Non-equilibrium QFT in cosmology: a case of resonant leptogenesis

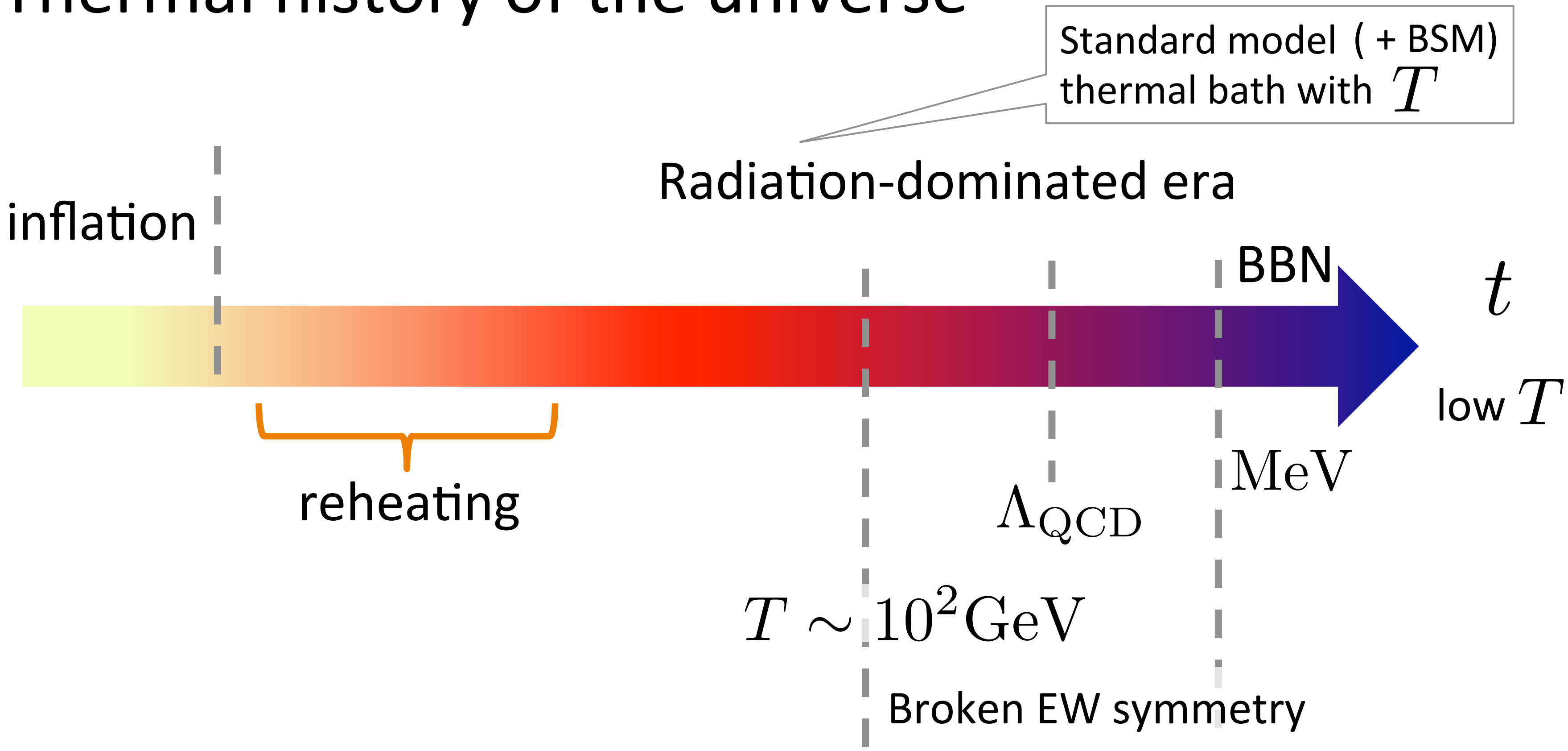
Kengo Shimada (LAPTh)

Journée Théorie du CPTGA, 12 mai 2016

Based on JHEP 1408, 043 arXiv:1404.4816[hep-ph] with S.Iso

JHEP 1404, 062 arXiv:1312.7680[hep-ph] with S.Iso and M.Yamanaka

1. Thermal history of the universe



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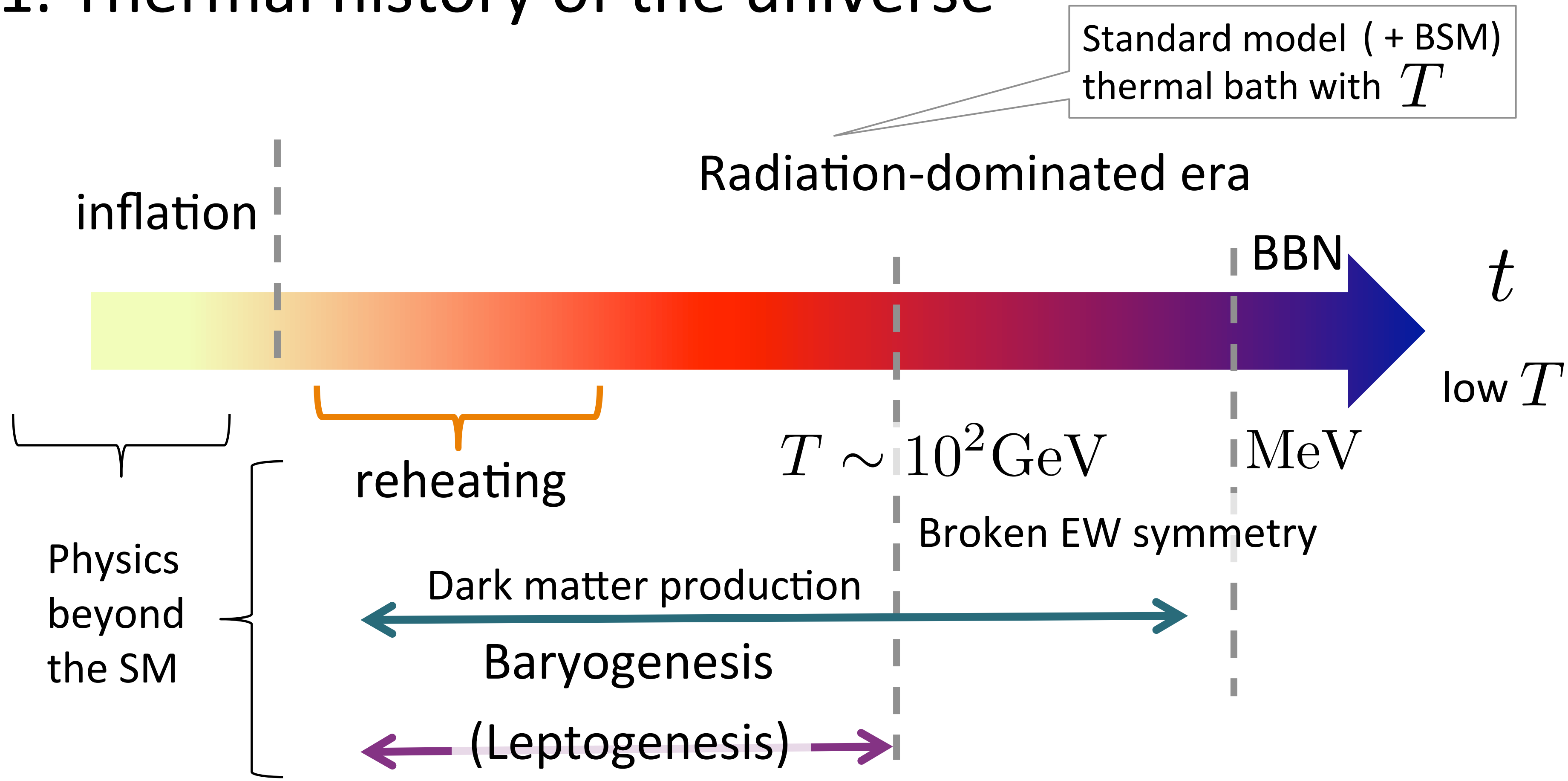

