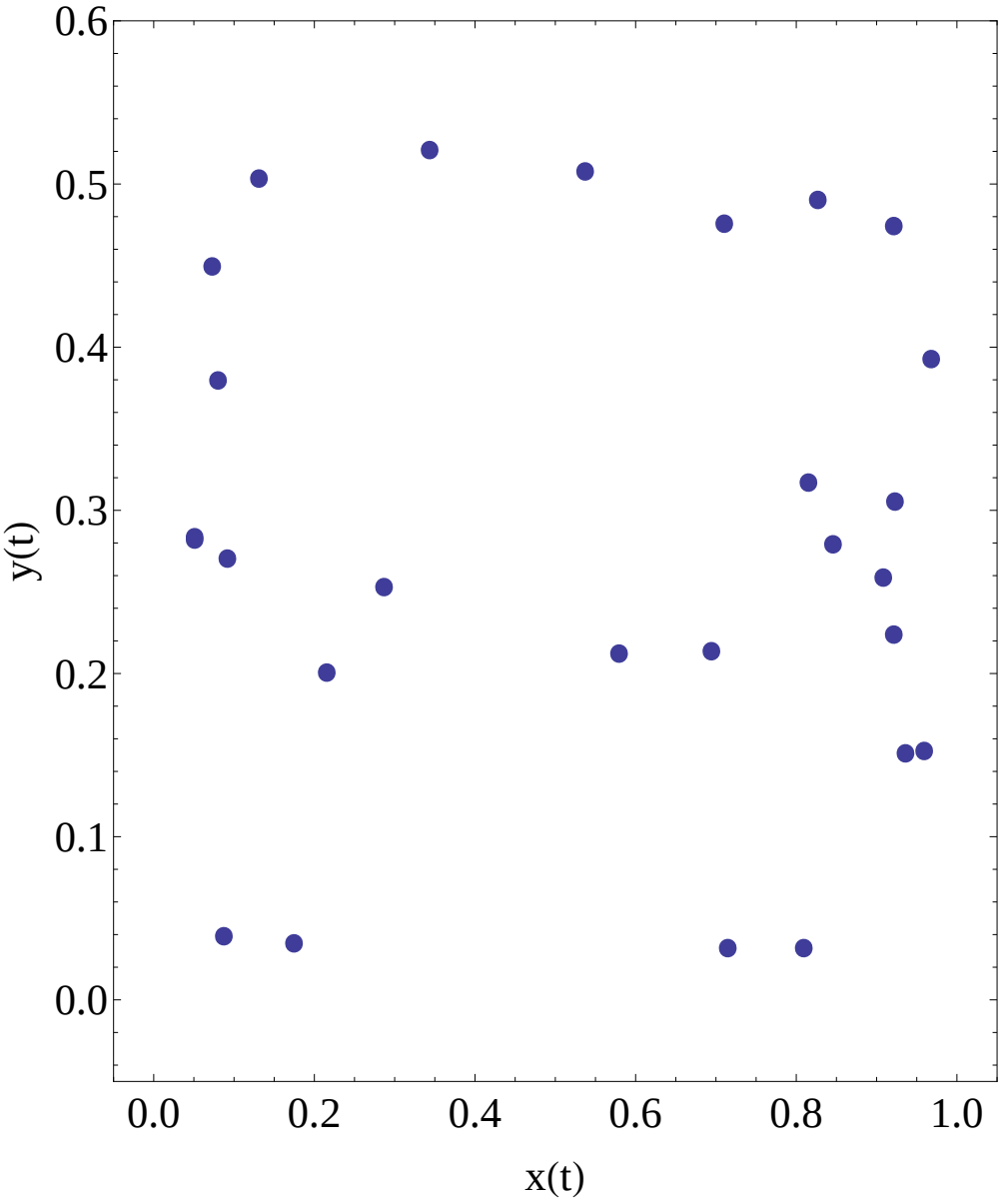
Overfitting



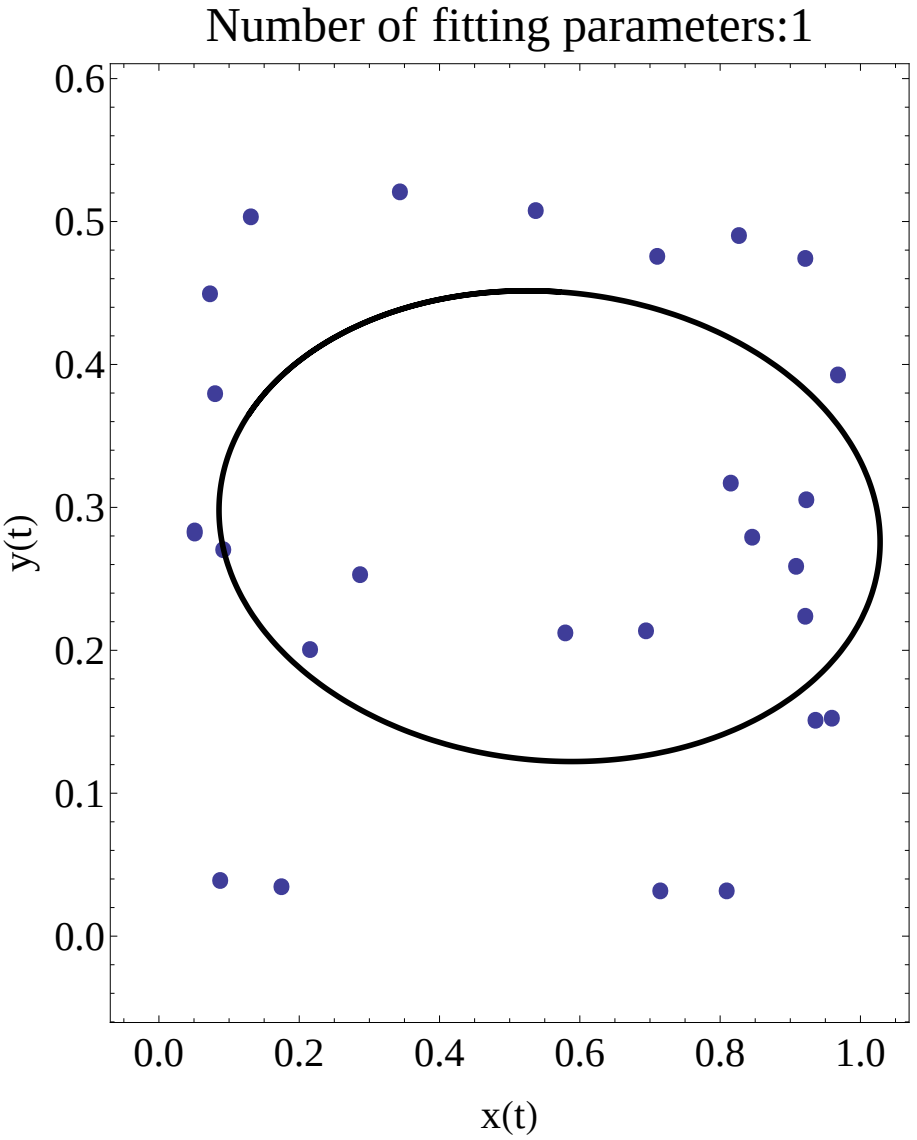
Overfitting



Overfitting



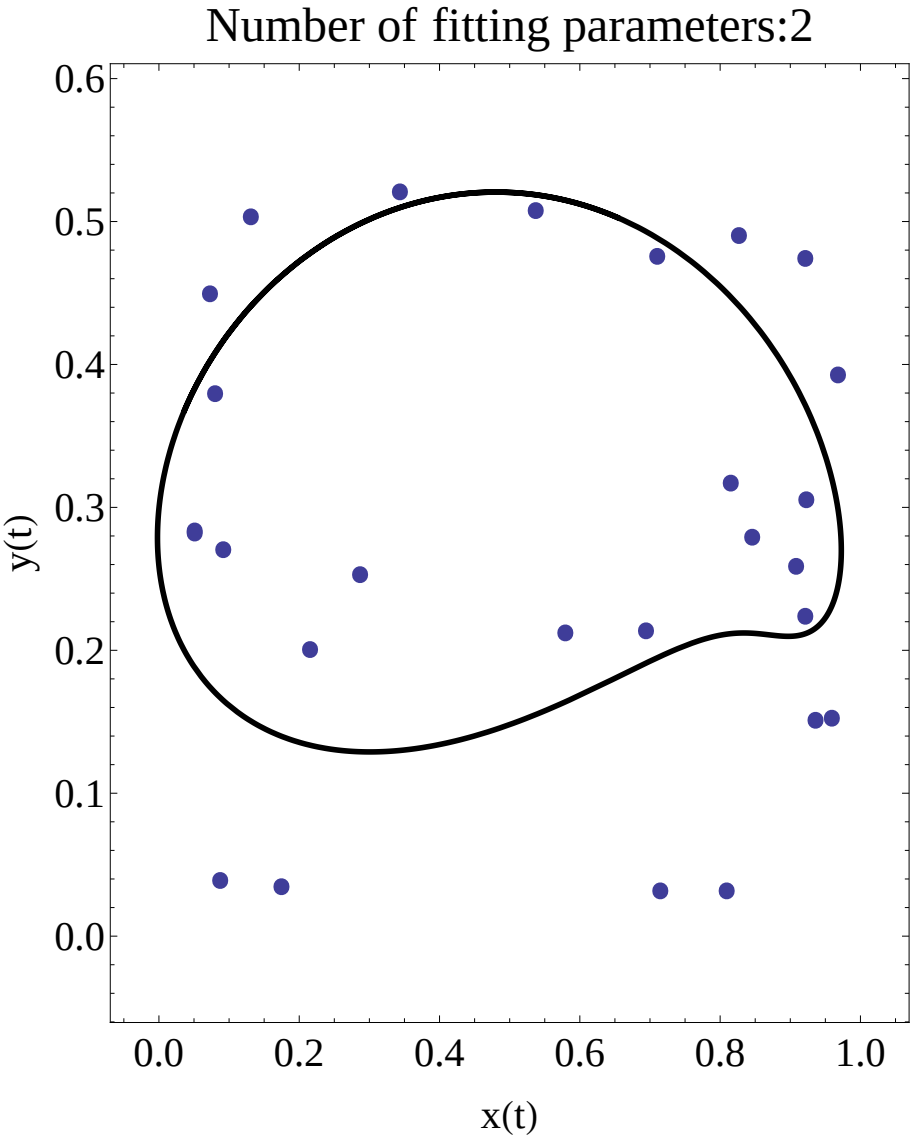
Overfitting



$$x(t)= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right]\right)$$

$$y(t)= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right]\right)$$

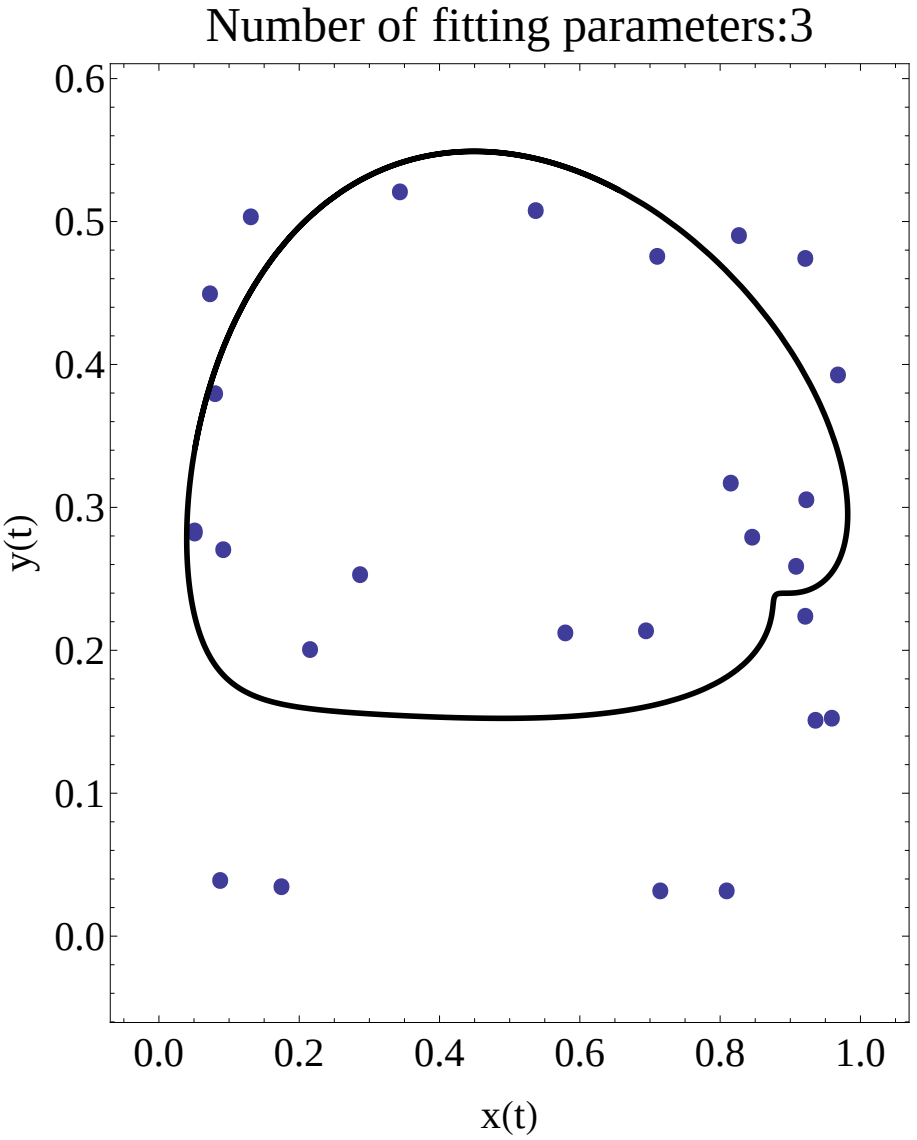
Overfitting



$$x(t)= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.027242\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right]\right)$$

$$y(t)= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right]\right)$$

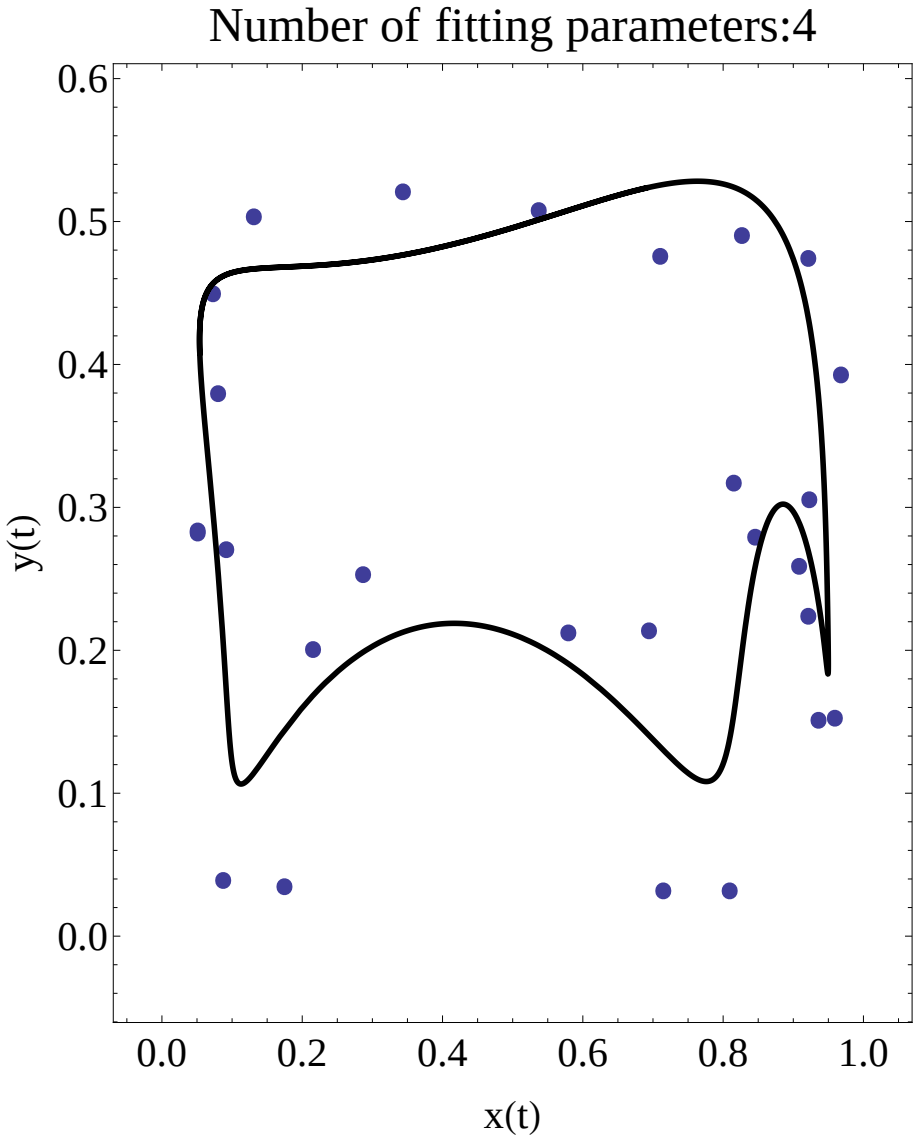
Overfitting



$$x(t)= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.0152492\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.027242\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] - 0.0435712\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right]\right)$$

