Interface dynamics with correlated noise: Emergent symmetries and non-universal observables

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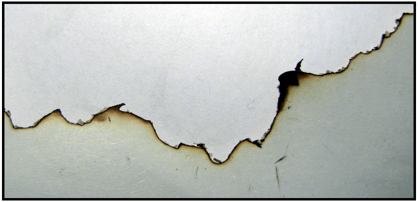
Interface dynamics

The **Kardar–Parisi–Zhang (KPZ)** equation is a model for the dynamics of interfaces with

- Non-Equilibrium scale invariance
- a mathematically exact solution

Motivation

The main objective is to understand the effect of a correlated noise on the dynamics of the Kardar–Parisi–Zhang (KPZ) steady-state.

In particular I plan on:

- Capturing the physics of the stationary state at **all** scales.
- Using the Functional Renormalisation Group (FRG)

Spatial correlations can be used to model

- existing microscopic correlations.
- a large scales driving mechanism.

Plan

Based on...

This exploits the formalism, approximation scheme and numerical code developed in

- L. Canet, arXiv:cond-mat/0509541v4 [cond-mat.stat-mech]
- L. Canet, H. Chaté, B. Delamotte, N. Wschebor, Phys.Rev.Lett.104:150601,2010, arXiv:0905.1025v2 [cond-mat.stat-mech]
- L. Canet, H. Chaté, B. Delamotte, N. Wschebor, Phys. Rev. E 84, 061128 (2011); Phys. Rev. E 86, E019904 (2012), arXiv:1107.2289v3 [cond-mat.stat-mech]
- T. Kloss, L. Canet, N. Wschebor, Phys. Rev. E 86, 051124 (2012), arXiv:1209.4650v2 [cond-mat.stat-mech]
- T. Kloss, L. Canet, B. Delamotte, N. Wschebor, Phys. Rev. E 89, 022108 (2014), arXiv:1312.6028v2 [cond-mat.stat-mech]
- T. Kloss, L. Canet, N. Wschebor, Phys. Rev. E 90, 062133 (2014), arXiv:1409.8314v2 [cond-mat.stat-mech]

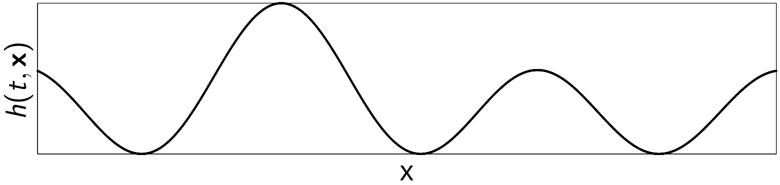
Kardar–Parisi–Zhang (KPZ) equation

A model for interface growth,

$$\partial_t h = \frac{\lambda}{2}\left[\boldsymbol{\nabla} h\right]^2 + \nu \nabla^2 h + \eta$$

with diffusion, perpendicular expansion and stochastic driving,

$$\langle \eta \rangle = 0 \quad \langle \eta(\mathbf{x}, t)\eta(\mathbf{x}', t')\rangle = D\,\delta(t - t')\,\delta(\mathbf{x} - \mathbf{x}')$$



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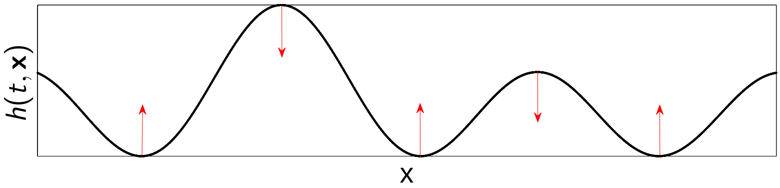

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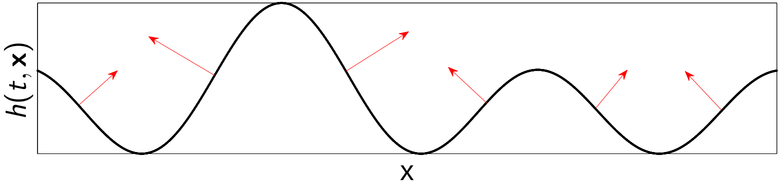

