

On the Structure of ^{32}Mg

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**Shapes and Symmetries in Nuclei: from Experiment to Theory
(SSNET) Workshop**
Gif sur Yvette, November 7th – 11th, 2016

Outline

Short Introduction

The $^{30}\text{Mg}(t,p)^{32}\text{Mg}$ Reaction

Fortune's analysis

The “Puzzle” Revisited

Three-level Mixing

^{33}Mg 1n KO à la Nilsson

Summary

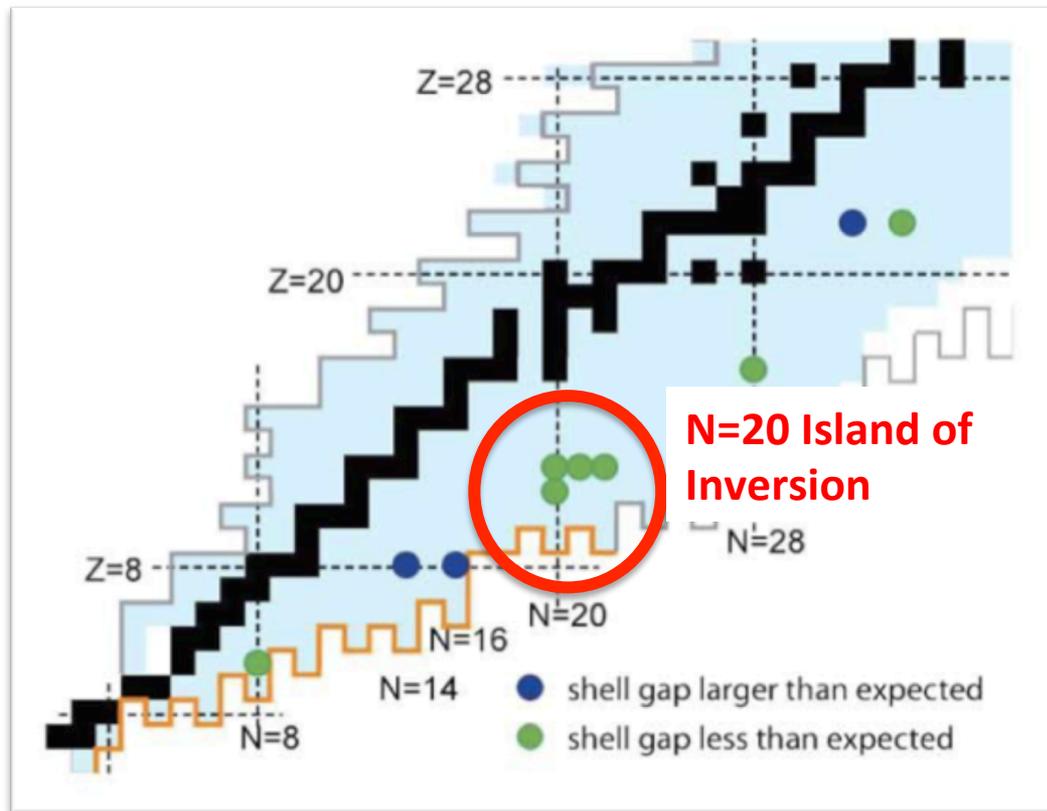
Evolution of Shell Structure and Collectivity

“Classic” magic numbers are generally correct only for stable and near stable isotopes

Experimental studies of new exotic isotopes has provided insight on the important role play by the tensor and 3-body forces in the changes in shell structure and collectivity:

A delicate balance between the monopole field and correlations.

R.V.F Janssens, Nature, Vol. 435, 2005.



^{32}Mg , at the center of this region, has been a subject of intense work for many years, both experimental and theoretical.

→ A clear fingerprint : Rotational ground state band

Cf. Heather Crawford's talk

The $^{30}\text{Mg}(t,p)^{32}\text{Mg}$ reaction

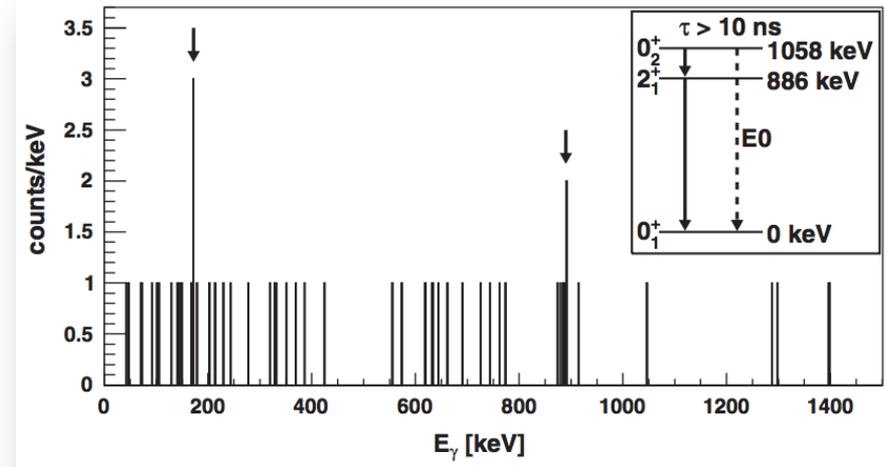
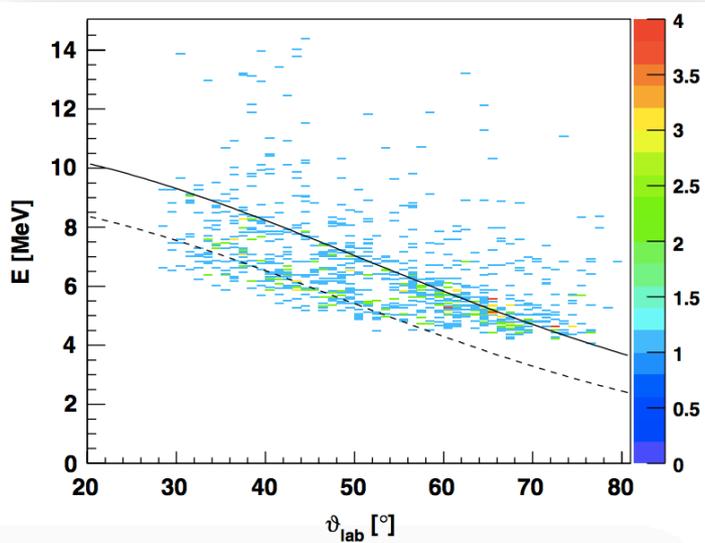
PRL **105**, 252501 (2010)

Selected for a **Viewpoint** in *Physics*
PHYSICAL REVIEW LETTERS

week ending
17 DECEMBER 2010

Discovery of the Shape Coexisting 0^+ State in ^{32}Mg by a Two Neutron Transfer Reaction

K. Wimmer,¹ T. Kröll,^{1,*} R. Krücken,¹ V. Bildstein,¹ R. Gernhäuser,¹ B. Bastin,² N. Bree,² J. Diriken,² P. Van Duppen,² M. Huyse,² N. Patronis,^{2,†} P. Vermaelen,² D. Voulot,³ J. Van de Walle,³ F. Wenander,³ L.M. Fraile,⁴ R. Chapman,⁵ B. Hadinia,⁵ R. Orlandi,⁵ J.F. Smith,⁵ R. Lutter,⁶ P.G. Thierolf,⁶ M. Labiche,⁷ A. Blazhev,⁸ M. Kalkühler,⁸ P. Reiter,⁸ M. Seidlitz,⁸ N. Warr,⁸ A.O. Macchiavelli,⁹ H.B. Jeppesen,⁹ E. Fiori,¹⁰ G. Georgiev,¹⁰ G. Schrieder,¹¹ S. Das Gupta,¹² G. Lo Bianco,¹² S. Nardelli,¹² J. Butterworth,¹³ J. Johansen,¹⁴ and K. Riisager¹⁴



