$$[P_\mu, P_\nu] = 0 \ ,$$

$$\Delta P_0 = P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 \ , \quad \Delta P_j = P_j \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes P_j \ ,$$

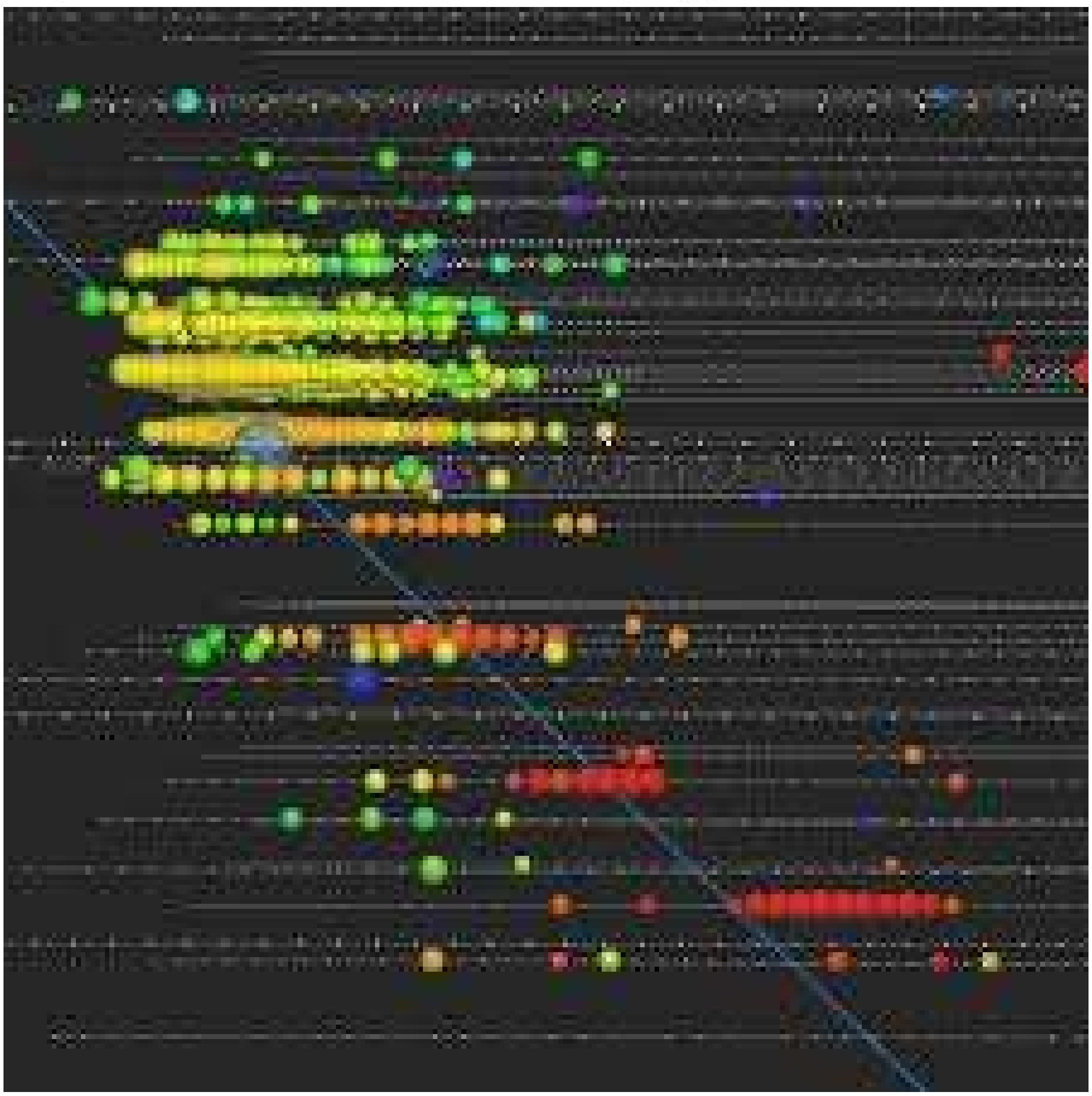
$$\varepsilon(P_\mu) = 0 \ , \qquad S(P_0) = -P_0 \ , \qquad S(P_j) = -e^{P_0/\kappa} P_j \ ,$$

