# Topics on lattice studies of the Higgs-Yukawa model

Composite models, electroweak physics and the LHC

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# The theory

$$S^{\text{cont}}[\bar{\psi}^{(c)}, \psi^{(c)}, \varphi^{(c)}] = \int d^4x \left\{ \frac{1}{2} \left(\partial_\mu \varphi^{(c)}\right)^\dagger \left(\partial^\mu \varphi^{(c)}\right) + \frac{1}{2} m_0^2 \varphi^{(c)\dagger} \varphi^{(c)} + \frac{\lambda_0}{4} \left(\varphi^{(c)\dagger}\varphi^{(c)}\right)^2 \right\}$$
$$+ \int d^4x \left\{ \overline{t^{(c)}} \partial\!\!\!/ t^{(c)} + \overline{b^{(c)}} \partial\!\!\!/ b^{(c)} + y_{b_0} \overline{\psi_L^{(c)}} \varphi^{(c)} b_R^{(c)} + y_{t_0} \overline{\psi_L^{(c)}} \tilde{\varphi}^{(c)} t_R^{(c)} + h.c. \right\}$$

Usual Higgs-Yukawa model with two flavours, degenerate Yukawa coupling, no gauge fields.

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$$a\varphi^{(\text{latt})} = \begin{pmatrix} \Phi^2 + i\Phi^1 \\ \Phi^4 - i\Phi^3 \end{pmatrix}$$

$$S_\Phi^{\text{latt}} = \sum_{\alpha=1}^{4} \left\{ -\sum_{x,\mu} \Phi_x^\alpha \Phi_{x+\hat{\mu}}^\alpha + \sum_x \left[ \frac{1}{2}(8 + \bar{m}_0^2) \Phi_x^\alpha \Phi_x^\alpha + \frac{\lambda_0}{4} \left(\Phi_x^\alpha \Phi_x^\alpha\right)^2 \right] \right\}$$

O(4) scalar sector

$$\Phi^\alpha = \sqrt{2\kappa}\phi^\alpha, \quad \lambda_0 = \frac{\hat{\lambda}}{\kappa^2}, \quad \bar{m}_0^2 = \frac{1 - 2\hat{\lambda} - 8\kappa}{\kappa}$$

$$S_\phi^{\text{latt}} = \sum_{\alpha=1}^{4} \left\{ -2\kappa \sum_{x,\mu} \phi_x^\alpha \phi_{x+\hat{\mu}}^\alpha + \sum_x \left[ \phi_x^\alpha \phi_x^\alpha + \hat{\lambda} \left(\phi_x^\alpha \phi_x^\alpha - 1\right)^2 \right] \right\}$$

"Ising form" and the hopping parameter

---

$$S_f^{\text{latt}} = \sum_x \bar{\psi}_x \left[ D^{ov} + P_+ \phi_x^\alpha \theta_\alpha^\dagger \text{diag}(\hat{y}_t, \hat{y}_b) \hat{P}_+ + P_- \text{diag}(\hat{y}_t, \hat{y}_b) \phi_x^\alpha \theta_\alpha \hat{P}_- \right] \psi_x$$

Lattice chiral fermions

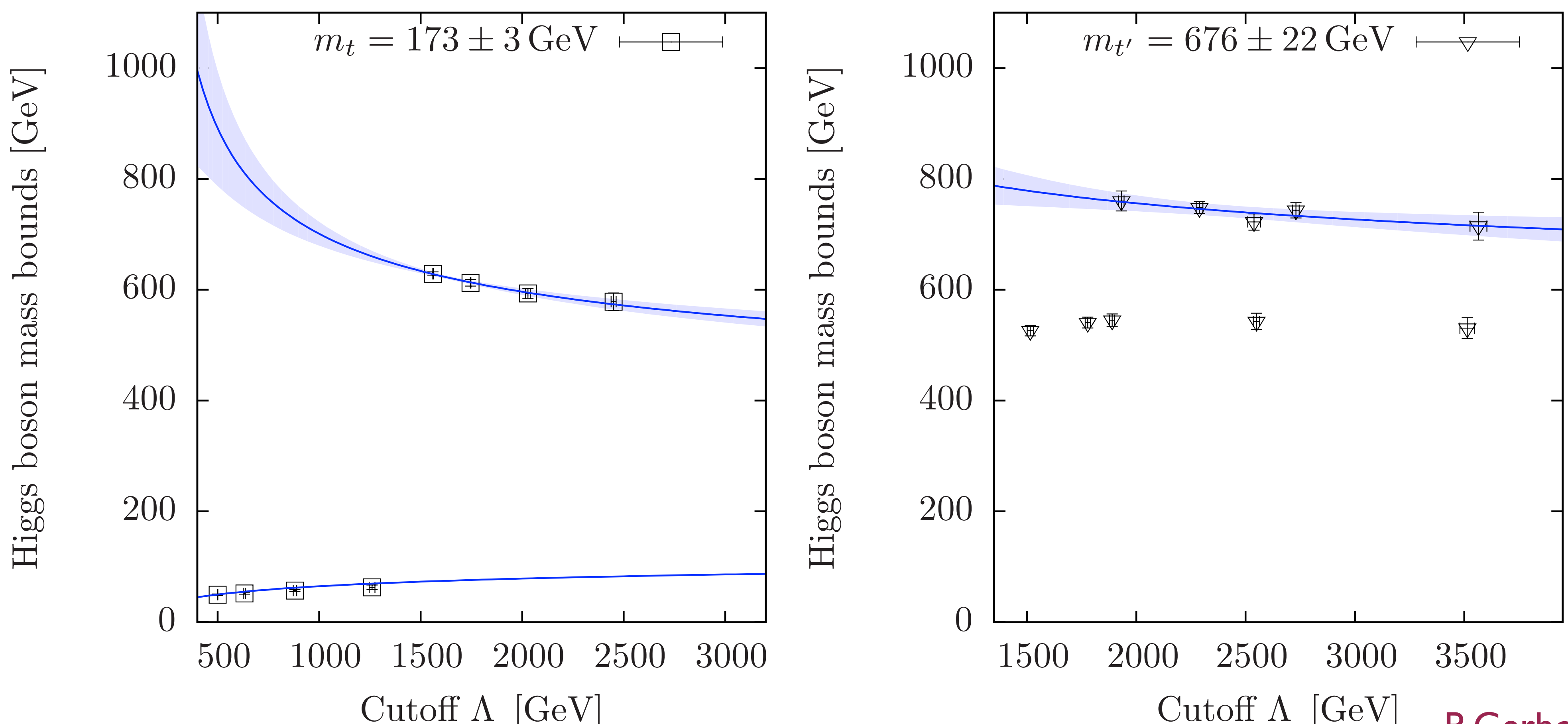
$$\theta_{1,2,3} = -i\tau_{1,2,3}, \; \theta_4 = 1_{2\times 2}$$

Also introduce $Y = y^2$

# What the lattice did in the near past

* In the far past (circa 1990) there were many exploratory studies.

# Higgs mass bounds v.s. fermion mass

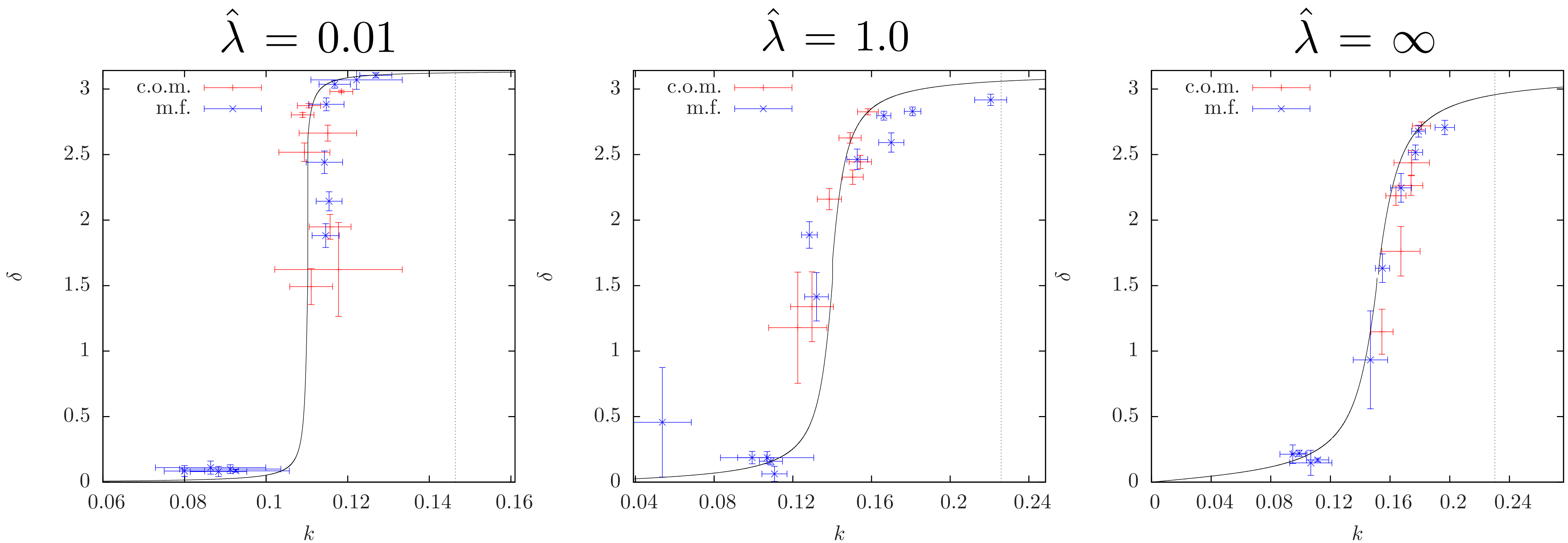



P. Gerhold and K. Jansen, 2011

* Constraints on the masses of extra-generation fermions.

# Higgs width (decay to Goldstone modes)

Study of the two-Goldstone elastic scattering *a'la* Luscher's method.




| $\hat{\lambda}$ | $aM_H^p$ | $aM_G^p$ | $m_t$ [GeV] | $av_R$ | $\Lambda$ [GeV] | $aM_H$ | $a\Gamma_H$ |
|---|---|---|---|---|---|---|---|
| 0.01 | 0.278(1) | 0.085(2) | 174(1) | 0.2786(3) | 883(1) | 0.278(3) | 0.0018(14) |
| 1.0 | 0.386(28) | 0.133(4) | 179(2) | 0.1637(5) | 1503(5) | 0.383(6) | 0.0169(4) |
| $\infty$ | 0.405(4) | 0.129(1) | 178(1) | 0.1539(2) | 1598(2) | 0.403(6) | 0.037(9) |

P. Gerhold, K. Jansen and Kallarackal, 2012

Staying below the inelastic threshold, implemented through coupling the scalar fields to external source.

# Outline for the rest of the talk

- General issue and strategy.
- Inclusion of a dimension-6 operator.
- Triviality of the 4-dimensional Higgs-Yukawa model.
    - ★ Strategy: finite-size scaling.
    - ★ Status of the numerical test.
- Outlook.

# General issue: The continuum limit

$$a \to 0 \quad \text{or} \quad \Lambda \to \infty$$

- Supercomputers only know "pure numbers".
- All couplings are rescaled to be in lattice units.
- For a theory with asymptotic freedom, and without additive mass renormalisation, e.g., QCD:

$$g_0^2(a) \xrightarrow{a\to 0} 0, \; am_0 \xrightarrow{a\to 0} 0$$

while

$$g_R^2(\mu, a) \stackrel{a\to 0}{=} \text{finite}, \; aM_R \xrightarrow{a\to 0} 0 \text{ with } M_R = \text{finite and} \ll \Lambda.$$

⇒ Keep lowering the dimensionless bare couplings.

# General issue: The continuum limit

$$a \to 0 \quad \text{or} \quad \Lambda \to \infty$$

- A trivial theory with additive mass renormalisation:

$$g_0^2(a) \overset{a\to 0}{\longrightarrow} \text{finite}, \; am_0 \overset{a\to 0}{\longrightarrow} \text{finite}$$

while

$$g_R^2(\mu, a) \overset{a\to 0}{=} 0, \; am_R \overset{a\to 0}{\longrightarrow} 0.$$

- In practice, we input the bare coupling:

$$g_0^2(a), \; am_0 = \text{arbitrary number.}$$

➡ Scanning in bare couplings, and keep the cut-off.

# General strategy: The continuum limit

$$a \to 0 \quad \text{or} \quad \Lambda \to \infty$$

- The key point is the separation of the scales.
- It can be achieved at 2nd-order bulk phase transitions:

$$\xi / a \longrightarrow \infty.$$

non-thermal

- Condensed matter physics:

  At fixed $a$, take $\xi \to \infty$.

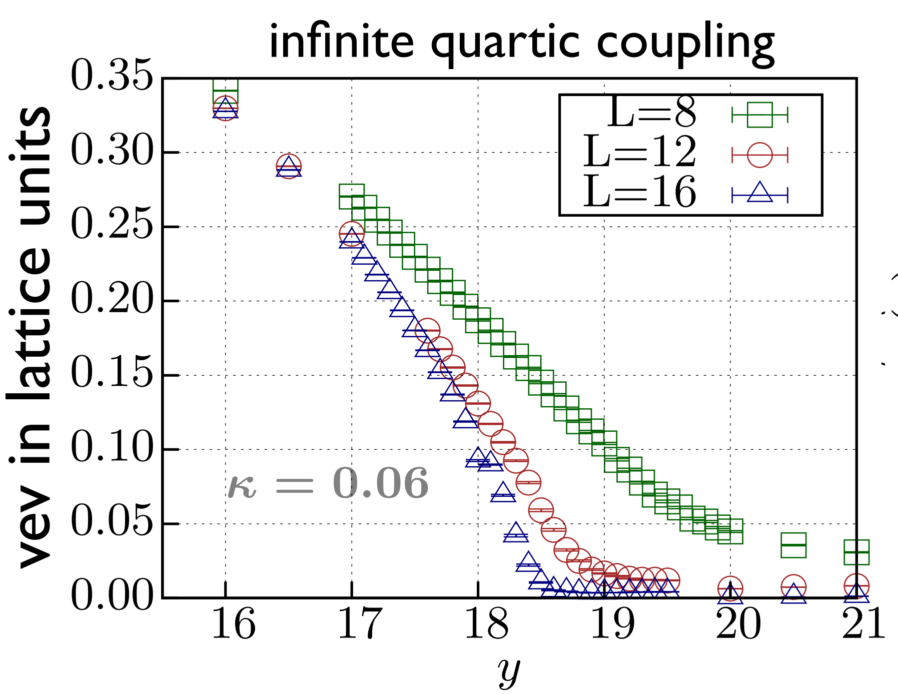
- For our purpose:

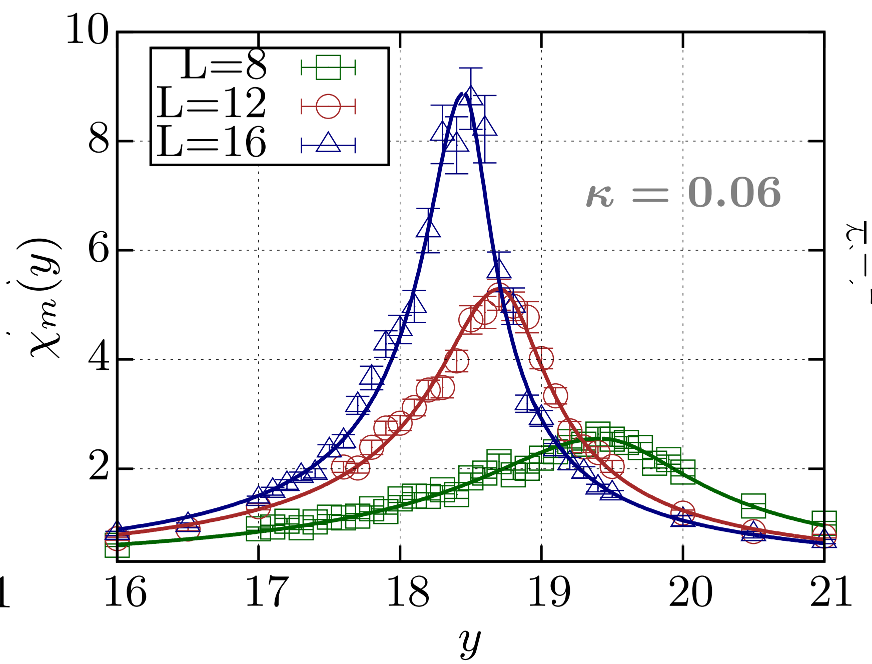
  At fixed $\xi$, take $a \to 0$.

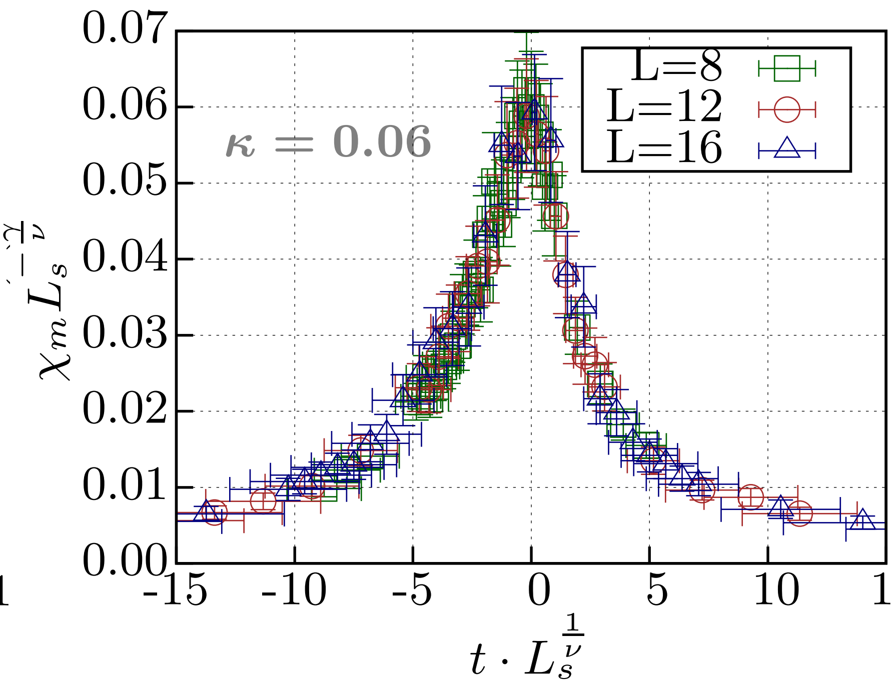
# General strategy: The continuum limit

$a \to 0$ and $\Lambda \to \infty$

Scan the bare parameter space.




