# Asymptotic Safety Beyond the Standard Model

**Kamila Kowalska**

TU Dortmund

based on work with A.Bond, G.Hiller and D.Litim

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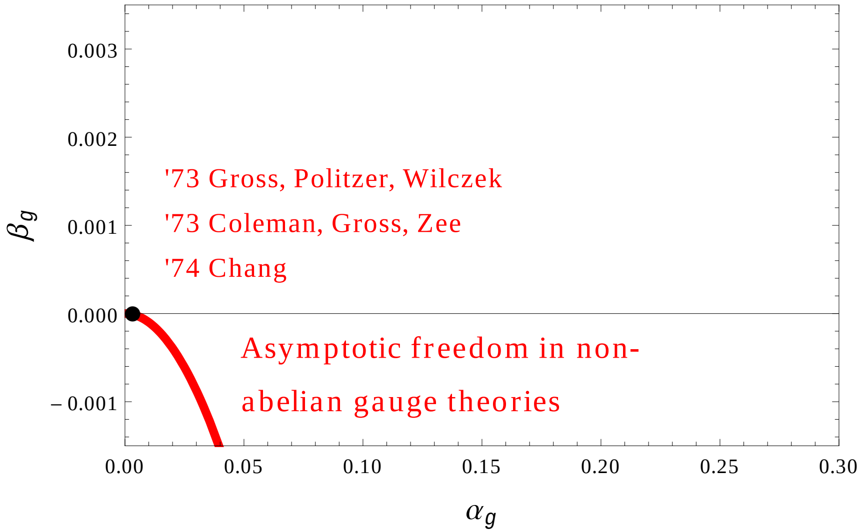
# Overview

# A brief history of asymptotic safety

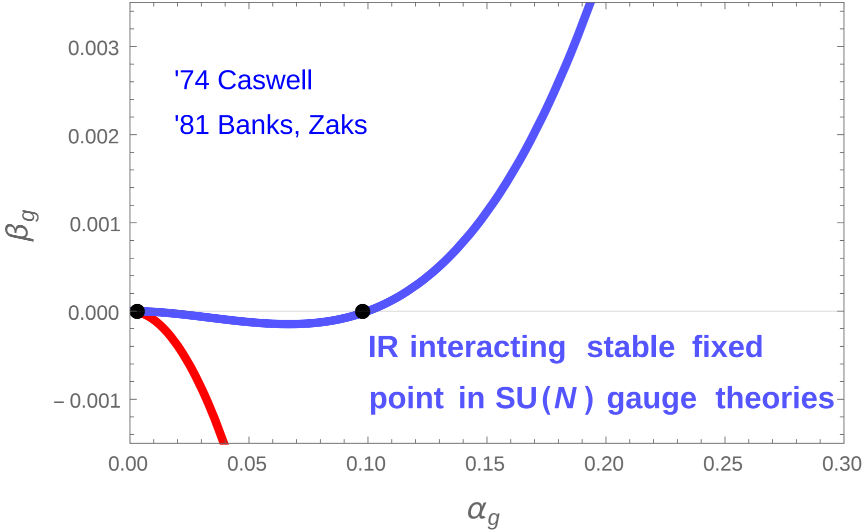
1971, Wilson: UV fixed points are central for Quantum Field Theory to be fundamental

