

Dirac-like plasmons in honeycomb arrays of interacting metallic nanoparticles

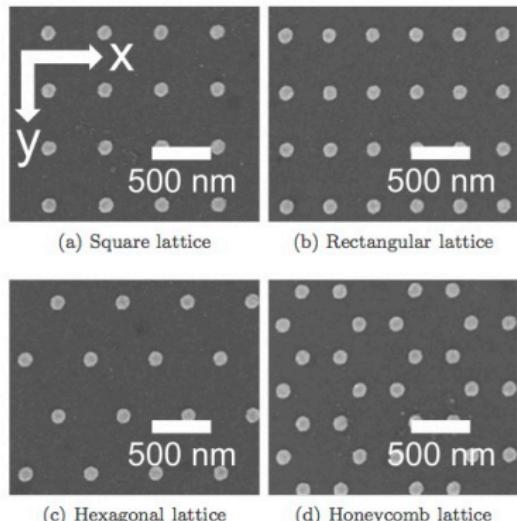
Inès Rachid & Mauricio Gómez Viloria

Supervisor: Guillaume Weick

May 23, 2016

IPCMS/Université de Strasbourg

Motivation



Different arrays of metallic nanoparticles. Humpfrey et al. PRB, 90, 2014

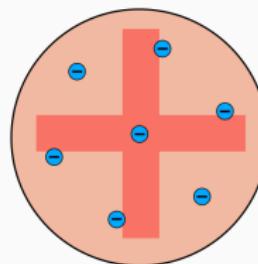
We are interested in

- Plasmonic metamaterials based on ordered metallic nanoparticle arrays
scale : 1-100nm
- Plasmonic analogue of graphene mimicking its unique electronic properties

Single nanoparticle in the jellium model

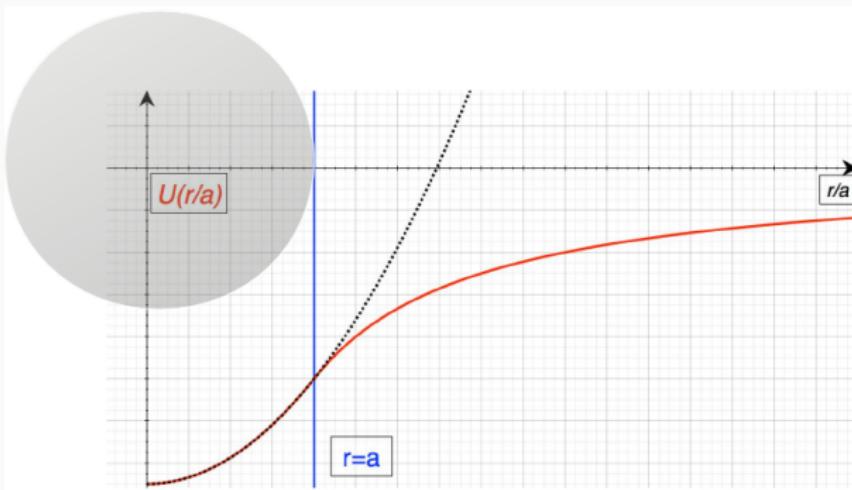
Spherical neutral metallic nanoparticle under:

- Born-Oppenheimer approx.
(free valence electrons and
fixed ions)
- Jellium model (continuous
positive charged density)



Wikimedia Commons

Single nanoparticle in the jellium model



$$H = \sum_{i=1}^N \left[\frac{P_i^2}{2m_e} + U(r_i) \right] + \frac{e^2}{2} \sum_{i \neq j}^N \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

$U(r_i)$: Electrostatic potential energy of an electron in the external field of a homogeneously positive charged sphere

LOCALIZED SURFACE PLASMON



Nanoparticle under an external field

Center of mass and relative coordinates

$$\left\{ \begin{array}{l} \vec{R} = \sum_{i=1}^N \vec{r}_i / N \\ \vec{P} = \sum_{i=1}^N \vec{p}_i \\ \vec{r}'_i = \vec{r}_i - \vec{R} \\ \vec{p}'_i = \vec{p}_i - \vec{P} / N \end{array} \right.$$