$$[N_j, P_k] = i\,\delta_{jk} \left(\tfrac{\kappa}{2}(1 - e^{-2P_0/\kappa}) + \tfrac{1}{2\kappa}|\vec{P}|^2 \right) - \tfrac{i}{\kappa} P_j P_k \ ,$$

$$[N_j, P_0] = i\,P_j \ , \quad [R_j, P_k] = i\,\epsilon_{jkl} P_l$$

$$\Delta N_k = N_k \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes N_k + \tfrac{i}{\kappa}\epsilon_{klm} P_l \otimes R_m \ , \quad \Delta R_j = R_j \otimes \mathbb{1} + \mathbb{1} \otimes R_j \ ,$$

$$\varepsilon(N_j) = 0 \ , \quad \varepsilon(R_k) = 0 \ , \quad S(N_j) = -e^{P_0/\kappa} N_j + \tfrac{i}{\kappa}\epsilon_{jkl} e^{\lambda P_0} P_k R_l \ , \quad S(R_k) = -R_k$$



Paris 29.11.2017

$$E_{QG} \sim E_{Planck} = 1.2 \cdot 10^{19}\,\mathrm{GeV} = \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}}$$

i.e. 10^{-35}meters ("Planck length")

mainly comes from observing that at the Planck scale

$$\lambda_C \sim \lambda_S$$

$$\lambda_c \equiv \frac{1}{M}$$

$$\lambda_s \equiv G_N M$$

Note that it is only rough order-of-magnitude estimate at best
in particular this estimate assumes that G does not run at all!!!!!!!!!
it most likely does run!!!

$$\frac{1}{M} = G_N M \Rightarrow M = E_p$$

$$\frac{1}{M} = G_N(M)\, M \Rightarrow M = ?$$

Planck length as the minimum allowed value for wavelengths:

- **suggested by several indirect arguments combining quantum mechanics and GR**
- **found in some detailed analyses of formalisms in use in the study of the QG problem**

But the minimum wavelength is the Planck length for which observer?

GAC, ModPhysLettA (1994)
PhysLettB (1996)

Other results from the 1990s (mainly from spacetime noncommutativity and LoopQG) provided "theoretical evidence" of Planck-scale modifications of the on-shell relation, in turn inviting us to scrutinize the fate of relativistic symmetries at the Planck scale

Toward the mid 1990s these observations led several researchers to work at the hypothesis that in order to address the quantum-gravity problem one should give up the relativity of observers (preferred-frame picture)

GAC+Ellis+Nanopoulos+Sarkar, Nature(1998)
Alfaro+Tecotl+Urrutia,PhysRevLett(1999)
Gambini+Pullin, PhysRevD(1999)
Schaefer,PhysRevLett(1999)

This would be "Planck-scale broken Lorentz symmetry"

[notice difference with SME (talk by Stecker): here looking for specific scenarios of Planck-scale Lorentz-symmetry breaking, in SME most general scenario of Lorentz-symmetry breaking is considered]

but from 2000 onwards together with broken Lorentz symmetry
there starts to be a literature on the possibility
of "Planck-scale deformations of Lorentz symmetry"
[jargon: "DSR", for "doubly-special", or "deformed-special", relativity]

GAC, grqc0012051, IntJournModPhysD11,35
hepth0012238,PhysLettB510,255
KowalskiGlikman,hepth0102098,PhysLettA286,391
Magueijo+Smolin,hepth0112090,PhysRevLett88,190403
grqc0207085,PhysRevD67,044017
GAC,grqc0207049,Nature418,34

change the laws of transformation between observers so that the new properties are observer-independent

* a law of minimum wavelength can be turned into a DSR law
* could be used also for properties other than minimum wavelength, such as deformed on-shellness, deformed uncertainty relations…

The notion of DSR-relativistic theories is best discussed in analogy with the transition from Galileian Relativity to Special Relativity

analogy with Galilean-SR transition

introduction to DSR case is easier starting from reconsidering the Galilean-SR transition (the SR-DSR transition would be closely analogous)

Galilean Relativity

on-shell/dispersion relation $$E = \frac{p^2}{2m} \quad (+m)$$

linear composition of momenta $$p_\mu^{(1)} \oplus p_\mu^{(2)} = p_\mu^{(1)} + p_\mu^{(2)}$$

linear composition of velocities $$\vec{V} \oplus \vec{V}_0 = \vec{V} + \vec{V}_0$$