Location

Scaling

Detailed scaling

# The constraint effective potential

Fukuda and Kyriakopoulos, 1985

- Phase structure is probed using the vev,

$$\hat{v} = a\varphi_c = \langle \hat{m} \rangle = \left\langle \frac{1}{V} \left| \sum_x \Phi_x^0 \right| \right\rangle .$$

- The constraint effective potential is a useful tool,

$$e^{-V U(\hat{v})} \sim \int \mathcal{D}\varphi \mathcal{D}\bar{\psi} \mathcal{D}\psi \; \delta\left(\varphi_0^0 - \varphi_c\right) \; e^{-S[\varphi, \bar{\psi}, \psi]},$$

$$\text{where } \varphi_0^0 = \frac{1}{V} \int d^4x \; \varphi^0 .$$

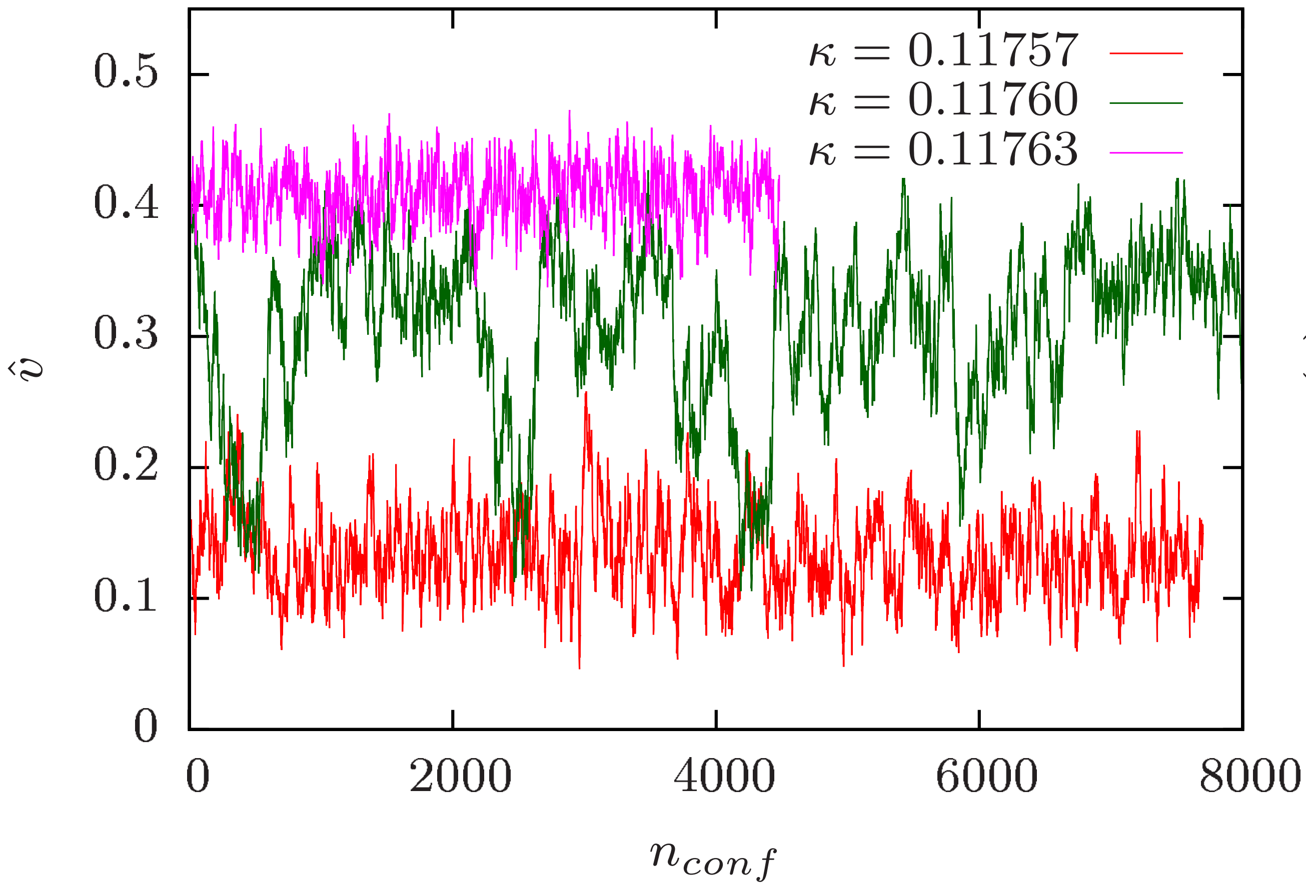
- Analytically calculated in perturbation theory.
- Numerically obtained by histograming $\hat{m}$.

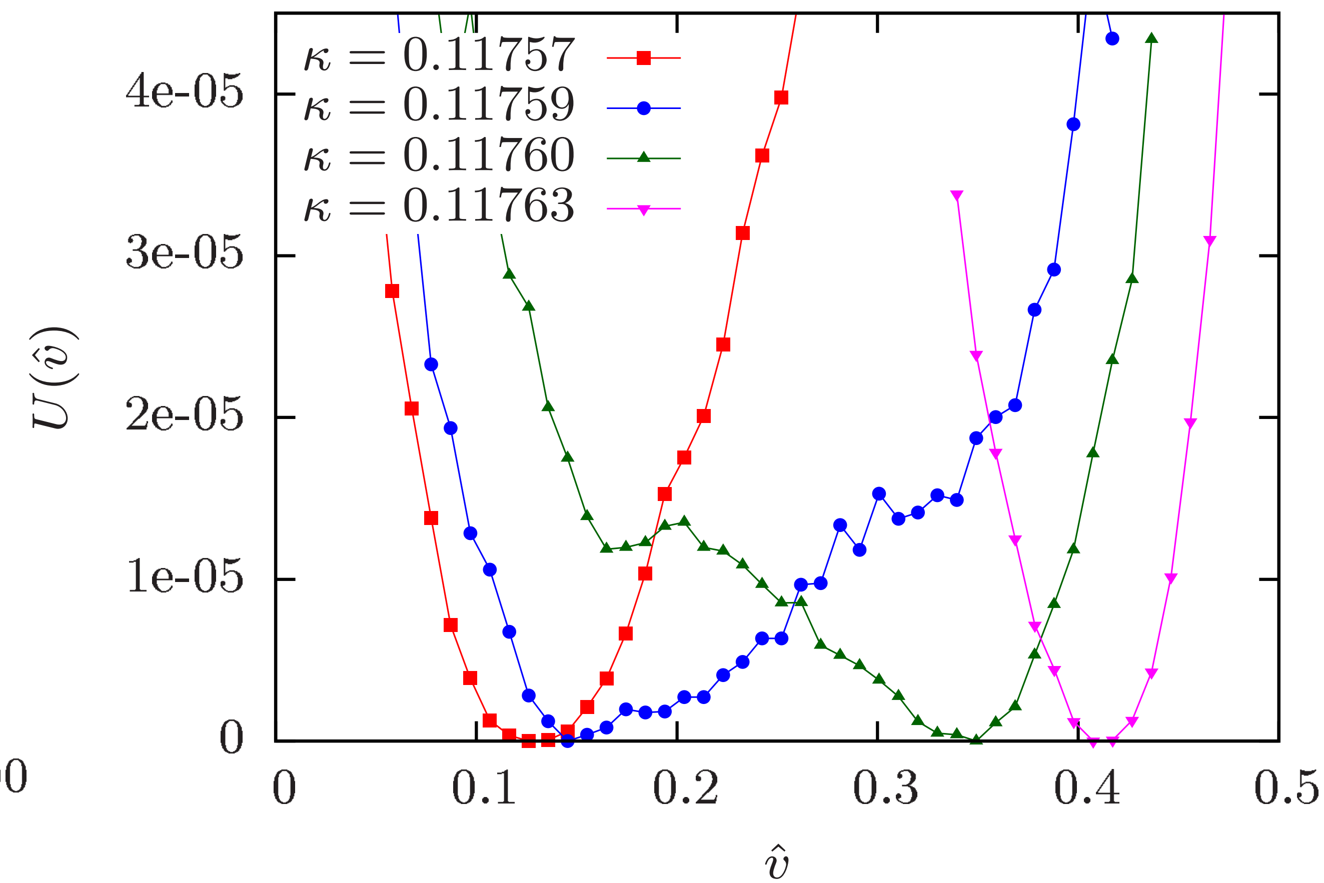
# Adding a dimension-six operator

$$\lambda_6 \left(\varphi^\dagger \varphi\right)^3$$

# With the dimension-6 operator

$$y = 173/246 \qquad \lambda_6 = 0.1 \text{ and } \lambda = -0.38$$

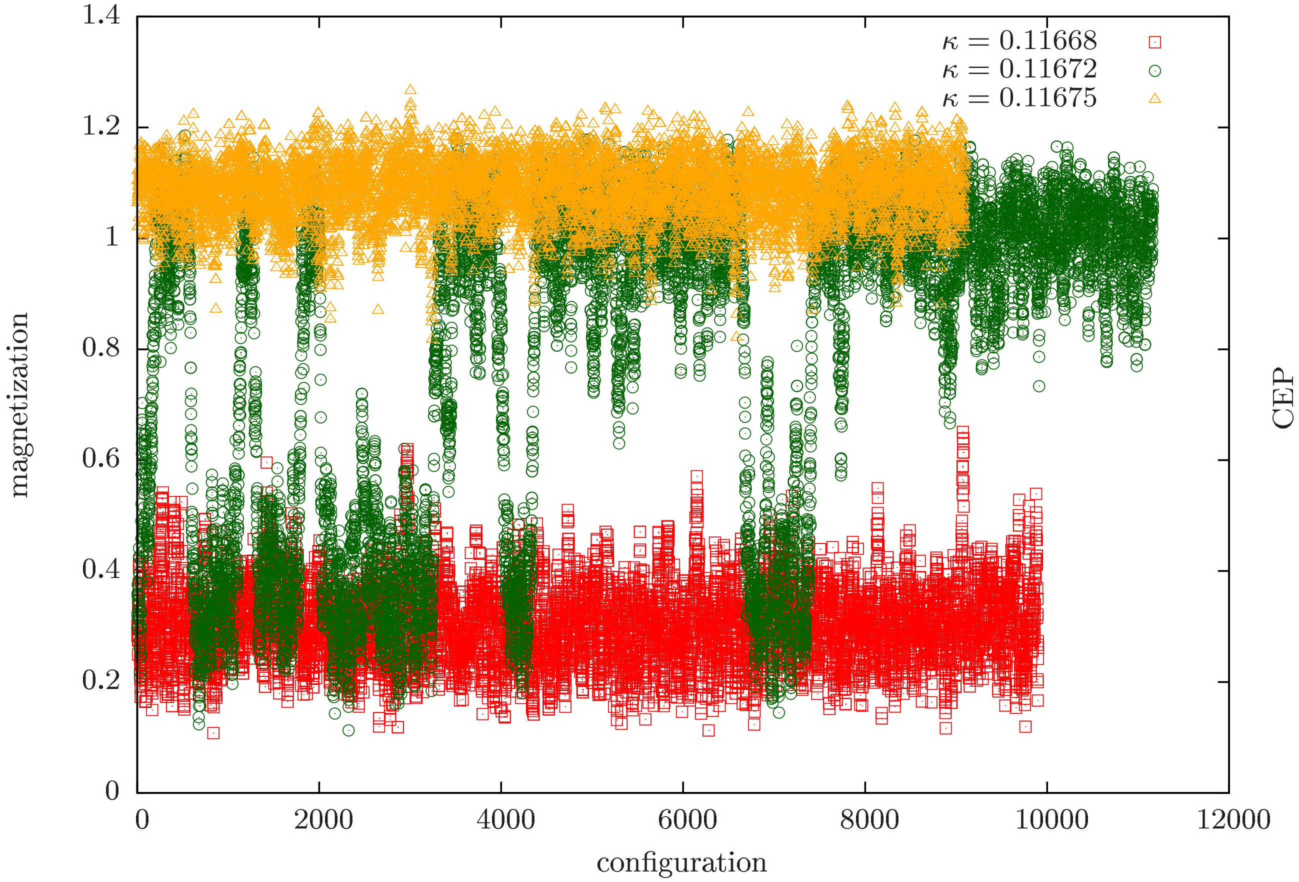


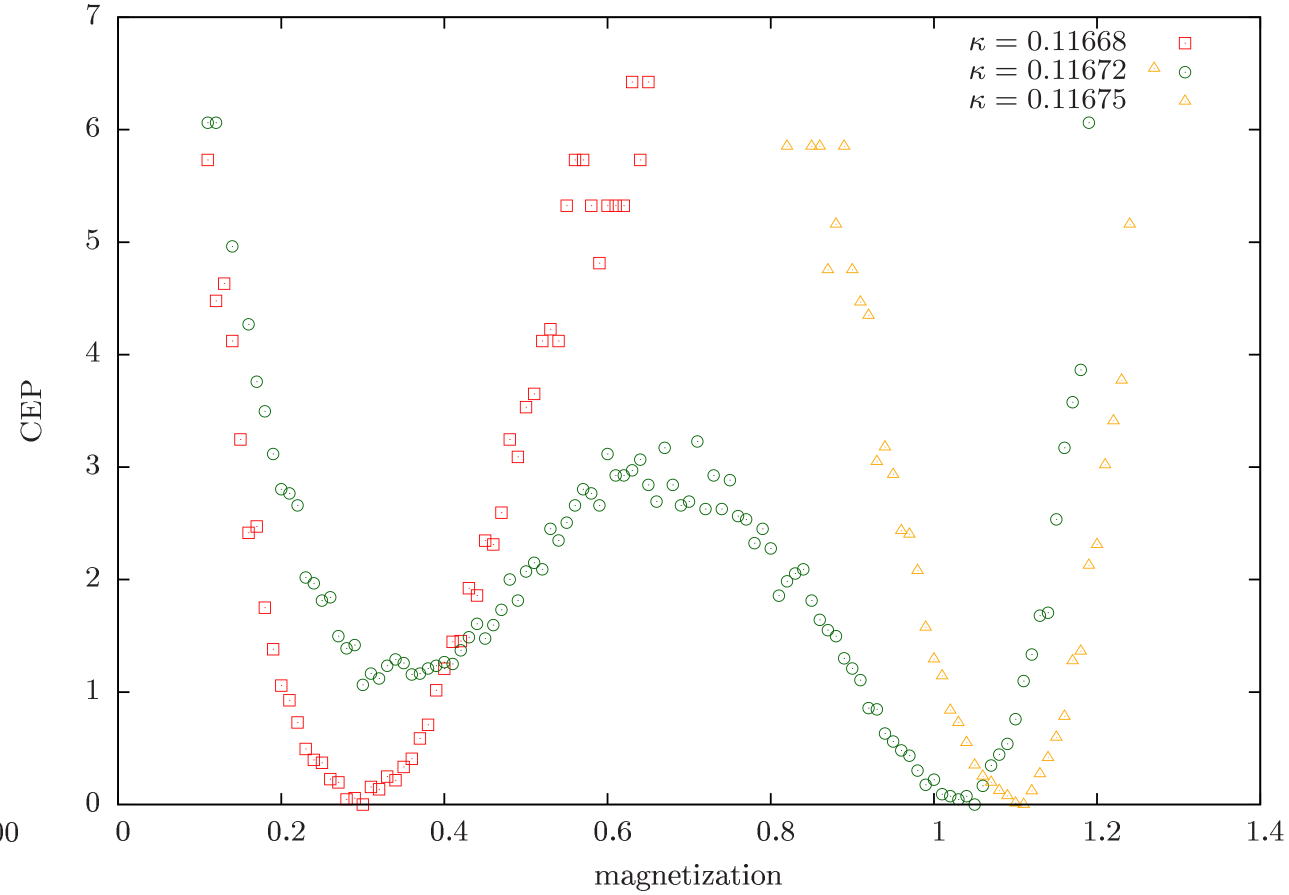




First-order phase transition?