The $^{30}\text{Mg}(t,p)^{32}\text{Mg}$ reaction

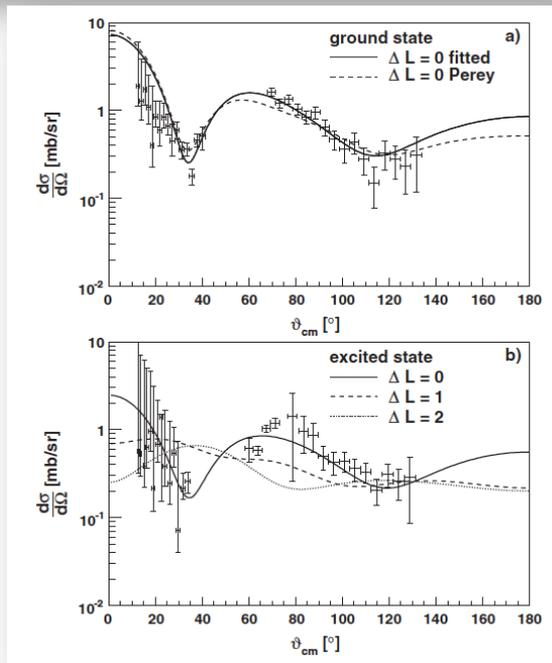
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$$E(0_2^+) = 1058(2) \text{ keV}$$

$$\sigma(\text{g.s.}) = 10.5(7) \text{ mb}$$

$$\sigma(0_2^+) = 6.5(5) \text{ mb}$$

$$\sigma(0_2^+)/\sigma(\text{g.s.}) = 0.62(6)$$

The $^{30}\text{Mg}(t,p)^{32}\text{Mg}$ “puzzle”

PHYSICAL REVIEW C **84**, 024327 (2011)

The puzzle of ^{32}Mg

H. T. Fortune

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia Pennsylvania, 19104, USA

(Received 2 April 2011; revised manuscript received 2 August 2011; published 29 August 2011)

An analysis of results of the $^{30}\text{Mg}(t,p)^{32}\text{Mg}$ reaction demonstrates that the ground state is the normal state and the excited 0^+ state is the intruder, contrary to popular belief. Additional experiments are suggested.

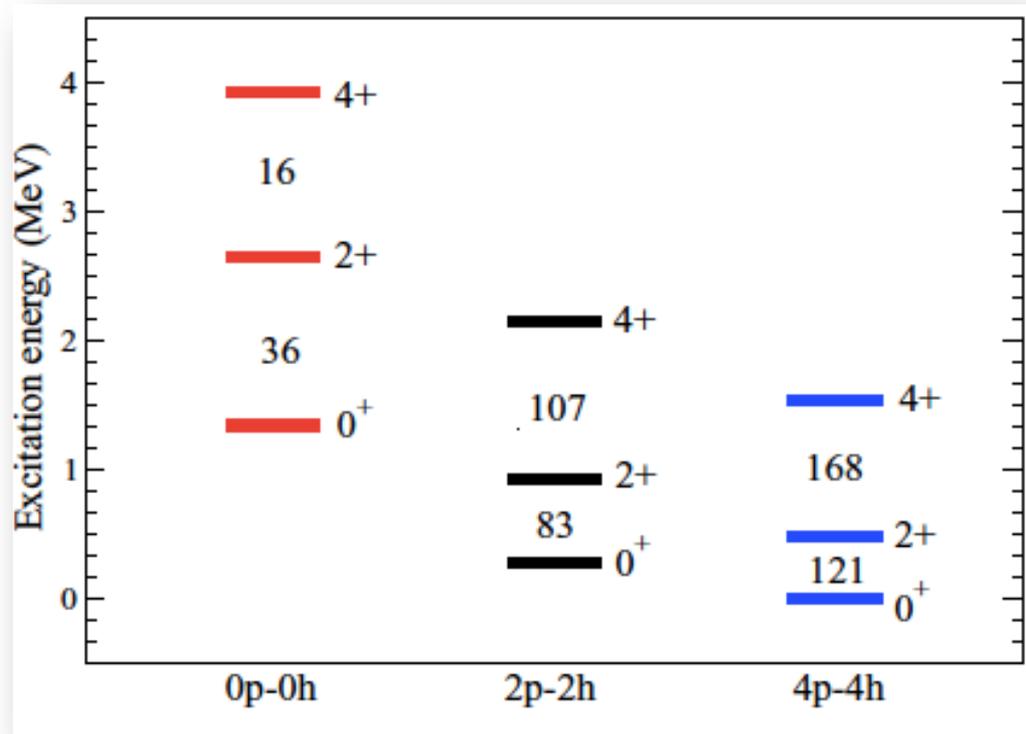
$$|0^+_{gs}\rangle = \alpha |sph\rangle + \beta |def\rangle$$

$$\underbrace{\hspace{15em}}_{\alpha^2 \approx 0.8 \quad \beta^2 \approx 0.2}$$



A paradigm shift: 2-Level to 3-Level Mixing ☺

Guided by shell model we consider the need for 3 configurations to describe the low-energy structure in ^{32}Mg .



Caurier, Nowacki, Poves PRC 90, 0914302 (2014). -- SDPF-U-MIX interaction

3-Level Mixing

Construct a mixing matrix of the form:

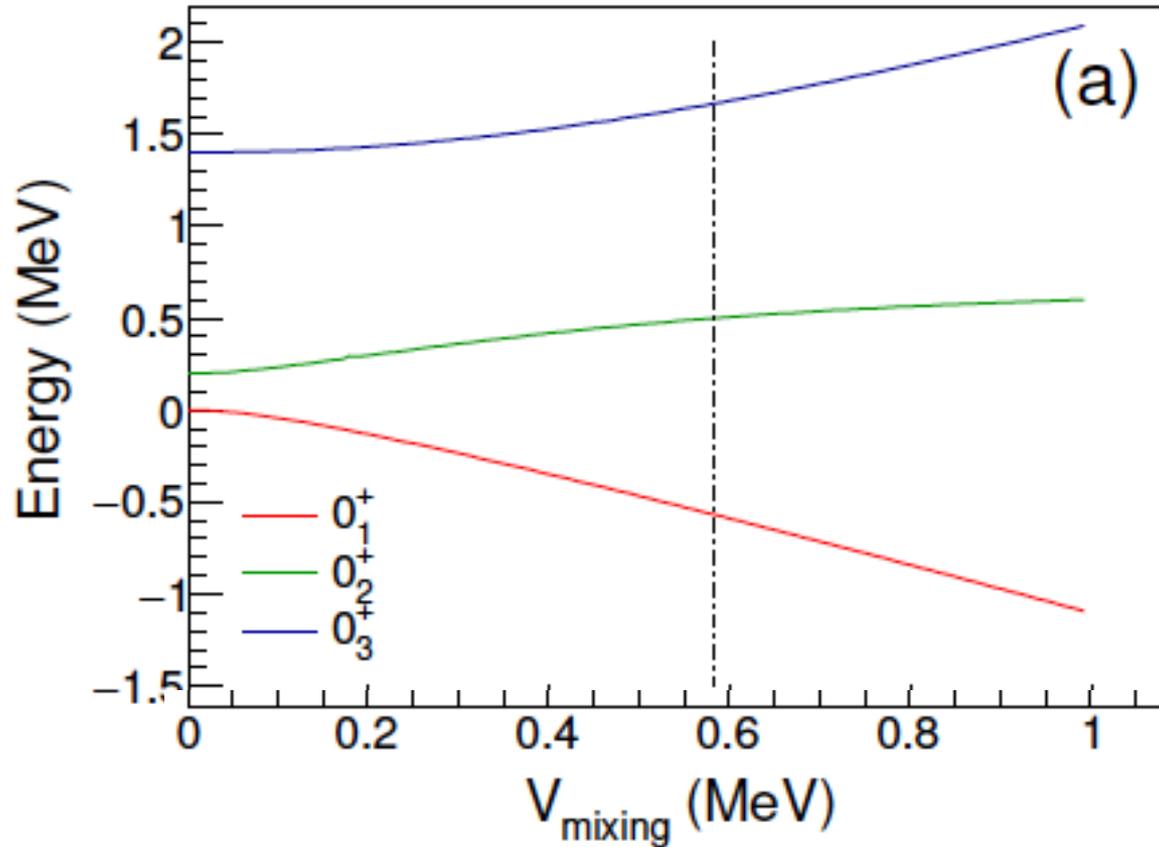
$$\begin{pmatrix} e_0 & -V & 0 \\ -V & e_2 & -V \\ 0 & -V & e_4 \end{pmatrix}$$