$$(t)= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] - 0.0254946\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] + 0.012766\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right]\right)$$

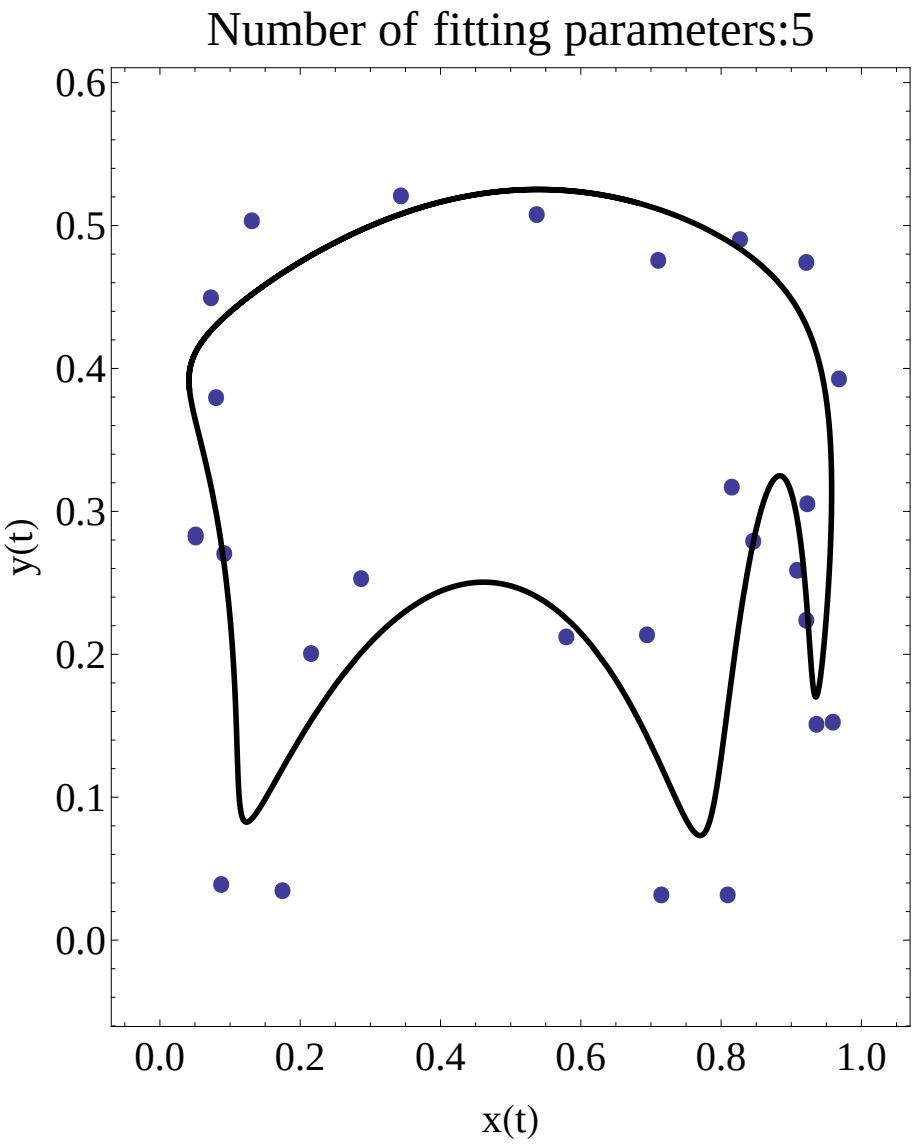
Overfitting



$$x(t)= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.0152492\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.00213549\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.027242\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] - 0.0435712\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.0354978\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right]\right)$$

$$y(t)= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] - 0.0254946\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.0657329\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] + 0.012766\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.00454687\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right]\right)$$