# With the dimension-6 operator

$y = 173/246$ $\lambda_6 = 0.1$ and $\lambda = -0.40$

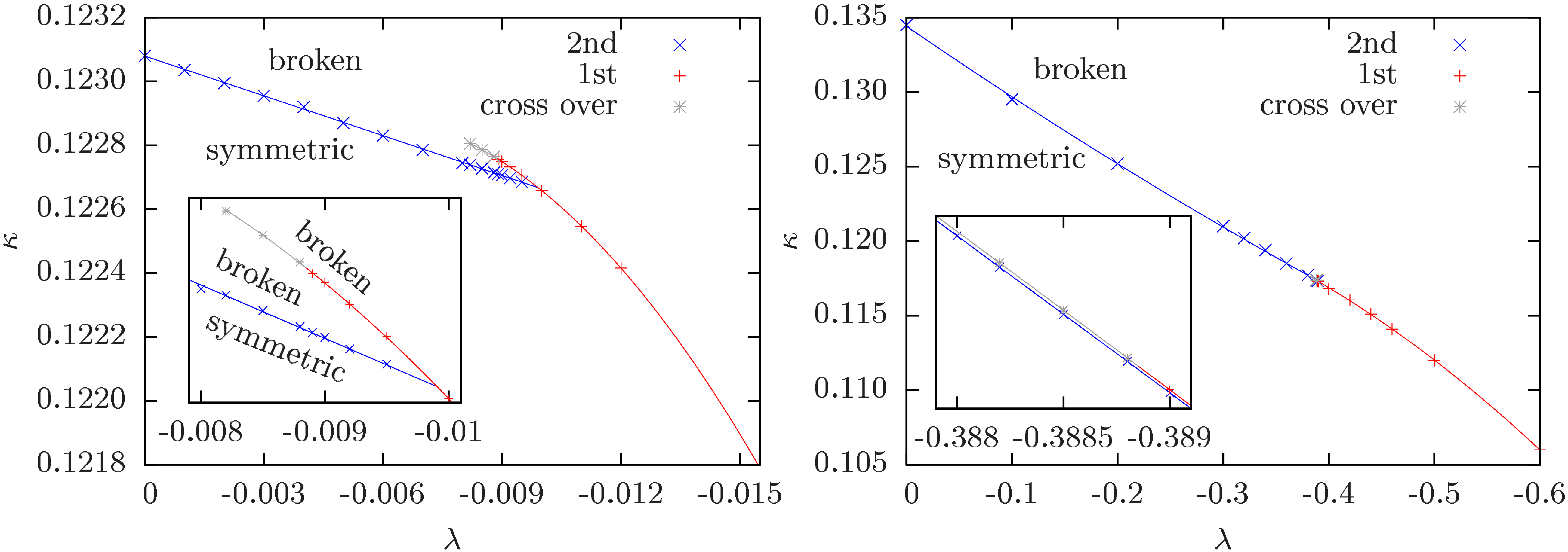



First-order phase transition observed

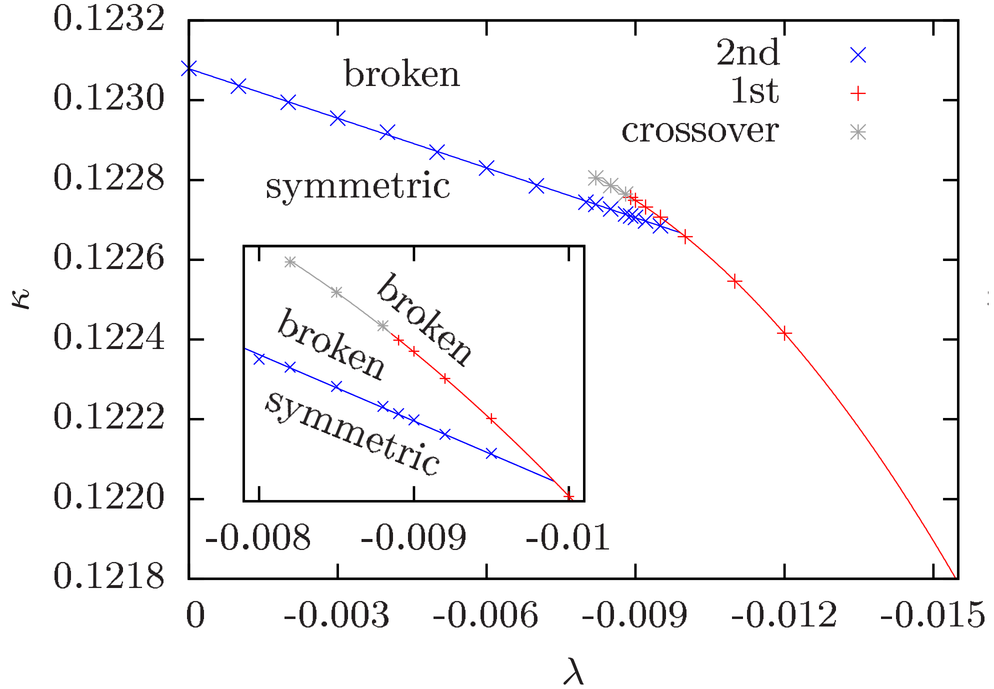
# The phase structure

$$y = 173/246$$



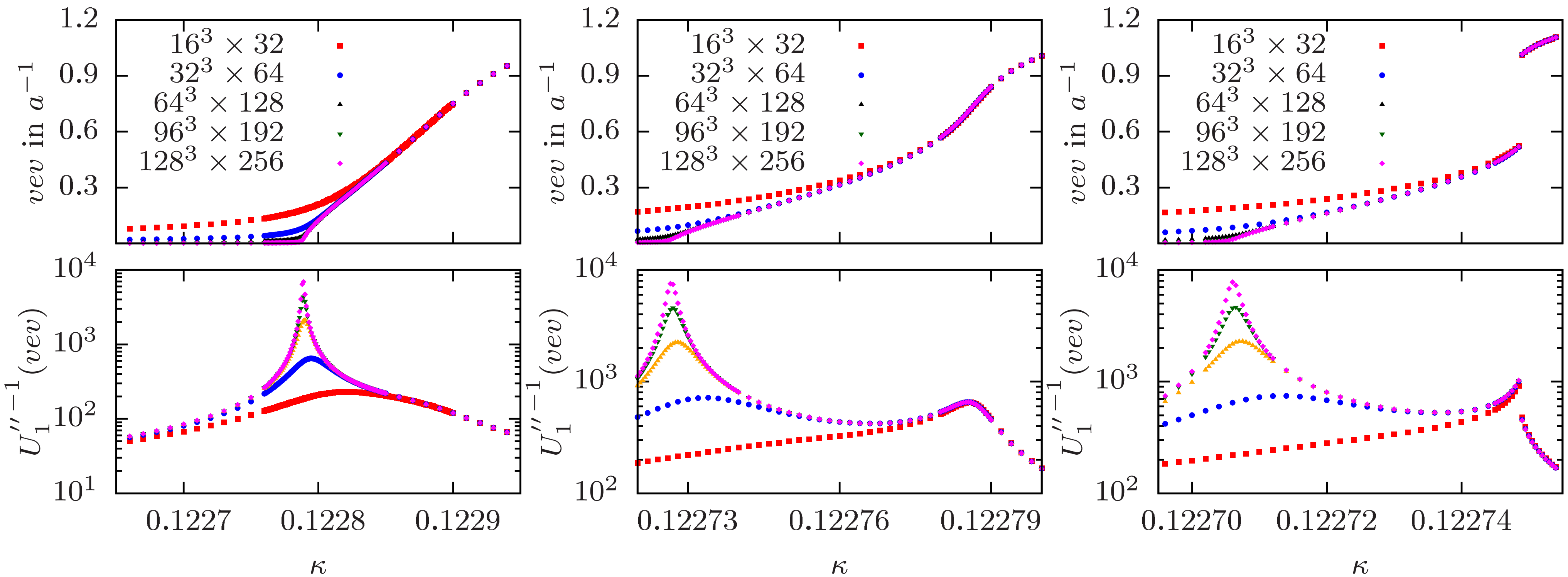

$\lambda_6 = 0.001$ $\lambda_6 = 0.1$

# The phase structure




$\lambda_6 = 0.001$

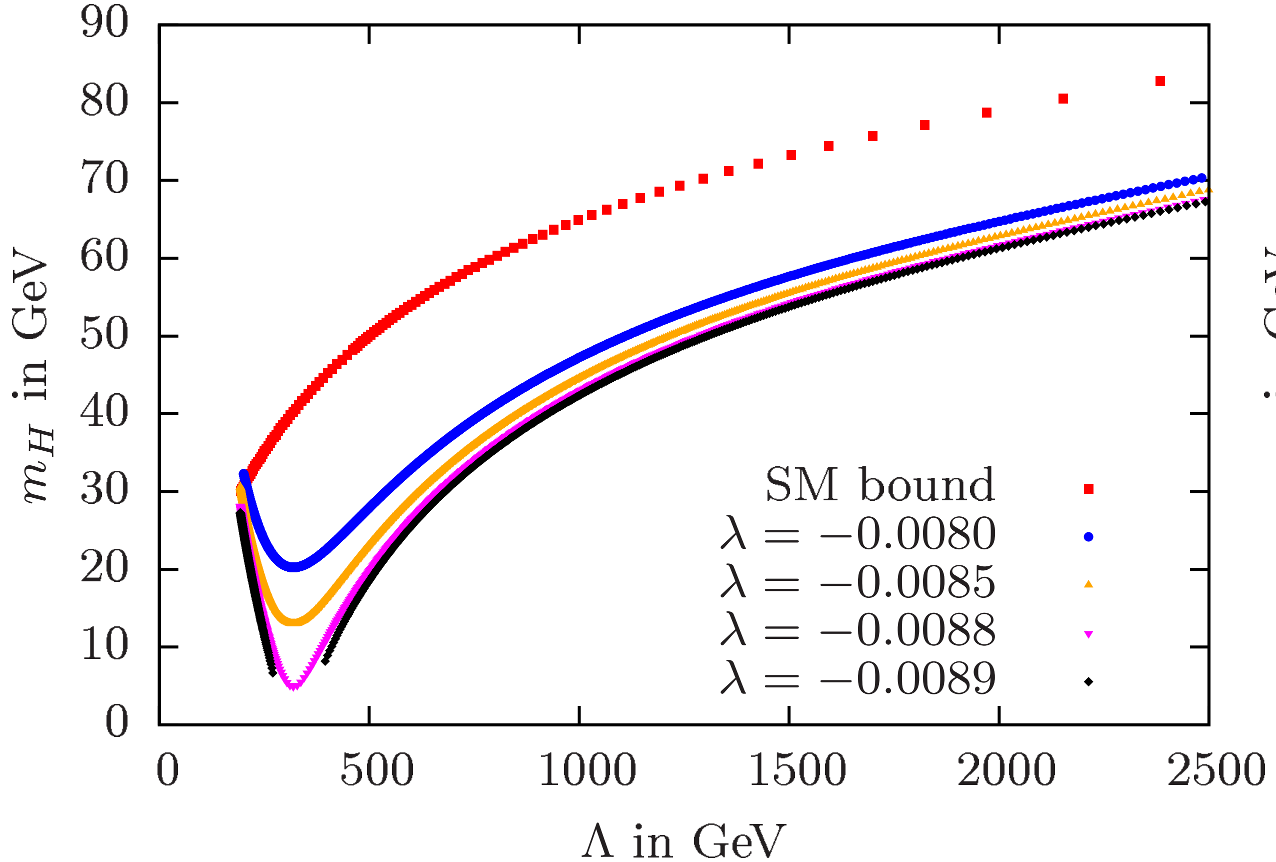
$y = 173/246$



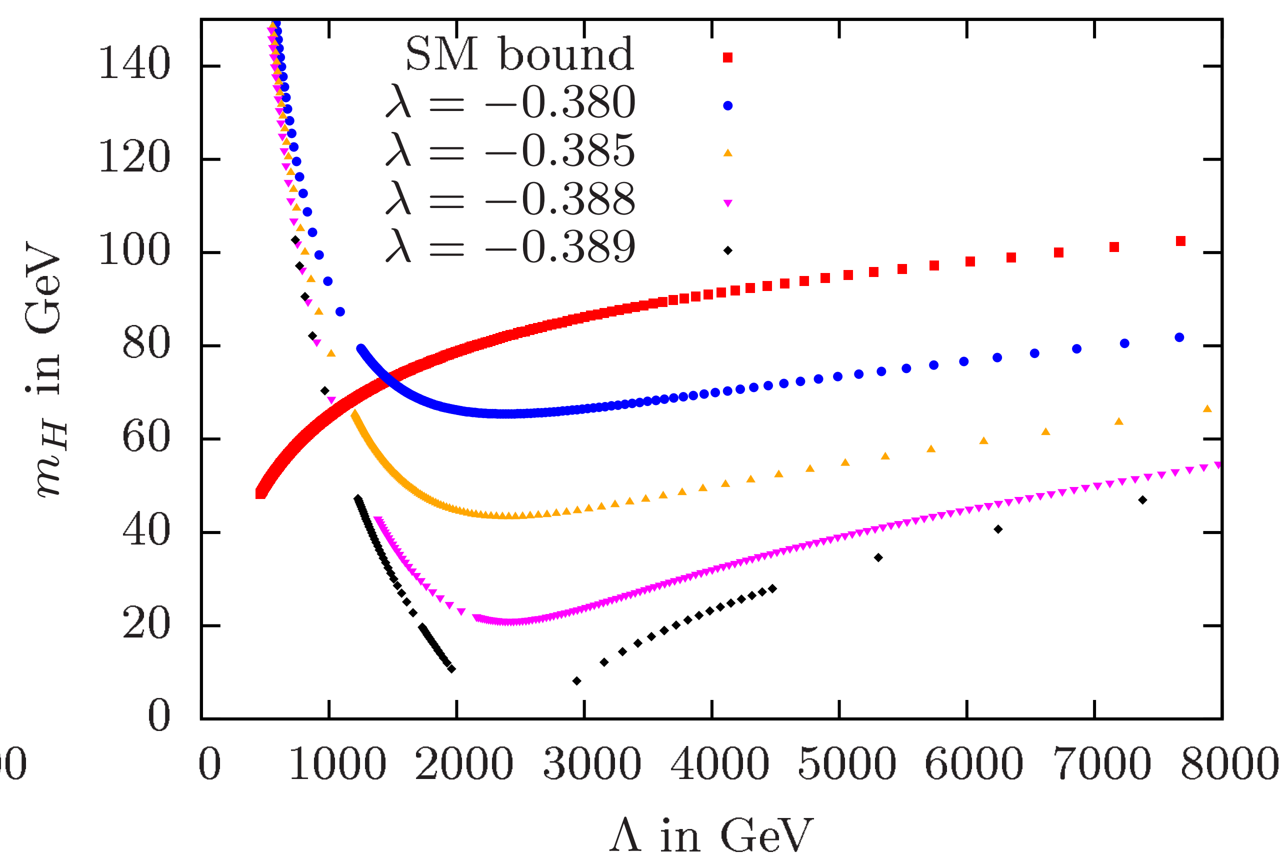

(a) $\lambda = -0.007$ (b) $\lambda = -0.0085$ (c) $\lambda = -0.009$

# The Higgs mass from the CEP

$$y = 173/246$$



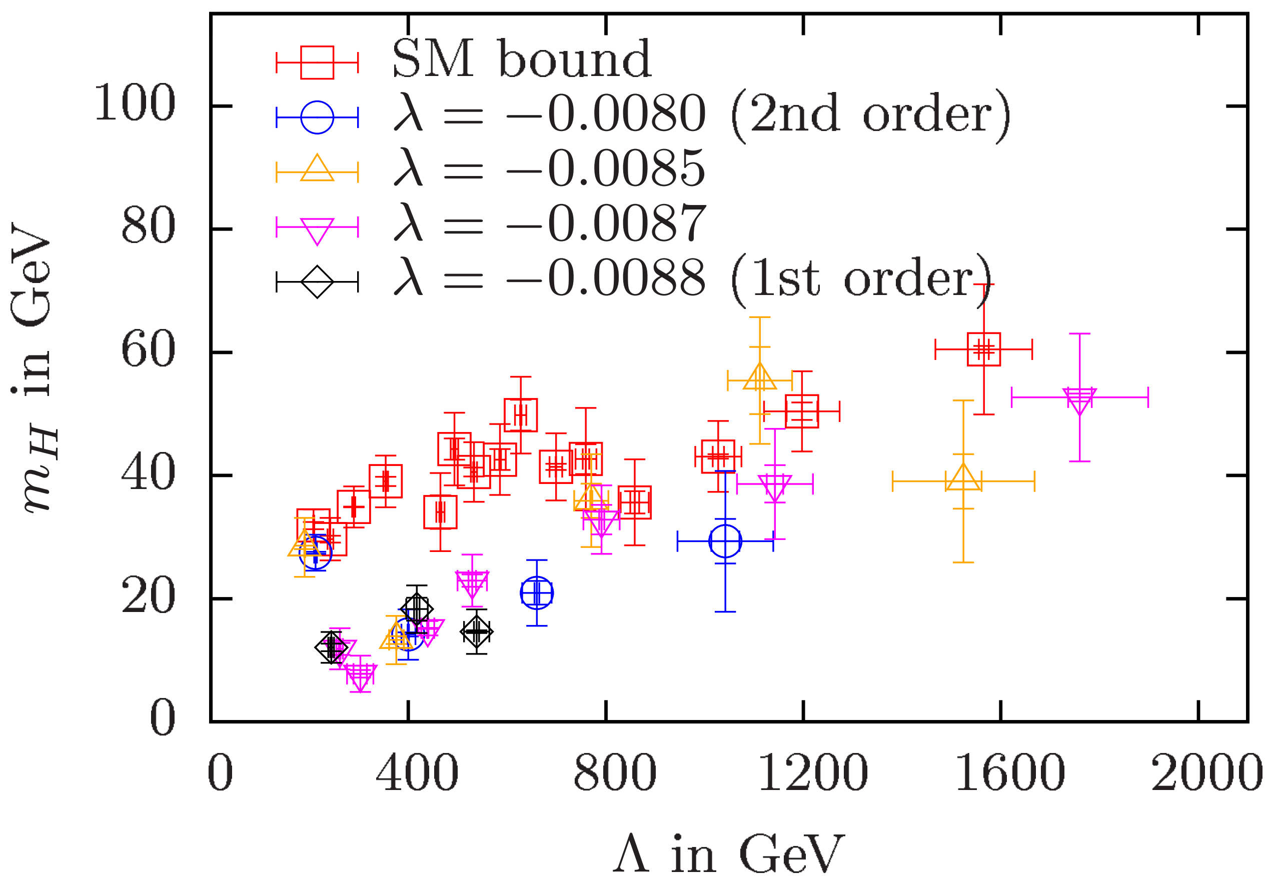

(a) $\lambda_6 = 0.001$

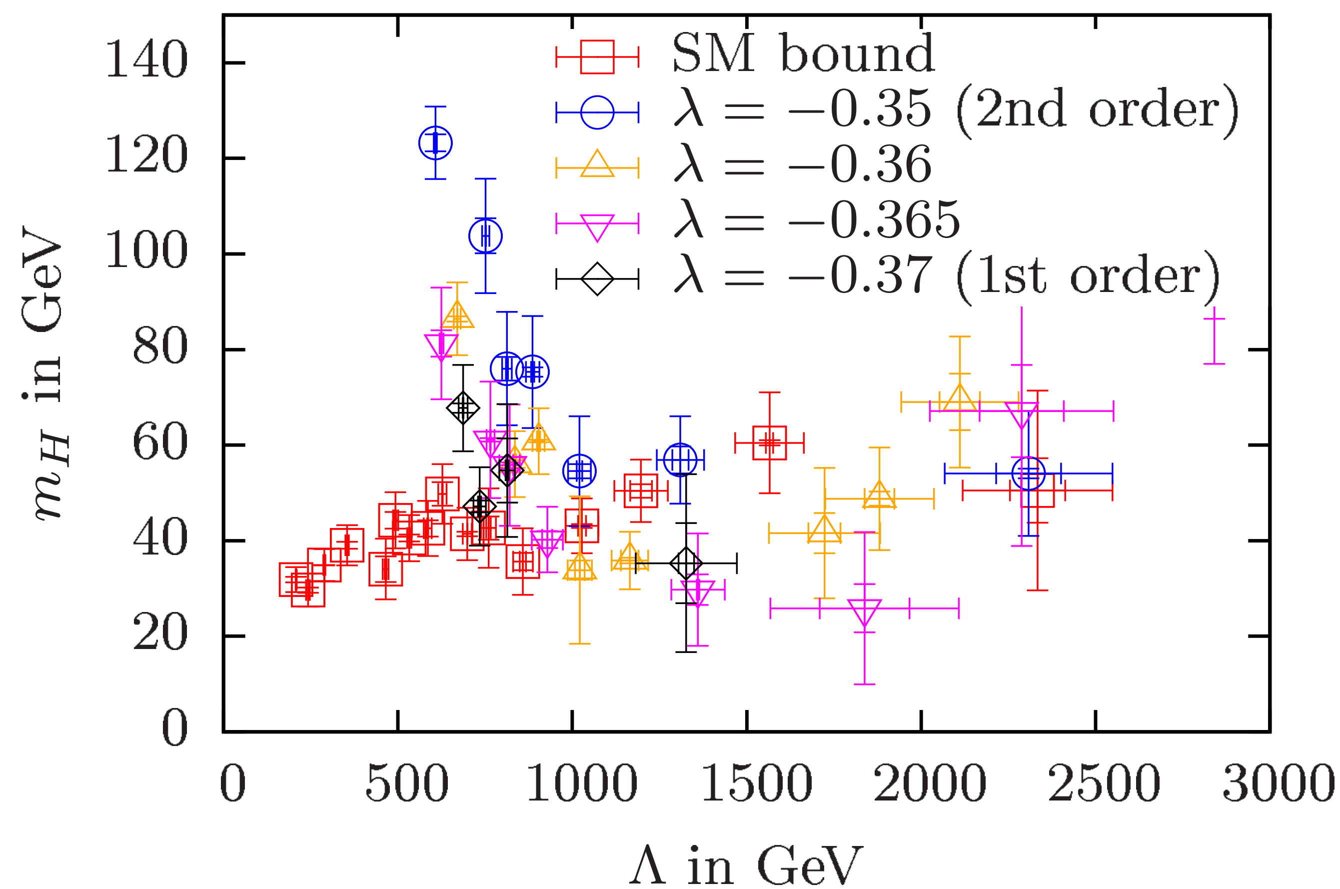



(b) $\lambda_6 = 0.1$

# The Higgs mass lower bounds

$$y = 173/246$$



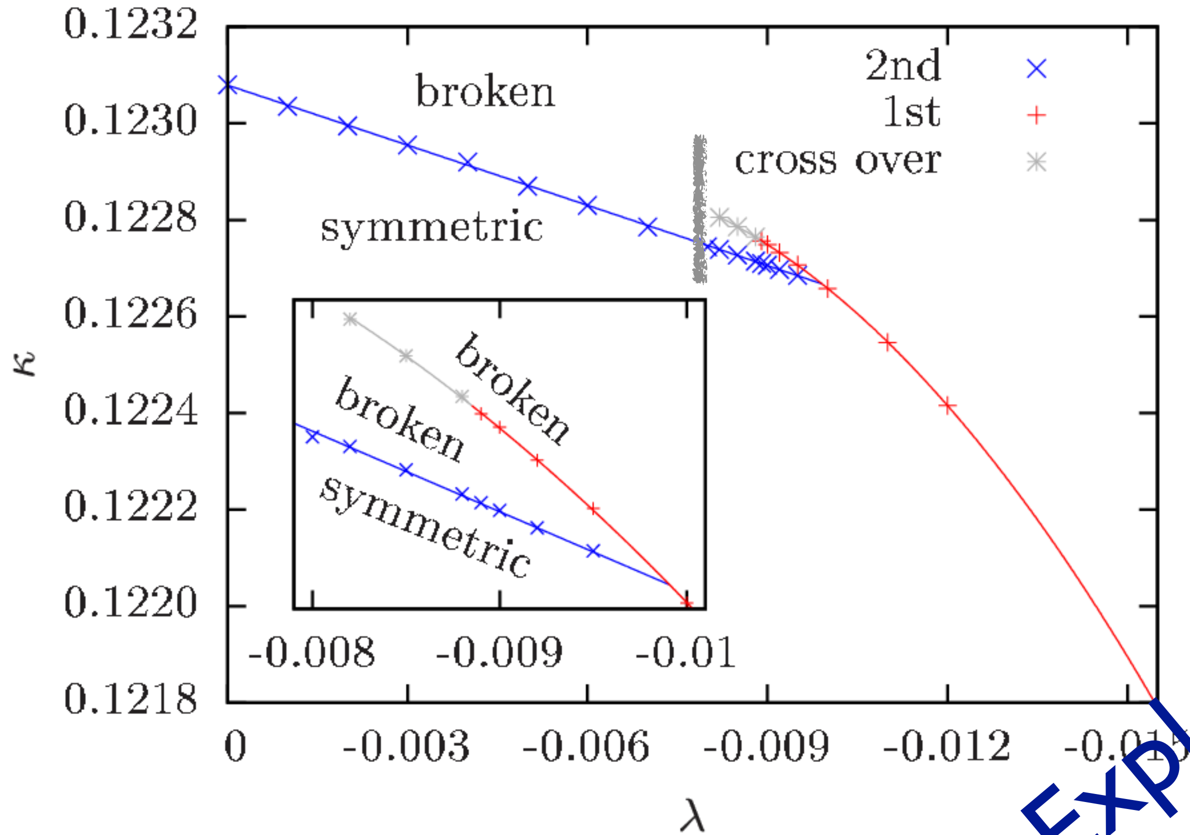



