The role of quantum measurement in stochastic thermodynamics

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Outline

- Classical concepts of thermodynamics
- A new framework for quantum thermodynamics
- Applications

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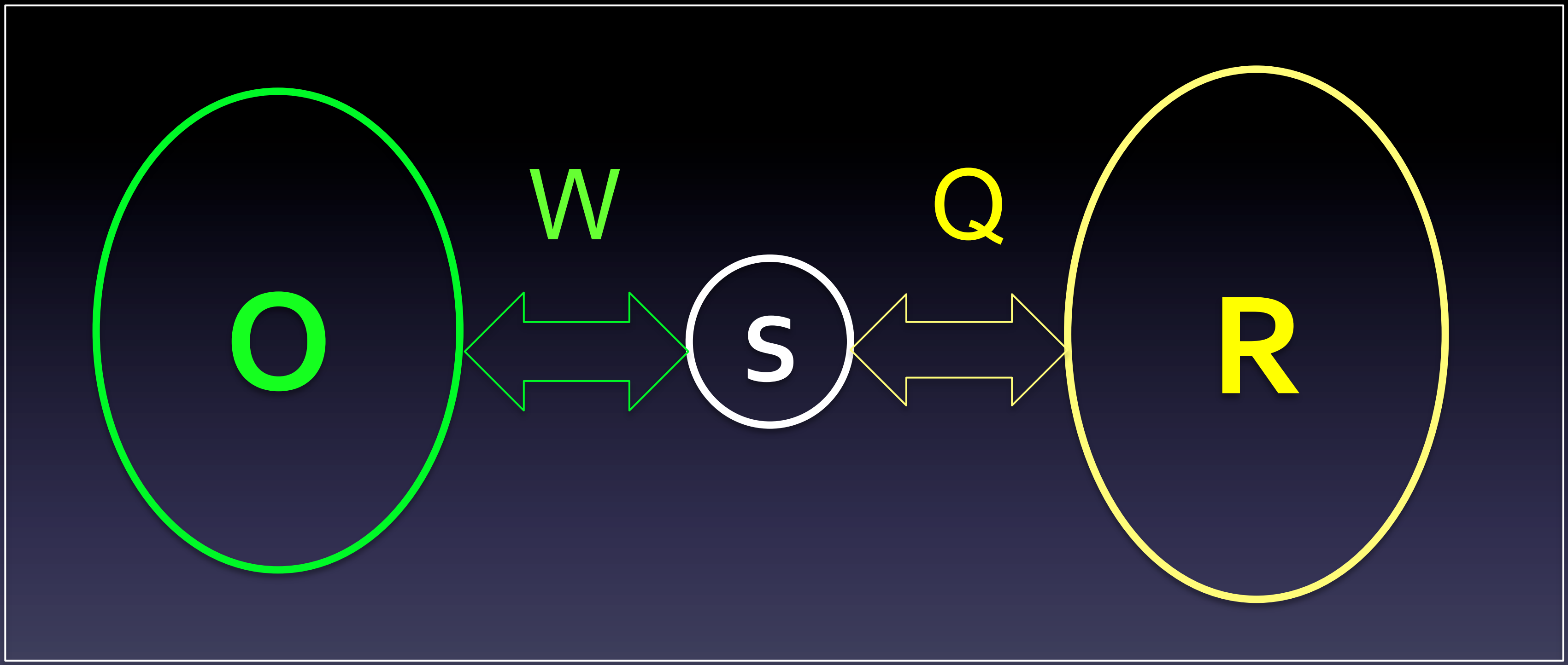
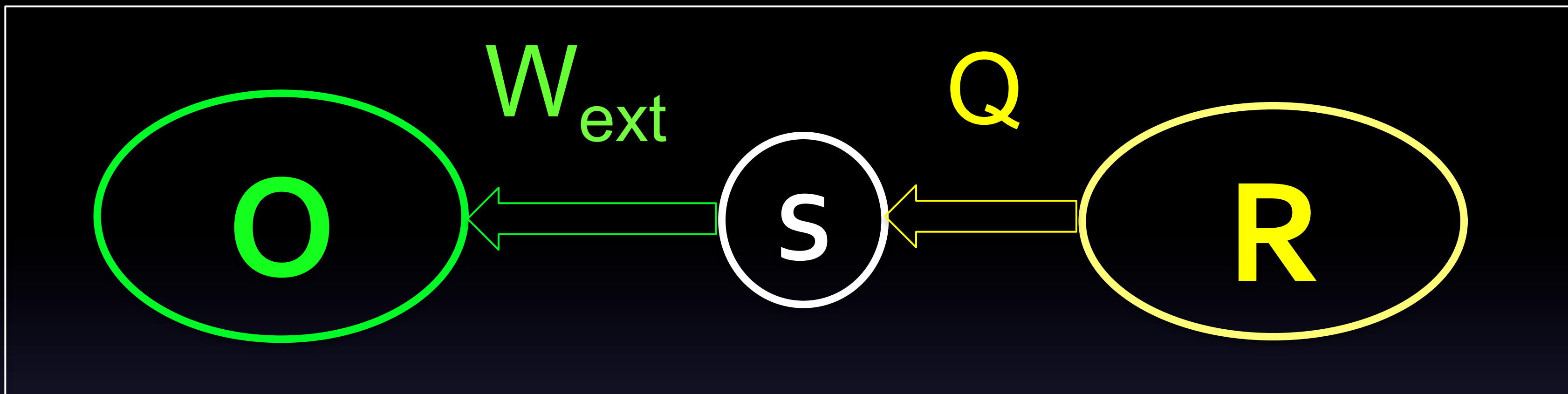
Classical thermodynamics

The heat engine paradigm



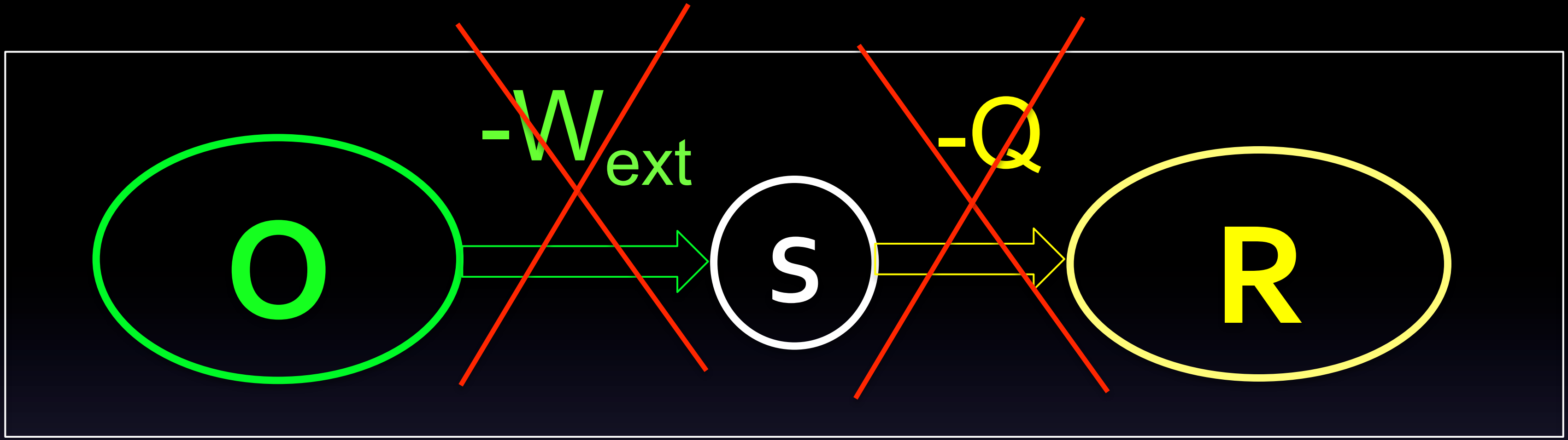
A working fluid S exchanges heat Q with a thermal reservoir R (temperature T) and work W with an operator O

From the applied side…



Heat is extracted from the bath and converted into « good » work W_{ext}

...to the fundamental one



A transformation is not always reversible

Thermodynamic irreversibility

$\Delta_i S = \Delta S_{sys} + \Delta S_R >= 0$

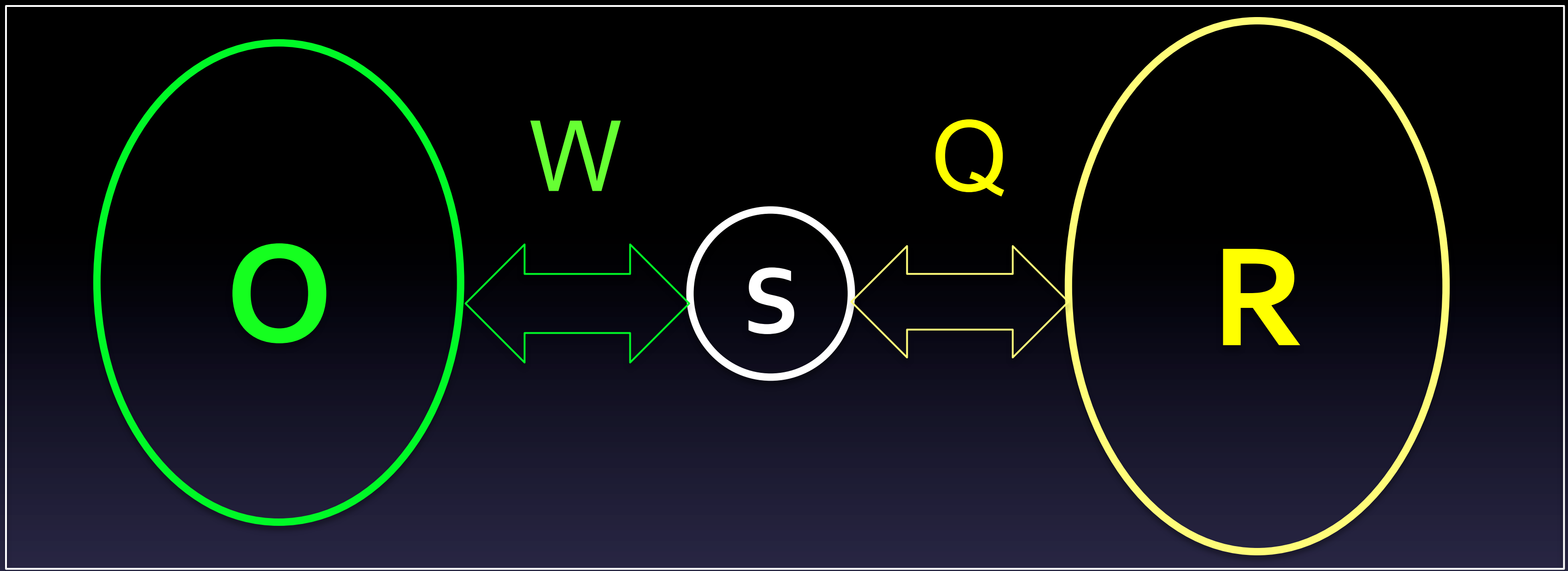
Classical thermodynamics

$$W_{ext} = -\Delta F - \Delta_i S/T$$

(For isoT processes)

Work extraction is optimal if it is reversible.

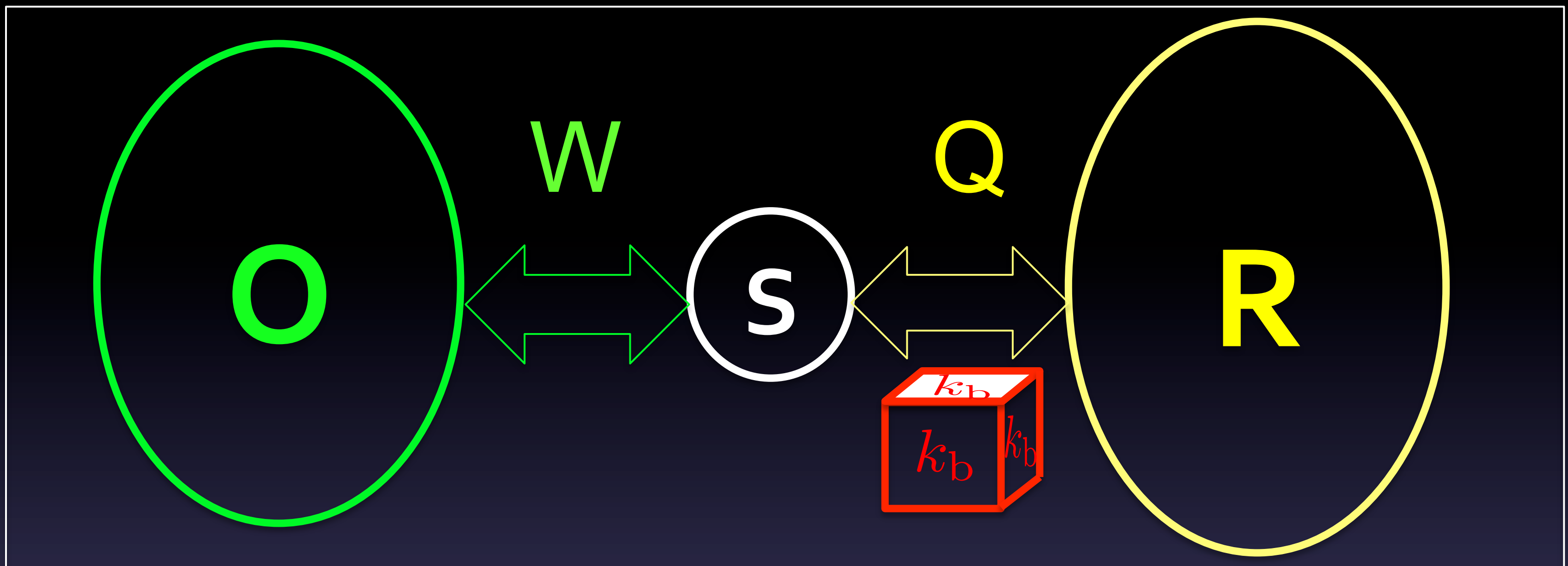
Down to the microscopic scale



Description of the system = ensemble of micro-states = the system's phase space

Why irreversibility, if the physical laws are reversible???