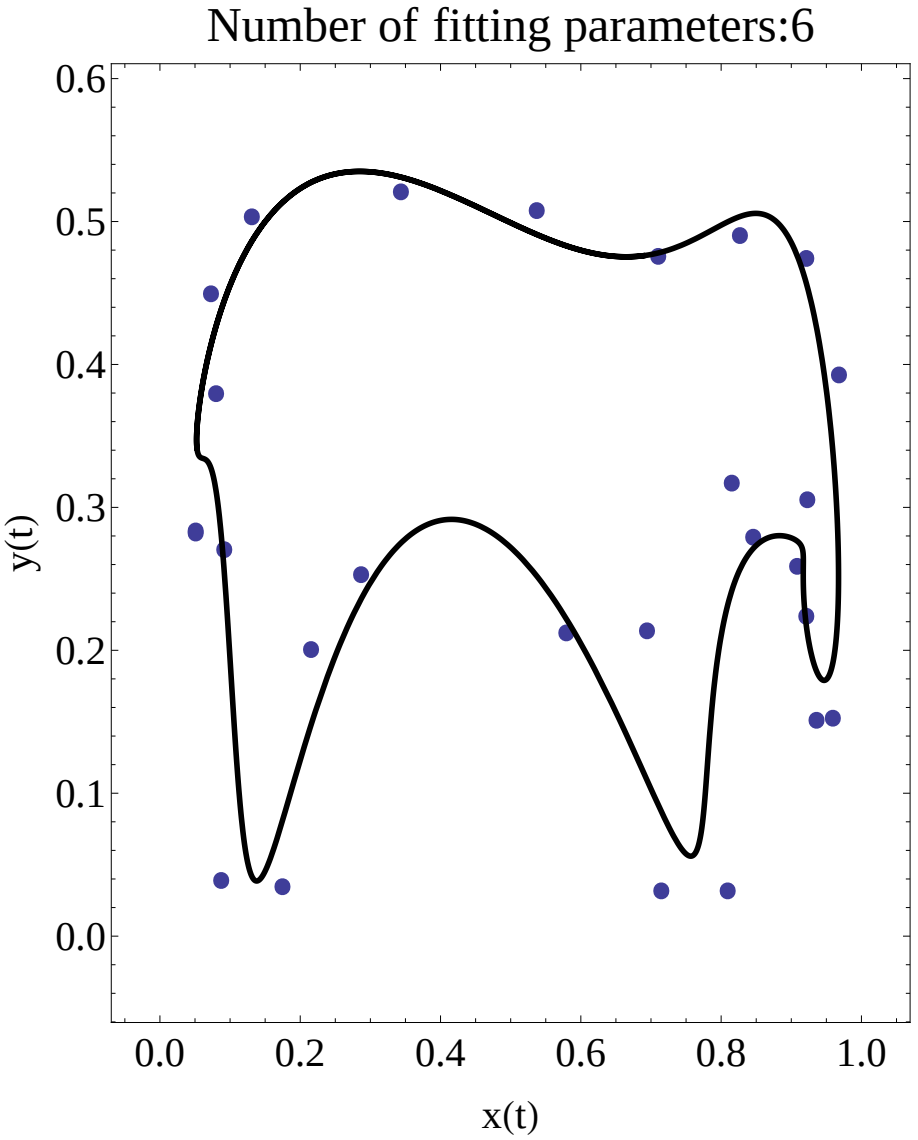
Overfitting



$$= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.0152492\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.00213549\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0120502\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.027242\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] - 0.0435712\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.0354978\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.00210601\,\text{Sin}\left[$$

$$= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] - 0.0254946\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.0657329\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0218874\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] + 0.012766\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.00454687\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.0275796\,\text{Sin}\left[$$

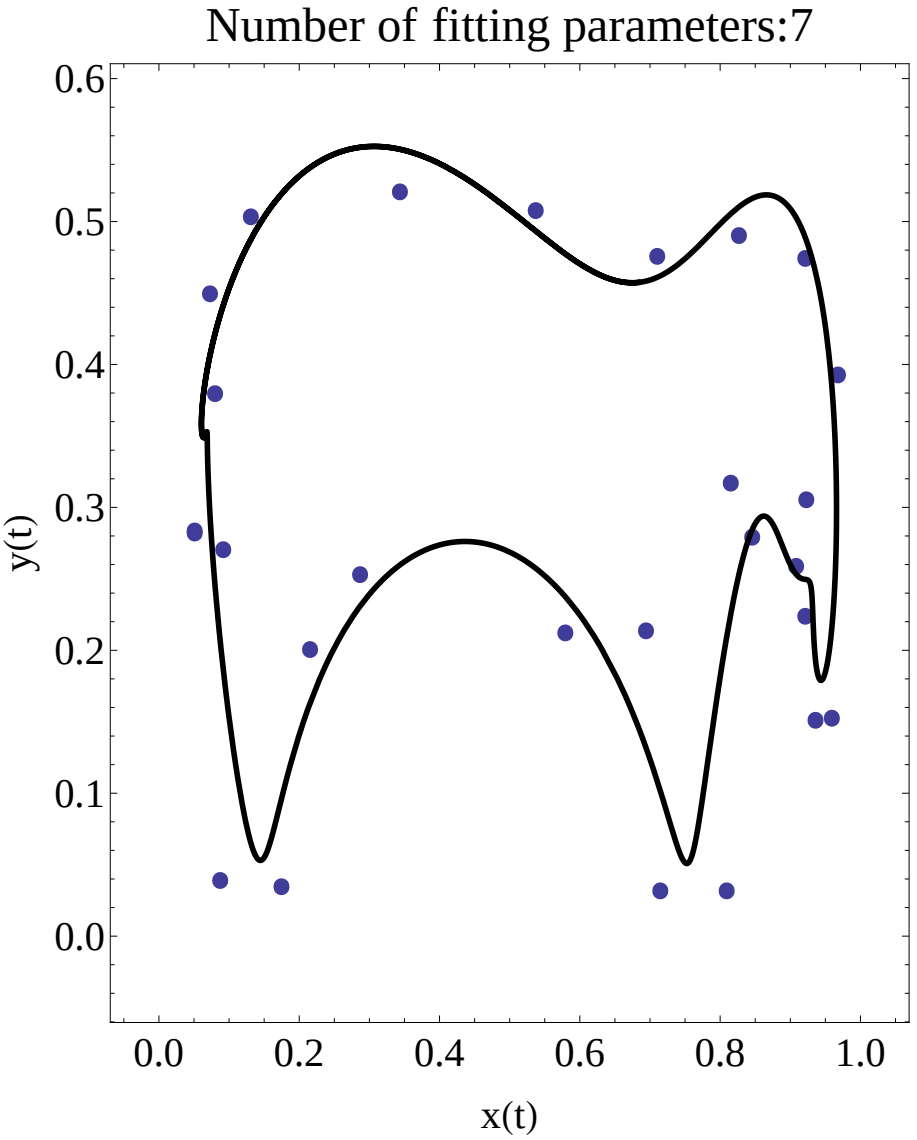
Overfitting



$$x(t)= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.0152492\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.00213549\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0120502\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] + 0.0107965\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.027242\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] - 0.0435712\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.0354978\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.00210601\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] + 0.00128555\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right]\right)$$

$$(t)= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] - 0.0254946\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.0657329\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0218874\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] - 0.0445885\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] + 0.012766\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.00454687\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.0275796\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] - 0.00924224\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right]\right)$$