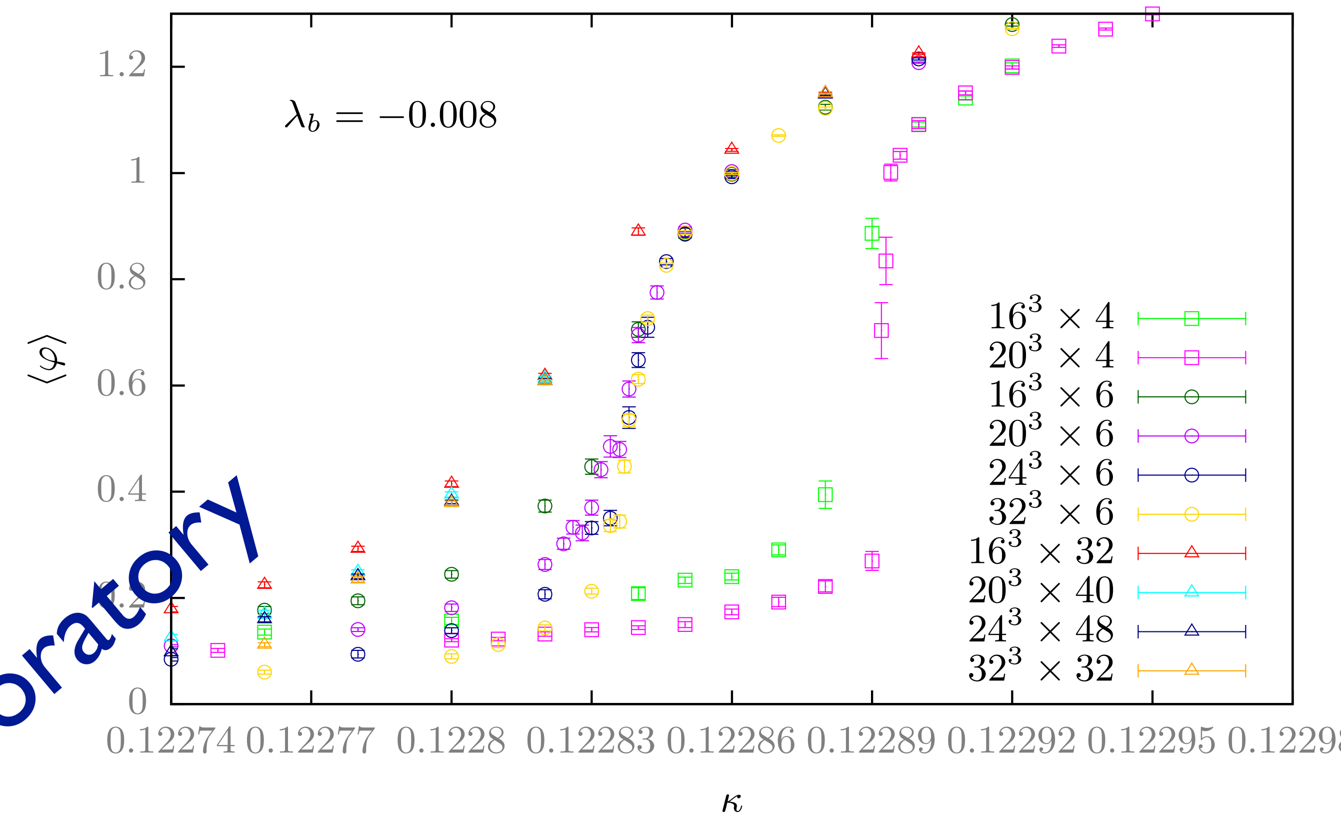

$\lambda_6 = 0.001$

$\lambda_6 = 0.1$

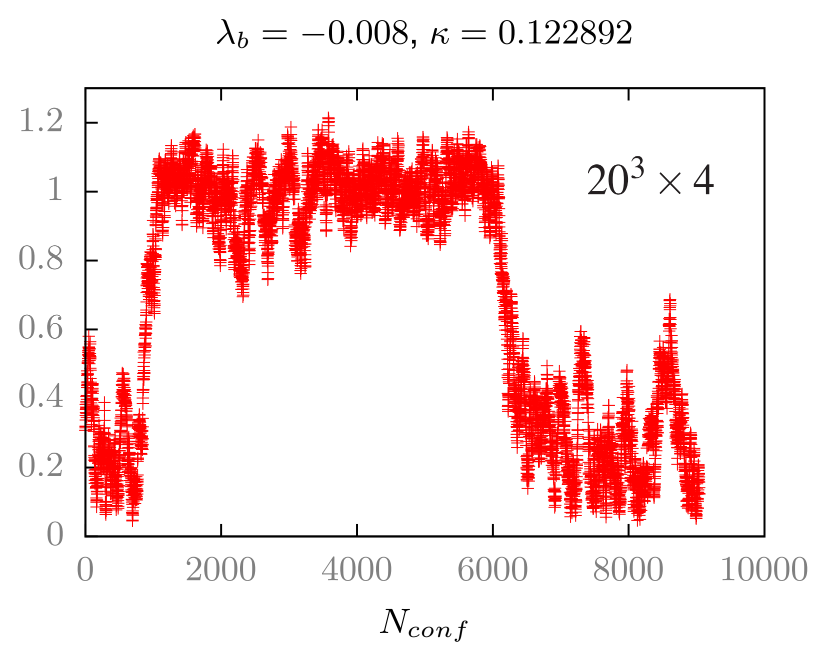
# Finite-temperature phase transition

$y = 173/246 \quad \text{bare } \lambda_6 = 0.001$




Exploratory




Parameter choice at which non-thermal transition is 2nd-order,
while thermal transition is 1st-order

# Triviality of the Higgs-Yukawa model

# Motivation

- Very little doubt that the pure-scalar sector of the SM is trivial.

M. Aizenman, PRL. 47 (1981)

J. Fröhlich, NPB 200 (1982)

M. Luscher and P. Weisz, PLB212 (1988), NPB 290 (1987), 295 (1988), 318 (1989)

M. Hoogervorst and U. Wolff, NPB 855 (2012)

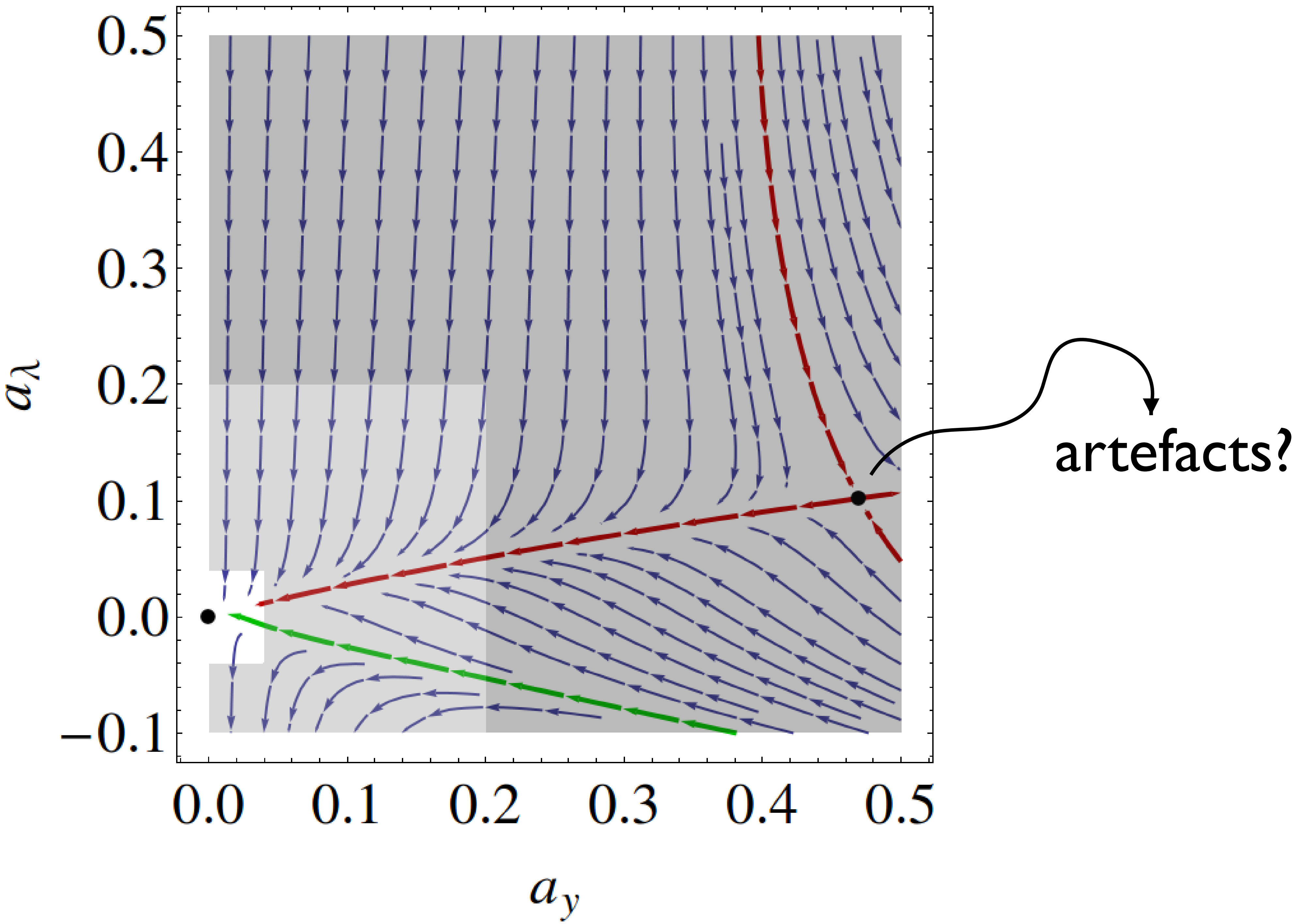
J. Sievert and U. Wolff, PLB 733 (2014) (High-precision study with large volumes)

...

- How about the Higgs-Yukawa sector?