Take e_0 (0p0h) = 1.4 MeV, e_2 (2p2h) = 0.2 MeV, and e_4 (4p4h) = 0.0 MeV.

Diagonalize to extract energies and 0^+ wavefunctions of the form:

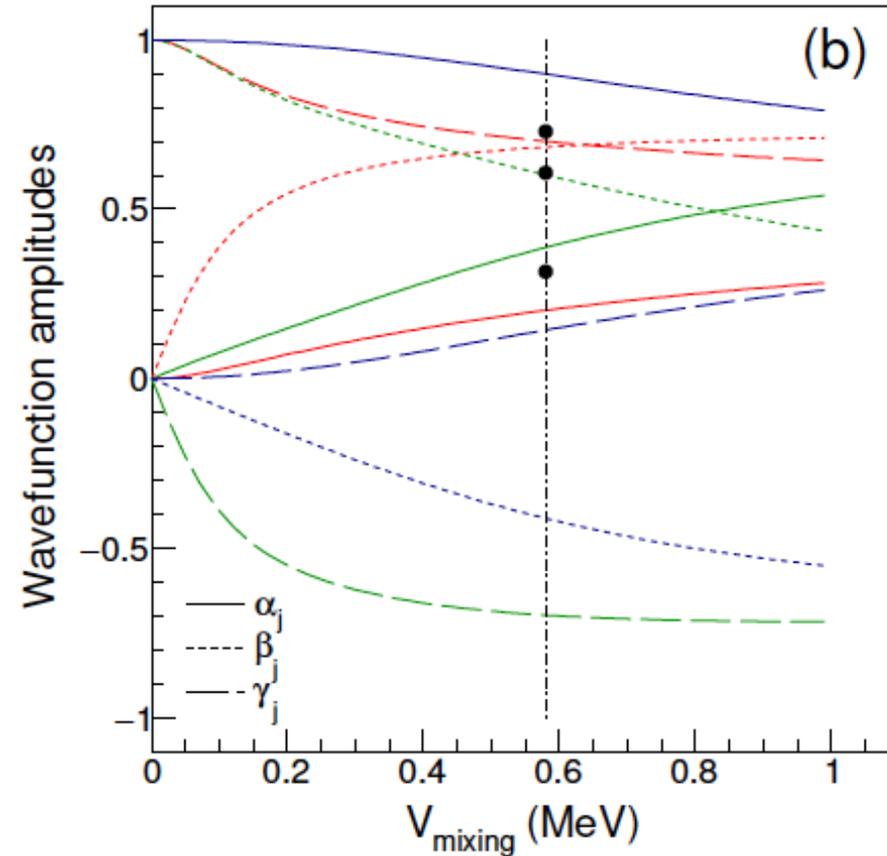
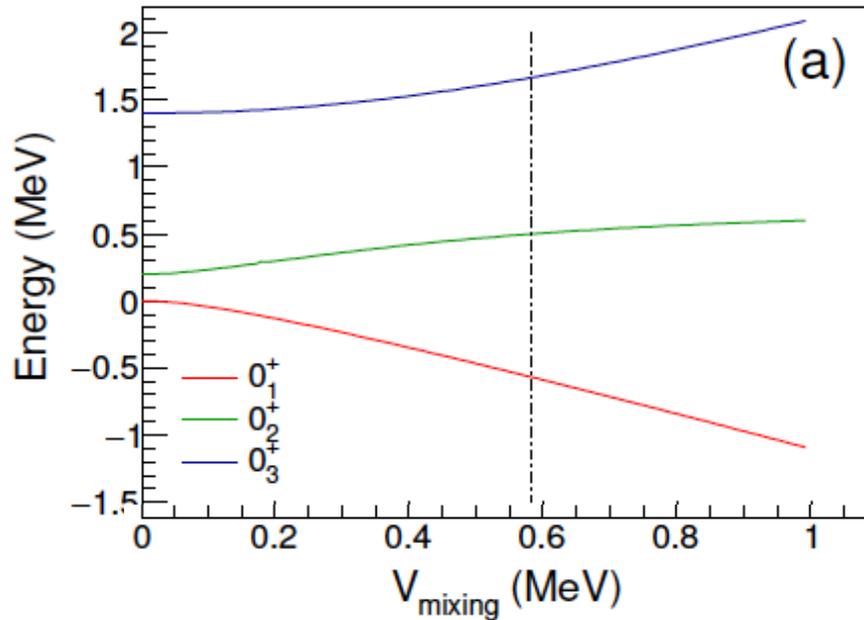
$$|0_j^+\rangle = \alpha_j |0p0h\rangle + \beta_j |2p2h\rangle + \gamma_j |4p4h\rangle$$