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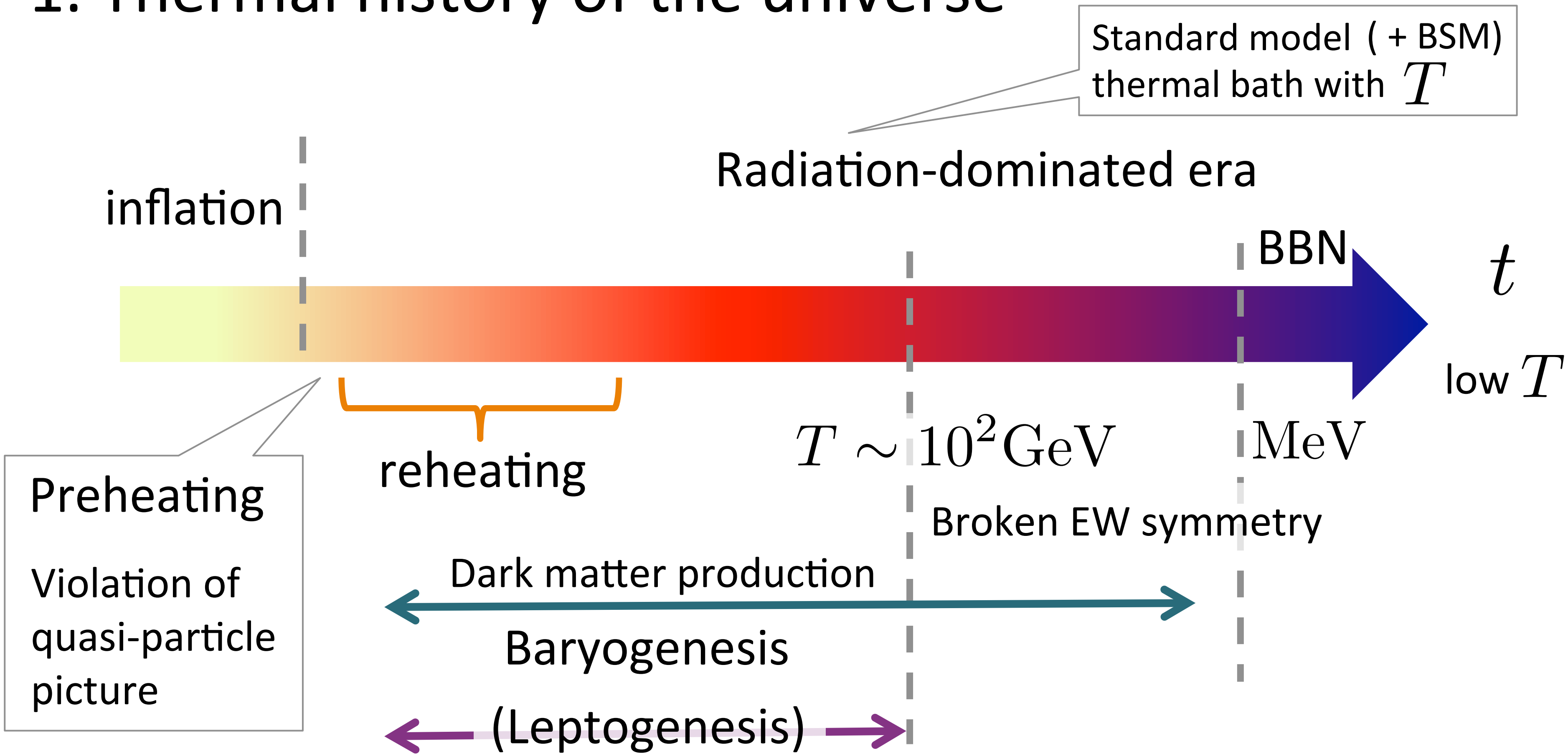

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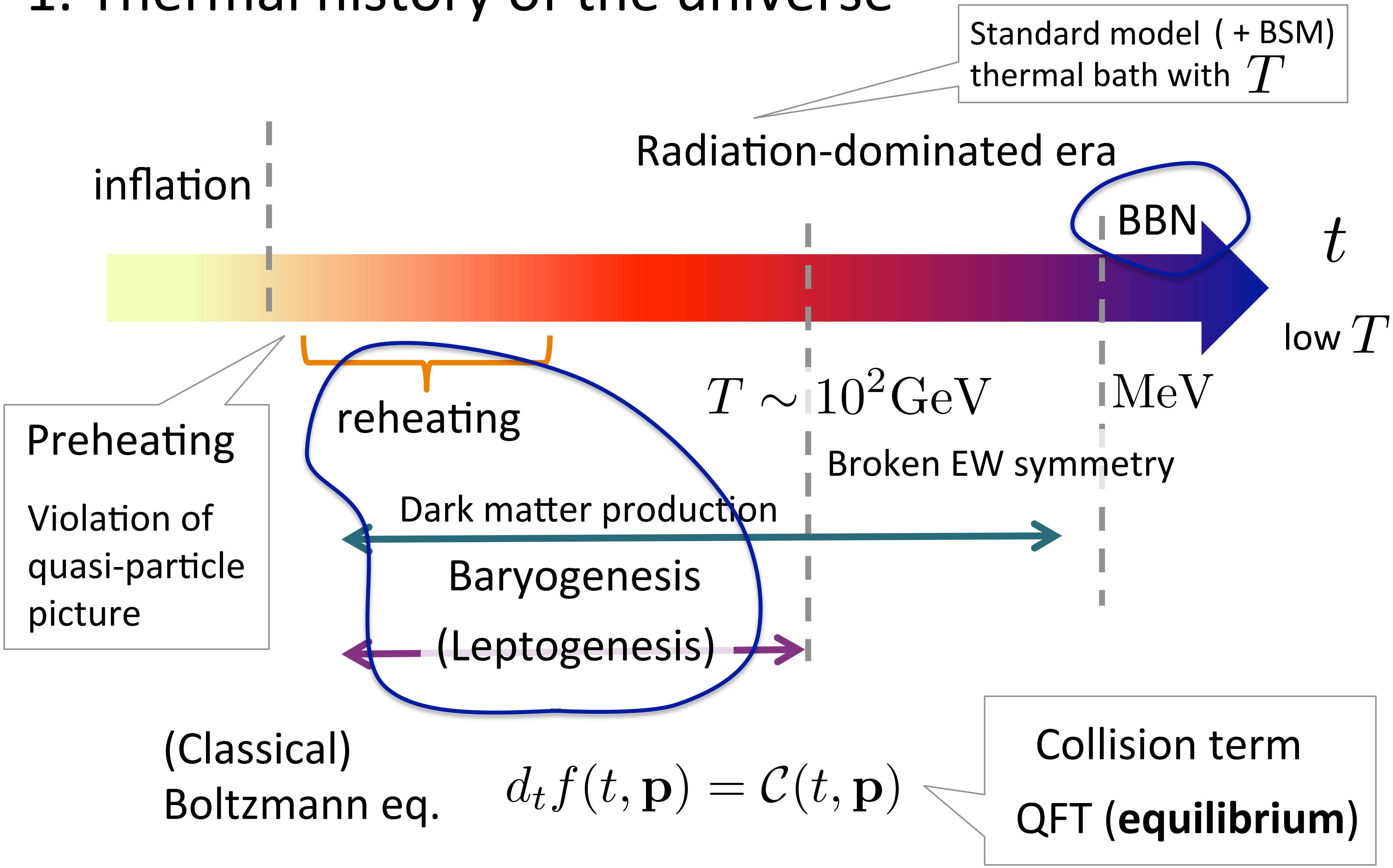


Even if the system is almost thermalized, (and quasi-particle picture is valid,)

Classical Boltzmann eq. + equilibrium QFT

$$d_t f(t, \mathbf{p}) = \mathcal{C}(t, \mathbf{p})$$

can be invalid because of

degenerate time scales.