Classical stochastic thermodynamics



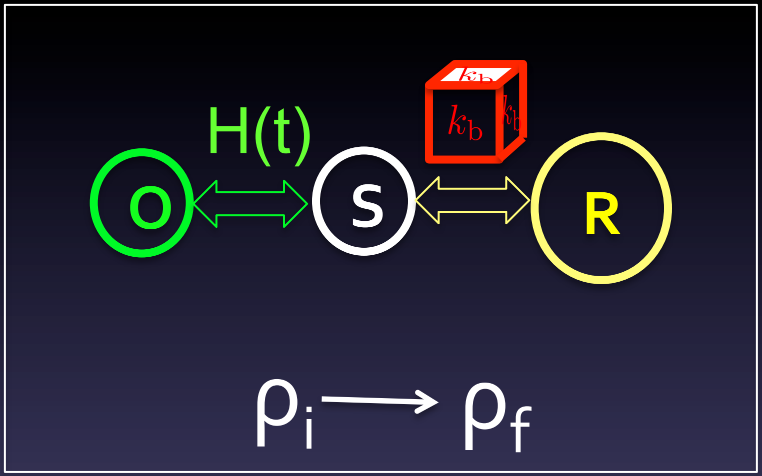
Evolution of the system : trajectories γ in the system's phase space

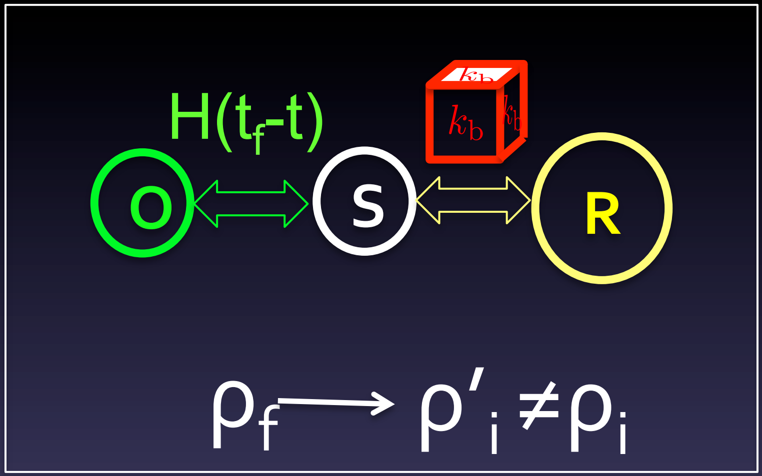
=> deterministic drive O

=> stochastic perturbation R

Classical irreversibility

The coupling to the stochastic entity breaks the microscopic reversibility of physical laws

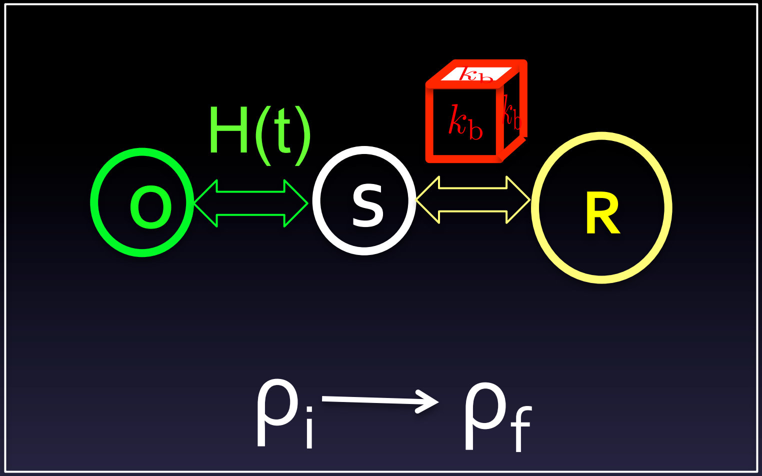


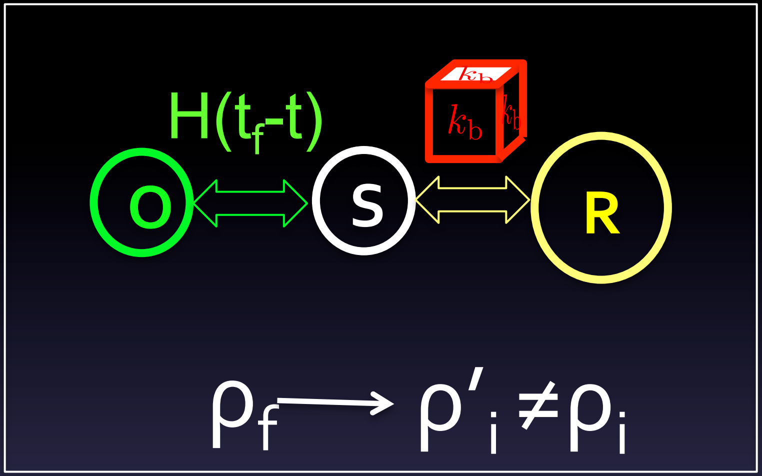


Experimental evidence:

- Apply a protocol H(t) from ρ_i to ρ_f
- Reverse the protocol $H(t_f\text{-}t)$, end up in ρ'_i
- If $\rho_i \neq \rho'_i$ the process is irreversible

Quantifying classical irreversibility (on average)



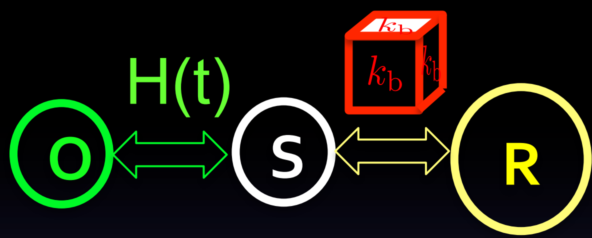


Average entropy production

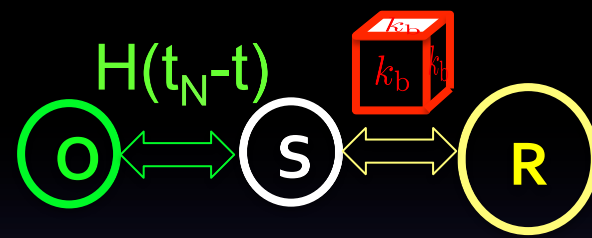
$<\Delta_i S> = D[\rho_i || \rho'_i]$

Mathematical distance between the initial and the final system's state

Classical fluctuation theorems



$\gamma=[\sigma_o \ldots \sigma_N], P_d[\gamma]$



$\gamma_r=[\sigma_N \ldots \sigma_o], P_r[\gamma_r]$

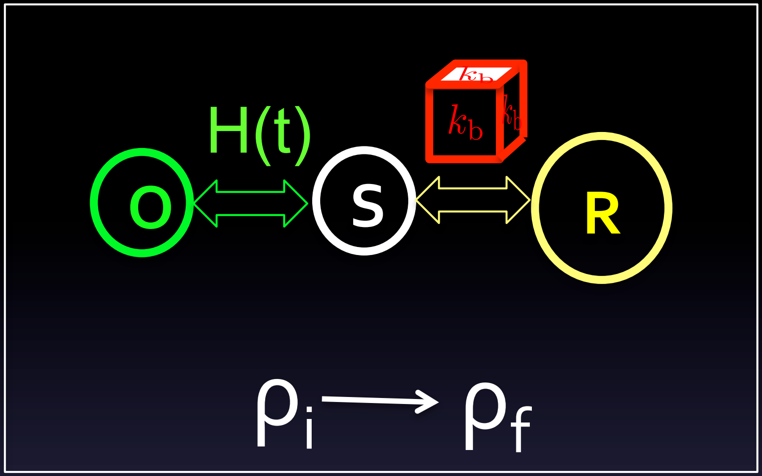
Entropy produced in a single trajectory γ

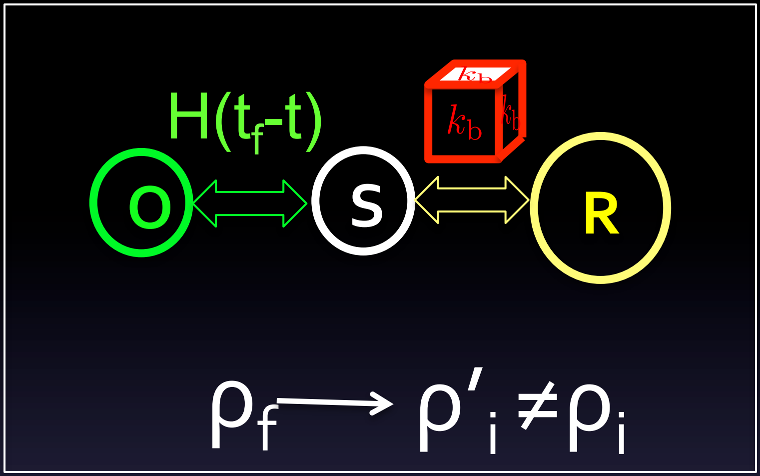
$\Delta_i s[\gamma]=\log(P_d[\gamma]/P_r[\gamma_r])$

By construction $\langle \exp(-\Delta_i s[\gamma]) \rangle_\gamma=1$

« Central fluctuation theorem »

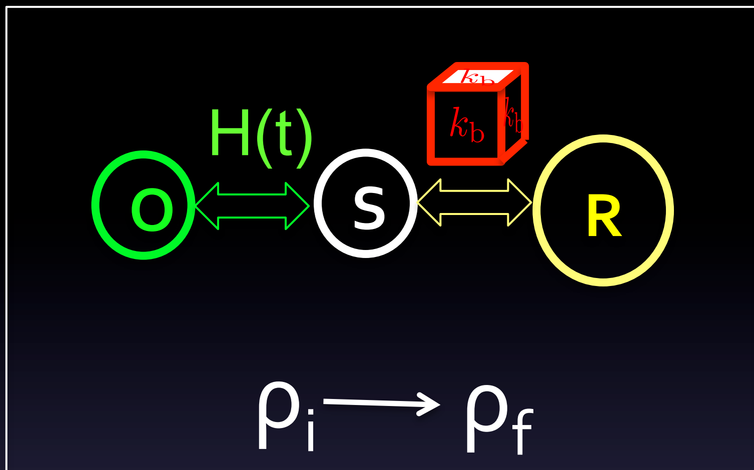
Classical Jarzynski's protocol

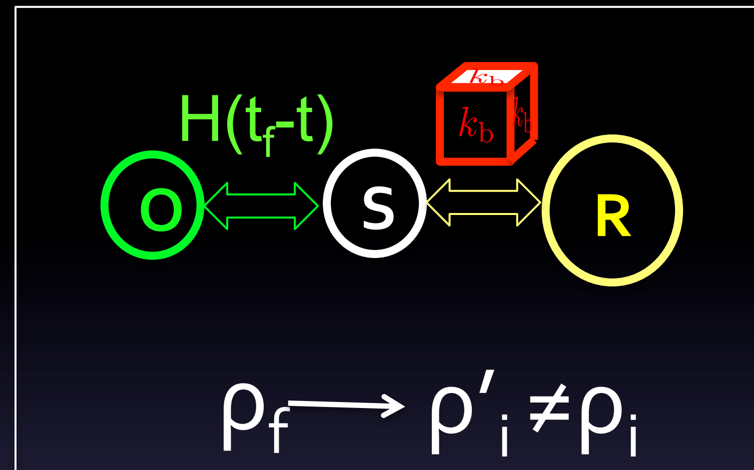




- Start from a thermal state $\rho_i = \exp(-\beta H(t_i))/Z_i$
- Drive it
- Wait for relaxation $\rho_f = \exp(-\beta H(t_f))/Z_f$
- Reverse the protocol
- On average: $<\Delta_i S> = (W - \Delta F)/T$ (isoT process)

Classical Jarzynski's equality



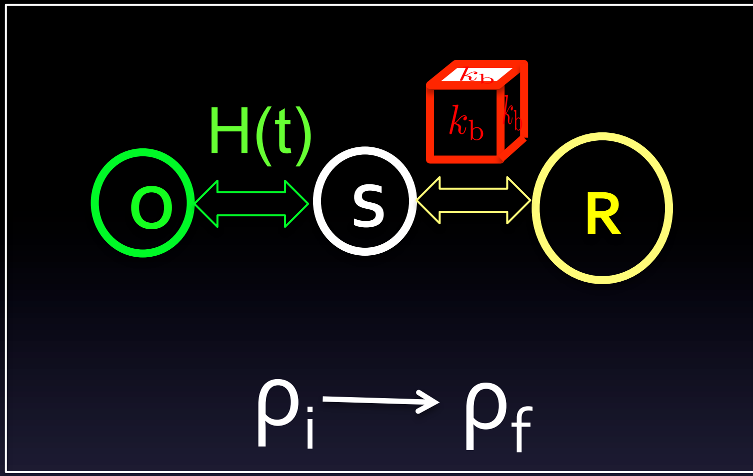


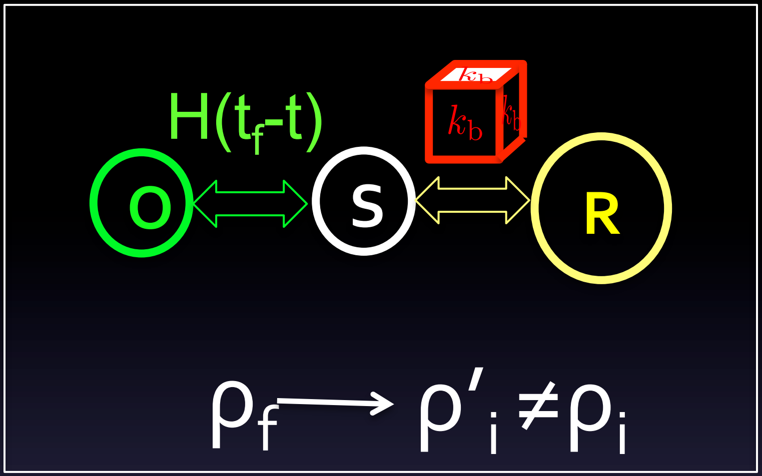
$$P_d[\gamma]/P_r[\gamma_r] = p_i[\sigma_o]/p_f[\sigma_N] * P_d[\gamma|\sigma_o]/P_r[\gamma_r|\sigma_N]$$

Boundary term

Conditional term

Classical Jarzynski's equality



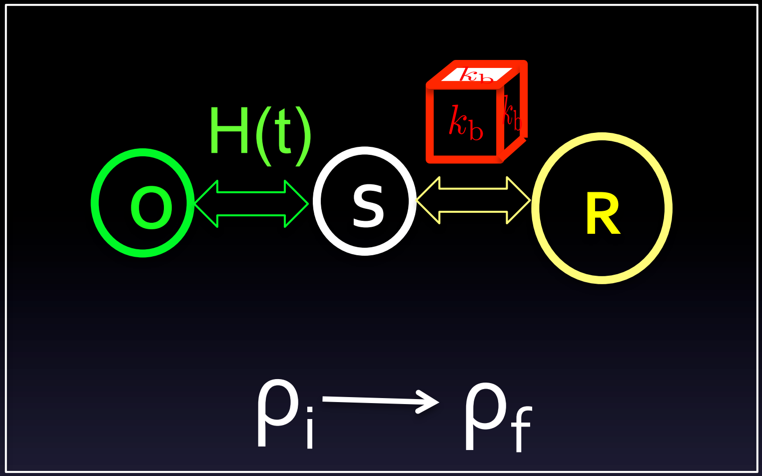


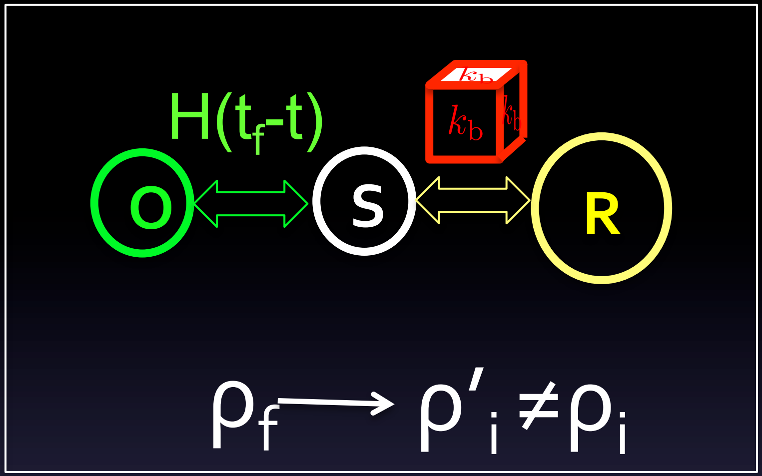
$$P_d[\gamma]/P_r[\gamma_r] = p_i[\sigma_o]/p_f[\sigma_N] * P_d[\gamma|\sigma_o]/P_r[\gamma_r|\sigma_N]$$