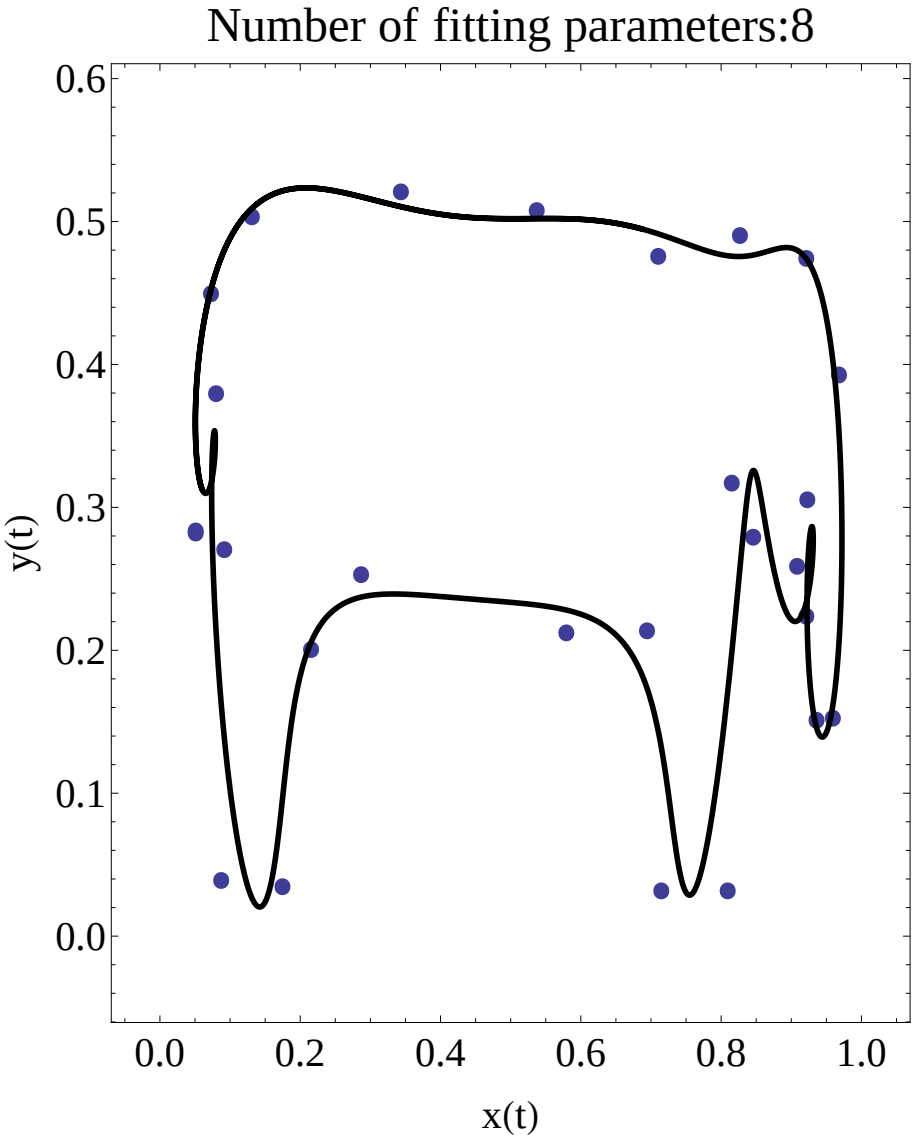
Overfitting



$$x(t)= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.0152492\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.00213549\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0120502\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] + 0.0107965\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.0112531\,\text{Cos}\left[\frac{14\,j\,\pi}{27}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.027242\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] - 0.0435712\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.0354978\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.00210601\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] + 0.00128555\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right] - 0.00223198\,\text{Sin}\left[\frac{14\,j\,\pi}{27}\right]\right)$$

$$y(t)= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] - 0.0254946\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.0657329\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0218874\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] - 0.0445885\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.00697783\,\text{Cos}\left[\frac{14\,j\,\pi}{27}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] + 0.012766\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.00454687\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.0275796\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] - 0.00924224\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right] - 0.0173974\,\text{Sin}\left[\frac{14\,j\,\pi}{27}\right]\right)$$

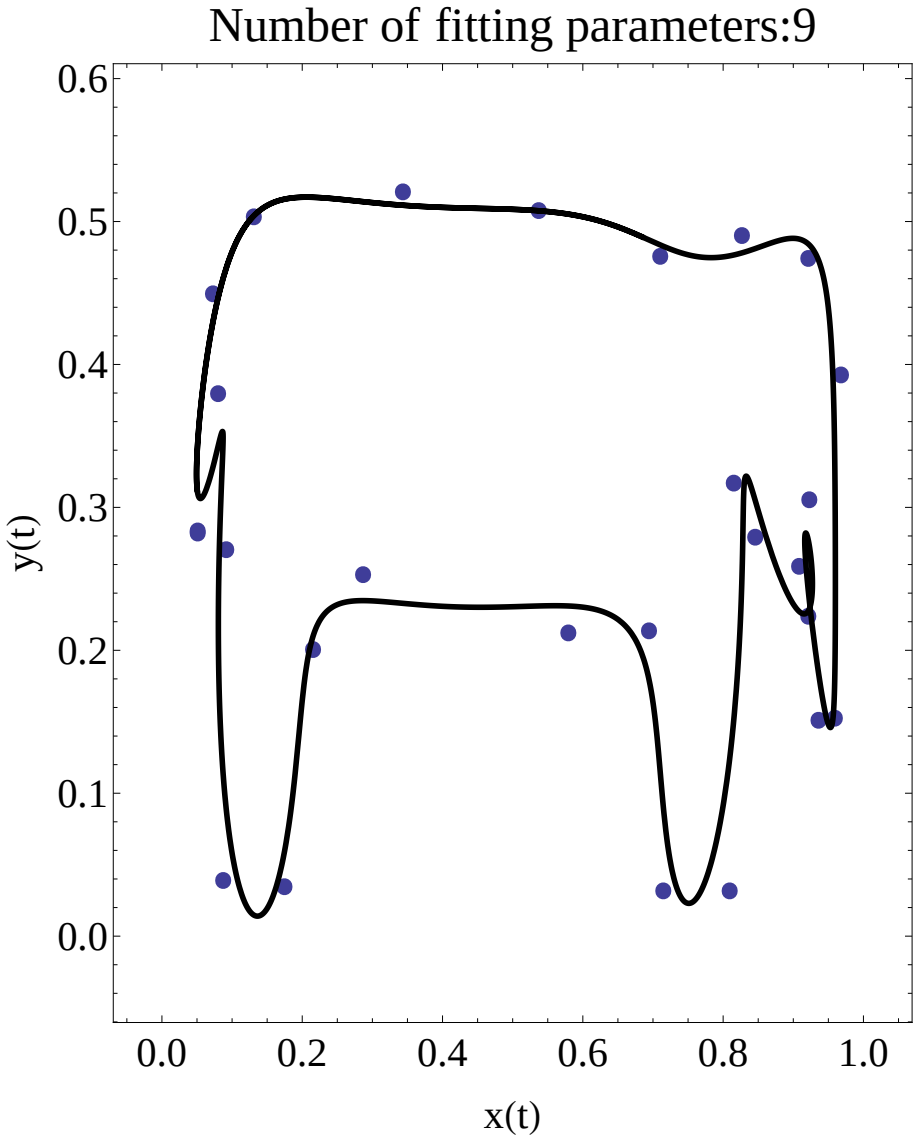
Overfitting



$$x(t)= \left(0.556831 - 0.427534 \cos\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282 \cos\left[\frac{4\,j\,\pi}{27}\right] + 0.0152492 \cos\left[\frac{2\,j\,\pi}{9}\right] + 0.00213549 \cos\left[\frac{8\,j\,\pi}{27}\right] - 0.0120502 \cos\left[\frac{10\,j\,\pi}{27}\right] + 0.0107965 \cos\left[\frac{4\,j\,\pi}{9}\right] + 0.0112531 \cos\left[\frac{14\,j\,\pi}{27}\right] - 0.00415648 \cos\left[\frac{16\,j\,\pi}{27}\right] + 0.198064 \sin\left[\frac{2\,j\,\pi}{27}\right] + 0.027242 \sin\left[\frac{4\,j\,\pi}{27}\right] - 0.0435712 \sin\left[\frac{2\,j\,\pi}{9}\right] - 0.0354978 \sin\left[\frac{8\,j\,\pi}{27}\right] - 0.00210601 \sin\left[\frac{10\,j\,\pi}{27}\right] + 0.00128555 \sin\left[\frac{4\,j\,\pi}{9}\right] - 0.00223198 \sin\left[\frac{14\,j\,\pi}{27}\right] - 0.00923493 \sin\left[\frac{16\,j\,\pi}{27}\right]\right)$$

$$y(t)= \left(0.286858 + 0.0791214 \cos\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559 \cos\left[\frac{4\,j\,\pi}{27}\right] - 0.0254946 \cos\left[\frac{2\,j\,\pi}{9}\right] + 0.0657329 \cos\left[\frac{8\,j\,\pi}{27}\right] - 0.0218874 \cos\left[\frac{10\,j\,\pi}{27}\right] - 0.0445885 \cos\left[\frac{4\,j\,\pi}{9}\right] + 0.00697783 \cos\left[\frac{14\,j\,\pi}{27}\right] - 0.0357556 \cos\left[\frac{16\,j\,\pi}{27}\right] + 0.144433 \sin\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055 \sin\left[\frac{4\,j\,\pi}{27}\right] + 0.012766 \sin\left[\frac{2\,j\,\pi}{9}\right] - 0.00454687 \sin\left[\frac{8\,j\,\pi}{27}\right] - 0.0275796 \sin\left[\frac{10\,j\,\pi}{27}\right] - 0.00924224 \sin\left[\frac{4\,j\,\pi}{9}\right] - 0.0173974 \sin\left[\frac{14\,j\,\pi}{27}\right] + 0.0184027 \sin\left[\frac{16\,j\,\pi}{27}\right]\right)$$

