Higgs particle signal at the LHC from Gauged Unparticles Model

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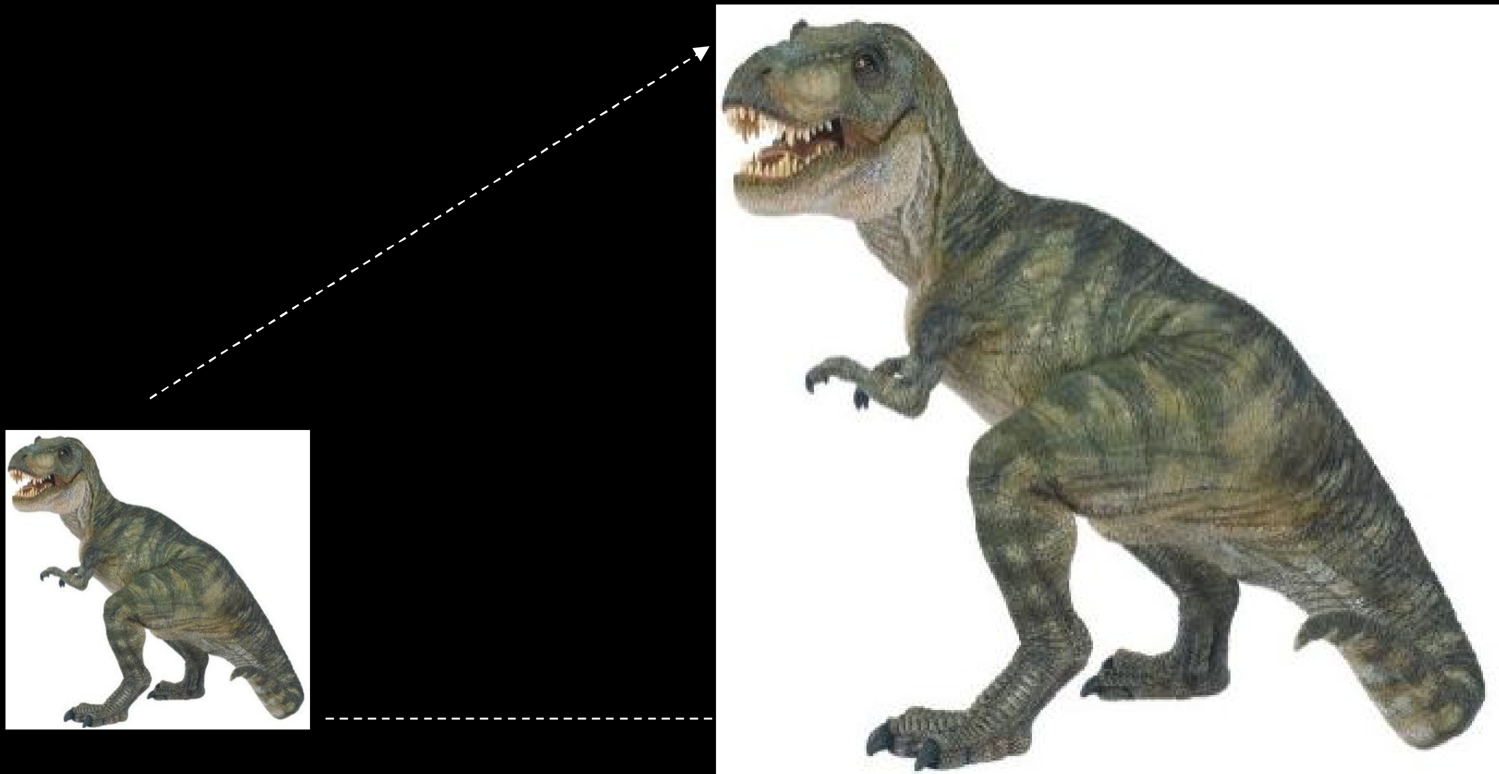
The idea of unparticles comes from the principle of scale invariance

The physics of a system remains the same regardless of a change of length (or equivalently energy).

Scale Transformation

$$\vec{x} \rightarrow \lambda \vec{x}$$



Scale and conformal transformations

S. Coleman et al have shown that under some conditions, a scale invariant theory is also comformally invariant

Conformal group: **Transformations that preserve the form of metric up to a factor.**

$$g_{\mu\nu}(x) \rightarrow \Omega^2(x) \times g_{\mu\nu}(x)$$

It preserve the angle between two 4 vectors

$$\frac{A^{\mu} B_{\mu}}{\sqrt{A^2 B^2}}$$

It includes Poincare transformations, scale transformations etc …

Conformal symmetry plays an important role in both critical phenomena and superstring theory. However, in particle physics and in four dimensional space-time it is broken explicitly by the masses of the particles

Georgi 's notion of unparticles

Based on

At high energies , they could exist stuff with non trivial scale invariance in the infrared regime such as the Banks-Zaks fields which as a component of the beyond the standard Model (SM) physics above the TeV scale.

Existence of a hidden sector (e.g. Banks-Zaks CFT theory) at high energy scale M but flows to an infrared fixed point at a low energy scale Λ_u through dimensional transmutation

Unparticles emerges below Λ_u

Georgi used the method of the low energy effective field theory to study the unparticles production and peculiar virtual effects in high energy processes.

Effective field theories used to decribe SM particles couplings with unparticles

O_{SM} ⟷ O_{BZ} $\underline{\mathrm{M}}$

$$\frac{1}{\mathrm{M}^{d_{SM}+d_{BZ}-d_u}} O_{SM} O_{BZ}$$

Transmutation of scale (changing to another form)

Flows to IR fixed point (use renormalization group equation)

$$C_u \frac{\Lambda_u^{d_{SM}-d_u}}{\mathrm{M}^{d_{SM}+d_{BZ}-d_u}} O_{SM} O_u$$

O_u $\underline{\Lambda_u}$

Unparticle operator O_u

emerges the scales with non integral scaling dimension d_u

$x \longrightarrow \lambda x$

$$O_u(\lambda x) \rightarrow \lambda^{-d_u} O_u(x)$$

Thus

Georgi's idea was based on a scenario that goes back to Banks and Zaks in 1982 which investigates a class of scale invariant fields later termed 'BZ fields'.

In the low energy limit, these could couple to the fields of the standard model with fairly unusual coupling of fractional dimension.

Examples of the $\mathcal{BZ}$ sector

- Banks & Zaks (1982): SU(3) YM with n massless fermions in e.g. fundamental representation

$$\beta(g) = -\left(\beta_0 \frac{g^3}{16\pi^2} + \beta_1 \frac{g^5}{(16\pi^2)^2} + 3 \text{ loops} \cdots\right)$$

$$\begin{array}{lll} \beta_0 = 11 - \frac{2}{3}n & \beta_0(n_0) = 0 & n_0 = 16.5 \\ \beta_1 = 102 - \frac{38}{3}n & \beta_1(n_1) = 0 & n_1 \simeq 8.05 \end{array}$$

If $n_1 < n < n_0$ (so $\beta_0 > 0$ & $\beta_1 < 0$) then keeping β_0 and β_1 one gets

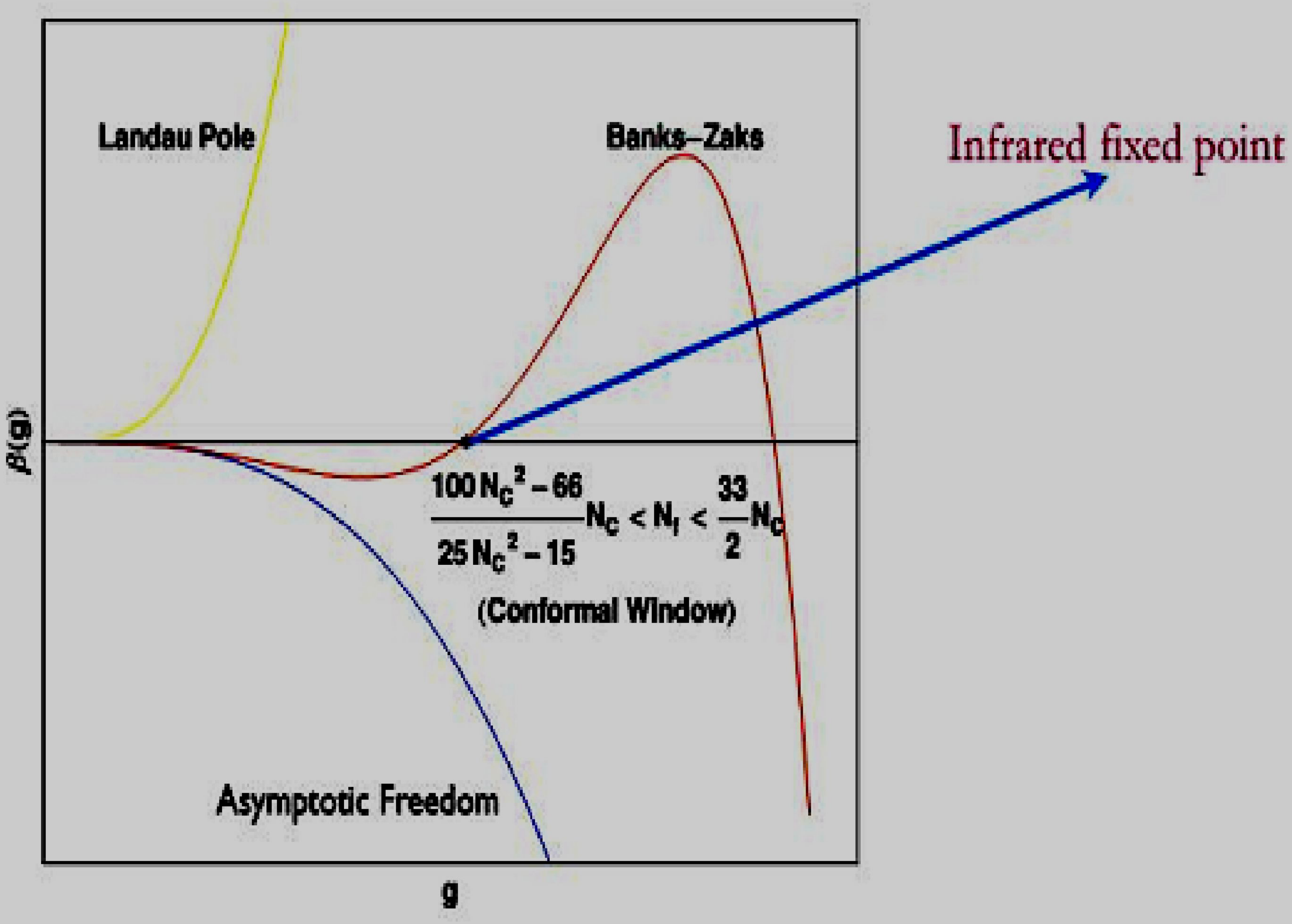
$$\beta(g_{IR}) = 0 \quad \text{for} \quad \frac{g_{IR}^2}{16\pi^2} = -\frac{33 - 2n}{306 - 38n}$$

Expanding n around n_0: $n = n_0\left(1 - \frac{\varepsilon}{11}\right)$

$$\frac{g_{IR}^2}{16\pi^2} \simeq \varepsilon \times 10^{-2}$$

Conclusions:

– If $g = g_{IR}$, then the low-energy theory is scale invariant with small anomalous scaling


Landau Pole
Banks–Zaks
Infrared fixed point
$\beta(g)$
$\frac{100 N_C^2 - 66}{25 N_C^2 - 15} N_C < N_f < \frac{33}{2} N_C$
(Conformal Window)
Asymptotic Freedom
g

The scenario again

$$\begin{array}{llll}
UV & CFT : \mathcal{BZ} \text{ fields} & & SM \text{ fields} \\
\downarrow & \searrow & & \swarrow \\
M_{\mathcal{U}} & & \frac{1}{M_{\mathcal{U}}^k} O_{\mathcal{BZ}} O_{\rm SM} & \\
\downarrow & & & \\
\Lambda_{\mathcal{U}} & \text{transmutation scale} & \downarrow & \\
\downarrow & & & \\
IR & \text{fixed point} & c_{\mathcal{U}} \frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}} - d_{\mathcal{U}}}}{M_{\mathcal{U}}^k} O_{\mathcal{U}} O_{\rm SM} &
\end{array}$$

$v^2 \equiv \langle |H|^2 \rangle \neq 0 \quad \Rightarrow \quad$ scaling violation at $\Lambda_{\not{\mathcal{U}}}$:

$$\Lambda_{\not{\mathcal{U}}}^{4-d_{\mathcal{U}}} = \left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{d_{\mathcal{BZ}} - d_{\mathcal{U}}} M_{\mathcal{U}}^{2-d_{\mathcal{U}}} v^2$$

- For $Q < \Lambda_{\not{\mathcal{U}}}$ unparticles become particles (we assume $\Lambda_{\not{\mathcal{U}}} < \Lambda_{\mathcal{U}}$).
- For $Q > \Lambda_{\not{\mathcal{U}}}$ unparticle effects could possibly be seen.
- Low energy experiments may not be sensitive to unparticles.

Some characteristics Unparticles?

