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Shapes and Symmetries in Nuclei: from Experiment to Theory (SSNET) Workshop
November 7th to 11th , 2016, I2BC auditorium (Building 21)
CNRS campus，Gif-sur-Yvette, France

Collective Hamiltonian for chiral mode

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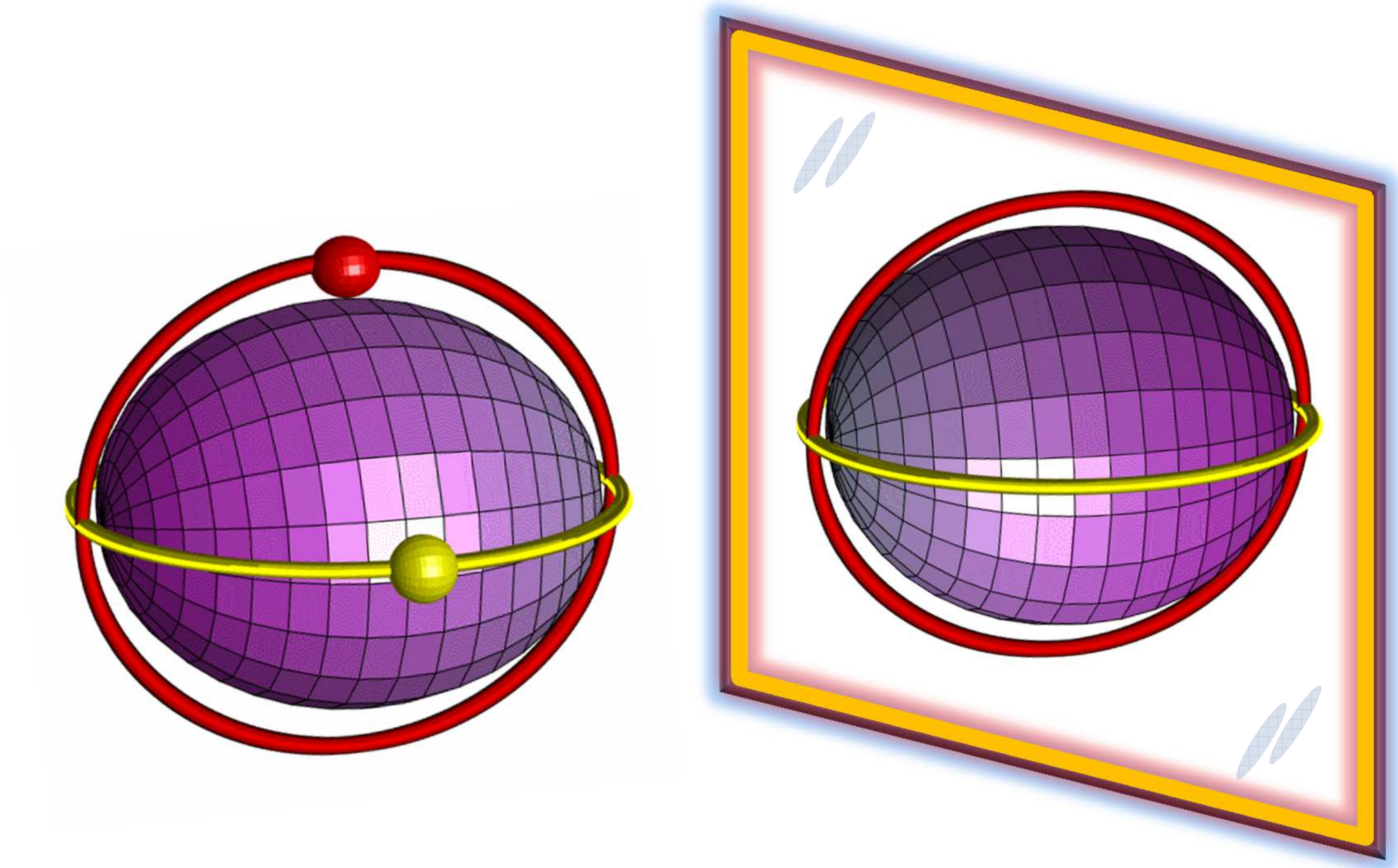
Outline

- Introduction
- Collective Hamiltonian
- One-dimensional
- Two-dimensional
- Summary and perspective

Chiral Rotation

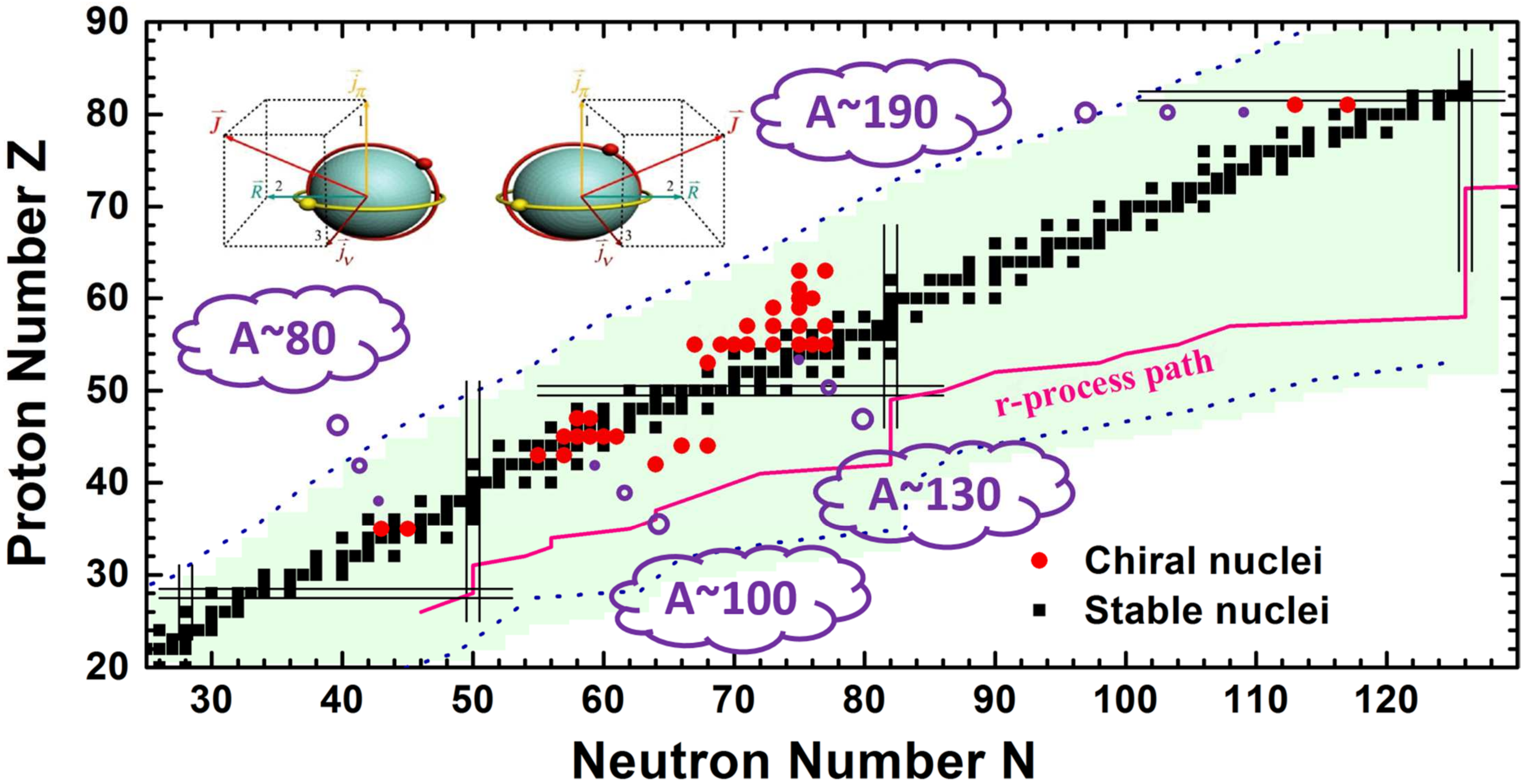


courtesy of X.H. Wu

Chiral symmetry breaking in intrinsic frame

Chiral Rotation: experiment

- **In the past two decades, the nuclear chirality in rotating triaxial nuclei have arouse much attention.** *Frauendorf&Meng, NPA 617, 131 (1997); Starosta2001PRL; Meng&Zhang, JPG 37, 064025 (2010); Meng&Chen&Zhang, IJMPE 23, 1430016 (2014); Meng&Zhao, PS 91, 053008 (2016)*



Chiral Rotation: theory

- Triaxial PRM
  - ✔ Lab frame; quantal model; with quantum tunneling;
  - ✖ Phenomenological

- **One-particle-one-hole PRM**

  Frauendorf_Meng 1997NPA; Koike2004PRL; Peng2003PRC; Qi2009PRC, CPL2010; Chen2010PRC; Zhang2016CPC

- **Two quasiparticles PRM**

  Starosta2002PRC; Koike2003PRC; Zhang2007PRC; Wang2007PRC, 2008PRC, 2010PRC; Qi2011CPL; Lawrie 2008PRC, 2010PLB;

- **n-particle-n-hole PRM**

  Qi2009PLB, 2011PRC; Ayangeakaa2013PRL; Lieder2014PRL; Kuti2014PRL; Liu2016PRL

- Other models
  - Projected shell model
  - IBFFM
  - Pairing truncated shell model

Bhat2012PLB, 2014PLB; Dar2015NPA
Brant2004PRC, 2008PRC; Tonev 2006PRL
Higashiyama2005PRC, 2007EPJA

Chiral Rotation: theory

- **Tilted axis cranking (TAC)**
  - ✔ **Intrinsic frame; microscopic; self-consistent; mean-field approximation**
  - ✖ **Semi-classical; no quantum tunneling;**

- ■ **Single-j model** *Frauendorf_Meng1997NPA;*
- ■ **Hybird Woods-Saxon and Nilsson model** *Dimitrov2000PRL*
- ■ **Skyrme Hartree-Fock model** *Olbratorwski2004PRL, 2006PRC*
- ■ **Covariant density function theory (CDFT)** *Madokoro2000PRC*

- **TAC + RPA**
  - ✔ **Beyond mean field;**
  - ✖ **Small amplitude harmonic vibration approximation;**

Mukhopadhyay2007PRL; Almehed2011PRC

Parity splitting in Collective H.

Helmholtz International Summer School
NUCLEAR THEORY AND ASTROPHYSICAL APPLICATIONS
Dubna, Russia. July 24 - August 2, 2011

- V. Jolos and P. von Brentano, Angular momentum dependence of the parity splitting in nuclei with octupole correlations, Phys. Rev. C49, R2301 (1994).
- R. V. Jolos and P. von Brentano, Rotational spectra andparity splitting in nuclei with strong octupole correla-tions, Nucl. Phys. A587, 377 (1995).
- R. V. Jolos and P. von Brentano, Parity splitting in thealternating parity bands of some actinide nuclei, Phys.Rev. C60, 064317 (1999).

An angular momentum dependence of the parity splitting in the alternating parity bands of nuclei with strong octupole correlations is considered based on **the model of the octupole motion in a one-dimensional potential well conserving axial symmetry**.

Chiral Rotation in Collective H.

- **Develop a collective Hamiltonian based on TAC model with particle-hole configuration to investigate the chiral rotation.**

  - **1-D Collective H. :** *Chen, Zhang, Zhao, Jolos, and Meng, PRC 87, 024314 (2013)*
  - **2-D Collective H. :** *Chen, Zhang, Zhao, Jolos, and Meng, PRC 94, 044301 (2016).*
  - **Based on TAC-CDFT including 1-D and 2-D:** *in progress*

This talk will focus on progresses of collective Hamiltonian for chiral rotation.