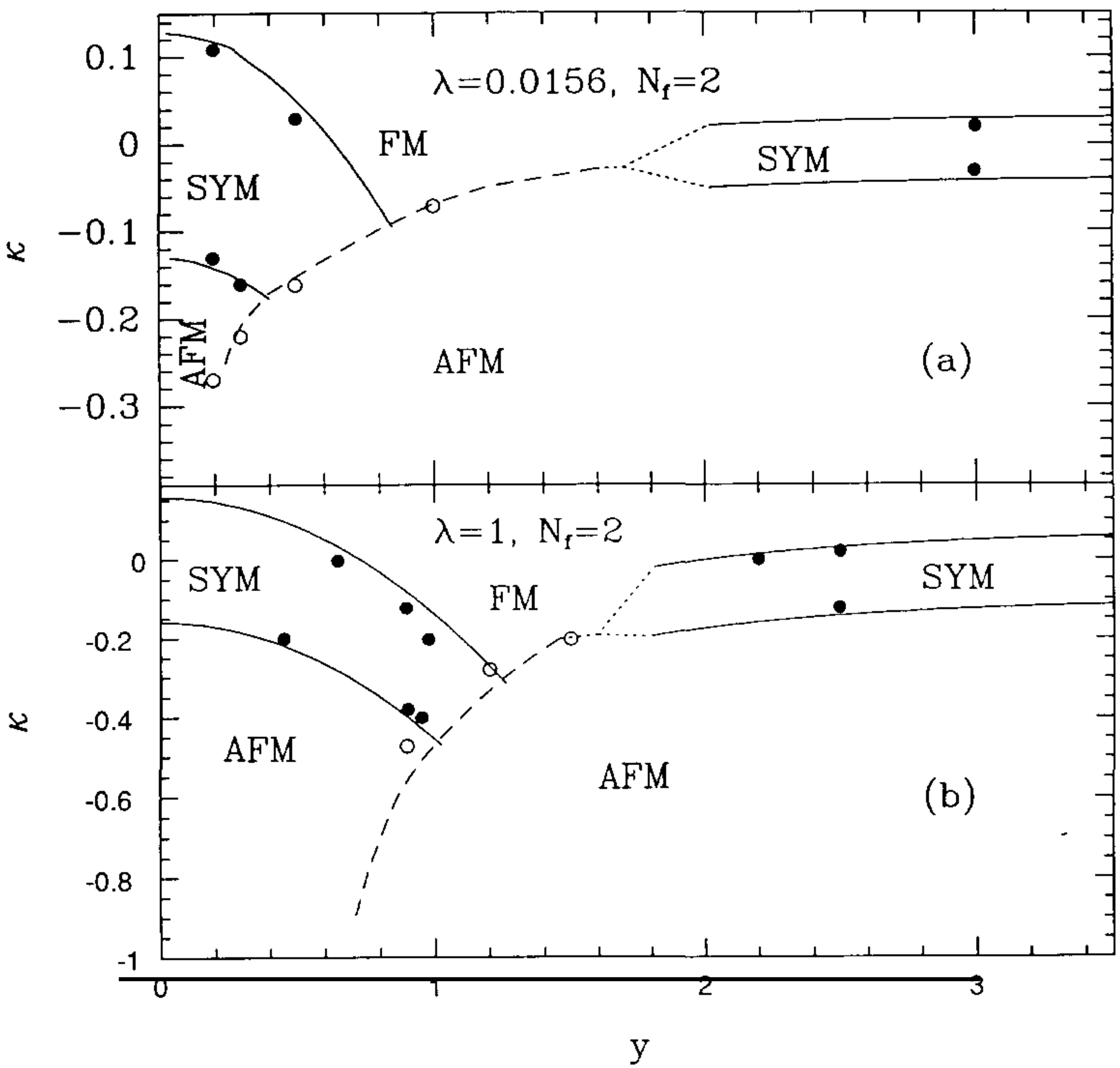
# Two-loop perturbation theory

$$a_y \equiv \frac{y^2}{(4\pi)^2}\ , \qquad a_\lambda \equiv \frac{\lambda}{(4\pi)^2}$$




E. Molgaard and R. Shrock, PRD 89 (2014)

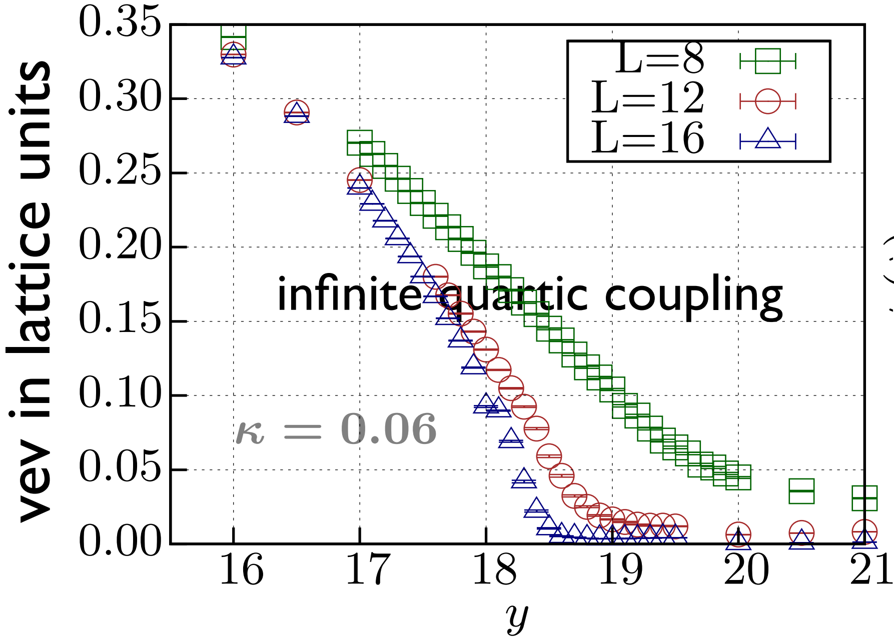
# Early work on the quantum phase structure

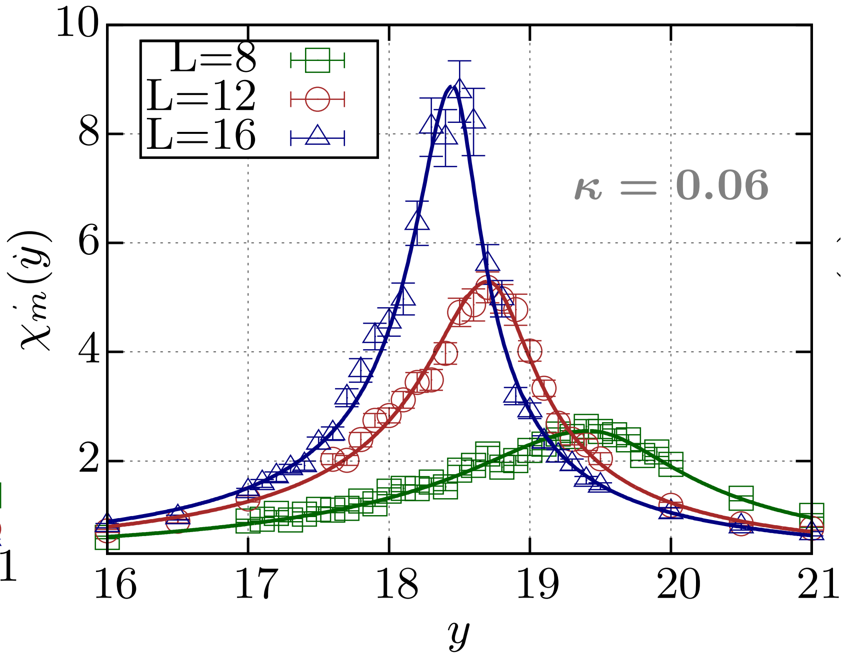


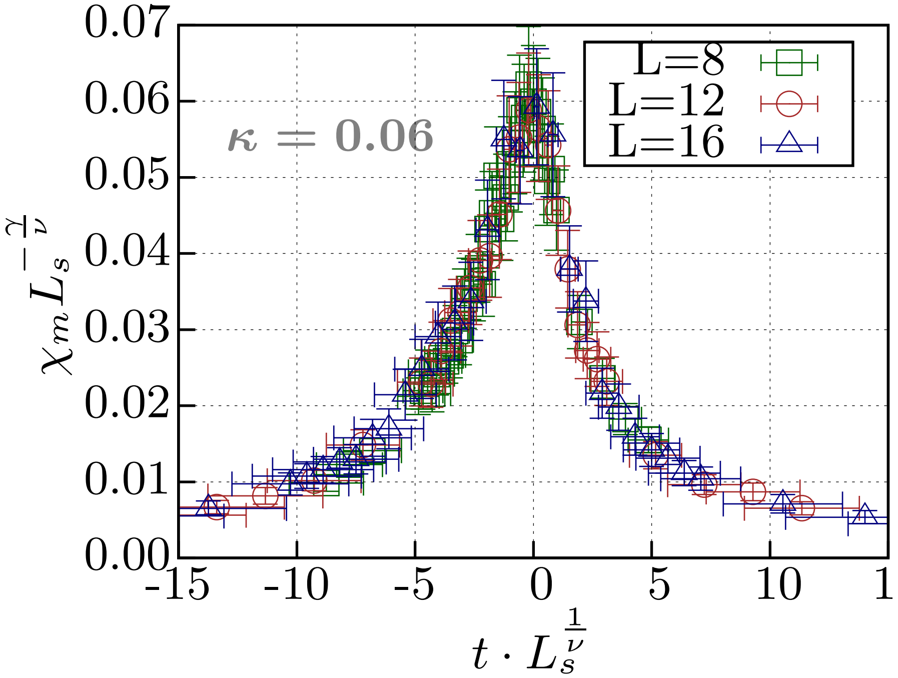

A. Hasenfratz, K. Jansen, Y. Shen, NPB 394 (1993)

# What we found previously

notice: strong (bare) Yukawa coupling




$t \sim \hat{m}_0^2 - \hat{m}_{\mathrm{crit}}^2$

Results from Binder's cumulant with a curve-collapse method

| | $T_c^{(L=\infty)}$ | $\nu$ | interval |
|---|---|---|---|
| $\kappa = 0.06$ | 18.147(24) | 0.550(1) | 17.4, 18.8 |
| $\kappa = 0.00$ | 16.667(27) | 0.525(6) | 16.0, 17.2 |
| O(4) | 0.3005(34) | 0.50000(3) | 0.294, 0.314 |

J. Bulava *et al.*, AHEP 2013

mean-field? → logarithmic corrections needed

# Which continuum limit?
# Strong or weak coupling?

(Finite-size scaling techniques)

For observables containing only the scalar fields:

$$\begin{aligned}\hat{M}_b\left[m_b^2, \lambda_b, Y_b; a, L\right] Z_\phi^{-D_M/2}(a,l) &= \hat{M}\left[m^2(\hat{l}), \lambda(\hat{l}), Y(\hat{l}); l, L\right], \\ &= \zeta_M(l,L)\hat{L}^{-D_M}\hat{M}\left[\hat{m}^2(\hat{L})\hat{L}^2, \lambda(\hat{L}), Y(\hat{L}); 1, 1\right]\end{aligned}$$

$$\zeta_M(l,L) = \exp\left(\int_l^L \gamma_M(\rho)\mathrm{d}\log\rho\right)$$

Fixed point: $\lambda(\hat{L}) \approx \lambda_*,\ Y(\hat{L}) \approx Y_*,\ \gamma_M \approx \gamma.$ $\longrightarrow$ Power law from $\zeta_M$.

Similar RG argument also results in the following power-law scaling:

$\hat{m}^2(\hat{L}) \sim \hat{L}^{1/\nu-2}$ where $1/\nu = 2 + \gamma_{m^2}$

*Naive* expectation for the mean-field FP

$\downarrow$

Logarithmic corrections are present

mass-square anomalous dimension

# Mean-field FSS for the Higgs-Yukawa model

Sequel of E. Brezin and J. Zinn-Justin, NPB 257 (1985)

$$Z = \int D\Phi\ D\bar{\Psi}\ D\Psi \exp\left(-S[\Phi,\bar{\Psi},\Psi]\right)$$

$\phi_a = \varphi_a + \chi_a$

$\varphi \to \left(sL^4\lambda(L^{-1})\right)^{-1/4}\varphi \equiv S^{-1/4}\varphi$

one-loop

$$Z = \mathcal{N}\Omega_{N-1}S^{-N/4}\int_0^\infty d\varphi\,\varphi^{N-1}\exp\left(-\frac{1}{2}z\varphi^2 - \varphi^4\right)$$

zero mode

$$\equiv \mathcal{N}\Omega_{N-1}S^{-N/4}\bar{\varphi}_{N-1}(z),$$

Logarithms

The scaling variable: $z = \sqrt{s}\overbrace{\hat{M}^2(L^{-1})\hat{L}^2\lambda(L^{-1})^{-1/2}}$

---

$$\bar{\varphi}_0 = \frac{\pi}{8}\exp\left(\frac{z^2}{32}\right)\sqrt{|z|}\left[I_{-1/4}\left(\frac{z^2}{32}\right) - \mathrm{Sgn}(z)I_{1/4}\left(\frac{z^2}{32}\right)\right],$$

$$\bar{\varphi}_1 = \frac{\sqrt{\pi}}{8}\exp\left(\frac{z^2}{16}\right)\left[1 - \mathrm{Sgn}(z)\,\mathrm{Erf}\left(\frac{|z|}{4}\right)\right],\quad \bar{\varphi}_{n+2} = -2\frac{\mathrm{d}}{\mathrm{d}z}\bar{\varphi}_n$$

Can compute $\langle\varphi^k\rangle$. $S^k\langle\varphi^k\rangle$ is a universal function of $z$.

# Mean-field FSS for the Higgs-Yukawa model

$$\hat{M}_b\left[m_b^2, \lambda_b, Y_b; a, L\right] Z_\phi^{-D_M/2}(a,l) = \hat{M}\left[m^2(\hat{l}), \lambda(\hat{l}), Y(\hat{l}); l, L\right],$$
$$= \zeta_M(l,L)\hat{L}^{-D_M}\hat{M}\left[\hat{m}^2(\hat{L})\hat{L}^2, \lambda(\hat{L}), Y(\hat{L}); 1, 1\right]$$

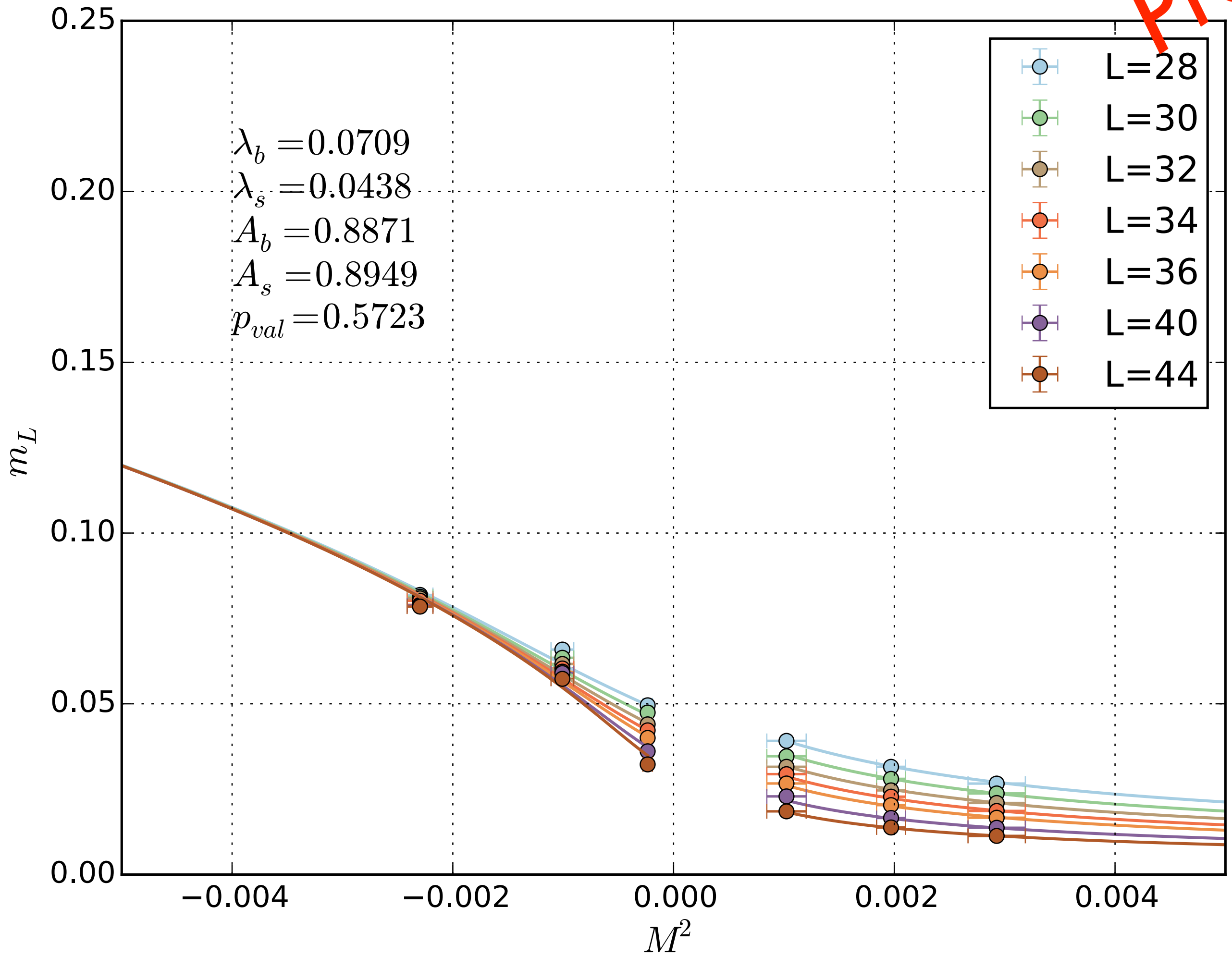
---

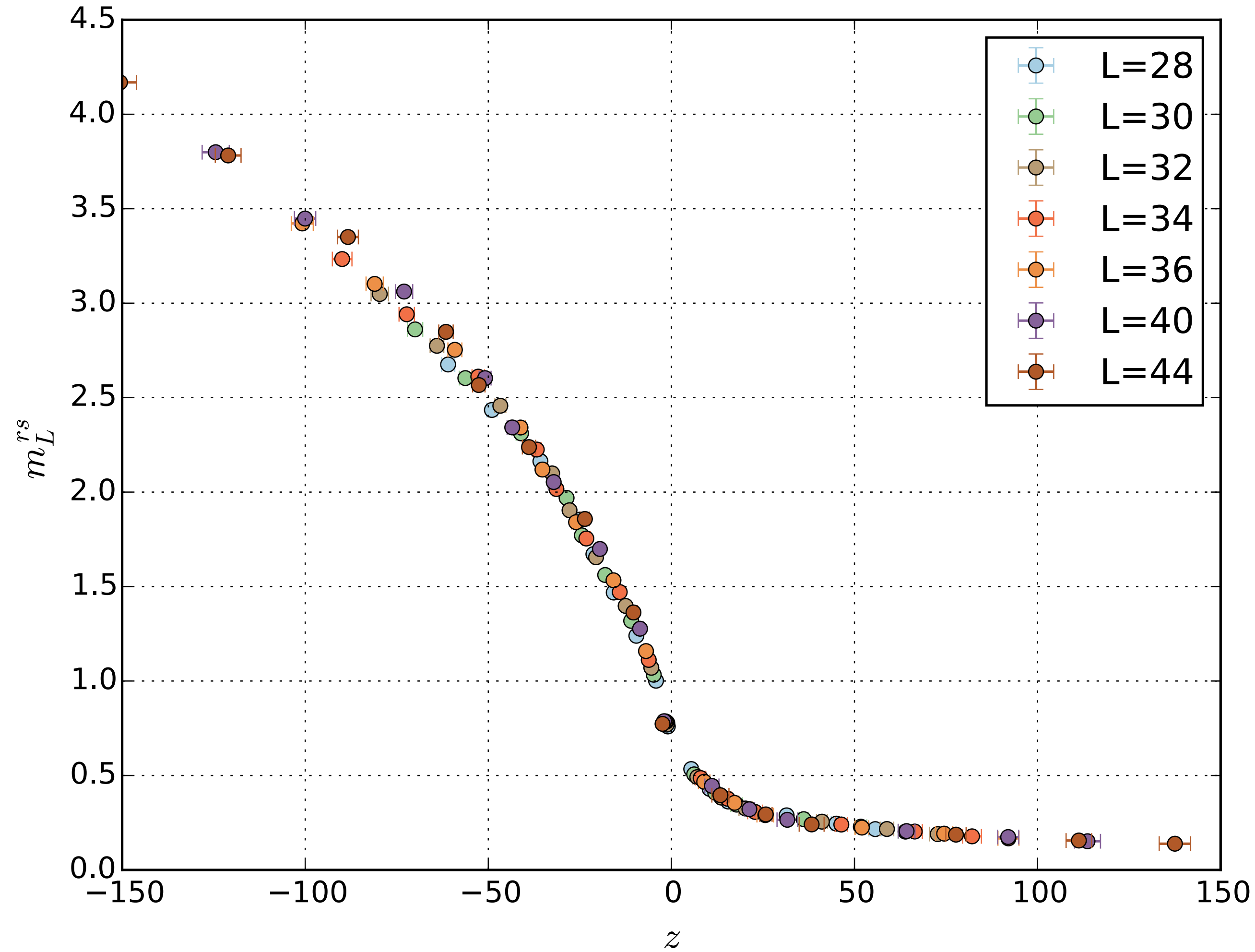
To obtain logarithmic corrections to the MF scaling