Unparticle Phase Space (Georgi, PRL, 98 (2007) 221601)

Phase space

- N massless particle phase space: $(p_1 + p_2 + \cdots + p_n)^2 = s^2$

$$dLIPS_n = A_n s^{n-2}, \quad A_n = \frac{16\pi^{5/2}}{(2\pi)^{2n}} \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n-1)\Gamma(2n)}$$

- By scale invariance

$$\Phi(P_U^2) = A_d \theta(P_U^0)\theta(P_U^2)(P_U^2)^{d-2}$$

- Identifies $d \to n$; $A_d \to A_n$
- Unparticle resembles a collection of d (non-integral) massless particles
- Unparticle stuff has continuous mass distribution – Infraparticles in QFT (Schroer, 1963, 2008)

Propagators

Unparticle propagator

The Feynman propagator:

$$i\Delta_F^{\mathcal{U}}(p^2) = \int d^4x e^{ipx} \langle 0|T\{O_{\mathcal{U}}(x)O_{\mathcal{U}}(0)\}|0\rangle = \int_0^\infty \frac{dm^2}{2\pi}\rho_{\mathcal{U}}(m^2)\frac{i}{p^2 - m^2 + i\varepsilon}$$

From the scaling properties $\rho_{\mathcal{U}}(m^2) = A_{d_{\mathcal{U}}}\theta(m^2)\left(m^2\right)^{d_{\mathcal{U}}-2}$, so

$$\Delta_F^{\mathcal{U}}(p^2) = \frac{A_{d_{\mathcal{U}}}}{2\sin(\pi d_{\mathcal{U}})}\frac{1}{(-p^2 - i\varepsilon)^{2-d_{\mathcal{U}}}}$$

Constraints on the dimension:

*The dimension is constrained by the unitarity of the conformal algebra. The unitarity imposes lower bounds on the dimension

- Scalar operators: $d \geq 1$
- Fermion operators: $d \geq 3/2$
- Gauge invariant vector operators: $d \geq 3$

*On the other hand we can also find upper bounds on the dimension.
the theory is ultraviolet sensitive; small changes at high energies radically alter the propagator.

*This has several
consequences. These imply that we should restrict ourselves to $d < 2$(for scalars)and ($d < 5/2$ for fermions).

Interaction with SM particles

Couplings of unparticles to the SM

Assumptions:

- $\mathcal{O}_\mathcal{U}$ in neutral under the SM gauge group
- $\dim(\mathcal{O}_{\rm SM}) \leq 4$

$$\mathcal{L}_{\rm int} = c_\mathcal{U} \frac{\Lambda_\mathcal{U}^{d_{\mathcal{BZ}}-d_\mathcal{U}}}{M_\mathcal{U}^k} \mathcal{O}_\mathcal{U} \mathcal{O}_{\rm SM} \quad \text{for} \quad k = d_{\rm SM} + d_{\mathcal{BZ}} - 4$$

- Scalar unparticles $\mathcal{O}_\mathcal{U}$
  - Couplings with gauge bosons

$$\lambda_{gg}\Lambda_\mathcal{U}^{-d_\mathcal{U}} G^{\mu\nu} G_{\mu\nu} \mathcal{O}_\mathcal{U},\ \lambda_{ww}\Lambda_\mathcal{U}^{-d_\mathcal{U}} W^{\mu\nu} W_{\mu\nu} \mathcal{O}_\mathcal{U},\ \lambda_{bb}\Lambda_\mathcal{U}^{-d_\mathcal{U}} B^{\mu\nu} B_{\mu\nu} \mathcal{O}_\mathcal{U},$$
$$\tilde{\lambda}_{gg}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \tilde{G}^{\mu\nu} G_{\mu\nu} \mathcal{O}_\mathcal{U},\ \tilde{\lambda}_{ww}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \tilde{W}^{\mu\nu} W_{\mu\nu} \mathcal{O}_\mathcal{U},\ \tilde{\lambda}_{bb}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \tilde{B}^{\mu\nu} B_{\mu\nu} \mathcal{O}_\mathcal{U},$$

  - Coupling with Higgs and Gauge bosons

$$\lambda_{hh}\Lambda_\mathcal{U}^{2-d_\mathcal{U}} H^\dagger H \mathcal{O}_\mathcal{U},\ \tilde{\lambda}_{hh}\Lambda_\mathcal{U}^{-d_\mathcal{U}} (H^\dagger D_\mu H)\partial^\mu \mathcal{O}_\mathcal{U}\ ,$$
$$\lambda_{4h}\Lambda_\mathcal{U}^{-d_\mathcal{U}} (H^\dagger H)^2 \mathcal{O}_\mathcal{U},\ \lambda_{dh}\Lambda_\mathcal{U}^{-d_\mathcal{U}} (D_\mu H)^\dagger (D^\mu H) \mathcal{O}_\mathcal{U},$$

  - Couplings with fermions and gauge bosons

$$\lambda_{QQ}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \bar{Q}_L \gamma_\mu D^\mu Q_L \mathcal{O}_\mathcal{U},\ \lambda_{UU}\Lambda_U^{-d_\mathcal{U}} \bar{U}_R \gamma_\mu D^\mu U_R \mathcal{O}_\mathcal{U},\ \lambda_{DD}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \bar{D}_R \gamma_\mu D^\mu D_R \mathcal{O}_\mathcal{U},$$
$$\lambda_{LL}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \bar{L}_L \gamma_\mu D^\mu L_L \mathcal{O}_\mathcal{U},\ \lambda_{EE}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \bar{E}_R \gamma_\mu D^\mu E_R \mathcal{O}_\mathcal{U},\ \lambda_{\nu\nu}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \bar{\nu}_R \gamma_\mu D^\mu \nu_R \mathcal{O}_\mathcal{U},$$
$$\tilde{\lambda}_{QQ}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \bar{Q}_L \gamma_\mu Q_L \partial^\mu \mathcal{O}_\mathcal{U},\ \tilde{\lambda}_{UU}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \bar{U}_R \gamma_\mu U_R \partial^\mu \mathcal{O}_\mathcal{U},\ \tilde{\lambda}_{DD}\Lambda_\mathcal{U}^{-d_\mathcal{U}} \bar{D}_R \gamma_\mu D_R \partial^\mu \mathcal{O}_\mathcal{U},$$

$$\bar{\lambda}_{LL}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\bar{L}_L\gamma_\mu L_L\partial^\mu O_{\mathcal{U}},\ \bar{\lambda}_{EE}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\bar{E}_R\gamma_\mu E_R\partial^\mu O_{\mathcal{U}},\ \bar{\lambda}_{RR}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\bar{\nu}_R\gamma_\mu \nu_R\partial^\mu O_{\mathcal{U}},$$
$$\lambda_{YR}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\bar{\nu}_R^C\nu_R O_{\mathcal{U}},$$

- Couplings with fermions and Higgs boson

$$\lambda_{YU}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\bar{Q}_L H U_R O_{\mathcal{U}},\ \lambda_{YD}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\bar{Q}_L\tilde{H}D_R O_{\mathcal{U}},$$
$$\lambda_{Y\nu}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\bar{L}_L H\nu_R O_{\mathcal{U}},\ \lambda_{YE}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}\bar{L}_L\tilde{H}E_R O_{\mathcal{U}},$$

- Vector unparticles $O_{\mathcal{U}}^\mu$
  - Couplings with fermions

$$\lambda'_{QQ}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\bar{Q}_L\gamma_\mu Q_L O_{\mathcal{U}}^\mu,\ \lambda'_{UU}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\bar{U}_R\gamma_\mu U_R O_{\mathcal{U}}^\mu,\ \lambda'_{DD}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\bar{D}_R\gamma_\mu D_R O_{\mathcal{U}}^\mu,$$
$$\lambda'_{LL}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\bar{L}_L\gamma_\mu L_L O_{\mathcal{U}}^\mu,\ \lambda'_{EE}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\bar{E}_R\gamma_\mu E_R O_{\mathcal{U}}^\mu,\ \lambda'_{RR}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}\bar{\nu}_R\gamma_\mu \nu_R O_{\mathcal{U}}^\mu,$$

  - Couplings with Higgs boson and Gauge bosons

$$\lambda'_{hh}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}(H^\dagger D_\mu H)O_{\mathcal{U}}^\mu\ ,\ \lambda'_{bO}\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}B_{\mu\nu}\partial^\mu O^\nu\ .$$

- Spinor unparticles $O_{\mathcal{U}}^s$

$$\lambda_{s\nu}\Lambda_{\mathcal{U}}^{5/2-d_{\mathcal{U}}}\bar{\nu}_R O_{\mathcal{U}}^s,\ \lambda_s\Lambda_{\mathcal{U}}^{3/2-d_{\mathcal{U}}}\bar{O}_{\mathcal{U}}^s L_L^i\epsilon_{ij}H^j\ .$$

Gauged Unparticles

Georgi suggested also → it will be interesting

↓

if unparticles carry standard model quantum numbers

↓

leading to more interesting phenomenology at LHC + ILC

Because of non trivial canonical dimensions of unparticles

Related gauge theory is non local

Complicated that unparticles be coupled to MS gauge fields

NB:

Study was made by many authors

we follow the approach of ref.

G. Cacciapaglia, G. Marandella and J. Terning, JHEP **0801**, 070 (2008)

Consider scalar unparticles

Parametrization of breakdown of scale invariance

via introd. of masse term μ_s In propagator $\Delta_{\mu_s}(k)$

$$\begin{aligned}\Delta_{\mathcal{U}_s}(k,\mu_s) &= \int d^4x e^{ikx}\langle 0|T\mathcal{U}_s(x)\mathcal{U}_s^\dagger(0)|0\rangle \\ &= \frac{iA(d_{\mathcal{U}_s})}{2\pi}\int_{\mu^2}^{\infty} dm^2 \frac{\rho(m^2-\mu_s^2)}{k^2-m^2+i\,\epsilon} \\ &\equiv \frac{A(d_{\mathcal{U}_s})}{2\sin(\pi d_{\mathcal{U}_s})}\frac{i}{\Sigma_0^s(k,\mu_s)}\end{aligned}$$

$$\Sigma_0^s(k,\mu_s) \equiv (\mu_s^2 - k^2 - i\,\epsilon)^{2-d_{\mathcal{U}_s}}$$

Effective action in momentum space corresponding to this propagator

$$
\begin{aligned}
S_0[\mathcal{U}_s] &= \int \frac{d^4k}{(2\pi)^4}\mathcal{U}_s^\dagger(k)\tilde{\Delta}_{\mathcal{U}_s}^{-1}(k,\mu_s)\mathcal{U}_s(k) \\
&= \frac{2\sin(\pi d_{\mathcal{U}_s})}{A(d_{\mathcal{U}_s})}\int \frac{d^4k}{(2\pi)^4}\mathcal{U}_s^\dagger(k)(\mu_s^2-k^2)^{2-d_{\mathcal{U}_s}}\mathcal{U}_s(k) \\
&= \frac{2\sin(\pi d_{\mathcal{U}_s})}{A(d_{\mathcal{U}_s})}\int \frac{d^4k}{(2\pi)^4}\mathcal{U}_s^\dagger(k)\Sigma_0^s(k,\mu_s)\mathcal{U}_s(k)\,.
\end{aligned}
$$

In order to gauge our theory we get the Fourier transform and get a non local action in configuration space

$$S_0[\mathcal{U}_s] = \int d^4x d^4y \mathcal{U}_s^\dagger(x) \tilde{\Delta}_{\mathcal{U}_s}^{-1}(x-y) \mathcal{U}_s(y)$$

$$\tilde{\Delta}_{\mathcal{U}_s}^{-1}(z) = \int \frac{d^4k}{(2\pi)^4} e^{ikz} \tilde{\Delta}_{\mathcal{U}_s}^{-1}(k)$$

Gauging this non local action

By Making it invariant under a gauge transformation

Via introd. gauge Invariant observable

between the 02 unparticles $\mu_s(x)$ $\mu_s(y)$

Called Wison line (loop)

$$S[\mathcal{U}_s] = \int d^4x d^4y \mathcal{U}_s^\dagger(x) \tilde{\Delta}_{\mathcal{U}_s}^{-1}(x-y) W(x,y) \mathcal{U}_s(y)$$

Wilson loop $W(x,y)$

Defined by a path ordred exponential of a gauge field A_μ^a

of gauge groups $SU(2), U(1), SU(3)$

Along a ligne C between x and y

$$W(x,y) = P \exp\left(-ig_{\mathcal{U}_s} T^a \int_{x,C}^{y} A_\mu^a(w) dw^\mu\right)$$

Transforms like $W(x,y) \rightarrow U^{-1}(x) W(x,y) U(y)$

Require that Wilson line verifies Mandelstam condition

$$\frac{\partial}{\partial x^{\mu}} W(x,y) = igT^{a} A_{\mu}^{a}(w) W(x,y)$$

To get Vertices

$$\begin{aligned} ig_{\mathcal{U}_s} \Gamma_s^{a\mu}(k;q) &= \frac{i\delta^3 S[\mathcal{U}_s]}{\delta A_{\mu}^{a}(q) \delta \mathcal{U}_s^{\dagger}(k+q) \delta \mathcal{U}_s(k)} \\ &= ig_{\mathcal{U}_s} T^{a} (2k+q)^{\mu} \Sigma_1^{s}(k;q) \end{aligned}$$

03 point Green function

$$ig^2_{\mathcal{U}_s}\Gamma^{ab\mu\nu}_s(k;q_1,q_2) = \frac{i\delta^3 S[\mathcal{U}_s]}{\delta A^a_\mu(q_1)\delta A^b_\nu(q_2)\delta\mathcal{U}^\dagger_s(k+q_1+q_2)\delta\mathcal{U}_s(k)}$$

$$\begin{aligned}
= \quad & ig^2_{\mathcal{U}_s}\Big[\{T^a,T^b\}g^{\mu\nu}\Sigma^s_1(k;q_1+q_2) \\
+ \quad & T^aT^b(2k+q_2)^\nu(2k+2q_2+q_1)^\mu\Sigma^s_2(k;q_2,q_1) \\
+ \quad & T^bT^a(2k+q_1)^\mu(2p+2q_1+q_2)^\nu\Sigma^s_2(k;q_1,q_2)\Big]
\end{aligned}$$

04 point Green function

$$\Sigma^s_1(k,q) = \frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})}\,\frac{\Sigma^s_0(k+q)-\Sigma^s_0(k)}{(k+q)^2-k^2}$$

$$\Sigma^s_2(k,q_1,q_2) = \frac{\Sigma^s_1(k,q_1+q_2)-\Sigma^s_1(k,q_1)}{(k+q_1+q_2)^2-(k+q_1)^2}\,.$$