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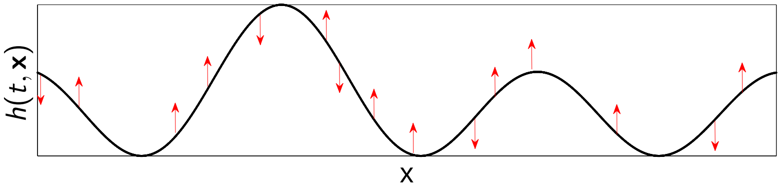


Set-up

$$\partial_t h = \frac{\lambda}{2}\left[\boldsymbol{\nabla} h\right]^2 + \nu \Delta h + \eta \,,$$

$$\langle \eta \rangle = 0 \,, \qquad \langle \eta(t,\mathbf{x})\eta(t',\mathbf{x}')\rangle = 2D\delta(t-t')\, R_\xi(\mathbf{x}-\mathbf{x}') \,.$$

The interface is propagating in a **correlated environment**.

$$R_\xi(\mathbf{r}) = \frac{1}{\left(\sqrt{2\pi}\xi\right)^d} \mathrm{e}^{-\frac{r^2}{2\xi^2}} \,, \qquad R_\xi(\mathbf{p}) = \mathrm{e}^{-\frac{\xi^2 p^2}{2}} \,.$$

Set-up

$$\partial_t h = \frac{1}{2}\left[\nabla h\right]^2 + \quad \Delta h + \eta\,,$$

$$\langle \eta \rangle = 0\,, \qquad \langle \eta(t,\mathbf{x})\eta(t',\mathbf{x}')\rangle = 2 \quad \delta(t-t')\,R_\xi(\mathbf{x}-\mathbf{x}')\,.$$

The interface is propagating in a **correlated environment**

$$R_\xi(\mathbf{r}) = \quad \frac{1}{\sqrt{2\pi}\;\xi}\,\mathrm{e}^{-\frac{r^2}{2\xi^2}}\,, \qquad\qquad R_\xi(\mathbf{p}) = \mathrm{e}^{-\frac{\xi^2 p^2}{2}}\,.$$

Choose $d = 1$ and pick simpler units: $\qquad \xi \to \xi\,\frac{2D\lambda^2}{\nu^3}\,.$

KPZ field theory

Stationary-state observables are generated by

$$Z[J] = \left\langle e^{\int_{t,\mathbf{x}} J(t,\mathbf{x}) h(t,\mathbf{x})} \right\rangle$$

KPZ field theory

Stationary-state observables are generated by

$$Z[J] = \langle \mathrm{e}^{\int_{t,\mathbf{x}} J(t,\mathbf{x})h(t,\mathbf{x})} \rangle \sim \int DhD\tilde{h}\, \mathrm{e}^{-S[h,\tilde{h}]+\int_{t,\mathbf{x}} J(t,\mathbf{x})h(t,\mathbf{x})}\,,$$

with

$$S = \int_{t,\mathbf{x}} \tilde{h}\left\{\partial_t h - \frac{1}{2}\left[\boldsymbol{\nabla} h\right]^2 - \nabla^2 h\right\} - \int_{t,\mathbf{x},\mathbf{y}} \tilde{h}(t,\mathbf{x})\tilde{h}(t,\mathbf{y})\, R_\xi(\mathbf{x}-\mathbf{y})\,.$$

Time-Reversal Symmetry

$$S = \int_{t,\mathbf{x}} \tilde{h} \left\{ \partial_t h - \quad \frac{1}{2} [\boldsymbol{\nabla} h]^2 - \quad \nabla^2 h \right\} - \quad \int_{t,\mathbf{x},\mathbf{y}} \tilde{h}(t,\mathbf{x}) \tilde{h}(t,\mathbf{y}) \, R_\xi(\mathbf{x}-\mathbf{y}) \, ,$$

is symmetric under

$$\left. \begin{array}{l} h'(t,\mathbf{x}) = -h(-t,\mathbf{x}) \\ \tilde{h}'(t,\mathbf{x}) = \tilde{h}(-t,\mathbf{x}) + \quad \nabla^2 h(-t,\mathbf{x}) \end{array} \right\} \quad \text{Time} \quad \text{Reversal}$$