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Microscopic basis

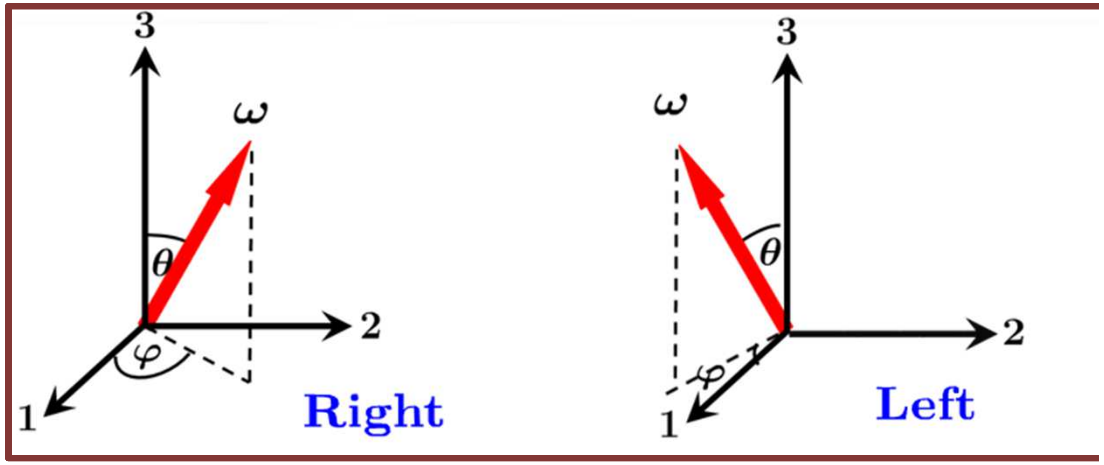
- **Microscopic derivation** Collective Hamiltionian can be obtained by
  - **Generate coordinate method** *Hill & Wheeler, PR 89, 1102 (1953); Ring & Schuck 1980*
  - **Adiabatic time-dependent Hartree-Fock (ATDHF) method** *Baranger & Kumar, NPA 122, 241 (1968); Ring & Schuck 1980*
  - **Adiabatic self-consistent collective coordinate method (ASCC)** *Marumori et al., PTP 64, 1294 (1980); Matsuo et al., PTP 103, 959 (2000); Hinohara et al., PRC 82, 064313 (2010); Matsuyanagi et al., JPG 37, 064018 (2010)*
    - **Starting point:** time-dependent Hartree-Fock (TDHF) equation
    - **Assumptions:** adiabatic approximation, i.e., the collective motion is slow or collective momenta are small (can be large)
    - **Procedure:** expand the TDHF equations with respect to the collective momenta up to second order

$$\mathcal{H}(q,p) = \langle \phi(q,p)|\hat{H}|\phi(q,p)\rangle = \frac{1}{2}\sum_{ij} B^{ij}(q)p_i p_j + V(q)$$

$$B^{ij}(q) = \left.\frac{\partial^2 \mathcal{H}}{\partial p_i \partial p_j}\right|_{p=0} \qquad V(q) = \mathcal{H}(q,p)|_{p=0}$$

1D Collective Hamiltonian

- **Collective coordinates:** the orientation angles of angular momentum (θ, φ)



- **For simplicity,** only one is considered here, and minimize automatically with the other

$$(\theta, \varphi) \longrightarrow \varphi$$

- **The classical form of a collective Hamiltonian in terms of φ as,**

$$\hat{H}_{\rm coll} = \hat{T}_{\rm kin}(\varphi) + V(\varphi) = \frac{1}{2}B\dot{\varphi}^2 + V(\varphi)$$

- **According to general Pauli quantization** *Pauli1933*

$$\hat{H}_{\rm coll} = \hat{T}_{\rm kin}(\varphi) + V(\varphi) = -\frac{\hbar^2}{2\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi}\frac{1}{\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi} + V(\varphi)$$

Collective potential $V(\varphi)$

- **For a system of a high-j proton and high-j neutron coupled to a triaxial rotor.**

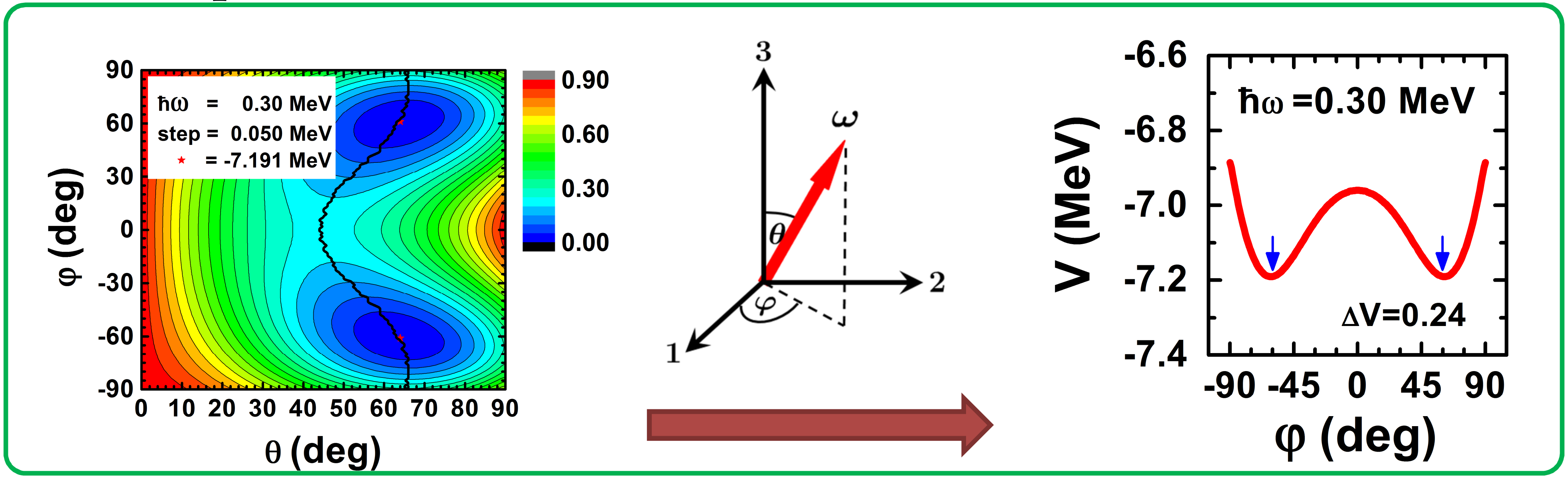
$$\hat{h}' = \hat{h}_{\rm def} - \boldsymbol{\omega}\cdot\hat{\boldsymbol{j}}, \quad \hat{h}_{\rm def} = \hat{h}^{\pi}_{\rm def} + \hat{h}^{\nu}_{\rm def}, \quad \boldsymbol{j} = \boldsymbol{j}_\pi + \boldsymbol{j}_\nu$$

$$\boldsymbol{\omega} = (\omega\sin\theta\cos\varphi, \omega\sin\theta\sin\varphi, \omega\cos\theta),$$

$$\hat{h}_{\rm def} = \frac{1}{2}C\left\{(\hat{j}_3^2 - \frac{j(j+1)}{3})\cos\gamma + \frac{1}{2\sqrt{3}}(\hat{j}_+^2 + \hat{j}_-^2)\sin\gamma\right\},$$

$$E'(\theta,\varphi) = \langle h'\rangle - \frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k\omega_k^2, \quad \mathcal{J}_k : \text{moments of inertia},$$

Minimizing the total Routhian with respect to θ for given φ, the collective potential $V(\varphi)$ is obtained.



Mass parameter

The mass parameter can be obtained by solving the adiabatic self-consistent collective coordinate equations, for a simple case where the Hamiltonian *H* does not contain two-body residual interaction.

2nd quantization

$$\hat{H}_M = \sum_\gamma \varepsilon_\gamma a_\gamma^\dagger a_\gamma$$

$$\hat{Q}_j(\bar{q}) = \sum_{\alpha\beta} Q^j_{\alpha\beta} a^\dagger_\alpha a_\beta,$$

$$\hat{P}_j(\bar{q}) = i\sum_{\alpha\beta} P^j_{\alpha\beta} a^\dagger_\alpha a_\beta.$$

$$\delta\langle\phi(\bar{q})| = \langle\phi(\bar{q})| a^\dagger_\beta a_\alpha$$

ASCC eqs

$$\delta\langle\phi(\bar{q})|\left[\hat{H}_M, \hat{Q}_j(\bar{q})\right] - \frac{1}{i}\sum_k B_{jk}(\bar{q})\hat{P}_k(\bar{q})|\phi(\bar{q})\rangle = 0$$

$$\langle\phi(\bar{q})|[\hat{Q}_j(\bar{q}), \hat{P}_k(\bar{q})]|\phi(\bar{q})\rangle = i\delta_{jk}$$

$$(\varepsilon_\alpha - \varepsilon_\beta)Q^j_{\alpha\beta} = \sum_k B_{jk}P^k_{\alpha\beta}$$

$$2\sum_{\alpha\beta} Q^j_{\beta\alpha}P^k_{\alpha\beta} = \delta_{jk}$$

$$B^{-1}_{mk} = -2\sum_{\alpha\beta}\frac{P^m_{\alpha\beta}P^k_{\beta\alpha}}{\varepsilon_\alpha - \varepsilon_\beta}$$

$$\hat{H}_M|\alpha\rangle = \varepsilon_\alpha|\alpha\rangle \qquad P^m_{\alpha\beta} = \langle\alpha|\frac{\partial}{\partial q_m}|\beta\rangle$$

$$\langle\alpha|[\hat{H}_M, \frac{\partial}{\partial q_m}]|\beta\rangle = (\varepsilon_\alpha - \varepsilon_\beta)\langle\alpha|\frac{\partial}{\partial q_m}|\beta\rangle$$

Cranking formula

$$B^{-1}_{mk} = 2\sum_{\alpha\beta}\frac{\langle\alpha|\frac{\partial\hat{H}_M}{\partial q_m}|\beta\rangle\langle\beta|\frac{\partial\hat{H}_M}{\partial q_k}|\alpha\rangle}{(\varepsilon_\alpha - \varepsilon_\beta)^3}$$

Mass parameter B(φ)

- **Mass parameter $B(\varphi)$ could be obtained from TAC.** *Chen2013PRC*

$$\boxed{B_{mk}^{-1} = 2\sum_{\alpha\beta} \frac{\langle\alpha|\frac{\partial \hat{H}_M}{\partial q_m}|\beta\rangle\langle\beta|\frac{\partial \hat{H}_M}{\partial q_k}|\alpha\rangle}{(\varepsilon_\alpha - \varepsilon_\beta)^3}}$$

$$q_{m,k} \rightarrow \varphi \qquad \hat{H}_M \rightarrow h' = h_0 - \boldsymbol{\omega}\cdot\boldsymbol{j}$$

$$B(\varphi) = 2\hbar^2 \sum_{l\neq 0} \frac{|\frac{\partial\vec{\omega}}{\partial\varphi}\langle l|\hat{\vec{j}}|0\rangle|^2}{(E_l - E_0)^3}$$

$|l\rangle$ **cranking single particle state**

E_l **cranking single particle energy**

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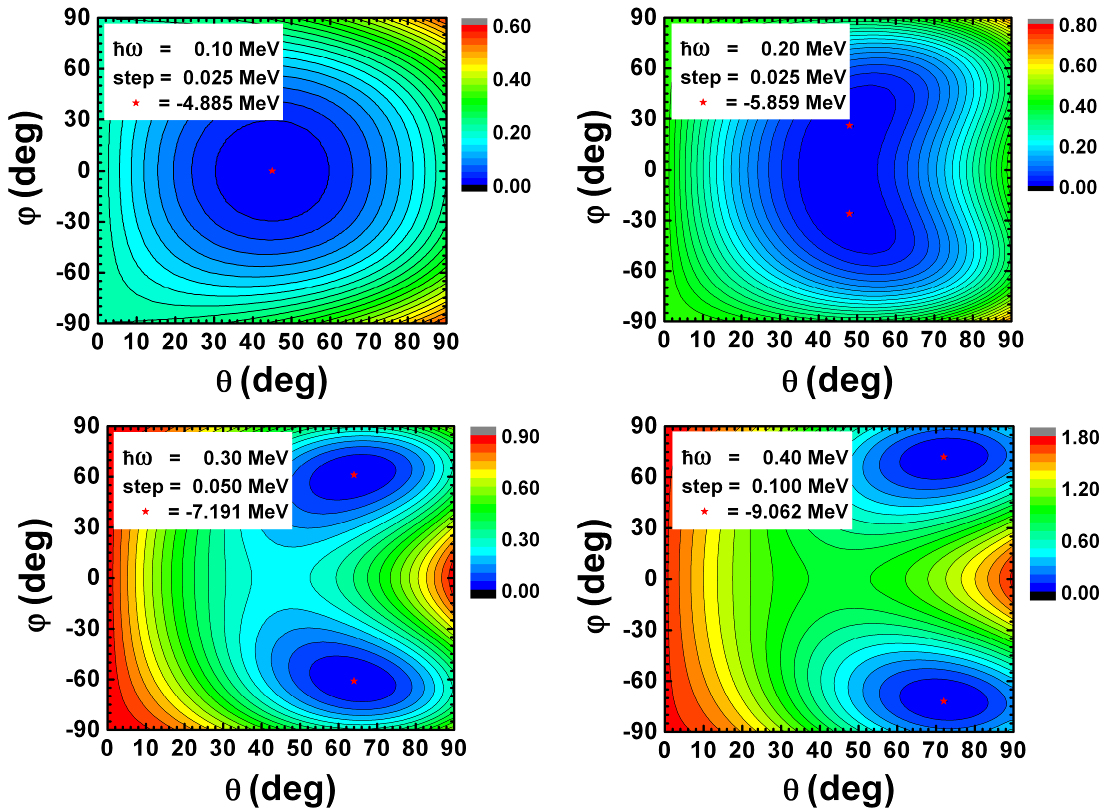
Numerical details