Consider fermionic unparticles

Similarly introd. Fermionic propagator

$$\Delta_{\mathcal{U}_f}(k^2) = \frac{A(d_{\mathcal{U}_f} - 1/2)}{2\cos(\pi d_{\mathcal{U}_f})} \frac{i}{(\not{k} - \mu_f)\Sigma_0^f(k, \mu_f)}$$

$$\Sigma_0^f(k, \mu_f) \equiv (\mu_f^2 - k^2 - i\,\epsilon)^{3/2 - d_{\mathcal{U}_f}}$$

$$S_0[\mathcal{U}_f] = \int \frac{d^4k}{(2\pi)^4} \overline{\mathcal{U}}_f(k)\tilde{\Delta}^{-1}_{\mathcal{U}_f}(k)\mathcal{U}_f(k)$$

$$\equiv \frac{2\cos(\pi d_{\mathcal{U}_f})}{A(d_{\mathcal{U}_f} - 1/2)} \int \frac{d^4k}{(2\pi)^4} \overline{\mathcal{U}}_f(k)(\not{k} - \mu_f)\Sigma_0^f(k, \mu)\mathcal{U}_f(k)$$

Gauging this non local action in configuration space

$$S[\mathcal{U}_f] = \frac{2\cos(\pi d_{\mathcal{U}_f})}{A(d_{\mathcal{U}_f} - 1/2)} \int d^4x d^4y \overline{\mathcal{U}}_f(x) \left(-i \not\partial \tilde{\Sigma}_0^{f,-1}(x-y)\right) W(x,y) \mathcal{U}_f(y)$$

For the vertex

$$A_\mu^a \mathcal{U}_f \overline{\mathcal{U}}_f$$

$$\begin{aligned} i g_{\mathcal{U}_f} \Gamma_f^{a\mu}(k;q) &= \frac{i\delta^3 S[\mathcal{U}_f]}{\delta A_\mu^a(q) \delta \overline{\mathcal{U}}_f(k+q) \delta \mathcal{U}_f(k)} \\ &= i \frac{g_{\mathcal{U}_f}}{2} T^a \left[\gamma^\mu \left(\Sigma_0^f(k+q) + \Sigma_0^f(k+q)\right) \right. \\ &\left. + (2\not k + \not q - 2\mu_f)(2k+q)^\mu \Sigma_1^f(k+q)\right] \end{aligned}$$

For the vertex $A^a_\mu A^b_\nu \mathcal{U}_f \overline{\mathcal{U}}_f$

[redacted]

$$ig^2_{\mathcal{U}_f}\Gamma_f^{ab\mu\nu}(k;q_1,q_2) = \frac{i\delta^3 S[\mathcal{U}_f]}{\delta A^a_\mu(q_1)\delta A^b_\nu(q_2)\delta\overline{\mathcal{U}}_f(k+q_1+q_2)\delta\mathcal{U}_f(k)}$$

[redacted]

$$+ \frac{\gamma^\mu}{2}\Gamma_f^{ab\nu}(k,q_2,q_1) + \frac{\gamma^\nu}{2}\Gamma_f^{ab\mu}(k;q_1,q_2)\Big\}$$

$$\begin{aligned} = &\; ig^2_{\mathcal{U}_f}\Big\{[\not{k} + (\not{q}_1 + \not{q}_2)/2 - \mu_f]\Big[\{T^a,T^b\}g^{\mu\nu}\Sigma^s_1(k;q_1+q_2) \\ + &\; T^aT^b(2k+q_2)^\nu(2k+2q_2+q_1)^\mu\Sigma^s_2(k;q_2,q_1) \\ + &\; T^bT^a(2k+q_1)^\mu(2p+2q_1+q_2)^\nu\Sigma^s_2(k;q_1,q_2)\Big] \end{aligned}$$

[redacted]

Where

[redacted]

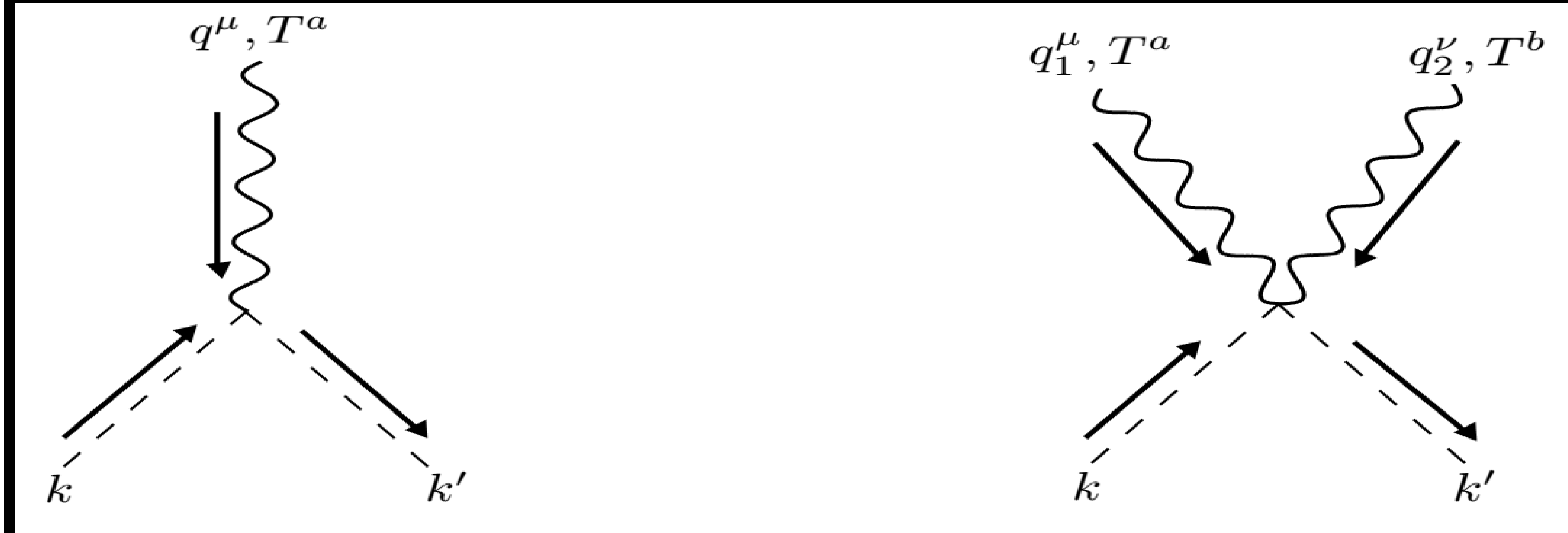
Fermionic form factors

$$\Sigma_1^f(k;q) = \frac{2\cos(d_{\mathcal{U}_f}\pi)}{A(d_{\mathcal{U}_f}-1/2)} \frac{\Sigma_0^f(k+q)-\Sigma_0^f(k)}{(k+q)^2-k^2}$$

$$\Sigma_2^f(k;q_1,q_2) = \frac{\Sigma_1^f(k,q_1+q_2)-\Sigma_1^f(k,q_1)}{(k+q_1+q_2)^2-(k+q_1)^2},$$

and

$$\begin{aligned}\Gamma_f^{ab\mu}(k;q_1,q_2) &= T^aT^b(2k+q_1)^\mu\Sigma_1^f(k,q_2) \\ &+ T^bT^a(2k+2q_2+q_1)^\mu\Sigma_1^f(k+q_2,q_1)\end{aligned}$$



Ward-Takahashi identities

Abelian case

$$
\begin{aligned}
i g_{\mathcal{U}_{s/f}} q_\mu \Gamma^\mu_{s/f}(k;q) &= i g_{\mathcal{U}_{s/f}} \left(\Delta^{-1}_{s/f}(k) - \Delta^{-1}_{s/f}(k+q) \right), \\
i g^2_{\mathcal{U}_{s/f}} q_{1\mu} \Gamma^{\mu\nu}_{s/f}(k; q_1, q_2) &= i g^2_{\mathcal{U}_{s/f}} \left[\Gamma^\nu_{s/f}(k+q_1; q_2) - G^\nu_{s/f}(k; q_2) \right]
\end{aligned}
$$

Non Abelian case

$$
\begin{aligned}
i g_{\mathcal{U}_{s/f}} q_\mu \Gamma^{a\mu}_{s/f}(k;q) &= i g_{\mathcal{U}_{s/f}} T^a \left[\Delta^{-1}_{s/f}(k) - \Delta^{-1}_{s/f}(k+q)\right], \\
i g^2_{\mathcal{U}_{s/f}} q_{1\mu} \Gamma^{a\mu,b\nu}_{s/f}(p,q_1,q_2) &= i g^2_{\mathcal{U}_{s/f}} \left[\Gamma^{b\nu}_{s/f}(k+q_1;q_2) T^a - T^a \Gamma^{b\nu}_{s/f}(k;q_2)\right] \\
&+ i g^2_{\mathcal{U}_{s/f}} f^{abc} \Gamma^{c\nu}_{s/f}(k;q_1+q_2).
\end{aligned}
$$

Asymptotic form

Vertex functions are dominated by infrared region of loop integrals which are strongly suppressed in the UV regime

To calculate corresponding loop diagrams (e.g vaccum polarization),

we will make first a Taylor expansion for form factors

$\Sigma_0^s(K+q)$

in the neighborhood of $y = 2k.q$

Then

$$\Sigma_1^s(k;q) \simeq \frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})} \frac{(-1)^{2-d_{\mathcal{U}_s}}(2-d_{\mathcal{U}_s})}{[(k+q)^2-\mu_s^2]^{d_{\mathcal{U}_s}-1}} \left[1 - \frac{(1-d_{\mathcal{U}_s})}{2(k'^2-\mu_s^2)} y + \cdots \right]$$

$$\Sigma_1^s(k;q) \simeq \frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})} \frac{(-1)^{2-d_{\mathcal{U}_s}}(2-d_{\mathcal{U}_s})}{[(k+q)^2-\mu_s^2]^{d_{\mathcal{U}_s}-1}}$$

$$\Sigma_2^s(k;q_1,q_2) \simeq -\frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})} \frac{(-1)^{2-d_{\mathcal{U}_s}}(2-d_{\mathcal{U}_s})(1-d_{\mathcal{U}_s})}{[(k+q_1+q_2)^2-\mu_s^2]^{d_{\mathcal{U}_s}-1}[(k+q_1)^2-\mu_s^2]}$$

$$ig_{\mathcal{U}_s}\Gamma_s^{a\mu}(k;q) \simeq ig_{\mathcal{U}_s}T^a(-1)^{2-d_{\mathcal{U}_s}}\frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})}\frac{(2-d_{\mathcal{U}_s})(2k+q)^\mu}{[(k+q)^2-\mu_s^2]^{d_{\mathcal{U}_s}-1}}$$

$$\begin{aligned} ig^2_{\mathcal{U}_s}\Gamma_s^{ab\mu\nu}(k;q_1,q_2) &\simeq ig^2_{\mathcal{U}_s}\frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})}\frac{(-1)^{2-d_{\mathcal{U}_s}}(2-d_{\mathcal{U}_s})}{[(k+q_1+q_2)^2-\mu_s^2]^{d_{\mathcal{U}_s}-1}}\Big[\{T^a,T^b\}g^{\mu\nu} \\ &- T^aT^b\frac{(1-d_{\mathcal{U}_s})(2k+q_2)^\nu(2k+2q_2+q_1)^\mu}{(k+q_2)^2-\mu_s^2} \\ &- T^bT^a\frac{(1-d_{\mathcal{U}_s})(2k+q_1)^\mu(2k+2q_1+q_2)^\nu}{(k+q_1)^2-\mu_s^2}\Big] \end{aligned}$$

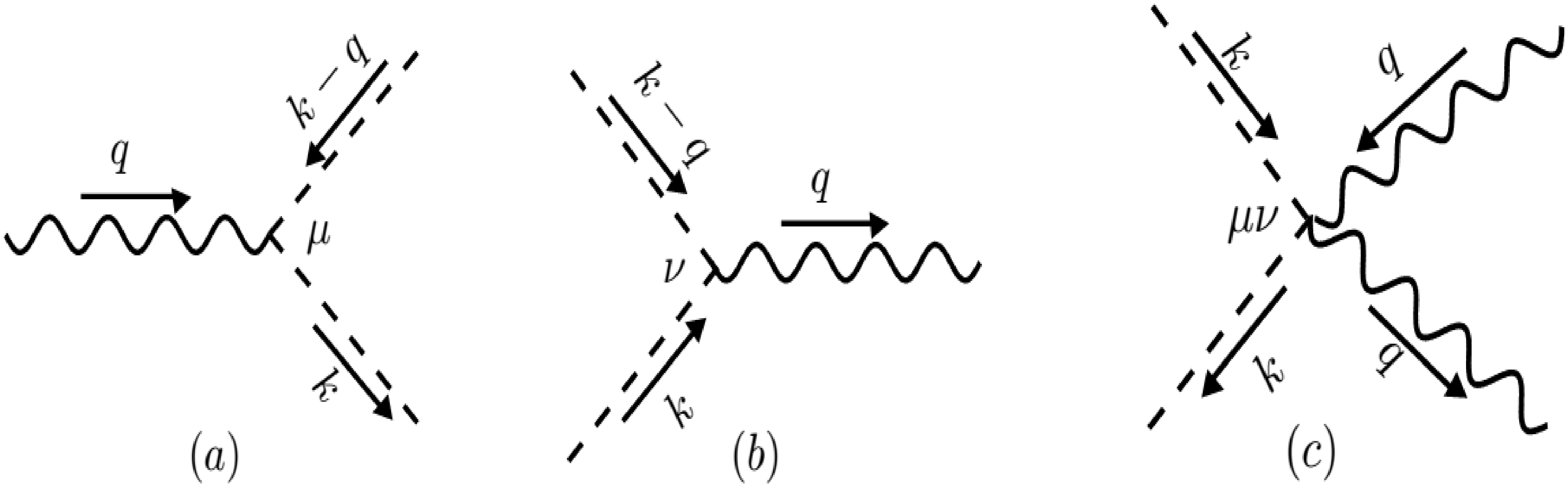
Bosonic case

$$
ig_{\mathcal{U}_f}\Gamma_f^{a\mu}(k;q) \simeq ig_{\mathcal{U}_f}\gamma^\mu T^a \frac{2\cos(d_{\mathcal{U}_f}\pi)}{A(d_{\mathcal{U}_f}-1/2)} \frac{(-1)^{3/2-d_{\mathcal{U}_f}}\, 2(2-d_{\mathcal{U}_f})}{\left[(k+q)^2-\mu_f^2\right]^{d_{\mathcal{U}_f}-3/2}}
$$

$$
\begin{aligned}
ig^2_{\mathcal{U}_f}\Gamma_f^{ab\mu\nu}(k;q_1,q_2) \simeq\ & ig^2_{\mathcal{U}_f} \frac{2\cos(d_{\mathcal{U}_f}\pi)}{A(d_{\mathcal{U}_f}-1/2)} \frac{(-1)^{3/2-d_{\mathcal{U}_f}}\, 2(3/2-d_{\mathcal{U}_f})}{\left[(k+q_1+q_2)^2-\mu_f^2\right]^{d_{\mathcal{U}_f}-1/2}} \\
\times\ & \left[\not{k}+(\not{q}_1+\not{q}_2)/2-\mu_f\right]\Big[\{T^a,T^b\}g^{\mu\nu} \\
-\ & T^aT^b\frac{(2k+q_2)^\nu(2k+2q_2+q_1)^\mu}{(k+q_2)^2-\mu_f^2} \\
-\ & T^bT^a\frac{(2k+q_1)^\mu(2k+2q_1+q_2)^\nu}{(k+q_1)^2-\mu_f^2}\Big]
\end{aligned}
$$