Time-Reversal Symmetry

$$S = \int_{t,\mathbf{x}} \tilde{h} \left\{ \partial_t h - \lambda \frac{1}{2} \left[\boldsymbol{\nabla} h \right]^2 - \nu \nabla^2 h \right\} - D \int_{t,\mathbf{x},\mathbf{y}} \tilde{h}(t,\mathbf{x}) \tilde{h}(t,\mathbf{y}) \, R_\xi(\mathbf{x}-\mathbf{y}) \, ,$$

is symmetric under

$$\left. \begin{array}{l} h'(t,\mathbf{x}) = -h(-t,\mathbf{x}) \\ \tilde{h}'(t,\mathbf{x}) = \tilde{h}(-t,\mathbf{x}) + \frac{\nu}{D} \nabla^2 h(-t,\mathbf{x}) \end{array} \right\} \quad \begin{array}{l} \text{Time Reversal only} \\ d = 1 \text{ and } \xi = 0 \end{array}$$

$$\xi = 0 \; \rightarrow \; R_\xi(\mathbf{r}) = \delta(\mathbf{r})$$

The Functional Renormalisation Group (FRG)

The FRG is a non-perturbative incarnation of the Renormalisation Group (RG). It focuses on constructing the $1PI$ effective action by gradually including fluctuations on increasingly large scales.

A momentum space cut-off scale k, is introduced and fluctuations with momenta larger than k are integrated out,

$$\mathrm{e}^{-\Gamma_k[h,\tilde{h}]} = \int \Pi_{p>k} \mathrm{d}h(p)\mathrm{d}\tilde{h}(p)\, \mathrm{e}^{-S[h,\tilde{h}]}\,.$$

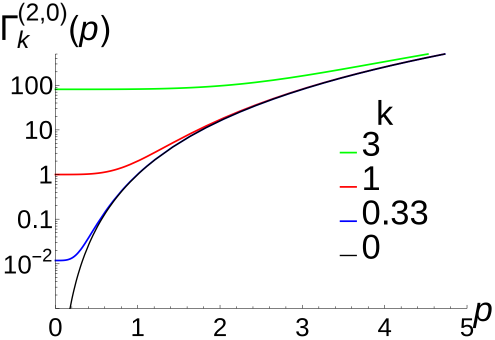
$\Gamma_k[h, \tilde{h}]$ interpolates between the bare action and the generating function of observables

$$\Gamma_{k\to\infty}[h, \tilde{h}] = S[h, \tilde{h}]$$
$$\Gamma_{k\to 0}[h, \tilde{h}] = \Gamma[h, \tilde{h}] = -\ln\left[Z[J(h)]\right]$$

Decoupling of scales

$\Gamma_k[h, \tilde{h}]$ provides

- observable quantities for $p \gg k$
- an effective microscopic action for $p \ll k$



$$\frac{\Gamma_k[h,\tilde{h}]}{\delta h(p)\delta h(-p)} = \Gamma_k^{(2,0)}(p)$$

$\Gamma^{(2,0)}(p)$ can be inferred from $\Gamma^{(2,0)}_{k \lesssim p}(p)$.

Approximation scheme

The action for the stationary state fluctuations is

$$S \;= \int_{t,\mathbf{x}} \left\{ \tilde{h} D_t h - \frac{1}{2} \left[\nabla^2 h \quad \tilde{h} + \tilde{h} \quad \nabla^2 h \right] \right\} - \int_{t,\mathbf{x}, \quad \mathbf{y}} \tilde{h}(t,\mathbf{x}) \tilde{h}(t \;, \mathbf{y}) \, R_\xi(\mathbf{x} - \mathbf{y}) \,,$$