⇓

Boundary term : $\exp(-\beta(\Delta U[\gamma]-\Delta F))$

Classical Jarzynski's equality



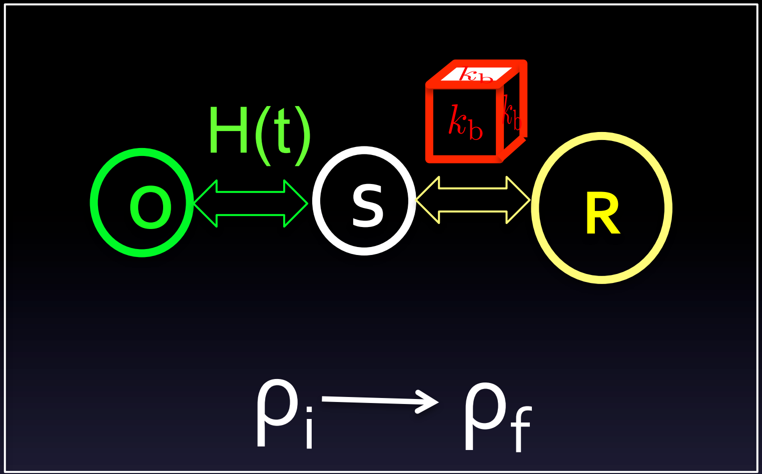


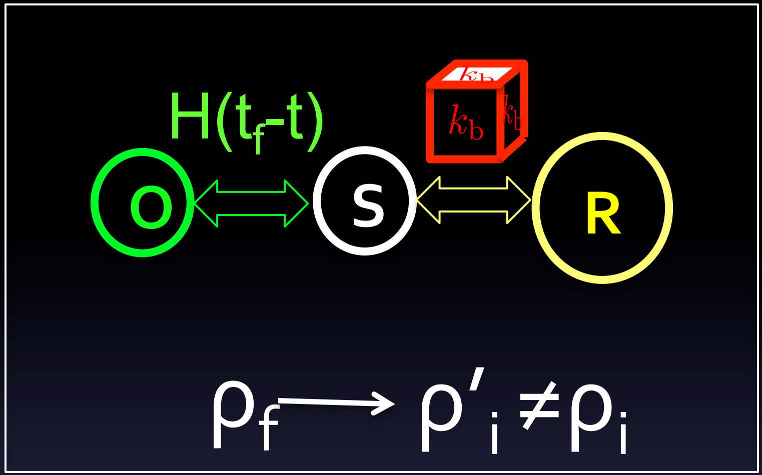
$$P_d[\gamma]/P_r[\gamma_r] = p_i[\sigma_o]/p_f[\sigma_N] * P_d[\gamma|\sigma_o]/P_r[\gamma_r|\sigma_N]$$

⇓

Conditional term : $\exp(-\beta Q[\gamma])$

Classical Jarzynski's equality





$\Delta_i s[\gamma] = \log(P_d[\gamma]/P_r[\gamma_r]) = \beta(W[\gamma]-\Delta F)$

Valid at the single trajectory level

$\langle \exp(-\beta(W[\gamma])\rangle_\gamma = \exp(-\beta\Delta F)$

Classical Jarzynski's equality

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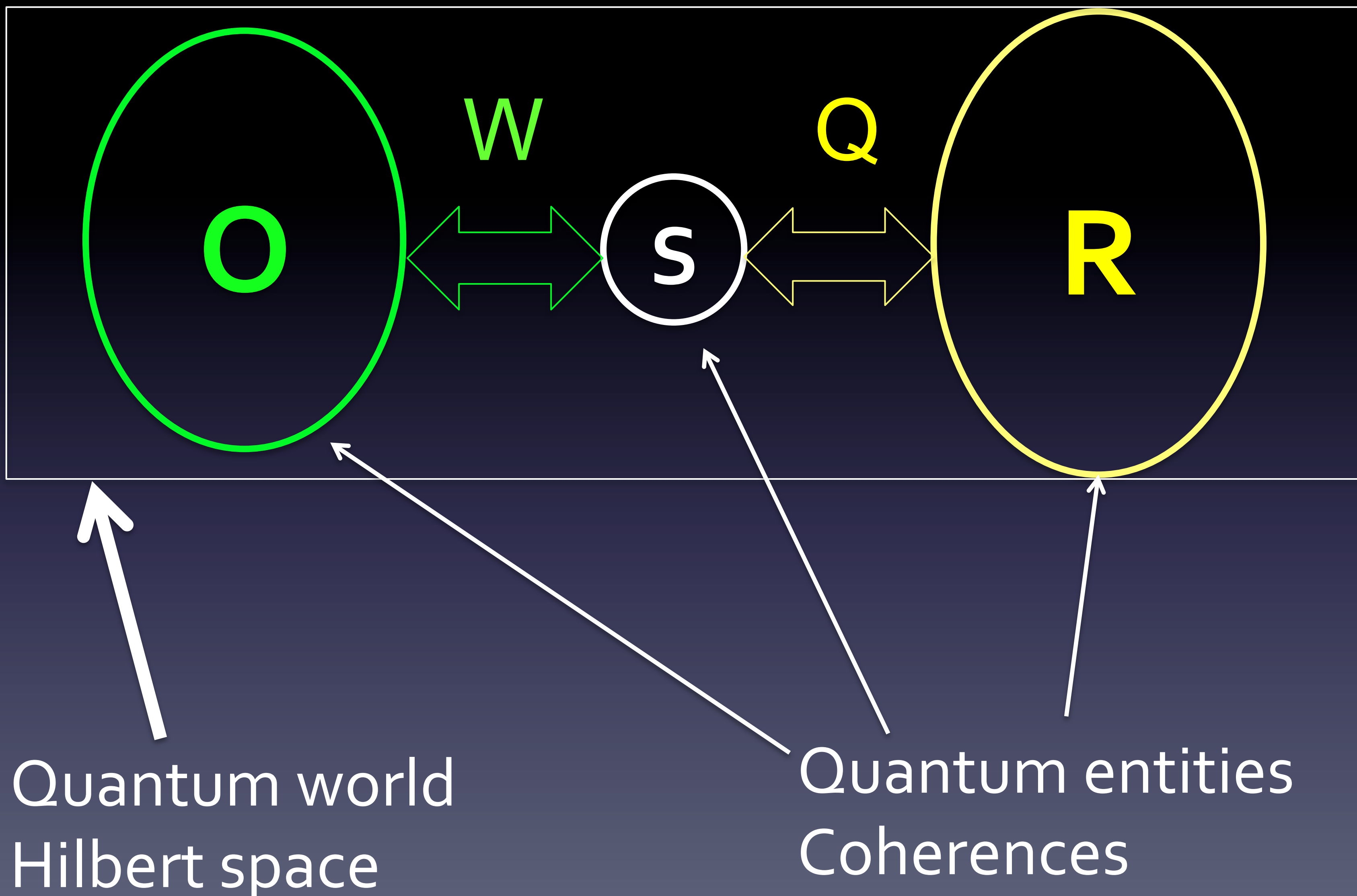
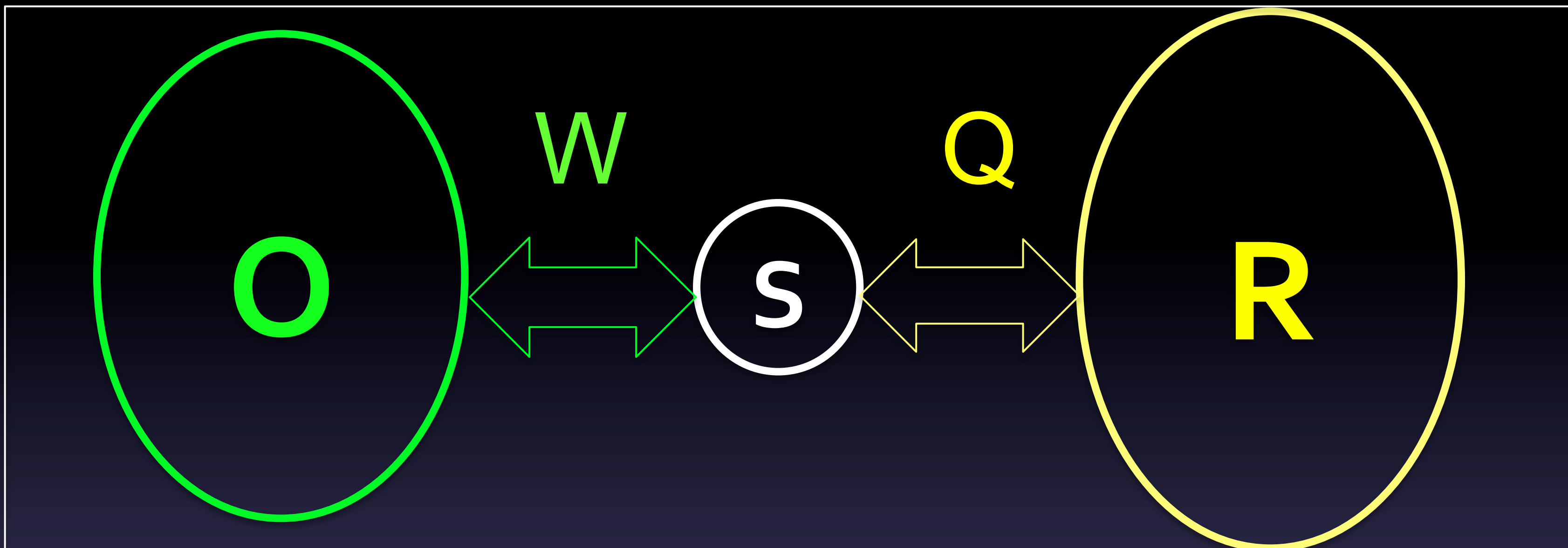
Quantum thermodynamics

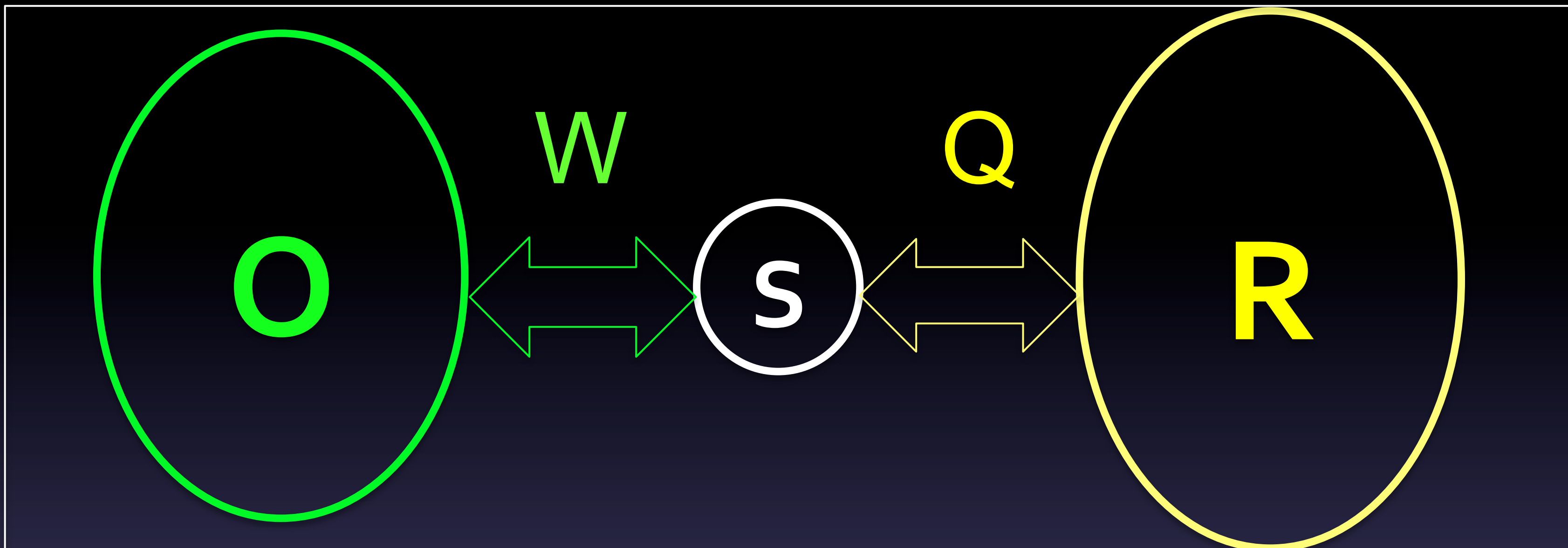


Quantum thermodynamics



Are coherence/entanglement energetic resources? Enhanced performances of quantum heat engines?
Nature of irreversibility in the quantum world?

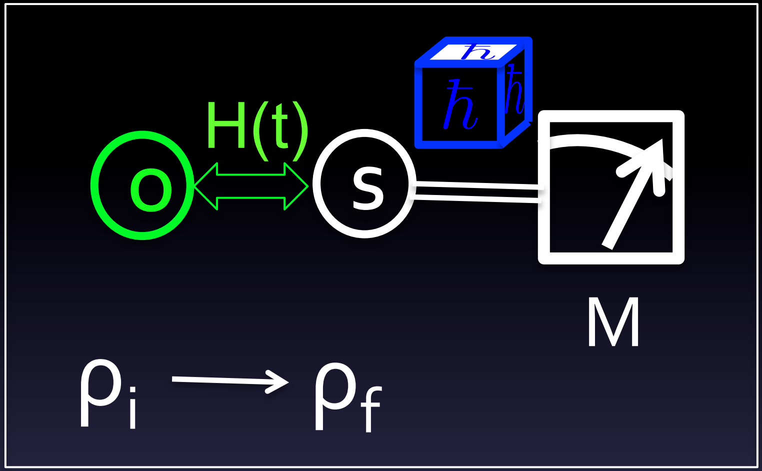
Quantum thermodynamics

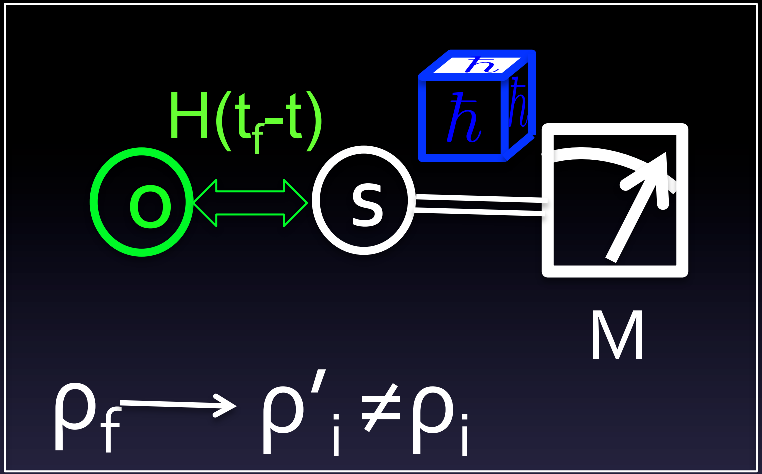


Are coherence/entanglement energetic resources? Enhanced performances of quantum heat engines?