Special Relativity

special-relativistic law of composition of momenta is still linear

$$p_\mu^{(1)} \oplus p_\mu^{(2)} = p_\mu^{(1)} + p_\mu^{(2)}$$

but the on-shell/dispersion relation takes the new form

$$E = \sqrt{p^2 + m^2}$$

of course (since c is invariant of the new theory) the special-relativistic boosts act nonlinearly on velocities (whereas Galilean boosts acted linearly on velocities)

and the special-relativistic law of composition of velocities is nonlinear, noncommutative and nonassociative

$$\mathbf{w} = \mathbf{v} \oplus \mathbf{u}$$

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \frac{1}{1 + \frac{v_1 u_1 + v_2 u_2 + v_3 u_3}{c^2}} \left\{ \left[1 + \frac{1}{c^2} \frac{\gamma_{\mathbf{v}}}{1 + \gamma_{\mathbf{v}}} (v_1 u_1 + v_2 u_2 + v_3 u_3) \right] \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \frac{1}{\gamma_{\mathbf{v}}} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \right\}$$

much undervalued in most textbooks, which only give composition of parallel velocities:

$$\frac{v + u}{1 + (vu/c^2)}$$

from Special Relativity to DSR

If there was an observer-independent scale E_P (inverse of length scale ℓ) then, for example, one could have a modified on-shell relation as relativistic law

$$m^2 = \Lambda(E, p; E_P) = E^2 - p^2 - \frac{E}{E_P} p^2 + O\left(\frac{E^4}{E_P{}^2}\right)$$

For suitable choice of $\Lambda(E,p;E_P)$ one can easily have a maximum allowed value of momentum, i.e. minimum wavelength ($p_{max}=E_P$ for $\ell=-1/E_P$ in the formula here shown)

$$\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2$$

it turns out that such laws could still be relativistic, part of a relativistic theory where not only c ("speed of massless particles in the infrared limit") but also E_P would be a nontrivial relativistic invariant

action of boosts on momenta must of course be deformed so that

$$[N_k, \Lambda(E, p; E_P)] = 0$$

then it turns out to be necessary to correspondingly deform the law composition of momenta

$$p_\mu^{(1)} \oplus p_\mu^{(2)} \neq p_\mu^{(1)} + p_\mu^{(2)}$$

minimum wavelength from noncommutativity:
the kappaMINKOWSKI noncommutative spacetime

$$[x_j, t] = i\lambda x_j \qquad [x_j, x_m] = 0$$

Lukierski+Nowicki+Ruegg+Tolstoy,PLB(1991)
Nowicki+Sorace+Tarlini,PLB(1993)
Majid+Ruegg,PLB (1994)
Lukierski+Ruegg+Zakrzewski, AnnPhys(1995)

evidently not invariant under «classical translations»

$$[x_j + a_j, x_0 + a_0] = [x_j, x_0] = i\lambda x_j \neq i\lambda(x_j + a_j)$$

but adding commutative numbers to the noncommutative coordinates of kappa-Minkowski is evidently not a reasonable thing….
Adopting in particular noncommutative translation parameters such that

$$[\varepsilon_j, \varepsilon_0] = 0 \qquad [\varepsilon_j, x_\mu] = 0 \qquad [\varepsilon_j, x_k] = 0 \qquad [\varepsilon_j, x_0] = i\lambda\varepsilon_j$$

Sitarz, PhysLettB349(1995)42 Majid+Oeckl, math.QA/9811054

then

$$[x_j + \varepsilon_j, x_0 + \varepsilon_0] = [x_j, x_0] + [\varepsilon_j, x_0] = i\lambda(x_j + \varepsilon_j)$$

boosts must adapt to these deformed translations, resulting in deformed mass Casimir

deformed boosts are such that there is a maximum momentum (minimum wavelength)

minimum wavelength from discreteness:
the simple case of a one-dimensional polymer

$$X = X_0 + n\lambda \qquad n \in \mathbb{Z}$$

evidently, because of discreteness, translation transformations reflect the fact that

$$f(\mathbf{X}) \longrightarrow f(\mathbf{X}) + \mathbf{d}f(\mathbf{X})$$

with

$$\frac{f(X+\lambda)dX - f(X)dX}{\lambda} = df(X)$$

boosts must adapt to these deformed translations, resulting in deformed mass Casimir

deformed boosts are such that there is a maximum momentum (minimum wavelength)