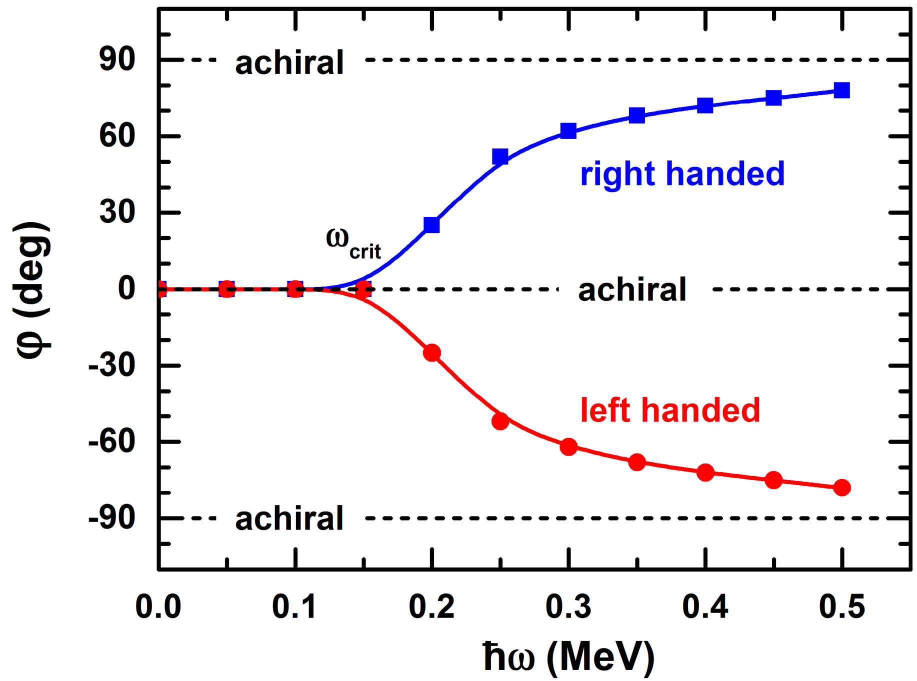
- **Configurations:** $\pi(1h_{11/2})^{1} \otimes \nu(1h_{11/2})^{-1}$
- **Single-*j* shell Hamiltonian coefficients:** $C_\pi = 0.25, \quad C_\nu = -0.25$
- **Triaxial deformation:** $\gamma = -30^\circ$
- **Moments of inertia:**

$$\mathcal{J}_k^{\mathrm{irr}} = \mathcal{J}_0^{\mathrm{irr}} \sin^2\left(\gamma - \frac{2\pi}{3}k\right), \quad \mathcal{J}_0^{\mathrm{irr}} = 40\ \hbar^2/\mathrm{MeV}$$

Frauendorf and Meng, Nucl. Phys. A 617, 131 (1997)

Total Routhian surfaces





- Total Routhian surfaces
  - Obtained by TAC for $\pi(1h_{11/2})^1 \otimes \nu(1h_{11/2})^{-1}$
  - Symmetrical about $\varphi = 0$; chiral solutions with $\pm|\varphi|$ are identical.
  - Minima: from $\varphi = 0$ to $\varphi \neq 0$; from one to two; critical at $\hbar\omega = 0.15$ MeV.
  - Rotating mode: from planar to aplanar to principal axis rotation.

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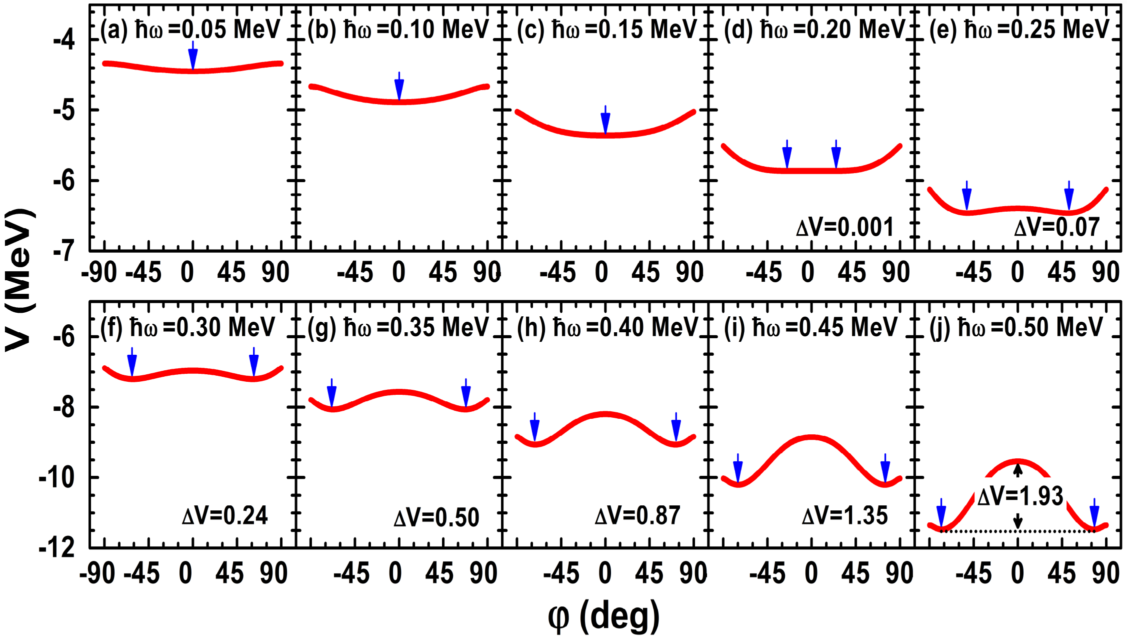
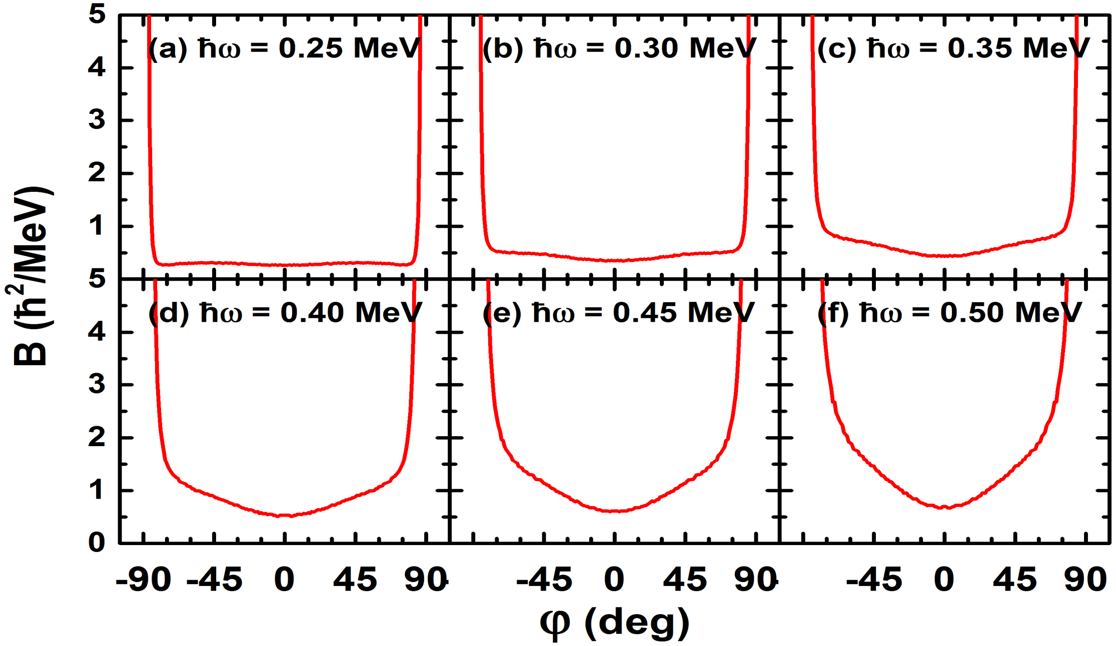
Collective potential and mass



- **Collective potential**
  - extracted from the Routhian

$$E'(\theta,\varphi) = \langle h - \boldsymbol{\omega}\cdot\boldsymbol{j}\rangle - \frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k\omega_k^2$$

  - symmetrical about $\boldsymbol{\varphi} = 0$
  - from planar to aplanar rotation
  - potential barrier increases