Nature of irreversibility in the quantum world?

Quantum measurement: a genuinely quantum source of irreversibility

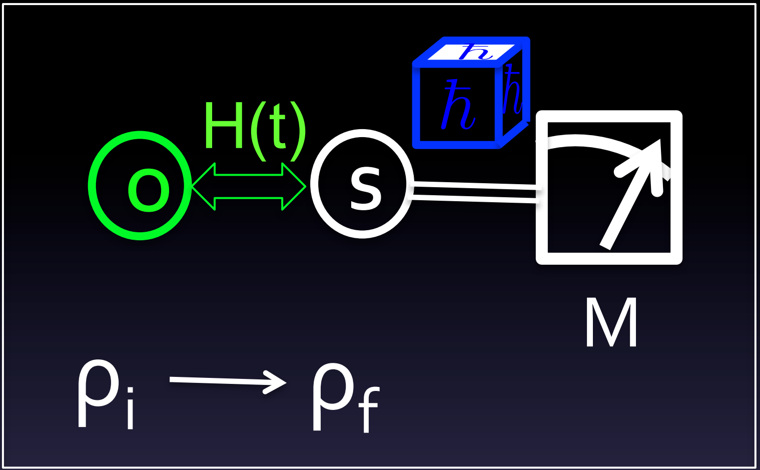


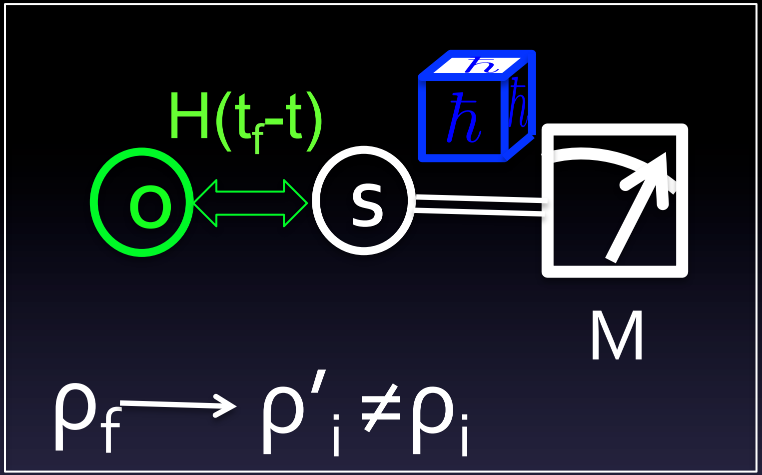


Experimental evidence:

- Start from a pure state $\rho_i = |\psi_i\rangle\langle\psi_i|$
- Apply a protocol H(t) + measure M at $\{t_i\}$
- Reverse the protocol $H(t_f-t)$, measure M at $\{t_{N-i}\}$
- $[H(t),M]\neq 0 \Rightarrow \rho_i \neq \rho_f \Rightarrow$ The process is irreversible

Entropy produced during a quantum measurement



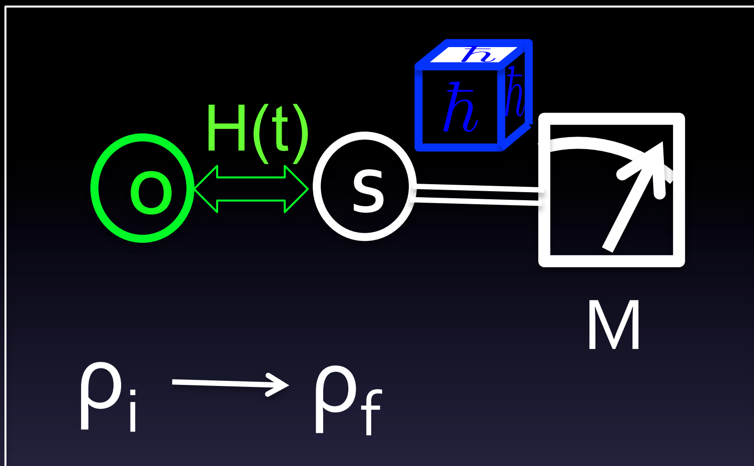


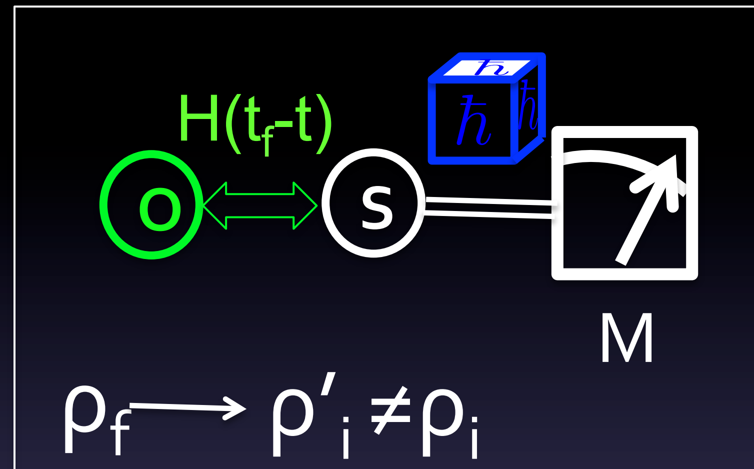
$$P_d[\gamma]/P_r[\gamma_r] = p_i[\sigma_o]/p_f[\sigma_N] * P_d[\gamma|\sigma_o]/P_r[\gamma_r|\sigma_N]$$

Boundary term

Conditional term

Entropy produced during a quantum measurement



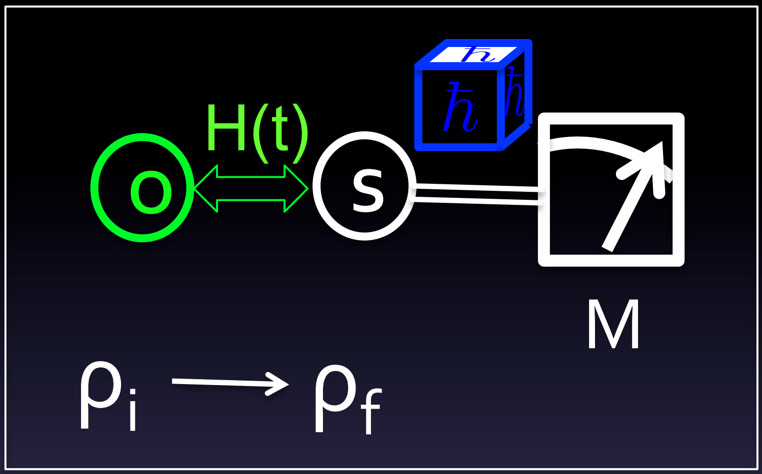


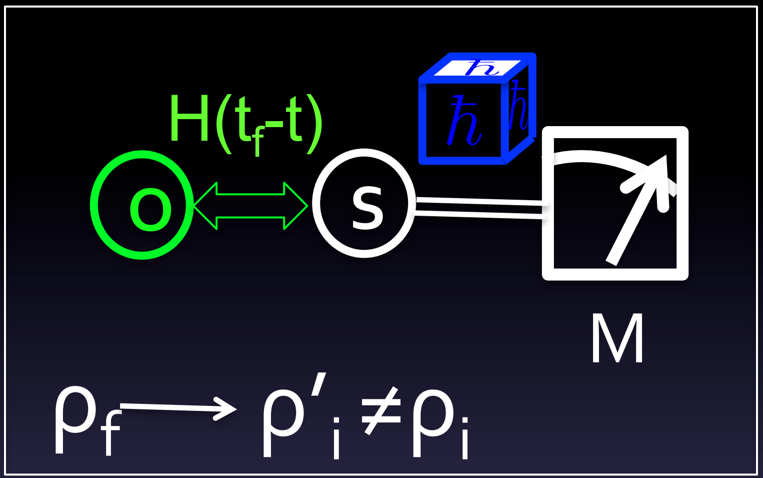
$$P_d[\gamma]/P_r[\gamma_r]=p_i[\sigma_o]/p_f[\sigma_N]*P_d[\gamma|\sigma_o]/P_r[\gamma_r|\sigma_N]$$

⇓

Boundary term: $p_i[\sigma_o]=1$ (pure state)

Entropy produced during a quantum measurement





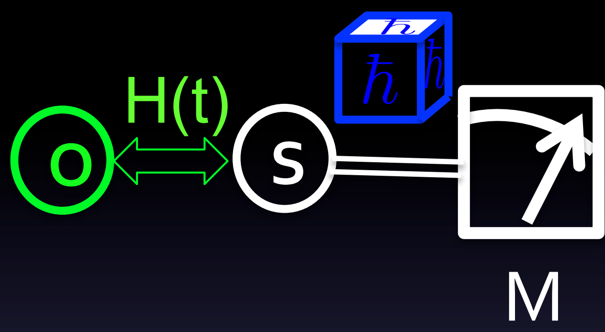
$$P_d[\gamma]/P_r[\gamma_r]=p_i[\sigma_o]/p_f[\sigma_N]*P_d[\gamma|\sigma_o]/P_r[\gamma_r|\sigma_N]$$

$$P_d[\gamma|\sigma_o]=\Pi_i|<\sigma_{i+1}|U(t_{i+1},t_i)|\sigma_i>|^2$$

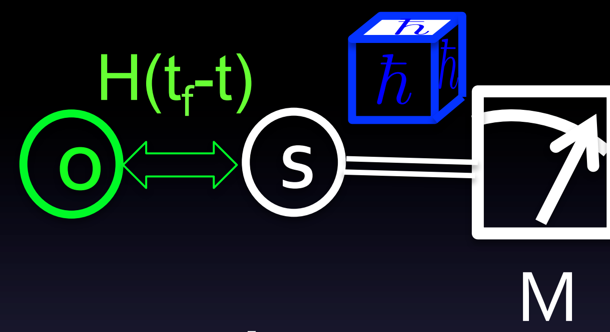
$$P_r[\gamma_r|\sigma_N]=\Pi_i|<\sigma_{i+1}|U(t_{i+1},t_i)|\sigma_i>|^2$$

Conditional term = 1

Entropy produced during a quantum measurement



$\rho_i \longrightarrow \rho_f$



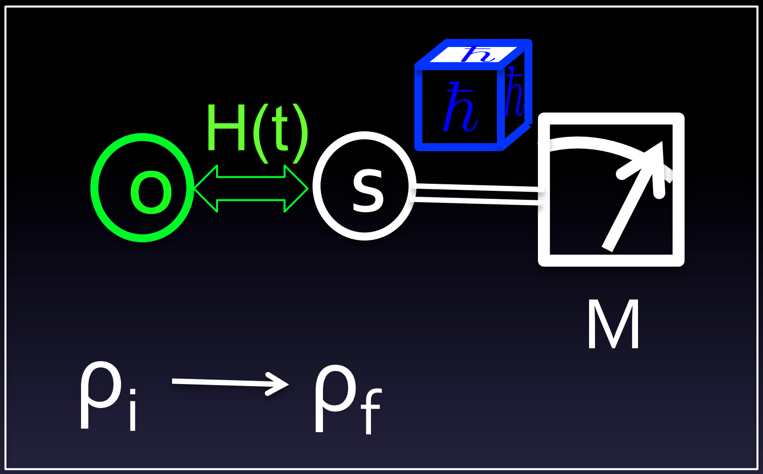
$\rho_f \longrightarrow \rho'_i \neq \rho_i$

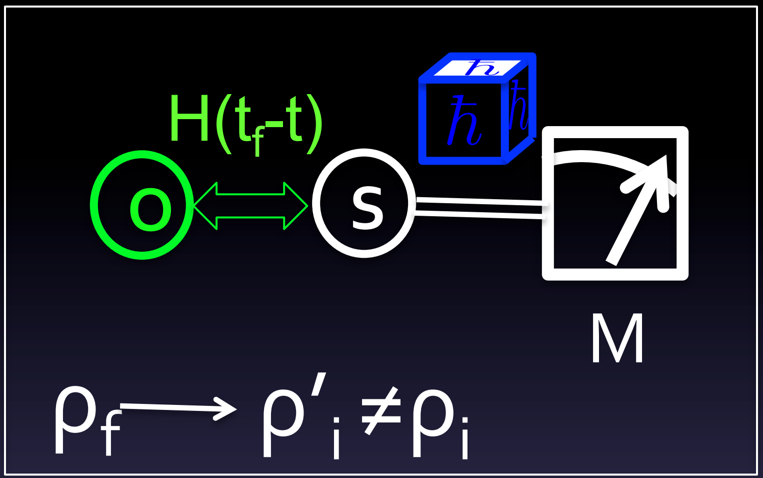
$$\Delta_i s[\gamma] = \log(P_d[\gamma]/P_r[\gamma_r]) = -\log(p_f[\sigma_N])$$

$$\langle \Delta_i s[\gamma] \rangle_\gamma = -\mathrm{Tr}[\rho_f \log(\rho_f)] = \Delta S_{VN}$$

Entropy production = Change of the system's Von Neumann's entropy

Energetic signature for quantum irreversibility?





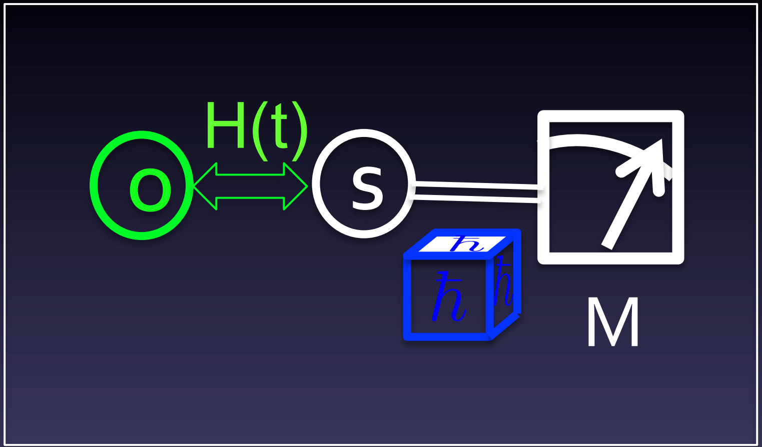
In the classical world, irreversibility affects work extraction = has some energetic imprint

Is there a quantum counterpart?