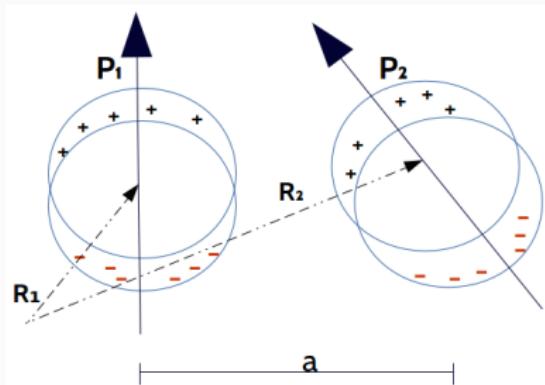
$$H = \frac{P^2}{2M} + \sum_{i=1}^N \frac{p_i'^2}{2m_e} + U(\vec{r}_i' + \vec{R}) + \frac{e^2}{2} \sum_{i \neq j} \frac{1}{|\vec{r}_i' - \vec{r}_j'|} ; \quad M = N m_e$$

For $|\vec{R}| < r_0$

$$H \approx \frac{P^2}{2M} + \frac{M \tilde{\omega}_M^2 R^2}{2} + H'_{rel} + H_c ; \quad \tilde{\omega}_M = \sqrt{\frac{N_{in} e^2}{m_e r_0^3}}$$

Dipole-dipole interaction between two nanoparticles

Near-field and quasistatic approximation $3r_0 < a \ll \lambda$

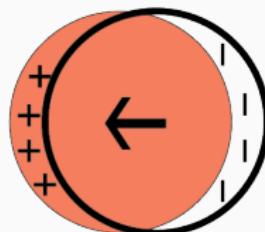
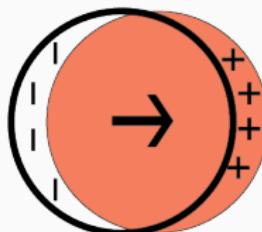
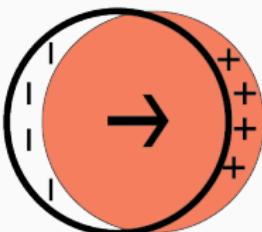
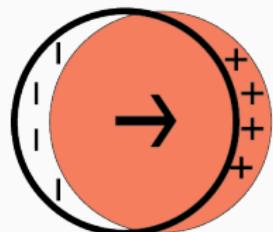
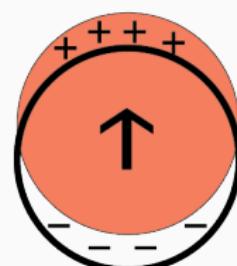
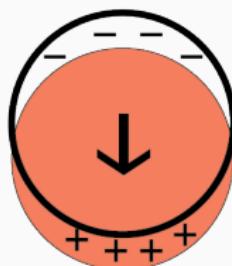
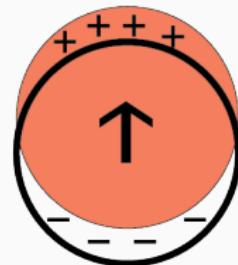
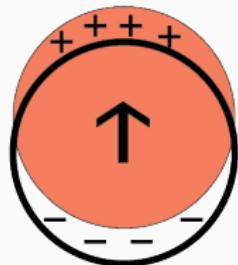


$$V_{d-d} = \frac{\vec{p}_1 \cdot \vec{p}_2 - 3(\vec{p}_1 \cdot \hat{n})(\vec{p}_2 \cdot \hat{n})}{|\vec{R}_1 - \vec{R}_2|^3}$$

$$\vec{P} = -eN\hbar(\vec{R})$$

DIMER : NORMAL MODES

Two normal modes for each direction of space:

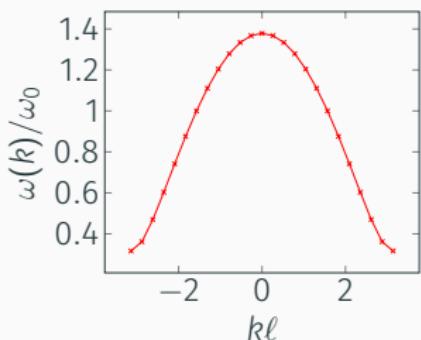


Transverse and longitudinal modes

LINEAR CHAIN OF PLASMONIC NANOPARTICLES

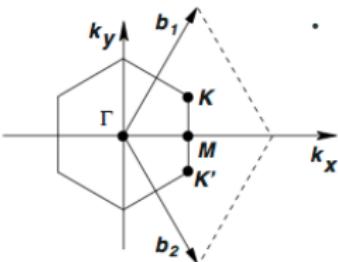
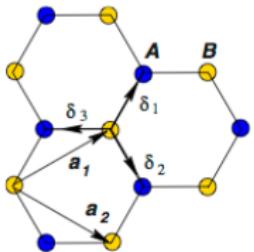


- Chain of infinite number of nanoparticles
- Interaction between LSPs: Nearest-neighbors dipole-dipole interaction \Rightarrow collective plasmonic modes
- Periodicity \Rightarrow Bloch theorem



$$\omega(k) = \omega_0 \sqrt{1 + 2\eta \cos(kl)}$$

HONEYCOMB LATTICE



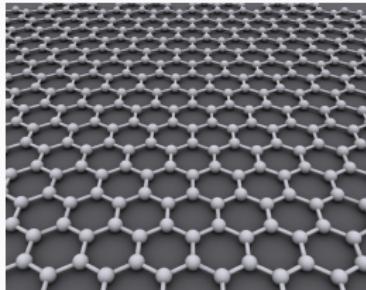
- 2D Hexagonal lattice
- Lattice with a basis
Triangular Bravais lattice with a two point basis

$$\vec{r} = n\vec{a}_1 + m\vec{a}_2$$

Castro Neto et al., Review of Modern Physics, 81, 2009

- Two different sublattices \Rightarrow two Bloch functions
- Hexagonal reciprocal lattice

GRAPHENE



Wikimedia Commons

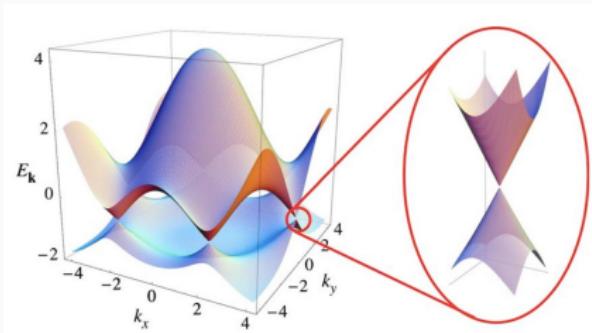
Tight-binding Hamiltonian
considering nearest neighbors
hopping

$$H = -t \sum_{\langle i,j \rangle} (a_i^\dagger b_j + H.c.)$$

Dispersion relation

$$E(\vec{k}) = \pm t \sqrt{3 + f(\vec{k})} \approx \hbar v_F |\vec{k}_{(K)}|$$

t : nearest-neighbor hopping
energy



ELECTRONIC PROPERTIES OF GRAPHENE

Relativistic effective Hamiltonian

$$H_{\text{eff}} = \pm \hbar v_F \vec{\sigma} \cdot \vec{k}$$

Fermi velocity: $v_F = c/300$

- Hamiltonian of pseudorelativistic particles
- Conserved helicity



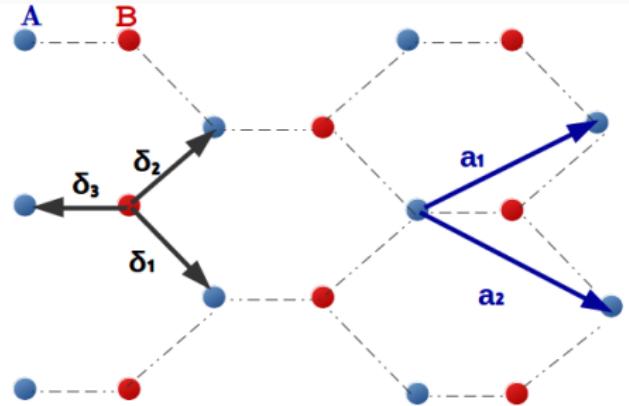
Ceci n'est pas un électron

- No backscattering
- Klein paradox: 100% Transmission
- \Rightarrow High mobility !

