Fourier description of exotic shapes of deformed and fissioning nuclei

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Program:

– Introduction

– Fourier decomposition of a nuclear liquid drop profile,

– Optimal parametrization of shapes of fissionning nuclei,

– Potential energy surfaces in the new deformation parameters sets,

– Summary

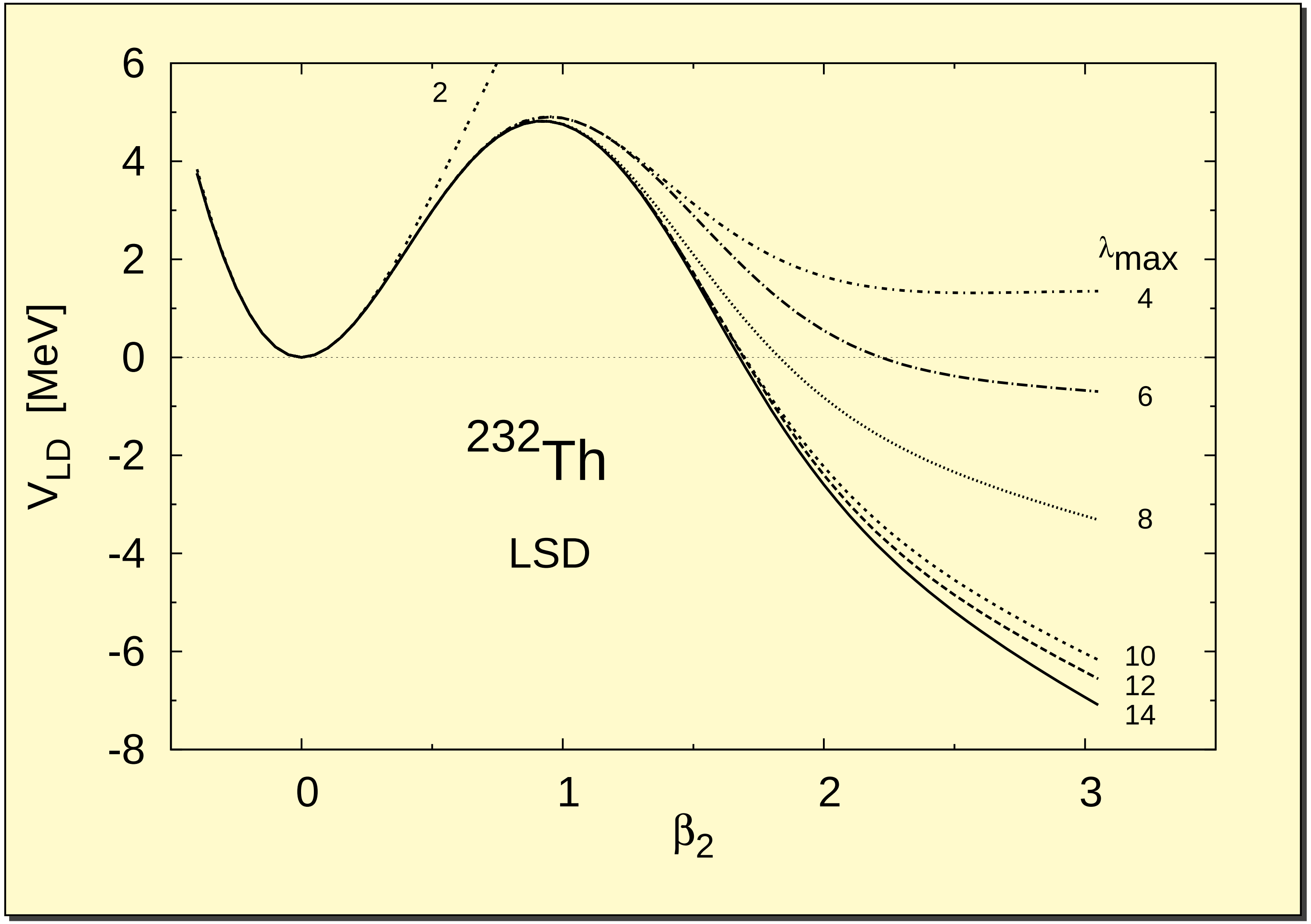
Different shape parametrisations of nuclei:

Proper and low dimensional description of shapes of fissioning nuclei is one of the most difficult task with which nuclear physicists are fighting since the first paper of Bohr and Wheeler on nuclear fission theory.

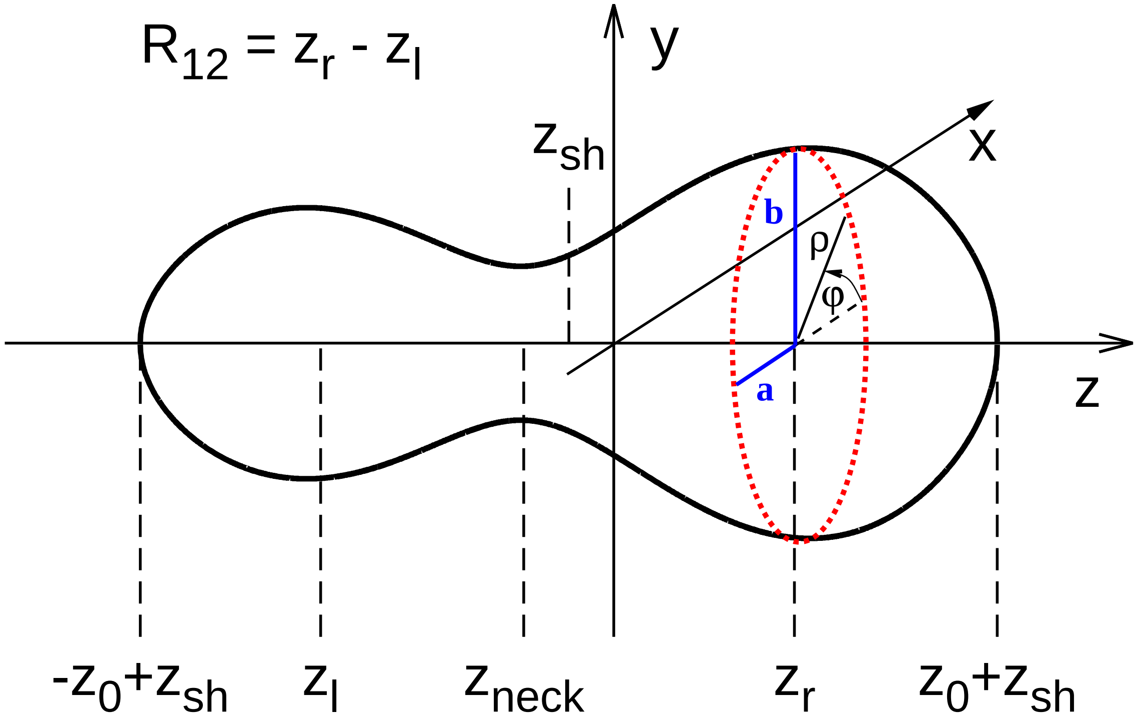
Among the most popular shape parametrisations are:

- classical expansion of nuclear surface in spherical harmonics series, proposed by Lord Rayleigh in 19th century,
- Quadratic Surfaces of Revolutions by J.R. Nix in 1969, used by P. Moller
- Cassini ovals with Pashkevich modifications, 1971,
- Funny-Hills (FH) parametrisation introduced by Brack at al. in 1972, and its modification (Gauss neck and nonaxiality) done in 2004,
- Trentalange, Koonin and Sierk expansion in the Lagrange polynomial series, 1980,

Lord Rayleigh expansion: $R(\theta) = R_0 \sum_{\lambda=0}^{\lambda_{max}} \beta_\lambda P_\lambda[cos(\theta)]$



Fourier expansion of nuclear shapes *



Nonaxial shapes:

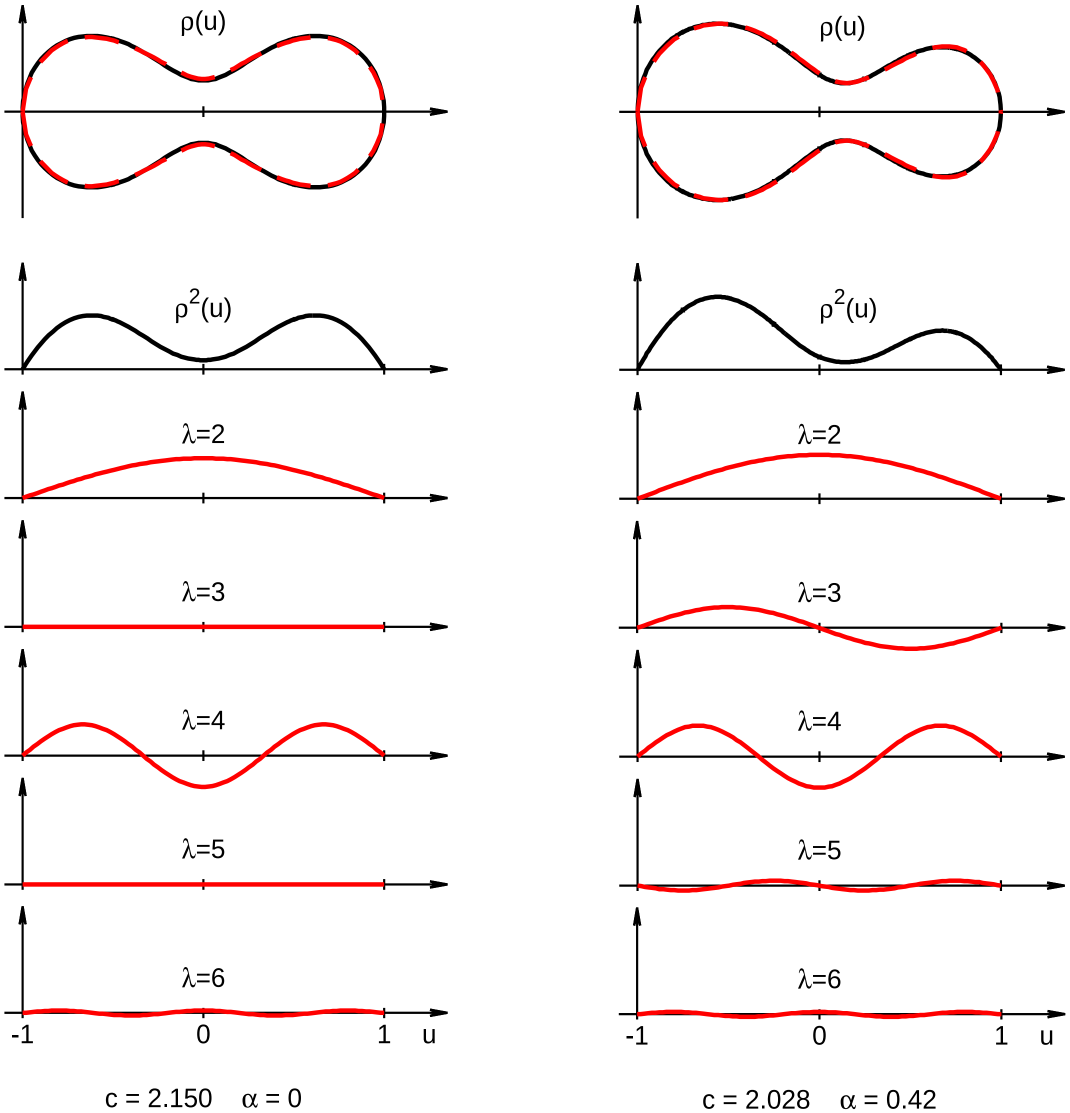
$$\eta = \frac{b-a}{a+b}$$

$$\rho_s^2(z) = a(z)b(z)$$

$$\boxed{\frac{\rho_s^2(z)}{R_0^2} = \sum_{n=1}^{\infty} \left[a_{2n} \cos\left(\frac{(2n-1)\pi}{2} \frac{z - z_{sh}}{z_0} \right) + a_{2n+1} \sin\left(\frac{2n\pi}{2} \frac{z - z_{sh}}{z_0} \right) \right]} \; ,$$

*K. Pomorski, B. Nerlo-Pomorska, J. Bartel, and C. Schmitt Acta Phys. Pol. B Supl. **8** (2015) 667.

Fourier expansion of nuclear shapes*



Here:

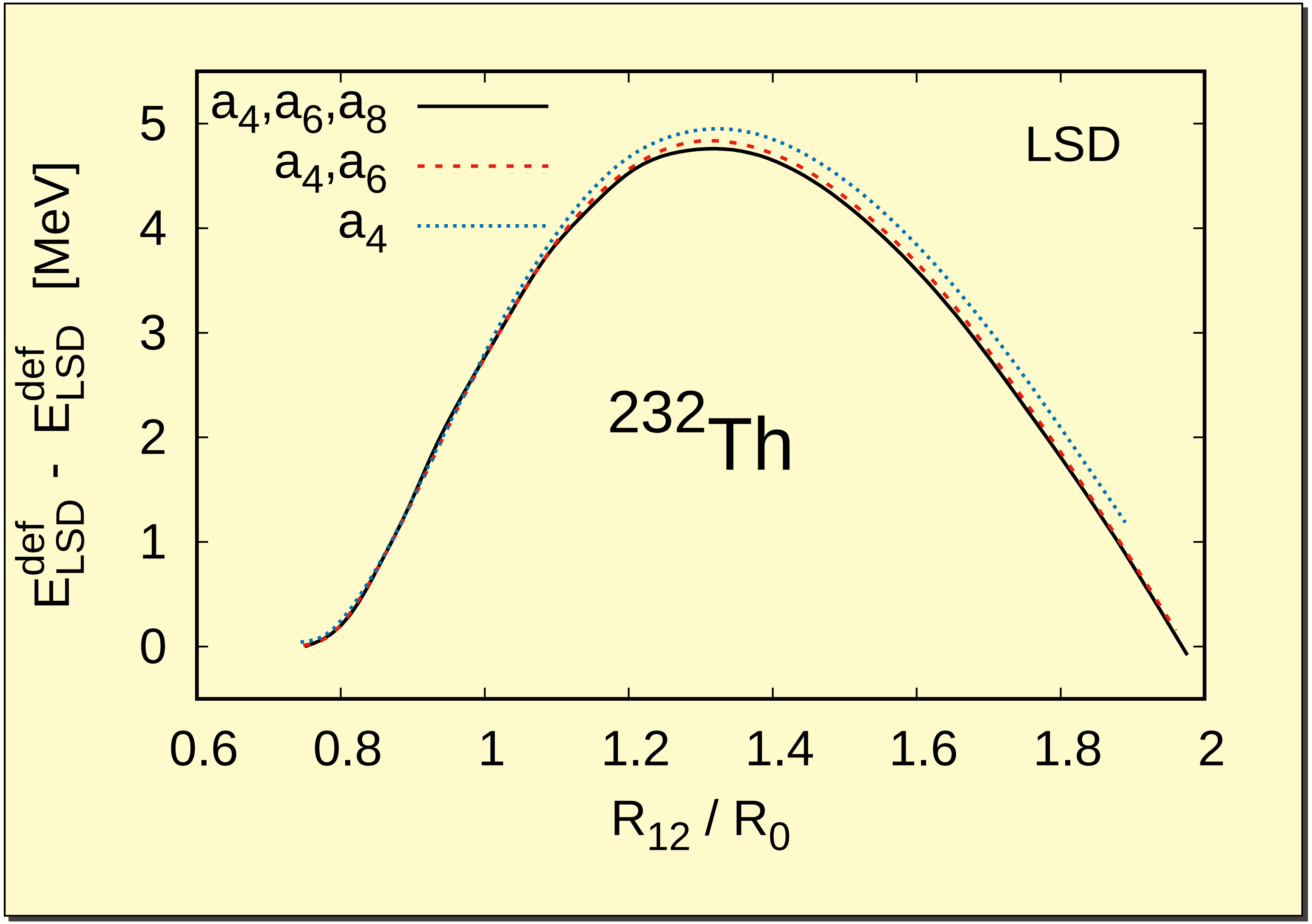
$$u = \frac{z - z_{\rm sh}}{z_0} \,,$$

$$c = \frac{z_0}{R_0} \,,$$

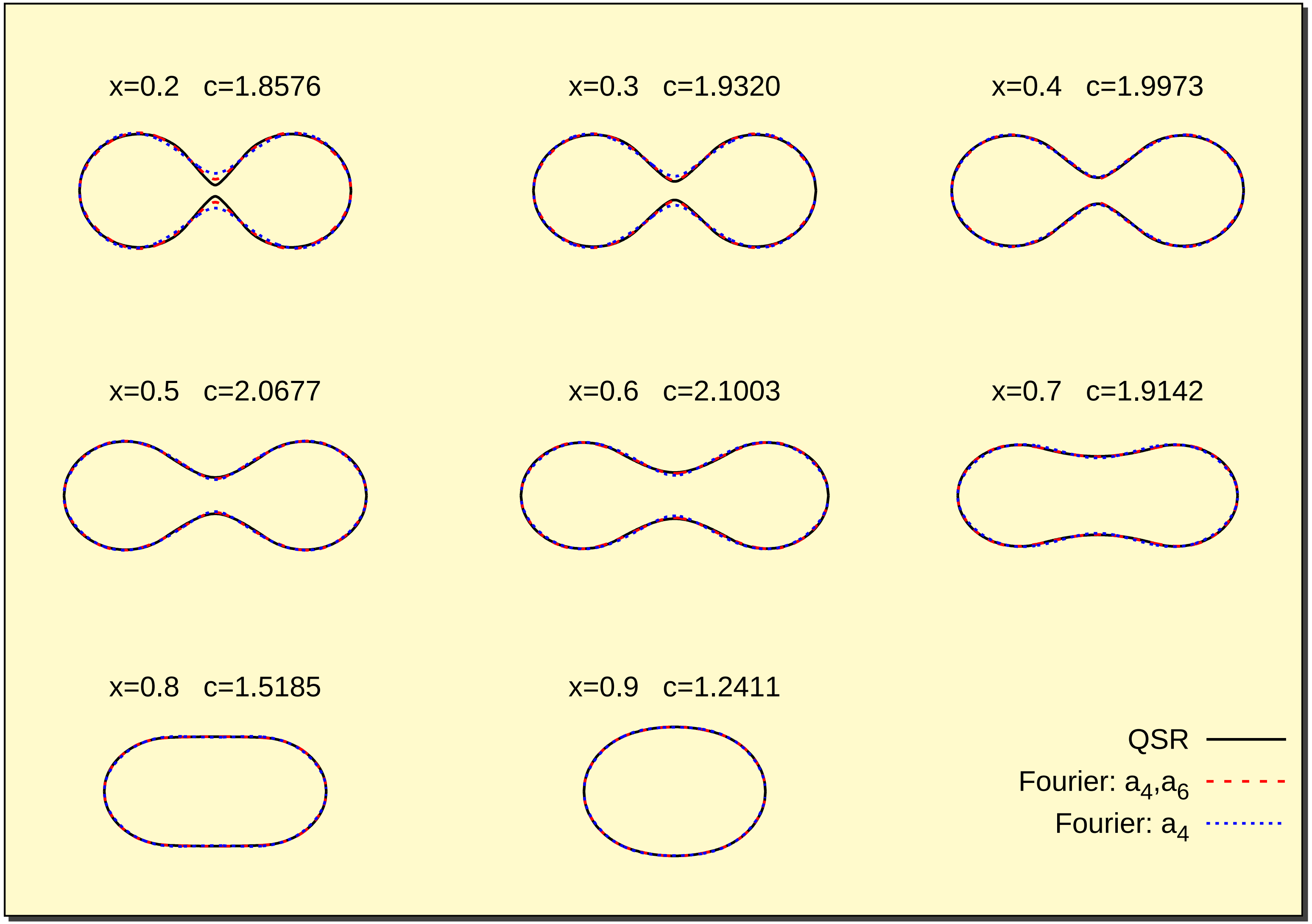
$$\alpha = \frac{M_l - M_r}{M_l + M_r} \,.$$

*Optimal shapes (in black) origin from [K.P. and F. Ivanyuk, PRC **79** (2009) 054327.]