Diagonalization Solution



- Constrain V_{mixing} by matching to experimental $E(0_2^+) = 1058(2)$ keV

Diagonalization Solution



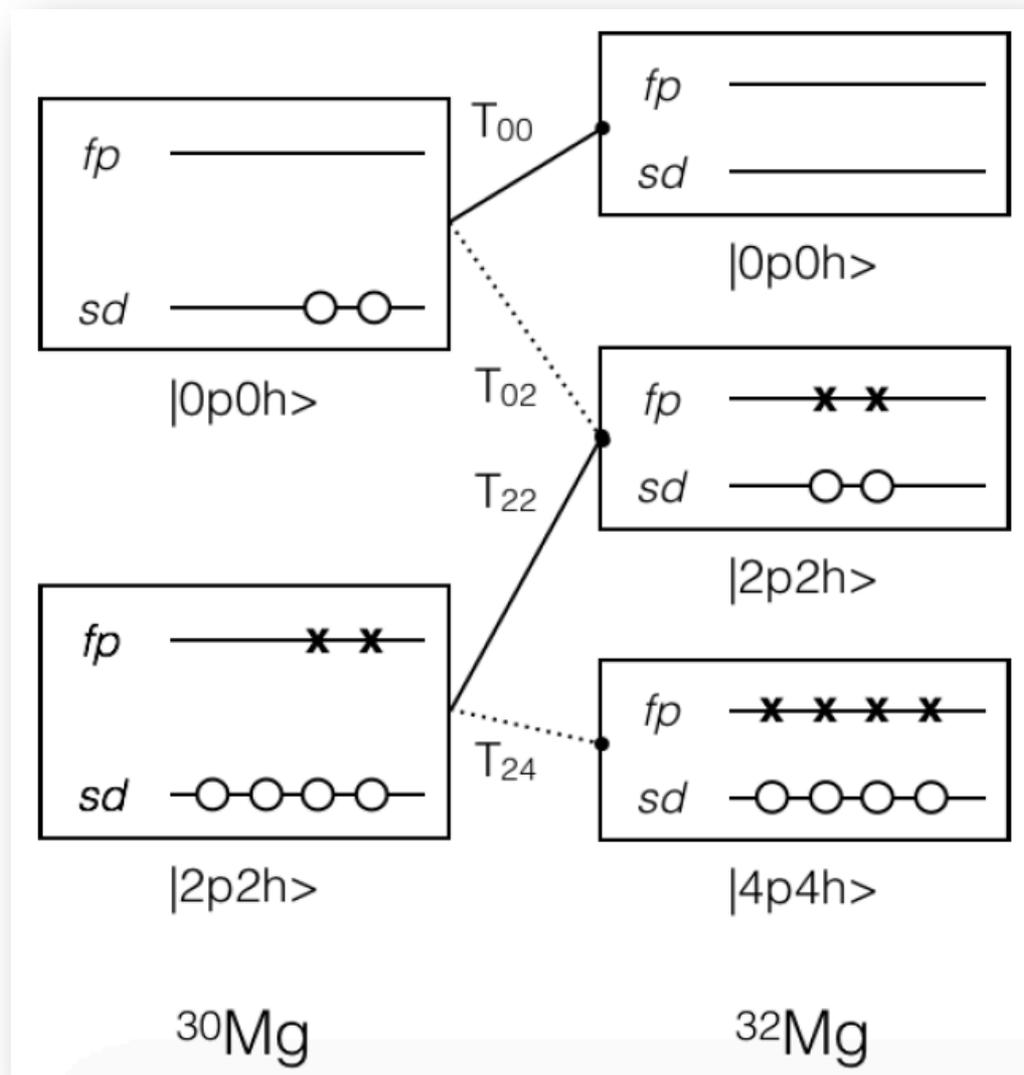
State	Excitation Energy [MeV]	α $ 0p0h\rangle$	β $ 2p2h\rangle$	γ $ 4p4h\rangle$
0_1^+	0.0	0.20	0.68	0.70
0_2^+	1.06	0.39	0.60	-0.70
0_3^+	2.22	0.90	-0.41	0.14

● SDPF-U-MIX interaction

The $^{30}\text{Mg}(t,p)^{32}\text{Mg}$ “puzzle”



Cross-Section Analysis



Cross-Section Analysis

$$|0_1^+(^{30}\text{Mg})\rangle = \epsilon|0p0h\rangle + \sqrt{1 - \epsilon^2}|2p2h\rangle$$

$$^{32}\text{Mg}: |0_j^+\rangle = \alpha_j|0p0h\rangle + \beta_j|2p2h\rangle + \gamma_j|4p4h\rangle$$



$$\sigma_{0_i^+} \propto (\epsilon\alpha_i T_{0,0} + \epsilon\beta_i T_{0,2} + \sqrt{1 - \epsilon^2}\beta_i T_{2,2} + \sqrt{1 - \epsilon^2}\gamma_i T_{2,4})^2$$



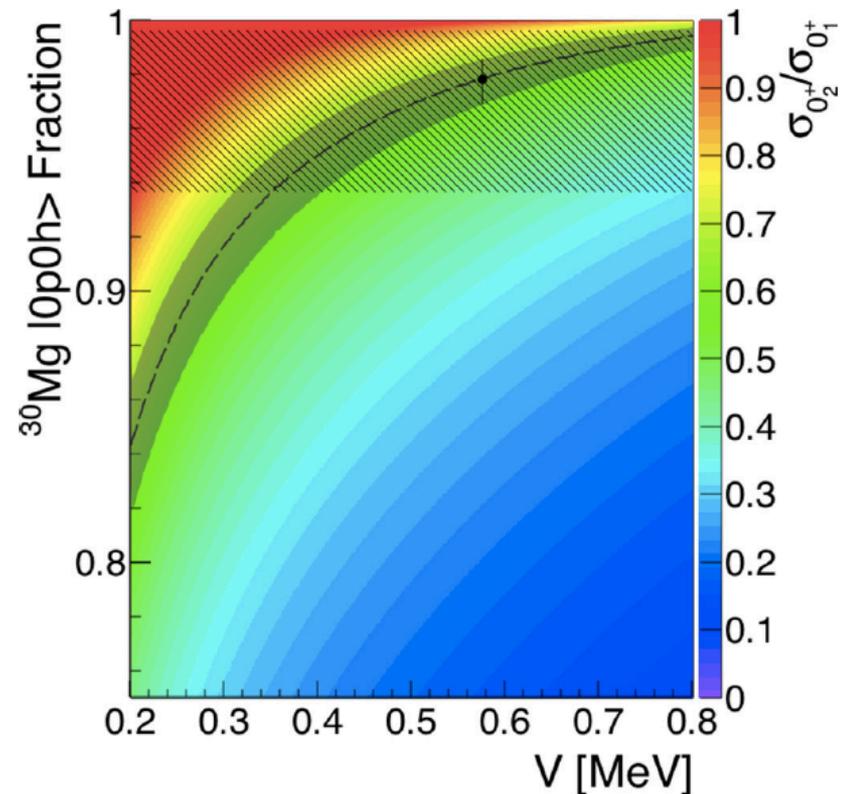
$$\boxed{T_{0,2} \ \& \ T_{2,4}} \text{---} \textcircled{T_{fp}} = R \times \textcircled{T_{sd}} \text{---} \boxed{T_{0,0} \ \& \ T_{2,2}}$$

Cross-Section Ratio and the ^{30}Mg Wavefunction

$$\frac{\sigma_{0_i^+}}{\sigma_{0_j^+}} = \left(\frac{\epsilon (\alpha_i + \beta_i R) + \sqrt{1 - \epsilon^2} (\beta_i + \gamma_i R)}{\epsilon (\alpha_j + \beta_j R) + \sqrt{1 - \epsilon^2} (\beta_j + \gamma_j R)} \right)^2$$

