Out-of-equilibrium dynamics in superconductors

Julia S. Meyer

with

Anton Bespalov (→ Nizhni-Novgorod),

Manuel Houzet, and **Yuli Nazarov** (TU Delft)

Journée Théorie CPTGA

LAPTh – May 12, 2016

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PHELIQS

How many quasiparticles can be in a superconductor?

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PHELIQS

A theoretical model to explain excess of quasiparticles in superconductors

Julia S. Meyer

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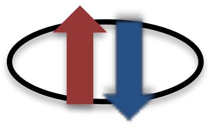
PHELIQS

Quasiparticles in superconductors

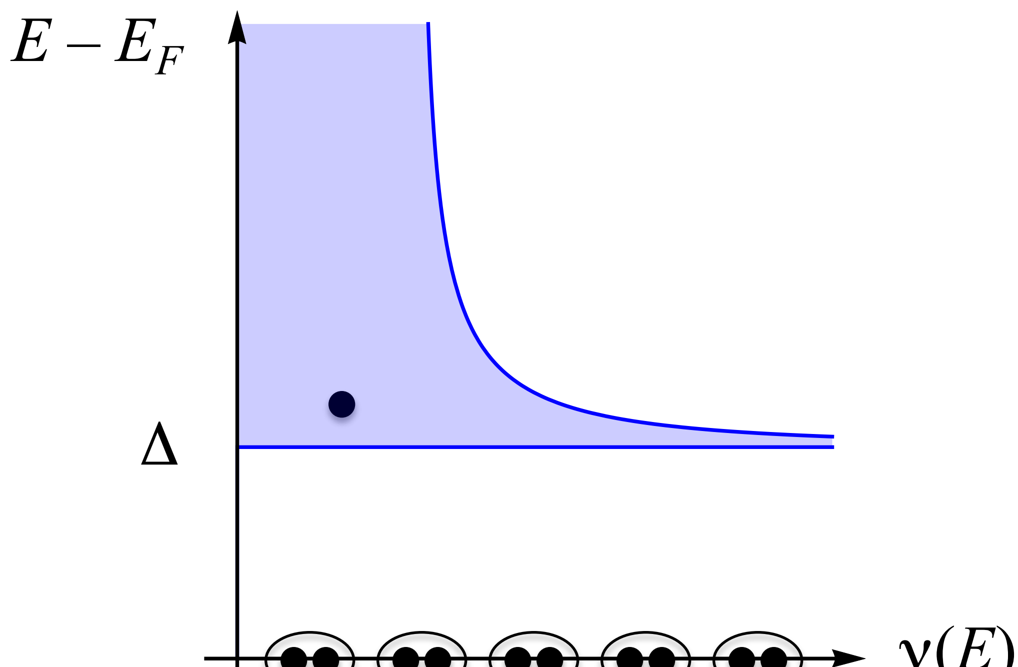
conventional superconductors (e.g. Aluminium):

- spin-singlet
- s-wave
- fully gapped density of states



at low temperatures (T ¿ Δ):
exponentially small
thermal concentration
of quasiparticles

$$c_{\mathrm{eq}}(T) \simeq \nu_0 \sqrt{8\pi k_B T \Delta}\, e^{-\Delta/k_B T}$$



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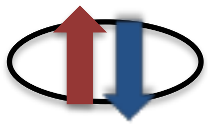

at low temperatures (T ¿ Δ):

exponentially small thermal concentration of quasiparticles

$$c_{\mathrm{eq}}(T) \simeq \nu_0 \sqrt{8\pi k_B T \Delta}\, e^{-\Delta/k_B T}$$

typical values for Al: $\Delta = 200\ \mu\mathrm{eV}$

$c(100\ \mathrm{mK})$ » $1\ \mu\mathrm{m}^{-3}$, $c(50\ \mathrm{mK})$ » $10^{-6}\ \mu\mathrm{m}^{-3}$, $c(10\ \mathrm{mK})$ » $10^{-51}\ \mu\mathrm{m}^{-3}$

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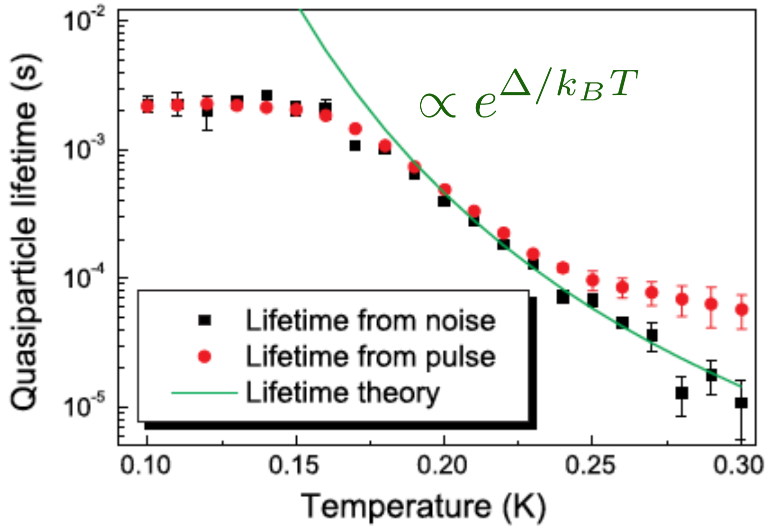
Experiment: Excess quasiparticles

residual quasiparticle concentration at low temperatures: c À c_{eq}

Experiment: Excess quasiparticles

saturation of the lifetime of superconducting resonators at low T



de Visser *et al.*, PRL 2011

theory: $\tau_r \simeq \frac{\tau_0}{2c} \frac{\nu_0 (k_B T_c)^3}{\Delta^2}$

τ_0 normal-metal electron-phonon relaxation rate at energy Δ

$\rightarrow$ residual quasiparticle concentration: $c \sim 25 - 55\,\mu\mathrm{m}^{-3}$