Convergence of the Fourier expansion

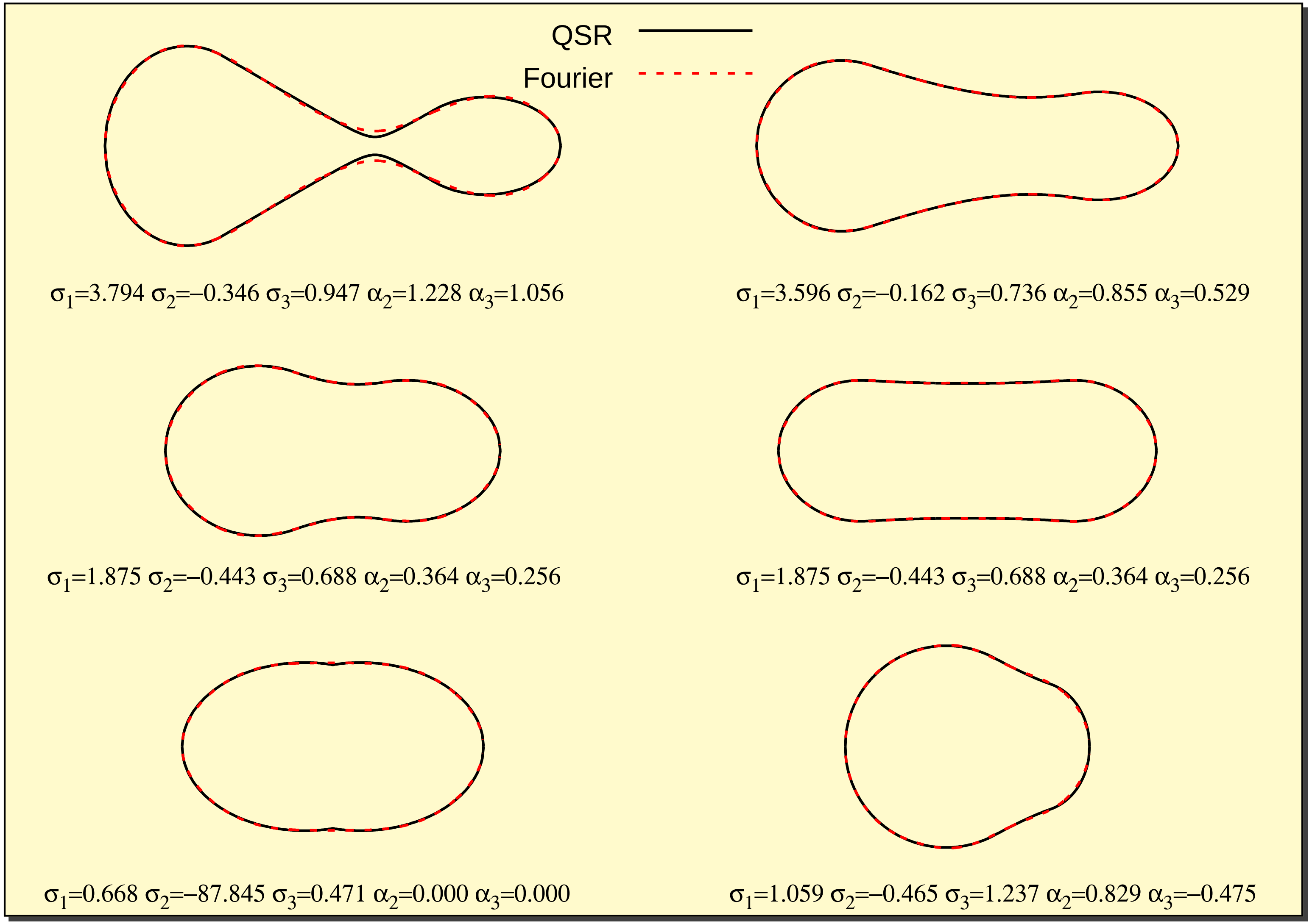


Shapes of nuclei in the LD saddle points *



*J.R. Nix, Nucl. Phys. **A130** (1969) 241. Here x and c are the fissility and elongation parameters.

QSR shapes * and the Fourier parametrisation



The QSR data was provided by dr Peter Moller from the LANL

The volume conservation condition leads to the following relation:

$$\frac{\pi}{3c} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{a_{2n}}{2n-1} \; .$$

where $c = z_0/R_0$ is the elongation parameter.
In case of odd-multipolarity deformations the coordinate z in ρ_S^2 must be shifted by:

$$z_{sh} = \frac{3c^2}{2\pi} R_0 \sum_n (-1)^n \frac{a_{2n+1}}{n} \; ,$$

in order to keep the centre of mass at $z = 0$. For symmetric fission the scission point is located at $z = 0$ which implies: $\sum\limits_{n=1}^{\infty} a_{2n} = 0$.
In presence of the odd-deformation the neck appears at:

$$z_{\rm neck} = z_{sh} + z_0 \cdot \frac{4}{\pi} \frac{\sum\limits_n n\, a_{2n+1}}{\sum\limits_n (2n-1)^2\, a_{2n}} \; ,$$

what has to be taken into account when one evaluates the relative distance between fragments: R_{12}.

Charged liquid drop potential energy surface

The potential energy of a deformed nucleus is given in the liquid drop (LD) model by:

$$\boxed{\frac{\Delta E(\mathrm{def})}{E_{\mathrm{surf}}(\mathrm{sph})} = [B_{\mathrm{surf}}(\mathrm{def}) - 1] + 2\chi[B_{\mathrm{Coul}}(\mathrm{def}) - 1] \ .}$$