An example: Resonant leptogenesis

Single framework of non-equilibrium QFT

1. Thermal history of the universe

2. Leptogenesis

3. Resonant leptogenesis

4. Kadanoff-Baym equation (non-equilibrium QFT)

5. Summary

2. Leptogenesis

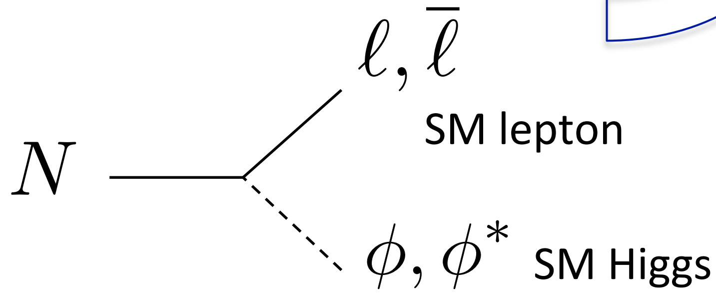
Fukugita, Yanagida (86)

- Baryon asymmetry of the universe

$$\eta_{\Delta B} \equiv \frac{n_B - n_{\overline{B}}}{n_\gamma} \approx 6 \times 10^{-10}$$

Right-handed neutrino (with Majorana mass)

$$N_i = \nu_{Ri} + \nu_{Ri}^c \quad i = 1, 2, 3$$



Hubble expansion

→ Out-of-equilibrium decay

→ ΔL → ΔB

CP violation

SU(2) Sphaleron

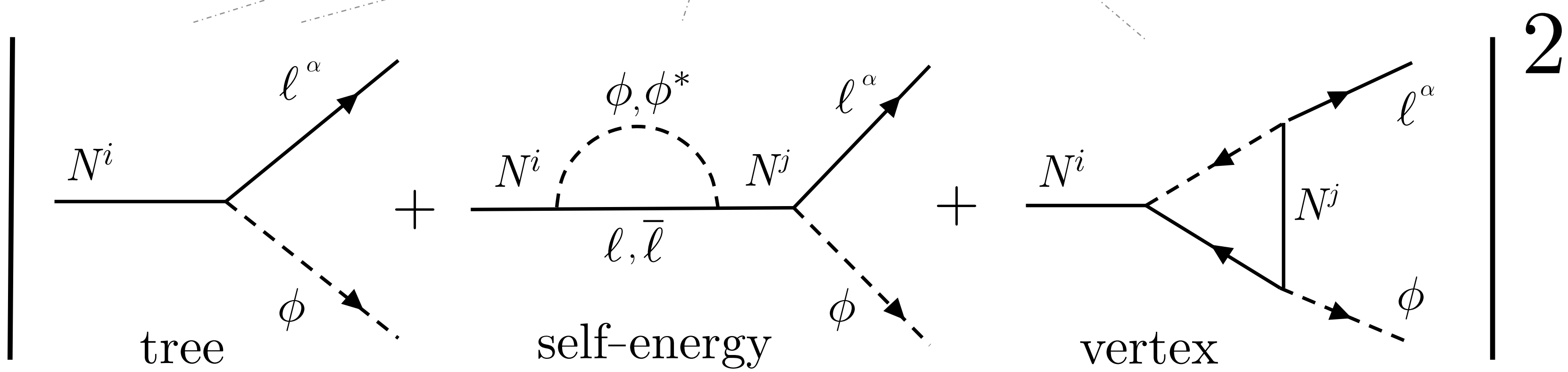
- Tiny SM neutrino mass (Seesaw mechanism)

2. Leptogenesis

Calculating **CP-violating parameter** $\varepsilon \equiv \dfrac{\Gamma_{N\to\ell\phi} - \Gamma_{N\to\bar{\ell}\phi^*}}{\Gamma_{N\to\ell\phi} + \Gamma_{N\to\bar{\ell}\phi^*}}$

in " equilibrium QFT "

(Lowest order) CP violation



decay

2. Leptogenesis

Calculating **CP-violating parameter** in " equilibrium QFT "

$$\varepsilon \equiv \frac{\Gamma_{N\to\ell\phi} - \Gamma_{N\to\bar{\ell}\phi^*}}{\Gamma_{N\to\ell\phi} + \Gamma_{N\to\bar{\ell}\phi^*}}$$

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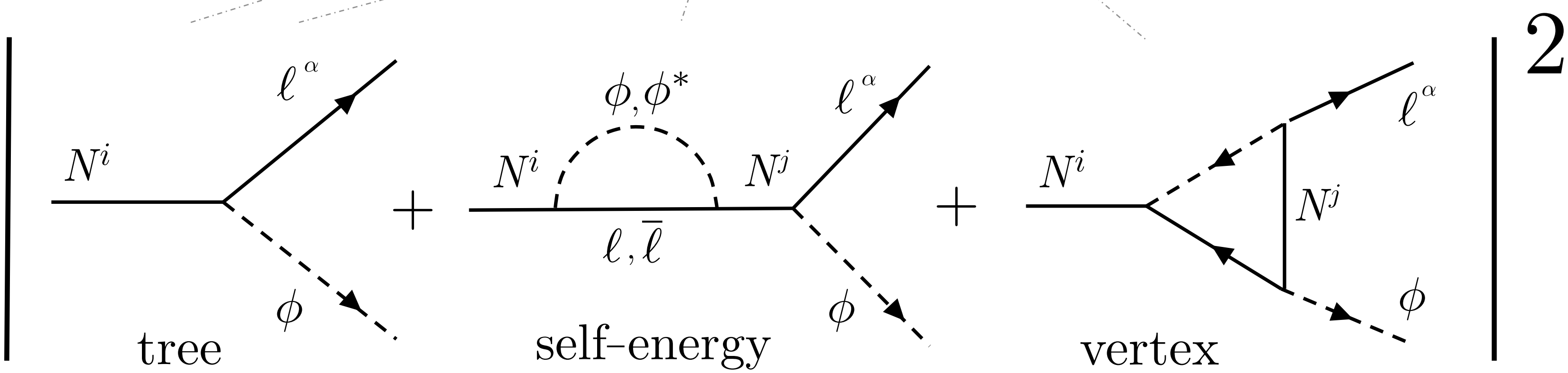
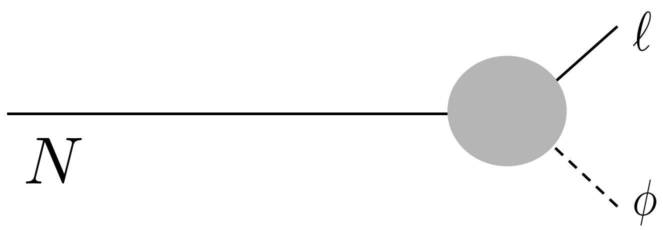


RH neutrino flavor oscillation

$$N_i \to N_{j\neq i}$$

dominates in **resonant case**

(Classical) Boltzmann equation + equilibrium QFT



Decay at t

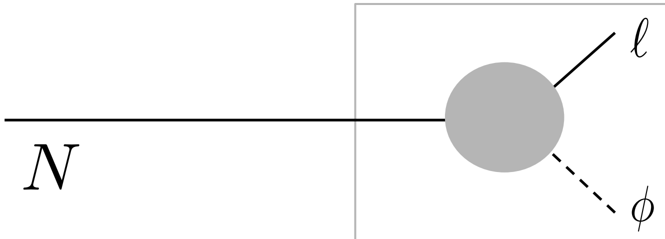
External line of
classical particle

On-shell mass eigenstate

Off-shell state

Quantum superposition of
different mass eigenstates

(Classical) Boltzmann equation + equilibrium QFT



Collision term at t

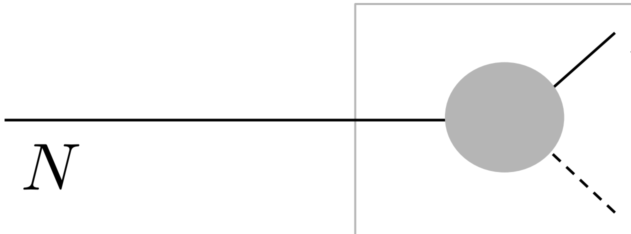
External line of classical particle

CP-violating parameter

$$\frac{dn_L}{dt} + 3Hn_L = \varepsilon \times \Gamma \times \delta n_N$$

Boltzmann equation of lepton number (momentum-integrated)