Fermionic case

Example Vaccum polarization



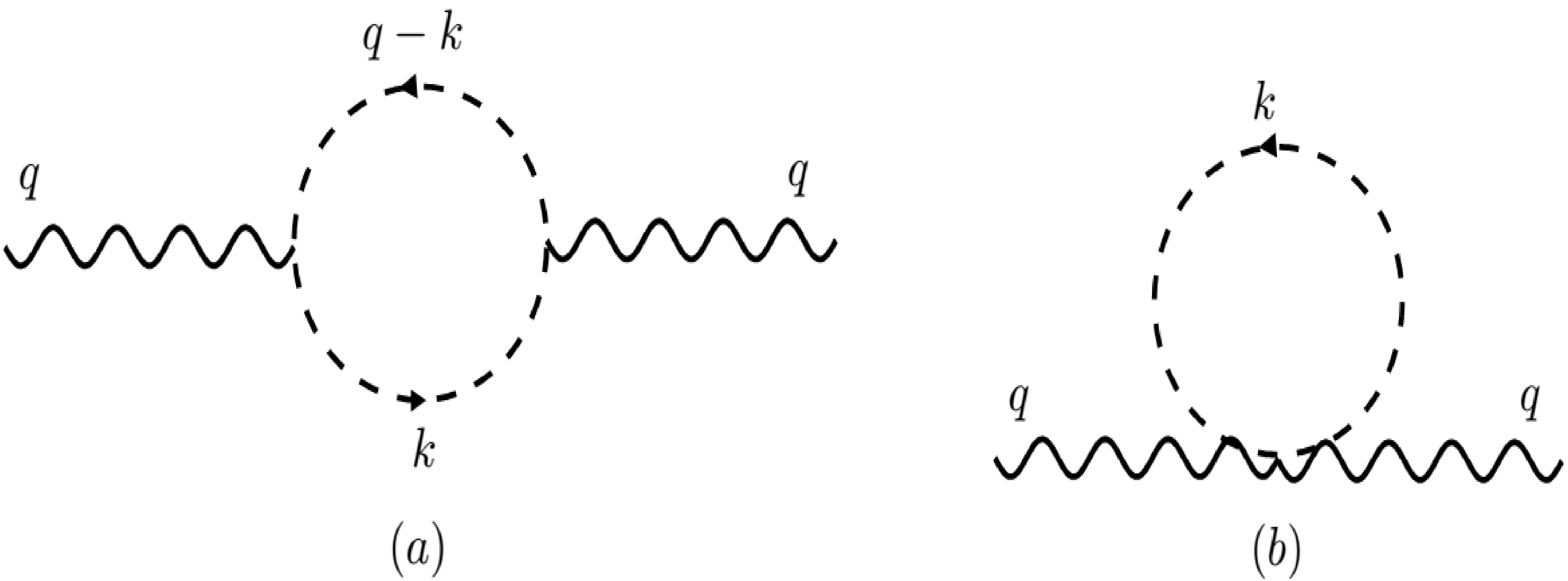
Which Generate the following vertices

[redacted]

$$
\begin{aligned}
ig_{\mathcal{U}_s}\Gamma^{\mu}_{\mathcal{U}_s}(k-q,q) &\simeq -ig_{\mathcal{U}_s}(-1)^{2-d_{\mathcal{U}_s}}\frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})}\frac{(2-d_{\mathcal{U}_s})(2k-q)^{\mu}}{(k^2-\mu_s^2)^{d_{\mathcal{U}_s}-1}}, \\
ig_{\mathcal{U}_s}\Gamma^{\nu}_{\mathcal{U}_s}(k,-q) &\simeq -ig_{\mathcal{U}_s}(-1)^{2-d_{\mathcal{U}_s}}\frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})}\frac{(2-d_{\mathcal{U}_s})(2k-q)^{\nu}}{[(k-q)^2-\mu_s^2]^{d_{\mathcal{U}_s}-1}} \\
ig^2_{\mathcal{U}_s}\Gamma^{\mu\nu}_{\mathcal{U}_s}(k,q,-q) &\simeq ig^2_{\mathcal{U}_s}(-1)^{2-d_{\mathcal{U}_s}}\frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})}\frac{(2-d_{\mathcal{U}_s})}{(k^2-\mu_s^2)^{d_{\mathcal{U}_s}-1}} \\
&\times \left[2g^{\mu\nu}+\frac{(1-d_{\mathcal{U}_s})(2k-q)^{\mu}(2k-q)^{\nu}}{[(k-q)^2-\mu_s^2]}\right].
\end{aligned}
$$

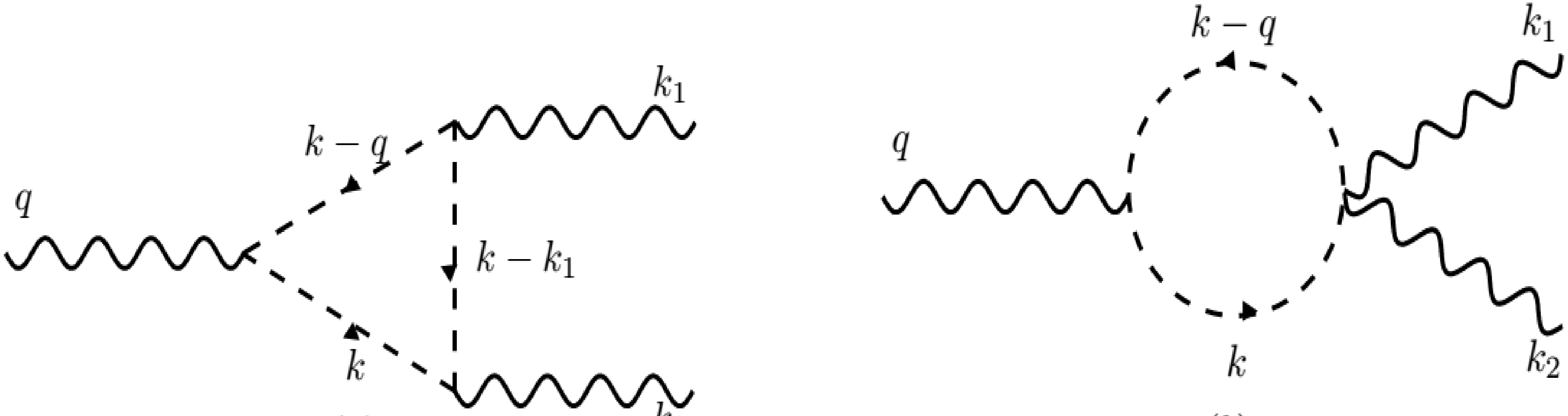
[redacted]

Total contribution to vaccum polarization diagram



$$
\begin{aligned}
i\Pi^{\mu\nu}_{\mathcal{U}_s}(k) &= (2-d_{\mathcal{U}_s})\int \frac{d^4k}{(2\pi)^4}\left[\frac{-2g^{\mu\nu}[(k-q)^2-\mu_s^2]+(2k-q)^\mu(2k-q)^\nu}{(k^2-\mu_s^2)[(k-q)^2-\mu_s^2]}\right. \\
&= (2-d_{\mathcal{U}_s})\,[\Pi^{\mu\nu}_s(k)]_{\mathrm{MS}}
\end{aligned}
$$

03 points Green functions



(a) (b)

Scalar vetices

$$
\begin{aligned}
\Gamma^{\mu}_{\mathcal{U}_s}(k-k_2, k_1+k_2) &\simeq (-1)^{1-d_{\mathcal{U}_s}} \frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})} \frac{(2-d_{\mathcal{U}_s})(2k+k_1-k_2}{[(k+k_1)^2-\mu_s^2]^{d_{\mathcal{U}_s}-1}} \\
\Gamma^{\mu}_{\mathcal{U}_s}(k+k_1, -k_1) &\simeq (-1)^{1-d_{\mathcal{U}_s}} \frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})} \frac{(2-d_{\mathcal{U}_s})(2k+k_1)^{\mu}}{(k^2-\mu_s^2)^{d_{\mathcal{U}_s}-1}}, \\
\Gamma^{\nu}_{\mathcal{U}_s}(k, -k_2) &\simeq (-1)^{1-d_{\mathcal{U}_s}} \frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})} \frac{(2-d_{\mathcal{U}_s})(2k-k_2)^{\nu}}{[(k-k_2)^2-\mu_s^2]^{d_{\mathcal{U}_s}-1}},
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^{\mu\nu}_{\mathcal{U}_s}(k+k_1,-k_1,-k_2) &\simeq (-1)^{1-d_{\mathcal{U}_s}}\frac{2\sin(d_{\mathcal{U}_s}\pi)}{A(d_{\mathcal{U}_s})}\frac{(2-d_{\mathcal{U}_s})}{[(k-k_2)^2-\mu_s^2]^{d_{\mathcal{U}_s}-1}} \\
&\times \left[-2g^{\mu\nu}-\frac{(1-d_{\mathcal{U}_s})(2k+k_1)^{\mu}(2k-k_2)^{\nu}}{[k^2-\mu_s^2]}\right]
\end{aligned}
$$

Similarly **Fermionic unparticles case**

$$ig_{\mathcal{U}_f}\Gamma^{\mu}_{\mathcal{U}_f}(k-q,q) \simeq ig_{\mathcal{U}_f}\gamma^{\mu}\frac{2\cos(d_{\mathcal{U}_f}\pi)}{A(d_{\mathcal{U}_f}-1/2)}\frac{(-1)^{3/2-d_{\mathcal{U}_f}}2(2-d_{\mathcal{U}_f})}{(k^2-\mu_f^2)^{d_{\mathcal{U}_f}-3/2}},$$

$$ig_{\mathcal{U}_f}\Gamma^{\nu}_{\mathcal{U}_f}(k,-q) \simeq ig_{\mathcal{U}_f}\gamma^{\mu}\frac{2\cos(d_{\mathcal{U}_f}\pi)}{A(d_{\mathcal{U}_f}-1/2)}\frac{(-1)^{3/2-d_{\mathcal{U}_f}}2(2-d_{\mathcal{U}_f})}{\left[(k-q)^2-\mu_f^2\right]^{d_{\mathcal{U}_f}-3/2}},$$

$$ig^2_{\mathcal{U}_f}\Gamma^{\mu\nu}_{\mathcal{U}_f}(k,q,-q) \simeq ig^2_{\mathcal{U}_f}\frac{2\cos(d_{\mathcal{U}_f}\pi)}{A(d_{\mathcal{U}_f}-1/2)}\frac{(-1)^{3/2-d_{\mathcal{U}_f}}2(2-d_{\mathcal{U}_f})}{(k^2-\mu_f^2)^{d_{\mathcal{U}_s}-1}}$$

$$\times\ (\not{k}-\mu_f)\left[2g^{\mu\nu}-\frac{(2k-q)^{\nu}(2k-q)^{\mu}}{(k-q)^2-\mu_f^2}\right].$$

Total contribution to vaccum polarization diagram

$$\Pi^{\mu\nu}_{\mathcal{U}_f}(k) \;=\; 4(2-d_{\mathcal{U}_f})^2\left[\Pi^{\mu\nu}_f(k)\right]_{\mathrm{MS}} + 2(3/2-d_{\mathcal{U}_f})\left[\Pi^{\mu\nu}_s(k)\right]_{\mathrm{MS}}$$

$$H \rightarrow \gamma + \gamma$$

Important channel

Study new physics BSM

+

Study of SM precision data

Will not calculate cross section

instead

Examine how do the unparticles states can affect the partial width of diphoton production of higgs boson decay