$$\beta_i \equiv \mu \frac{d\alpha_i}{d\mu}\Big|_{\alpha_i^*} = 0$$

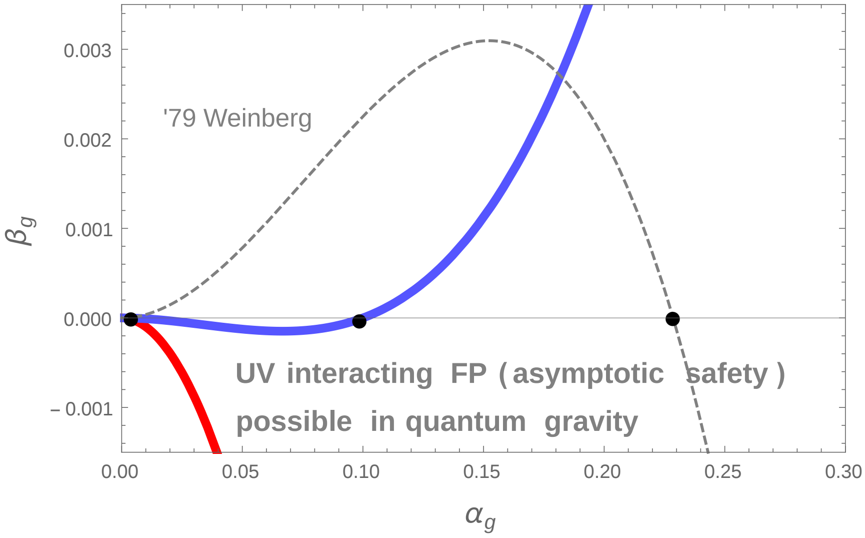
# A brief history of asymptotic safety



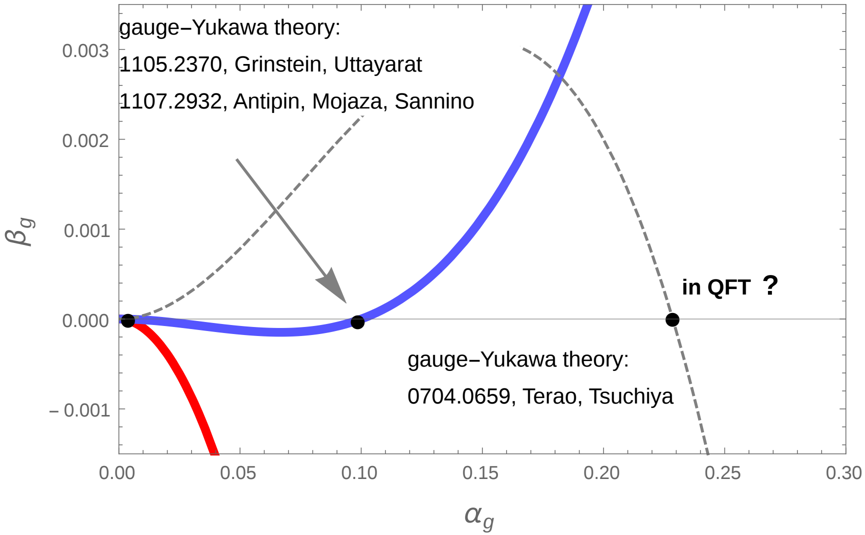
# A brief history of asymptotic safety



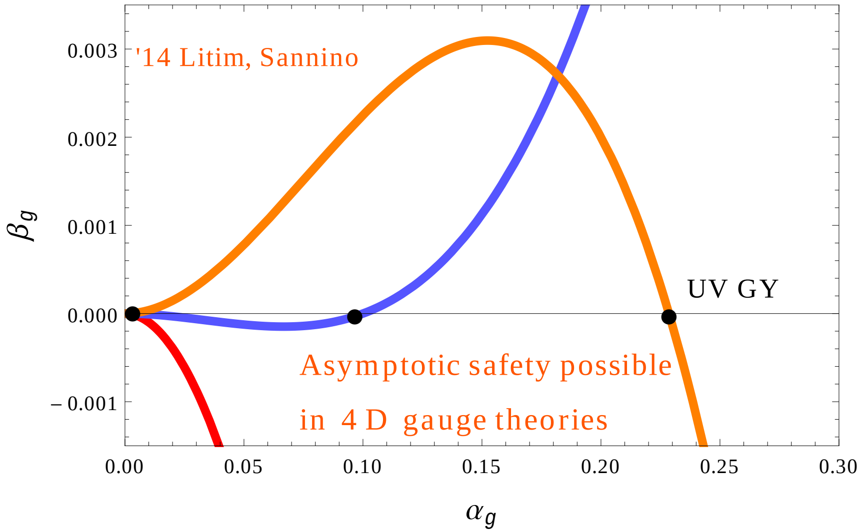
# A brief history of asymptotic safety



# A brief history of asymptotic safety



# A brief history of asymptotic safety



# Asymptotic Safety in Gauge-Yukawa theory

**Gauge-Yukawa model:**

$$\mathcal{L} \sim \mathrm{Tr}\left(\bar{\psi} i\gamma^\mu D_\mu \psi\right) + \mathrm{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right) - \frac{1}{2}\mathrm{Tr} F^{\mu\nu} F_{\mu\nu}$$
$$- \; y(\bar{\psi}_{Li} S_{ij} \psi_{Rj} + \bar{\psi}_{Ri} S^\dagger_{ij} \psi_{Lj}) - \lambda_u \mathrm{Tr}\left[(S^\dagger S)^2\right] - \lambda_v \left[\mathrm{Tr}(S^\dagger S)\right]^2$$

- $N_f$ flavors of $\psi_i$ ($i = 1, ..., N_f$)
- $\psi_i$ in fundamental of $SU(N_c)$
- $\psi = \psi_L + \psi_R$
- S is a $N_f \times N_f$ complex matrix

# Asymptotic Safety in Gauge-Yukawa theory

The set of RGEs for gauge ($SU(N_c)$) and Yukawa couplings:

$$\beta_g = \frac{d\alpha_g}{d\ln\mu} = \alpha_g^2(-B + C\alpha_g - D\alpha_y),$$
$$\beta_y = \frac{d\alpha_y}{d\ln\mu} = \alpha_y(E\alpha_y - F\alpha_g)$$

(where $\alpha_g = \frac{g^2}{(4\pi)^2}$, $\alpha_y = \frac{y^2}{(4\pi)^2}$)

- $B > 0$ (asymptotic freedom) or $B < 0$ (asymptotic safety)
- $C > 0$ if $B < 0$ in any QFT (proof in *Bond, Litim, arXiv:1608.00519)*
- $D, E, F > 0$ for any quantum field theory

# Asymptotic Safety in Gauge-Yukawa theory

Three types of fixed points possible:

- $(\alpha_g^*, \alpha_y^*) = (0, 0)$

  Gaussian fixed point, always exists, UV $(B > 0)$ or IR $(B < 0)$.

- $(\alpha_g^*, \alpha_y^*) = (B/C, 0)$

  Caswell-Banks-Zaks fixed point, ALWAYS IR fixed point.

  Interacting UV fixed point ONLY with Yukawas.

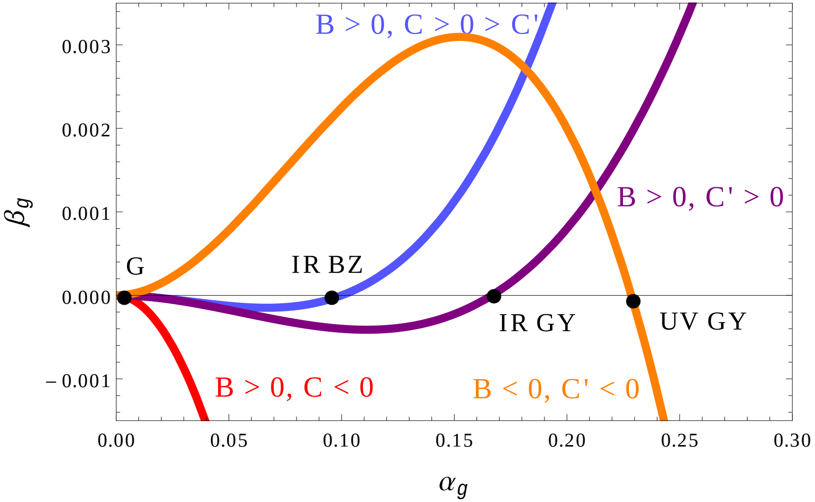
- $(\alpha_g^*, \alpha_y^*) = \left(\frac{B}{C'}, \frac{BF}{C'E}\right)$, where $C' = C - D\frac{F}{E}$

  Fully interacting gauge-Yukawa fixed point:

  - IR $(B > 0$ and $C^{'} > 0)$
  - UV $(B < 0$ and $C^{'} < 0)$

  Asymptotic Safety: $B < 0$ and $CE - DF < 0$.