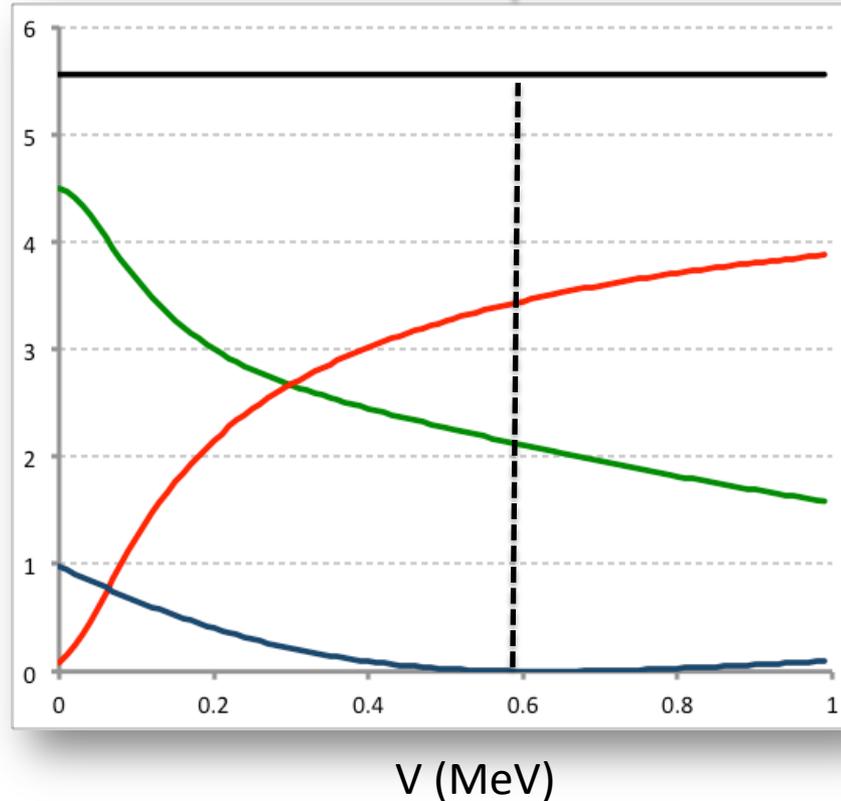
- Constrain the ^{30}Mg ground state wavefunction based on the cross-section ratio $\sigma(0_2^+)/\sigma(\text{g.s.}) = 0.62(6)$
- ^{32}Mg is dominated by 2p2h and 4p4h components (>95%) while ^{30}Mg is predominantly 0p0h, in agreement with experiment

(W. Schwerdtfeger *et al.*, Phys. Rev. Lett. 103, 012501 (2009).)



Sum Rule

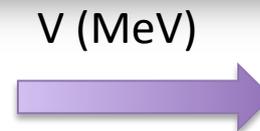
$$\sigma_{0_i^+} = \sigma_{sd} * (\epsilon\alpha_i + \epsilon\beta_i R + \sqrt{1 - \epsilon^2\beta_i} + \sqrt{1 - \epsilon^2\gamma_i} R)^2$$



Total

- 0₁⁺
- 0₂⁺
- 0₃⁺

$$(\sigma_{0_1^+} + \sigma_{0_2^+} + \sigma_{0_3^+}) = 17.0(9) \text{ mb}$$



$$\sigma_{sd} = 3.1 \text{ mb}$$

B(E2) Analysis

Construct a similar mixing matrix for the 2⁺ states:

$$\begin{pmatrix} e_0 & -V_2 & 0 \\ -V_2 & e_2 & -V_2 \\ 0 & -V_2 & e_4 \end{pmatrix}$$

Diagonalize adjusting V_2 to reproduce the 2⁺ energy of ³²Mg.

Solved 0⁺ states with $V = 0.576$

Energy levels: -0.561 MeV, 0.498 MeV, 1.663 MeV

$|0^+\rangle(1) = 0.201|0p0h\rangle + 0.683|2p2h\rangle + 0.702|4p4h\rangle$

Solved 2⁺ states with $V_2 = 0.306$

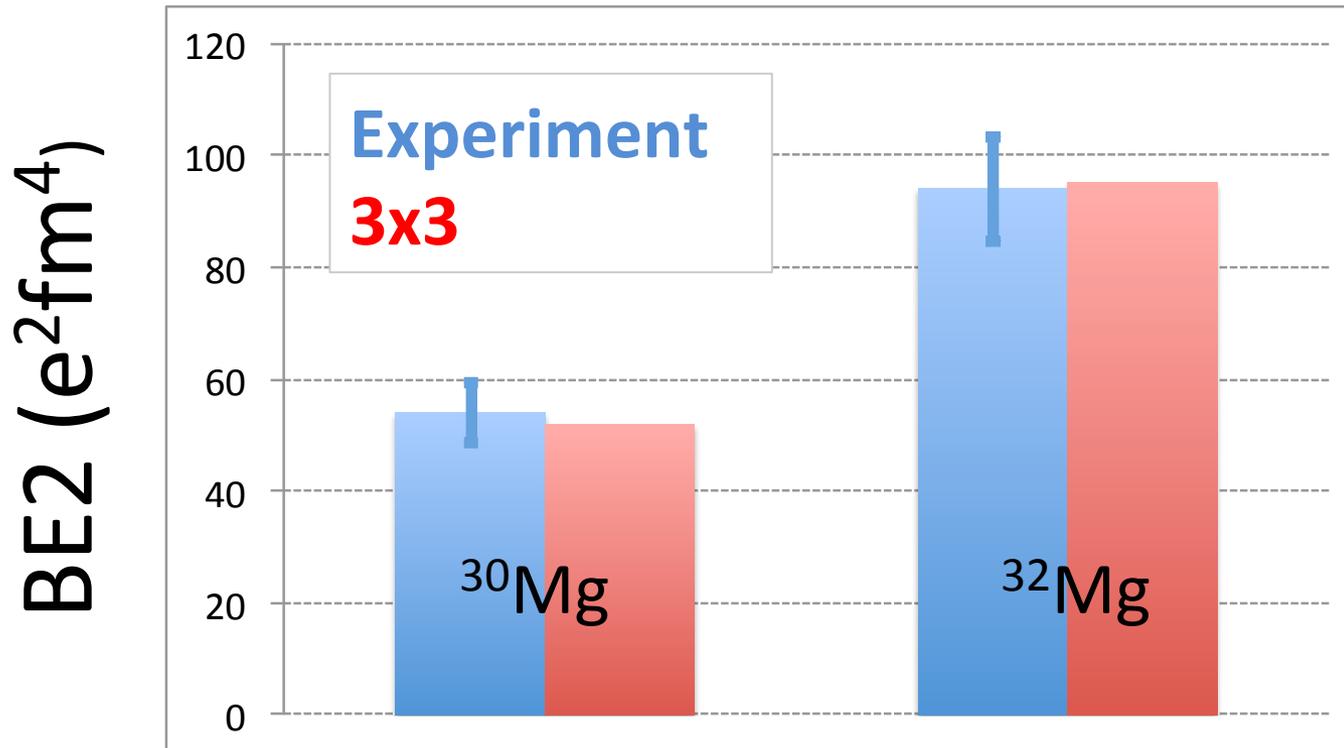
Energy levels: 0.325 MeV, 1.022 MeV, 2.713 MeV