A new paradigm for stochastic thermodynamics: Thermodynamics without bath

Replace the thermal source of stochasticity by a genuinely quantum one = a measuring apparatus

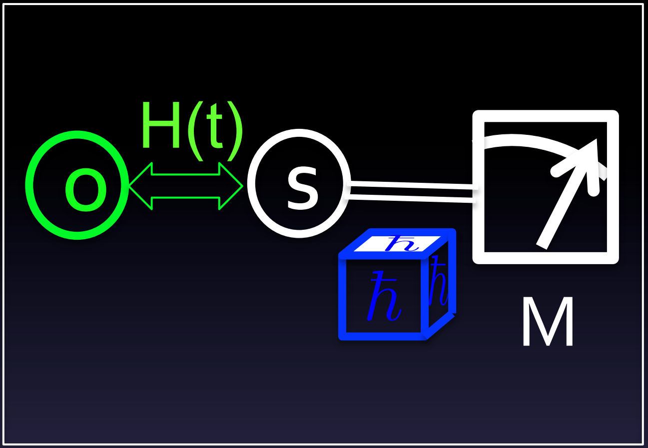
...and proceed by analogy



- Stochastic trajectories=>Quantum trajectories
- Stochastic thermodynamic quantities U, W, Q => Quantum thermodynamic quantities

Elementary model for quantum trajectories

- S driven and measured each t_i
- M: stochastic outcomes $\{k(t_1), k(t_2), \ldots k(t_N)\}$
- Each t_i: S is projected on $|k(t_i)>$: Measurement induced back-action=quantum jump



- S: $\{G_t(t-t_1)|k(t_1)>; G_t(t-t_2)|k(t_2)>, \ldots G_t(t-t_N)k(t_N)> \}$
- G_t = evolution operator

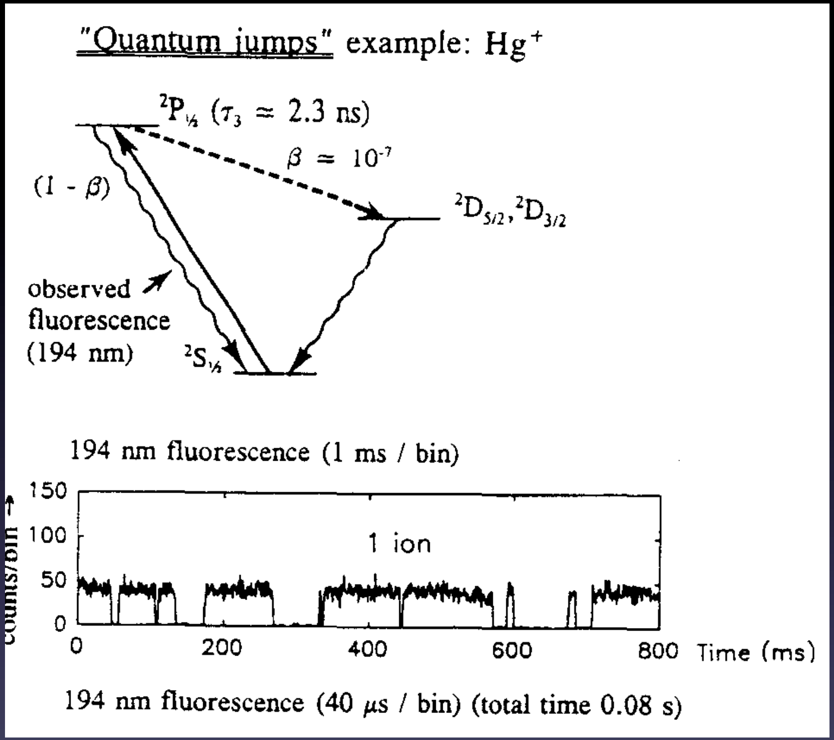
=> Elementary quantum trajectory, where stochasticity is quantum and not thermal

Quantum trajectories are real!!!

Quantum jumps first observed with trapped ions (80ies)…

…2007: Cavity Quantum ElectroDynamics, life and death of a photon

…2013: circuit Quantum ElectroDynamics…



Necessary condition to observe a quantum trajectory: Have a high efficiency detection scheme

Internal energy

$U(t) = \langle\psi(t)|H(t)|\psi(t)\rangle$

$|\psi(t)\rangle$ = quantum state of the system

Requirements:

- Know the applied Hamiltonian $H(t)$: *OK*
- Know the quantum trajectory $|\psi(t)\rangle$: *OK*
  => $U(t)$ can be reconstructed with experimental data
- Attribute some « energy » to a state, which is not a Hamiltonian eigenstate: *looks weird, but high gain…*

Internal energy

$$U(t) = \langle\psi(t)|H(t)|\psi(t)\rangle$$

$|\psi(t)\rangle$ = quantum state of the system

What do we gain with this approach?

⇒ Internal energy is defined at any time, for any state, including quantum superpositions of energy states

⇒ Effect of the measurement visible on internal energy => Energetic imprint of quantum measurement/ of quantum irreversibility

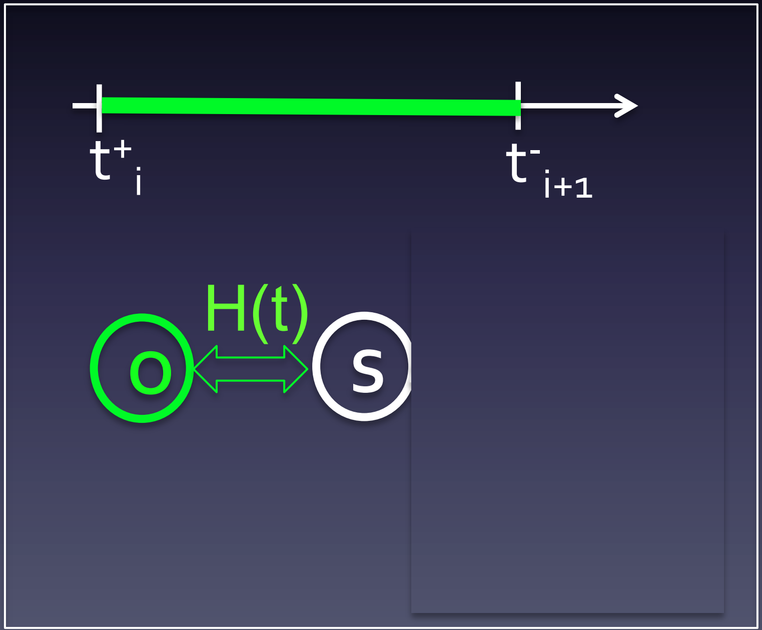
Definition of work

Work is provided to the system during the deterministic evolution

$W(t_i \text{->} t_{i+1}) = U(t^-_{i+1}) - U(t^+_i)$

Due to the time-dependence of the Hamiltonian

Corresponds to the mean energy exchanged with the battery

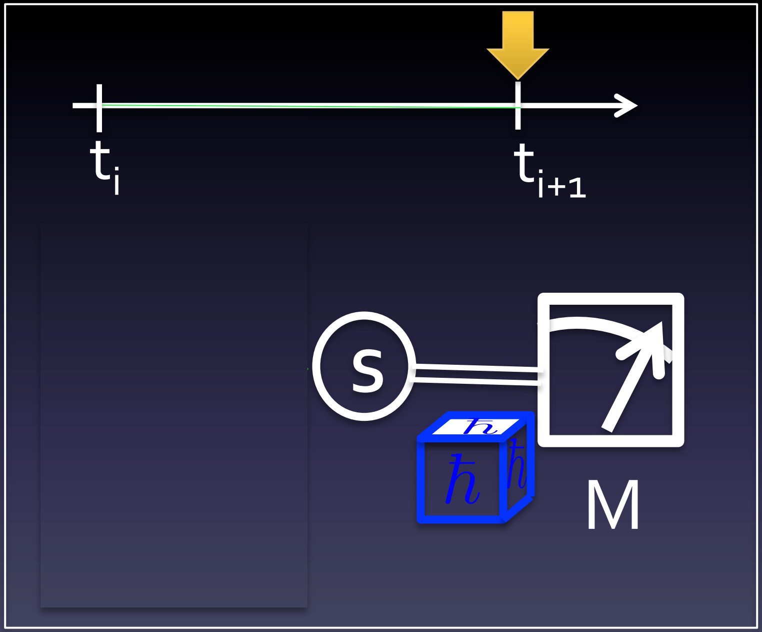


Definition of heat

Heat is exchanged during the stochastic quantum jumps
$Q(t_i)=U(t^+_i)-U(t^-_i)$

= Energy fluctuations induced by measurement back-action

= Only appear if the state has coherences in the measurement basis



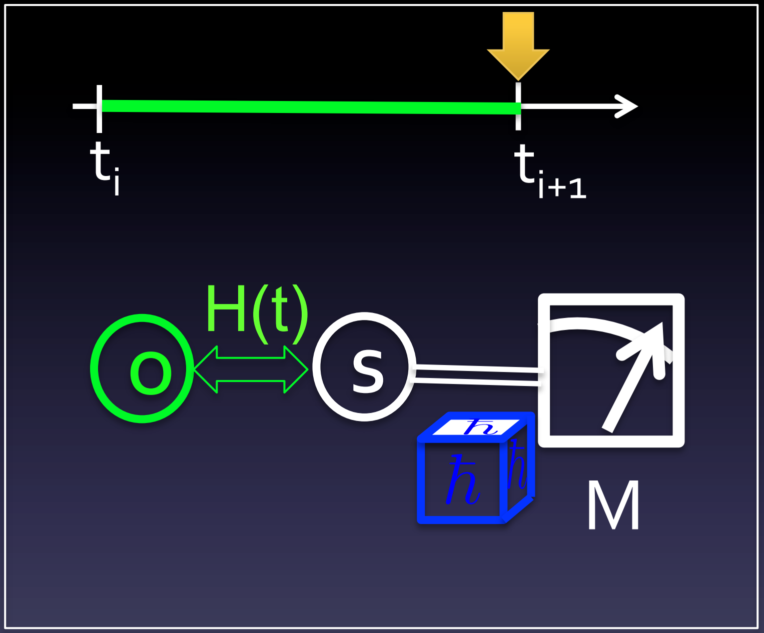
Purely quantum term:
« Quantum heat » Q_q

First principle

$$\Delta U[\gamma]=W+Q_q[\gamma]$$

Work deterministically exchanged during Hamiltonian evolution

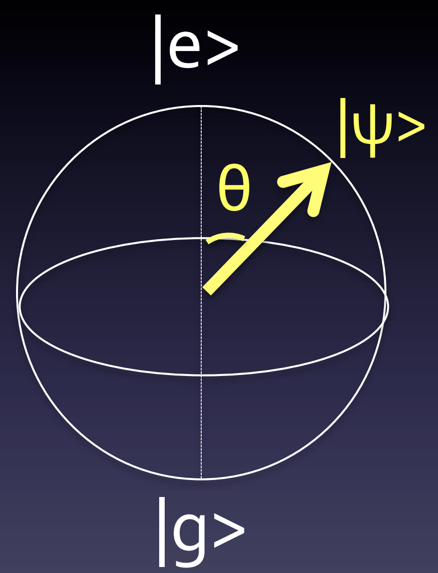
Heat stochastically exchanged during quantum measurement



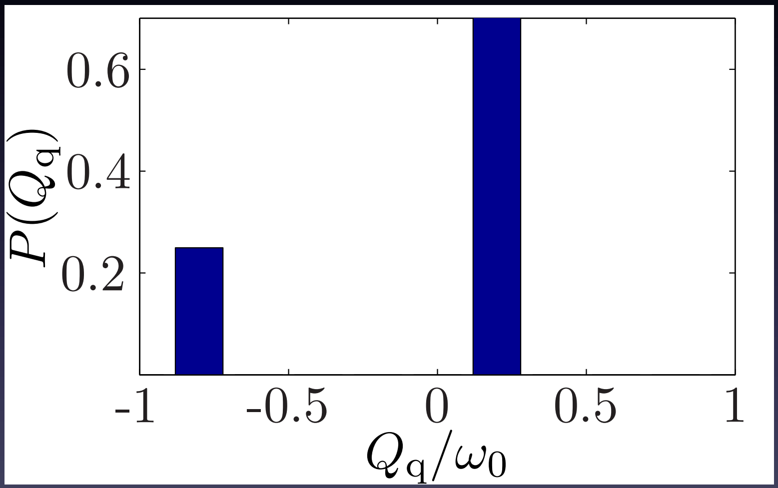
Time-resolved perspective on energy exchanges

Properties of quantum heat

A Qubit is prepared in the state |ψ> and measured in the {|e>;|g>} basis: $\Delta U[\gamma] = Q_q[\gamma]$



Q_q distribution



The distribution of quantum heat has non-zero components iff θ≠0 ⇔ The state has coherences in the measurement basis

Properties of quantum heat

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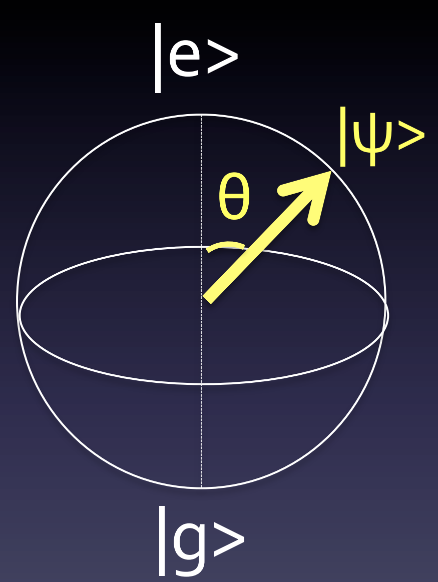
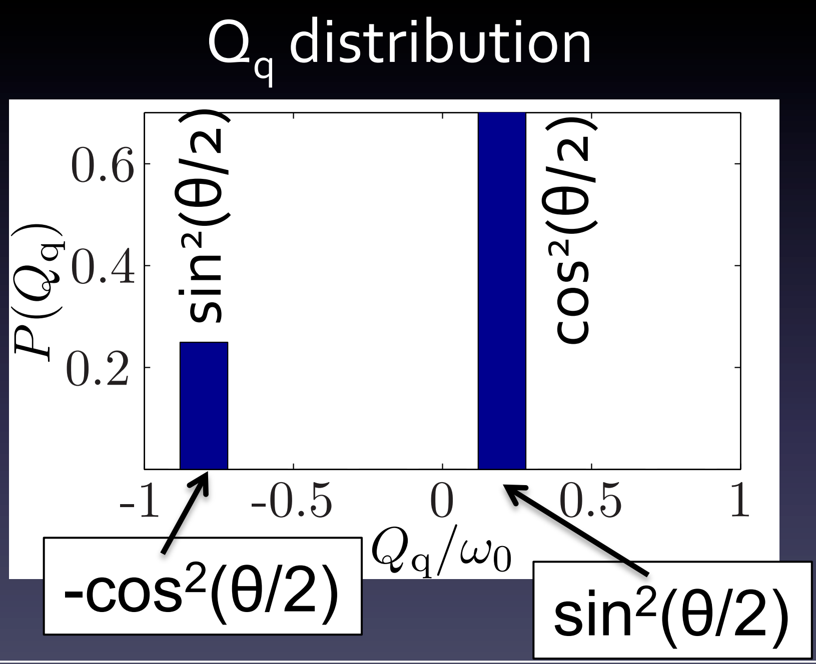
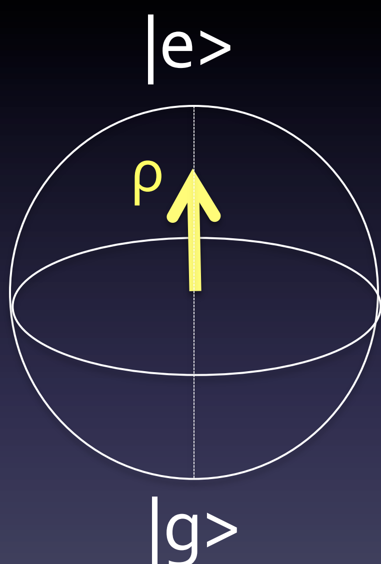


