Gravitational Instability in the Cosmic Web

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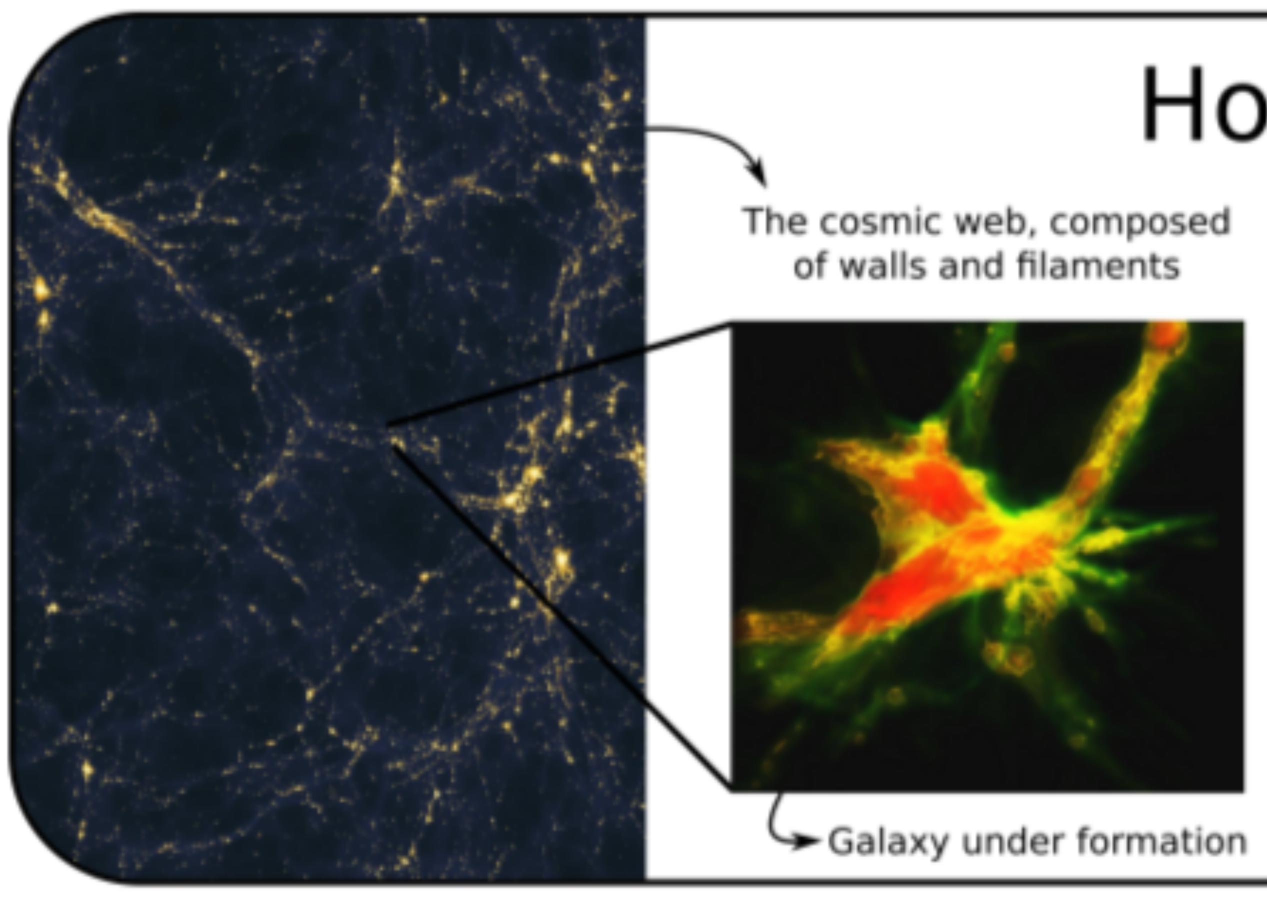
How do structures in the Universe form?



Cosmological numerical simulations and observations suggest that matter in the Universe is distributed in a web-like structure, called the **cosmic web**. Galaxy clusters are at the intersection of filaments, and filaments themselves correspond to the intersection of planar structures, called walls.