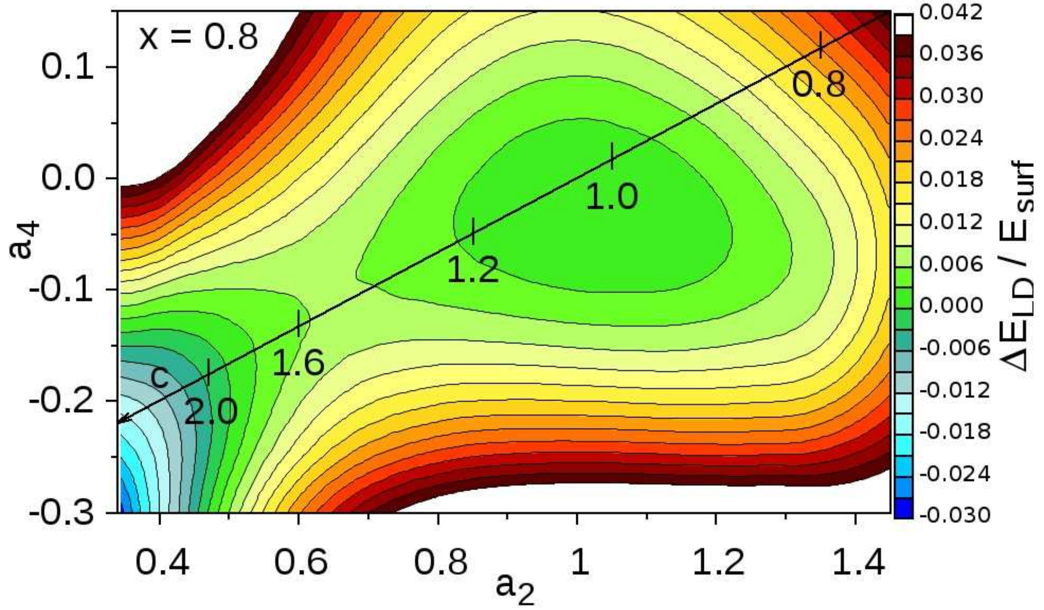
Here $\Delta E(\mathrm{def}) = E_{\mathrm{LD}}(\mathrm{def}) - E_{\mathrm{LD}}(\mathrm{sph})$ is the difference between the energies of the deformed the spherical nucleus.

The factor χ is the fissility parameter:

$$\chi = \frac{E_{\mathrm{Coul}}(\mathrm{sph})}{2E_{\mathrm{surf}}(\mathrm{sph})} \ ,$$

where $E_{\mathrm{surf}}(\mathrm{sph})$ and $E_{\mathrm{Coul}}(\mathrm{sph})$ are respectively the surface and Coulomb energies of the spherical nucleus.

Potential energy surface in the (a_2, a_4) plane



Optimal coordinates:

$$q_2 = a_2^{(0)}/a_2 - a_2/a_2^{(0)} \; ,$$

$$q_3 = a_3 \; ,$$

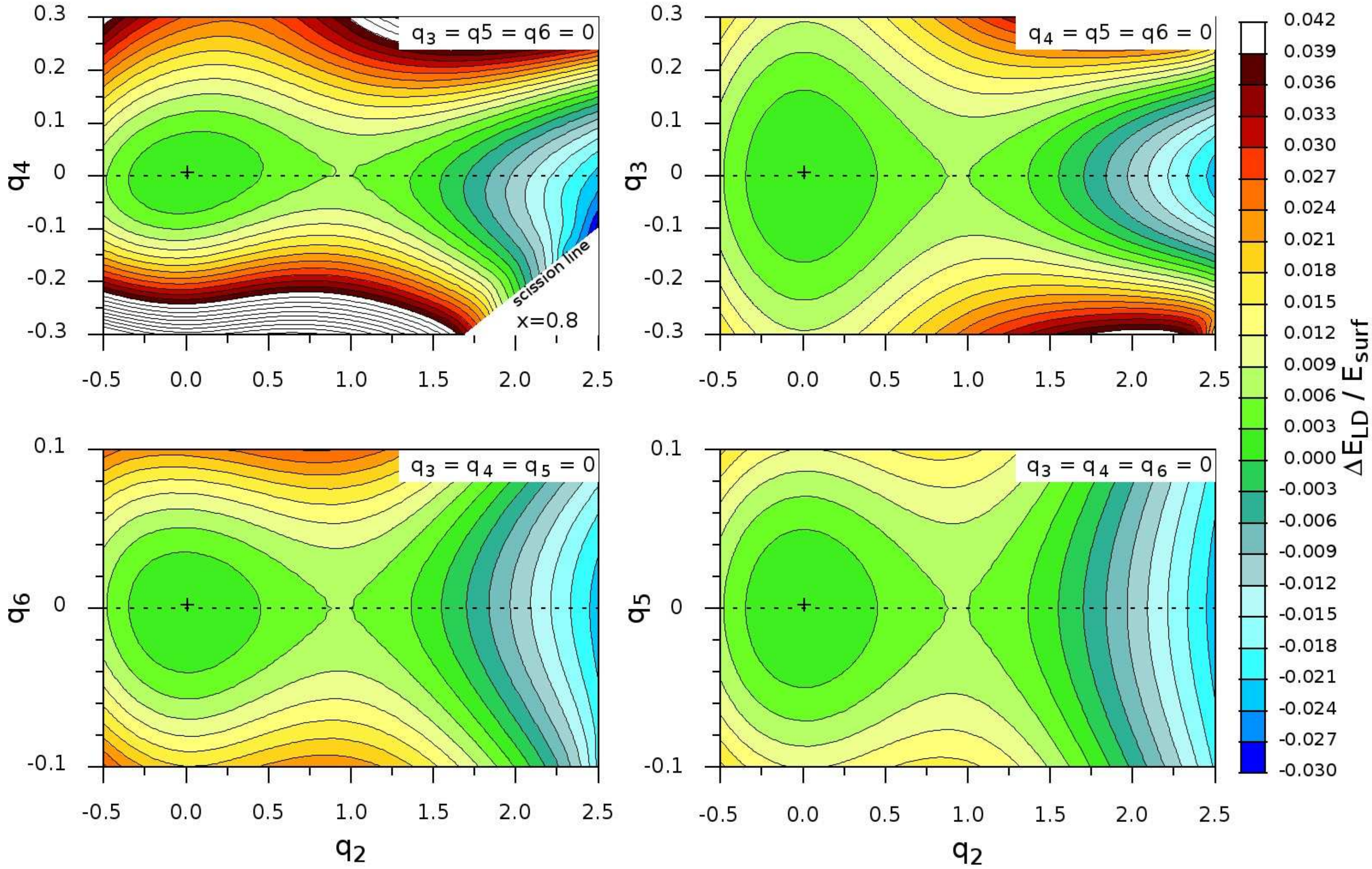
$$q_4 = a_4 + \sqrt{(q_2/9)^2 + (a_4^{(0)})^2} \; ,$$