- Integrate one-loop RGE in perturbation theory from $l$ to $L$.
  - ★ Three couplings and the wavefunction renormalisation.
  - ★ Additive scalar mass renormalisation.
  - ★ Also need to specify $l$.
- Identify the scalar (Higgs) pole mass, $m_{\rm P}$, as $1/l$.
  - ★ Can be extracted in lattice units from the lattice data.
  - ★ Run to $1/L$ using perturbation theory. (only need to know $m_P L$)
  - ★ Need the hierarchy $a/L < am_P << 1$.
- Reduces the number of fit parameters by three!

# Numerical test in the O(4) scale model
# Scaling of magnetisation(scalar vev)

Preliminary

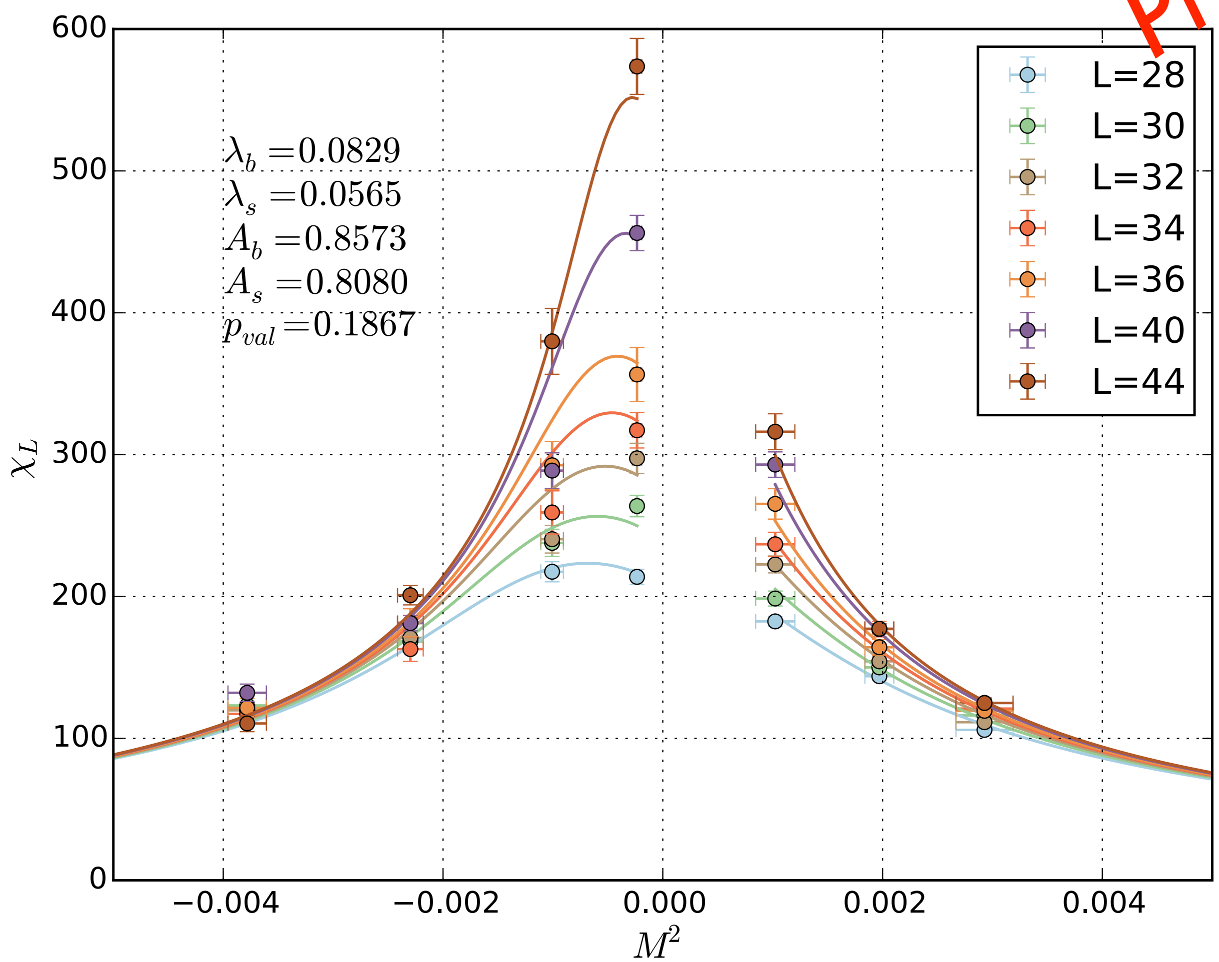


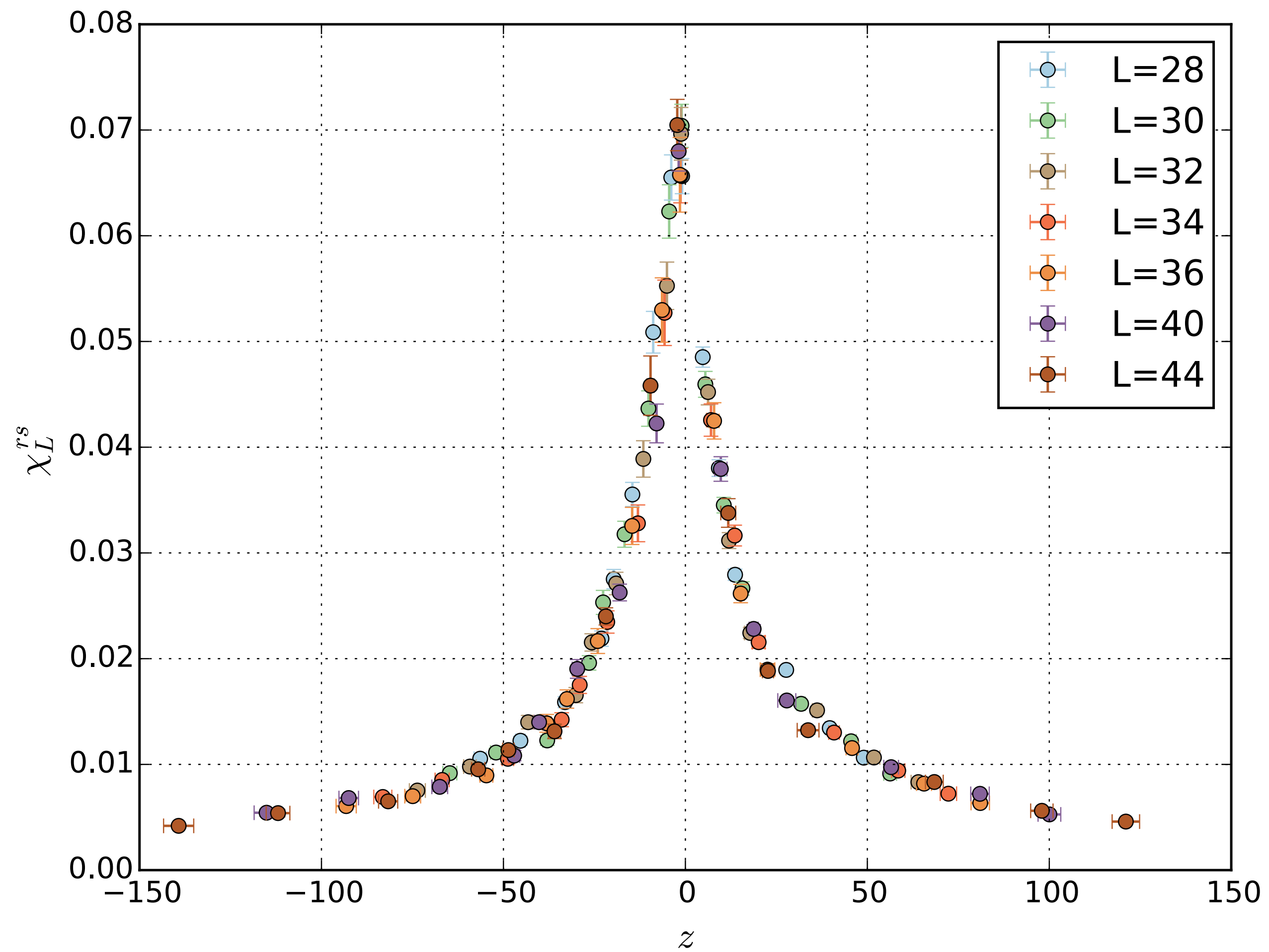




Notice: fit without logarithms is almost as good.

# Numerical test in the O(4) scale model
# Scaling of the susceptibility

Preliminary

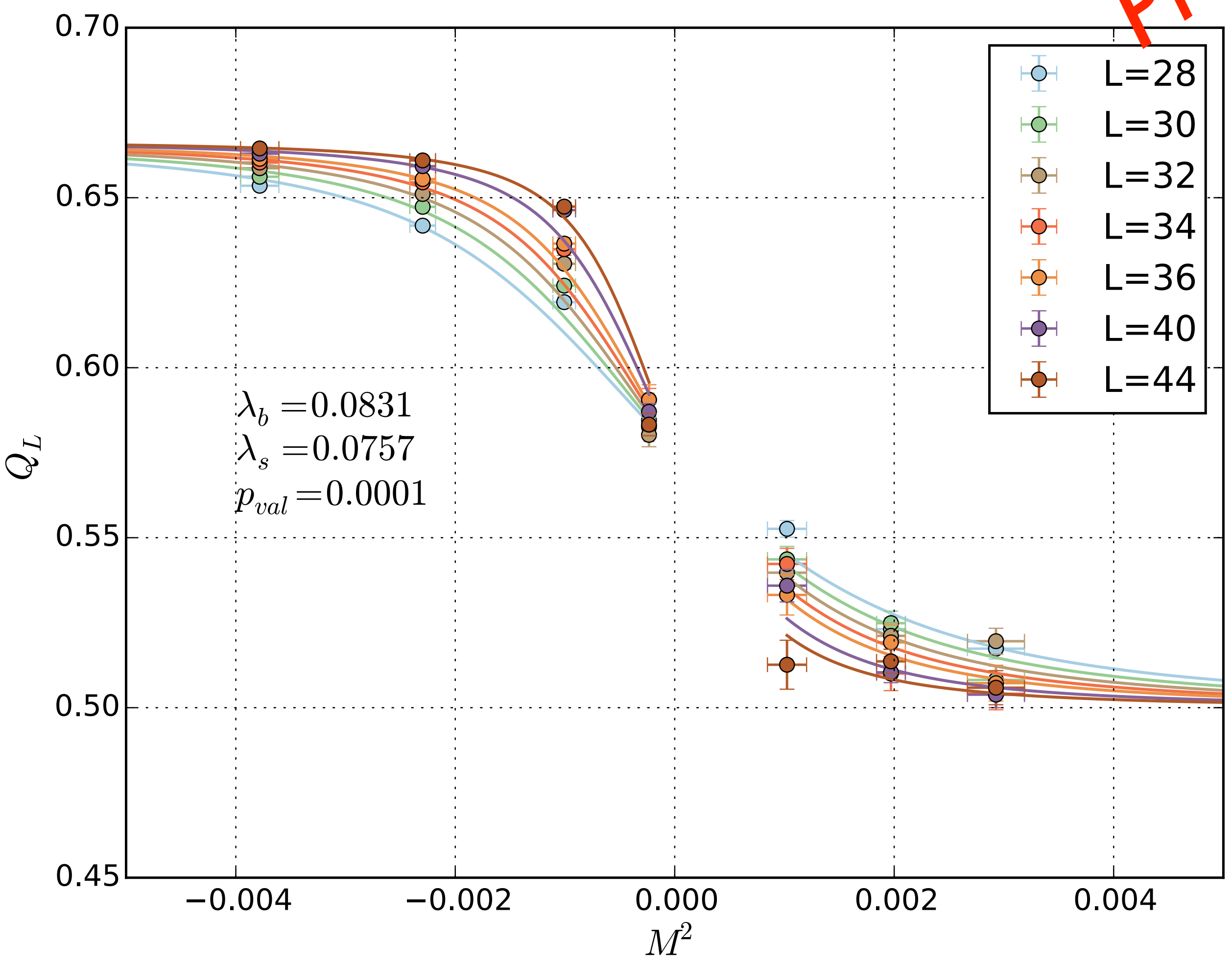


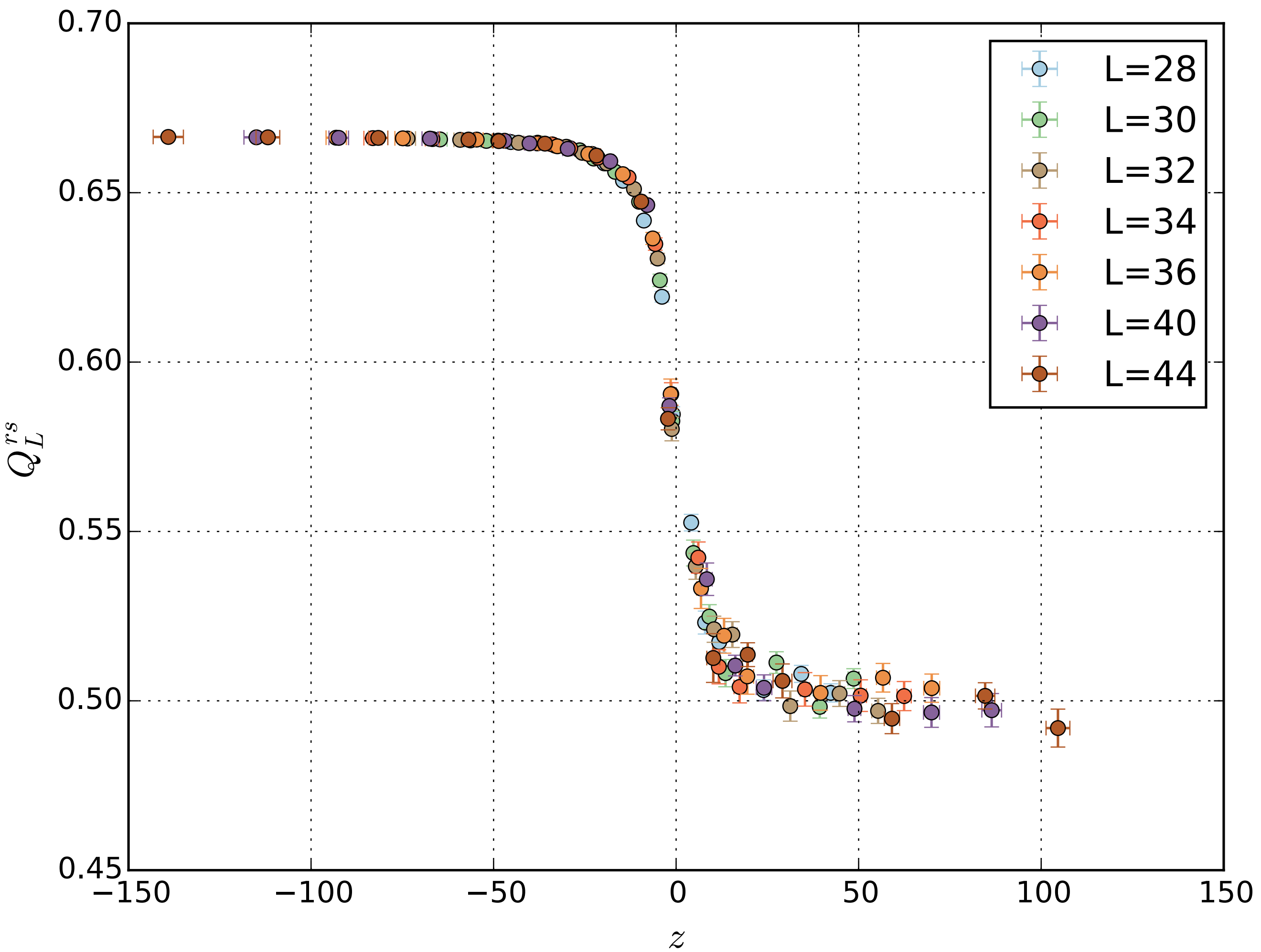




Notice: fit without logarithms is almost as good.