$|2^+\rangle(1) = 0.065|0p0h\rangle + 0.495|2p2h\rangle + 0.866|4p4h\rangle$

B(E2) Analysis

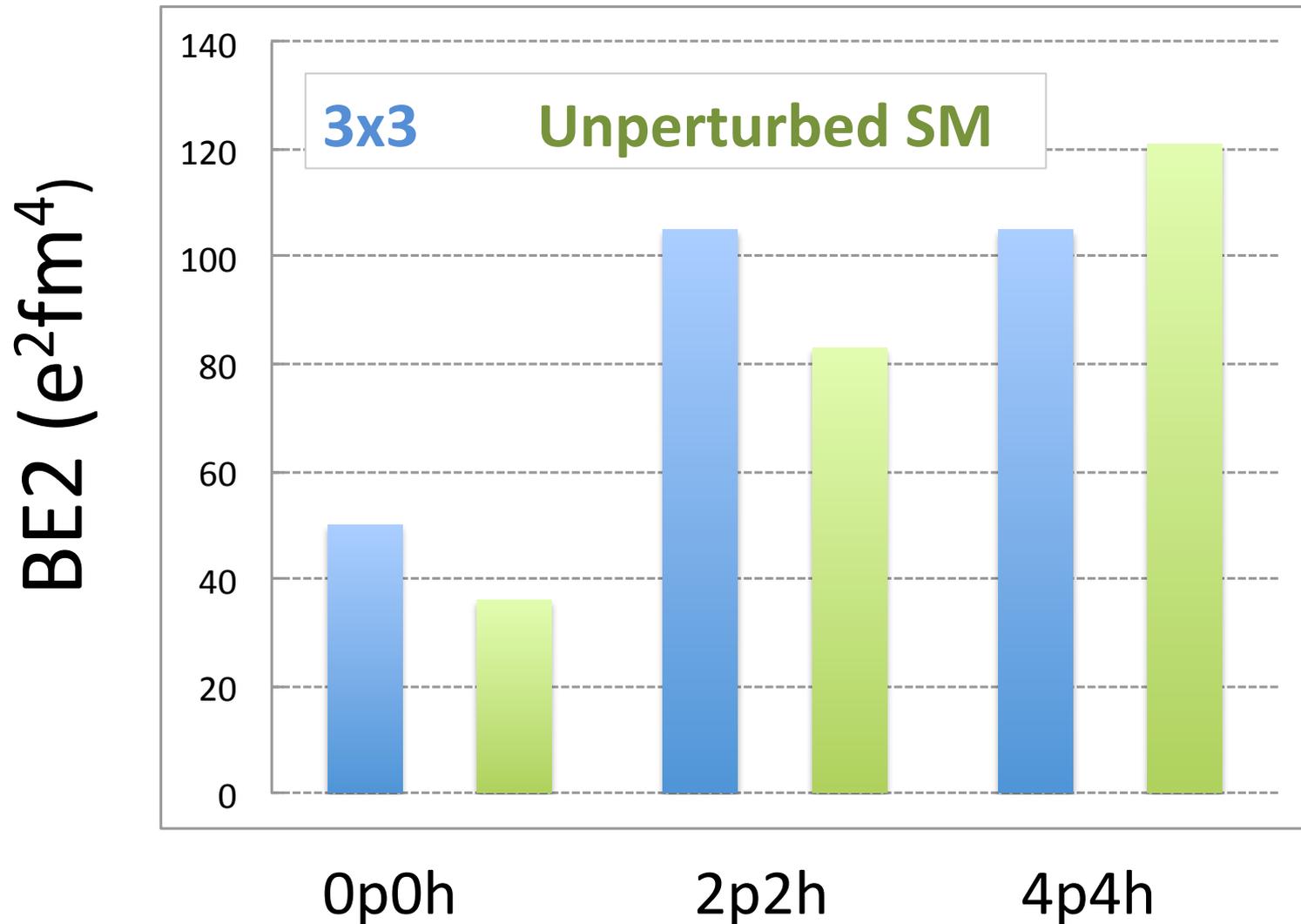
Following Rick's advice

Eye² – Fit 😊



$$B(E2)_{0p0h}, \quad B(E2)_{2p2h} = B(E2)_{4p4h}$$

B(E2) Analysis



$^{33}\text{Mg} - 1n \text{ KO}$

Physics Letters B 685 (2010) 253–257



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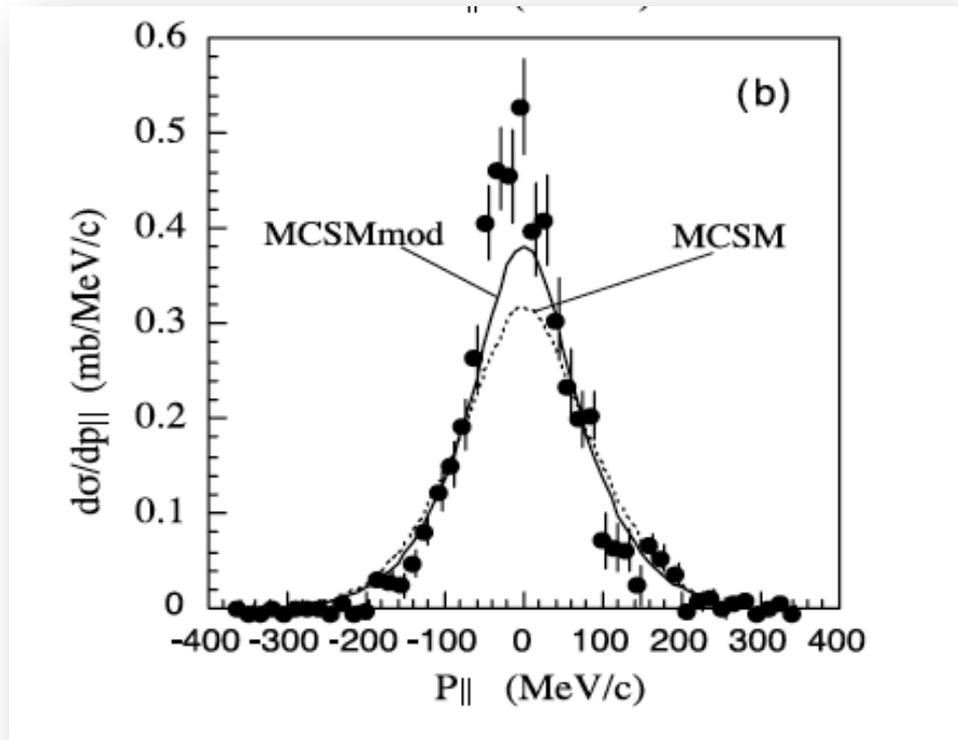
www.elsevier.com/locate/physletb



Structure of ^{33}Mg sheds new light on the $N = 20$ island of inversion

R. Kanungo^{a,*}, C. Nociforo^b, A. Prochazka^{b,c}, Y. Utsuno^d, T. Aumann^b, D. Boutin^c, D. Cortina-Gil^e, B. Davids^f, M. Diakaki^g, F. Farinon^{b,c}, H. Geissel^b, R. Gernhäuser^h, J. Gerl^b, R. Janikⁱ, B. Jonson^j, B. Kindler^b, R. Knöbel^{b,c}, R. Krücken^h, M. Lantz^j, H. Lenske^c, Y. Litvinov^{b,k}, K. Mahata^b, P. Maierbeck^h, A. Musumarra^{l,m}, T. Nilsson^j, T. Otsukaⁿ, C. Perro^a, C. Scheidenberger^b, B. Sitarⁱ, P. Strmenⁱ, B. Sun^b, I. Szarkaⁱ, I. Tanihata^o, H. Weick^b, M. Winkler^b