PLASMONIC ANALOG OF GRAPHENE

Conditions:

- Nearest-neighbors interactions
- Dipole-dipole interaction
- Normal modes solution



$$H_0 = \sum_{s=A,B} \sum_{\vec{R}_s} \left(\frac{P_s^2}{2M} + \frac{M\omega_0^2 h_s^2(\vec{R}_s)}{2} \right)$$

$$H_{\text{int}} = M\omega_0^2 \eta \sum_{\vec{R}_B} \sum_{j=1}^3 h_B(\vec{R}_B) h_A(\vec{R}_B + \vec{\delta}_j); \quad \eta = \left(\frac{r}{a} \right)^3$$

SECOND QUANTIZATION AND BLOCH TRANSFORMATION

Anihilation and creation operators

$$\begin{aligned}a_{\vec{R}} &= \sqrt{M\omega_0/2\hbar}h_A(\vec{R}) + iP_A/\sqrt{2\hbar M\omega_0} \\b_{\vec{R}} &= \sqrt{M\omega_0/2\hbar}h_B(\vec{R}) + iP_B/\sqrt{2\hbar M\omega_0} \\[b_{\vec{R}}, a_{\vec{R}'}^\dagger] &= 0 ; [a_{\vec{R}}, a_{\vec{R}'}^\dagger] = \delta_{\vec{R}, \vec{R}'} ; [b_{\vec{R}}, b_{\vec{R}'}^\dagger] = \delta_{\vec{R}, \vec{R}'}\end{aligned}$$

Second Quantization Hamiltonian

$$H_0 = \hbar\omega_0 \sum_{\vec{R}_A} a_{\vec{R}_A}^\dagger a_{\vec{R}_A} + \hbar\omega_0 \sum_{\vec{R}_B} b_{\vec{R}_B}^\dagger b_{\vec{R}_B}$$

$$H_{\text{int}} = \hbar\omega_0(\eta/2) \sum_{\vec{R}_B} \sum_{j=1}^3 b_{\vec{R}_B}^\dagger (a_{\vec{R}_B + \vec{\delta}_j}^\dagger + a_{\vec{R}_B + \vec{\delta}_j}) + \text{h.c.}$$

Translational symmetry

$$H = \sum_{\vec{q}} \hbar\omega_0(a_{\vec{q}}^\dagger a_{\vec{q}} + b_{\vec{q}}^\dagger b_{\vec{q}}) + \hbar\omega_0(\eta/2)f_{\vec{q}}b_{\vec{q}}^\dagger(a_{\vec{q}}^\dagger + a_{\vec{q}}) + \text{h.c.}$$

$$f_{\vec{q}} = \sum_{j=1}^3 e^{i\vec{\delta}_j \cdot \vec{q}}$$

BOSONIC BOGOLIUBOV TRANSFORMATION

- Diagonalisation of quadratic Hamiltonians

$$H = \varepsilon(a^\dagger a + b^\dagger b) + \chi(a^\dagger b^\dagger + ba)$$

$$H = \sqrt{\varepsilon^2 - \chi^2}(\alpha_1^\dagger \alpha_1 + \alpha_2^\dagger \alpha_2) - \varepsilon + \sqrt{\varepsilon^2 - \chi^2}$$

- Mix creation and annihilation operators

$$\begin{pmatrix} a \\ b^\dagger \end{pmatrix} = \begin{pmatrix} \cosh \vartheta & \sinh \vartheta \\ \sinh \vartheta & \cosh \vartheta \end{pmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \alpha_2^\dagger \end{pmatrix}$$

- Preserve bosonic commutation relations

$$[a^\dagger, b^\dagger] = 0 \quad \cosh \vartheta^2 - \sinh \vartheta^2 = 1$$

$$[a, a^\dagger] = \cosh \vartheta^2 [\alpha_1, \alpha_1^\dagger] - \sinh \vartheta^2 [\alpha_2, \alpha_2^\dagger] = 1$$