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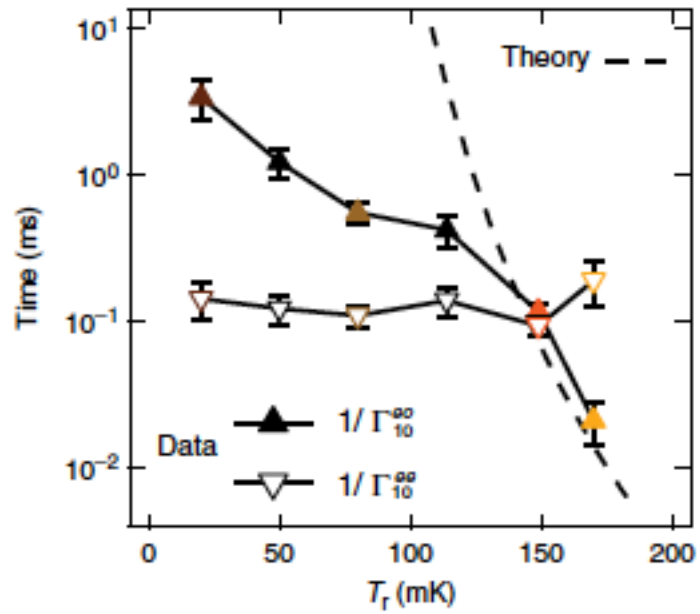
Experiment: Excess quasiparticles

saturation of the coherence time of superconducting qubits at low T



$\propto e^{\Delta/k_B T}$

Ristè *et al.*,
Nat. Commun. 2013

theory: $\Gamma^{eo} \simeq \frac{c}{\pi \nu_0}\sqrt{\frac{2\omega_{01}}{\Delta}}$

ω_{01} qubit frequency

→ residual quasiparticle concentration: $c \sim 0.04\,\mu\mathrm{m}^{-3}$

Main results

observation:

excess quasiparticles in virtually all superconducting devices which limits their performances

our work: generation-recombination model

$\rightarrow$ residual quasiparticle concentration

- for delocalized quasiparticles above the superconducting gap

  $$c \propto \sqrt{A}$$

- for localized quasiparticles

  at mesoscopic fluctuations of the gap edge

  $$c \propto \frac{1}{\ln^3(1/A)}$$

) **poor efficiency of shielding**

where A generation rate due to non-equilibrium agent

Outline

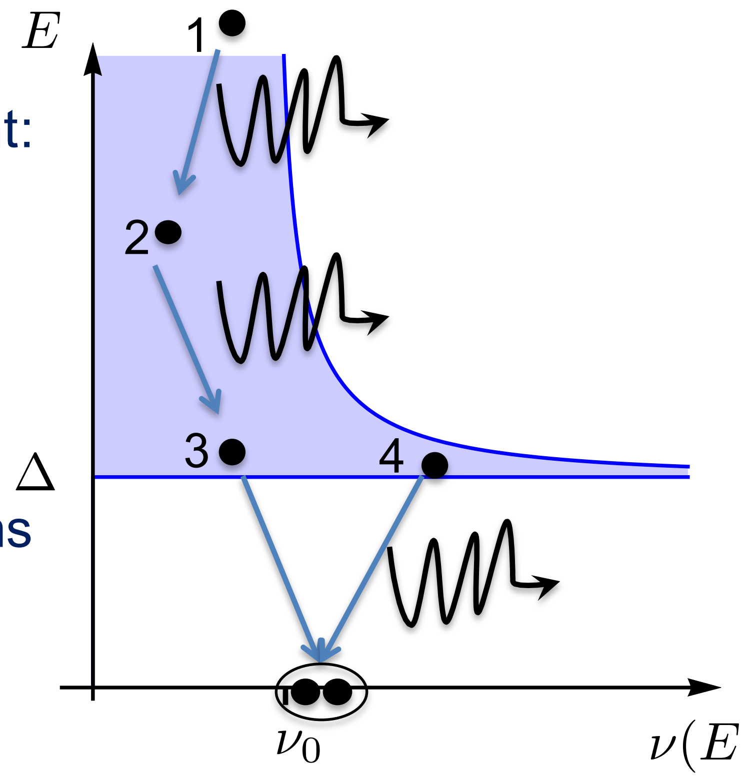
- Motivation
- Generation/recombination model of delocalized quasiparticles
- Effects of disorder
- **Our work:** Extremely slow relaxation of localized quasiparticles
- Conclusion & Perspectives

Non-equilibrium quasiparticles

- generation due to a non-equilibrium agent:
  - EM and blackbody radiation
  - cosmic rays
  - natural radioactivity
  - …

- fast energy relaxation by emitting phonons
- slow annihilation of two quasiparticles near the gap edge with rate

$$\Gamma_{34} = \bar{\Gamma} \int d\mathbf{r}\, p_3(\mathbf{r}) p_4(\mathbf{r})$$



balance between generation (rate A) and annihilation for delocalized quasiparticles near gap edge:

$$A = \bar{\Gamma} c^2 \implies c = \sqrt{A/\bar{\Gamma}}$$

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Non-equilibrium quasiparticles

- generation due to a non-equilibrium agent:
  - EM and blackbody radiation
  - cosmic rays
  - natural radioactivity
  - …

material constant:

$$\bar{\Gamma} = 8\frac{(\Delta/k_B T_c)^3}{\tau_0 \nu_0 \Delta}$$

$$\approx 40\,\mathrm{s}^{-1}\mu\mathrm{m}^3$$

in Al

- fast energy relaxation by emitting phonons
- slow annihilation of two quasiparticles

  near the gap edge with rate

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Disordered superconductors

ℓ mean free path
ξ superconducting coherence length

- clean metal $\ell > \xi$:

  pairing of electrons with opposite spins and momenta
- dirty metal $\ell < \xi$:

  pairing of electrons in time-reversed states

"Anderson theorem" (mean field):