$^{33}\text{Mg} - 1n \text{ KO}$

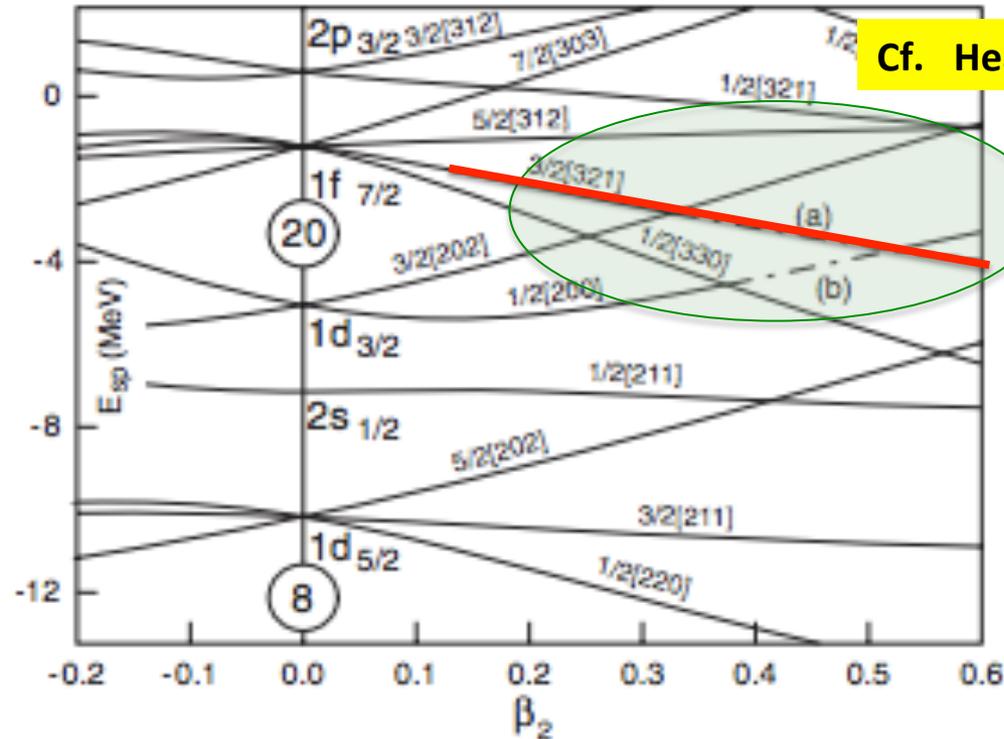


Longitudinal momentum distribution from the one-neutron removal reaction on a C target at 898 MeV/A. Experiment performed at the FRS, GSI.

An increased contribution from the $2p_{3/2}$ orbital is required to explain the observation showing its lowering compared to existing model predictions.

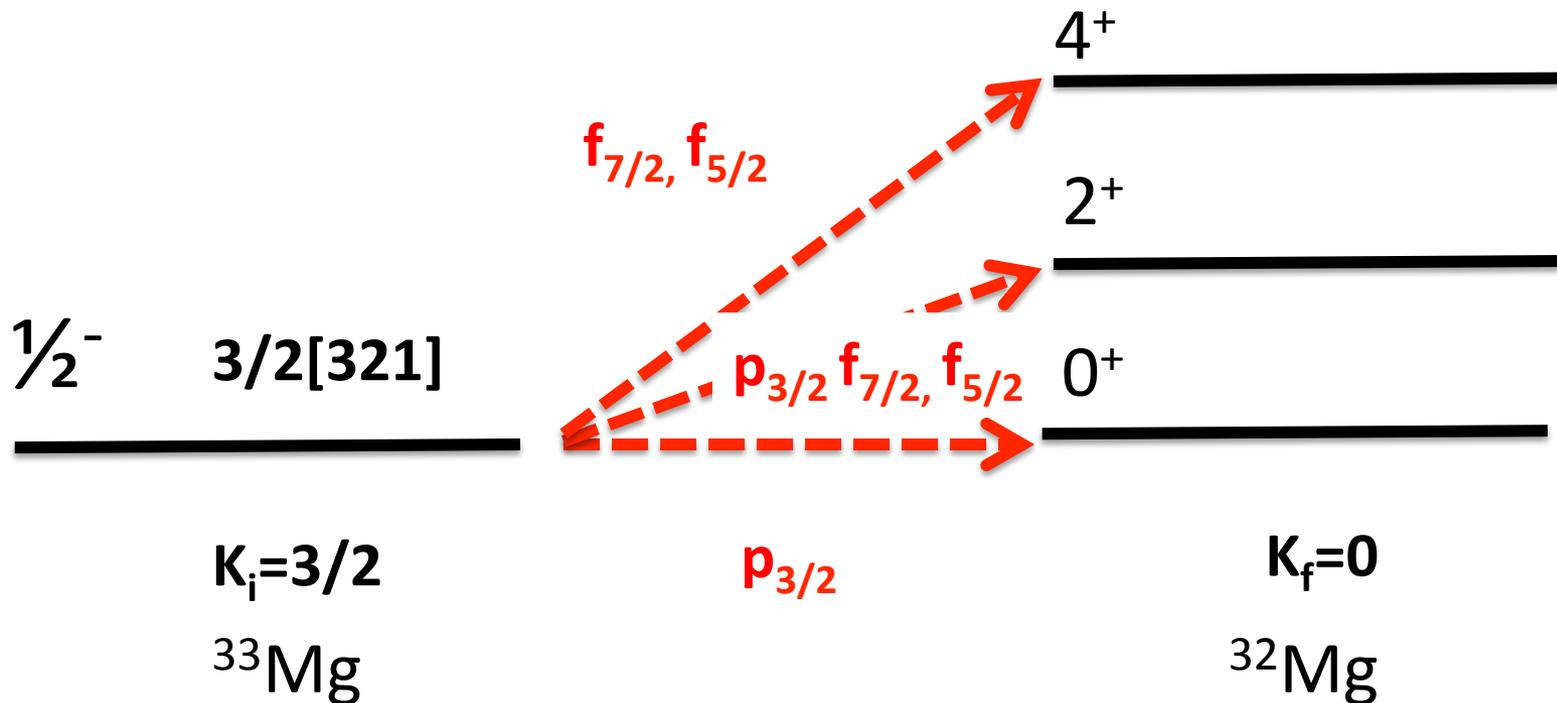
^{33}Mg -1n KO à la Nilsson

Assume ground state of ^{33}Mg is the $3/2[321]$ neutron Nilsson level



$$|K=3/2\rangle = C_{3/2,3/2} |p_{3/2}\rangle + C_{5/2,3/2} |f_{5/2}\rangle + C_{7/2,3/2} |f_{7/2}\rangle$$