with

$$D_t h = \partial_t h - \frac{1}{2} \left(\boldsymbol{\nabla} h \right)^2 \;.$$

Approximation scheme

The ansatz for the flowing effective action is

$$\Gamma_k = \int_{t,\mathbf{x}} \left\{ \tilde{h} D_t h - \frac{1}{2}\left[\nabla^2 h f_k^{\nu} \tilde{h} + \tilde{h} f_k^{\nu} \nabla^2 h \right] \right\} - \int_{t,\mathbf{x},t',\mathbf{y}} \tilde{h}(t,\mathbf{x}) \tilde{h}(t',\mathbf{y}) f_k^D \,,$$

with effective driving and dissipation

$$D_t h = \partial_t h - \frac{1}{2} (\boldsymbol{\nabla} h)^2 \,, \quad f_k^X = f_k^X(-\tilde{D}_t^2, -\nabla^2) \,, \quad \tilde{D}_t = \partial_t - \boldsymbol{\nabla} h \cdot \boldsymbol{\nabla} \,.$$

The RG flow

When Γ_k is expressed in terms of variables that are rescaled with k,

$$\hat{f}_k^D(\hat{\omega}, \hat{\mathbf{p}}) = \frac{f_k^D(\omega, \mathbf{p})}{D_k}\,, \qquad \hat{f}_k^\nu(\hat{\omega}, \hat{\mathbf{p}}) = \frac{f_k^\nu(\omega, \mathbf{p})}{\nu_k}\,,$$

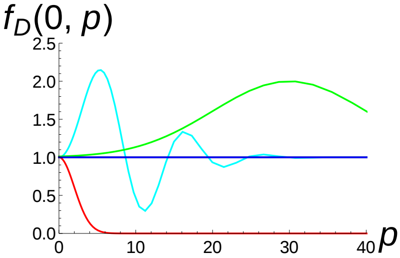
$$\hat{\mathbf{p}} = \frac{p}{k}\,, \qquad \hat{\omega} = \frac{\omega}{k^2\nu_k}\,,$$

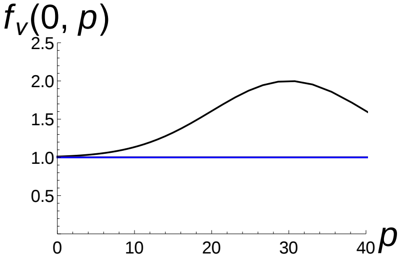
$$D_k = f_k^D(0, \mathbf{0})\,, \qquad \nu_k = f_k^\nu(0, \mathbf{0})\,,$$

the RG flow is expressed in terms of dimensionless objects and k drops out of the flow equations.

In particular a fixed point of the RG flow signals scale invariance.

A film of the RG flow





Different models correspond to different RG flow initial conditions.

	$f^D_\Lambda(\omega, \mathbf{p})$	$f^\nu_\Lambda(\omega, \mathbf{p})$
	1	1
	$e^{-(p/2)^2/2}$	1
	$e^{-[(p-30)/10]^2/2}$	1
	1	$1 + e^{-[(p-30)/10]^2/2}$
	$1 + \frac{4}{3}\, e^{-(p/10)^2/2} \sin\left[\pi(\sqrt{(p/5)^2+1} - 1)\right]$	1

KPZ fixed point

The ($\xi = 0$) KPZ fixed point seems to be reached for almost any choice of $R_\xi(r)$.

The large scale physics is universal and, up to normalisation factors does **not depend on $\boldsymbol{\xi}$**.