# Numerical test in the O(4) scale model
# Scaling of Binder's cumulant

Preliminary



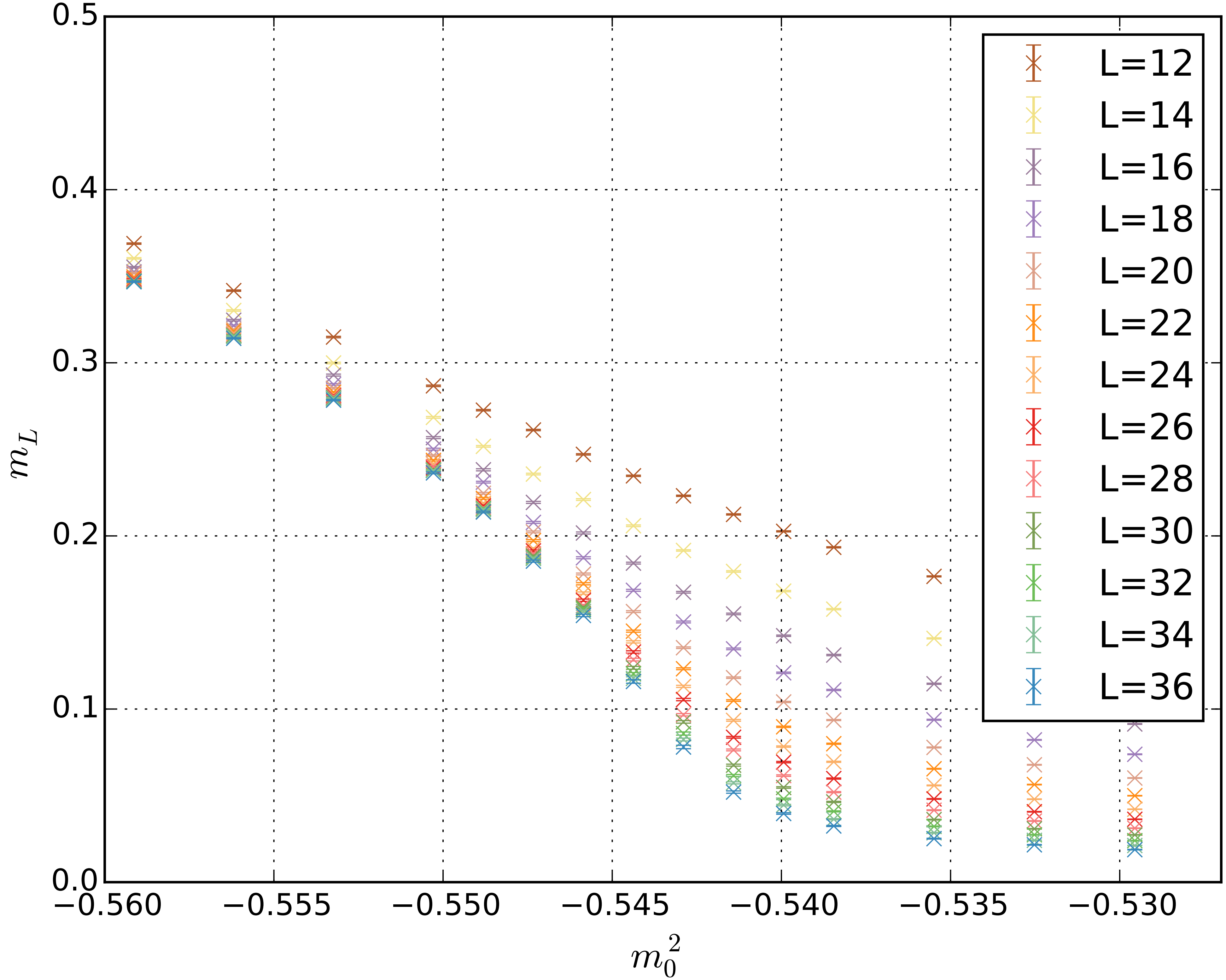



# Conclusion and outlook

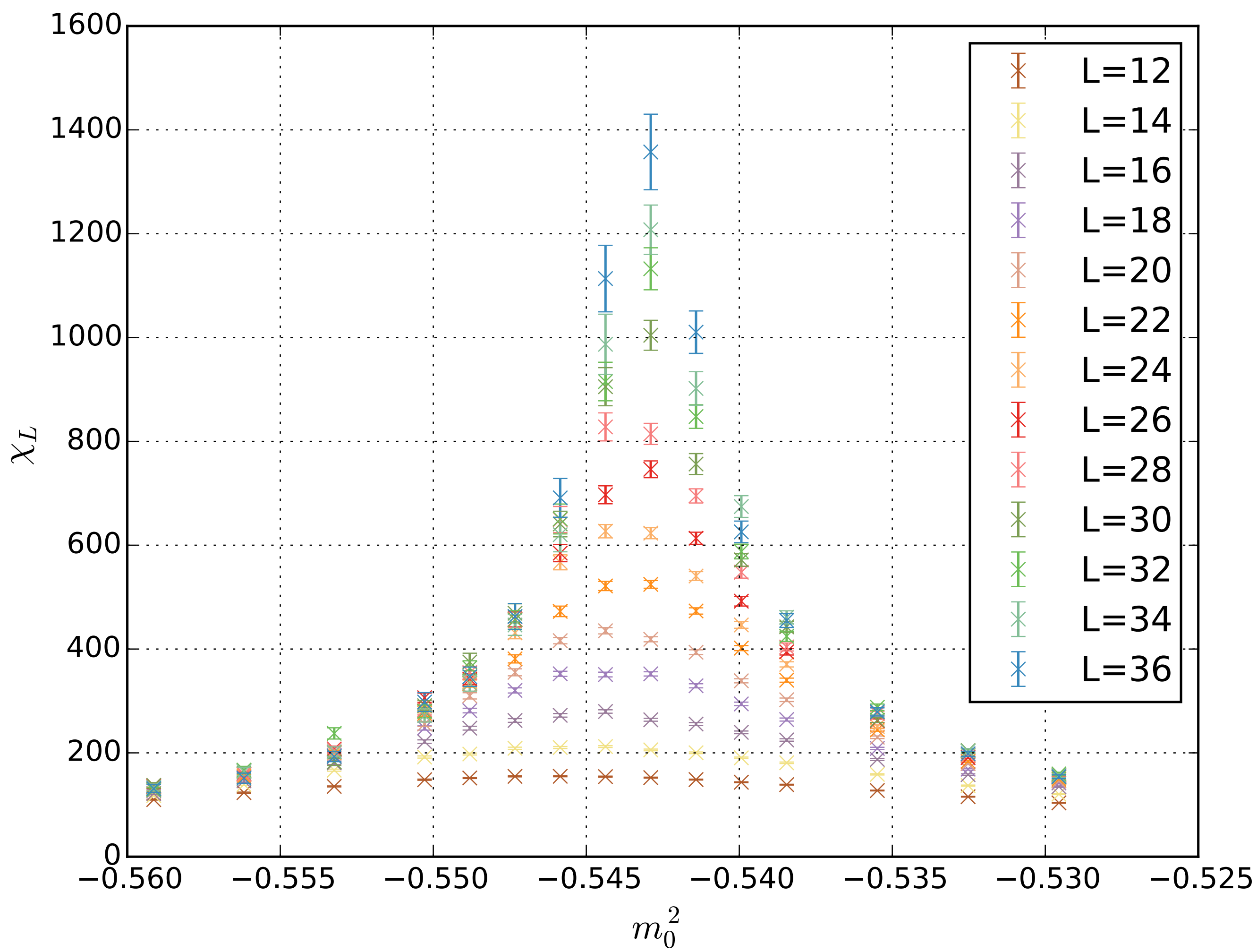
- The lattice can play a role in the study of the Higgs-Yukawa sector of the Standard Model and beyond.
- We investigated the effects of a dimension-6 operator.
  - ★ It can be non-negligible in the Higgs-boson mass.
  - ★ It can induce first-order thermal phase transition at high cutoff.
  - ★ Other quantities, such as the Higgs-boson width can be studied.
- We derived the logarithmic scaling formulae for the Higgs-Yukawa model near the mean-field fixed point.
  - ★ Numerical test in the O(4) scalar model shows good fits.
  - ★ However, fits without logarithms are almost as good.
  - ★ We are generating data at larger volumes for this test.