# Asymptotic Safety in Gauge-Yukawa theory



# Asymptotic Safety in Gauge-Yukawa theory

## Critical exponents:

linearization of the RG flow in vicinity of the FP:

$$\beta_i = \sum_j M_{ij}(\alpha_j - \alpha_j^*) + \mathcal{O}(\alpha_j^2)$$

where stability matrix is defined as $M_{ij} = \partial\beta_i/\partial\alpha_j|_*$.

Properties of the FP (scaling of the couplings near the FP) determined by eigenvalues $\lambda_k$ of $M$:

- $Re(\lambda_k) > 0$ irrelevant direction $\rightarrow \delta\alpha_i \sim \mu^{\lambda_k}$ increasing with $\mu$
- $Re(\lambda_k) < 0$ relevant direction $\rightarrow \delta\alpha_i \sim \mu^{\lambda_k}$ decreasing with $\mu$
- $Re(\lambda_k) = 0$ marginal direction $\rightarrow \delta\alpha_i \sim \log(\mu)$

# Asymptotic Safety in Gauge-Yukawa theory

## Critical surface:

In the vicinity of the UV fixed point:

$$\alpha_g(\mu) = \alpha_g^* + \sum_n c_n V_g^n \left(\frac{\mu}{\Lambda}\right)^{\lambda_n}$$

$$\alpha_y(\mu) = \alpha_y^* + \sum_n c_n V_y^n \left(\frac{\mu}{\Lambda}\right)^{\lambda_n}$$

and $V_i$ - eigenvectors of $M$.

So for the relevant eigendirection one gets:

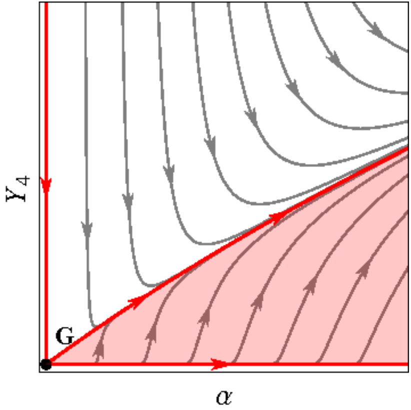
$$\alpha_y(\alpha_g) = \alpha_y^* + (V_y^1)^{-1}(\alpha_g - \alpha_g^*)$$

The UV fixed point can be reached only along a critical direction

# Phase diagrams

## Asymptotic freedom ($B > 0$ and $C < 0$):

Bond, Litim, arXiv:1608.00519

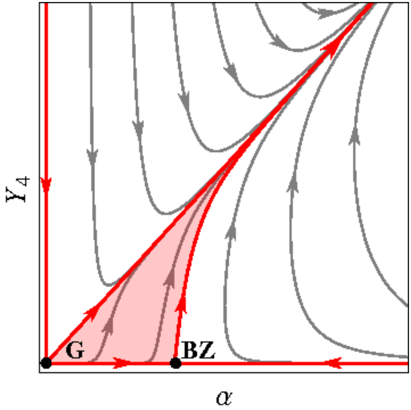



In the vicinity of the FP the running logarithmically slow.

# Phase diagrams

**Asymptotic freedom + Banks-Zaks FP ($B > 0$ and $C > 0 > C'$):**

Bond, Litim, arXiv:1608.00519

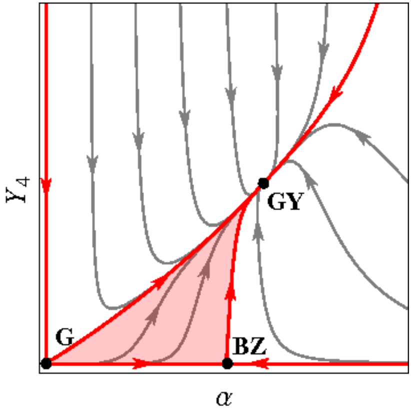



$\lambda_1 = -\frac{BF}{C}$, $\lambda_2 = \frac{B^2}{C}$ $\rightarrow$ always one relevant and one irrelevant direction