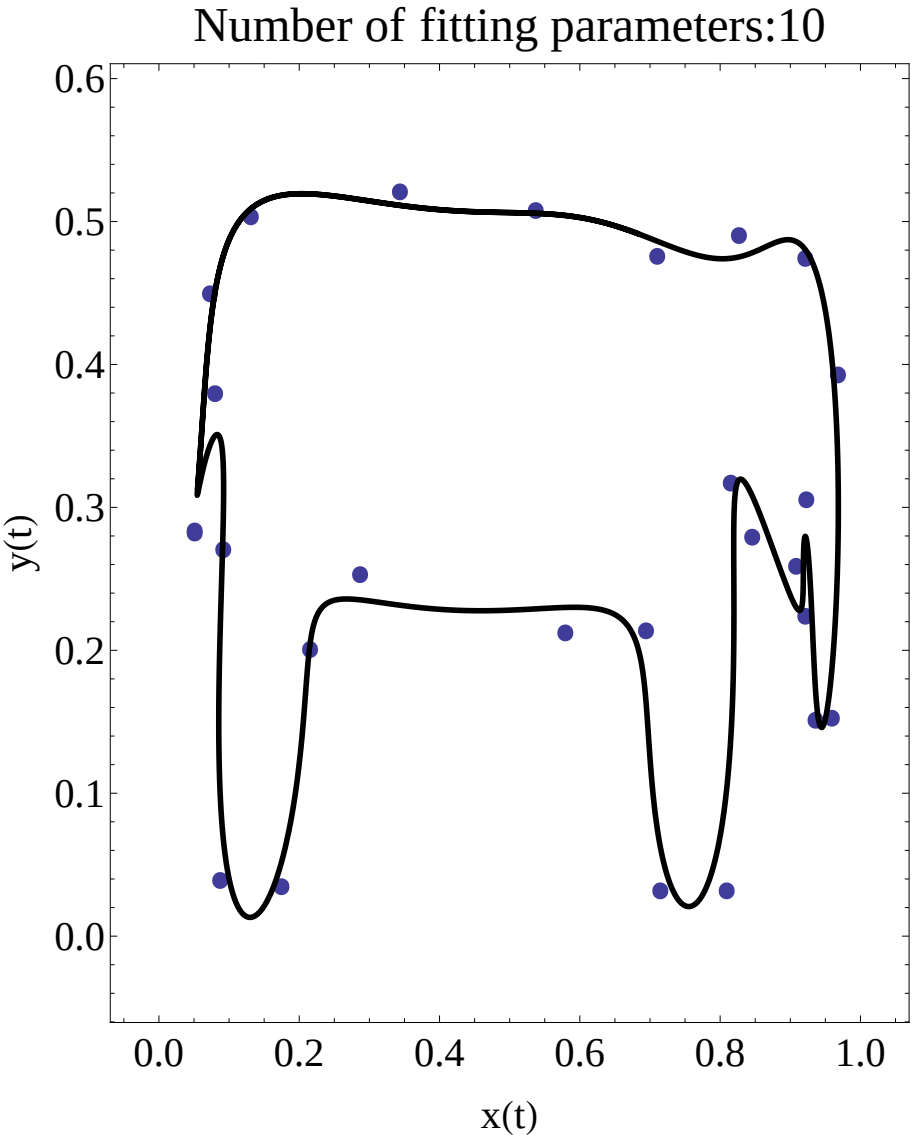
Overfitting



$$x(t)= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.0152492\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.00213549\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0120502\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] + 0.0107965\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.0112531\,\text{Cos}\left[\frac{14\,j\,\pi}{27}\right] - 0.00415648\,\text{Cos}\left[\frac{16\,j\,\pi}{27}\right] - 0.00958542\,\text{Cos}\left[\frac{2\,j\,\pi}{3}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.027242\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] - 0.0435712\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.0354978\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.00210601\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] + 0.00128555\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right] - 0.00223198\,\text{Sin}\left[\frac{14\,j\,\pi}{27}\right] - 0.00923493\,\text{Sin}\left[\frac{16\,j\,\pi}{27}\right] + 0.00769094\,\text{Sin}\left[\frac{2\,j\,\pi}{3}\right]\right)$$

$$(t)= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] - 0.0254946\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.0657329\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0218874\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] - 0.0445885\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.00697783\,\text{Cos}\left[\frac{14\,j\,\pi}{27}\right] - 0.0357556\,\text{Cos}\left[\frac{16\,j\,\pi}{27}\right] - 0.000429811\,\text{Cos}\left[\frac{2\,j\,\pi}{3}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] + 0.012766\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.00454687\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.0275796\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] - 0.00924224\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right] - 0.0173974\,\text{Sin}\left[\frac{14\,j\,\pi}{27}\right] + 0.0184027\,\text{Sin}\left[\frac{16\,j\,\pi}{27}\right] + 0.00668781\,\text{Sin}\left[\frac{2\,j\,\pi}{3}\right]\right)$$

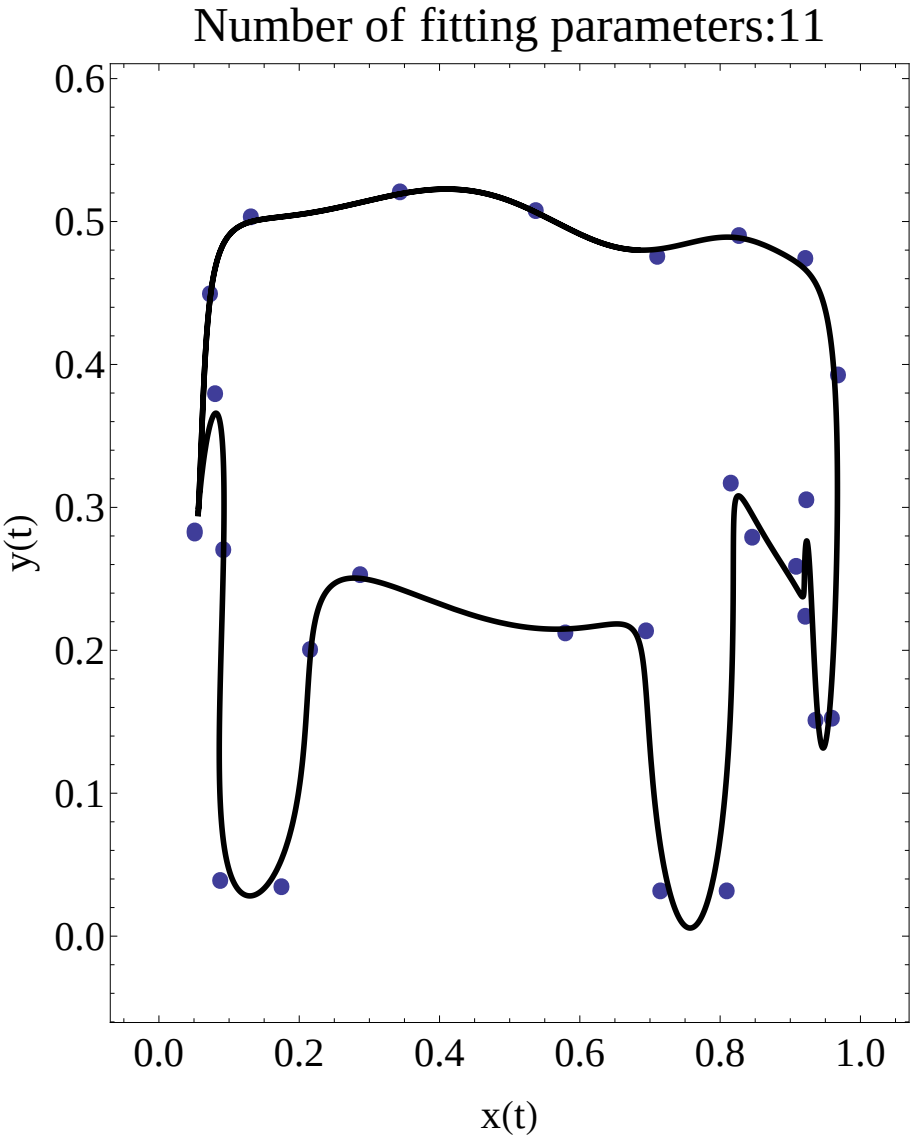
Overfitting



$$x(t)= \left(0.556831 - 0.427534 \cos\left[\frac{2 j \pi}{27}\right] - 0.0927282 \cos\left[\frac{4 j \pi}{27}\right] + 0.0152492 \cos\left[\frac{2 j \pi}{9}\right] + 0.00213549 \cos\left[\frac{8 j \pi}{27}\right] - 0.0120502 \cos\left[\frac{10 j \pi}{27}\right] + 0.0107965 \cos\left[\frac{4 j \pi}{9}\right] + 0.0112531 \cos\left[\frac{14 j \pi}{27}\right] - 0.00415648 \cos\left[\frac{16 j \pi}{27}\right] - 0.00958542 \cos\left[\frac{2 j \pi}{3}\right] + 0.00508887 \cos\left[\frac{20 j \pi}{27}\right] + 0.198064 \sin\left[\frac{2 j \pi}{27}\right] + 0.027242 \sin\left[\frac{4 j \pi}{27}\right] - 0.0435712 \sin\left[\frac{2 j \pi}{9}\right] - 0.0354978 \sin\left[\frac{8 j \pi}{27}\right] - 0.00210601 \sin\left[\frac{10 j \pi}{27}\right] + 0.00128555 \sin\left[\frac{4 j \pi}{9}\right] - 0.00223198 \sin\left[\frac{14 j \pi}{27}\right] - 0.00923493 \sin\left[\frac{16 j \pi}{27}\right] + 0.00769094 \sin\left[\frac{2 j \pi}{3}\right] + 0.00724309 \sin\left[\frac{20 j \pi}{27}\right]\right)$$

$$y(t)= \left(0.286858 + 0.0791214 \cos\left[\frac{2 j \pi}{27}\right] + 0.00210559 \cos\left[\frac{4 j \pi}{27}\right] - 0.0254946 \cos\left[\frac{2 j \pi}{9}\right] + 0.0657329 \cos\left[\frac{8 j \pi}{27}\right] - 0.0218874 \cos\left[\frac{10 j \pi}{27}\right] - 0.0445885 \cos\left[\frac{4 j \pi}{9}\right] + 0.00697783 \cos\left[\frac{14 j \pi}{27}\right] - 0.0357556 \cos\left[\frac{16 j \pi}{27}\right] - 0.000429811 \cos\left[\frac{2 j \pi}{3}\right] + 0.00174577 \cos\left[\frac{20 j \pi}{27}\right] + 0.144433 \sin\left[\frac{2 j \pi}{27}\right] + 0.0737055 \sin\left[\frac{4 j \pi}{27}\right] + 0.012766 \sin\left[\frac{2 j \pi}{9}\right] - 0.00454687 \sin\left[\frac{8 j \pi}{27}\right] - 0.0275796 \sin\left[\frac{10 j \pi}{27}\right] - 0.00924224 \sin\left[\frac{4 j \pi}{9}\right] - 0.0173974 \sin\left[\frac{14 j \pi}{27}\right] + 0.0184027 \sin\left[\frac{16 j \pi}{27}\right] + 0.00668781 \sin\left[\frac{2 j \pi}{3}\right] - 0.00171511 \sin\left[\frac{20 j \pi}{27}\right]\right)$$