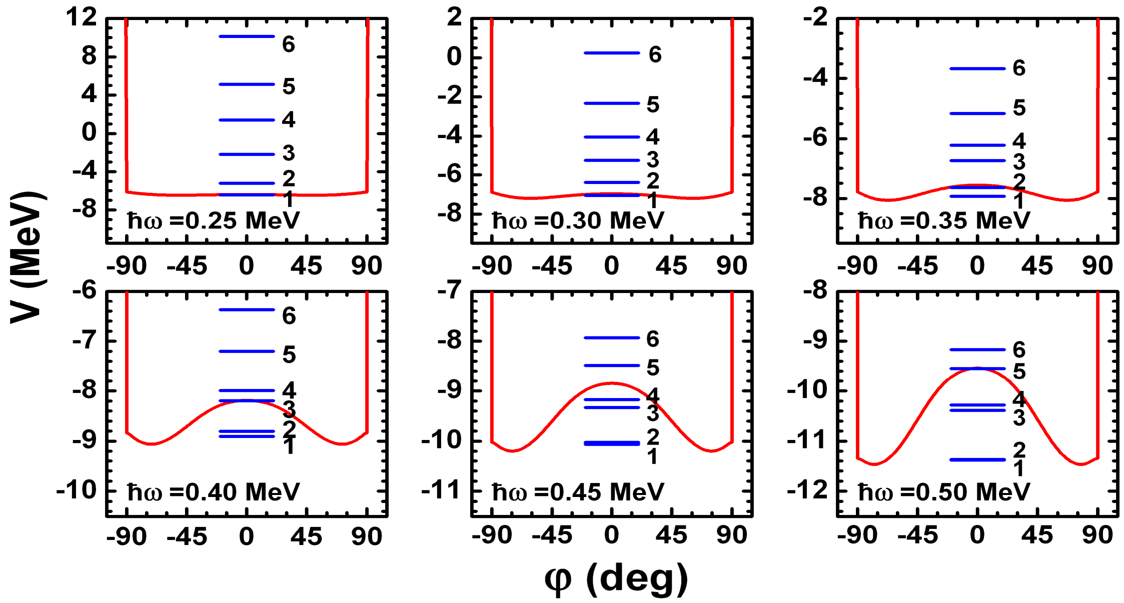


- **Mass parameter**
  - obtained from TAC by cranking formula

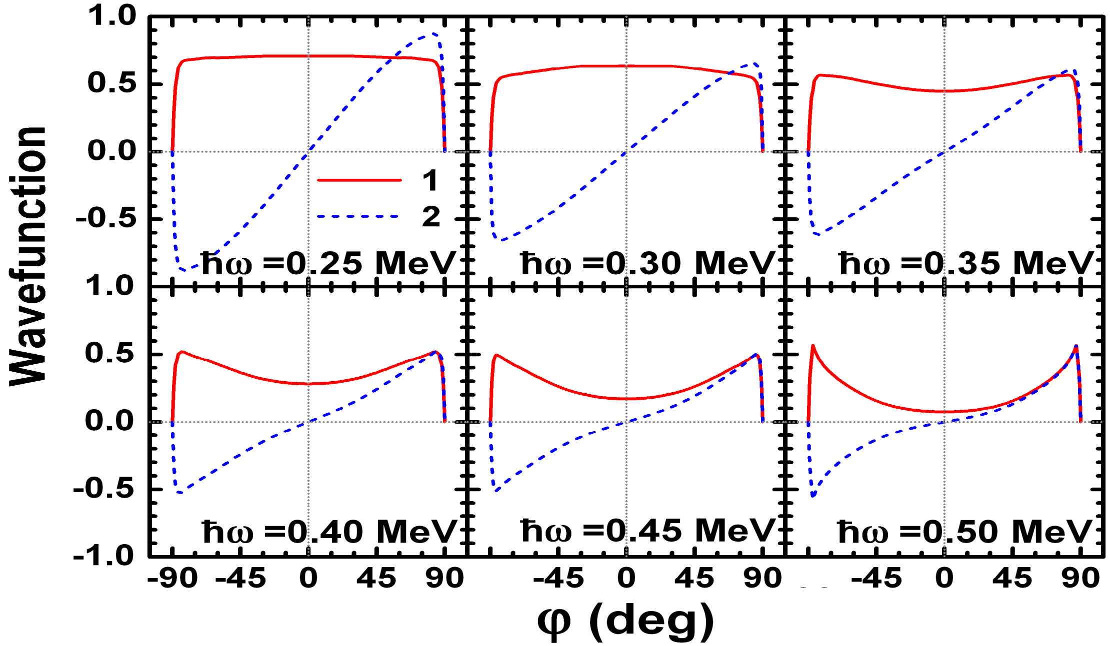
$$B(\varphi) = 2\hbar^2\sum_{l\neq 0}\frac{|\frac{\partial\vec{\omega}}{\partial\varphi}\langle l|\hat{\vec{j}}|0\rangle|^2}{(E_l - E_0)^3}$$

  - symmetrical about $\boldsymbol{\varphi} = 0$
  - sharp increase at boundary

Collective energy and wave function

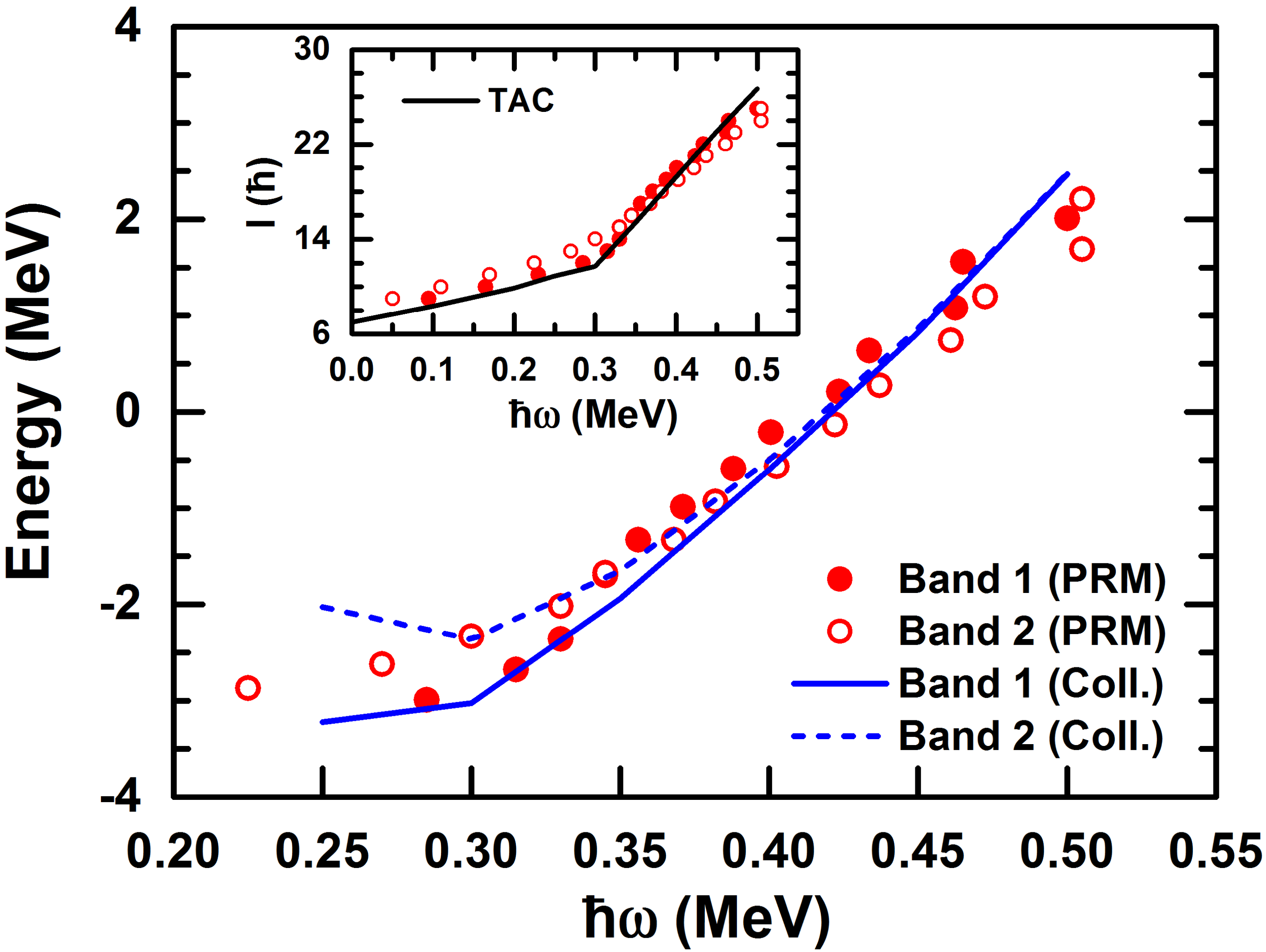


- **Energy levels**
  - energy levels become paired
  - tunneling is suppressed
  - MχD appears



- **Wave function**
  - symmetric for level 1 and antisymmetric for level 2
  - chiral symmetry broken in the aplanar TAC solutions is restored
  - from chiral vibration to chiral rotation

Compared with PRM



- Collective Hamiltonian gives the partner band and well reproduce the PRM results.
- The success of the collective Hamiltonian guarantees its application for TAC density functional calculations and prediction.

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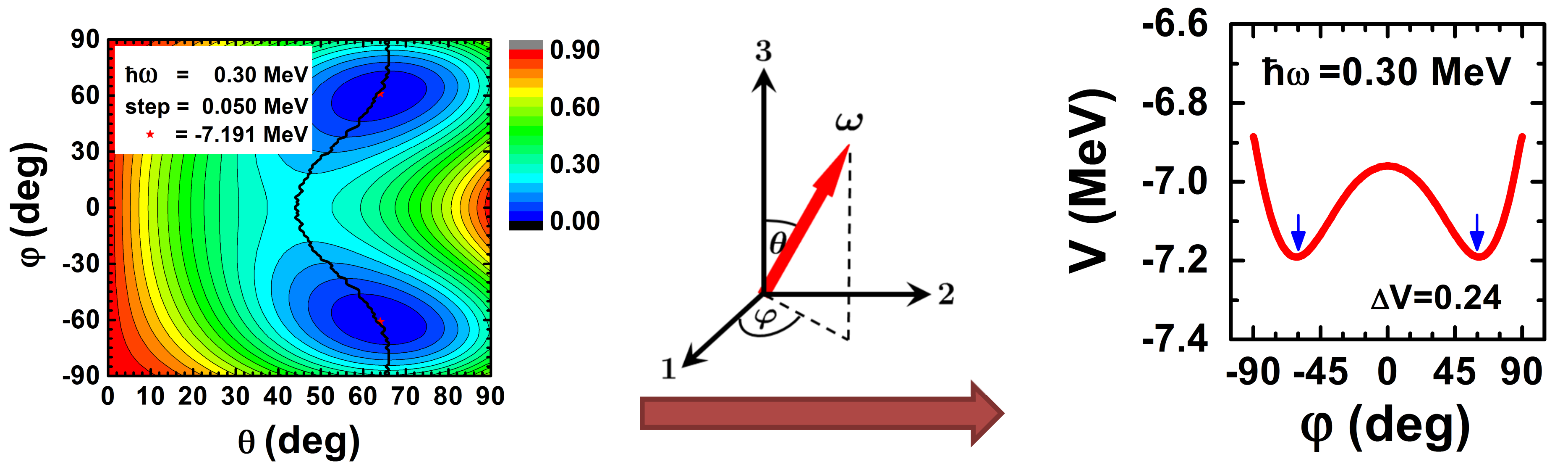
Motivation

- Previously, only ***φ*……** *PRC 87, 024314 (2013); PRC 90, 044306 (2014)*



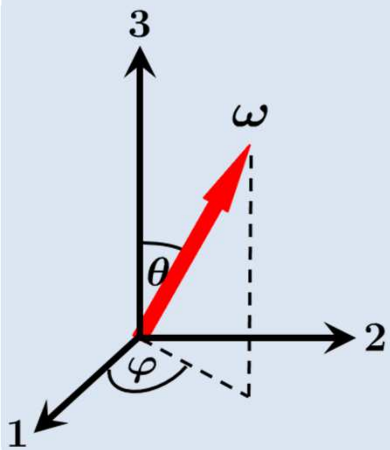
- **It is interesting to further consider the *θ* ……**

In the following, a two dimensional collective Hamiltonian (2DCH), which includes the full dynamical motions of nuclear orientations (*θ*, *φ*), will be constructed.

Chen, Zhang, Zhao, Jolos, and Meng, PRC 94, 044301 (2016).

2D collective Hamiltonian

- **Establishment of 2DCH**
  - **Collective coordinate:** **orientation angles** $(\boldsymbol{\theta}, \boldsymbol{\varphi})$
  - **Collective Hamiltonian** $\boldsymbol{H}$**:**



Classical form: $\mathcal{H}_{\rm coll} = \frac{1}{2}B_{\varphi\varphi}\dot{\varphi}^2 + \frac{1}{2}B_{\theta\theta}\dot{\theta}^2 + \frac{1}{2}B_{\theta\varphi}\dot{\theta}\dot{\varphi} + \frac{1}{2}B_{\varphi\theta}\dot{\varphi}\dot{\theta} + \mathcal{V}(\theta,\varphi)$

Quantal form:

$$\begin{aligned}\hat{H}_{\rm coll} &= \hat{T}_{\rm kin}(\theta,\varphi) + V(\theta,\varphi)\\ &= -\frac{\hbar^2}{2\sqrt{w}}\left[\frac{\partial}{\partial\varphi}\frac{B_{\theta\theta}}{\sqrt{w}}\frac{\partial}{\partial\varphi} - \frac{\partial}{\partial\varphi}\frac{B_{\varphi\theta}}{\sqrt{w}}\frac{\partial}{\partial\theta} - \frac{\partial}{\partial\theta}\frac{B_{\theta\varphi}}{\sqrt{w}}\frac{\partial}{\partial\varphi} + \frac{\partial}{\partial\theta}\frac{B_{\varphi\varphi}}{\sqrt{w}}\frac{\partial}{\partial\theta}\right] + V(\theta,\varphi)\end{aligned}$$

with $w = B_{\varphi\varphi}B_{\theta\theta} - B_{\varphi\theta}B_{\theta\varphi}$

Collective parameters

- **Collective parameters**

  - **Collective potential $V(\theta, \varphi)$: based on TAC**

$$V(\theta,\varphi) = \langle \hat{h}' \rangle - \frac{1}{2}\sum_{k=1}^{3} \mathcal{J}_k \omega_k^2$$

$$V(\theta,\varphi) = \langle \hat{h}' \rangle - \frac{1}{2}\sum_{k=1}^{3} \mathcal{J}_k \omega_k^2$$

$$\hat{h}_{\rm def}^{\pi(\nu)} = \frac{1}{2}C_{\pi(\nu)}\left\{(\hat{j}_3^2 - \frac{j(j+1)}{3})\cos\gamma + \frac{1}{2\sqrt{3}}(\hat{j}_+^2 + \hat{j}_-^2)\sin\gamma\right\}.$$