# Phase diagrams

**AF + BZ + gauge-Yuakwa FP ($B > 0$ and $C > C' > 0$):**

Bond, Litim, arXiv:1608.00519

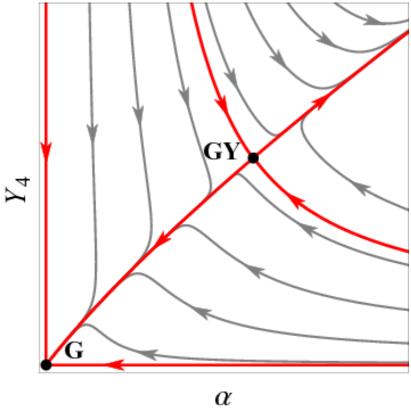



$\lambda_1 = \frac{B^2}{C'}$, $\lambda_2 = \frac{BF}{C'}$ $\rightarrow$ two relevant directions for the IR GY

# Phase diagrams

## Asymptotic safety ($B < 0$ and $C' < 0$):

Bond, Litim, arXiv:1608.00519




$\lambda_1 = \frac{B^2}{C'}$, $\lambda_2 = \frac{BF}{C'}$ $\rightarrow$ one relevant and one irrelevant direction

NO Landau Poles!

# An explicit example

Litim, Sannino, JHEP 1412, 178 (2014) (arXiv:1406.2337)

$$B = \frac{22}{3} N_c - \frac{4}{3} N_f, \quad C = -\frac{68}{3} N_c^2 + 2 N_f \left( \frac{N_c^2 - 1}{N_c} + \frac{10}{3} N_c \right), \quad D = 2 N_f^2,$$

$$E = 2\left(N_f + N_c\right), \quad F = 6 \frac{N_c^2 - 1}{N_c}.$$

How to make $C^{'} < 0$?

$$C^{'} = C - D\frac{F}{E} = -\frac{68}{3} N_c^2 + \frac{2}{3} N_f \left[ \frac{13 N_c^2 - 3}{N_c} - 9 \frac{N_f}{N_f + N_c} \frac{N_c^2 - 1}{N_c} \right]$$

Particular flavour sector essential to make $C^{'} < 0$ when $B < 0$.

# A challenge

**Asymptotically Safe Standard Model+**

# QCD

Let $\psi_i$ transform as $R_3$ of $SU(3)_c$

$$B = 14 - \frac{8n_f}{3} S_2(R_3)$$

Asymptotic freedom is lost $(B < 0)$;

$$n_f \geq \frac{21}{4S_2(R_3)}$$

| $R_3$ | $S_2(R_3)$ | $n_{AS}$ |
|---|---|---|
| **3** | 1/2 | 11 |
| **6** | 5/2 | 3 |
| **8** | 3 | 2 |
| **10** | 15/2 | 1 |
| **15** | 10 | - |
| **15′** | 35/2 | - |

Conclusion 1: large number of new degrees of freedom needed.

# QCD

Asymptotic safety and physicality ($CE - DF < 0$):

$$C = -52 + 4n_f S_2(R_3)(2C_2(R_3) + 10), \quad D = \frac{n_f^2}{2} C_2(R_3)\mathrm{dim}(R_3),$$