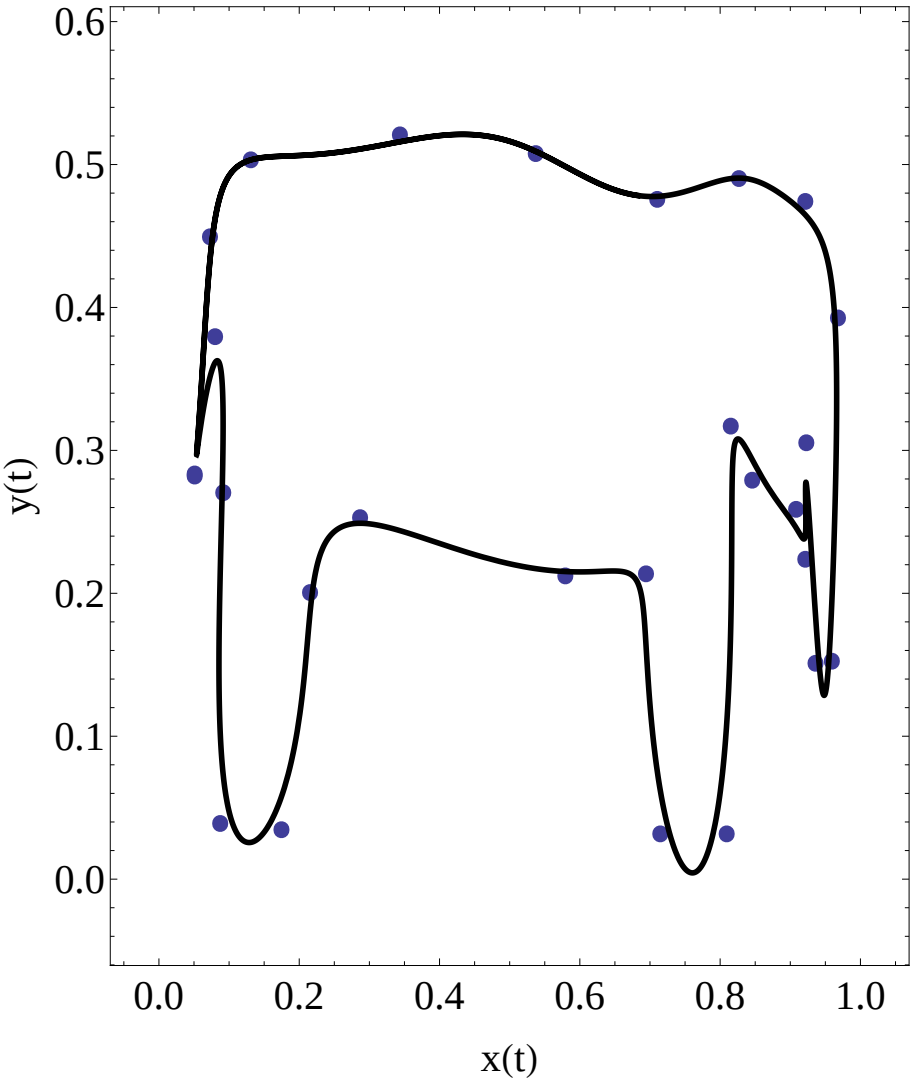
Overfitting



$$x(t)= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.0152492\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.00213549\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0120502\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] + 0.0107965\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.0112531\,\text{Cos}\left[\frac{14\,j\,\pi}{27}\right] - 0.00415648\,\text{Cos}\left[\frac{16\,j\,\pi}{27}\right] - 0.00958542\,\text{Cos}\left[\frac{2\,j\,\pi}{3}\right] + 0.00508887\,\text{Cos}\left[\frac{20\,j\,\pi}{27}\right] + 0.00125267\,\text{Cos}\left[\frac{22\,j\,\pi}{27}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.027242\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] - 0.0435712\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.0354978\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.00210601\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] + 0.00128555\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right] - 0.00223198\,\text{Sin}\left[\frac{14\,j\,\pi}{27}\right] - 0.00923493\,\text{Sin}\left[\frac{16\,j\,\pi}{27}\right] + 0.00769094\,\text{Sin}\left[\frac{2\,j\,\pi}{3}\right] + 0.00724309\,\text{Sin}\left[\frac{20\,j\,\pi}{27}\right] - 0.000161075\,\text{Sin}\left[\frac{22\,j\,\pi}{27}\right]\right)$$

$$y(t)= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] - 0.0254946\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.0657329\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0218874\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] - 0.0445885\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.00697783\,\text{Cos}\left[\frac{14\,j\,\pi}{27}\right] - 0.0357556\,\text{Cos}\left[\frac{16\,j\,\pi}{27}\right] - 0.000429811\,\text{Cos}\left[\frac{2\,j\,\pi}{3}\right] + 0.00174577\,\text{Cos}\left[\frac{20\,j\,\pi}{27}\right] - 0.0136406\,\text{Cos}\left[\frac{22\,j\,\pi}{27}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] + 0.012766\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.00454687\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.0275796\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] - 0.00924224\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right] - 0.0173974\,\text{Sin}\left[\frac{14\,j\,\pi}{27}\right] + 0.0184027\,\text{Sin}\left[\frac{16\,j\,\pi}{27}\right] + 0.00668781\,\text{Sin}\left[\frac{2\,j\,\pi}{3}\right] - 0.00171511\,\text{Sin}\left[\frac{20\,j\,\pi}{27}\right] + 0.00638932\,\text{Sin}\left[\frac{22\,j\,\pi}{27}\right]\right)$$

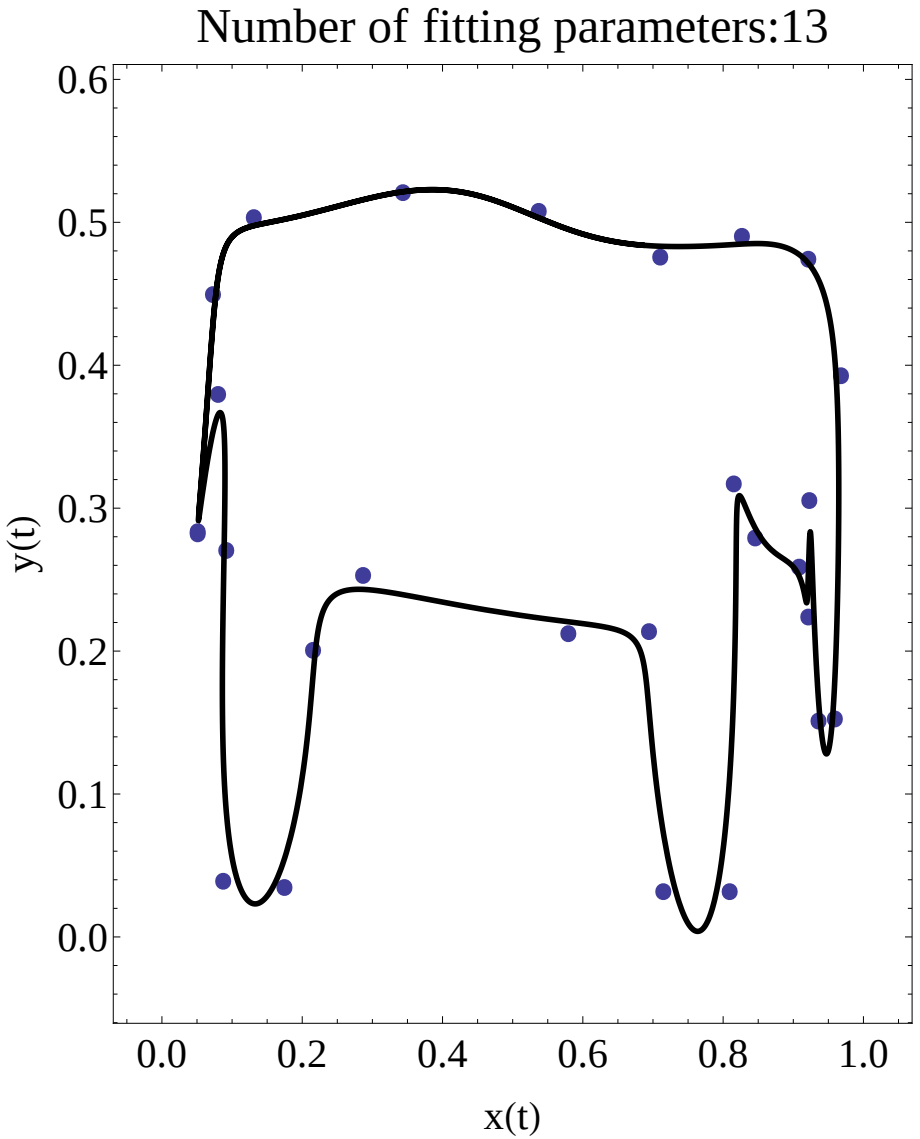
Overfitting



$$x(t)= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.0152492\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.00213549\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0120502\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] + 0.0107965\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.0112531\,\text{Cos}\left[\frac{14\,j\,\pi}{27}\right] - 0.00415648\,\text{Cos}\left[\frac{16\,j\,\pi}{27}\right] - 0.00958542\,\text{Cos}\left[\frac{2\,j\,\pi}{3}\right] + 0.00508887\,\text{Cos}\left[\frac{20\,j\,\pi}{27}\right] + 0.00125267\,\text{Cos}\left[\frac{22\,j\,\pi}{27}\right] - 0.0019295\,\text{Cos}\left[\frac{8\,j\,\pi}{9}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.027242\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] - 0.0435712\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.0354978\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.00210601\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] + 0.00128555\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right] - 0.00223198\,\text{Sin}\left[\frac{14\,j\,\pi}{27}\right] - 0.00923493\,\text{Sin}\left[\frac{16\,j\,\pi}{27}\right] + 0.00769094\,\text{Sin}\left[\frac{2\,j\,\pi}{3}\right] + 0.00724309\,\text{Sin}\left[\frac{20\,j\,\pi}{27}\right] - 0.000161075\,\text{Sin}\left[\frac{22\,j\,\pi}{27}\right] + 0.00070835\,\text{Sin}\left[\frac{8\,j\,\pi}{9}\right]\right)$$

$$(t)= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] - 0.0254946\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.0657329\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0218874\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] - 0.0445885\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.00697783\,\text{Cos}\left[\frac{14\,j\,\pi}{27}\right] - 0.0357556\,\text{Cos}\left[\frac{16\,j\,\pi}{27}\right] - 0.000429811\,\text{Cos}\left[\frac{2\,j\,\pi}{3}\right] + 0.00174577\,\text{Cos}\left[\frac{20\,j\,\pi}{27}\right] - 0.0136406\,\text{Cos}\left[\frac{22\,j\,\pi}{27}\right] + 0.00236546\,\text{Cos}\left[\frac{8\,j\,\pi}{9}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] + 0.012766\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.00454687\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.0275796\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] - 0.00924224\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right] - 0.0173974\,\text{Sin}\left[\frac{14\,j\,\pi}{27}\right] + 0.0184027\,\text{Sin}\left[\frac{16\,j\,\pi}{27}\right] + 0.00668781\,\text{Sin}\left[\frac{2\,j\,\pi}{3}\right] - 0.00171511\,\text{Sin}\left[\frac{20\,j\,\pi}{27}\right] + 0.00638932\,\text{Sin}\left[\frac{22\,j\,\pi}{27}\right] - 0.00215222\,\text{Sin}\left[\frac{8\,j\,\pi}{9}\right]\right)$$