$$q_5 = a_5 - (q_2 - 2)a_3/10 \; ,$$

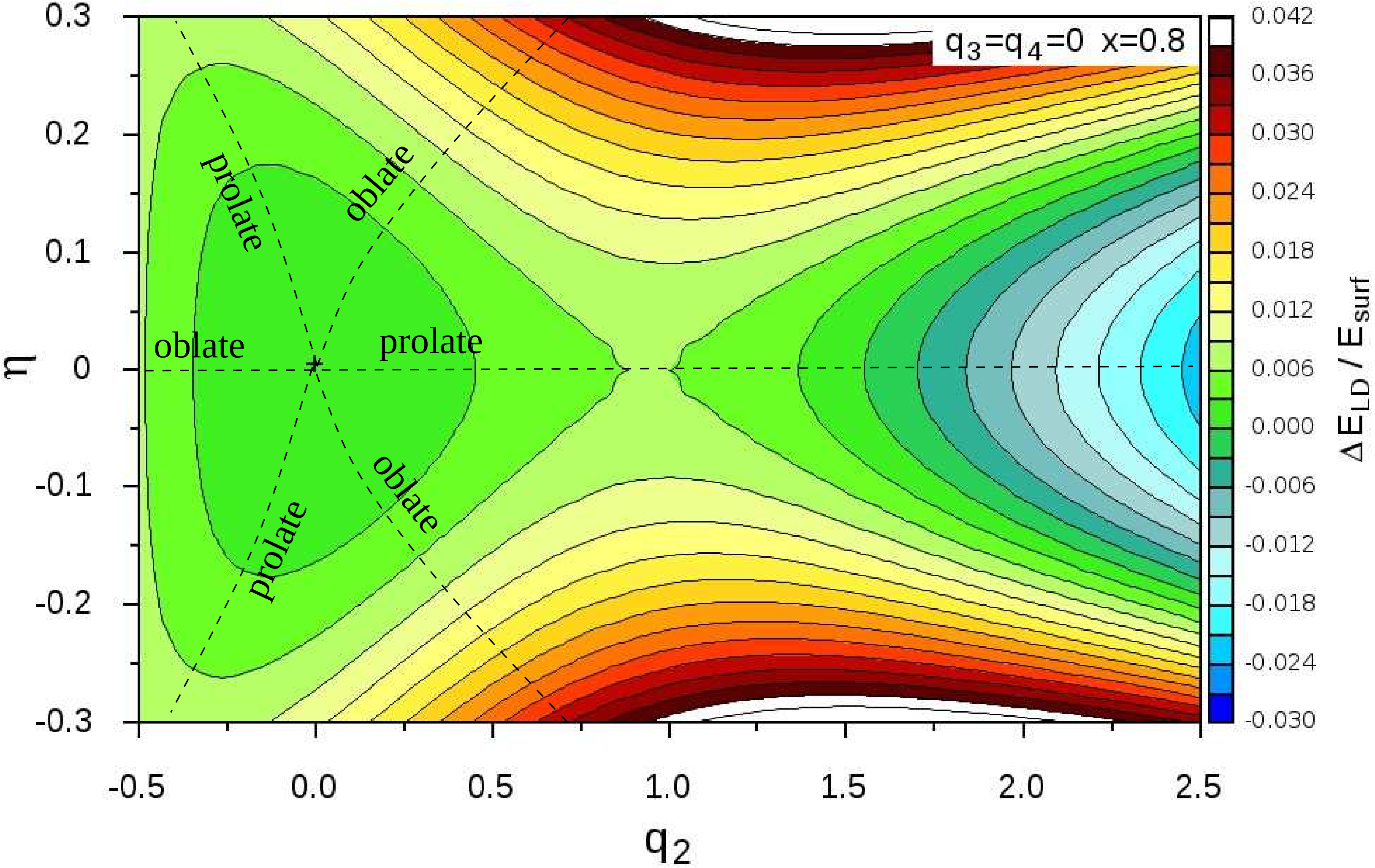
$$q_6 = a_6 - \sqrt{(q_2/100)^2 + (a_6^{(0)})^2} \; .$$

Here $a_2^{(0)} = 1.03205$, $a_4^{(0)} = -0.03822$, and $a_6^{(0)} = 0.00826$ are the Fourier expansion coefficients of a sphere. q_2 describes the elongation of a nucleus, q_3 its left-right asymmetry, and q_4 formation of the neck.

Examples of the LSD potential energy surfaces:



LSD potential surface surface on the (q_2, η) plane:



Potential energy surfaces of fissioning nuclei

Main assumptions of our model:

– macroscopic-microscopic approximation of nuclear energy,

– Lublin-Strasbourg-Drop,

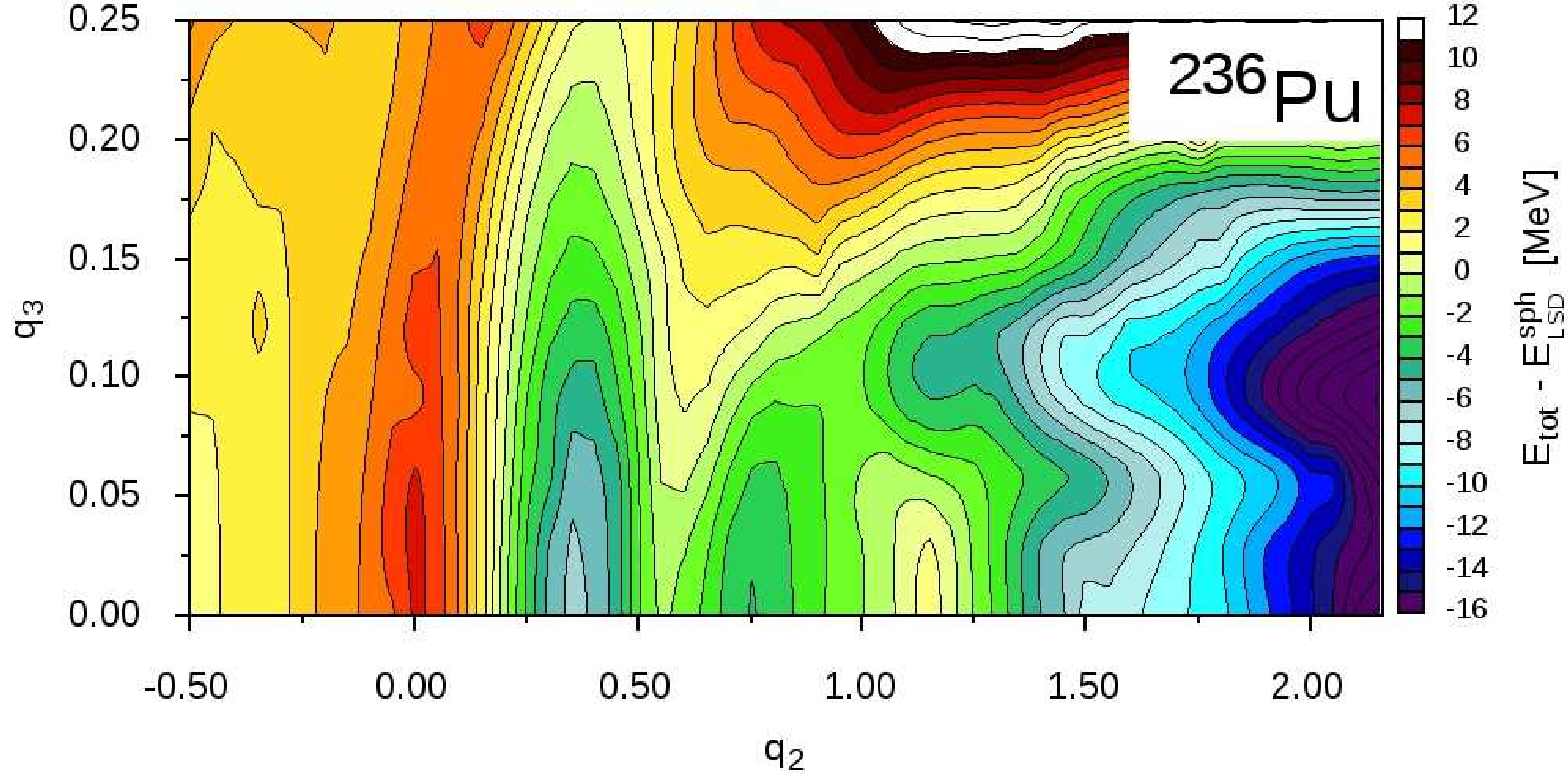
– Yukawa-folded single-particle potential,