Two-Point correlation function

The stationary-state two-point correlation function is

$$G_\xi(\tau, \mathbf{r}) = \langle h(t+\tau, \mathbf{x}+\mathbf{r}) h(t, \mathbf{x}) \rangle - \langle h(t+\tau, \mathbf{x}+\mathbf{r}) \rangle \langle h(t, \mathbf{x}) \rangle \,.$$

The FRG provides directly its Fourier transform

$$G_\xi(\omega, \mathbf{p}) = \int_{\tau, \mathbf{r}} \mathrm{e}^{\mathrm{i}(\omega t - \mathbf{p}\cdot\mathbf{r})} G_\xi(\tau, \mathbf{r}) = \lim_{k \to 0} \frac{2 f_k^D(\omega, \mathbf{p})}{\omega^2 + \left[f_k^\nu(\omega, \mathbf{p}) p^2 \right]^2} \,.$$

Infrared (IR) data collapse

Large scale physics is described by the usual KPZ fixed point. Then

$$G_\xi(\omega, \mathbf{p}) = p^{-7/2}\, G_\xi\left(\frac{\omega}{p^{3/2}}\right) \quad \text{for } p \ll 1/\xi \text{ and } \omega \ll (1/\xi)^{3/2}\,.$$

$G(x)$ is universal up to normalisation factors

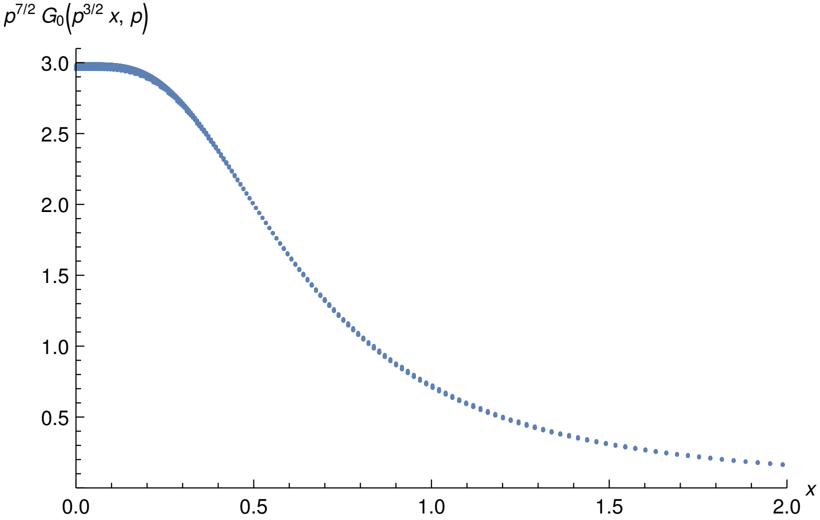
$$G_\xi(x) = \alpha_\xi\, G_0(\beta_\xi x)\,.$$

IR data collapse

This can be checked numerically.

- Plot $p^{7/2}\, G_0(x\, p^{3/2}, p)$ against $x = \omega / p^{3/2}$.
-
-
-
-
-

IR data collapse



IR data collapse

This can be checked numerically.

- Plot $p^{7/2}\, G_0(x\, p^{3/2}, p)$ against $x = \omega / p^{3/2}$.
- **Extract $\mathbf{G_0(x)}$.**
-
-
-
-

IR data collapse

This can be checked numerically.

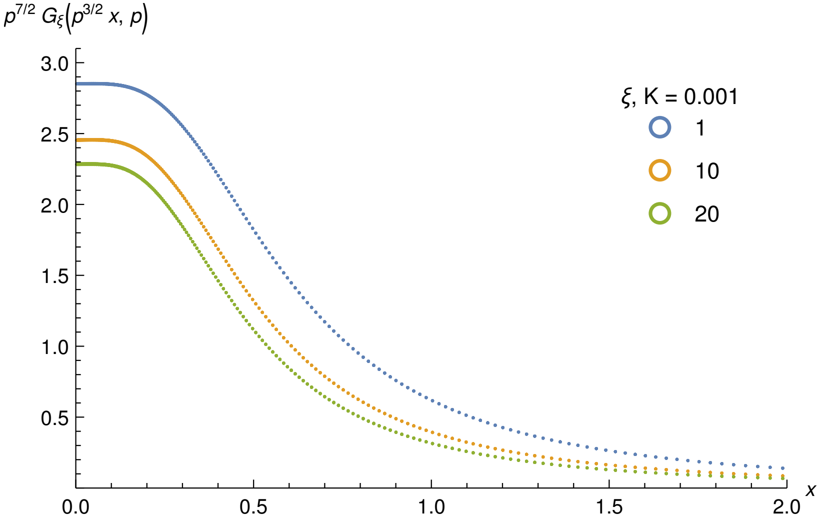
- Plot $p^{7/2}\, G_0(x\, p^{3/2}, p)$ against $x = \omega / p^{3/2}$.
- **Extract $\mathbf{G_0}(\mathbf{x})$.**
- Pick a momentum cut-off K.
-
-
-

IR data collapse

This can be checked numerically.