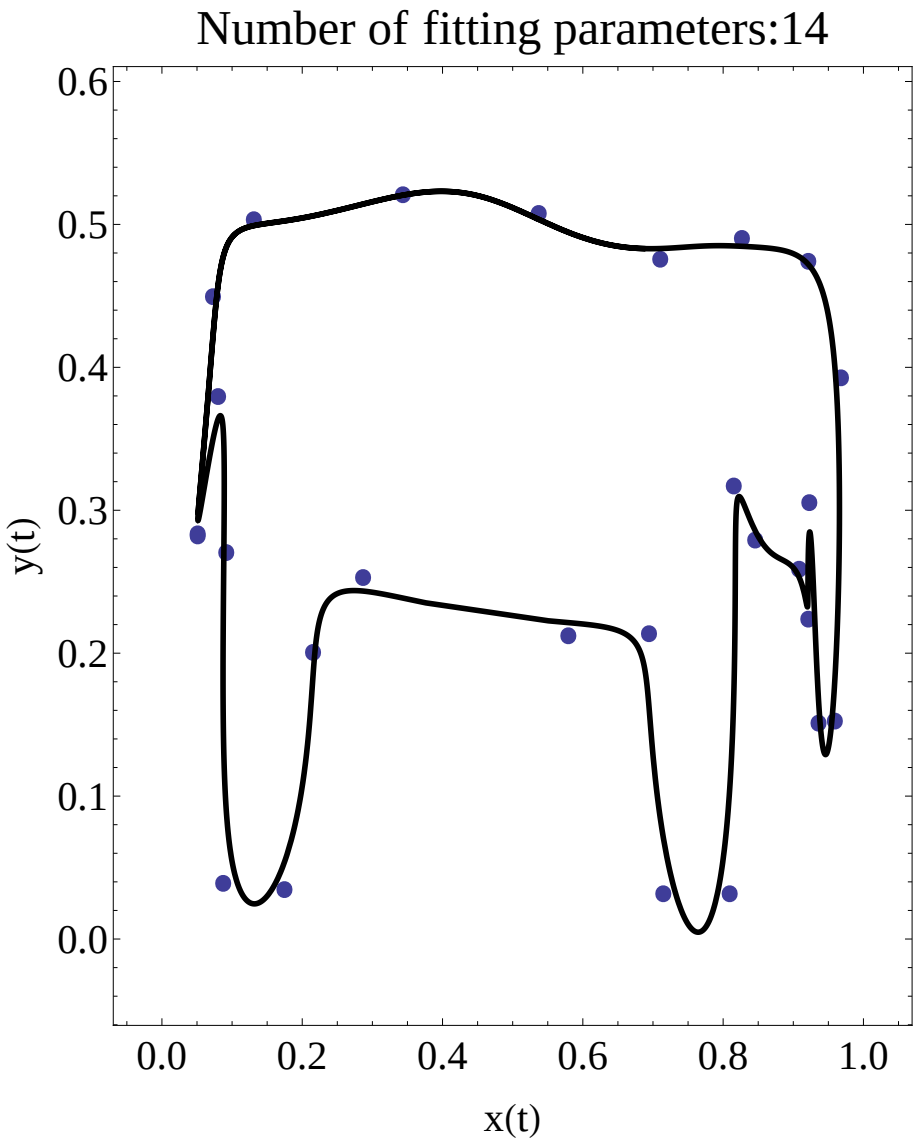
Overfitting



$$x(t)= \left(0.556831 - 0.427534 \cos\left[\frac{2 j \pi}{27}\right] - 0.0927282 \cos\left[\frac{4 j \pi}{27}\right] + 0.0152492 \cos\left[\frac{2 j \pi}{9}\right] + 0.00213549 \cos\left[\frac{8 j \pi}{27}\right] - 0.0120502 \cos\left[\frac{10 j \pi}{27}\right] + 0.0107965 \cos\left[\frac{4 j \pi}{9}\right] + 0.0112531 \cos\left[\frac{14 j \pi}{27}\right] - 0.00415648 \cos\left[\frac{16 j \pi}{27}\right] - 0.00958542 \cos\left[\frac{2 j \pi}{3}\right] + 0.00508887 \cos\left[\frac{20 j \pi}{27}\right] + 0.00125267 \cos\left[\frac{22 j \pi}{27}\right] - 0.0019295 \cos\left[\frac{8 j \pi}{9}\right] - 0.00179849 \cos\left[\frac{26 j \pi}{27}\right] + 0.198064 \sin\left[\frac{2 j \pi}{27}\right] + 0.027242 \sin\left[\frac{4 j \pi}{27}\right] - 0.0435712 \sin\left[\frac{2 j \pi}{9}\right] - 0.0354978 \sin\left[\frac{8 j \pi}{27}\right] - 0.00210601 \sin\left[\frac{10 j \pi}{27}\right] + 0.00128555 \sin\left[\frac{4 j \pi}{9}\right] - 0.00223198 \sin\left[\frac{14 j \pi}{27}\right] - 0.00923493 \sin\left[\frac{16 j \pi}{27}\right] + 0.00769094 \sin\left[\frac{2 j \pi}{3}\right] + 0.00724309 \sin\left[\frac{20 j \pi}{27}\right] - 0.000161075 \sin\left[\frac{22 j \pi}{27}\right] + 0.00070835 \sin\left[\frac{8 j \pi}{9}\right] + 0.000336465 \sin\left[\frac{26 j \pi}{27}\right]\right)$$

$$y(t)= \left(0.286858 + 0.0791214 \cos\left[\frac{2 j \pi}{27}\right] + 0.00210559 \cos\left[\frac{4 j \pi}{27}\right] - 0.0254946 \cos\left[\frac{2 j \pi}{9}\right] + 0.0657329 \cos\left[\frac{8 j \pi}{27}\right] - 0.0218874 \cos\left[\frac{10 j \pi}{27}\right] - 0.0445885 \cos\left[\frac{4 j \pi}{9}\right] + 0.00697783 \cos\left[\frac{14 j \pi}{27}\right] - 0.0357556 \cos\left[\frac{16 j \pi}{27}\right] - 0.000429811 \cos\left[\frac{2 j \pi}{3}\right] + 0.00174577 \cos\left[\frac{20 j \pi}{27}\right] - 0.0136406 \cos\left[\frac{22 j \pi}{27}\right] + 0.00236546 \cos\left[\frac{8 j \pi}{9}\right] - 0.00550866 \cos\left[\frac{26 j \pi}{27}\right] + 0.144433 \sin\left[\frac{2 j \pi}{27}\right] + 0.0737055 \sin\left[\frac{4 j \pi}{27}\right] + 0.012766 \sin\left[\frac{2 j \pi}{9}\right] - 0.00454687 \sin\left[\frac{8 j \pi}{27}\right] - 0.0275796 \sin\left[\frac{10 j \pi}{27}\right] - 0.00924224 \sin\left[\frac{4 j \pi}{9}\right] - 0.0173974 \sin\left[\frac{14 j \pi}{27}\right] + 0.0184027 \sin\left[\frac{16 j \pi}{27}\right] + 0.00668781 \sin\left[\frac{2 j \pi}{3}\right] - 0.00171511 \sin\left[\frac{20 j \pi}{27}\right] + 0.00638932 \sin\left[\frac{22 j \pi}{27}\right] - 0.00215222 \sin\left[\frac{8 j \pi}{9}\right] + 0.00198997 \sin\left[\frac{26 j \pi}{27}\right]\right)$$

Overfitting



$$x(t)= \left(0.556831 - 0.427534\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] - 0.0927282\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] + 0.0152492\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.00213549\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0120502\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] + 0.0107965\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.0112531\,\text{Cos}\left[\frac{14\,j\,\pi}{27}\right] - 0.00415648\,\text{Cos}\left[\frac{16\,j\,\pi}{27}\right] - 0.00958542\,\text{Cos}\left[\frac{2\,j\,\pi}{3}\right] + 0.00508887\,\text{Cos}\left[\frac{20\,j\,\pi}{27}\right] + 0.00125267\,\text{Cos}\left[\frac{22\,j\,\pi}{27}\right] - 0.0019295\,\text{Cos}\left[\frac{8\,j\,\pi}{9}\right] - 0.00179849\,\text{Cos}\left[\frac{26\,j\,\pi}{27}\right] - 0.00105706\,\text{Cos}\left[\frac{28\,j\,\pi}{27}\right] + 0.198064\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.027242\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] - 0.0435712\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.0354978\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.00210601\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] + 0.00128555\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right] - 0.00223198\,\text{Sin}\left[\frac{14\,j\,\pi}{27}\right] - 0.00923493\,\text{Sin}\left[\frac{16\,j\,\pi}{27}\right] + 0.00769094\,\text{Sin}\left[\frac{2\,j\,\pi}{3}\right] + 0.00724309\,\text{Sin}\left[\frac{20\,j\,\pi}{27}\right] - 0.000161075\,\text{Sin}\left[\frac{22\,j\,\pi}{27}\right] + 0.00070835\,\text{Sin}\left[\frac{8\,j\,\pi}{9}\right] + 0.000336465\,\text{Sin}\left[\frac{26\,j\,\pi}{27}\right] - 0.000230555\,\text{Sin}\left[\frac{28\,j\,\pi}{27}\right]\right)$$