$$E = 2(n_f + \mathrm{dim}(R_3)), \quad F = 12C_2(R_3).$$

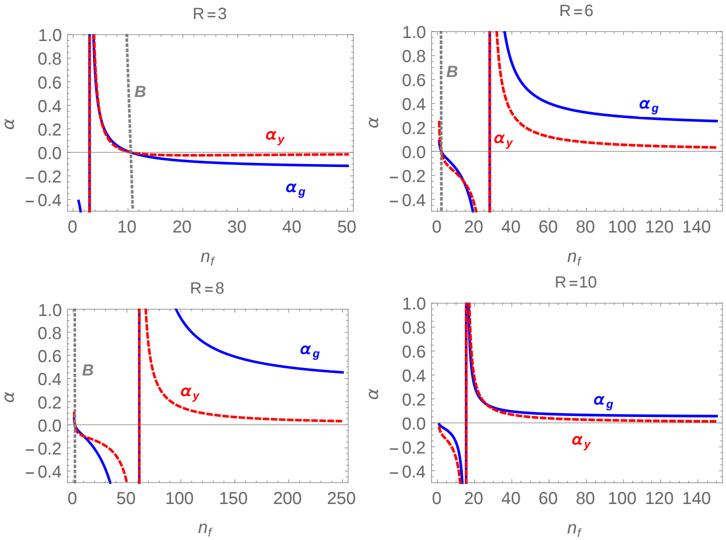
$$2n_f^2 C_2(R_3)\mathrm{dim}(R_3)(5 - 2C_2(R_3)) - 108\,(n_f + \mathrm{dim}(R_3)) < 0$$

| $R_3$ | $C_2(R_3)$ | $n_{AS}$ |
|---|---|---|
| **3** | 4/3 | - |
| **6** | 10/3 | 37 |
| **8** | 3 | 95 |
| **10** | 6 | 17 |
| **15** | 16/3 | 30 |
| **15′** | 28/3 | 17 |

Conclusion 2: $SU(3)$ reps. larger than **3** needed.

# QCD

Fixed point values vs. $n_f$ and $R_3$:



# QCD

## Matching onto SM

The UV fixed point $(\alpha_3^*, \alpha_y^*)$ has one relevant eigendirection $\rightarrow$ 1D critical surface given by $\alpha_y(\alpha_3)$.

$\alpha_3(\mu_X)$ is known $\rightarrow$ on the CS also $\alpha_y(\mu_X)$ is known

Conclusion 3: prediction for the value of $y$ (for arbitrary $\mu_X$).

# $SU(3)_c \times SU(2)_L$

Let $\psi_i$ transform as $R_3$ of $SU(3)_c$ and as $R_2$ of $SU(2)_L$.

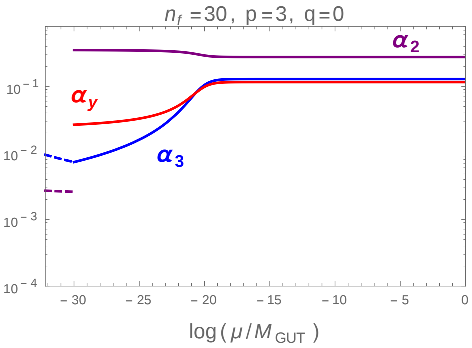
$$\frac{d\alpha_3}{d\ln\mu} = \alpha_3^2(-B_3 + C_3\alpha_3 + C_{32}\alpha_2 - D_3\alpha_y),$$
$$\frac{d\alpha_2}{d\ln\mu} = \alpha_2^2(-B_2 + C_2\alpha_2 + C_{23}\alpha_3 - D_2\alpha_y),$$
$$\frac{d\alpha_y}{d\ln\mu} = \alpha_y(E\alpha_y - F_3\alpha_3 - F_2\alpha_2).$$

| $\psi(\mathbf{R_3}, \mathbf{R_2}, \mathbf{0})$ | | | $R_2 = \mathbf{1}$ | | $R_2 = \mathbf{2}$ | | $R_2 = \mathbf{3}$ | |
|---|---|---|---|---|---|---|---|---|
| $R_3$ | $(p,q)$ | $S_2(R_3)$ | $N_{\rm AF}$ | $N_{\rm AS}$ | $N_{\rm AF}$ | $N_{\rm AS}$ | $N_{\rm AF}$ | $N_{\rm AS}$ |
| **3** | (1,0) | 1/2 | 11 | – | 6 | – | 4 | – |
| **6** | (2,0) | 5/2 | 3 | 37 | 1 | 77 | – | 116 |
| **8** | (1,1) | 3 | 2 | 95 | – | 198 | – | 299 |
| **10** | (3,0) | 15/2 | 1 | 17 | – | 34 | – | 51 |
| **15** | (2,1) | 10 | – | 30 | – | 60 | – | 90 |
| **15′** | (4,0) | 35/2 | – | 17 | – | 33 | – | 50 |