DIAGONALISATION

$$H = \sum_{\vec{q}} \hbar\omega_0(a_{\vec{q}}^\dagger a_{\vec{q}} + b_{\vec{q}}^\dagger b_{\vec{q}}) + \hbar\omega_0(\eta/2)f_{\vec{q}}b_{\vec{q}}^\dagger(a_{\vec{q}}^\dagger + a_{\vec{q}}) + \text{h.c.}$$

$$f_{\vec{q}} = \sum_{j=1}^3 e^{i\vec{\delta}_j \cdot \vec{q}}$$

DIAGONALISATION

$$H = \sum_{\vec{q}} \hbar\omega_0(a_{\vec{q}}^\dagger a_{\vec{q}} + b_{\vec{q}}^\dagger b_{\vec{q}}) + \hbar\omega_0(\eta/2)f_{\vec{q}}b_{\vec{q}}^\dagger(a_{\vec{q}}^\dagger + a_{\vec{q}}) + \text{h.c.}$$

$$f_{\vec{q}} = \sum_{j=1}^3 e^{i\vec{\delta}_j \cdot \vec{q}}$$

Bogoliubov # 1

$$\alpha_{\vec{q}}^\pm = \frac{1}{\sqrt{2}} \left(\frac{f_{\vec{q}}}{|f_{\vec{q}}|} a_{\vec{q}} \pm b_{\vec{q}} \right)$$

DIAGONALISATION

$$H = \sum_{\vec{q}} \hbar \omega_0 (a_{\vec{q}}^\dagger a_{\vec{q}} + b_{\vec{q}}^\dagger b_{\vec{q}}) + \hbar \omega_0 (\eta/2) f_{\vec{q}} b_{\vec{q}}^\dagger (a_{\vec{q}}^\dagger + a_{\vec{q}}) + \text{h.c.}$$

$$f_{\vec{q}} = \sum_{j=1}^3 e^{i \vec{\delta}_j \cdot \vec{q}}$$

Bogoliubov #1

$$\alpha_{\vec{q}}^{\pm} = \frac{1}{\sqrt{2}} \left(\frac{f_{\vec{q}}}{|f_{\vec{q}}|} a_{\vec{q}} \pm b_{\vec{q}} \right)$$

Bogoliubov #2

$$\beta_{\vec{q}}^{\pm} = \cosh(\vartheta_{\vec{q}}^{\pm}) \alpha_{\vec{q}}^{\pm} - \sinh(\vartheta_{\vec{q}}^{\pm}) \alpha_{-\vec{q}}^{\pm\dagger}$$

$$\begin{aligned} \cosh \vartheta_{\vec{q}}^{\pm} &= 2^{1/2} [(1 \pm \eta |f_{\vec{q}}|/2)/(1 \pm \eta |f_{\vec{q}}|) + 1]^{1/2} \\ \sinh \vartheta_{\vec{q}}^{\pm} &= \mp 2^{1/2} [(1 \pm \eta |f_{\vec{q}}|/2)/(1 \pm \eta |f_{\vec{q}}|) - 1]^{1/2} \end{aligned}$$

$$H = \hbar \omega_0 \sum_{\tau=\pm} \sum_{\vec{q}} (1 + \frac{\eta}{2} \tau |f_{\vec{q}}|) \alpha_{\vec{q}}^{\tau\dagger} \alpha_{\vec{q}}^{\tau} + \tau \frac{\eta}{4} f_{\vec{q}} (\alpha_{\vec{q}}^{\tau\dagger} \alpha_{-\vec{q}}^{\tau} + \text{h.c.})$$

DIAGONALISATION

$$H = \sum_{\vec{q}} \hbar\omega_0 (a_{\vec{q}}^\dagger a_{\vec{q}} + b_{\vec{q}}^\dagger b_{\vec{q}}) + \hbar\omega_0 (\eta/2) f_{\vec{q}} b_{\vec{q}}^\dagger (a_{\vec{q}}^\dagger + a_{\vec{q}}) + \text{h.c.}$$

$$f_{\vec{q}} = \sum_{j=1}^3 e^{i\vec{\delta}_j \cdot \vec{q}}$$

Bogoliubov #1

$$\alpha_{\vec{q}}^{\pm} = \frac{1}{\sqrt{2}} \left(\frac{f_{\vec{q}}}{|f_{\vec{q}}|} a_{\vec{q}} \pm b_{\vec{q}} \right)$$