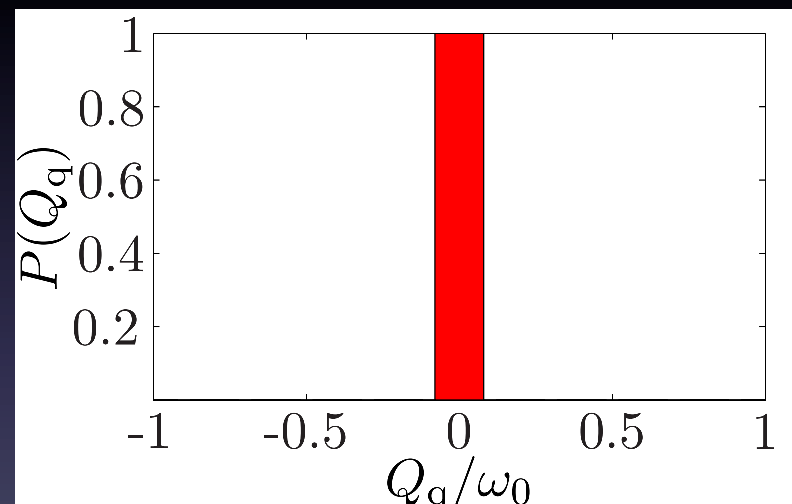


The mean quantum heat (obtained by summing up the components) is zero $\langle Q_q \rangle = 0$

Properties of quantum heat

A Qubit is prepared in the mixed state $\rho=p|e\rangle\langle e|+(1-p)|g\rangle\langle g|$, and measured in the $\{|e\rangle;|g\rangle\}$ basis

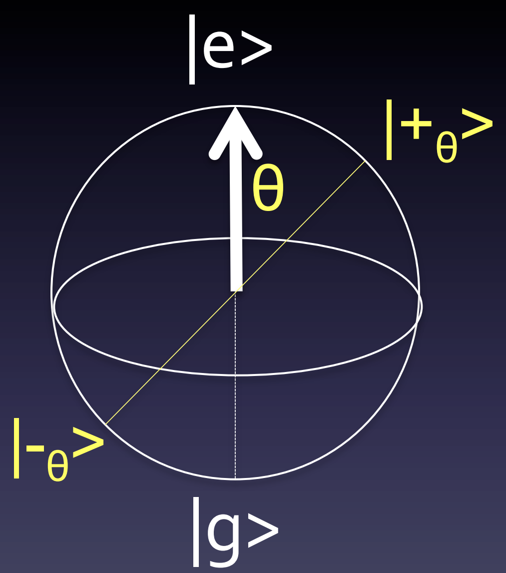


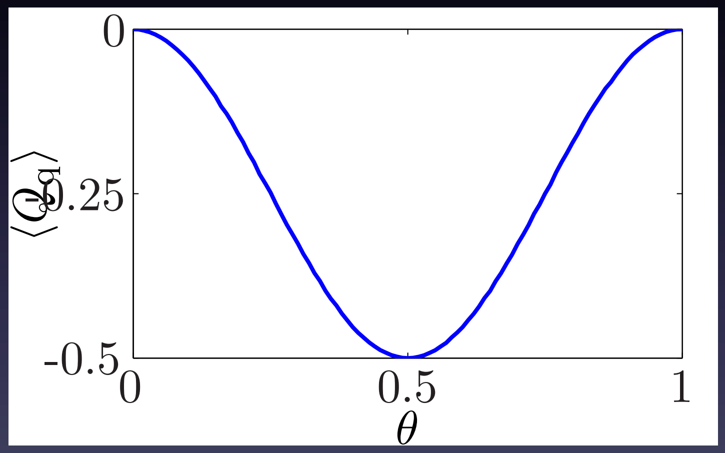


No measurement back-action => $\Delta U[\gamma] = Q_q[\gamma]=0$ =>
The quantum heat is reduced to a null component.

Properties of quantum heat

A Qubit is prepared in the state |e> and measured in the $\{|+_\theta>, |-_\theta>\}$ basis

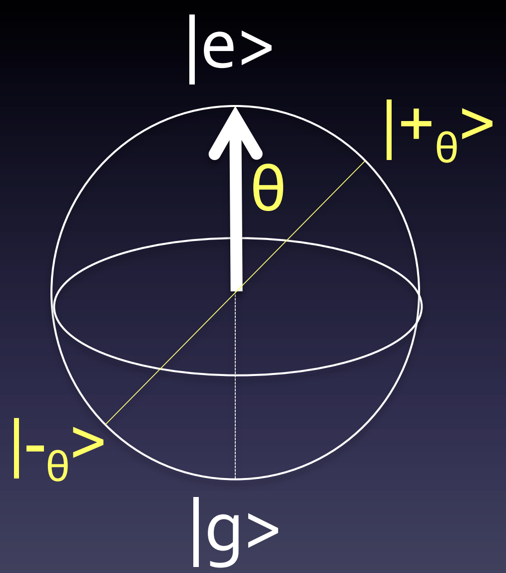


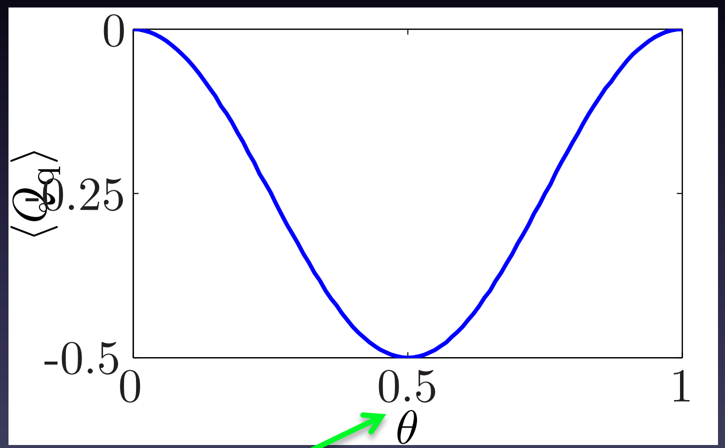


The mean quantum heat oscillates as a function of the measurement basis orientation

Properties of quantum heat

A Qubit is prepared in the state |e> and measured in the $\{|+_\theta>, |-_\theta>\}$ basis





There is a mean energy exchange between the system and the measurement channel

Where does the quantum heat comes from?

First answer: It is always possible to provide a thermodynamic energy balance specific to each situation, involving for instance the classical source used to prepare the coherences, or the microscopic structure of the measurement channel

Where does the quantum heat comes from?

Second answer: Quantum heat is a natural byproduct of the standard quantum formalism and measurement postulate. It is just another name for energetic quantum fluctuations, that we use here to build a new framework for quantum thermodynamics

Outline

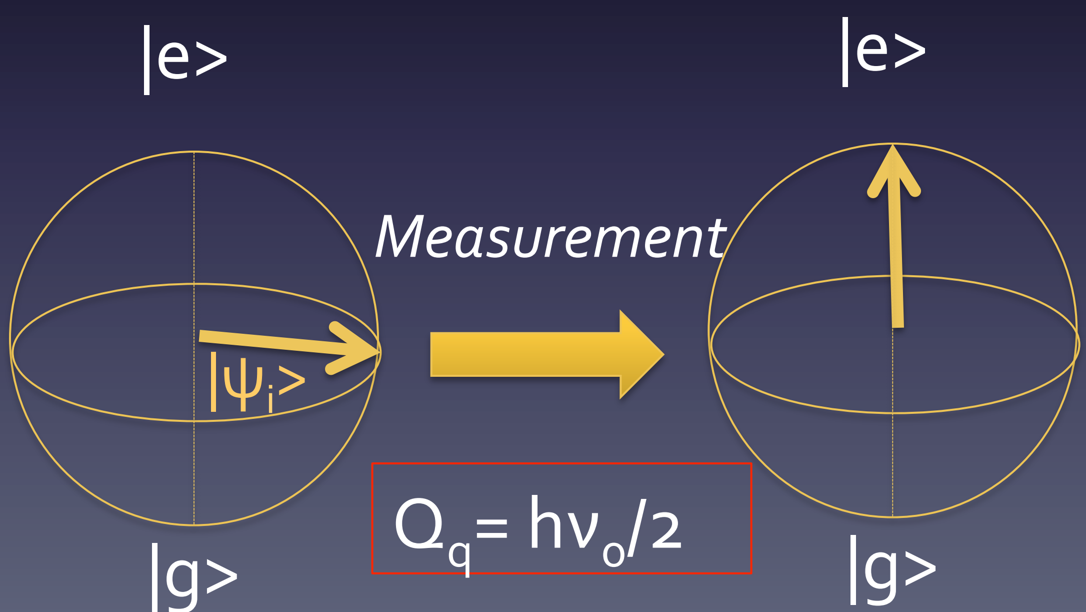
- Classical concepts of thermodynamics
- A new framework for quantum thermodynamics
- Applications

Operational approach to quantum heat

Elementary feedback stabilizing protocol

A Qubit is prepared in $|\psi_i\rangle = (|e\rangle+|g\rangle)/\sqrt{2}$
Measurement projects in $|\psi_f\rangle=|e\rangle$ or $|g\rangle$

$Q_q=\pm h\nu_o/2$

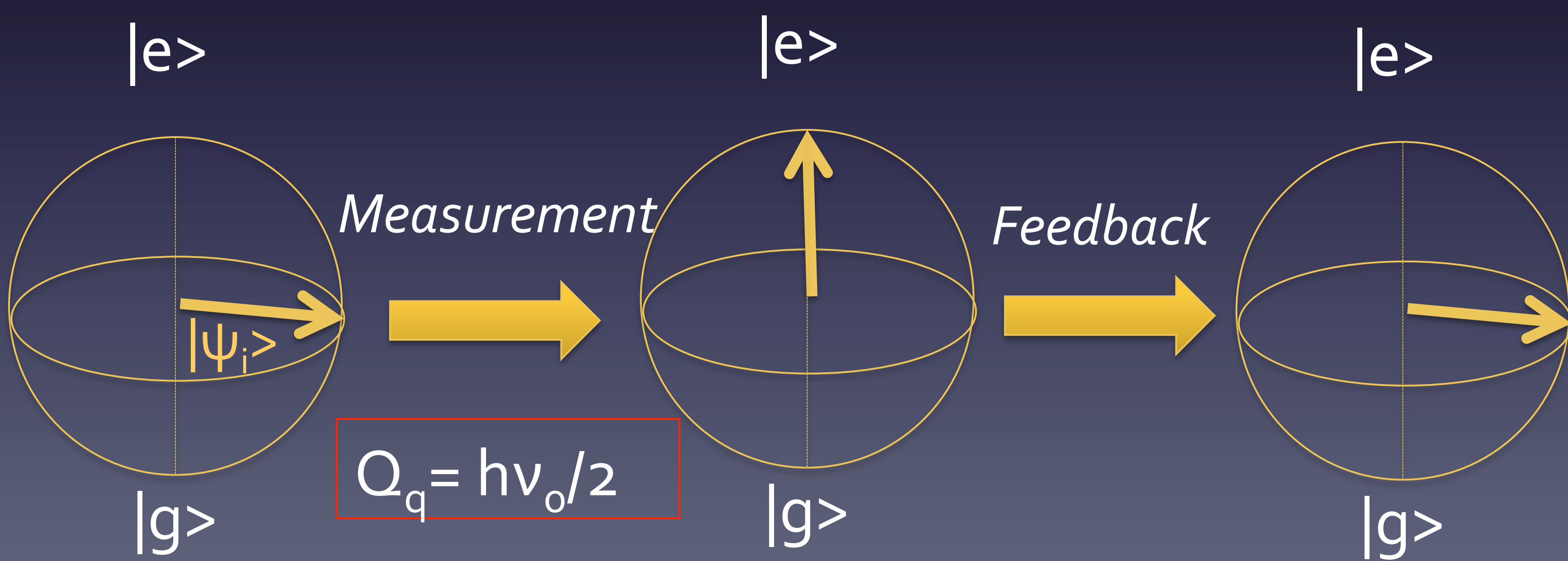


Operational approach to quantum heat

Elementary feedback stabilizing protocol

Goal: Stabilize $|\psi_i\rangle = (|e\rangle+|g\rangle)/\sqrt{2}$

Protocol: Read the measurement result, and use a feedback source to re-prepare $|\psi_i\rangle = (|e\rangle+|g\rangle)/\sqrt{2}$