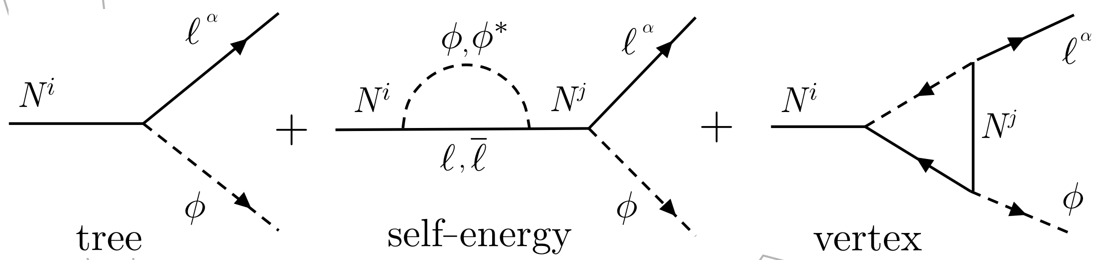
(Classical) Boltzmann equation + equilibrium QFT



Collision term at t

Markovian eq.

External line of classical particle



Matrix element calculated in equilibrium QFT

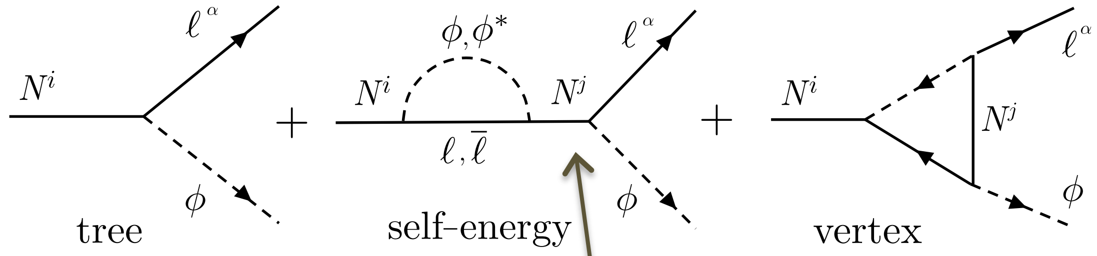
CP-violating parameter

$$\frac{dn_L}{dt} + 3Hn_L = \varepsilon \times \Gamma \times \delta n_N$$

Boltzmann equation of lepton number (momentum-integrated)

3. Resonant leptogenesis

Pilaftsis (97)



(Quantum) Time scale of Flavor oscillation

$$1/\Delta M$$

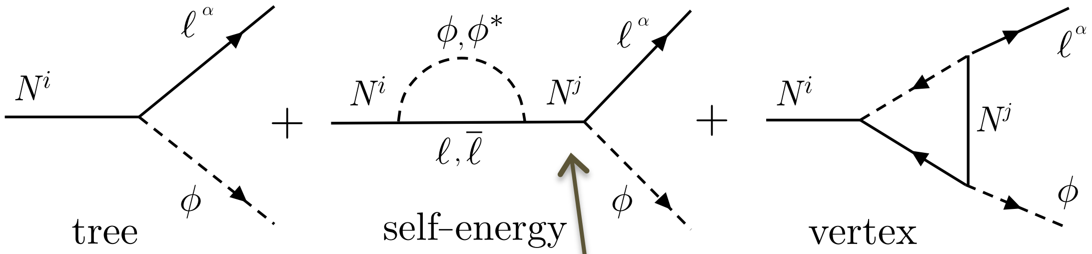
$M_1 \simeq M_2$: Degenerate mass spectrum

$$\frac{1}{q^2 - M_j^2} = \frac{1}{M_i^2 - M_j^2}$$

enhances the CP-violating parameter ε

3. Resonant leptogenesis

Pilaftsis (97)



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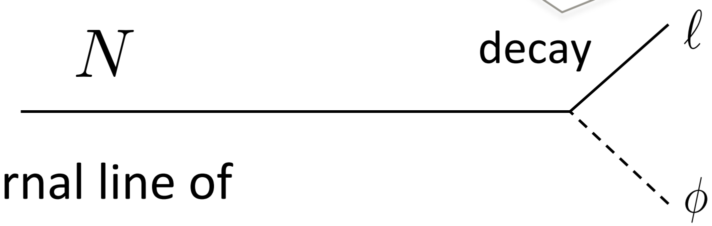
Conventional approach



time →

External line of classical particle

3. Resonant leptogenesis

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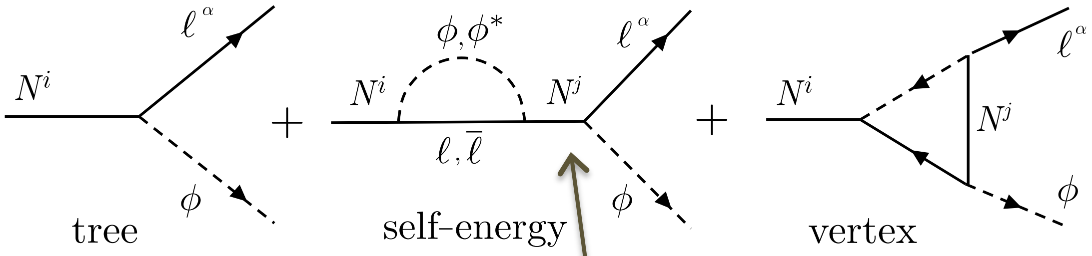

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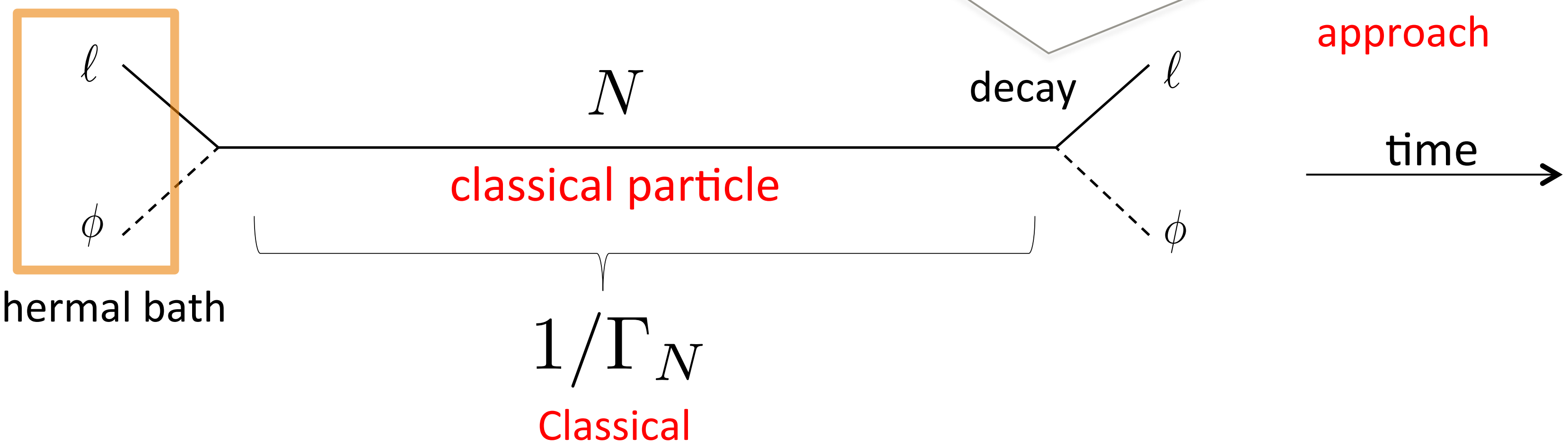