  - **Mass parameter $B(\theta, \varphi)$: cranking formula**

$$B_{\varphi\varphi} = 2\sum_{mi} \frac{\left|\langle m|\frac{\partial\boldsymbol{\omega}}{\partial\varphi}\cdot\hat{\boldsymbol{j}}|i\rangle\right|^2}{(\varepsilon_m - \varepsilon_i)^3}, \quad B_{\theta\theta} = 2\sum_{mi} \frac{\left|\langle m|\frac{\partial\boldsymbol{\omega}}{\partial\theta}\cdot\hat{\boldsymbol{j}}|i\rangle\right|^2}{(\varepsilon_m - \varepsilon_i)^3},$$

$$B_{\varphi\theta} = B_{\theta\varphi} = 2\sum_{mi} \frac{\langle m|\frac{\partial\boldsymbol{\omega}}{\partial\varphi}\cdot\hat{\boldsymbol{j}}|i\rangle\langle i|\frac{\partial\boldsymbol{\omega}}{\partial\theta}\cdot\hat{\boldsymbol{j}}|m\rangle}{(\varepsilon_m - \varepsilon_i)^3}.$$

***m*: particle**
***i*: hole**

Numerical details

- **Configurations:** $\pi(1h_{11/2})^1 \otimes \nu(1h_{11/2})^{-1}$
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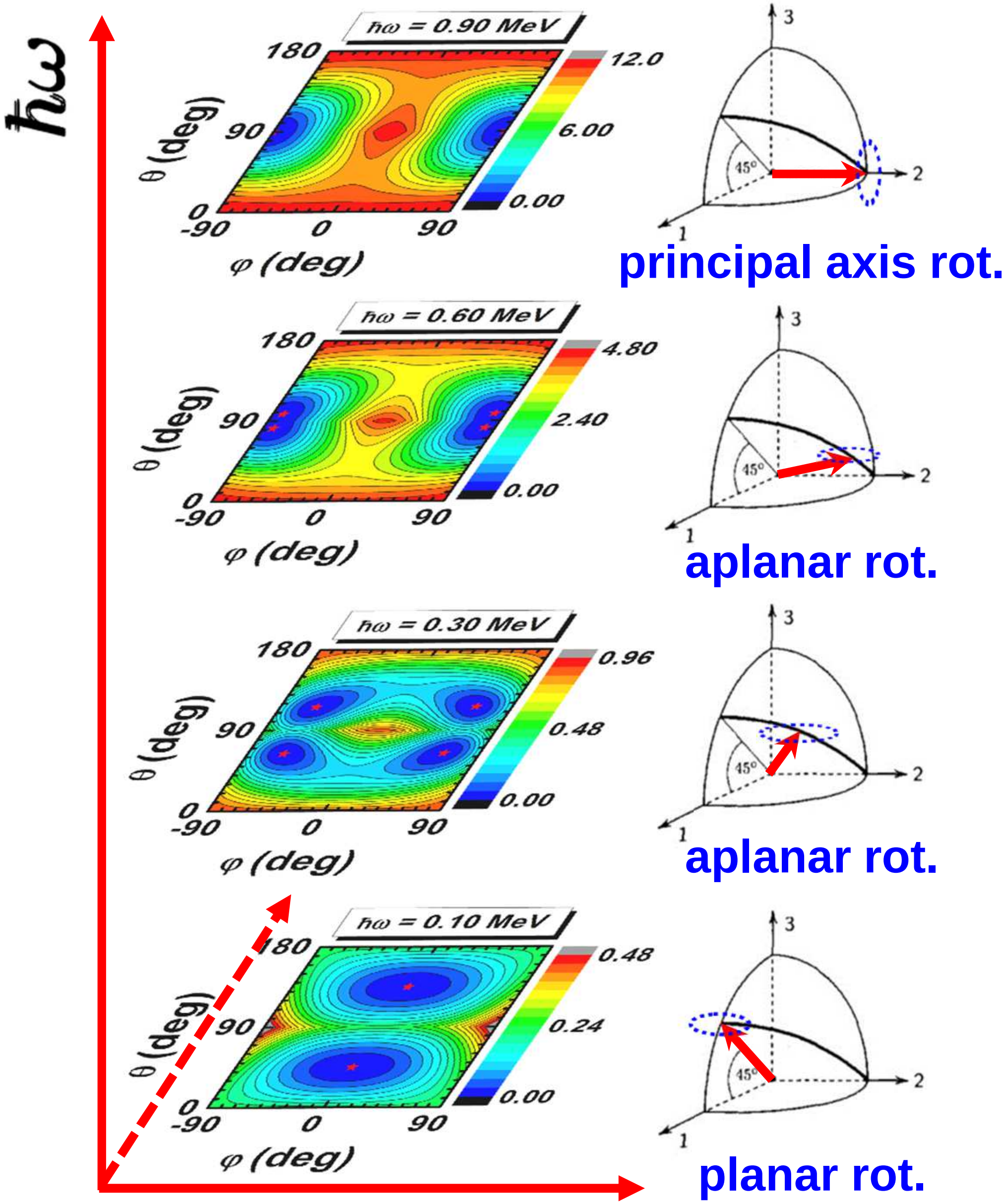
Frauendorf and Meng, Nucl. Phys. A 617, 131 (1997)

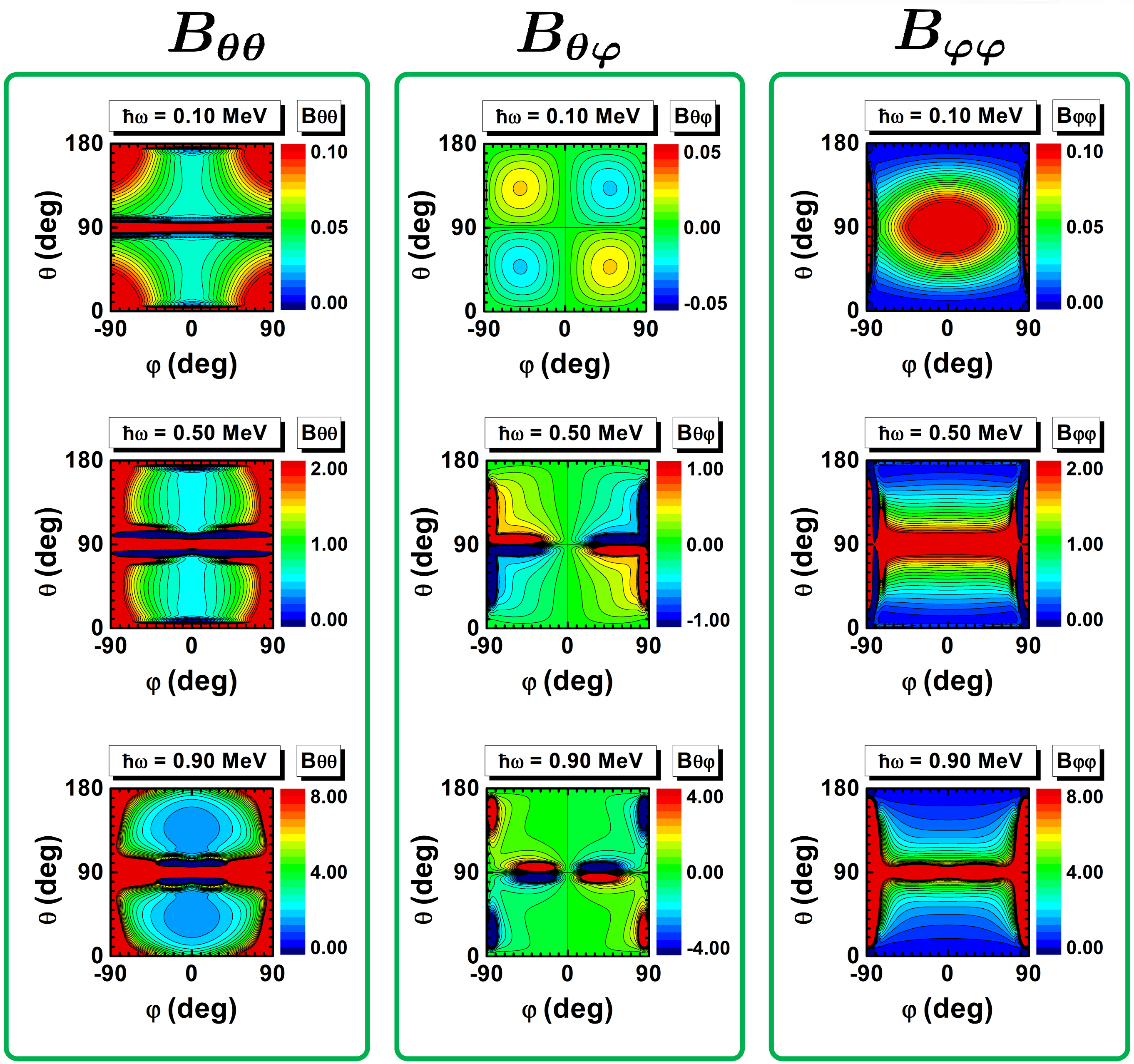
Collective potential



$$E'(\theta,\varphi) = \langle \hat{h}' \rangle - \frac{1}{2}\sum_{k=1}^{3} \mathcal{J}_k \omega_k^2$$

- **All the potential energy surfaces are symmetric with respect to the $\varphi = 0$ and $\theta = 90$ lines.**
- **With the increase of frequency, the minima in the potential energy surfaces change from $\varphi = 0$ to $\varphi \neq 0$: planar to aplanar rotation. At $\omega \sim 0.9$ MeV, the minima locates at ($\theta = 90, \varphi = 90$): principal axis rotation.**

Mass parameters



- With $\varphi \to -\varphi$, $\theta \to \pi - \theta$, the $B_{\theta\theta}$ and $B_{\varphi\varphi}$ are symmetric, while $B_{\theta\varphi}$ is antisymmetric.
- All the mass parameters generally increase with increasing frequency.
- $B_{\theta\theta}$ and $B_{\varphi\varphi}$ behave differently in the (θ, φ) plane.
- $B_{\theta\theta}$ and $B_{\varphi\varphi}$ are considerably large at θ~90, φ~±90

Collective energy

Diagonalization of the 2D CH yields the energy and wave functions at each cranking frequency.