Operational approach to quantum heat

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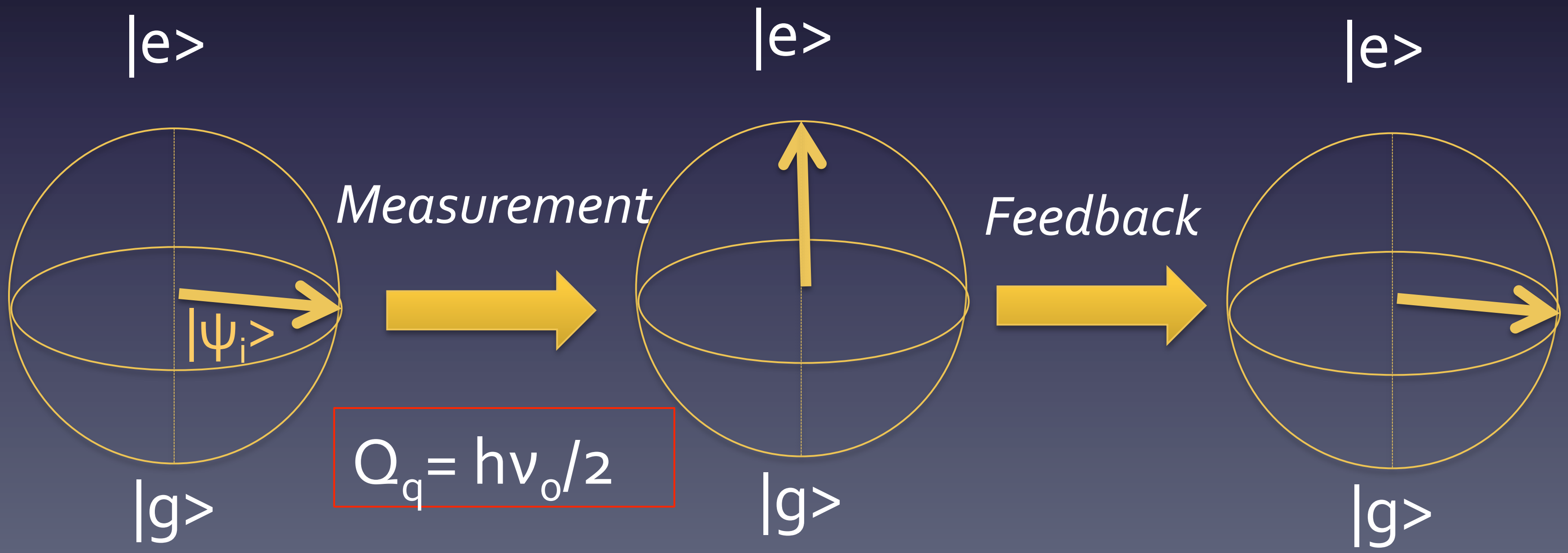
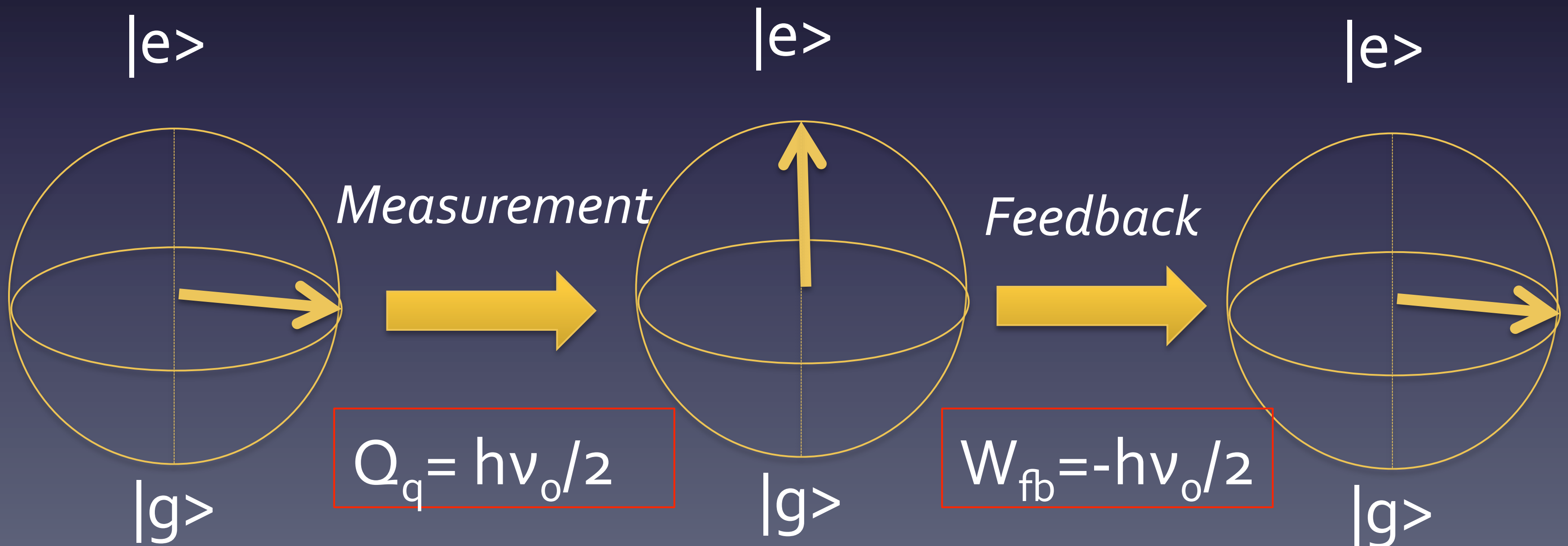


Operational approach to quantum heat

The feedback work spent to stabilize the superposition exactly compensates the quantum heat: $W_{fb} = -Q_q$
The quantum heat is a real, physical quantity = energy required for the feedback protocol



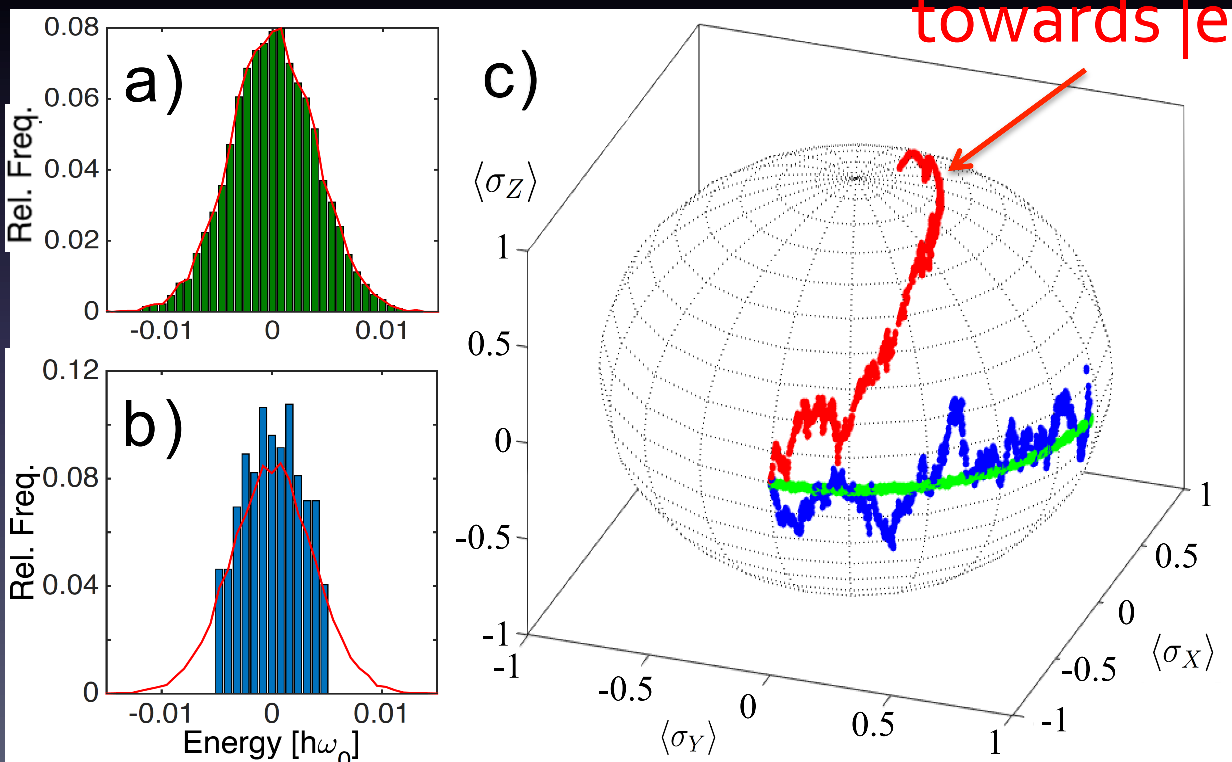
$Q_q = h\nu_o/2$

$W_{fb} = -h\nu_o/2$

A realistic feedback stabilizing protocol

A Qubit weakly monitored in the {|e>;|g>} basis + coupled to a feedback loop

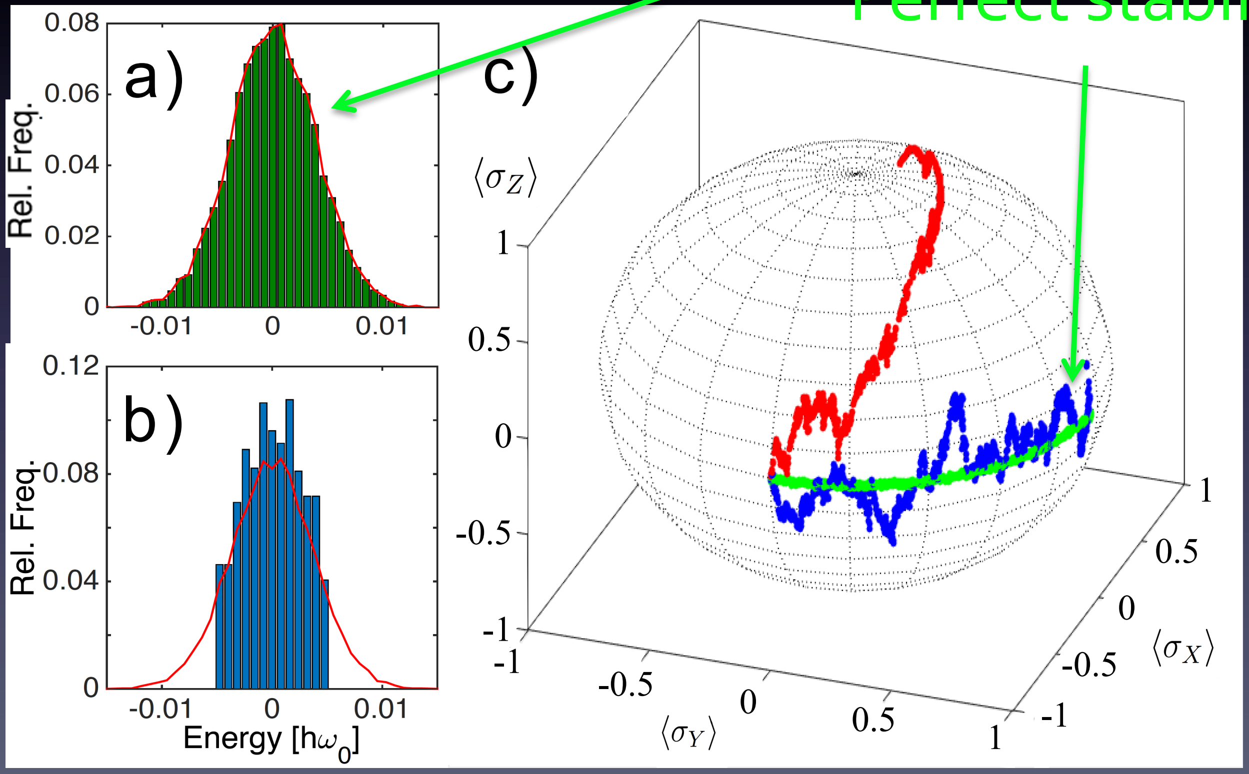
No feedback
Decoherence
towards |e> or |g>



A realistic feedback stabilizing protocol

A Qubit weakly monitored in the {|e>;|g>} basis + coupled to a feedback loop

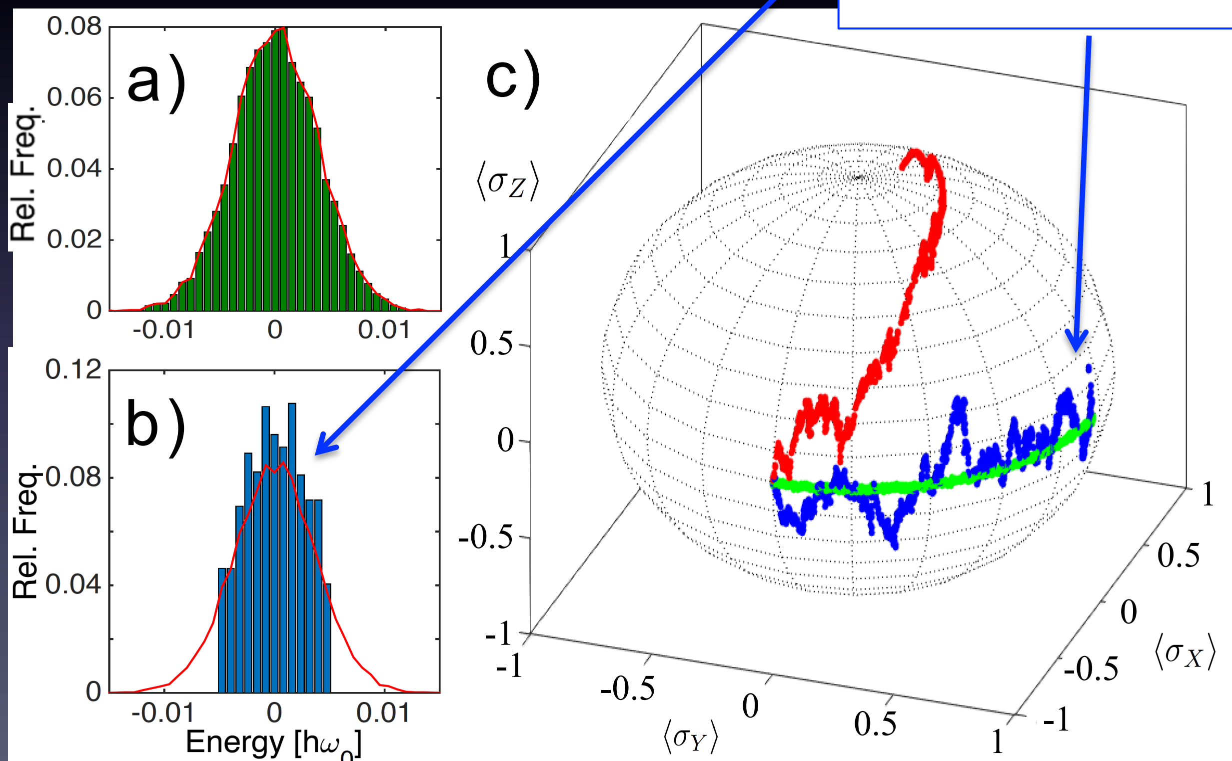
Perfect feedback
Perfect stabilization



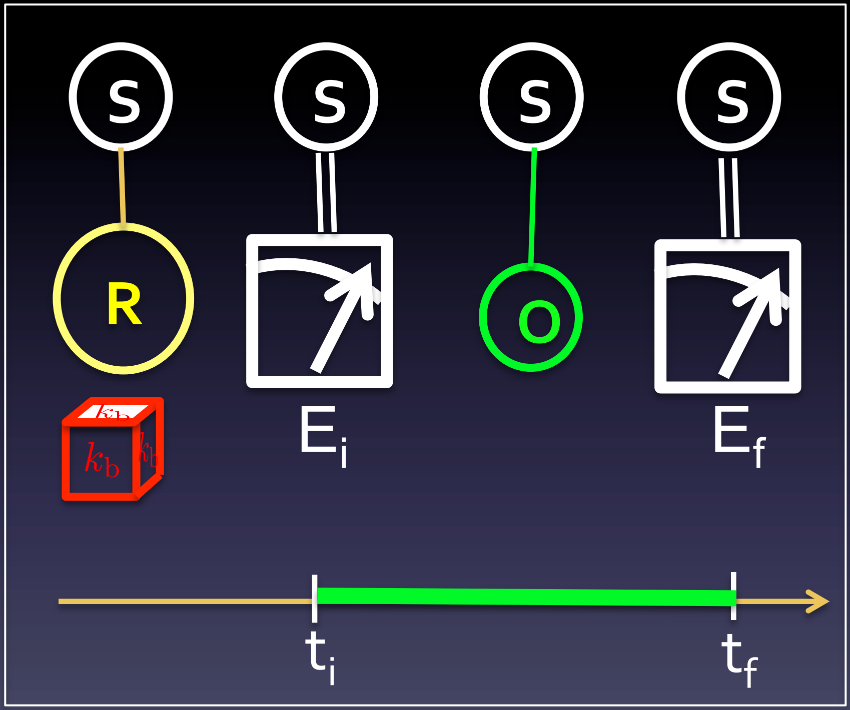
A realistic feedback stabilizing protocol