# $SU(3)_c \times SU(2)_L$: $\psi$ are $SU(2)$ singlets

- $\alpha_3^*, \alpha_2^*, \alpha_y^* \neq 0$

  one negative critical exponent $\rightarrow$ 1D critical surface (2 couplings to match)

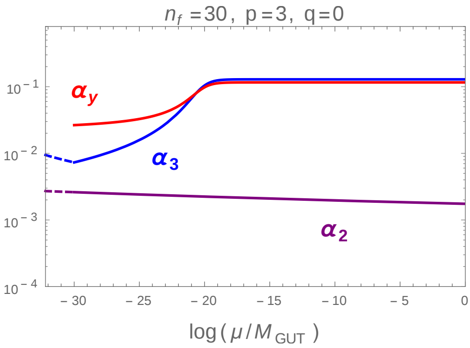



NO solution compatible with the SM.

# $SU(3)_c \times SU(2)_L$: $\psi$ are $SU(2)$ singlets

- $\alpha_3^*, \alpha_y^* \neq 0, \alpha_2^* = 0$

  one negative critical exponent and $\alpha_2(\mu) = 0$

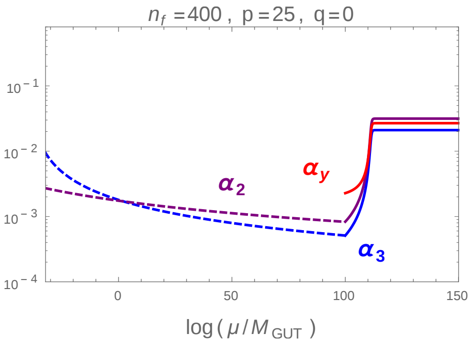



Offers good solutions for arbitrary representation and $n_f$.

# $SU(3)_c \times SU(2)_L$: $\psi$ are $SU(2)$ doublets

- $\alpha_3^*, \alpha_2^*, \alpha_y^* \neq 0$

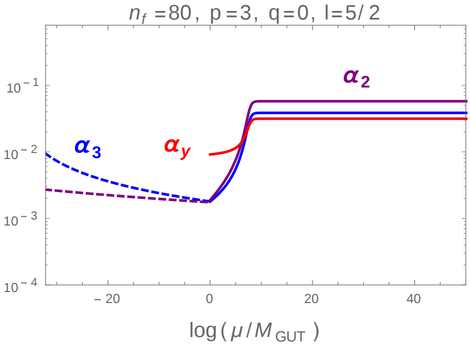
  one negative critical exponent $\to$ 1D critical surface (2 couplings to match)




Solutions for very large reps. and $n_f$, matching scale above $M_{GUT}$.

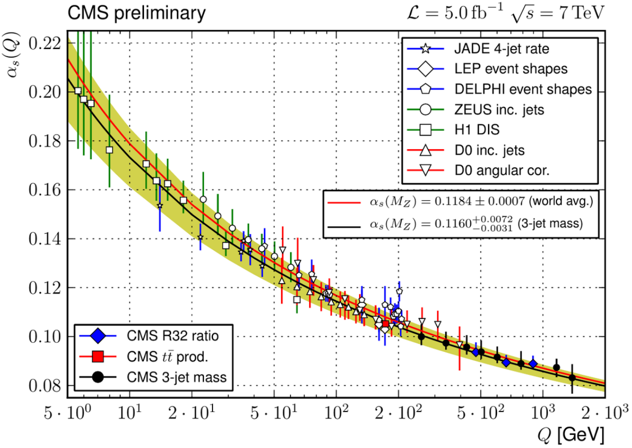
- $\alpha_3^*, \alpha_2^*, \alpha_y^* \neq 0$

  one negative critical exponent $\rightarrow$ 1D critical surface (2 couplings to match)




Solutions for moderate $n_f$, matching scale at $M_{GUT}$.