3. Resonant leptogenesis

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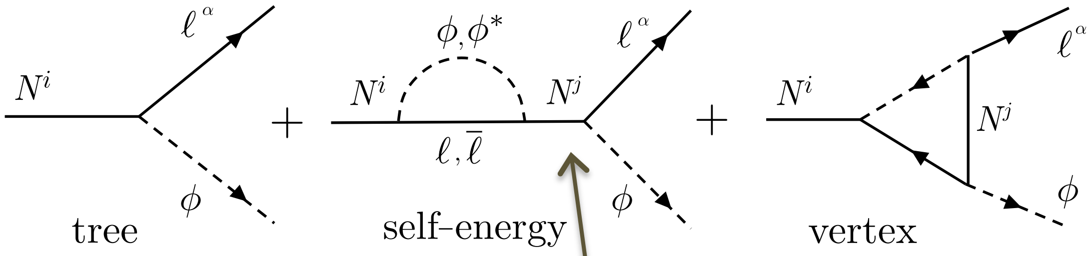

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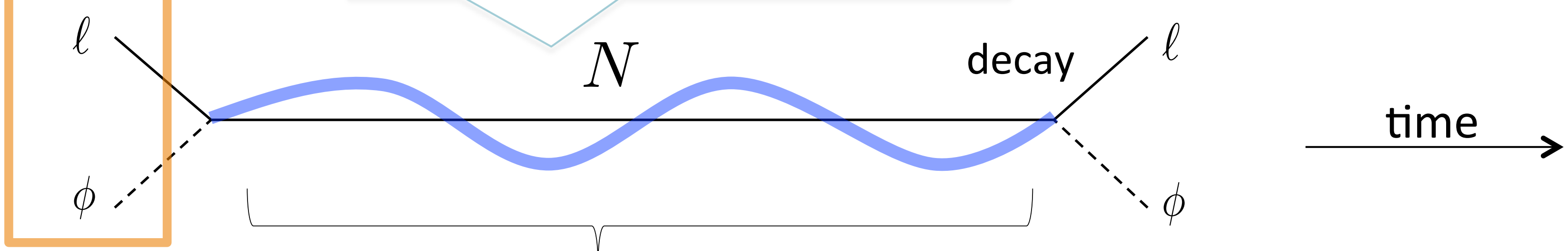
$$1/\Delta M$$

$M_1 \simeq M_2$: Degenerate mass spectrum

$$\frac{1}{q^2 - M_j^2} = \frac{1}{M_i^2 - M_j^2}$$

"External line" cannot be classical



SM thermal bath

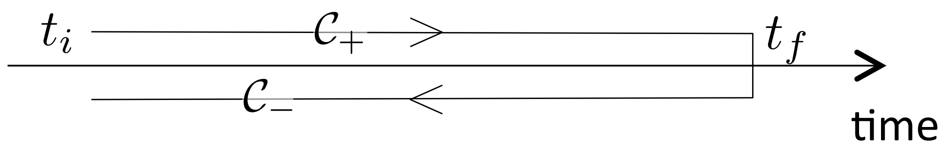
$$1/\Gamma_N \sim 1/\Delta M$$

Classical Quantum

Conventional Boltzmann eq. breaks down

3. Kadanoff-Baym equation

SD eq. on CTP (in-in formalism) $G^{-1} = G_0^{-1} - \Pi$



1PI

2PI formalism

Kadanoff-Baym eq. : Self-consistent equation of **Full propagator**

3. Kadanoff-Baym equation

SD eq. on CTP (in-in formalism) $G^{-1} = G_0^{-1} - \Pi$

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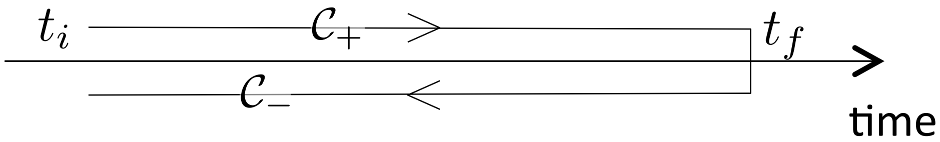

Kadanoff-Baym eq. : Self-consistent equation of **Full propagator**

$$(i\partial\!\!\!/_x - M)G_R(x,y) - \int d^4z\ \Pi_R(x,z)G_R(z,y) = -\delta^4(x-y)$$

$$(i\partial\!\!\!/_x - M)G_{\lessgtr}(x,y) - \int d^4z\ \Pi_R(x,z)G_{\lessgtr}(z,y) = \int d^4z\ \Pi_{\lessgtr}(x,z)G_A(z,y)$$

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SD eq. on CTP (in-in formalism) $G^{-1} = G_0^{-1} - \Pi$

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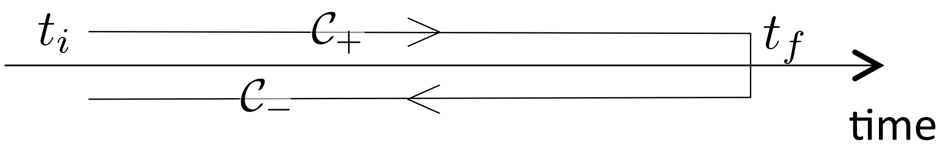


Kadanoff-Baym eq. : Self-consistent equation of **Full propagator**

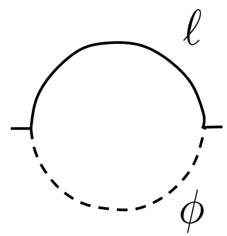
Spectrum of the system

$$(i\partial\!\!\!/_x - M)G_R(x,y) - \int d^4z\, \Pi_R(x,z)G_R(z,y) = -\delta^4(x-y)$$

$$(i\partial\!\!\!/_x - M)G_{\lessgtr}(x,y) - \int d^4z\, \Pi_R(x,z)G_{\lessgtr}(z,y) = \int d^4z\, \Pi_{\lessgtr}(x,z)G_A(z,y)$$

Retarded propagator

$$G_R(x,y) \equiv \theta(x^0 - y^0)G_\rho(x,y)$$
$$= \theta(x^0 - y^0)i\langle\{\psi(x), \overline{\psi}(y)\}\rangle$$

$\Pi \sim$ 

1PI

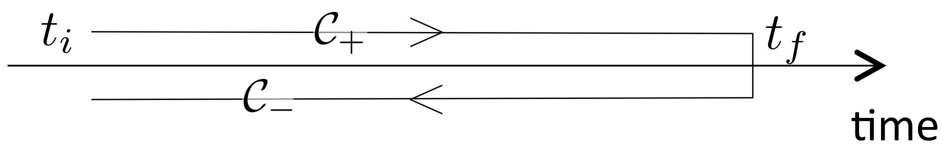
Width , Correction to the mass

3. Kadanoff-Baym equation

SD eq. on CTP (in-in formalism) $G^{-1} = G_0^{-1} - \Pi$

1PI

2PI formalism



Kadanoff-Baym eq. : Self-consistent equation of **Full propagator**

Spectrum of the system

1PI self-energy ➡ Width , Correction to the mass

$$(i\partial\!\!\!/_x - M)G_R(x,y) - \int d^4z\ \Pi_R(x,z)G_R(z,y) = -\delta^4(x-y)$$

$$(i\partial\!\!\!/_x - M)G_{\lessgtr}(x,y) - \int d^4z\ \Pi_R(x,z)G_{\lessgtr}(z,y) = \int d^4z\ \Pi_{\lessgtr}(x,z)G_A(z,y)$$

State of the system
➡ distribution func. f

$$G_>(x,y) \equiv \langle \psi(x)\overline{\psi}(y)\rangle$$

Wightman function

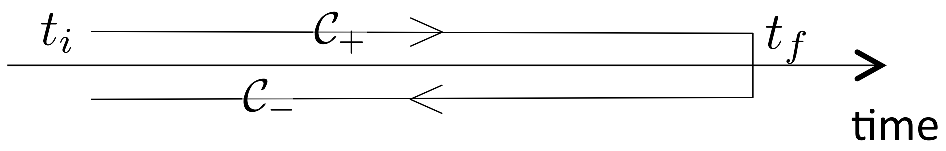
$$\hat{a}\hat{a}^\dagger = 1 - \hat{N}$$