Δ is unaffected by non-magnetic disorder

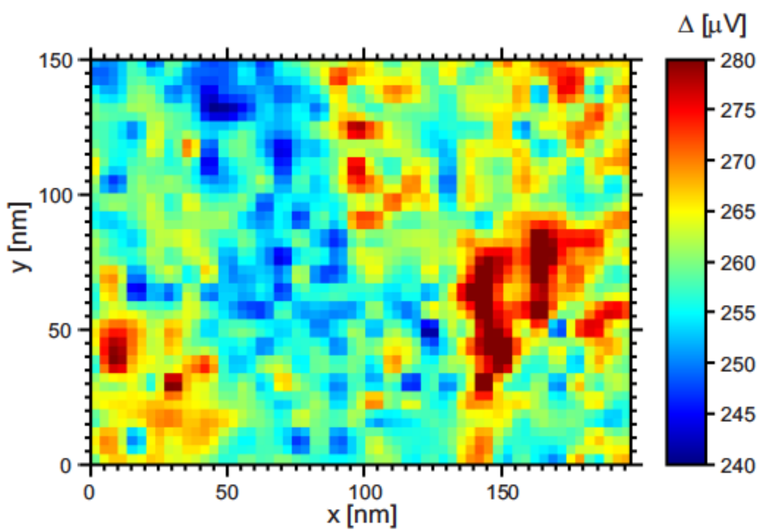
and remains spatially uniform

Abrikosov-Gorkov 1958

Anderson 1959

at larger disorder:

Sacépé *et al.*,
PRL 2008



STM study
of TiN films

Disordered superconductors

mesoscopic fluctuations of the gap:

Larkin and Ovchinnikov, JETP 1972

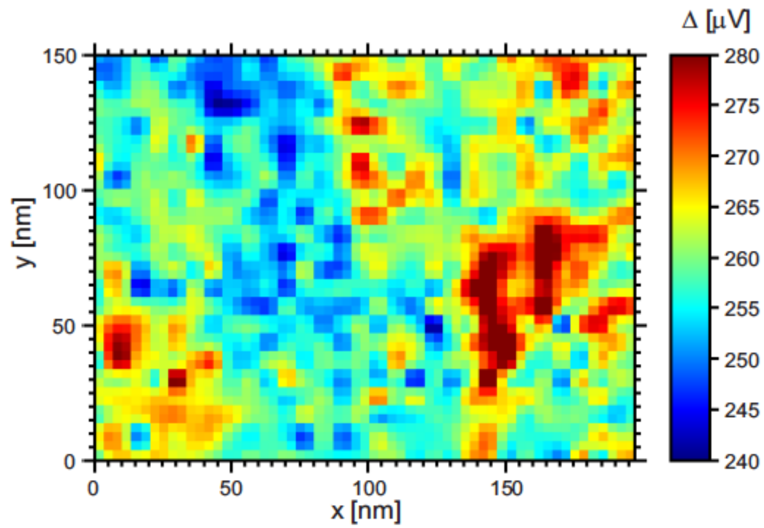
$$\delta\Delta(\mathbf{r}) = \Delta(\mathbf{r}) - \Delta$$

$$\langle \delta\Delta(\mathbf{r})\delta\Delta(\mathbf{r}')\rangle = (\delta\Delta)^2\delta(\mathbf{r}-\mathbf{r}')$$ with correlation radius $<\xi$

magnitude: $\frac{(\delta\Delta)^2}{\Delta^2\xi^3} \sim \frac{1}{g^2} \ll 1$,

where g dimensionless conductance on the scale ξ

(Al: g » 10^4)



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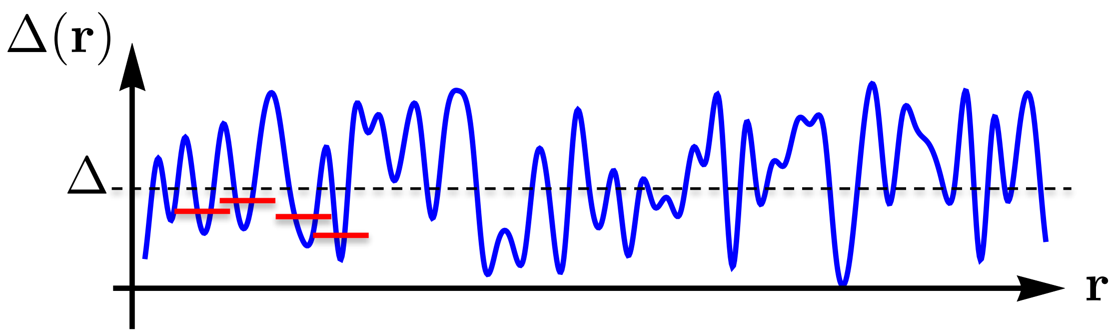

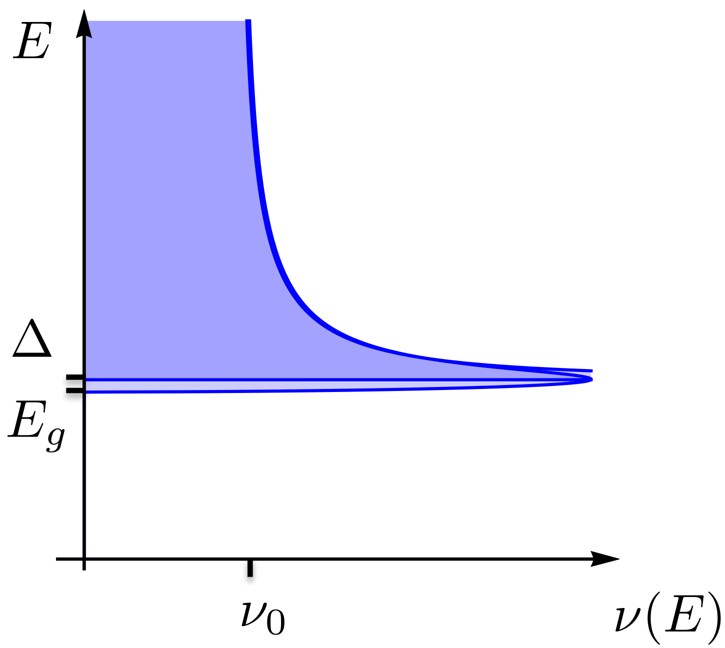

- reduced gap

  due to overlapping bound states

  $\varepsilon_g = \Delta - E_g \gg \Delta / g^{4/3}$
- rounding of the BCS singularity