Effective lagrangian describing coupling of higgs boson to photons

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_{em}}{8\pi v} \mathcal{A}^{\gamma\gamma}_{SM} h F_{\mu\nu} F^{\mu\nu}$$

$$v \simeq 246 \ \text{GeV}$$

$$\mathcal{A}^{\gamma\gamma}_{SM} = 4/3 A_t(\tau_t) + A_W(\tau_W)$$

Loop functions

$$A_t(\tau_t) = \frac{3}{2\tau_t^2}\left[\tau_t + (\tau_t - 1) f(\tau_t)\right],$$

$$A_W(\tau_W) = \frac{1}{7\tau_W^2}\left[3\tau_W + 2\tau_W^2 + 3(2\tau_W - 1) f(\tau_W)\right]$$

$$f(\tau) = \begin{cases} \arcsin^2 \sqrt{\tau} & \text{pour } \tau \leq 1 \\ -\frac{1}{4}\left(\ln \frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi\right)^2 & \text{pour } \tau > 1 \end{cases}$$

$$\tau_i = m_h^2/4m_i^2$$

$$\mathcal{A}_{SM}^{\gamma\gamma} = -6.48$$

$$m_h \simeq 125 \text{ GeV}$$

$$A_t(\tau_t) \to +1.8$$

$$A_W(\tau_W) \to -8.3$$

Effective lagrangian SM+Unparticles

$$\mathcal{L}_{\rm eff} = \frac{\alpha}{8\pi v}\left(\mathcal{A}_{SM}^{\gamma\gamma} + \sum_i d(r_i) Q_i^2 A_{NP}\right) h F_{\mu\nu} F^{\mu\nu}$$

"i" stands for unparticle states in loop

Q_i electric charges

$d(r_i)$ Dimension of representation r_i

A_{NP}^i Contribution of gauged particles

Assume that **Higgs production by gluon fusion**

$gg \to h$

Is not affected by new physics

Ratio $R_{\gamma\gamma}$

$$R_{\gamma\gamma} = \frac{[\sigma(pp \to h) \times Br(h \to \gamma\gamma)]_{\text{observé}}}{[\sigma(pp \to h) \times Br(h \to \gamma\gamma)]_{\text{MS}}}$$

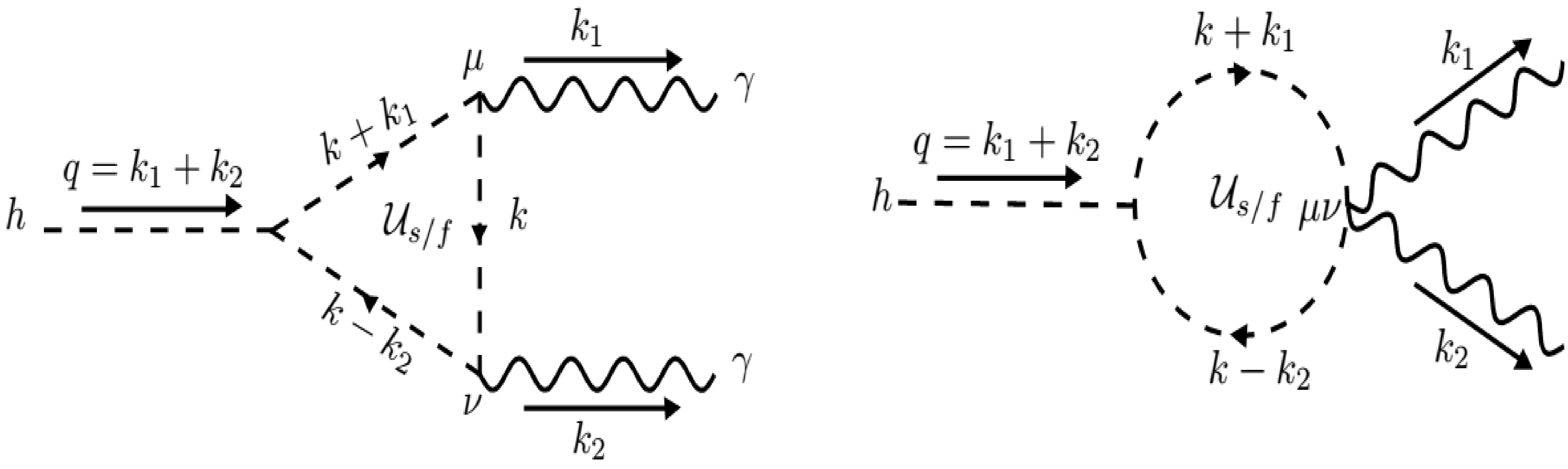
Can be simplified to

$$\begin{aligned} R_{\gamma\gamma} &= \frac{\sigma(pp \to h)_{MS+NP}}{\sigma(pp \to h)_{MS}} \times \frac{\Gamma(h \to \gamma\gamma)_{MS+NP}}{\Gamma(h \to \gamma\gamma)_{SM}} \\ &\simeq \frac{\Gamma(h \to \gamma\gamma)_{MS+NP}}{\Gamma(h \to \gamma\gamma)_{MS}} \\ &= \left| 1 + \frac{\sum_i d(r_i) Q_i^2 A_{i,NP}}{A_1(\tau_W) + N_c Q_t^2 A_{1/2}(\tau_t)} \right|^2 \end{aligned}$$

Formalism

$$\mathcal{L} \supset -\frac{\lambda_{H\mathcal{U}_s}}{(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}_s}-1}} H^\dagger H\, \mathcal{U}_s^\dagger \mathcal{U}_s - \frac{\lambda_{H\mathcal{U}_f}}{(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}_f}-1}} H^\dagger H\, \overline{\mathcal{U}}_f \mathcal{U}_f$$

Main diagrams



(a) (b)

Couplings after EWSB

$$g_{h\mathcal{U}_s^\dagger\mathcal{U}_s} = \lambda_{H\mathcal{U}_s} v^2/2\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}_s}-2}, \qquad g_{h\bar{\mathcal{U}}_f\mathcal{U}_f} = \lambda_{H\mathcal{U}_f} v^2/2\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}_f}-2}$$

Then

$$R_{\gamma\gamma} = \left|1 + \frac{\lambda_{H\mathcal{U}_i}}{2\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}_i}-2}} \frac{v^2}{(\mu_i^2)^{2-d_{\mathcal{U}_i}}} \frac{Q_{\mathcal{U}_i}^2 d(r_{\mathcal{U}_i}) A_{\mathcal{U}_i}}{A(\tau_W) + N_c Q_t^2 A(\tau_t)}\right|^2$$

Notice:

If $\lambda_{Hu_i} > 0$ constructive contribution

$\lambda_{Hu_i} < 0$ destructive contribution

Bosonic contribution

$$i\mathcal{M}_s = -4\pi\alpha_{em}\frac{2(\mu^2_{\mathcal{U}_s})^{2-d_{\mathcal{U}_s}}}{v}Q^2_{\mathcal{U}_s}\mathcal{M}^{\mu\nu}_s\,\epsilon^*_\mu(k_1)\,\epsilon^*_\nu(k_2)$$

$$2\mathcal{M}^{\mu\nu,a}_s = (2-d_{\mathcal{U}_s})\frac{A(d_{\mathcal{U}_s})}{2\sin(d_{\mathcal{U}_s}\pi)}(-1)^{2-d_{\mathcal{U}_s}}2\Pi^{\mu\nu,a}_s$$

$$\mathcal{M}^{\mu\nu,b}_s = (2-d_{\mathcal{U}_s})\frac{A(d_{\mathcal{U}_s})}{2\sin(d_{\mathcal{U}_s}\pi)}(-1)^{2-d_{\mathcal{U}_s}}\Pi^{\mu\nu,b}_s\,,$$

$$\begin{aligned}\Pi^{\mu\nu}_s &= 2\Pi^{\mu\nu,a}_s + \Pi^{\mu\nu,b}_s\\ &= \int\frac{d^4k}{(2\pi)^4}\frac{-2(k^2-\mu^2_s)g^{\mu\nu}+2(2k+k_1)^\mu(2k-k_2)^\nu}{(k^2-\mu^2_s)[(k-k_2)^2-\mu^2_s][(k+k_1)^2-\mu^2_s]^{2-d_{\mathcal{U}_s}}}\\ &+ \int\frac{d^4k}{(2\pi)^4}\frac{(1-d_{\mathcal{U}_s})(2k+k_1)^\mu(2k-k_2)^\nu}{(k^2-\mu^2_s)[(k-k_2)^2-\mu^2_s][(k+k_1)^2-\mu^2_s]^{2-d_{\mathcal{U}_s}}}\end{aligned}$$

$$
\begin{aligned}
\Pi_s^{\mu\nu} &= \frac{2\Gamma(4-d_{\mathcal{U}_s})}{\Gamma(d_{\mathcal{U}_s})\Gamma(2-d_{\mathcal{U}_s})^2}\int_0^1 dy\int_0^{1-y} dz\left(\frac{1-y-z}{yz}\right)^{d_{\mathcal{U}_s}-1} \\
&\times \int\frac{d^4k}{(2\pi)^4}\left[\frac{-(k^2-\mu_s^2)g^{\mu\nu}+(2k+k_1)^\mu(2k-k_2)^\nu}{[(k+k_1y-k_2z)^2-\Delta_s^2]^{4-d_{\mathcal{U}_s}}}\right. \\
&+ \left.\frac{(1-d_{\mathcal{U}_s})}{2}\frac{(2k+k_1)^\mu(2k-k_2)^\nu}{[(k+k_1y-k_2z)^2-\Delta_s^2]^{4-d_{\mathcal{U}_s}}}\right],
\end{aligned}
$$

$$
\Delta_s^2 = \mu_s^2 - 2k_1\cdot k_2yz = \mu_s^2 - m_h^2yz
$$

Simplified to

$$
\begin{aligned}
\Pi_s^{\mu\nu} & \\
&= i\frac{(-1)^{2-d_{\mathcal{U}_s}}}{(4\pi)^2}\frac{2(\mu_s^2)^{d_{\mathcal{U}_s}-2}}{\Gamma(d_{\mathcal{U}_s})\Gamma(2-d_{\mathcal{U}_s})}\int_0^1 dy\int_0^{1-y} dz\left(\frac{1-y-z}{yz}\right)^{d_{\mathcal{U}_s}-1} \\
&\times \frac{4yz}{(1-4\tau_{\mathcal{U}_s}yz)^{2-d_{\mathcal{U}_s}}}[(k_1^\nu k_2^\mu - k_1\cdot k_2 g^{\mu\nu}) + (1-d_{\mathcal{U}_s})g^{\mu\nu}(\ldots)],
\end{aligned}
$$

Finally

$$iM_s = i\frac{\alpha_{em}}{2\pi v} Q^2_{\mathcal{U}_s} A^s_{\mathcal{U}}(d_{\mathcal{U}_s}, \tau_{\mathcal{U}_s})(k_1^\nu k_2^\mu - k_1 \cdot k_2 g^{\mu\nu})\, \epsilon^*_\mu(k_1)\, \epsilon^*_\nu(k_2)$$

$$\begin{aligned} A^s_{\mathcal{U}}(d_{\mathcal{U}_s}, \tau_{\mathcal{U}_s}) &= -\frac{A(d_{\mathcal{U}_s})}{2\sin(d_{\mathcal{U}_s}\pi)} \frac{(2-d_{\mathcal{U}_s})}{\Gamma(d_{\mathcal{U}_s})\Gamma(2-d_{\mathcal{U}_s})} \\ &\times \int_0^1 dy \int_0^{1-y} dz \left(\frac{1-y-z}{yz}\right)^{d_{\mathcal{U}_s}-1} \frac{4yz}{(1-4\tau_{\mathcal{U}_s} yz)^{2-d_{\mathcal{U}_s}}} \end{aligned}$$

In practice when $\tau_{\mathcal{U}_s} \ll 1$

$$\tau_{\mathcal{U}_s} = m_h^2/4\mu_s^2$$

Simlpified to

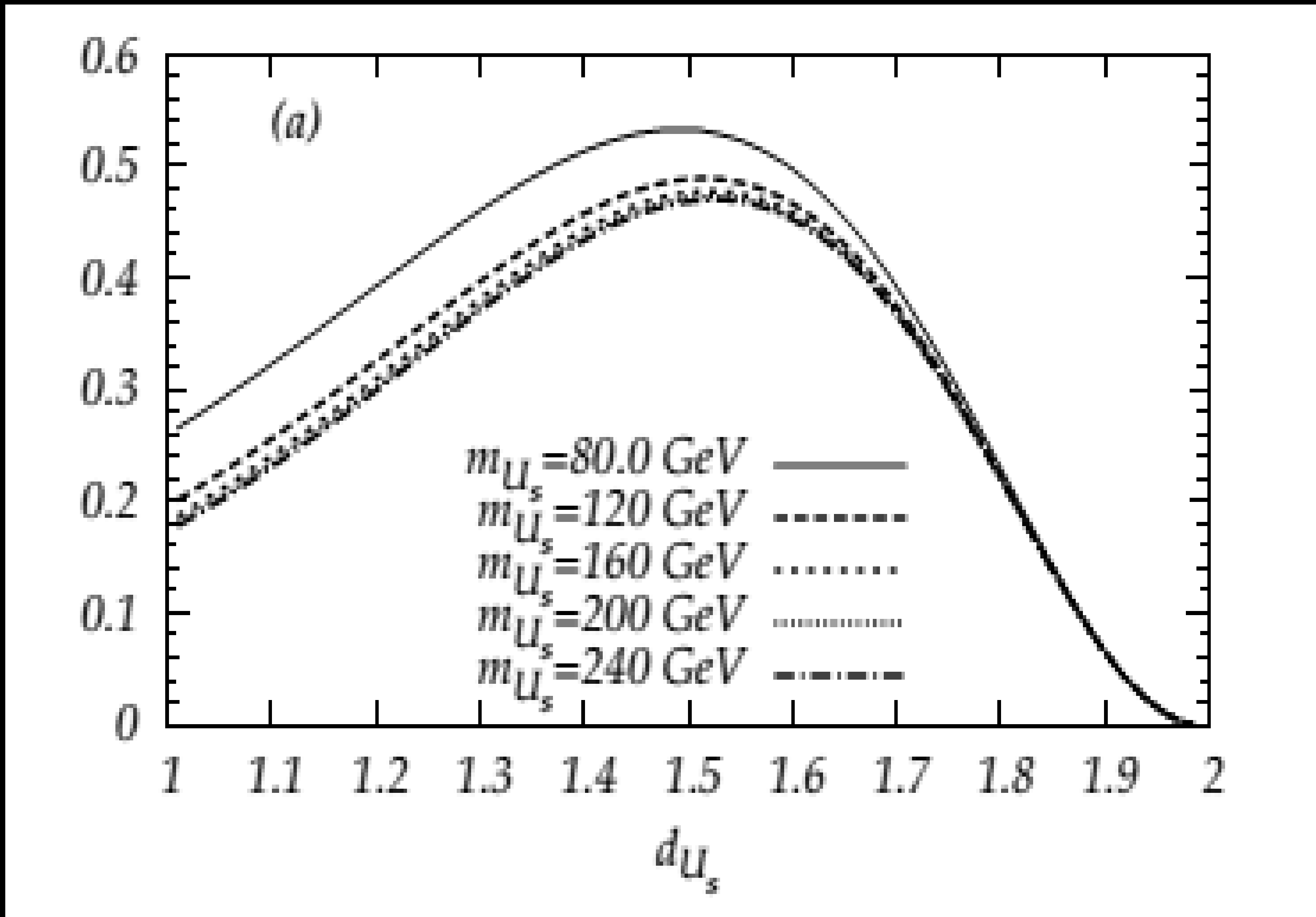
Asymptotic form

$$\begin{aligned} A_{\mathcal{U}_s}(d_{\mathcal{U}_s}, \tau_{\mathcal{U}_s}) &= \frac{A(d_{\mathcal{U}_s})}{2\sin(\pi d_{\mathcal{U}_s})\Gamma(d_{\mathcal{U}_s})\Gamma(2-d_{\mathcal{U}_s})} \\ &\times [\alpha B(3-d_{\mathcal{U}_s},3)B(3-d_{\mathcal{U}_s},d_{\mathcal{U}_s}) \\ &+ \beta B(4-d_{\mathcal{U}_s},4)B(4-d_{\mathcal{U}_s},d_{\mathcal{U}_s})] \end{aligned}$$