3. Kadanoff-Baym equation

SD eq. on CTP (in-in formalism) $G^{-1} = G_0^{-1} - \Pi$

1PI

2PI formalism



Kadanoff-Baym eq. : Self-consistent equation of **Full propagator**

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integrate

$$G_{\lessgtr}(x,y) = -\int d^4u d^4v G_R(x,u)\Pi_{\lessgtr}(u,v)G_A(v,y)$$

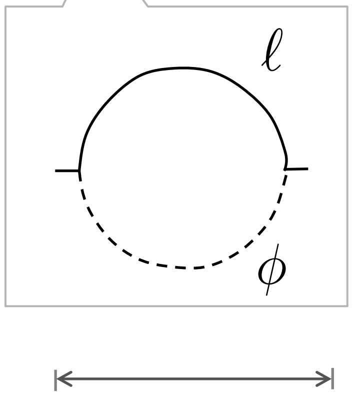
3. Kadanoff-Baym equation

$$t \simeq x^0 \simeq y^0 \qquad \tau \simeq u^0 \simeq v^0$$

$$G_{\lessgtr}(t) = -\int^t d\tau \; G_R(t,\tau)\Pi_{\lessgtr}(\tau)G_A(\tau,t)$$

$\sim n_N$

Number density at t



$1/\Gamma_{\ell,\phi}$

RH neutrino's

Wightman function

3. Kadanoff-Baym equation

$$t \simeq x^0 \simeq y^0 \quad > \quad \tau \simeq u^0 \simeq v^0$$

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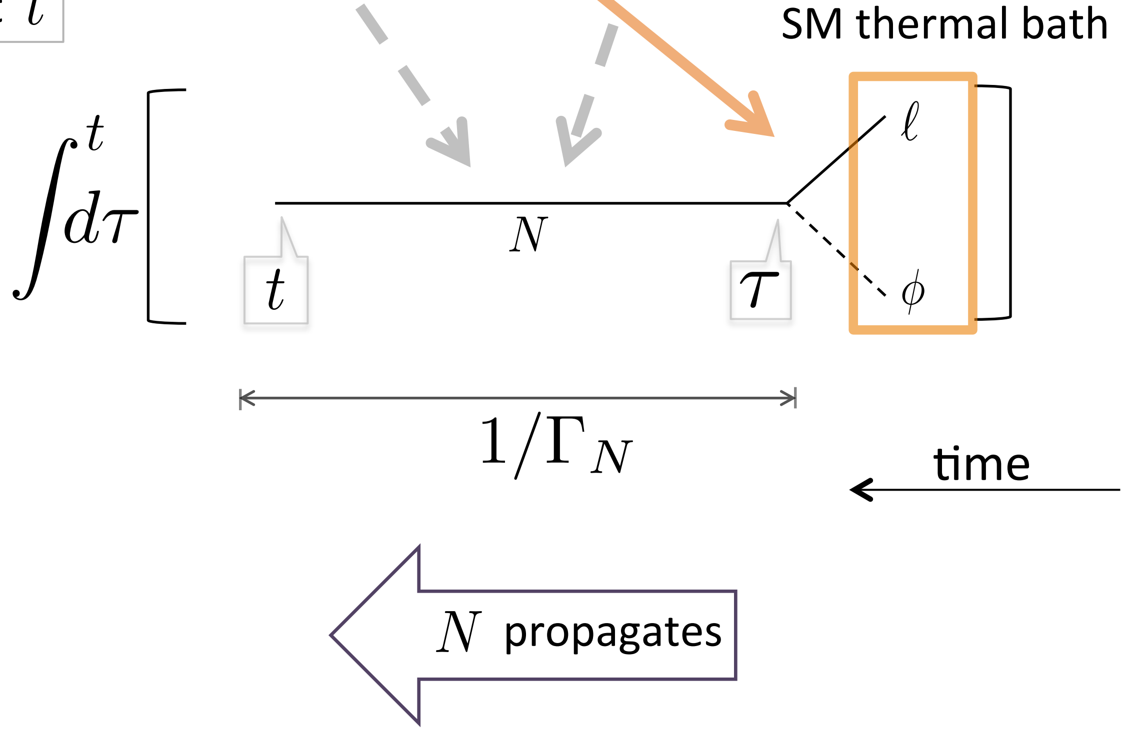

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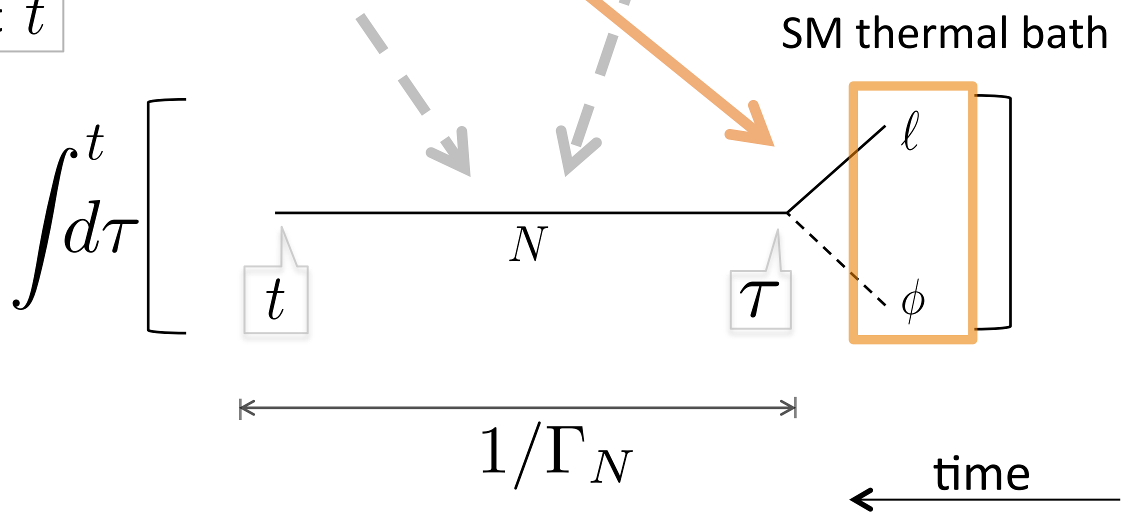

$\delta n_N(t)$

Deviation from
local equilibrium value

Low temperature ← High temperature

3. Kadanoff-Baym equation

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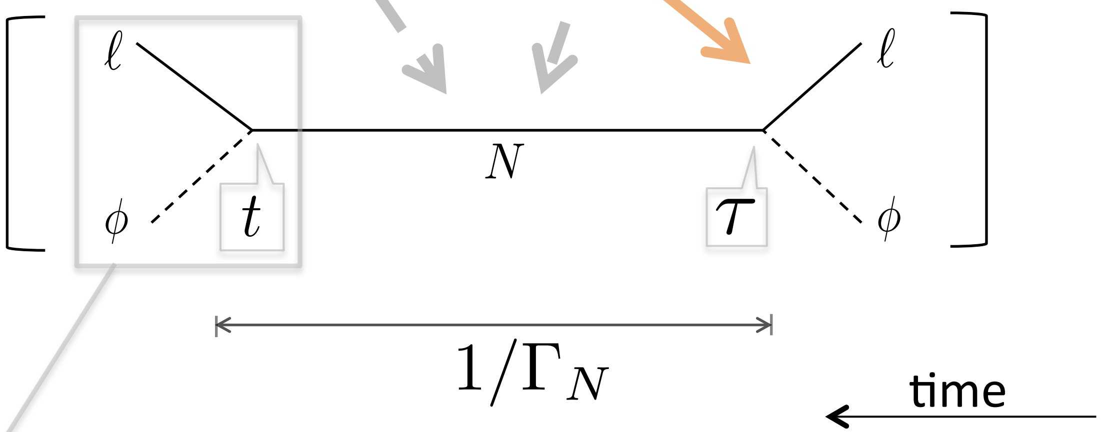
$$G_{\lessgtr}(t) = -\int^t d\tau\; G_R(t,\tau)\Pi_{\lessgtr}(\tau)G_A(\tau,t)$$