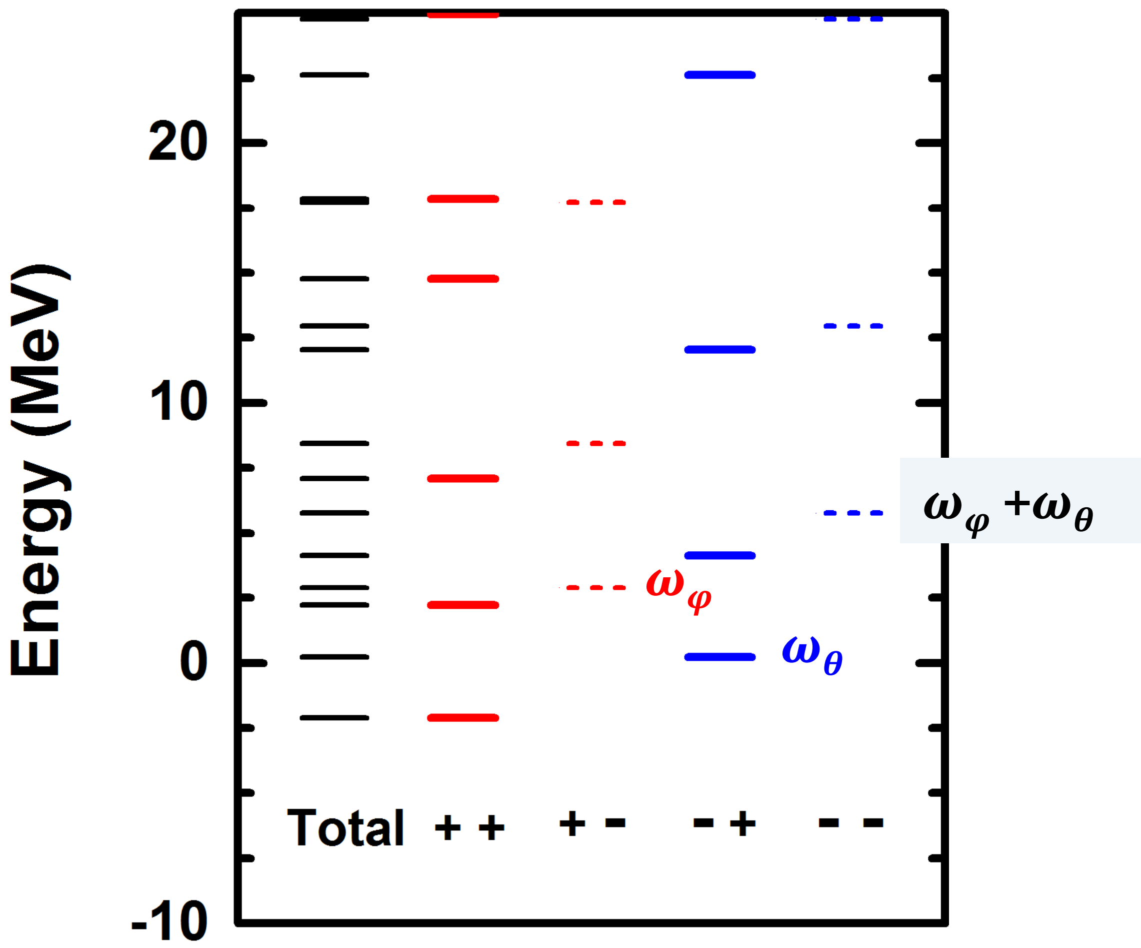
ħω=0.30 MeV

- **The energy levels are grouped:**

$$(P_\theta, P_\varphi) = (++), (+-), (-+), (--)$$

- **The energy levels in different groups are associated with different phonon excitation modes.**

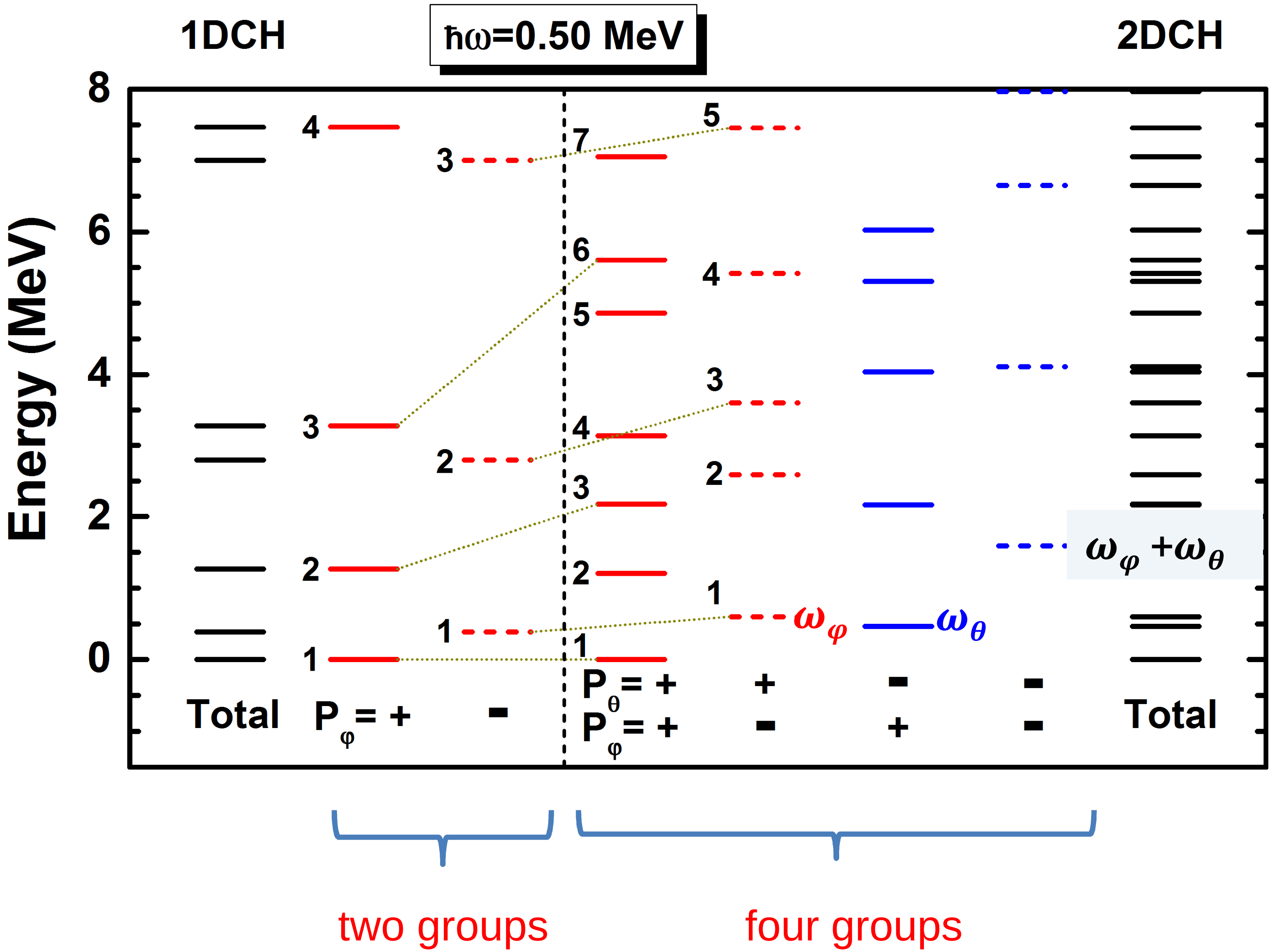
- **Generally ω_φ differs from ω_θ**



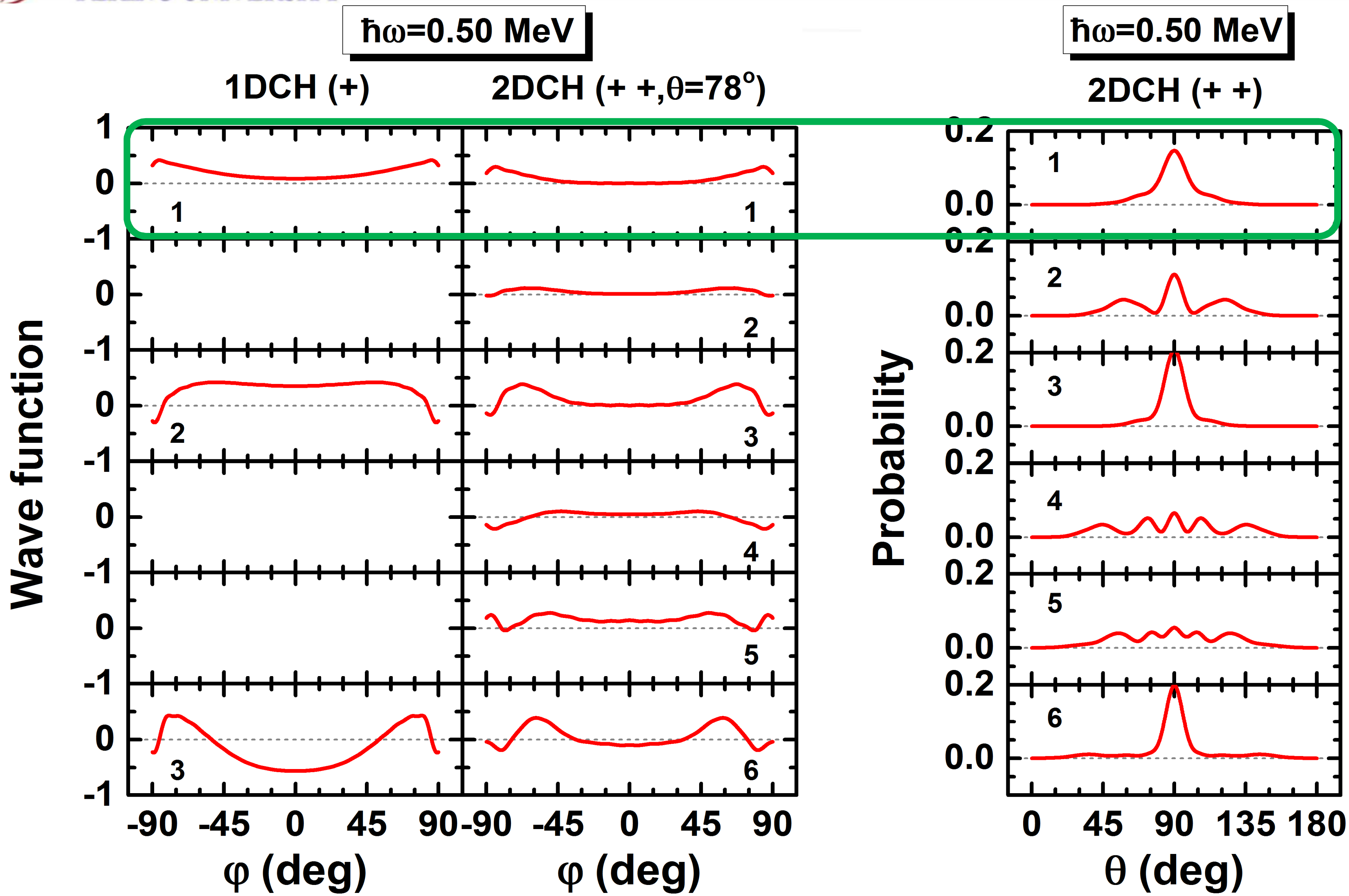
$$(P_\theta, P_\varphi) = (++), (+-), (-+), (--)$$

Comparison with 1D energy

■ Only P_θ=+ has 1D counterpart.