$$\alpha = 4(2-d_{\mathcal{U}_s})$$

$$\beta = 16(2-d_{\mathcal{U}_s})^2\tau_{\mathcal{U}_s}$$

$B(a,b)$ Beta function



0.6
0.5
0.4
0.3
0.2
0.1
0
(a)
m_{U_s}=80.0 GeV
m_{U_s}=120 GeV
m_{U_s}=160 GeV
m_{U_s}=200 GeV
m_{U_s}=240 GeV
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
d_{U_s}

Discussion

$\mu_{u_s} \in 80\ GeV - 250\ GeV$

A_{u_s} Very sensitive to canonical scale dimension d_{u_i}

Vanishes in the limit $d_{u_i} \to 2$

reflecting decoupling of scalar unparticles From Higgs sector

Natural: Can be explained by the fact that scalar unparticles becomes non dynamical when $d_{u_i} \to 2$ (see contribution vaccum polarization)

When μ_{u_s} approaches the lower limit of CSB scale v

A_{u_s} Becomes less sensitive to μ_{u_s}

This can be explained by considering the limit $\tau_{u_s} \ll 1$

$\mu_{u_s} \sim v$ (Use previous asymptotic form)

Were coefficient β vanishes

and A_{u_s} Is dominated only by term proportional to α

fermionic contribution

$$i\mathcal{M}_f^a = 4\pi\alpha_{em} Q_{\mathcal{U}_f}^2 \frac{2(\mu_f^2)^{2-d_{\mathcal{U}_f}}}{v} \mathcal{M}_f^{\mu\nu,a}\, \epsilon_\mu^*(k_1)\, \epsilon_\nu^*(k_2)$$

$$i\mathcal{M}_f^b = -4\pi\alpha_{em} Q_{\mathcal{U}_f}^2 \frac{2(\mu_f^2)^{2-d_{\mathcal{U}_f}}}{v} \mathcal{M}_f^{\mu\nu,b}\, \epsilon_\mu^*(k_1)\, \epsilon_\nu^*(k_2)$$

Where

$$\mathcal{M}_f^{\mu\nu,a} = (-1)^{3/2-d_{\mathcal{U}_f}} \frac{A(d_{\mathcal{U}_f} - 1/2)}{2\cos(d_{\mathcal{U}_f}\pi)} 2(2 - d_{\mathcal{U}_f})^2 \Pi_f^{\mu\nu,a}$$

$$\Pi_f^{\mu\nu,a} = \int \frac{d^4k}{(2\pi)^4} \frac{N^{\mu\nu}}{(k^2 - \mu_f^2)[(k-k_2)^2 - \mu_f^2][(k+k_1)^2 - \mu_f^2]^{\frac{5}{2}-d_{\mathcal{U}_f}}}$$

$$
\begin{aligned}
N^{\mu\nu} &= Tr\{\gamma^{\mu}(\not{k}+\not{k}_1+\mu_f)(\not{k}-\not{k}_2+\mu_f)\gamma^{\nu}(\not{k}+\mu_f)\} \\
&= 8\mu_f\left[4k^{\mu}k^{\nu}-k^2g^{\mu\nu}+k_1^{\nu}k_2^{\mu}-2k^{\mu}k_2^{\nu}-2k^{\mu}k_1^{\nu}-k_1^{\mu}k_2^{\nu}+g^{\mu\nu}(\mu_f^2-k_1\cdot k_2)\right]
\end{aligned}
$$

$$
\mathcal{M}_f^{\mu\nu,b} = (-1)^{3/2-d_{\mathcal{U}_f}}\frac{A(d_{\mathcal{U}_f}-1/2)}{2\cos(d_{\mathcal{U}_f}\pi)}2(3/2-d_{\mathcal{U}_f})\Pi_f^{\mu\nu,b}
$$

$$
\Pi_f^{\mu\nu,b} = \int\frac{d^4k}{(2\pi)^4}\frac{G^{\mu\nu}}{(k^2-\mu_f^2)[(k-k_2)^2-\mu_f^2]^2[(k+k_1)^2-\mu_f^2]^{\frac{5}{2}-d_{\mathcal{U}_f}}}
$$

$$
\begin{aligned}
G^{\mu\nu} &= Tr\{(\not{k}+\not{k}_1+\mu_f)(\not{k}-\not{k}_2+\mu_f)(\not{k}-\not{k}_2-\mu_f)\} \\
&\times\left[2(k^2-\mu_f^2)g^{\mu\nu}-2(2k+k_1)^{\mu}(2k-k_2)^{\nu}\right] \\
&= 8\mu_f[(k-k_2)^2-\mu_f^2]\left[(k^2-\mu_f^2)g^{\mu\nu}-(2k+k_1)^{\mu}(2k-k_2)^{\nu}\right]
\end{aligned}
$$

$$\mathcal{M}_f = -\frac{\alpha_{em}}{2\pi v} Q_{\mathcal{U}_f}^2 A_{\mathcal{U}}^f(d_{\mathcal{U}_f}, \tau_{\mathcal{U}_f})(k_1^\nu k_2^\mu - k_1 \cdot k_2 g^{\mu\nu})\, \epsilon_\mu^*(k_1)\, \epsilon_\nu^*(k_2)$$

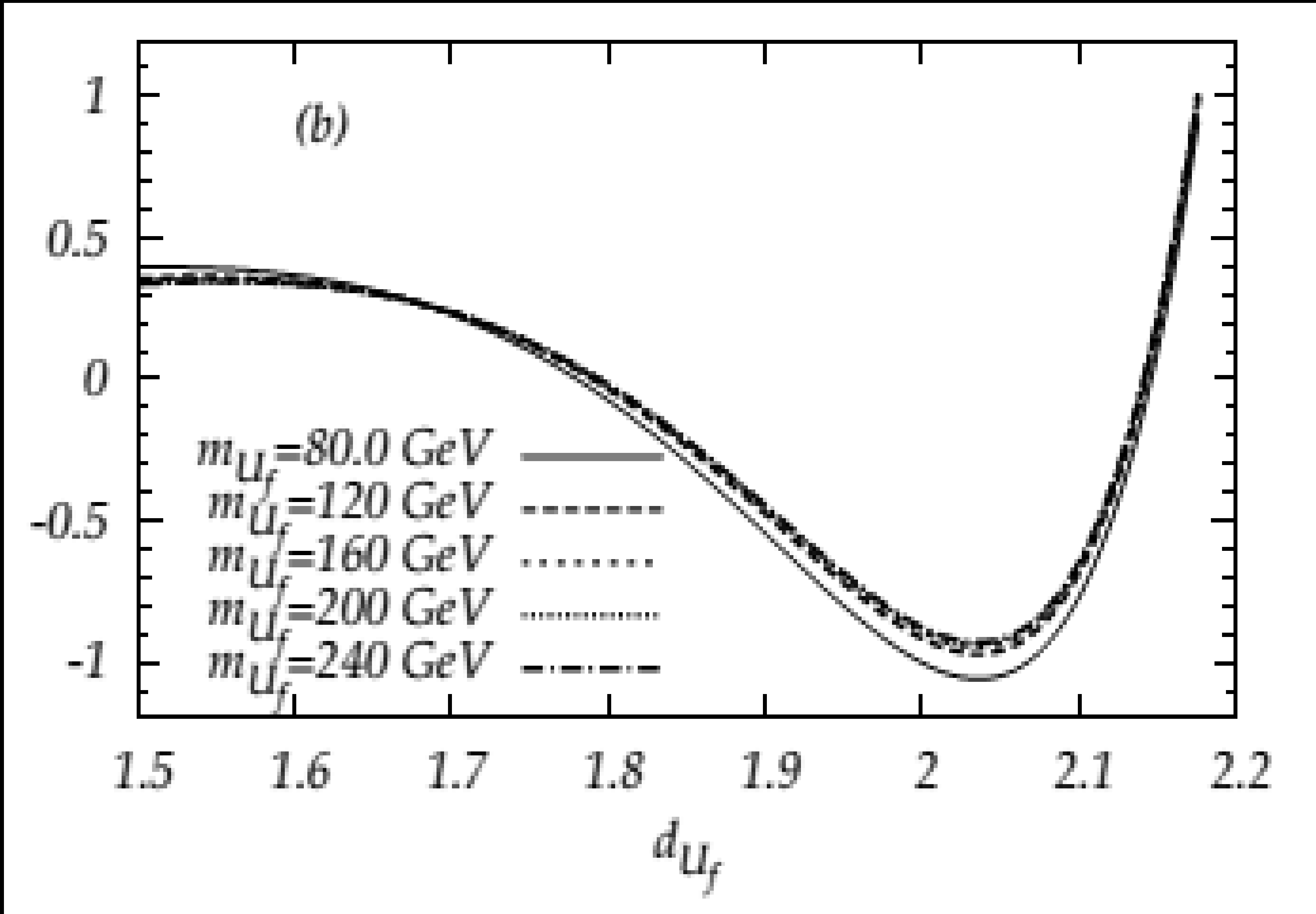
Finally

$$\begin{aligned} A_{\mathcal{U}}^f(d_{\mathcal{U}_f}, \tau_{\mathcal{U}_f}) &= \frac{A(d_{\mathcal{U}_f} - 1/2)}{2\cos(\pi d_{\mathcal{U}_f})} \frac{1}{\Gamma(d_{\mathcal{U}_f} - 1/2)\Gamma(5/2 - d_{\mathcal{U}_f})} \\ &\times \int_0^1 dy \int_0^{1-y} dz \left(\frac{1-y-z}{yz}\right)^{3/2 - d_{\mathcal{U}_f}} \\ &\times \frac{4(2-d_{\mathcal{U}_f})^2(1-4yz) + 2(3/2 - d_{\mathcal{U}_f})4yz}{\left(1 - 4\tau_{\mathcal{U}_f} yz\right)^{5/2 - d_{\mathcal{U}_f}}} \end{aligned}$$

When $\tau_{\mathcal{U}_f} \ll 1$

$$
\begin{aligned}
A_{\mathcal{U}_f}(d_{\mathcal{U}_f}, \tau_{\mathcal{U}_f}) \;&=\; \frac{A(d_{\mathcal{U}_f} - 1/2)}{2\cos(\pi d_{\mathcal{U}_f})} \frac{1}{\Gamma(d_{\mathcal{U}_f} - 1/2)\Gamma(5/2 - d_{\mathcal{U}_f})} \\
&\times\; [\gamma B(5/2 - d_{\mathcal{U}_f}, 2) B(5/2 - d_{\mathcal{U}_f}, d_{\mathcal{U}_f} - 1/2) \\
&+\; \delta B(7/2 - d_{\mathcal{U}_f}, 3) B(7/2 - d_{\mathcal{U}_f}, d_{\mathcal{U}_f} - 1/2) \\
&+\; \eta B(9/2 - d_{\mathcal{U}_f}, 4) B(9/2 - d_{\mathcal{U}_f}, d_{\mathcal{U}_f} - 1/2)]
\end{aligned}
$$

$$
\begin{aligned}
\gamma &= 4(2 - d_{\mathcal{U}_f})^2 \,, \\
\delta &= 8\{2(2 - d_{\mathcal{U}_f})^2[(5/2 - d_{\mathcal{U}_f})\tau_{\mathcal{U}_f} - 1] + (3/2 - d_{\mathcal{U}_f})\} \\
\eta &= 32(5/2 - d_{\mathcal{U}_f})\tau_{\mathcal{U}_f}[-2(2 - d_{\mathcal{U}_f})^2 + (3/2 - d_{\mathcal{U}_f})] \,.
\end{aligned}
$$



(b)
1
0.5
0
-0.5
-1
1.5
1.6
1.7
1.8
1.9
2
2.1
2.2
d_{U_f}
m_{U_f}=80.0 GeV
m_{U_f}=120 GeV
m_{U_f}=160 GeV
m_{U_f}=200 GeV
m_{U_f}=240 GeV