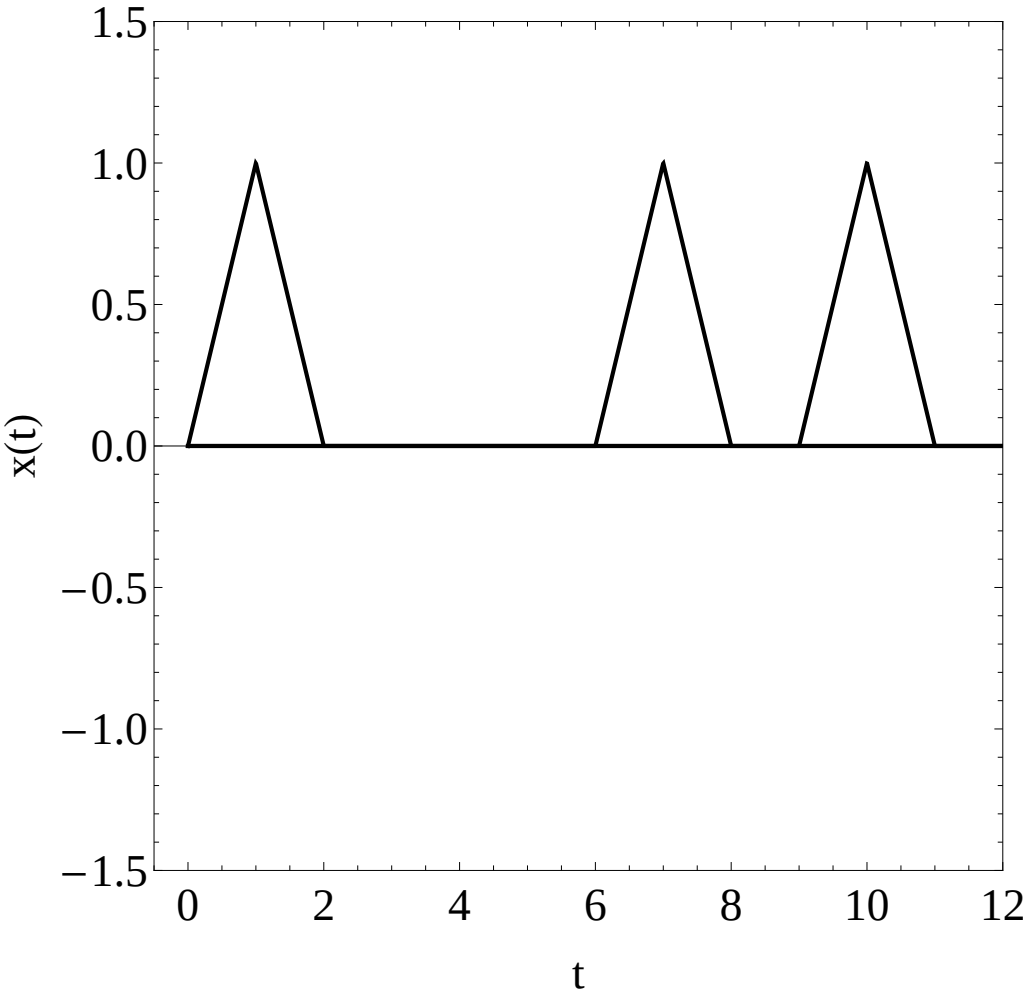
$$y(t)= \left(0.286858 + 0.0791214\,\text{Cos}\left[\frac{2\,j\,\pi}{27}\right] + 0.00210559\,\text{Cos}\left[\frac{4\,j\,\pi}{27}\right] - 0.0254946\,\text{Cos}\left[\frac{2\,j\,\pi}{9}\right] + 0.0657329\,\text{Cos}\left[\frac{8\,j\,\pi}{27}\right] - 0.0218874\,\text{Cos}\left[\frac{10\,j\,\pi}{27}\right] - 0.0445885\,\text{Cos}\left[\frac{4\,j\,\pi}{9}\right] + 0.00697783\,\text{Cos}\left[\frac{14\,j\,\pi}{27}\right] - 0.0357556\,\text{Cos}\left[\frac{16\,j\,\pi}{27}\right] - 0.000429811\,\text{Cos}\left[\frac{2\,j\,\pi}{3}\right] + 0.00174577\,\text{Cos}\left[\frac{20\,j\,\pi}{27}\right] - 0.0136406\,\text{Cos}\left[\frac{22\,j\,\pi}{27}\right] + 0.00236546\,\text{Cos}\left[\frac{8\,j\,\pi}{9}\right] - 0.00550866\,\text{Cos}\left[\frac{26\,j\,\pi}{27}\right] + 0.00141671\,\text{Cos}\left[\frac{28\,j\,\pi}{27}\right] + 0.144433\,\text{Sin}\left[\frac{2\,j\,\pi}{27}\right] + 0.0737055\,\text{Sin}\left[\frac{4\,j\,\pi}{27}\right] + 0.012766\,\text{Sin}\left[\frac{2\,j\,\pi}{9}\right] - 0.00454687\,\text{Sin}\left[\frac{8\,j\,\pi}{27}\right] - 0.0275796\,\text{Sin}\left[\frac{10\,j\,\pi}{27}\right] - 0.00924224\,\text{Sin}\left[\frac{4\,j\,\pi}{9}\right] - 0.0173974\,\text{Sin}\left[\frac{14\,j\,\pi}{27}\right] + 0.0184027\,\text{Sin}\left[\frac{16\,j\,\pi}{27}\right] + 0.00668781\,\text{Sin}\left[\frac{2\,j\,\pi}{3}\right] - 0.00171511\,\text{Sin}\left[\frac{20\,j\,\pi}{27}\right] + 0.00638932\,\text{Sin}\left[\frac{22\,j\,\pi}{27}\right] - 0.00215222\,\text{Sin}\left[\frac{8\,j\,\pi}{9}\right] + 0.00198997\,\text{Sin}\left[\frac{26\,j\,\pi}{27}\right] - 0.000476597\,\text{Sin}\left[\frac{28\,j\,\pi}{27}\right]\right)$$

Conclusions: Astrophysical Time Series

- Data sets are problematic.
- Time series methods are affected.
- Extensive Monte Carlo simulations are mandatory.
- Astrophysical time series DO contain information about the BH environment.
- Simultaneous observations across the e/m spectrum are needed.

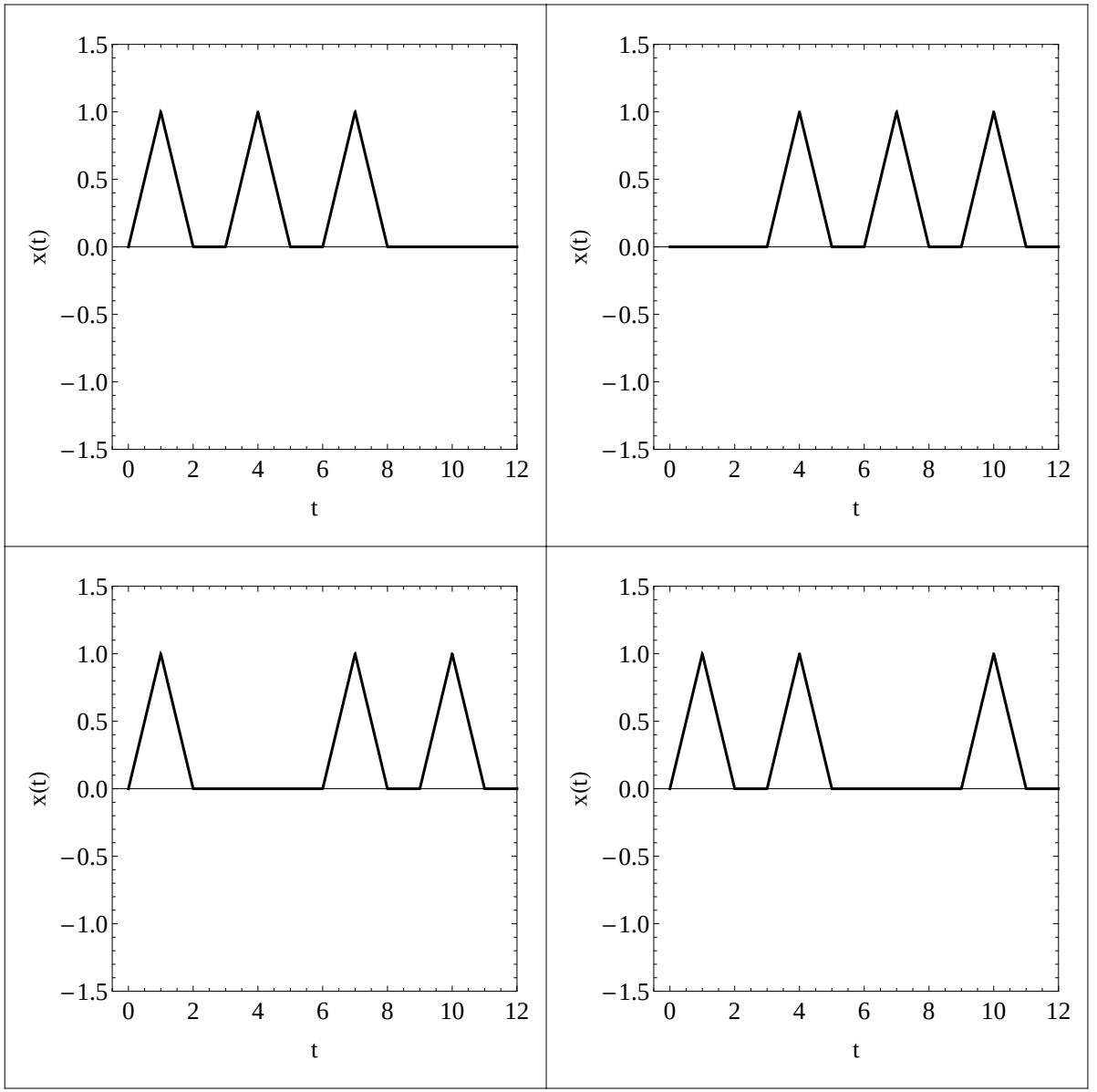
Thanks you!

Additional material



Additional material

Bursty’ chance coincidence=25%



Additional material

‘Gaussian’ chance coincidence≃3%