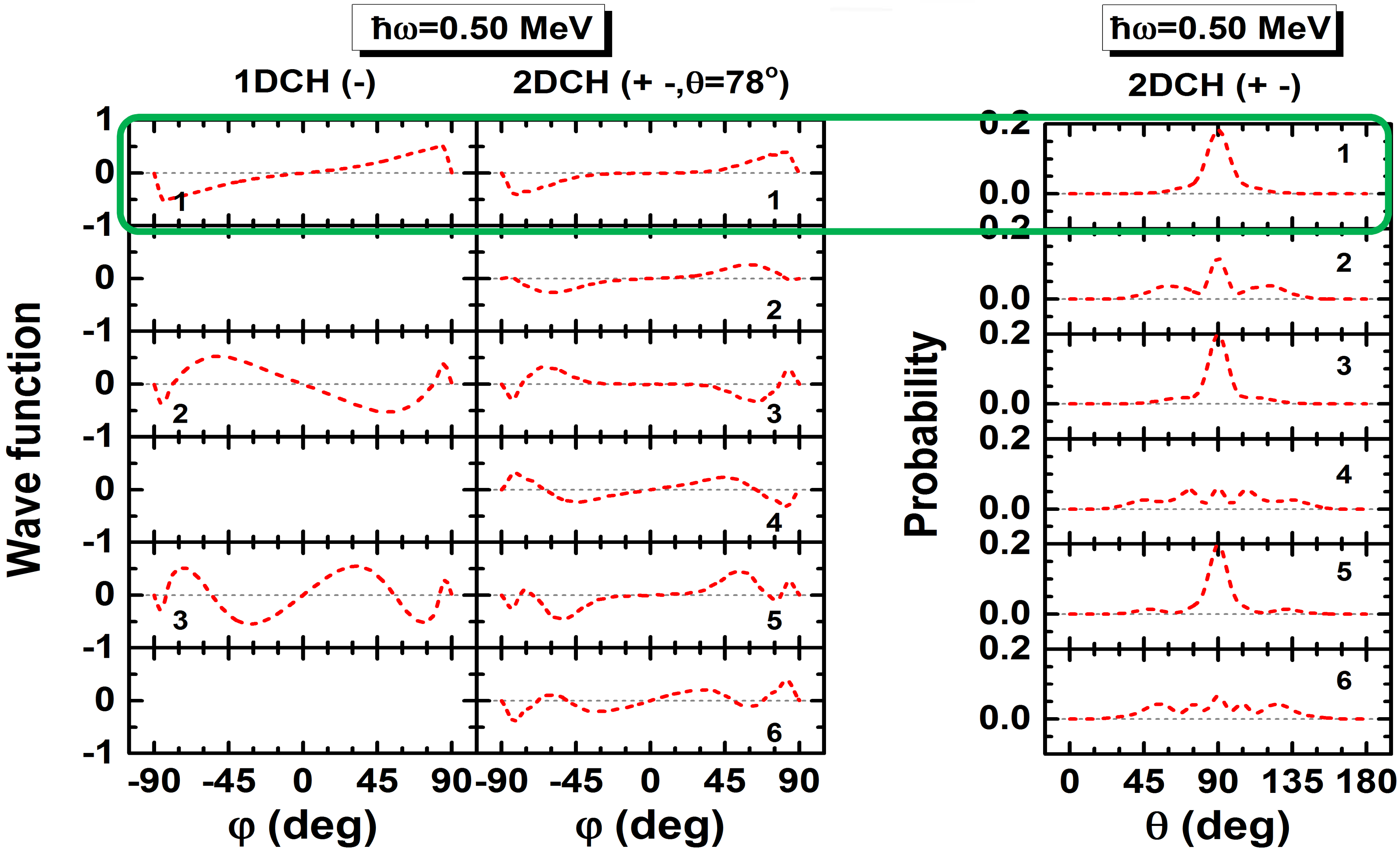
Comparison with 1D wave function



similar w.f. in φ

Only zero-phonon mode in θ has 1D corresponding

Comparison with 1D wave function

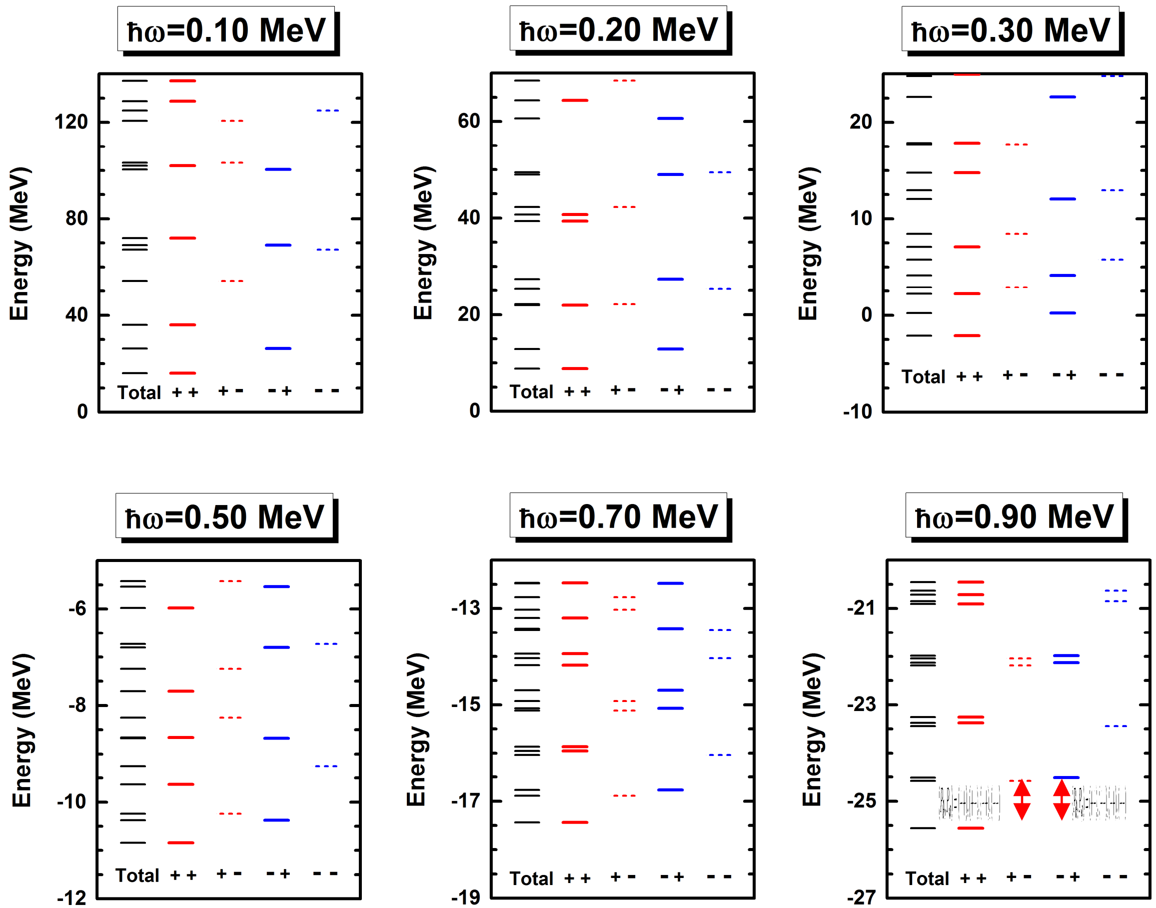


similar w.f. in φ

Only zero-phonon mode in θ has 1D corresponding

Collective energy

Diagonalization of the 2D CH yields the energy and wave functions at each cranking frequency.



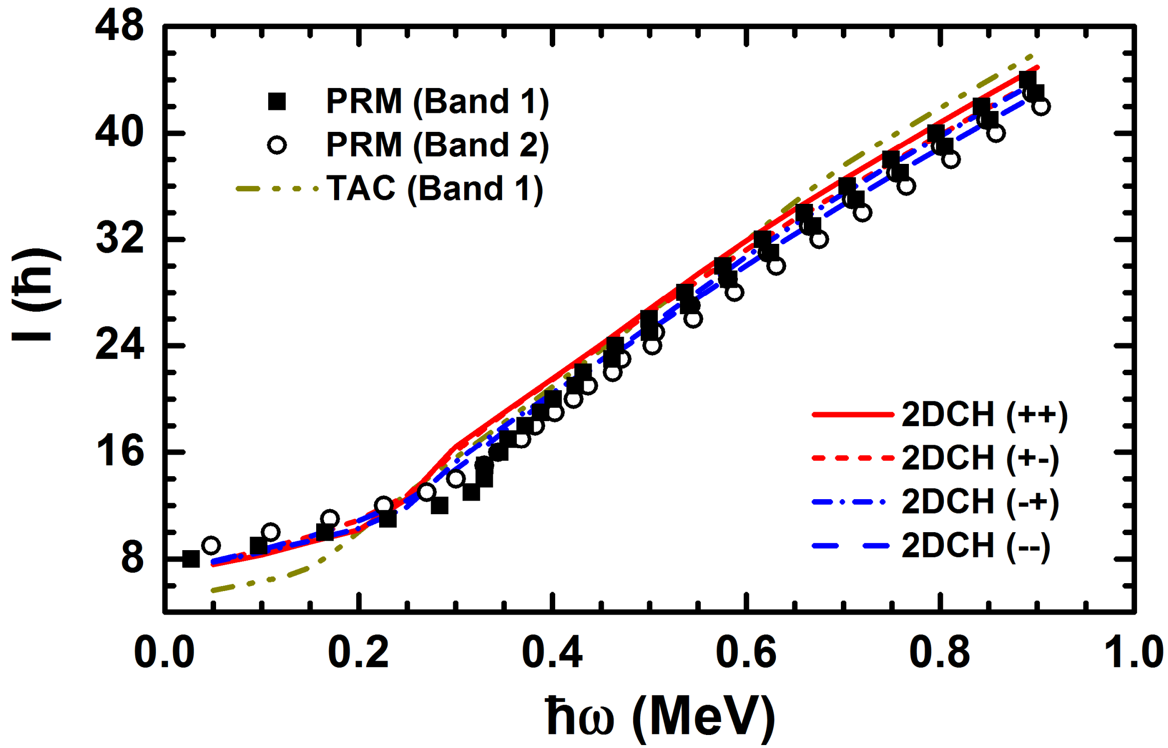
$$(P_\theta, P_\varphi) = (++), (+-), (-+), (--)$$

- **The energy levels are grouped:** $(P_\theta, P_\varphi) = (++), (+-), (-+), (--)$
- **The energy levels in different groups are associated with different phonon excitation modes.**
- **Sparsely distributed at low rotational frequencies.**
- **ω_φ is larger than ω_θ at low frequencies, while become nearly degenerate at high frequencies.**

Comparison of I-ω relation

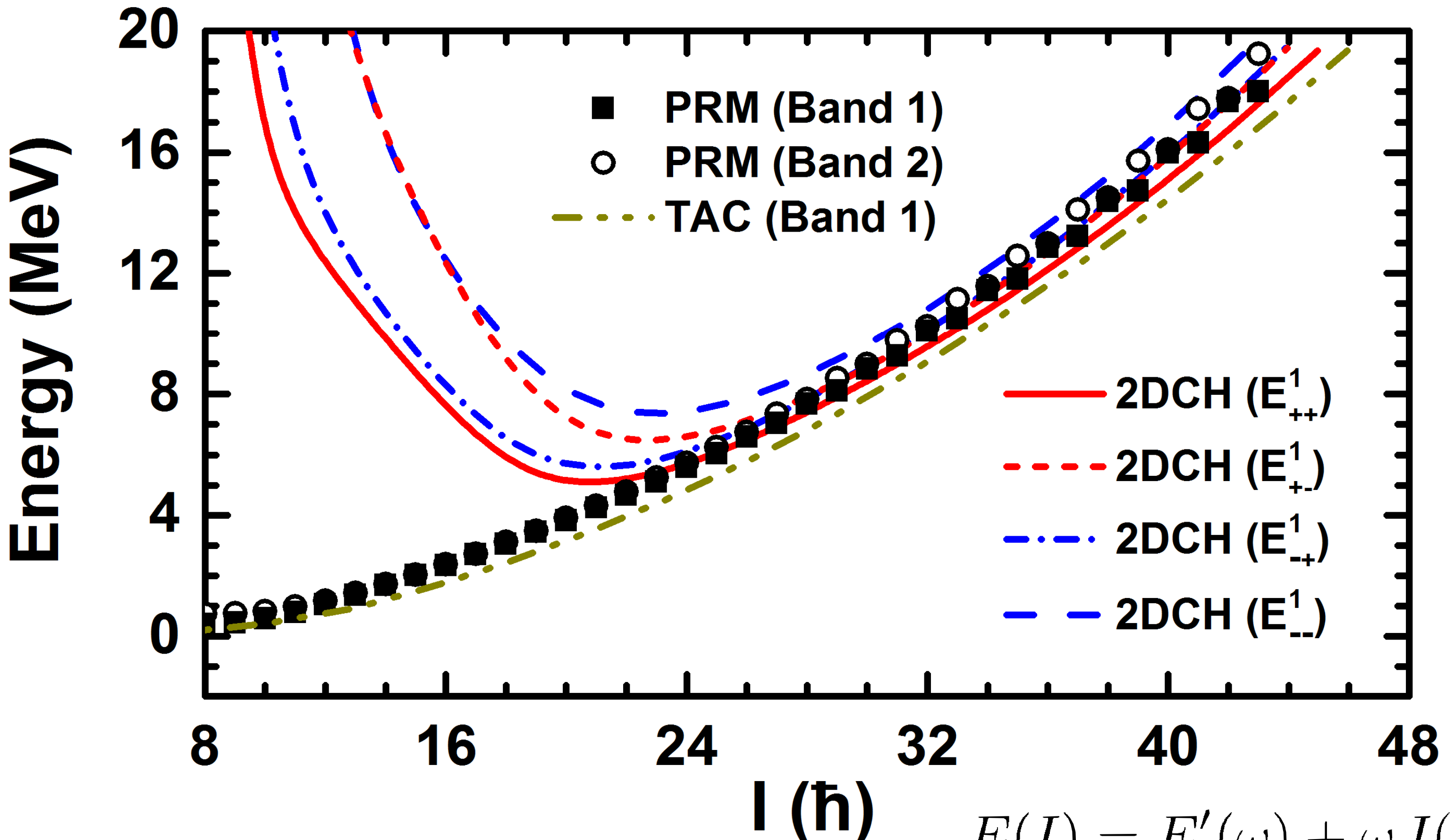
To examine the reliability of the collective Hamiltonian, the collective Hamiltonian results are compared with PRM solution.

$$\langle J\rangle_{\text{coll}} = \int_0^{\pi} d\theta \int_{-\pi/2}^{\pi/2} d\varphi |\Psi_{\text{coll}}(\theta,\varphi)|^2 J_{\text{TAC}}(\theta,\varphi)$$



- I-ω relation is reproduced.
- A kink at $\hbar\omega$=0.20 MeV in TAC becomes smooth in 2D Collective Hamiltonian.