It was recently realized that this sort of theoretical frameworks a la kappa-Minkowski (with DSR-deformed relativistic laws) may be connected to an old idea advocated by Max Born

one of the first papers on the quantum gravity problem was a paper by Max Born [*Proc.R.Soc.Lond.*A165,29(1938)] centered on the dual role within quantum mechanics between momenta and spacetime coordinates (Born reciprocity)

$$p_\mu \leftrightarrow x^\mu$$

Born argued that it might be impossible to unify gravity and quantum theory unless we make room for curvature of momentum space

this idea of curvature of momentum space had no influence on quantum-gravity research for several decades, but recently:

momentum space for certain models based on spacetime noncommutativity was shown to be curved

some "perspectives" on Loop Quantum Gravity have also advocated curvature of momentum space

and perhaps most importantly we now know that the only quantum gravity we actually can solve, which is 3D quantum gravity, definitely has curved momentum space

GAC+Matassa+Mercati+Rosati, PhysicalReviewLetters106,071301 (2011)
GAC+Freidel+KowalskiGlikman+Smolin, PhysRevD84,084010 (2011)
Carmona+Cortes+Mercati, PhysRevD84,084010 (2011)
GAC, PhysicalReviewLetters111,101301 (2013)

in 3D quantum gravity

see, e.g., Freidel+Livine, PhysRevLett96,221301(2006)

consider a matter field ϕ coupled to gravity,

$$Z = \int Dg \int D\phi \, e^{iS[\phi,g]+iS_{GR}[g]}, \qquad (1)$$

where g is the space-time metric, $S_{GR}[g]$ the Einstein gravity action and $S[\phi, g]$ the action defining the dynamics of ϕ in the metric g.

integrate out the quantum gravity fluctuations and derive an *effective action* for ϕ taking into account the quantum gravity correction:

$$Z = \int D\phi e^{iS_{eff}[\phi]}.$$

the effective action obtained through this constructive procedure gives matter fields in a noncommutative spacetime (similar to, but not exactly given by, kappa-Minkowski) and with curved momentum space, as signalled in particular by the deformed on-shellness

(anti-deSitter momentum space)

$$\cos(E) - e^{\ell E} \frac{\sin(E)}{E} P^2 = \cos(m)$$

GAC+Freidel+KowalskiGlikman+Smolin, PhysRevD84,084010 (2011)

mass of a particle with four-momentum p_μ is determined by the metric geodesic distance on momentum space from p_μ to the origin of momentum space

$$m^2 = d_\ell^2(p,0) = \int dt \sqrt{g^{\mu\nu}(\gamma^{[A;p]}(t))\dot{\gamma}_\mu^{[A;p]}(t)\dot{\gamma}_\nu^{[A;p]}(t)}$$

where $\gamma^{[A;p]}{}_\mu$ is the metric geodesic connecting the point p_μ to the origin of momentum space

$$\frac{d^2\gamma_\lambda^{[A]}(t)}{dt^2} + A^{\mu\nu}{}_\lambda \frac{d\gamma_\mu^{[A]}(t)}{dt}\frac{d\gamma_\nu^{[A]}(t)}{dt} = 0$$ **with $A^{\mu\nu}{}_\lambda$ the Levi-Civita connection**

the affine connection on momentum space determines the law of composition of momenta, and it might not be the Levi-Civita connection of the metric on momentum space (it is not in 3D quantum gravity and in all cases based on noncommutative geometry, where momentum space is a group manifold)