A Qubit weakly monitored in the {|e>;|g>} basis + coupled to a feedback loop

The feedback source has a bounded power
Imperfect stabilization



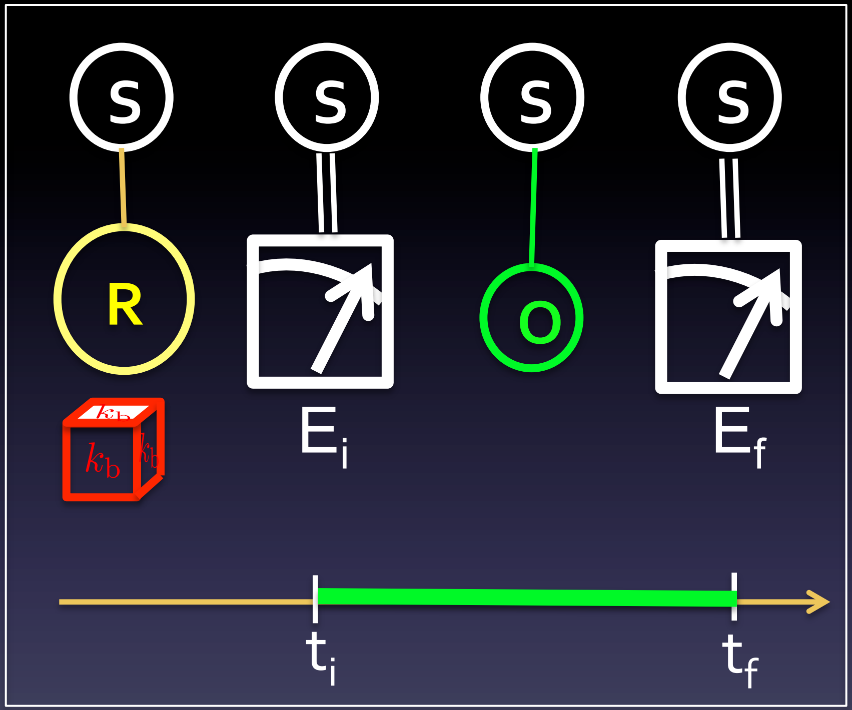
The quantum Jarzynski's equality



Protocol:

- t_i^- : Thermalize the system
- t_i: Measure energy E_i, decouple from the bath
- t_i->t_f: Drive the system H(t)
- t_f: Measure energy E_f

The quantum Jarzynski's equality



Always true:

$\Delta U[\gamma]=E_f-E_i$

$\langle \exp(-\Delta U[\gamma]) \rangle_\gamma = \exp(-\beta\Delta F)$

With standard definition:

$\Delta U[\gamma]=W[\gamma]$ (Two-points measurement protocol)

$\langle \exp(-W[\gamma]) \rangle_\gamma = \exp(-\beta\Delta F)$

Classical Jarzynski=Quantum Jarzynski???

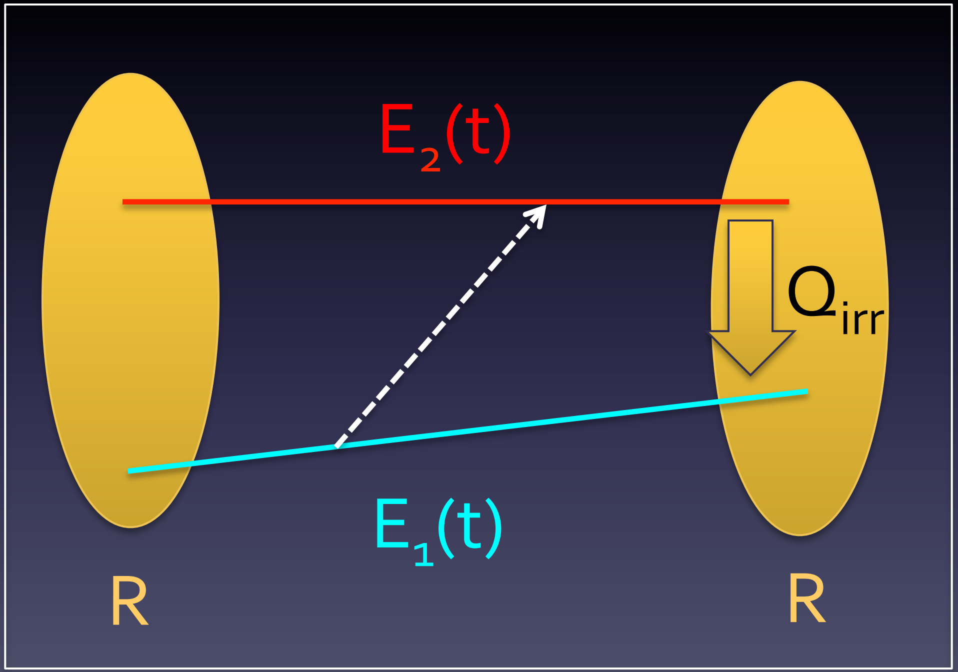
Nothing specific in quantum FT???

Irreversibility(ies) in Jarzynski's experiment

Always true:
$\Delta_i s[\gamma]=\beta(\Delta U[\gamma]-\Delta F)$

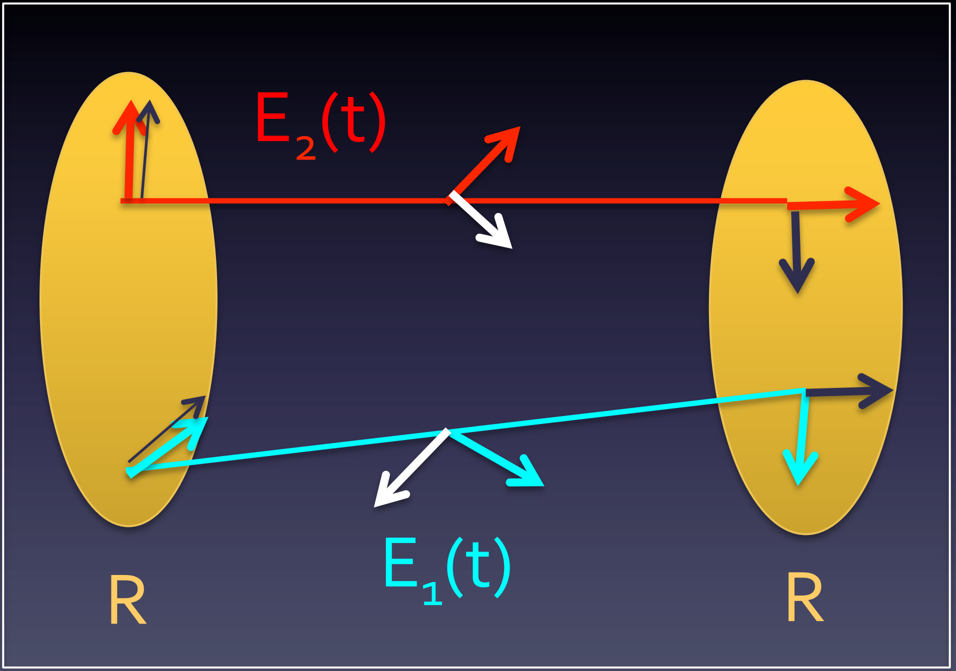
Current approach
$\Delta_i s[\gamma]=\beta(W[\gamma]-\Delta F)$
$\Delta_i s[\gamma]=-\beta Q_{irr}[\gamma]$

Measures « classical irreversibility » = departure from thermal equilibrium



Irreversibility(ies) in Jarzynski's experiment

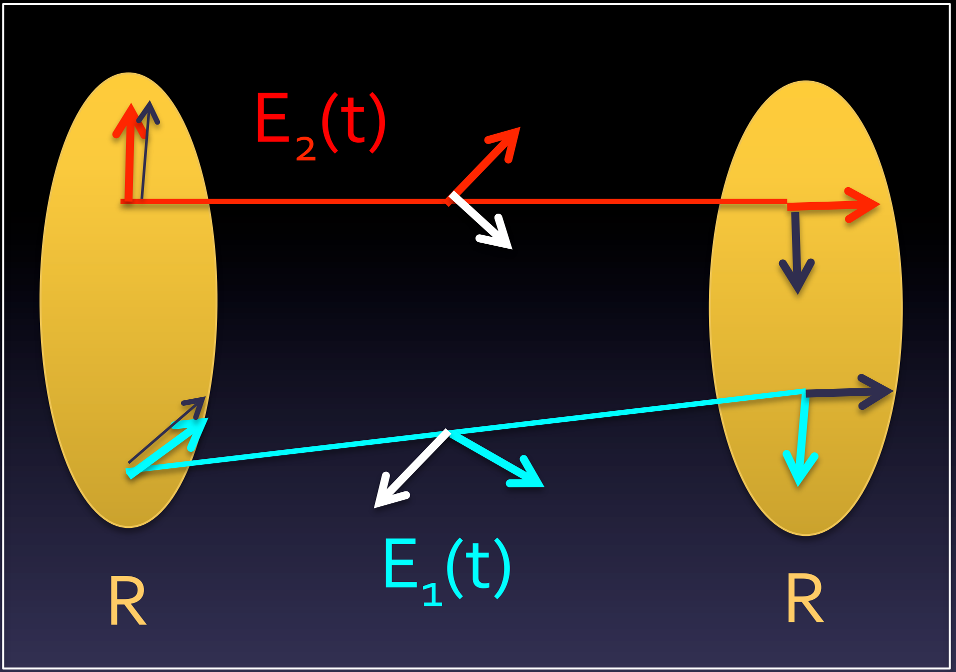
But Jarzynski's experiment also involves **quantum irreversibility** due to the very act of measuring
Hidden in current expressions of entropy production
But this is typically the kind of deviations we are looking for…



Revisiting Jarzynski

Always true:
$\Delta_i S[\gamma]=\beta(\Delta U[\gamma]-\Delta F)$

Our approach:
$\Delta_i S[\gamma]=\beta(W+Q_q[\gamma]-\Delta F)$



=> $\beta Q_q[\gamma]$: Genuinely quantum component in entropy production, due to quantum fluctuations

$\Rightarrow \beta Q_q[\gamma]=0$ if adiabatic transformation

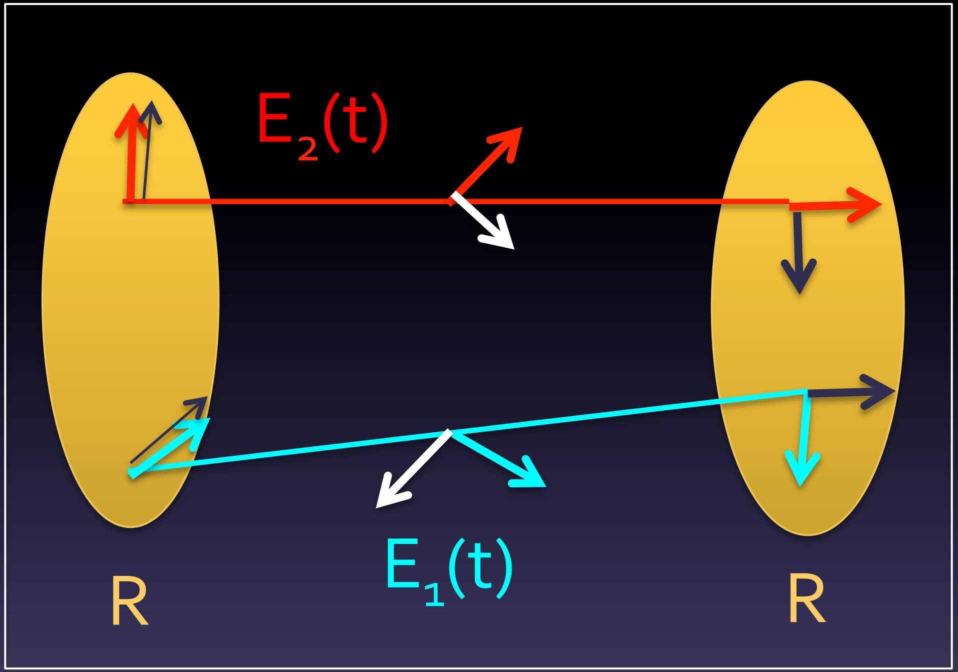
Revisiting Jarzynski

Always true:

$\Delta_i S[\gamma]=\beta(\Delta U[\gamma]-\Delta F)$

Our approach:

$\Delta_i S[\gamma]=\beta(W+Q_q[\gamma]-\Delta F)$



$\Rightarrow$ « Filters » quantum irreversibility

$\Rightarrow$ Relates energetic and entropic signatures

$\Rightarrow$ Quantum deviation to FT

Conclusions

A new formalism for stochastic quantum thermodynamics

- Thermal imprint of q measurement
- Treat on equal footing thermal and quantum stochasticity
- Quantum optics brings new insights into FTs
- Quantum thermodynamics brings new insights into q trajectories

Outlooks

- Influence of quantum irreversibility on work extraction in quantum engines/New kinds of « measurement driven » engines
- Thermodynamics of decoherence and elementary quantum processes
- Experimental platforms for quantum thermodynamics

$\Rightarrow$Engineered environments and circuit QED (Collaboration France/Brazil/Australia)

$\Rightarrow$Opto-mechanical devices

Microscopic interpretation

A Qubit prepared in $|\psi_i\rangle = (|e\rangle+|g\rangle)/\sqrt{2}$ is actually still entangled with a classical source

$|\Psi_i\rangle = (|e,\alpha_e\rangle+|g,\alpha_g\rangle)/\sqrt{2}$, with $|\alpha_e|^2 = |\alpha_g|^2 -1$

Energy is delocalized between the Qubit and the source

Measuring localizes the energy

=> $Q_q = -h\nu_o/2$ in the source: « Lost work »

=> $Q_q = h\nu_o/2$ in the Qubit: « Gained work »

Microscopic interpretation is nice, but…

The source is not necessarily accessible.

Quantum heat allows developing a consistent local thermodynamics, based on the observables of the system alone.

=> In the following, I keep the quantum heat as it is = an energetic signature of coherence erasure