– Strutinsky shell-correction method,

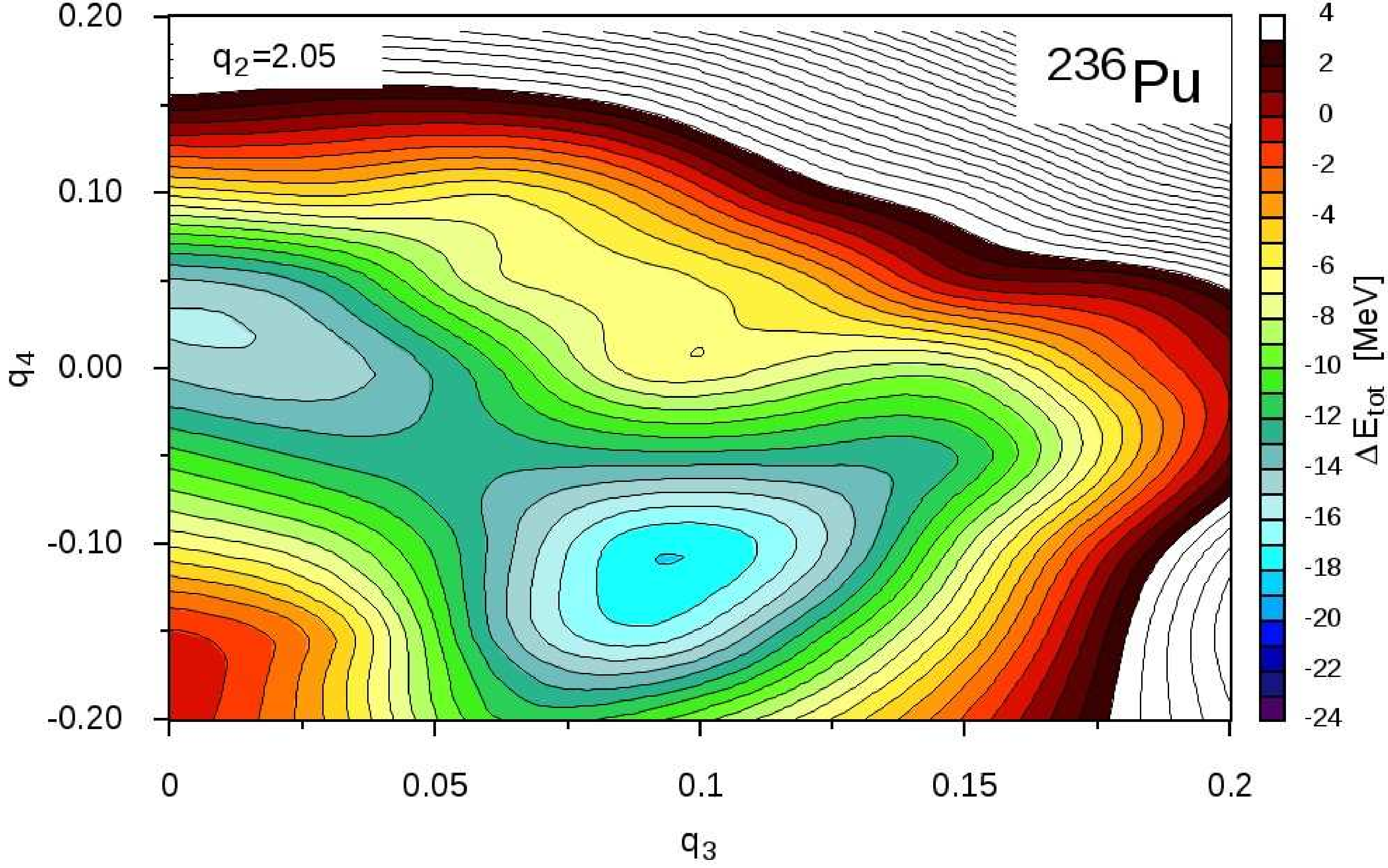
– BCS monopole pairing force,

– Fourier parametrisation of nuclear shapes,

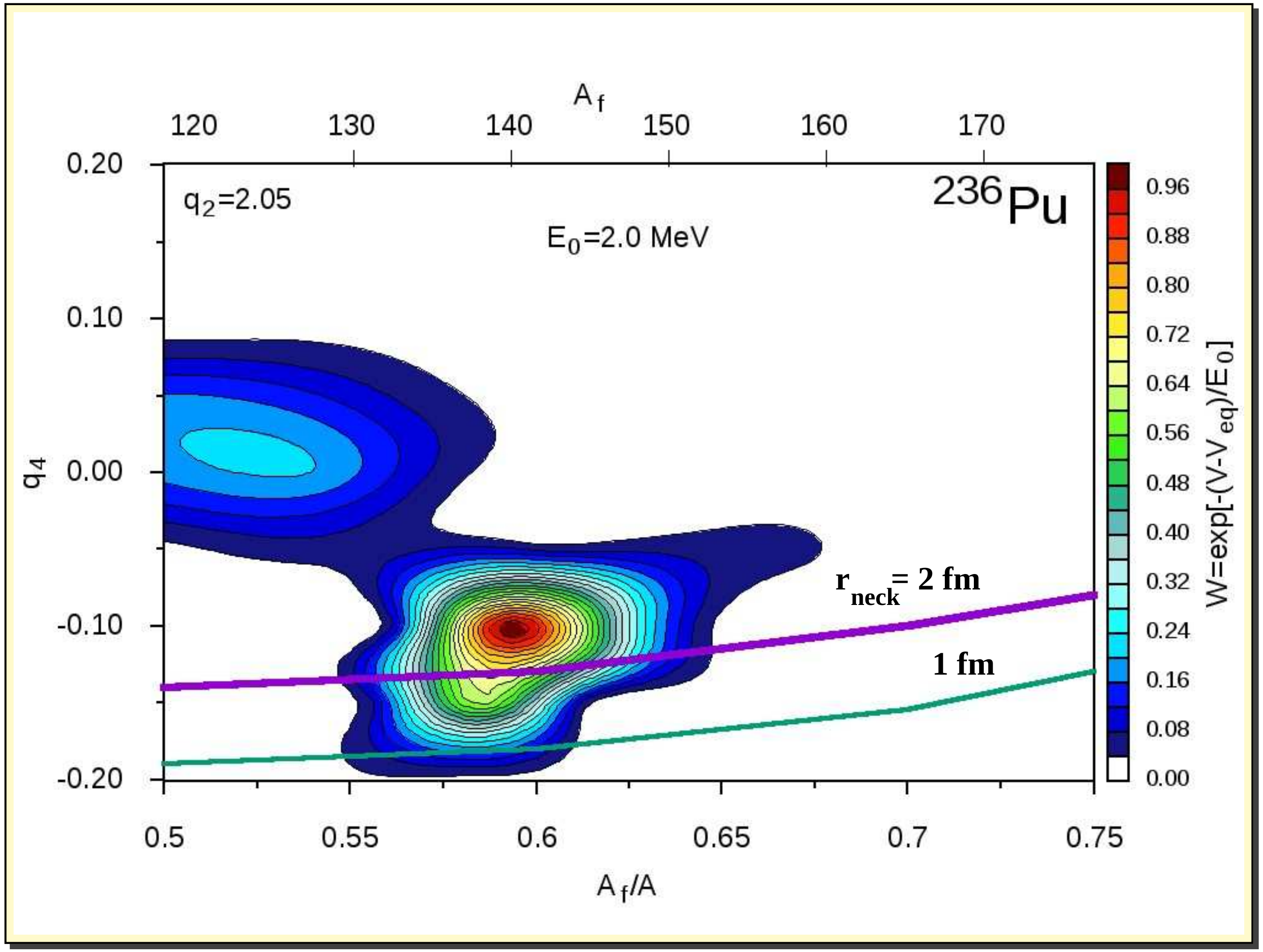
Potential energy surface on the (q_2, q_3) plane



PES cross-section at $q_2 = 2.05$

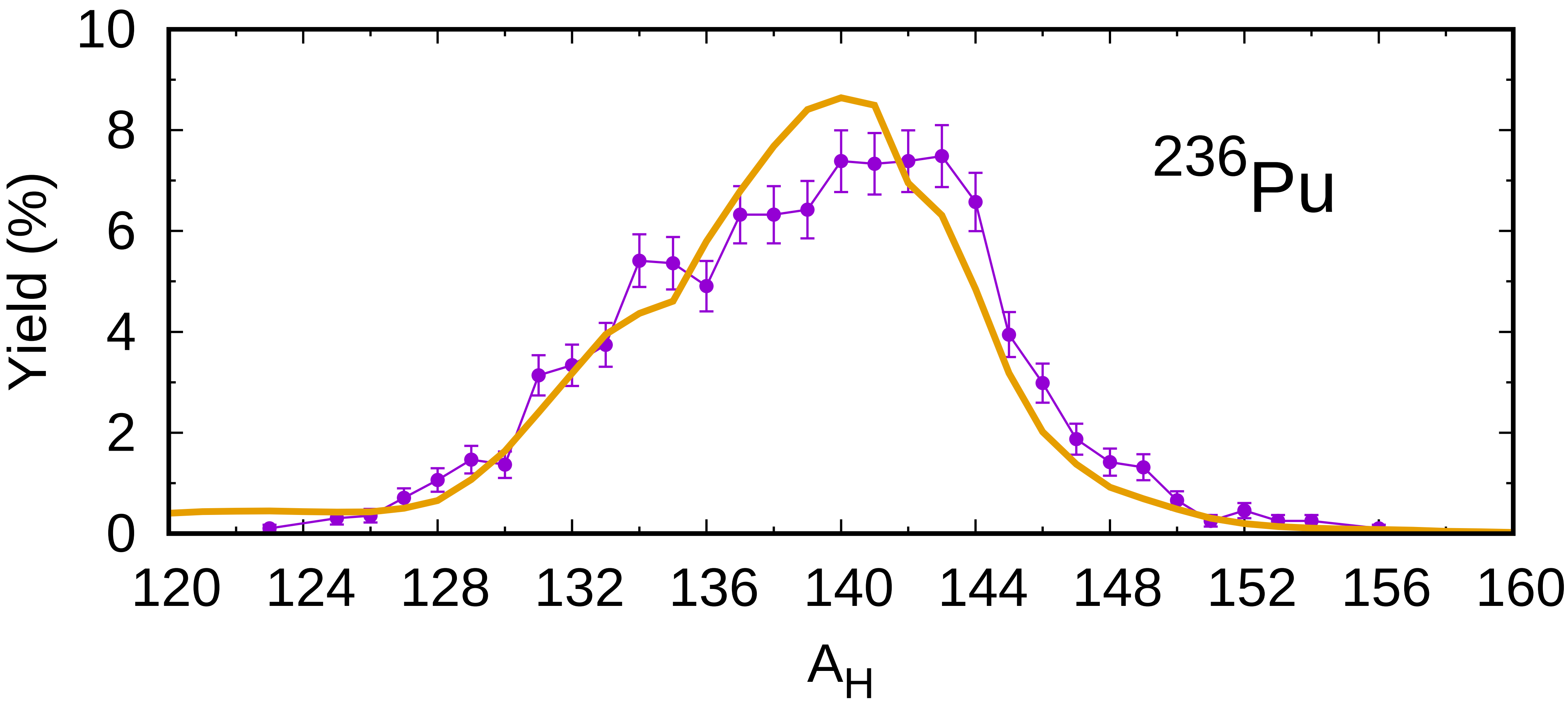


Wigner function for the cross-section at $q_2 = 2.05$



$$W(A_{\mathrm{f}}, q_4) = \exp\left(\frac{V(A_{\mathrm{f}}, q_4) - V_{\mathrm{min}}}{E_0}\right)$$

Fission fragment mass yield of ^{236}Pu



Experimental data from L. Dématté et al, Nucl. Phys. **A617**, 331 (1997).

Conclusions:

- New, rapidly convergent Fourier expansion of nuclear shape is proposed,
- An effective set of the deformation parameters to describe the nuclear fission process was found,
- Three dimensional model which couples the fission, neck and mass asymmetry modes is able to describe the main features of the fragment mass yield.

Thank you for your attention!

Short announcement:

The **24$^{\mathrm{th}}$ Nuclear Physics Workshop** in Kazimierz Dolny will be held

from September 20$^{\mathrm{th}}$ to 24$^{\mathrm{th}}$ 2017.

You all are cordially invited to participate in this every year meeting of theoreticians and experimentalists.