Walls and filaments are not homogeneous. The overdense **fragments** they contain at early stages of cosmic history probably evolved into the galaxies we see today.

But what is the physical **origin** of these fragments? Is it primordial or did they form later, in the cosmic web?

Method

1. Self-gravitating cylinder and/or plane at hydrostatic equilibrium: Profile $\rho_0(\vec{r})$
2. Perturbations: Lagrangian displacement $\vec{\xi}$ (such that $\vec{v}_1 = \partial_t \vec{\xi}$)
3. Stability: Normal mode analysis

Intuitively:

	stable	Unstable
displacement $\vec{\xi}$	→	→
force $\vec{F}$	←	→
eigenvalue ω^2	> 0	< 0

energy

Formally:

Linearized momentum conservation and Poisson equation:

$$\begin{cases} \rho_0 \partial_t \vec{v}_1 = \vec{F} \\ \text{with } \vec{F} = -\vec{\nabla} p_1 + \rho_1 \vec{g}_0 + \rho_0 \vec{g}_1 \\ \vec{\nabla} \cdot \vec{g}_1 = -4\pi G \rho_1 \end{cases} \xrightarrow[\vec{\xi}(\vec{r},t) = \hat{\xi}(\vec{r}) e^{-i\omega t}]{\text{normal modes}} \boxed{-\rho_0 \omega^2 \hat{\xi} = \vec{F}(\hat{\xi})} \quad (*)$$

Aim:

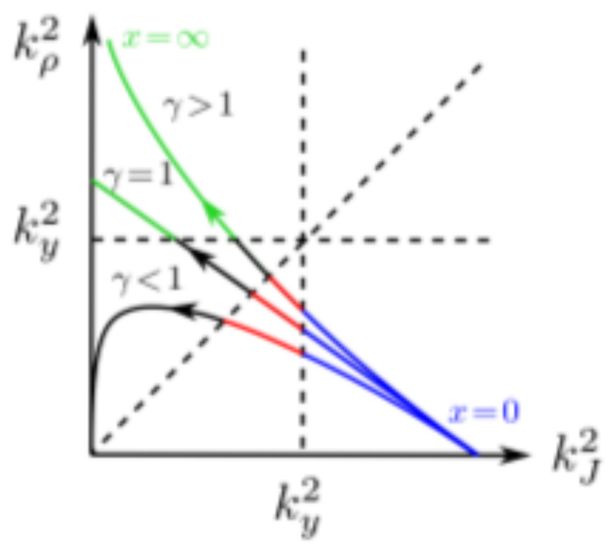
Find the sign of ω^2 (stability criterion)
and its value (growth rate of the instability)
as a function of scale (size of fragments)

Example: Cosmic Walls

Intuitively: **(I)** Two characteristic lengths:

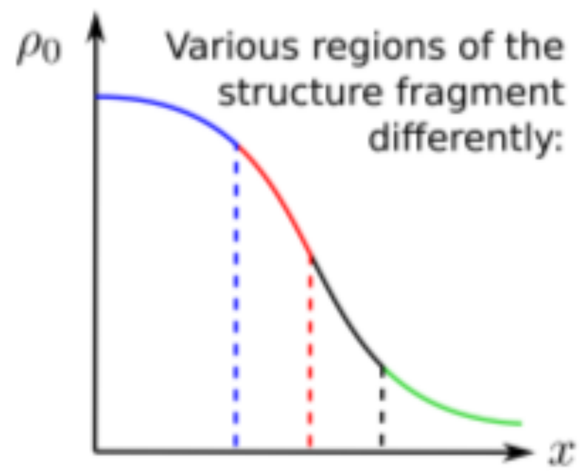
Local gradient: $k_\rho(x) = \left| \frac{\rho_0'}{\rho_0} \right|$ & Local Jeans length: $k_J^{-1}(x) = \frac{c_a}{\sqrt{4\pi G \rho_0}}$

For various polytropes: $(p_0 \propto \rho_0^\gamma)$



Ordering depends on position x:

$k_\rho < k_y < k_J$
$k_\rho < k_J < k_y$
$k_J < k_\rho < k_y$
$k_J < k_y < k_\rho$

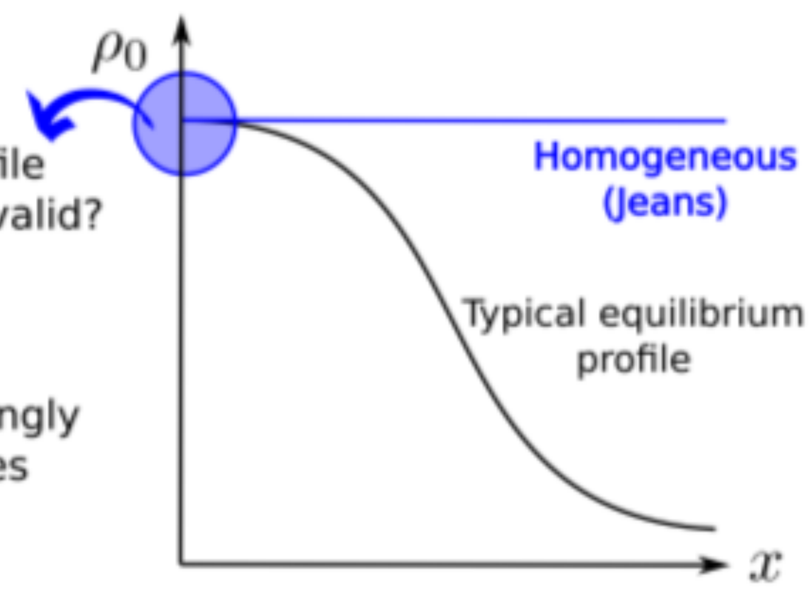


(II) Beware: Intuition may be deceiving...

Here: locally uniform profile hence usual Jeans criterion valid?

NO !

Because perturbations strongly depend on the derivatives of ρ_0 and c_a^2 at $x=0$!



Formally: In this case: $\vec{\xi} = \left(\hat{\xi}_x(x)\hat{x} + \hat{\xi}_y(x)\hat{y} \right) e^{i(k_y y - \omega t)}$

Homogeneous: (ρ_0 uniform) From equation $(*)$ we get:

$$c_a^2 \ \hat{\xi}_x^{(4)} + (\omega^2 + \omega_0^2 - 2\omega_y^2) \ \hat{\xi}_x'' - k_y^2(\omega^2 + \omega_0^2 - \omega_y^2) \ \hat{\xi}_x = 0$$

Constant coefficients $\Rightarrow \hat{\xi}_x \propto e^{i k_x x}$

$\Rightarrow$ Usual Jeans criterion:

$$\omega^2 = c_a^2 (k^2 - k_J^2)$$

Competition: acoustic pressure ($c_a^2 k^2$) — gravitational collapse (k_J^2)

Stratified profile: $(\rho_0 = \rho_0(x))$ From equation $(*)$ we get:

$$\sum_{i=0}^{4} A_i \hat{\xi}_x^{(i)} = 0$$

where the $A_i \propto \prod_n (\omega^2 - \omega_n^2)$ exhibit the characteristic frequencies $\omega_n^2(x)$ of the dynamics.

Tools for the analysis:
- Spectral study (e.g singularities)
- Stability criteria (e.g Suydam)
- Local dispersion relations
- WKB resolutions: quantification (boundary conditions)

Conclusion & Perspectives

The stratification:

- Modifies stability criteria
- With some regions stabilized and others destabilized

$\Rightarrow$ **Fragments may form in the Cosmic Web**

From now on, this approach is ideally suited to include:

- ✓ Cosmological expansion
- ✓ Dark matter: External potential, then bi-fluid description
- ✓ Flows (stationary)
- ✓ Magnetic field
- ✓ Convection

Credits: Cosmic web: Andrew Pontzen, Fabio Governato. Protogalaxy: Greif et al, MNRAS, **387**, 1021 (2008)