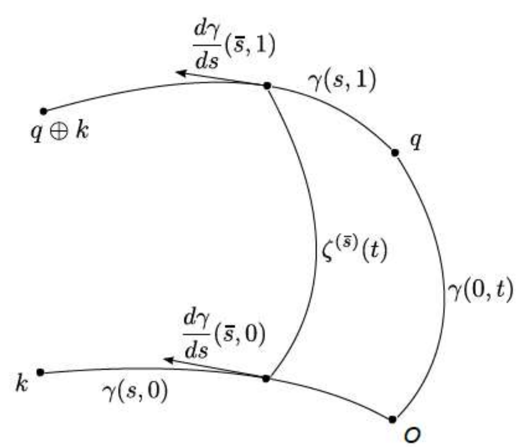


Figure 1. We determine the law of composition of momenta from the affine connection by associating to the points q and k of momentum space the connection geodesics $\gamma^{(q)}$ and $\gamma^{(k)}$ which connect them to the origin of momentum space. We then introduce a third curve $\bar{\gamma}(s)$, which we call the parallel transport of $\gamma^{(k)}(s)$ along $\gamma^{(q)}(t)$, such that for any given value $\bar{s}$ of the parameter s one has that the tangent vector $\frac{d}{ds}\bar{\gamma}(\bar{s})$ is the parallel transport of the tangent vector $\frac{d}{ds}\gamma^{(k)}(\bar{s})$ along the geodesic connecting $\gamma^{(k)}(\bar{s})$ to $\bar{\gamma}(\bar{s})$. Then the composition law is defined as the extremal point of $\bar{\gamma}$, that is $q \oplus_\ell k = \bar{\gamma}(1)$.

This could have been just a futile "geometric interpretation" but it is proving useful

It establishes valuable similarities between different theories.

In particular theories with curved momentum spaces can still be relativistic, but this requires that momentum space is <u>maximally symmetric</u> (dS/anti-dS cases discussed above)

GAC,arXiv:11105081, PhysRevD85,084034

and the relativistic symmetries are a "deformat of ordinary special-relativistic symmetries, examples of the above-mentioned <u>DSR-relativistic theories</u>

GAC, grqc0012051, IntJournModPhysD11,35
GAC, hepth0012238,PhysLettB510,255
KowalskiGlikman,hepth0102098,PhysLettA286,391
Magueijo+Smolin,hepth0112090,PhysRevLett88,190403
Magueijo+Smolin,grqc0207085,PhysRevD67,044017
GAC,grqc0207049,Nature418,34

…and is proving valuable for phenomenology.

Much studied opportunity for phenomenology comes from fact that several pictures of quantum spacetime predict that the speed of photons is energy dependent.

Calculation of the energy dependence in a given model used to be lengthy and cumbersome. We now understand those results as dual redshift on Planck-scale-curved momentum spaces:

these results so far are fully understood for the case of
[maximally symmetric curved momentum space] ⊗ [flat spacetime]

it turns out that there is a duality between this and the familiar case of
[maximally-symmetric curved spacetime] ⊗ [flat momentum space]

In particular,
ordinary redshift in deSitter spacetime implies in particular that massless particles emitted with same energy but at different times from a distant source reach the detector with different energy

dual redshift in deSitter momentum space implies that massless particles emitted simultaneously but with different energies from a distant source reach the detector at different times

GAC+Barcaroli+Gubitosi+Loret,
Classical&QuantumGravity30,235002 (2013)
GAC+Matassa+Mercati+Rosati,
PhysicalReviewLetters106,071301 (2011)

dual redshift on Planck-scale-curved momentum spaces (but with flat spacetime) produces time-of-arrival effects which at leading order are of the form ($n \in \{1,2\}$)

$$\Delta T = \left(\frac{E}{E_P} \right)^n T$$

and could be described in terms of an energy-dependent "physical velocity" of ultrarelativistic particles

$$\mathrm{v} = c + s_{\pm} \left(\frac{E}{E_P} \right)^n c$$

<u>these are very small effects but (at least for the case n=1) they could cumulate to an observably large ΔT if the distances travelled T are cosmological and the energies E are reasonably high (GeV and higher)!!!</u>
GRBs are ideally suited for testing this:
cosmological distances (established in 1997)
photons (and neutrinos) emitted nearly simultaneously
with rather high energies (GeV…..TeV…100 TeV…)

GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature393,763(1998)
GAC, NaturePhysics10,254(2014)

large variety of phenomenological models

*** quantum-gravity scale could be bigger or smaller than E_{planck}**

*** can be brokenSR or deformedSR**

- **notice that no quantum-spacetime picture has been shown rigorously to lead to brokenSR**
- **notice that threshold anomalies (e.g. anomalous transparency… $\gamma\gamma \rightarrow e^+e^-$) are only possible with brokenSR (protected by a theorem in any deformedSR scenario, GAC,PhysRevD85,084034)**
- **for time-of-flight analyses techniques borrowed from propagation of light in media might not apply to deformedSR**

***the redshift dependence may be different from the Jacob-Piran ansatz**

***the effects can be spin/helicity/polarization dependent**

***the effects can be particle-type dependent (different for photons and neutrinos)**

***the effects should be fuzzy but theory work at present only provides essentially the deformation of the lightcone, without being able to establish the fuzziness of the deformed lightcone**