- Plot $p^{7/2}\, G_0(x\, p^{3/2}, p)$ against $x = \omega / p^{3/2}$.
- **Extract $\mathbf{G_0(x)}$.**
- Pick a momentum cut-off K.
- Plot $p^{7/2}\, G_\xi(x\, p^{3/2}, p)$ **for $\mathbf{p < K} \lesssim 1/\xi$** and $\omega < K^{3/2}$.
-
-

IR data collapse



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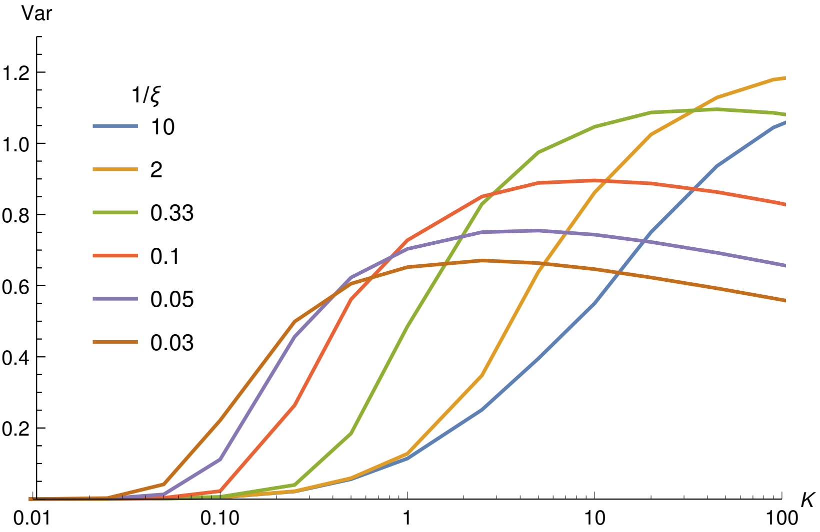
- Plot $p^{7/2}\, G_0(x\, p^{3/2}, p)$ against $x = \omega/p^{3/2}$.
- **Extract** $\mathbf{G_0(x)}$.
- Pick a momentum cut-off K.
- Plot $p^{7/2}\, G_\xi(x\, p^{3/2}, p)$ **for** $\mathbf{p < K} \lesssim 1/\xi$ and $\omega < K^{3/2}$.
- **Fit this with** $\alpha_\xi\, \mathbf{G_0}(\beta_\xi \mathbf{x})$.
- The variance of the fit tells if the scaling function is $G_0(x)$.

IR data collapse



Equal time correlation functions

The equal time two-point function

$$\bar{C}(\mathbf{r}) = \langle [h(t, \mathbf{x} + \mathbf{r}) - h(t, \mathbf{x}) - \langle h(t, \mathbf{x} + \mathbf{r}) \rangle + \langle h(t, \mathbf{x}) \rangle]^2 \rangle$$