Bogoliubov #2

$$\beta_{\vec{q}}^{\pm} = \cosh(\vartheta_{\vec{q}}^{\pm}) \alpha_{\vec{q}}^{\pm} - \sinh(\vartheta_{\vec{q}}^{\pm}) \alpha_{-\vec{q}}^{\pm\dagger}$$

$$\cdot \cosh \vartheta_{\vec{q}}^{\pm} = 2^{1/2} [(1 \pm \eta |f_{\vec{q}}|/2)/(1 \pm \eta |f_{\vec{q}}|) + 1]^{1/2}$$

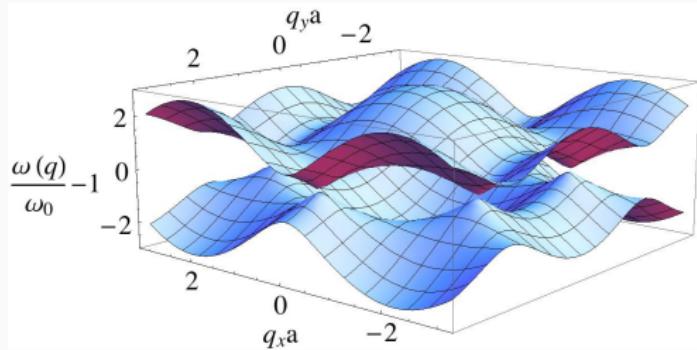
$$\cdot \sinh \vartheta_{\vec{q}}^{\pm} = \mp 2^{1/2} [(1 \pm \eta |f_{\vec{q}}|/2)/(1 \pm \eta |f_{\vec{q}}|) - 1]^{1/2}$$

$$H = \hbar\omega_0 \sum_{\tau=\pm} \sum_{\vec{q}} (1 + \frac{\eta}{2} \tau |f_{\vec{q}}|) \alpha_{\vec{q}}^{\tau\dagger} \alpha_{\vec{q}}^{\tau} + \tau \frac{\eta}{4} f_{\vec{q}} (\alpha_{\vec{q}}^{\tau\dagger} \alpha_{-\vec{q}}^{\tau} + \text{h.c.})$$

Diagonalized Hamiltonian

$$H = \sum_{\tau=\pm} \sum_{\vec{q}} \hbar\omega_{\vec{q}}^{\tau} \beta_{\vec{q}}^{\tau\dagger} \beta_{\vec{q}}^{\tau}$$

SPECTRUM AND PSEUDORELATIVISTIC PROPERTIES



$$\omega_{\vec{q}}^{\pm} = \omega_0 \sqrt{1 \pm \eta |f_{\vec{q}}|}$$

$$\omega_{\vec{q}}^{\pm} \approx \omega_0 \pm v ||\vec{k}||$$

Near K point:

$$H \approx \hbar \omega_0 \begin{pmatrix} a_{\vec{k}}^\dagger & b_{\vec{k}}^\dagger \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a_{\vec{k}} \\ b_{\vec{k}} \end{pmatrix} + \hbar v \begin{pmatrix} a_{\vec{k}}^\dagger & b_{\vec{k}}^\dagger \end{pmatrix} \begin{pmatrix} 0 & k_x + ik_y \\ k_x - ik_y & 0 \end{pmatrix} \begin{pmatrix} a_{\vec{k}} \\ b_{\vec{k}} \end{pmatrix}$$

$$H_{\text{eff}} = \hbar \omega_0 \mathbb{I} + \hbar v \vec{\sigma} \cdot \vec{k}$$

Relativistic Hamiltonian for bosonic massless particles

GRAPHENE VS ARTIFICIAL PLASMONIC GRAPHENE

GRAPHENE	Plasmonic graphene	
Fermions (electrons)	Bosons (plasmons)	
Electron hopping between orbitals	Dipole-dipole interaction between nanoparticles	
$b_R^\dagger a_{R+e_j}$	$b_R^\dagger a_{R+e_j}$	$b_R^\dagger a_{R+e_j}^\dagger$

SUMMARY

We analyzed...

- Jellium model for nanoparticles
- Different arrays of nanoparticles under dipole-dipole interaction
- Analysis of a plasmonic honeycomb lattice
- Comparison with graphene

Further research

- Klein paradox in plasmonic graphene
- Dissipation and damping
- Other geometries