Disordered superconductors

mesoscopic fluctuations of the gap:

Larkin and Ovchinnikov, JETP 1972

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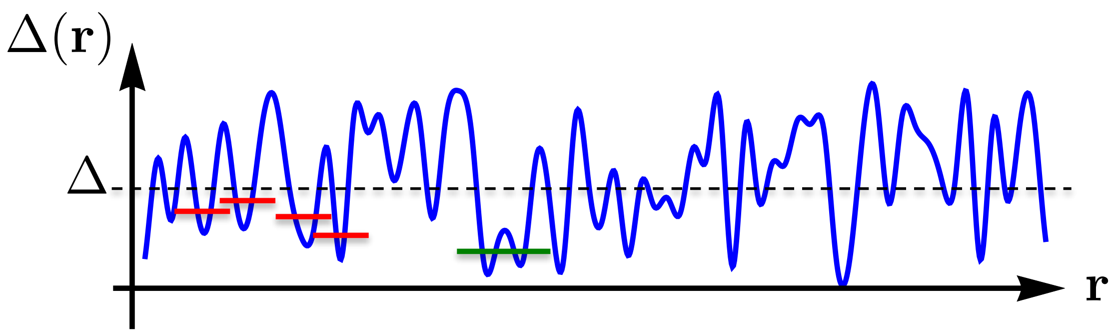
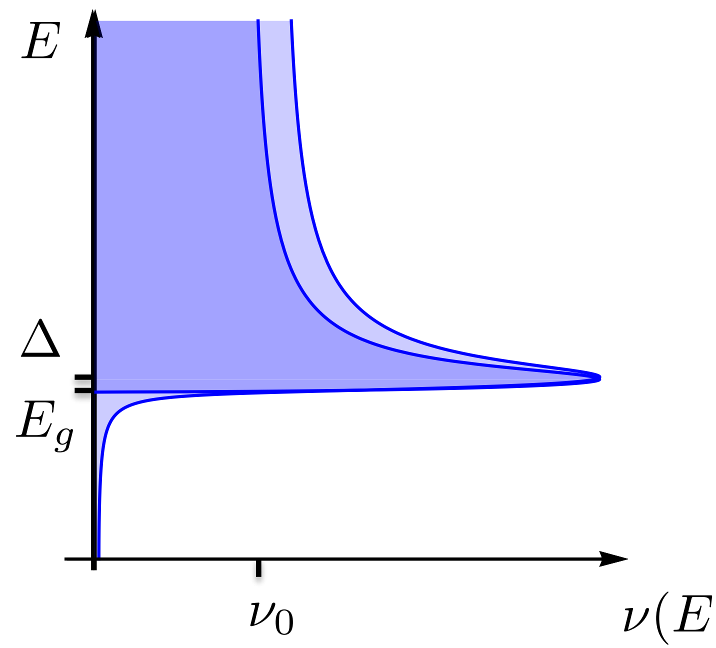


- rare optimal fluctuations

  generate localized tail states

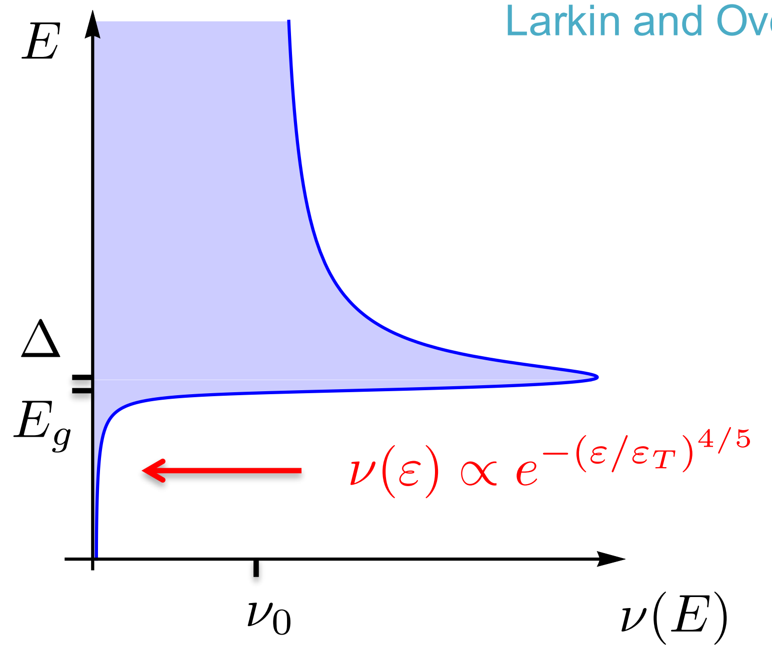
  with energies $E < E_g$



Disordered superconductors

mesoscopic fluctuations of the gap:

Larkin and Ovchinnikov, JETP 1972



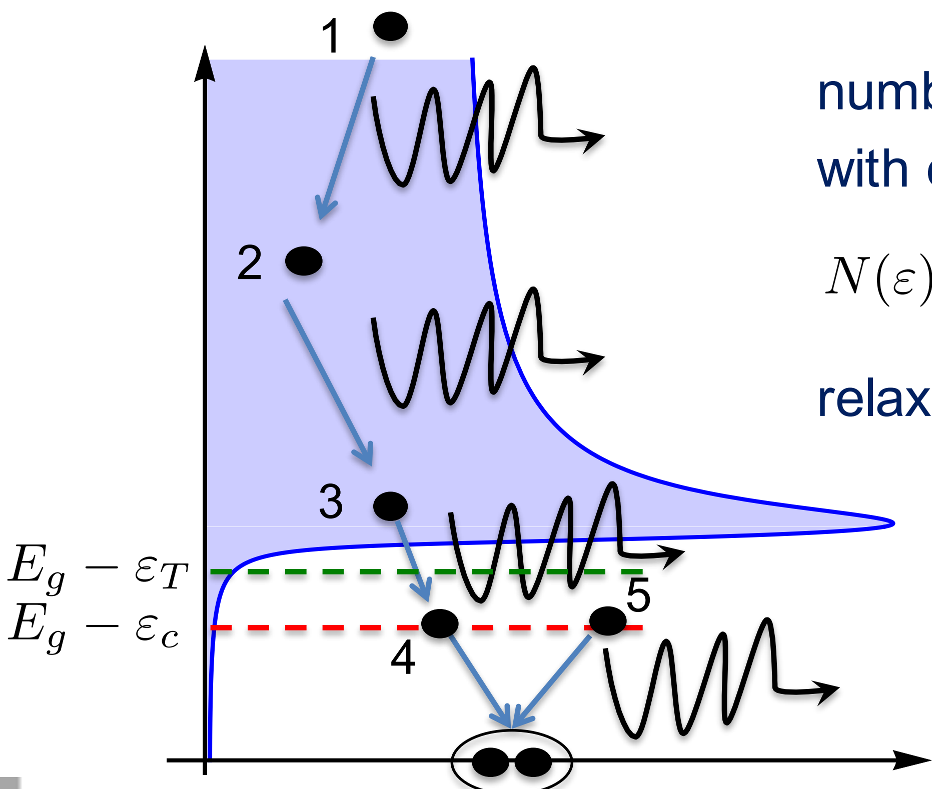
typical fluctuation $\epsilon = E_g - E$:

$\varepsilon^2 \sim (\delta\Delta)^2/L^3(\varepsilon)$ with $L(\varepsilon) \sim \xi(\Delta/\varepsilon)^{1/4}$

$\rightarrow$ characteristic energy scale $\epsilon_T \sim [(\delta\Delta)^2/(\Delta^2\xi^3)]^{4/5}\Delta \sim \Delta/g^{8/5}$

Bottleneck for relaxation

fast energy relaxation by emitting phonons does not stop at Δ: quasiparticles relax into the localized tail states



number of overlapping states with energy lower than ε:

$$N(\varepsilon) = L^3(\varepsilon) \int_{\varepsilon}^{\infty} d\varepsilon' \nu(\varepsilon')$$

relaxation stops when

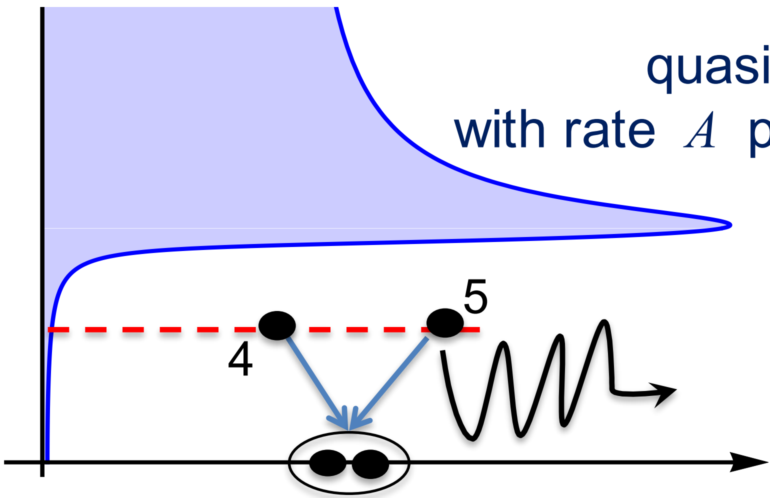
no more overlapping states with lower energy are available

$N(\varepsilon_c) \sim 1$

at ε_c » $\varepsilon_T (\ln g)^{4/5}$ À ε_T

localization radius at ε_c: r_c » few ξ

Generation/recombination model for localized states



quasiparticles are generated at random points with rate A per unit volume

- they keep their positions
- they annihilate pairwise with the rate

$$\Gamma(\mathbf{R}) = \bar{\Gamma} \int d\mathbf{r}\, p_c(\mathbf{r}) p_c(\mathbf{r}+\mathbf{R}) \propto \frac{\bar{\Gamma}}{r_c^3} e^{-R/r_c}$$

most probable shape of the localized state at energy ε_c

balance between generation and annihilation
$\rightarrow$ typical distance r between quasiparticles:

- dense limit r ¿ r_c: $A = \bar{\Gamma} c^2 \implies c = \sqrt{A/\bar{\Gamma}}$
- dilute limit r À r_c: $A r^3 \sim \frac{\bar{\Gamma}}{r_c^3} e^{-r/r_c} \implies c \propto \frac{1}{r_c^3 \ln^3\left(\frac{\bar{\Gamma}}{A r_c^6}\right)}$

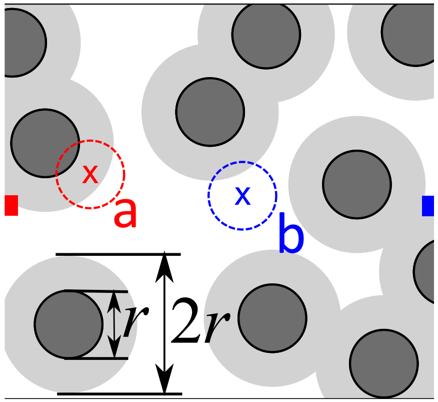
Simplified model: Bursting bubbles

characteristic length scale $\frac{r}{r_c} \approx \ln\left(\frac{\bar{\Gamma}}{A r_c^6}\right)$

annihilation rate varies very quickly with distance d_{ij} between two quasiparticles:

- rapid annihilation, if $d_{ij} < r$
- slow annihilation, if $d_{ij} > r$

→ describe quasiparticles as bubbles with radius $r/2$ that cannot overlap



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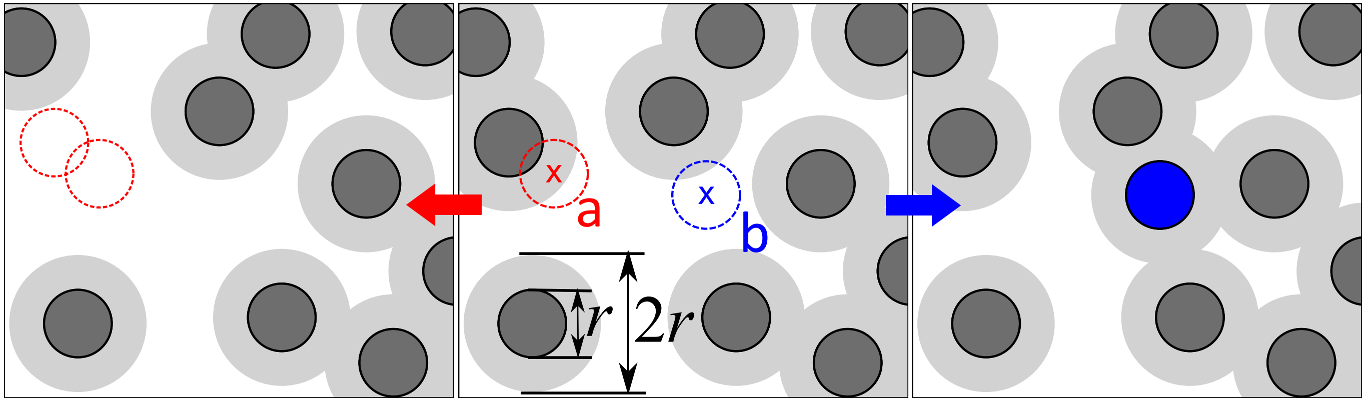


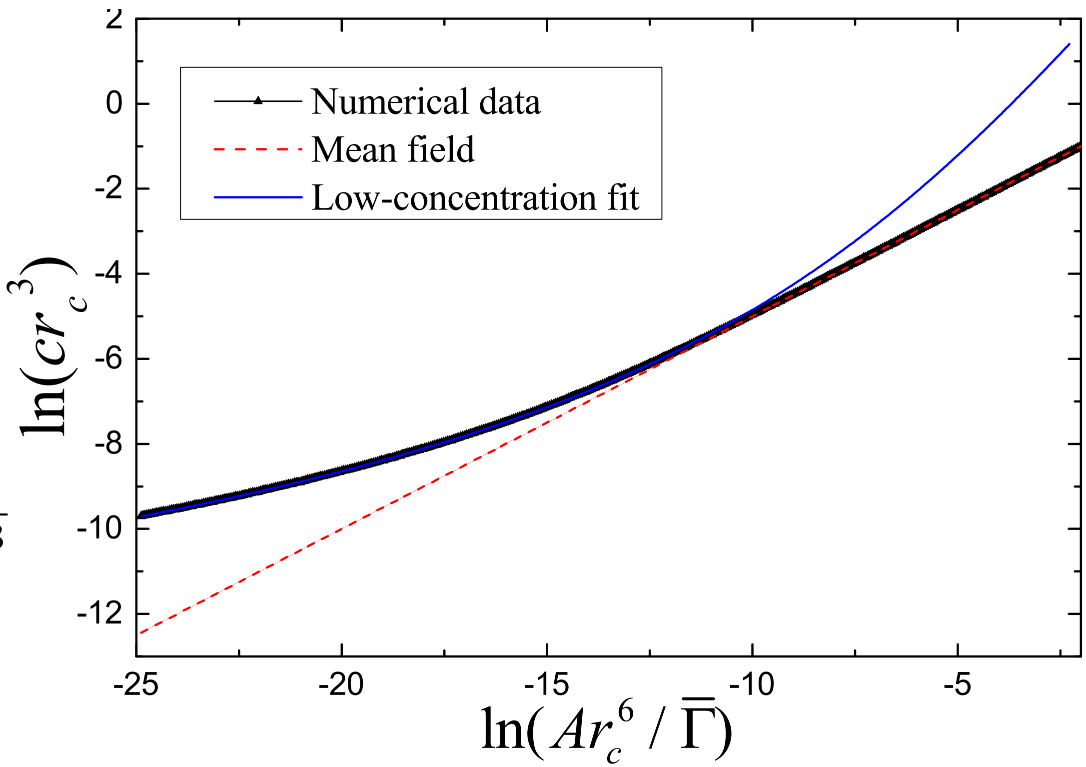
quasiparticle concentration: $c = \frac{C_p}{(4\pi/3)r^3}$

with packing coefficient C_p

- naïve estimate: $C_p = 0.5$
- simulation: $C_p \approx 0.605 \pm 0.008$

Full dynamical simulation

use annihilation rate $\Gamma(\mathbf{R}) = \bar{\Gamma} \int d\mathbf{r}\, p_c(\mathbf{r}) p_c(\mathbf{r}+\mathbf{R})$



$$cr_c^3 = \sqrt{Ar_c^6/\bar{\Gamma}}$$

dense limit

$$c = \frac{C_p}{(4\pi/3)r^3}$$

dilute limit

with improved estimate for r:

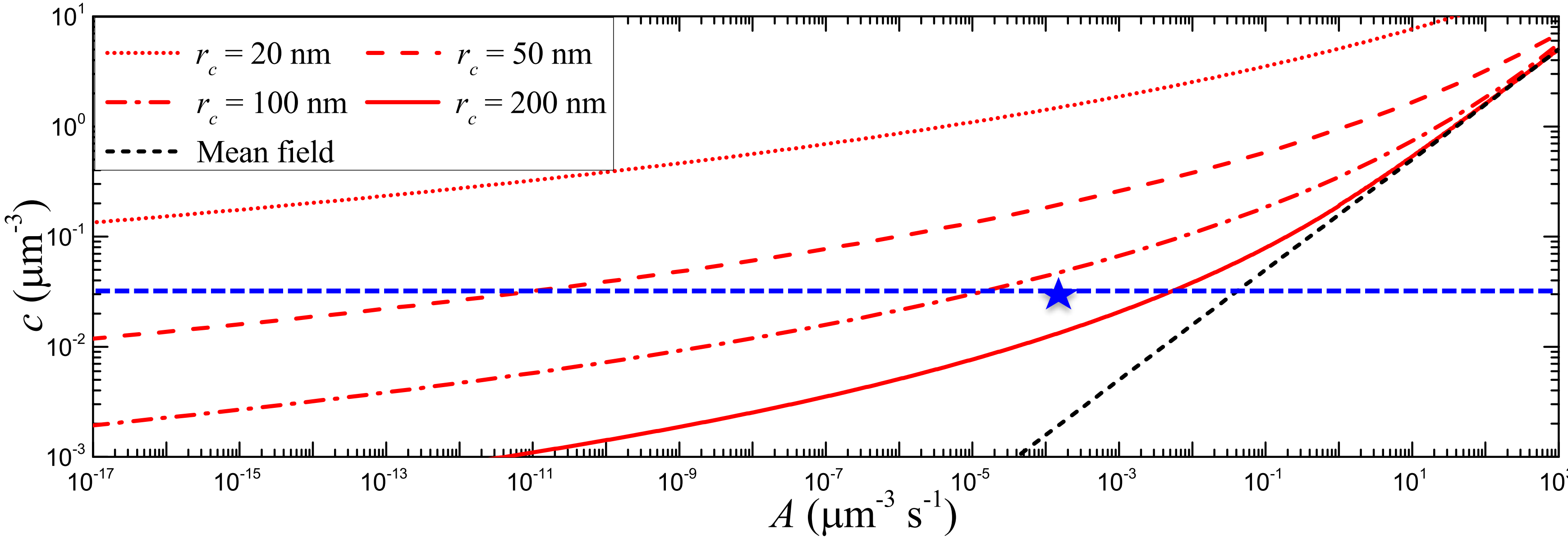
$$Ar^3 = \frac{\bar{\Gamma}}{r_c^3} b \left(\frac{r}{r_c}\right)^{\beta} e^{-r/r_c}$$

where b=0.008 and β=0.41 from fit

Results for Aluminium

$\bar{\Gamma} = 40\,\mathrm{s}^{-1}\mu\mathrm{m}^3$

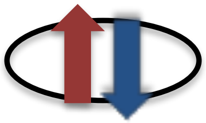
different values for r_c correspond to different disorder strengths:



cosmic radiation at sea level dominated by muons with:

- mean energy in the GeV range
- flux of 1 muon/cm^2/min $\rightarrow A \sim 10^{-4}\mathrm{s}^{-1}\mu\mathrm{m}^{-1}$
- stopping power in Al of » 1 MeV/cm

Spin?

Cooper pair: 

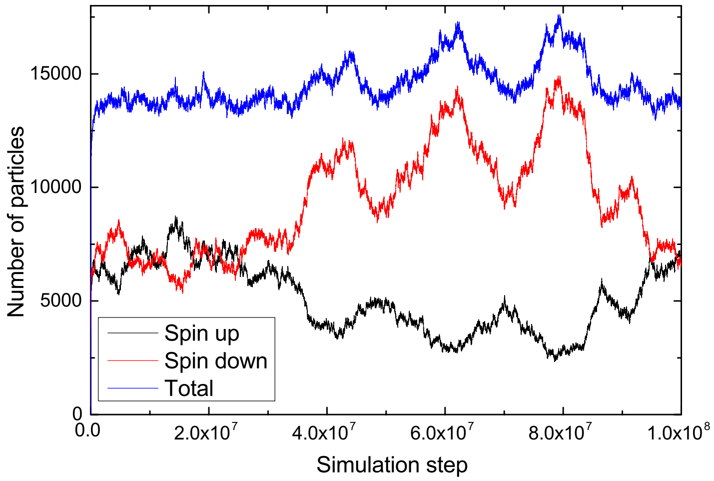
→ 2 quasiparticle can only annihilate, if they are in a singlet state

so far: bursting bubbles model with classical spin

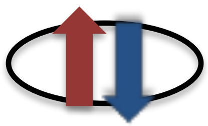
- without spin-flip:

$$C_p \approx 2.19 \pm 0.05$$

but large fluctuations …



Spin?

Cooper pair: 

$\rightarrow$ 2 quasiparticle can only annihilate, if they are in a singlet state

so far: bursting bubbles model with classical spin

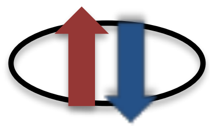
- without spin-flip:
  - probability to find no spin-up/spin-down quasiparticle in a sphere with radius r: $p_{\uparrow/\downarrow} = e^{-4\pi r^3 c_{\uparrow/\downarrow}/3}$
  - in equilibrium: $p_\uparrow p_\downarrow = (1 - p_\uparrow)(1 - p_\downarrow)$

$$\frac{2\pi}{3} c r^3 = \ln\left[2\cosh\left(\frac{2\pi}{3} P c r^3\right)\right]$$

with $c = c_\uparrow + c_\downarrow\,,\quad P = (c_\uparrow - c_\downarrow)/c$

) $C_p(P=0) = 2 \ln 2$ ¼ 1.39

Spin?