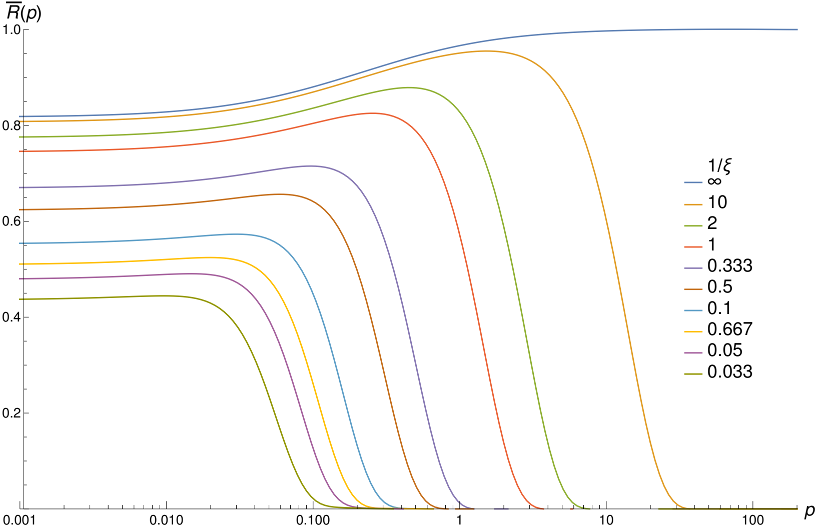
measures the fluctuations of h.

$\bar{R}(\mathbf{r}) = \frac{1}{2} \nabla^2 \bar{C}(\mathbf{r})$ measures the fluctuations of $\mathbf{u} = \boldsymbol{\nabla} h$.

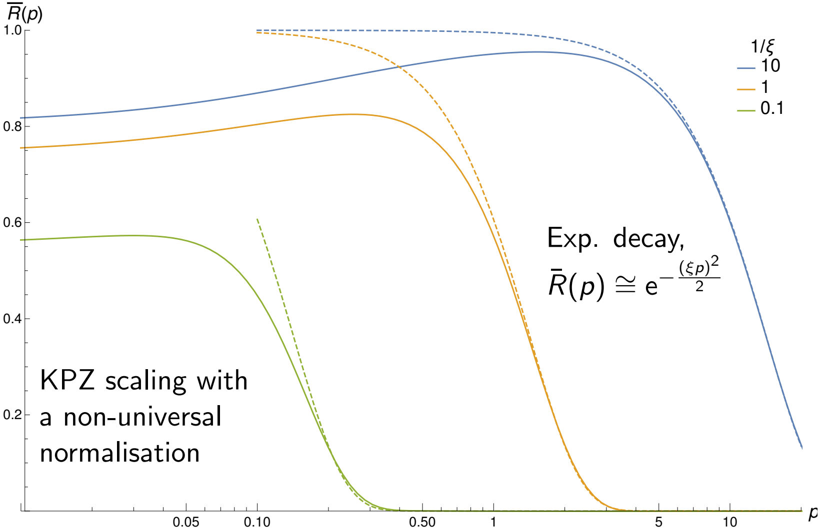
I computed the Fourier transform of $\bar{R}(\mathbf{r})$,

$$\bar{R}(\mathbf{p}) = p^2 \int_\omega G_\xi(\omega, \mathbf{p}) \, .$$

Equal time correlation functions

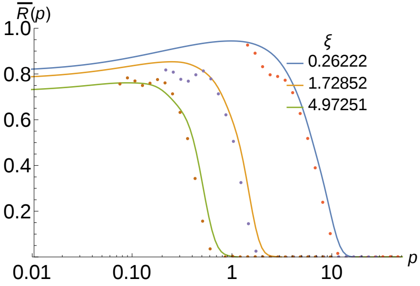


Equal time correlation functions



Equal time correlation functions

The FRG results can be compared to direct numerical simulations



This contains a correlation-time: **Galilee symmetry is emergent!**

Elisabeth Agoritsas, Vivien Lecomte, and Thierry Giamarchi, Phys. Rev. E 87, 062405

$\tilde{D}(\xi)$

$\tilde{D}(\xi)$ is defined through

$$\tilde{D}(\xi) = \int_{\mathbf{r}} \bar{R}(r)\,, \qquad \text{or} \qquad \bar{C}(\mathbf{r}) = \tilde{D}(\xi)\,|r|_{\xi}\ .$$