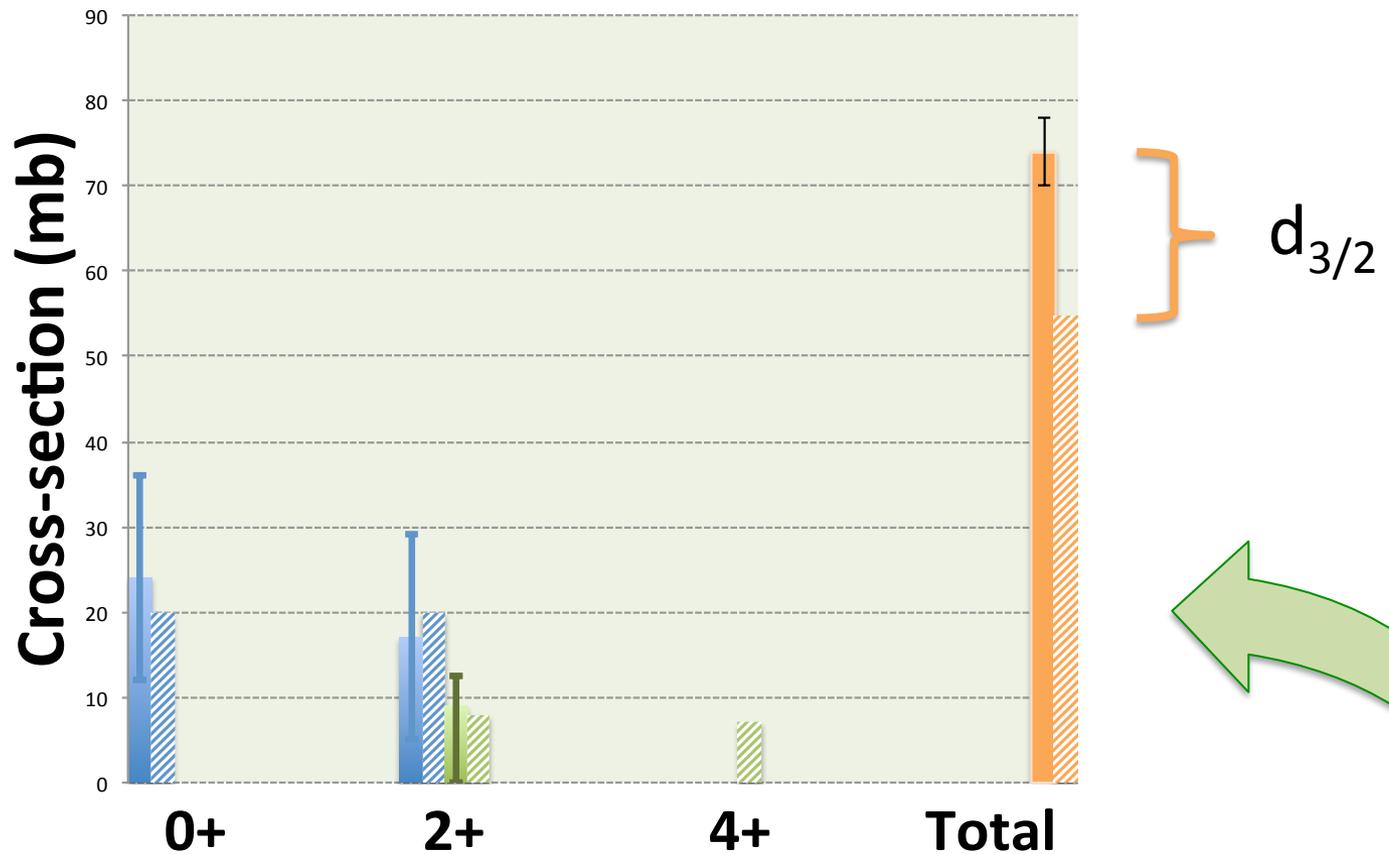
$^{33}\text{Mg} - 1n \text{ KO à la Nilsson}$



$$\left(\frac{d\sigma}{d\omega}\right)_s = N \sum_{j,l} g^2 \langle I_i j K_i \Delta K | I_f K_f \rangle^2 C_{jl}^2 \sigma_l(\theta)$$

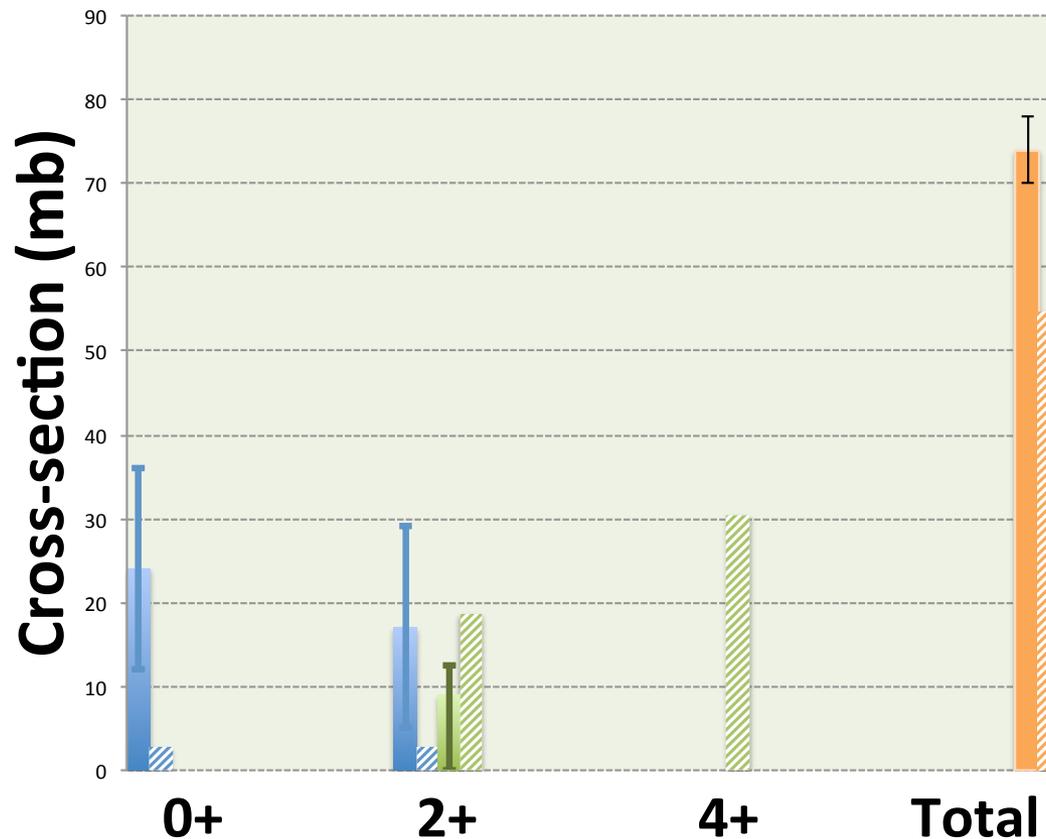
B. Elbek and P. Tjom, *Advances in Nuclear Physics* Vol.3 pp 259-323 (1969)

$^{33}\text{Mg} - 1n$ KO à la Nilsson



$$|K=3/2\rangle = 0.85 |p_{3/2}\rangle + 0.30 |f_{5/2}\rangle + 0.44 |f_{7/2}\rangle$$

$^{33}\text{Mg} - 1n$ KO à la Nilsson



$$|K=3/2\rangle = 0.32 |p_{3/2}\rangle + 0.22 |f_{5/2}\rangle + 0.92 |f_{7/2}\rangle$$

Chi's
Wavefunction

Summary

Inclusion of the third state, namely 4p4h configuration resolves the “puzzle” of ^{32}Mg discussed by Fortune, and the ground state emerges naturally as dominated at the 95% level by intruder (2p2h and 4p4h) configurations.

Within a simple three-level model, self-consistent solutions exist that provide good agreement with the experimental excitation energy of the 0^+_2 state, the cross-section ratio, summed cross-sections and $B(E2)$'s.

These scenarios also indicate a ^{30}Mg ground-state dominated by the 0p0h component, in line with experimental evidence and shell-model expectations .

Analysis of the ^{33}Mg -1n KO data in the Nilsson (strong coupling) limit provides a consistent description of the measured cross-sections.

Acknowledgements

PHYSICAL REVIEW C 00, 001300(R) (2016)

To appear soon !

$^{30}\text{Mg}(t, p)^{32}\text{Mg}$ “puzzle” reexamined

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This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under Contract No. DE-AC02-05CH11231 (LBNL), by the U.S. NSF Grant No. PHY-1404442, and by the Spanish Ministry of Ciencia e Innovacion under Grant FPA2014-57196 and Program Centros de Excelencia Severo Ochoa - SEV-2012-0249.

Acknowledgements

Merci Beaucoup!