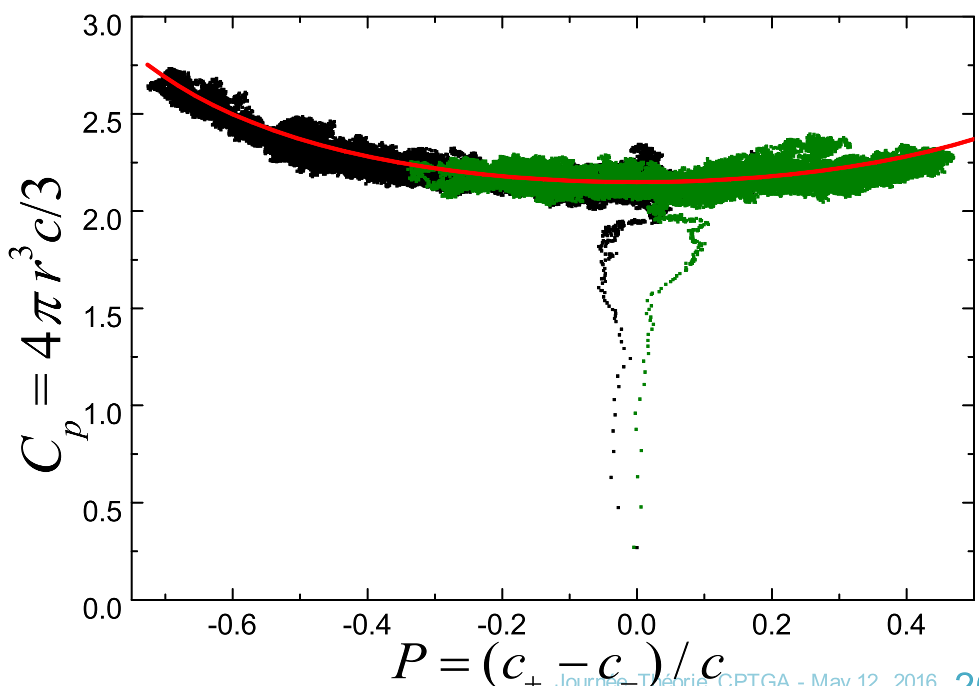
Cooper pair: 

→ 2 quasiparticle can only annihilate, if they are in a singlet state

so far: bursting bubbles model with classical spin

- without spin-flip:

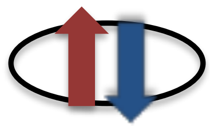
$C_p(P\!=\!0)$ increased
due to correlations
in quasiparticle positions



$$\frac{2\pi}{3}\frac{cr^3}{1.55} = \ln\left[2\cosh\left(\frac{2\pi}{3}P\frac{cr^3}{1.55}\right)\right]$$

fits the simulation

Spin?

Cooper pair: 

→ 2 quasiparticles can only annihilate, if they are in a singlet state

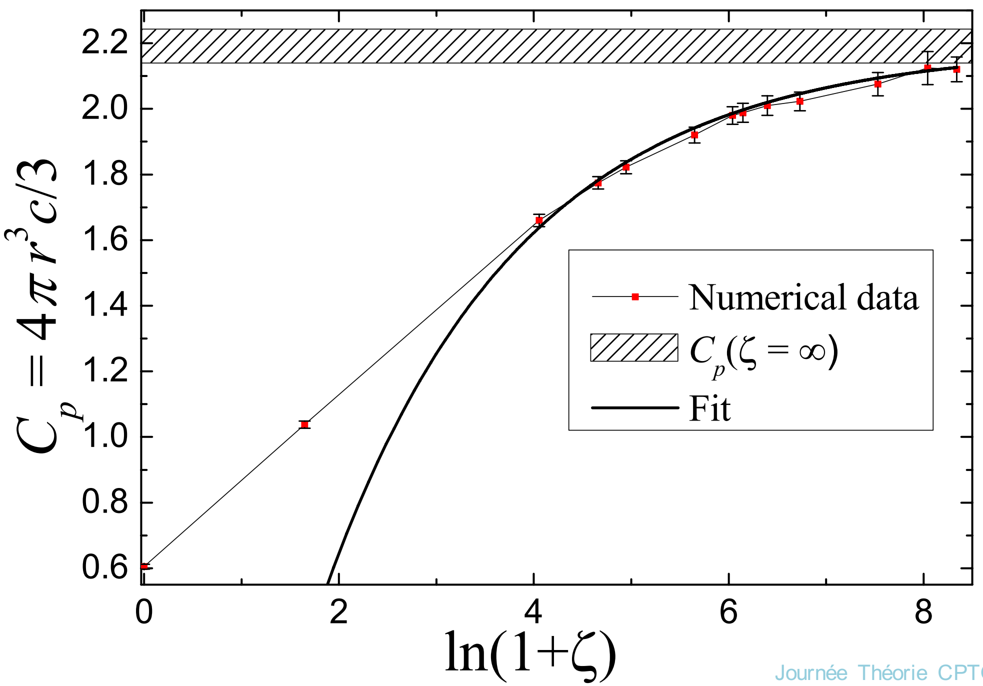
so far: bursting bubbles model with classical spin

- with spin-flip

  rate τ_{sf}:

$$\zeta = 4\pi A r^3 \tau_{\rm sf}/3$$

spinless result recovered in the limit $\zeta \to 0$



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Summary

extremely slow annihilation of quasiparticles trapped in localized states

→ large concentration of excess quasiparticles

in moderately disordered superconductors

- strategies to reduce their concentration
  - cleaner superconductors
  - shielding of the relevant non-equilibrium source
  - excite quasiparticles to delocalized states ?
- physical observables?
  - EM absorption …
  - role of large space and time fluctuations?

Ref: A. Bespalov *et al.*, arXiv:1603.04273

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THANK YOU!