$\sim n_N$
Number density at t

$$\int^t d\tau \left[\quad \right]$$



Collision term in
the Boltzmann eq. of **lepton number**

$$\frac{dn_L}{dt} + 3Hn_L = \varepsilon \times \Gamma \times \delta n_N$$

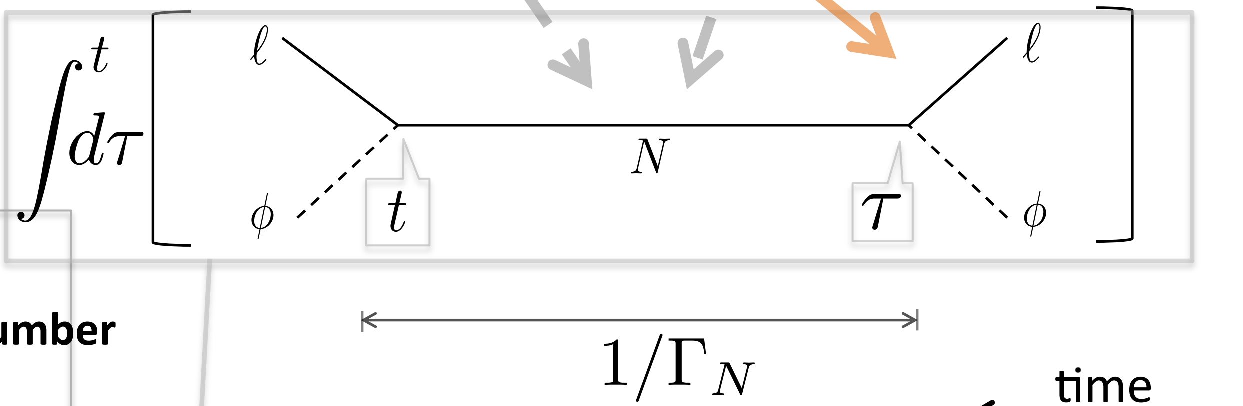
Conventional
approach

3. Kadanoff-Baym equation

$$t \simeq x^0 \simeq y^0 \;\; > \;\; \tau \simeq u^0 \simeq v^0$$

$$G_{\lessgtr}(t) = - \int^t d\tau \; G_R(t,\tau)\Pi_{\lessgtr}(\tau)G_A(\tau,t)$$

$\sim n_N$
Number density at t



Collision term in
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$$\frac{dn_L}{dt} + 3Hn_L = \varepsilon \times \Gamma \times \delta n_N$$

Non-Markovian

"Effective" Markovian eq.

Time integration

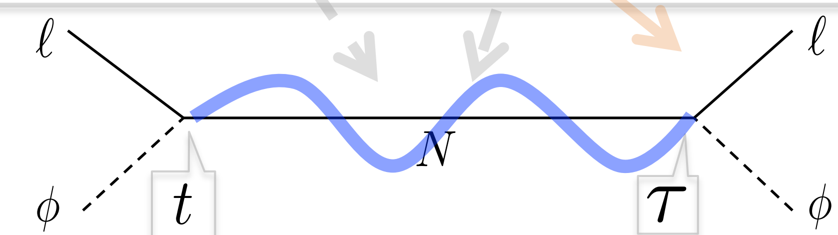
3. Kadanoff-Baym equation

$$t \simeq x^0 \simeq y^0 \;>\; \tau \simeq u^0 \simeq v^0$$

$$G_{\lessgtr}(t) = -\int^t d\tau\; G_R(t,\tau)\Pi_{\lessgtr}(\tau)G_A(\tau,t)$$

$\sim n_N$
Number density at t

$$\int^t d\tau \Big[\quad\Big]$$



Collision term in
the Boltzmann eq. of **lepton number**

$$\frac{dn_L}{dt} + 3Hn_L = \varepsilon \times \Gamma \times \delta n_N$$

Taking into account
full quantum effects,

$$R_{ij} = M_i\Gamma_i + M_j\Gamma_j$$

Garny et al. (2013) KS et al. (2014)

$$\varepsilon_i = \frac{\Im(h^\dagger h)^2_{ij}}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + R_{ij}^2}$$

cf. equilibrium QFT $R_{ij} = M_i\Gamma_i - M_j\Gamma_j$

: Plumacher, Buchmuller (98)

Summary

- Degeneracy of time scales makes

  the conventional approach invalid.

  Classical Boltzmann eq. + equilibrium QFT

- In the case of resonant leptogenesis ($M_1 \simeq M_2$),

$$1/\Gamma_N \sim 1/\Delta M \; .$$

Lifetime Flavor oscillation

- CP-violating parameter is obtained
  in the **single** framework (non-equilibrium QFT).

- Kadanoff-Baym eq. (→ Density matrix formalism)

Thank you

Density matrix formalism

Sigl, Raffelt (93)

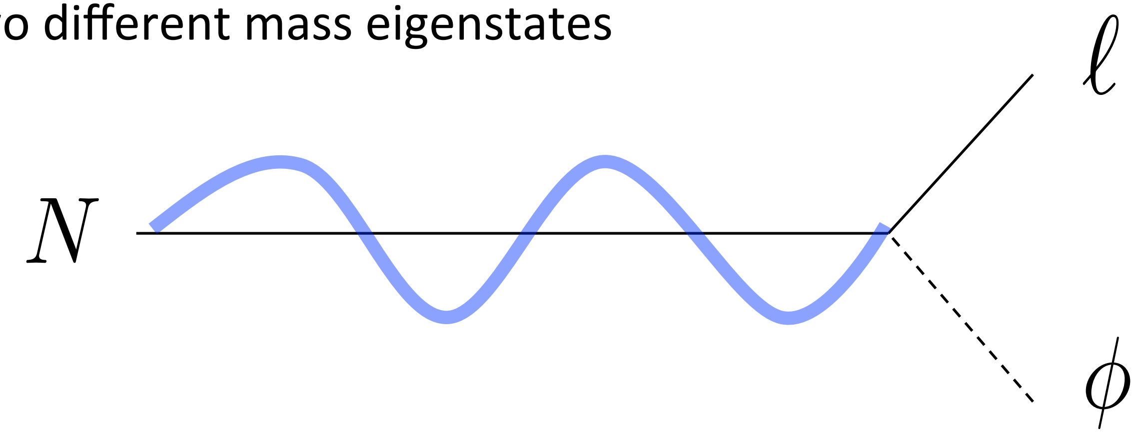
Quantum extension of Boltzmann eq. (Markovian)

Classical distribution function f

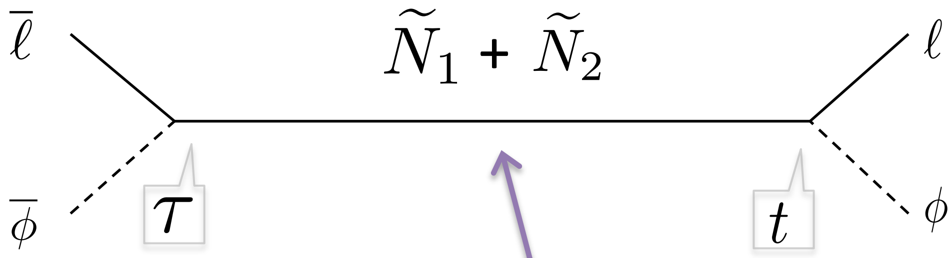
Matrix-valued extension

$$\begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix}$$

Quantum superposition of two different mass eigenstates



Non-Markovian eq.



Kadanoff-Baym eq.