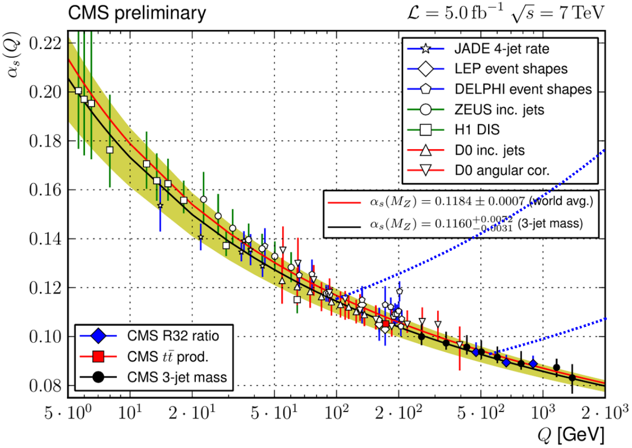
## Running of the SM gauge couplings



# Experimental signatures

## Running of the SM gauge couplings

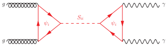
$SU(3)$ AS, $SU(2)$ AF, $R_3 = 10$, $n_f = 30$



# Experimental signatures

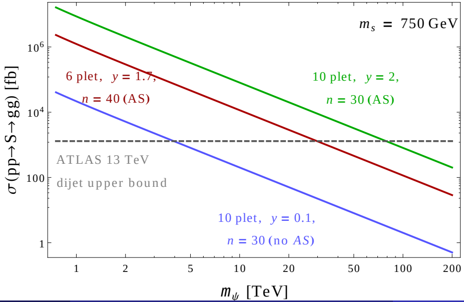
## Diboson spectra

if $m_S \leq m_\psi$ than loop decays $S \to gg, \gamma\gamma, ZZ, Z\gamma, WW$



Strong limits from the dijet search, ex. 13 TeV ATLAS $\sigma_{jj} \times A \times BR < 1.3\,\mathrm{pb}$

$$\Gamma_{gg} = \frac{\alpha_s^2 m_S^3}{32\pi^3} \left| \sum_{i=1}^{n_f} \frac{y S_2(R_3)}{M_\psi} A_{1/2}(x) \right|^2$$

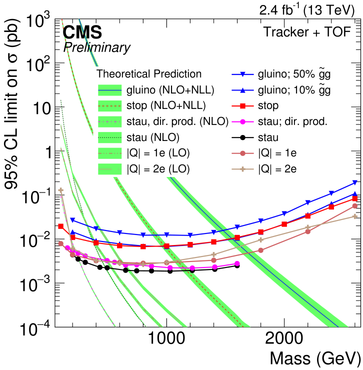


# Experimental signatures

## R-hadron searches

if $m_S \geq m_\psi$ and $Y_\psi = 0$ than $\psi$ can be stable $\rightarrow$ R-hadrons can be formed

$$\psi\bar{\psi},\ \psi_6 qq,\ \psi_8 q\bar{q},\ \psi_{10} qqq, \ldots.$$



# Next steps

- **Investigate perturbativity by going to higher order, or even non-perturbatively.**

  If $B/C^{'} \ll 1$ than the results are exact, but how small $\ll 1$?

- **Explicit inclusion of masses (bare ones, and through sym. breaking).**
  Mass term form $\psi$ breaks the chiral "flavor" symmetry
  $SU_L(n_f) \times SU_L(n_f)$.

- **Scalar potential, and portals, impact on Higgs.**

- **Role of hypercharge.**

# Summary

- Yukawa couplings offer the **ONLY** dynamical mechanism to obtain interacting fixed points in gauge theories.
- This mechanism is unique and universal.
- To make the SM asymptotically safe new fermions in reps. of $SU(3)$ higher then 3 are required.
- Matching with the SM possible if $SU(2)_L$ remains asymptotically free.
- Possible collider implications for (multi)-TeV scale matching.