Discussion

A_{u_f} Contrary to scalar case, does not vanish in the limit $d_{u_f} \sim 5/2$

As a consequence

spin1/2 unparticles doe not decouple from higgs sector in this critical limit

Can be explained from e.g self energy contribution

Moreover, when d_{u_f} Approaches value 5/2

spinoriel propagator Δ_{u_f} becomes very sensitive to UV regime

and non local fermionic gauged action can not be used

A_{u_f} Very sensitive to canonical scale dimension d_{u_f}

A_{u_f} Changes sign in $1.5 < d_{u_f} < 2.5$

Phenomenology different from scalar case

Can be constructive or destructive contribution

Very important

Main result:

Sign of fermionic contribution changes as a function of d_{u_f}

A_{u_f} Takes larger values for $d_{u_f} \sim 5/2$

A_{u_f} Becomes less sensitive to u_f

For values of u_f Approaching the upper limit of CBS v

This can be explained by considering the limit $\tau_{u_f \ll 1}$

(Use previous asymptotic form) $\mu_{u_f} \sim v$

Some constraints on coupling

Most renormalizable scalar potential

$$V(H,\mathcal{U}_s) \supset \frac{\lambda_{H\mathcal{U}_s}}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}_s}-2}} H^\dagger H\, \mathcal{U}_s^\dagger \mathcal{U}_s + (\mu_s^2)^{2-d_{\mathcal{U}_s}} \mathcal{U}_s^\dagger \mathcal{U}_s + \frac{\lambda_{\mathcal{U}_s}}{(\Lambda_{\mathcal{U}}^2)^{2d_{\mathcal{U}_s}-2}} |\mathcal{U}_s^\dagger \mathcal{U}_s|^2 - \mu_H^2 |H^\dagger H| + \lambda_H |H^\dagger H|^2$$

with $\lambda_{H\mathcal{U}_s} < 0$

Stability of scalar potential $\Rightarrow$ $|\lambda_{H\mathcal{U}_s}| \le 2\sqrt{\lambda_H \lambda_{\mathcal{U}_s}}$

for $m_h \simeq 125$ GeV $\Rightarrow$ $\lambda_H = m_h^2/2v^2$ $\quad \lambda_H \simeq 0.13$

$$|\lambda_{H\mathcal{U}_s}| \le 2\sqrt{\lambda_H \lambda_{\mathcal{U}_s}} = 0.72\sqrt{\lambda_{\mathcal{U}_s}}$$

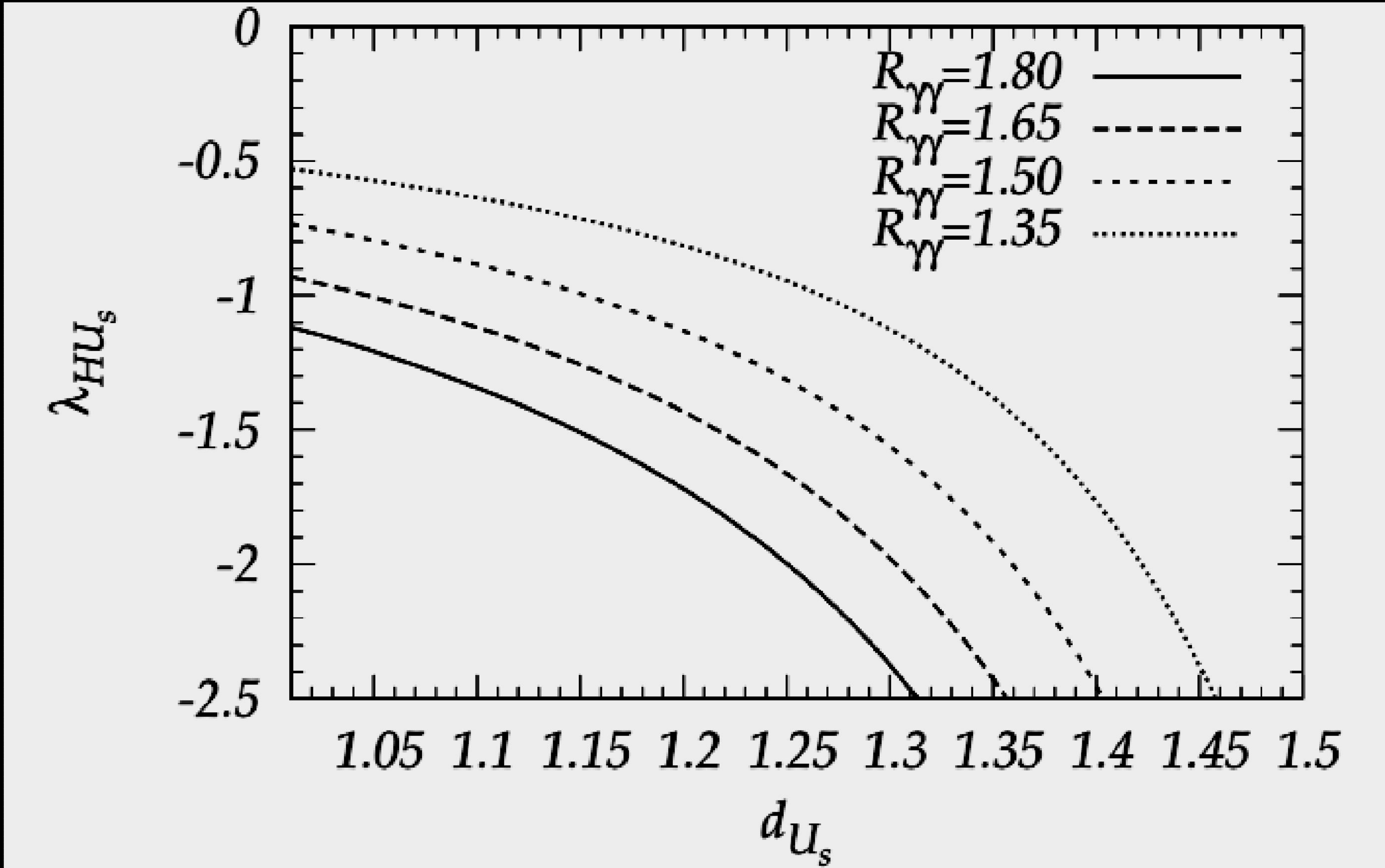
Perturbativity (unitarity)

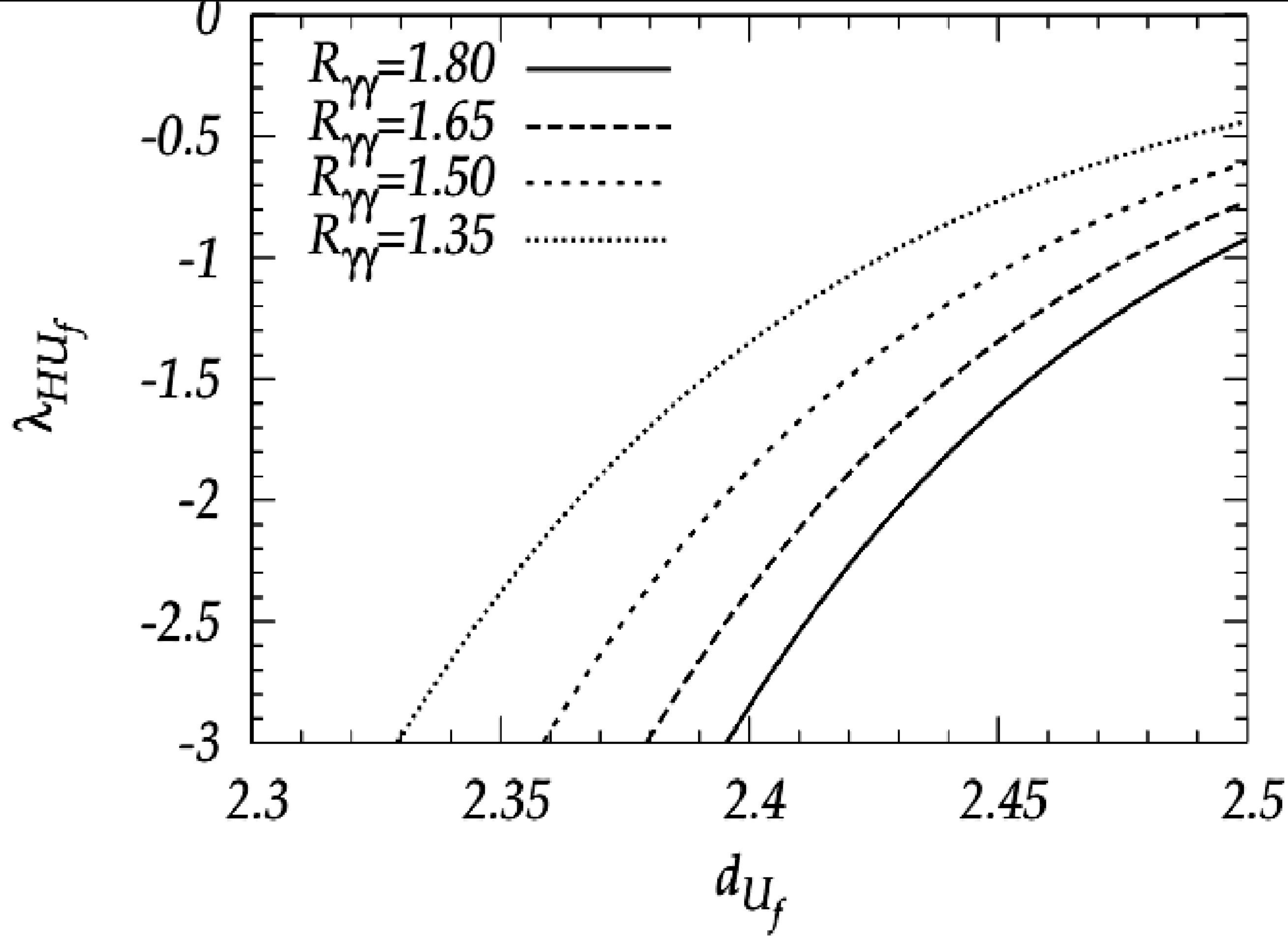
$$\lambda_{\mathcal{U}_s}/4\pi \sim \mathcal{O}(1)$$

Thus

$$-2.5 \lesssim \lambda_{H\mathcal{U}_s} < 0$$

Numerical results





$R_{\gamma\gamma}$=1.80
$R_{\gamma\gamma}$=1.65
$R_{\gamma\gamma}$=1.50
$R_{\gamma\gamma}$=1.35
0
-0.5
-1
-1.5
-2
-2.5
-3
λ_{HU_f}
2.3
2.35
2.4
2.45
2.5
d_{U_f}

Discussion

Contour lines in $(d_{u_s}, \lambda_{Hu_s})$ and $(d_{u_f}, \lambda_{Hu_f})$ planes

For fixed values $\mu_{u_s} = 100GeV$ $\mu_{u_f} = 500GeV$

$\Lambda_{u_s} = \Lambda_{u_f} = 1\,TeV$ $Q_{u_s} = 1$ $Q_{u_f} = 2$

$R_{\gamma\gamma} = 1.8$ Possible for $\rightarrow$ $-2.5 \lesssim \lambda_{Hu_s} \lesssim -1.1$ $-3 \lesssim \lambda_{Hu_f} \lesssim -1.35$

$1 \lesssim d_{u_s} \lesssim 1.31$ $2.395 \lesssim d_{u_f} \lesssim 2.5$

$R_{\gamma\gamma} = 1.65$ Possible for $\rightarrow$ $-2.5 \lesssim \lambda_{Hu_s} \lesssim -0.9$ $-3 \lesssim \lambda_{Hu_f} \lesssim -0.8$

$1 \lesssim d_{u_s} \lesssim 1.36$ $2.38 \lesssim d_{u_f} \lesssim 2.5$

TABLE I. Tuning the gauge unparticle parameters to fit the the diphoton Higgs decay rate $R_{\gamma\gamma}$ to data reported by the ATLAS and CMS experiments.