Quantum superposition
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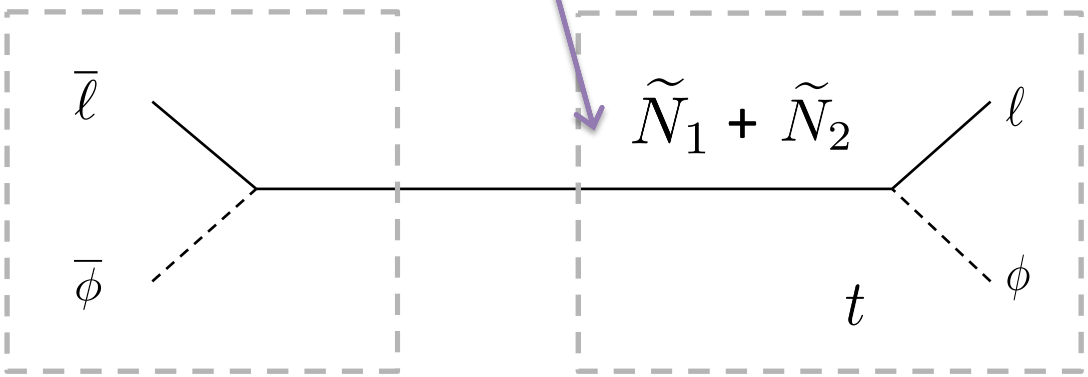
Markovian eq.



KB eq. ➡
Density matrix Formalism

Leptogenesis

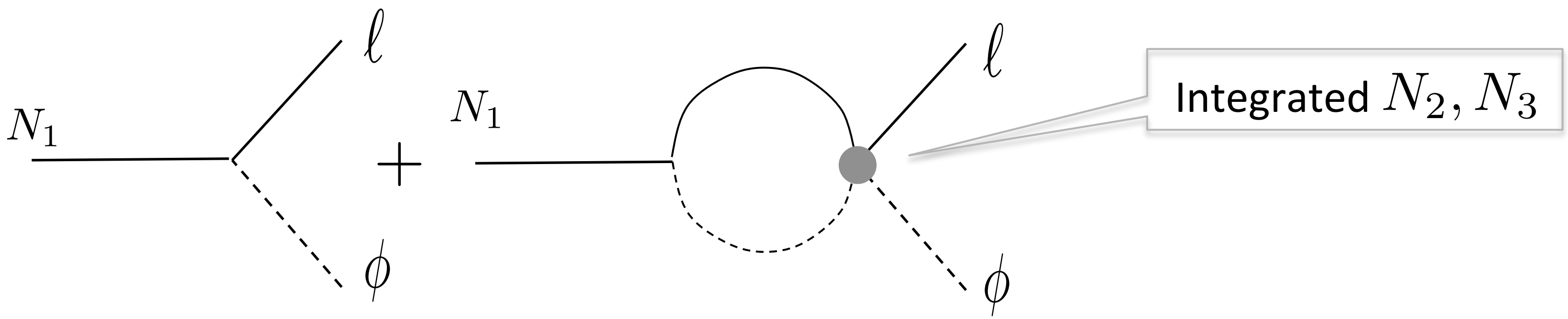
- Sakharov Conditions

(symmetric initial condition)	Standard Model	+ Right-handed Neutrino
Baryon number violation	Sphaleron process ~~$B+L$~~	$\Delta L \rightarrow \Delta B$ Sphaleron
C and CP violation	Weak interaction CKM matrix too small	Yukawa coupling $h_{\alpha i}$
Out of equilibrium dynamics	Strongly 1st order EW phase transition too small	Large Majorana mass

$$\mathcal{L}_{int} = -h_{\alpha i}\bar{\ell}_\alpha \widetilde{\phi} N_i - h^\dagger_{i\alpha}\overline{N}_i \ell_\alpha \widetilde{\phi}^\dagger$$

Leptogenesis

- Simplest model (decay of the lightest RH neutrino N_1) $M_1 \ll M_{2,3}$



$$\varepsilon \equiv \frac{\Gamma_{N\to\ell\phi} - \Gamma_{N\to\overline{\ell\phi}}}{\Gamma_{N\to\ell\phi} + \Gamma_{N\to\overline{\ell\phi}}}$$: CP-violating parameter

$$= \sum_{j=2,3} \frac{\mathrm{Im}[(h^\dagger h)_{1j}]^2}{8\pi (h^\dagger h)_{11}} \frac{3M_1}{2M_j} = \frac{3M_1 \mathrm{Im}(h^\dagger m_\nu h^*)_{11}}{16\pi \langle\phi\rangle^2 (h^\dagger h)_{11}}$$

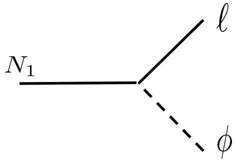
➡ Lower bound

$$M_1 \gtrsim 10^9 \mathrm{GeV}$$

$$m_\nu \simeq \langle\phi\rangle^2 h \frac{1}{M_N} h^T$$

Seesaw

Leptogenesis

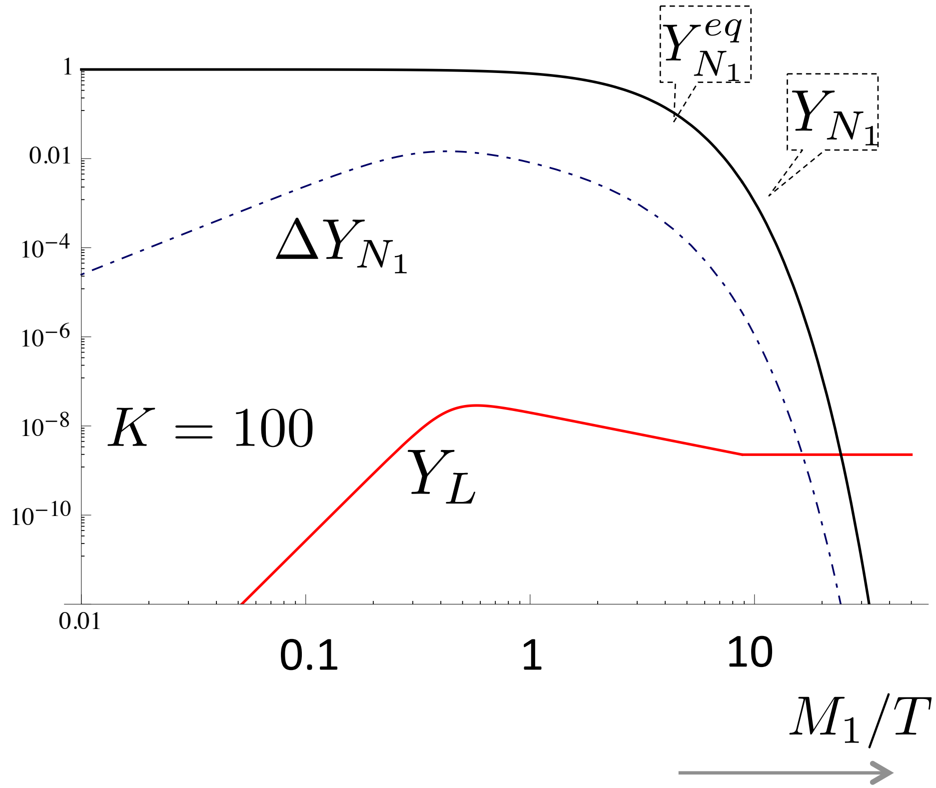


Thermal bath

Decay width (interaction rate) → **Relaxation** toward equilibrium

$$K \equiv \frac{\Gamma_{N_1}}{H|_{T=M_1}} \gg 1$$

Expanding rate → **Departure** from equilibrium



(Integrated) Boltzmann eq.

$$\frac{dY_{N_1}}{dt} = -D(Y_{N_1} - Y_{N_1}^{eq})$$

+ Thermal initial condition

CP-violating parameter

$$\frac{dY_L}{dt} = \varepsilon D(Y_{N_1} - Y_{N_1}^{eq}) - WY_L$$

RIS-subtraction