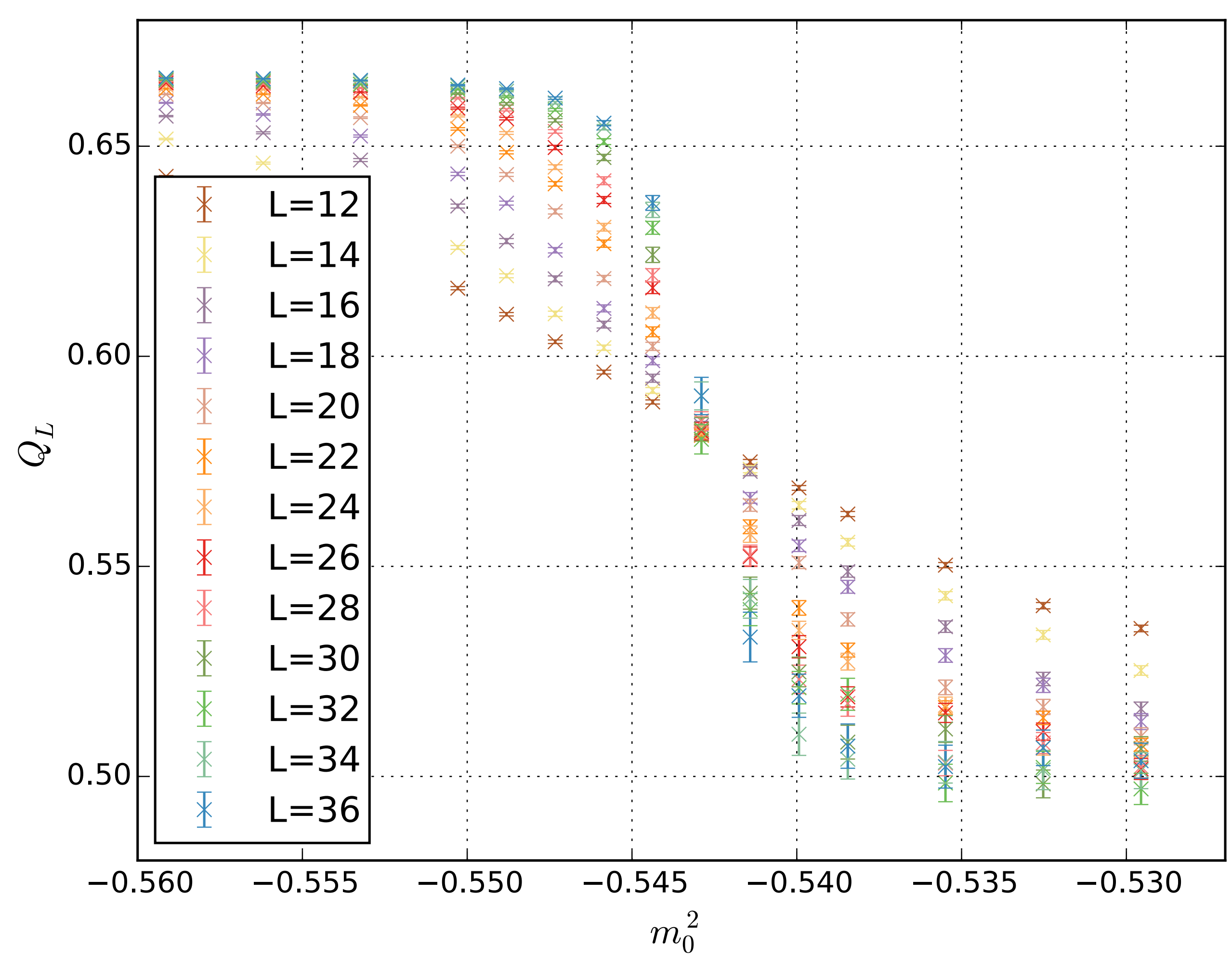
# Backup slides

# First scanning, the magnetisation (VEV)



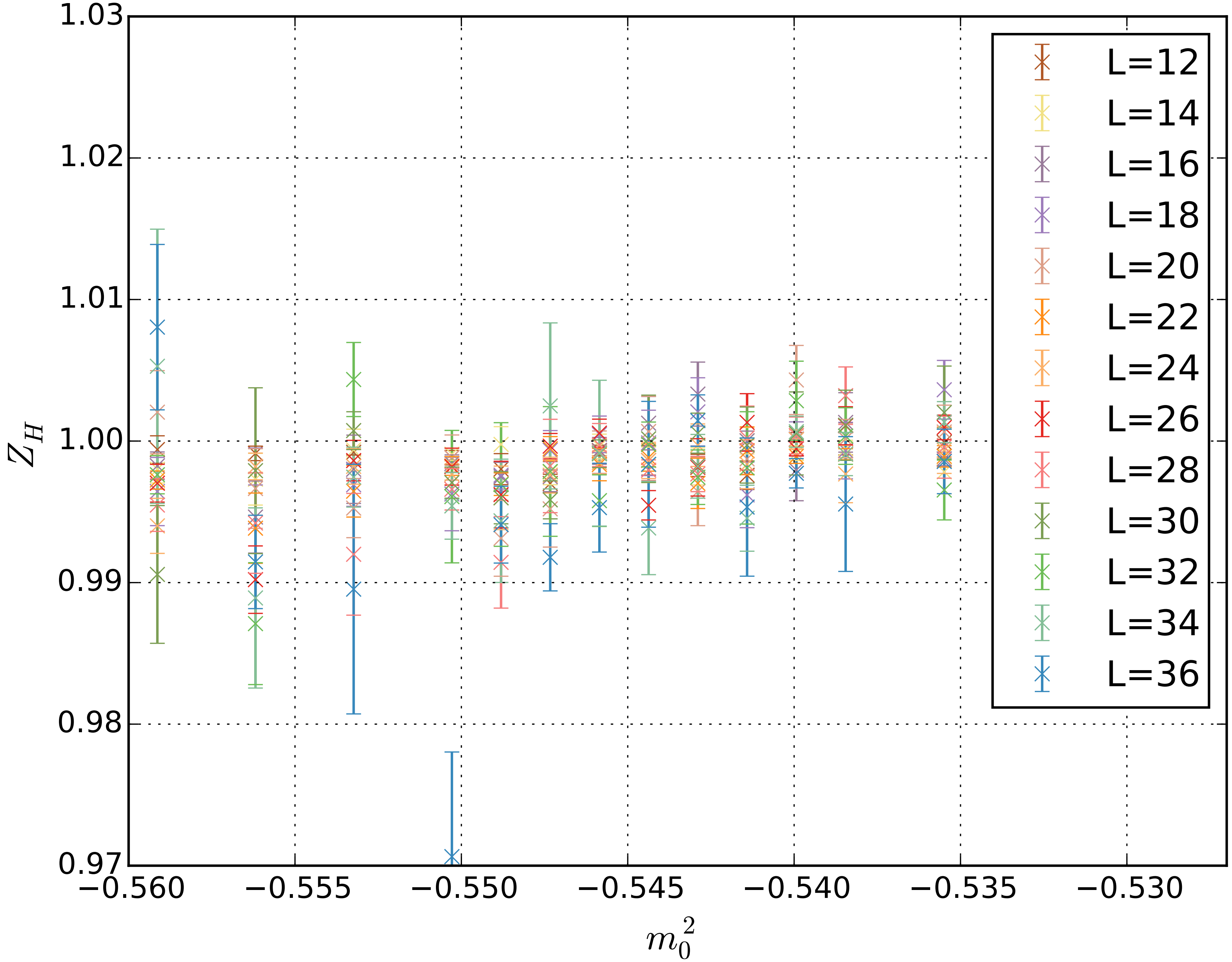
# Susceptibility and Binder's cumulant



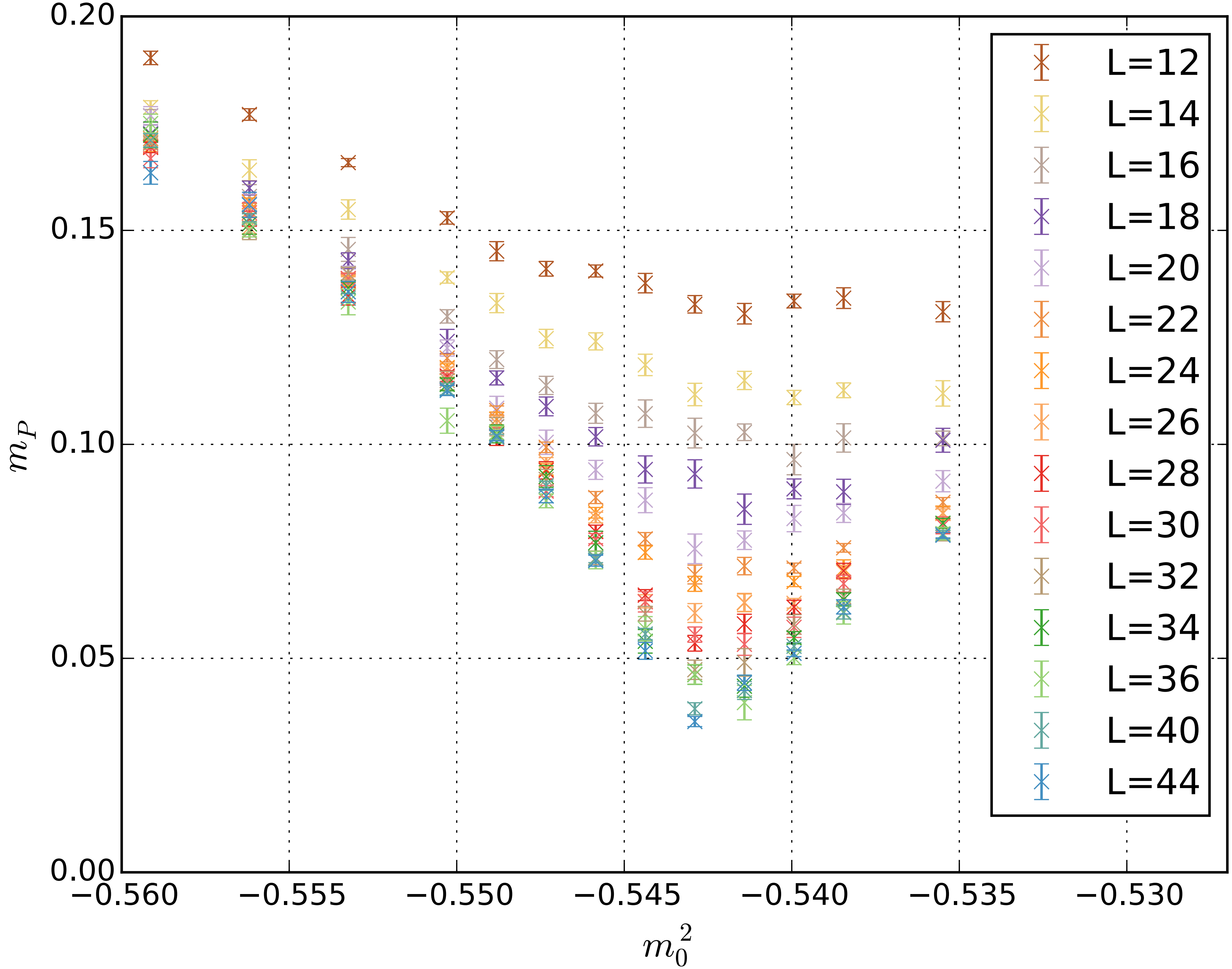

Next step: add data and work with large volumes

# Scalar wavefunction renormalisation

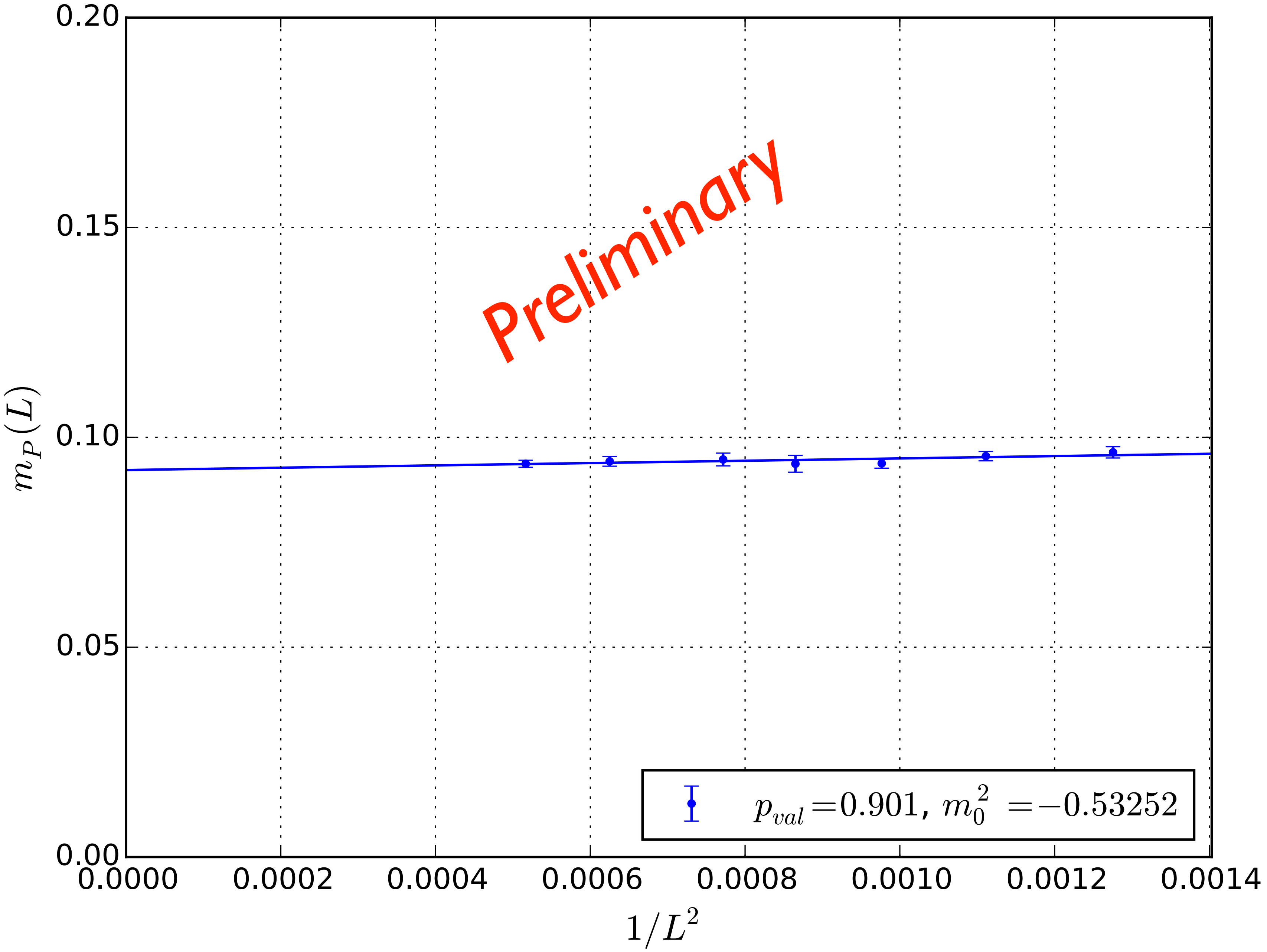



Note: one-loop perturbation theory gives unity.

# The pole mass

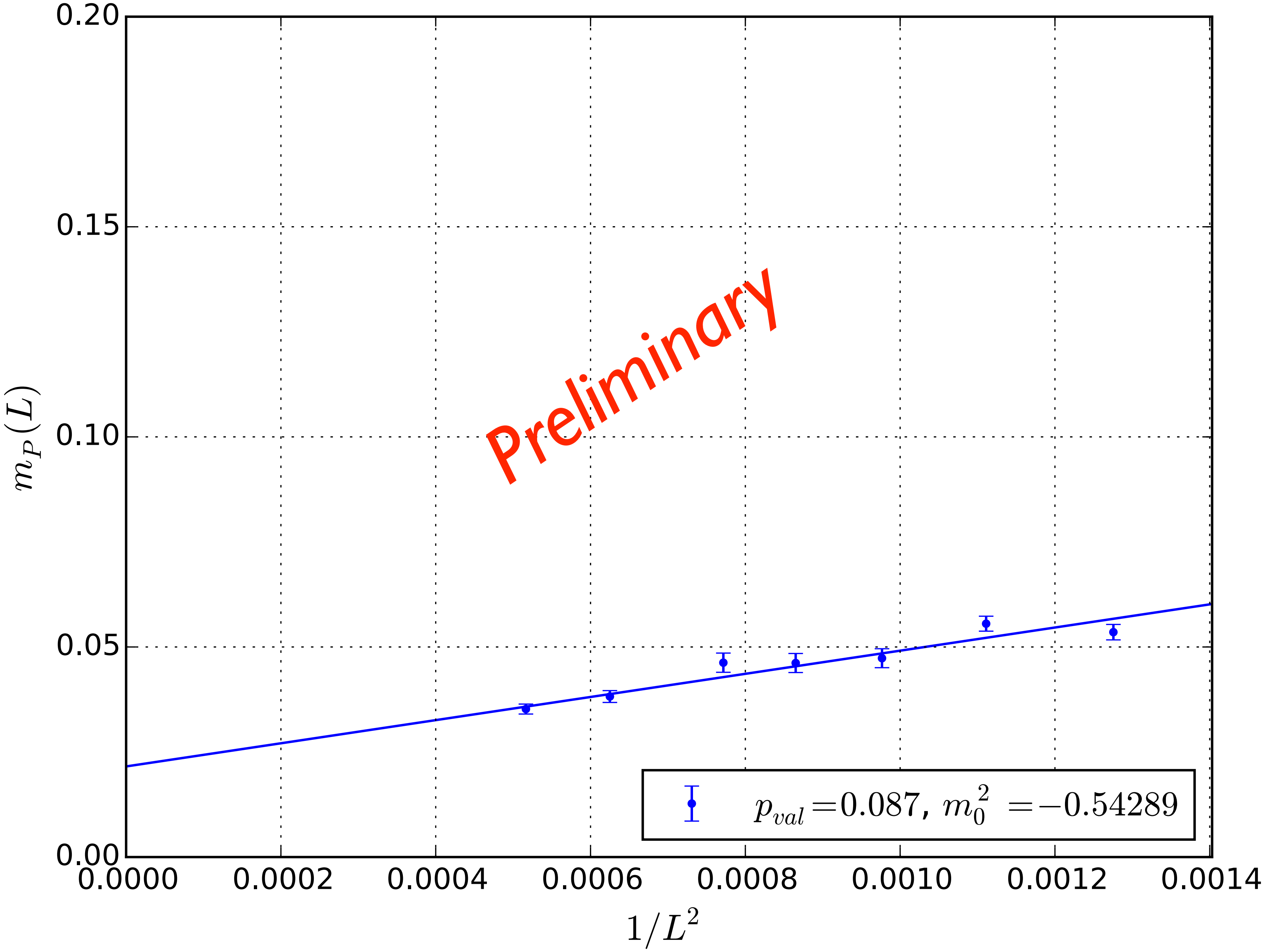


# Extrapolating away, in pole mass, the around-the-world volume effects



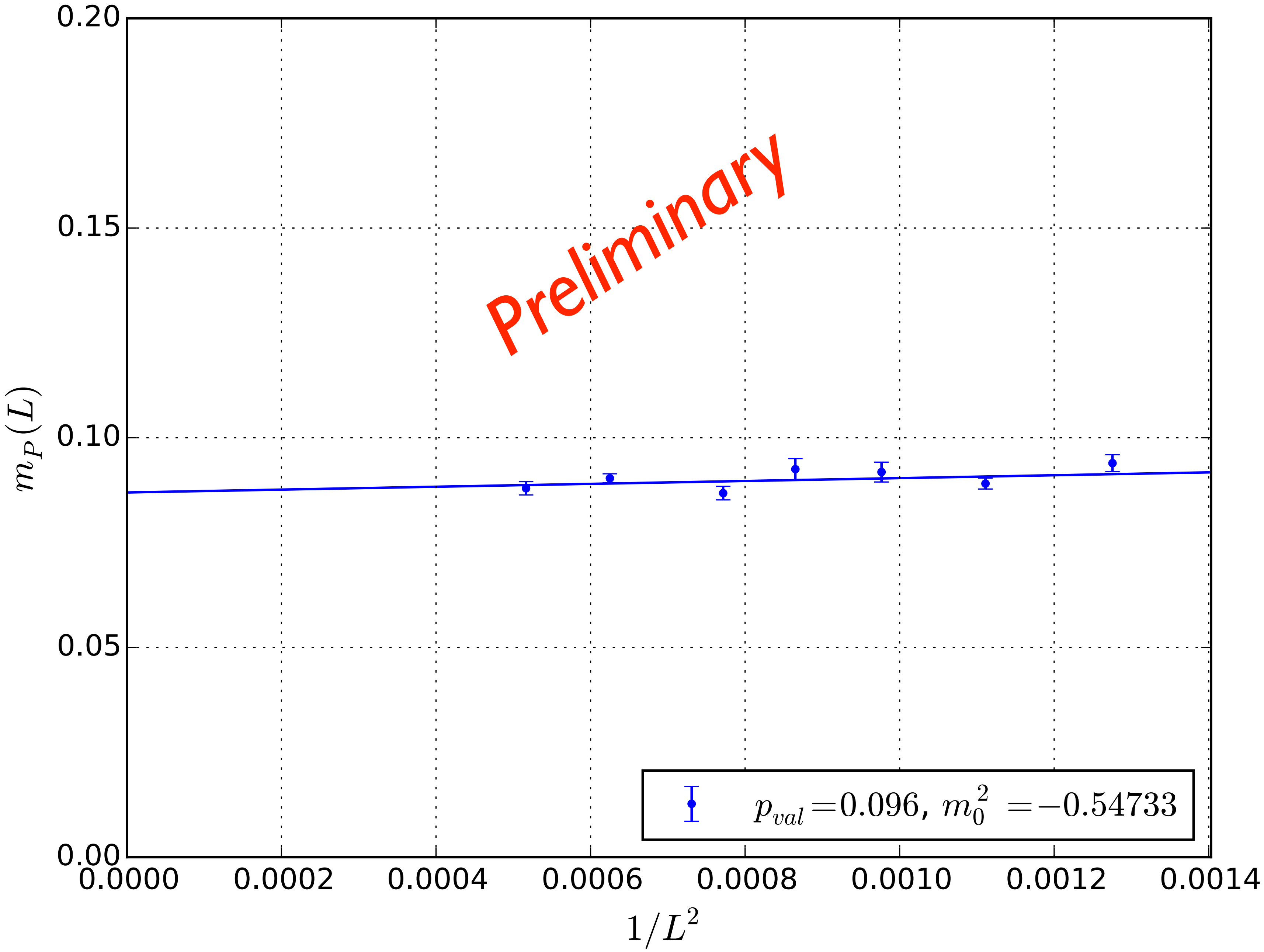

symmetric phase

# Extrapolating away, in pole mass, the around-the-world volume effects



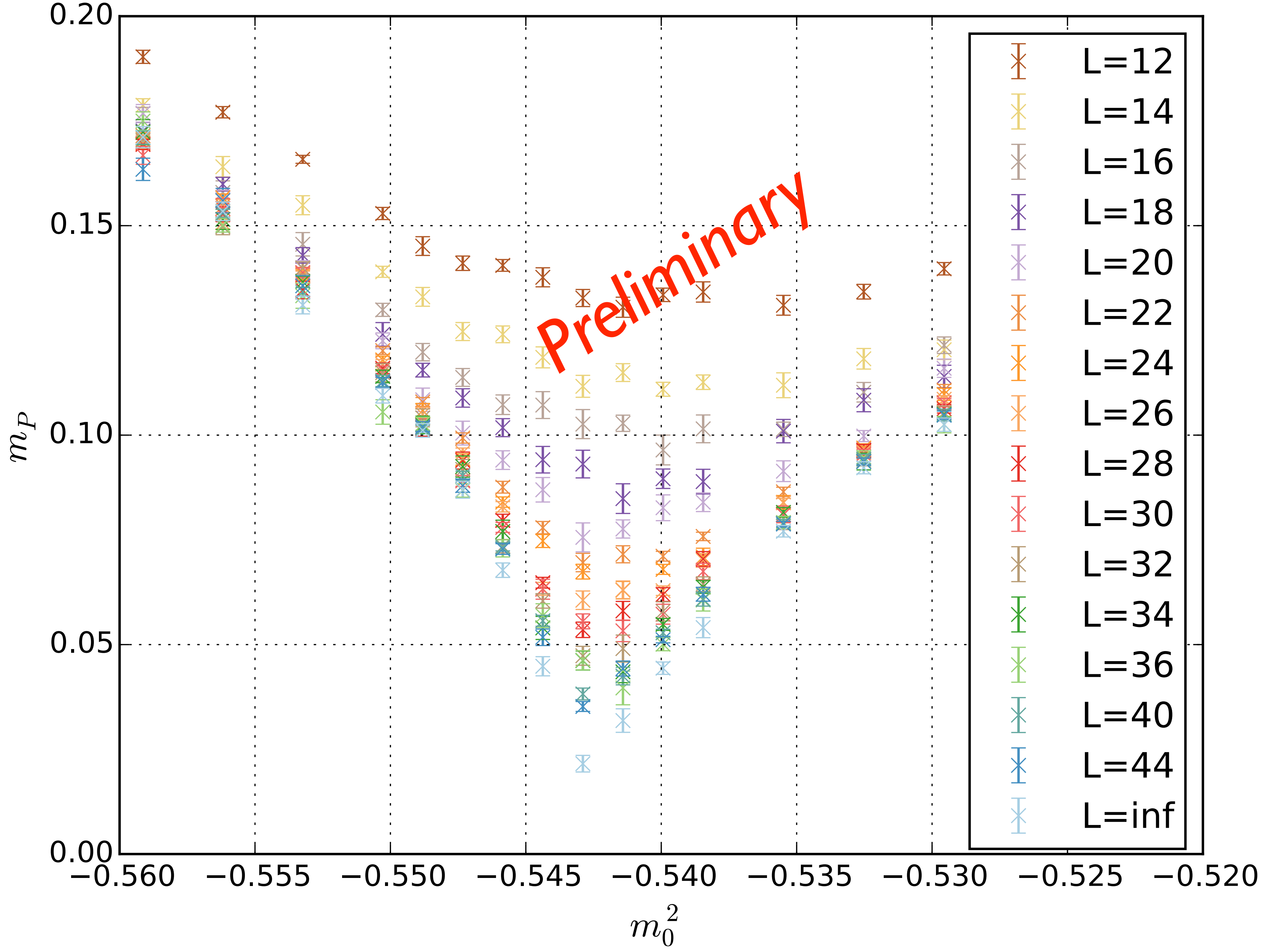

near the critical point

# Extrapolating away, in pole mass, the around-the-world volume effects




broken phase

# The extrapolated pole mass




Extrapolation performed with data at the seven largest volumes