$R_{\gamma\gamma}$	$d_{\mathcal{U}_s}$	$d_{\mathcal{U}_f}$	$Q_{\mathcal{U}_s}$	$Q_{\mathcal{U}_f}$
ATLAS	1-2	2.15-2.2	± 1	± 1
$\simeq$	1.3,1.7	~1.5-1.6	± 2	± 1
$1.65 \pm 0.24^{+0.25}_{-0.18}$	1.5	1.8	± 2	± 2
	1.8	1.7	± 2	± 2
CMS	1.1,1.8	2	± 1	± 1
$\simeq$	1.9	2.1	± 2	± 1
$0.78^{+0.28}_{-0.26}$	1.1,1.8	1.9	± 2	± 2

Phenomenological analysis of correlations between the channels

$H \longrightarrow \gamma\gamma$ and $H \longrightarrow Z\gamma$

If unparticles carry SM EW charges

Then expect

Deviation from SM of Production mode $H \longrightarrow Z\gamma$

Correlation between $H \longrightarrow \gamma\gamma$ and $H \longrightarrow Z\gamma$

Is interesting $\Rightarrow$ Because it is model depend

Thus A way to distinguish between our model and BSM models

Lagrangian

$$\mathcal{L}_{\mathcal{U}_s} = (D_\mu \mathcal{U}_s)^\dagger (D^\mu \mathcal{U}_s) - \mu_s^2 \mathcal{U}_s^\dagger \mathcal{U}_s - \lambda_{H\mathcal{U}_s} (H^\dagger H)(S^\dagger \mathcal{U}_s) - \tilde{\lambda}_{H\mathcal{U}_s} (H^\dagger T^a H)(\mathcal{U}_s^\dagger T^a \mathcal{U}_s)$$

Where

$$D_\mu \mathcal{U}_s = (\partial_\mu + ieQ_{\mathcal{U}_s} A_\mu + ieg_{Z\mathcal{U}_s\mathcal{U}_s} Z_\mu)\mathcal{U}_s$$

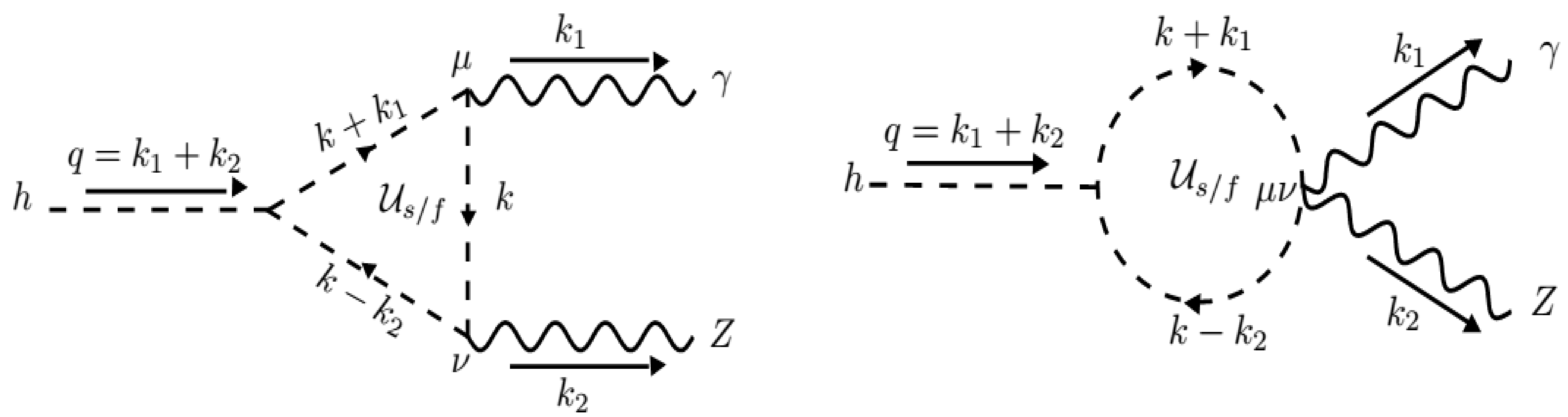
$$g_{ZSS} = (T_3^{(\mathcal{U}_s)} - Q_{\mathcal{U}_s} s_W^2)/(s_W c_W)$$

$$s_W = \sin\theta_W \text{ and } c_W = \cos\theta_W$$

θ_W Weinberg angle

H= doublet of SM

$H \to Z\gamma$



(a) (b)

amplitude

$$
\begin{aligned}
\mathcal{M} &= \left(\mathcal{M}_1^{\mu\nu} + \mathcal{M}_2^{\mu\nu} + \mathcal{M}_3^{\mu\nu}\right) \epsilon_\mu^{A*}(k_1)\, \epsilon_\nu^{Z*}(k_2) \\
&= e^2 Q g_{Z\mathcal{U}_s\mathcal{U}_s} \lambda_{H\mathcal{U}_s} v \left[\Pi_1^{\mu\nu}(k) + \Pi_2^{\mu\nu}(k) - 2\Pi_3^{\mu\nu}(k)\right] \epsilon_\mu^{A*}(k_1)\, \epsilon_\nu^{Z*}(k_2) \\
&= 2e^2 Q g_{Z\mathcal{U}_s\mathcal{U}_s} \lambda_{H\mathcal{U}_s} v 2(2 - d_{\mathcal{U}_s}) \frac{A(d_{\mathcal{U}_s})}{2\sin \pi d_{\mathcal{U}_s}} \left[\Pi_1^{\mu\nu}(k) - \Pi_3^{\mu\nu}(k)\right] \epsilon_\mu^{A*}(k_1)\, \epsilon_\nu^{Z*}(k_2)
\end{aligned}
$$

$$\Pi_1^{\mu\nu}(k) = \int \frac{d^4k}{(2\pi)^4} \frac{(2-d_{\mathcal{U}_s})(2k+k_1)^\mu (2k-k_2)^\nu}{(k^2-\mu_s^2)[(k+k_2)^2-\mu_s^2][(k+k_1)^2-\mu_s^2]^{1-d_{\mathcal{U}_s}}}$$

$$\Pi_2^{\mu\nu}(k) = \int \frac{d^4k}{(2\pi)^4} \frac{(2-d_{\mathcal{U}_s})(2k-k_1)^\mu (2k+k_2)^\nu}{(k^2-\mu_s^2)[(k+k_2)^2-\mu_s^2][(k-k_1)^2-\mu_s^2]^{1-d_{\mathcal{U}_s}}}$$

$$\Pi_3^{\mu\nu}(k) = \int \frac{d^4k}{(2\pi)^4} \frac{2(k^2-\mu_s^2)g^{\mu\nu} + 2(1-d_{\mathcal{U}_s})(k+k_1)^\mu (k-k_2)^\nu}{[(k+k_1)^2-\mu_s^2)][(k-k_2)^2-\mu_s^2]} \,.$$

$Q = T_3 + Y/2$ $\qquad \tau_{\mathcal{U}_s} = 4\mu_s^2/m_h^2$ $\qquad \lambda_{\mathcal{U}_s} = 4\mu_s^2/m_Z^2$

Result

$$\Gamma(h \to Z\gamma) = \frac{\alpha^2}{512\pi^3} m_h^3 \left(1 - \frac{m_Z^2}{m_h^2}\right)^3 \left| A_{SM}^{Z\gamma} - \frac{\lambda_{H\mathcal{U}_s} v}{\mu_s^2} \left(2 \sum_{T_3} Q_S \cdot g_{Z\mathcal{U}_s\mathcal{U}_s}\right) A_0^{Z\gamma}(\tau_{\mathcal{U}_s}, \lambda_{\mathcal{U}_s}) \right|^2$$

where

$$\mathcal{A}_{SM}^{Z\gamma} = \frac{2}{v}\left[\cot\theta_W A_1^{Z\gamma}(\tau_W,\lambda_W) + N_c \frac{(2Q_t)(T_3^{(t)} - 2Q_t s_W^2)}{s_W c_W} A_{1/2}^{Z\gamma}(\tau_t,\lambda_t)\right]$$

$$\tau_i = 4m_i^2/m_h^2 \qquad \lambda_i = 4m_i^2/m_Z^2$$

Ratio R

$$R_{Z\gamma} \equiv \frac{\Gamma(h\to Z\gamma)}{\Gamma_{SM}(h\to Z\gamma)} = \left|1 - \tilde{N}\lambda_{HS}\frac{v^2}{m_S^2}\left(2\sum_{T_3} Q_S \cdot g_{ZSS}\right)\frac{A_0^{Z\gamma}(\tau_S,\lambda_S)}{v\mathcal{A}_{SM}^{Z\gamma}}\right|^2$$

$\tilde{N}$ Degeneracy of the multiplet

Notice that

$$R_{\gamma\gamma} \equiv \frac{\Gamma(h\to\gamma\gamma)}{\Gamma_{SM}(h\to\gamma\gamma)} = \left|1 + \tilde{N}\frac{\lambda_{HS}}{2}\frac{v^2}{m_S^2}\left(\sum_{T_3} Q_S^2\right)\frac{A_0^{\gamma\gamma}(\tau_S)}{A_1^{\gamma\gamma}(\tau_W) + N_c Q_t^2 A_{1/2}^{\gamma\gamma}(\tau_t)}\right|^2$$

Main results

According to EW gauge symmetry

One expect that

particle responsible for observed enhancement in diphoton channel

Could also lead to modification of Z Gamma rate

Z Gamma amplitude — Depends on electric charge + isospin

Gamma Gamma amplitude — Depends only on electric charge

Thus

Combined analysis of decay widths in these modes can provide us with important information on EW quantum numbers of unparticles inside the loop

Singlets and doublets scalars

$$(\mathrm{I}): \quad g^{(1)}_{ZSS} = \frac{-Q_S s^2_W}{s_W c_W}$$

$$(\mathrm{II}): \quad g^{(2)}_{ZSS} = \frac{1}{s_W c_W}\left(\frac{1}{2} - Q_S s^2_W\right)$$

$Q_{u_s} = 1$ Corresponds to

$$\left\{ \begin{array}{l} (T, Y) = (0.2) \\ (T, Y) = (1/2.1) \end{array} \right.$$

Due to different representation of isospin T
Even if $Q_{u_s} = 1$ is the same

→ Enhancement

↓

In Gamma Gamma and Z Gamma modes will be different because

Case I

Case II

Has small suppression

Has an enhancement

Enhancement of gamma gamma mode by constructive interference

Requires that

$$\lambda_{Hu_s} < 0$$

Notice that $A_0^{\gamma\gamma}(\tau_s)$ and $A_{SM}^{\gamma\gamma}$ have opposite sign

In this case Z gamma mode

Depends only on sign of $Q_s g_{Zss}$

As $\lambda_{Hu_s} < 0$

Corresponding to both positive values of $A_0^{Z\gamma}(\tau_s)$ and $A_{SM}^{Z\gamma}$

In physical region of interest

As a result

$R_{Z\gamma}$

Is enhanced if $Q_s g_{Zss} > 0$

Suppressed if $Q_s g_{Zss} < 0$

NB:

$$Q_S \cdot g_{ZSS} = \frac{1}{s_W c_W}\left(T_3 + \frac{Y}{2}\right)\left(T_3 c_W^2 - \frac{Y s_W^2}{2}\right)$$

When weak isospin $T \gg Y$ ⇒ $Q_s g_{Zss} > 0$

⇒ Enhancement in $Z\gamma$ Channel/SM

When weak isospin $T \ll Y$ ⇒ $Q_s g_{Zss} < 0$

⇒ Suppression in $Z\gamma$ Channel/SM

New 750 GeV resonnance at LHC

Work in progress

Our starting model economical 331 model

+

Gauged unparticles idea

THANK YOU

$h \to \gamma\gamma$

$$
\begin{aligned}
A_0^{\gamma\gamma}(x) &= -x^2\left[\frac{1}{x} - f\left(\frac{1}{x}\right)\right] \,, \\
A_{1/2}^{\gamma\gamma}(x) &= 2x^2\left[\frac{1}{x} + \left(\frac{1}{x} - 1\right) f\left(\frac{1}{x}\right)\right] \,, \\
A_1^{\gamma\gamma}(x) &= -x^2\left[\frac{2}{x^2} + \frac{3}{x} + 3\left(\frac{2}{x} - 1\right) f\left(\frac{1}{x}\right)\right]
\end{aligned}
$$

$$
f(x) = \arcsin^2\sqrt{x} \,.
$$

$h \to Z\gamma$

$$\Gamma(h \to Z\gamma) = \frac{G_F \alpha_{em} m_h^3}{16\sqrt{2}\pi^3} \left(1 - \frac{m_Z^2}{m_h^2}\right)^3 |A_W + A_F|^2$$

$$\begin{aligned} A_W &= -\cos\theta_W \left\{ \left[\left(1 + \frac{2}{\tau_W}\right) \tan^2\theta_W - \left(5 + \frac{2}{\tau_W}\right) \right] I_1(\tau_W, \lambda_W) \right. \\ &+ \left. 4(3 - \tan^2\theta_W) I_2(\tau_W, \lambda_W) \right\} \end{aligned}$$

$$A_F = \sum_f N_{Cf} \frac{-2e_f (T_{3f} - 2e_f \sin^2\theta_W)}{\sin\theta_W \cos\theta_W} \left[I_1(\tau_F, \lambda_F) - I_2(\tau_F, \lambda_F) \right]$$

[redacted]

$$\tau_F = \frac{4m_f^2}{m_h^2}, \quad \lambda_F = \frac{4m_f^2}{m_Z^2}, \quad \tau_W = \frac{4m_W^2}{m_h^2}, \quad \lambda_W = \frac{4m_W^2}{m_Z^2}$$

[redacted]

$$I_1(x,y) = \frac{xy}{2(x-y)} + \frac{x^2y^2}{2(x-y)^2}[f(x) - f(y)] + \frac{x^2y^2}{(x-y)^2}[g(x) - g(y)]$$

[redacted]

$$I_1(x,y) = -\frac{xy}{2(x-y)}[f(x) - f(y)]$$

[redacted]

[redacted]

$$g(\tau) = \begin{cases} \sqrt{\tau - 1}\sin^{-1}(1/\sqrt{\tau}) & \text{pour } \tau \geq 1 \\ \frac{1}{2}\sqrt{1-\tau}\left[\ln\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right) - i\pi\right] & \text{pour } \tau < 1 \end{cases}$$

[redacted]

$$A_0^{\gamma Z}(x,y) \quad = \quad \mathcal{I}_1(x,y)$$

[redacted]

$$A_{1/2}^{\gamma Z}(x,y) \quad = \quad 4\left[\mathcal{I}_1(x,y) - \mathcal{I}_2(x,y)\right]$$

[redacted]

$$A_1^{\gamma Z}(x,y) \quad = \quad 4\left(3 - \frac{s_W^2}{c_W^2}\right)\mathcal{I}_2(x,y) + \left[\left(1+\frac{2}{x}\right)\frac{s_W^2}{c_W^2} - \left(5+\frac{2}{x}\right)\right]\mathcal{I}_1(x,y)$$

[redacted]

$$\mathcal{I}_1(x,y) \quad = \quad \frac{xy}{2(x-y)} + \frac{x^2y^2}{2(x-y)^2}\left[f\left(\frac{1}{x}\right) - f\left(\frac{1}{y}\right)\right] + \frac{x^2y}{(x-y)^2}\left[g\left(\frac{1}{x}\right) - g\left(\frac{1}{y}\right)\right]$$

[redacted]

$$I_2(x,y) = -\frac{xy}{2(x-y)}\left[f\left(\frac{1}{x}\right) - f\left(\frac{1}{y}\right)\right]$$

$$g(x) = \sqrt{\frac{1}{x} - 1}\,\arcsin\sqrt{x}$$