It is

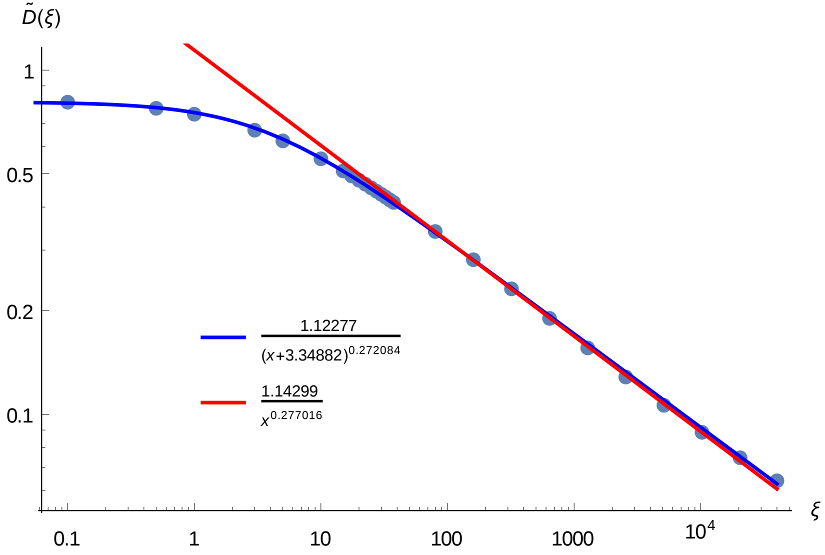
- easily observable.
- a measure of the large scale effects of ξ.

It is an exact result that $\tilde{D}(0) = 1\,,$

and it was predicted $\tilde{D}(\xi) \sim \frac{1}{\xi^{1/3}}\,,$ for $\xi \gg 1\,.$

Elisabeth Agoritsas, Vivien Lecomte and Thierry Giamarchi, Phys. Rev. E 87 042406 (2013)

$\tilde{D}(\xi)$



$\tilde{D}(\xi)$
1
0.5
0.2
0.1
0.1
1
10
100
1000
10^4
ξ
$\frac{1.12277}{(x+3.34882)^{0.272084}}$
$\frac{1.14299}{x^{0.277016}}$

Conclusions

- The TR symmetry is **emergent** at large spatial scales.
- Up to normalisation factors, the IR physics can be extracted from the known $\xi = 0$ case.
- The FRG gives access to non-universal quantities.

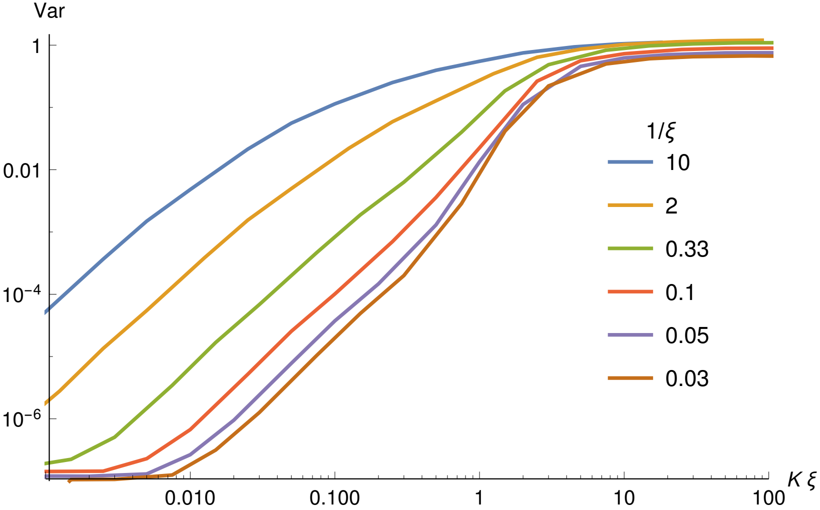
Straightforward extensions include:

- Computing three-point correlation functions
- Arbitrary $R_\xi(r)$

Interesting extensions include:

- Burgers turbulence at large ξ.
  - Energy cascade
  - Multi-scaling
- Anisotropic disorder $R \to R_{\boldsymbol{\xi}}(\mathbf{r})$

Supplementary material



Supplementary material

$$G_\xi(x) = \alpha_\xi \, G_0(\beta_\xi \, x)$$