Comparison of energy



$$E(I) = E'(\omega) + \omega J(\omega)$$

- Energy at high spin is well reproduced.
- Energy at low spin differs from those by PRM and TAC due to smaller mass parameter.

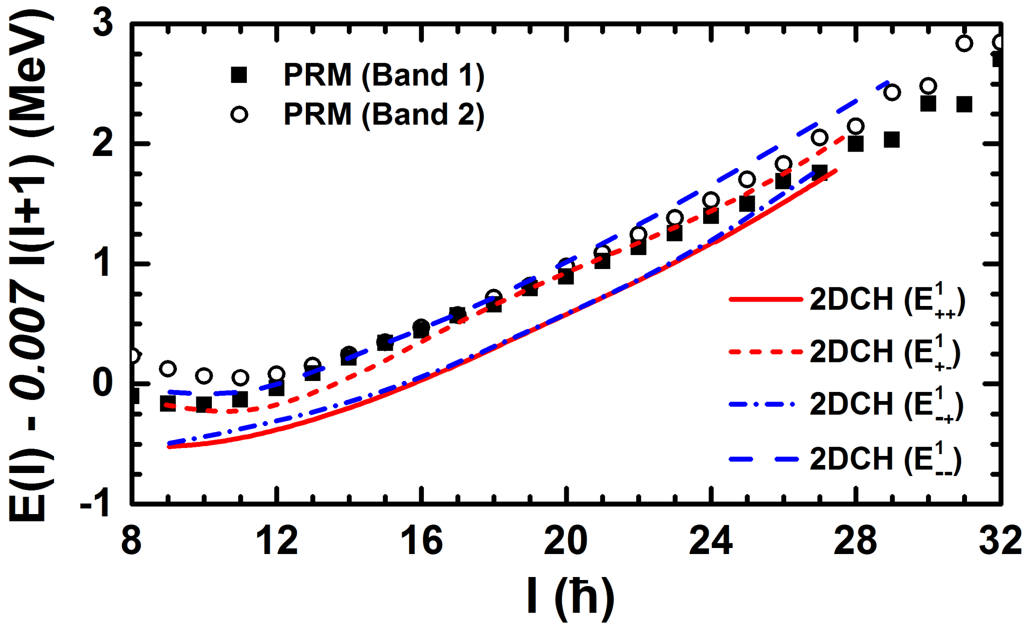
Comparison of energy

- **The mass parameter with vibrational frequencies:** *Chen2016PRC*

$$B_{\theta\theta}=2\sum_{mi}\frac{(\varepsilon_m-\varepsilon_i)\left|\langle m|\frac{\partial\boldsymbol{\omega}}{\partial\theta}\cdot\hat{\boldsymbol{j}}|i\rangle\right|^2}{[(\varepsilon_m-\varepsilon_i)^2-\hbar^2\Omega_\theta^2]^2},\quad B_{\varphi\varphi}=2\sum_{mi}\frac{(\varepsilon_m-\varepsilon_i)\left|\langle m|\frac{\partial\boldsymbol{\omega}}{\partial\theta}\cdot\hat{\boldsymbol{j}}|i\rangle\right|^2}{[(\varepsilon_m-\varepsilon_i)^2-\hbar^2\Omega_\theta^2]^2},$$

$$B_{\theta\varphi}=B_{\varphi\theta}=2\sum_{mi}\frac{(\varepsilon_m-\varepsilon_i)\langle m|\frac{\partial\boldsymbol{\omega}}{\partial\theta}\cdot\hat{\boldsymbol{j}}|i\rangle\langle i|\frac{\partial\boldsymbol{\omega}}{\partial\varphi}\cdot\hat{\boldsymbol{j}}|m\rangle}{[(\varepsilon_m-\varepsilon_i)^2-\hbar^2\Omega_\theta^2][(\varepsilon_m-\varepsilon_i)^2-\hbar^2\Omega_\varphi^2]}.$$

$$\hbar\Omega_\theta=\hbar\Omega_\varphi=1.1\ \mathrm{MeV}$$



After including the vibrational frequency in the calculations of the mass parameter, energy spectra at low spins are well reproduced.

But, how to determinate the vibrational frequencies self-consistently?

Self-consistent mass parameter

- **Starting point:** *Marshalek&Rasmussen1963NPA*

$$H_M(\alpha(t)) = \sum_i \left[\frac{p_i^2}{2m} + V_{\rm sp}(x_i, y_i, z_i; \sigma_i; \alpha(t))\right] - \frac{1}{2}\sum_{ij} G_{ij}$$

- **To treat harmonic vibrations, expanding the Hamiltonian H_M to 2nd order:**

$$H_M \approx H(\alpha_0) + \frac{\partial H}{\partial \alpha}\bigg|_{\alpha=\alpha_0}(\alpha-\alpha_0) + \frac{1}{2}\frac{\partial^2 H}{\partial \alpha^2}\bigg|_{\alpha=\alpha_0}(\alpha-\alpha_0)^2$$

- **Assumptions:** $\alpha - \alpha_0 = A\sin(\Omega t + \delta), \quad \delta = 0$

- **The problem is to solve the time-dependent Schrodinger equation corresponding to the Hamiltonian H_M to order A^2, by the usual method of variation of constants, but without the adiabatic approximation. To replace this approximation, a sensible constraint is needed, which determines Ω.**

- **Constraint:**

$$\frac{\partial}{\partial t}\langle\Psi(t)|H_M(t)|\Psi(t)\rangle = \langle\Psi(t)|\frac{\partial H_M}{\partial t}|\Psi(t)\rangle \equiv 0$$

$$H_M(t)|\Psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle$$

Self-consistent mass parameter

- **Perturbation treatment:**

$$\Psi(t) = \exp(-iE_0 t/\hbar)\sum_j b_j(t)|j;\alpha_0\rangle \qquad H(\alpha_0)|j;\alpha_0\rangle = E_j(\alpha_0)|j;\alpha_0\rangle$$

- **Coefficients of the expansion to first order:**

$$\begin{cases} b_j = \frac{i}{2}A\langle j|\frac{\partial H}{\partial\alpha}|0\rangle\left[\frac{\exp(i\Omega t)}{\varepsilon_j+\hbar\omega} - \frac{\exp(-i\Omega t)}{\varepsilon_j - \hbar\Omega}\right] \\ \quad = \frac{-\langle j|\frac{\partial H}{\partial\alpha}|0\rangle}{\varepsilon_j^2-(\hbar\Omega)^2}[\varepsilon_j(\alpha-\alpha_0)+i\hbar\dot{\alpha}], \quad j\neq 0, \qquad \varepsilon_j = E_j - E_0 \\ b_0 = 1 - \frac{1}{2}\sum_j \frac{|\langle j|\frac{\partial H}{\partial\alpha}|0\rangle|^2}{[\varepsilon_j^2-(\hbar\Omega)^2]^2}[\varepsilon_j^2(\alpha-\alpha_0)^2+\hbar^2\dot{\alpha}^2], \end{cases}$$

- **Perturbation increment in energy to second order:**

$$\delta E = \langle\Psi(t)|H_M|\Psi(t)\rangle - E_0$$

$$= \sum_{j\neq 0}|b_j|^2\varepsilon_j + \sum_{j\neq 0}\left(b_j\langle 0|\frac{\partial H}{\partial\alpha}|j\rangle(\alpha-\alpha_0)+\text{c.c.}\right) + \langle 0|\frac{1}{2}\frac{\partial^2 H}{\partial\alpha^2}|0\rangle(\alpha-\alpha_0)^2,$$

Self-consistent mass parameter

- **Perturbation increment in energy to second order:**

$$\delta E = A^2\Big\{(\hbar\Omega)^2\sum_j \frac{|\langle j|\frac{\partial H}{\partial\alpha}|0\rangle|^2\varepsilon_j}{[\varepsilon_j^2-(\hbar\Omega)^2]^2} + \sin^2\Omega t\Big[\langle 0|\frac{1}{2}\frac{\partial^2 H}{\partial\alpha^2}|0\rangle - \sum_j \frac{|\langle j|\frac{\partial H}{\partial\alpha}|0\rangle|^2\varepsilon_j}{\varepsilon_j^2-(\hbar\Omega)^2}\Big]\Big\}.$$

- **To satisfy the constraint condition that δE be independent of time,**

$$\sum_j \frac{|\langle j|\frac{\partial H}{\partial\alpha}|0\rangle|^2\varepsilon_j}{\varepsilon_j^2-(\hbar\Omega)^2} = \langle 0|\frac{1}{2}\frac{\partial^2 H}{\partial\alpha^2}|0\rangle.$$

Determinate Ω

- **Thus, the perturbation energy:** $\delta E = \frac{1}{2}A^2\Omega^2 B(\Omega)$

$$B(\Omega) = 2\hbar^2\sum_j \frac{|\langle j|\frac{\partial H}{\partial\alpha}|0\rangle|^2\varepsilon_j}{[\varepsilon_j^2-(\hbar\Omega)^2]^2}$$

Mass parameter

$$C(\Omega) = \Omega^2 B(\Omega)$$

Force constant

Self-consistent mass parameter

- **Applications to the tilted axis cranking model:**

$$\underbrace{\sum_{l\neq 0}\frac{|\langle l|\frac{\partial H}{\partial\alpha}|0\rangle|^2\varepsilon_l}{\varepsilon_l^2-(\hbar\Omega)^2}=\langle 0|\frac{1}{2}\frac{\partial^2 H}{\partial\alpha^2}|0\rangle.}$$

$$\alpha\rightarrow\varphi \qquad H\rightarrow h'=h_0-\boldsymbol{\omega}\cdot\boldsymbol{j} \qquad \alpha\rightarrow\theta$$

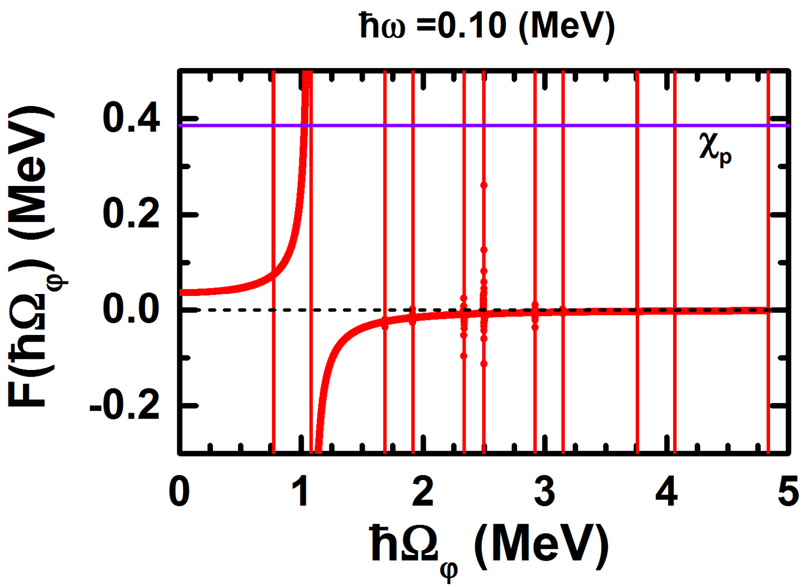
$$\boxed{\sum_{l\neq 0}\frac{|\langle l|\frac{\partial h'}{\partial\varphi}|0\rangle|^2\varepsilon_l}{\varepsilon_l^2-(\hbar\Omega_\varphi)^2}=\langle 0|\frac{1}{2}\frac{\partial^2 h'}{\partial\varphi^2}|0\rangle.}$$

$$\boxed{\sum_{l\neq 0}\frac{|\langle l|\frac{\partial h'}{\partial\theta}|0\rangle|^2\varepsilon_l}{\varepsilon_l^2-(\hbar\Omega_\theta)^2}=\langle 0|\frac{1}{2}\frac{\partial^2 h'}{\partial\theta^2}|0\rangle.}$$

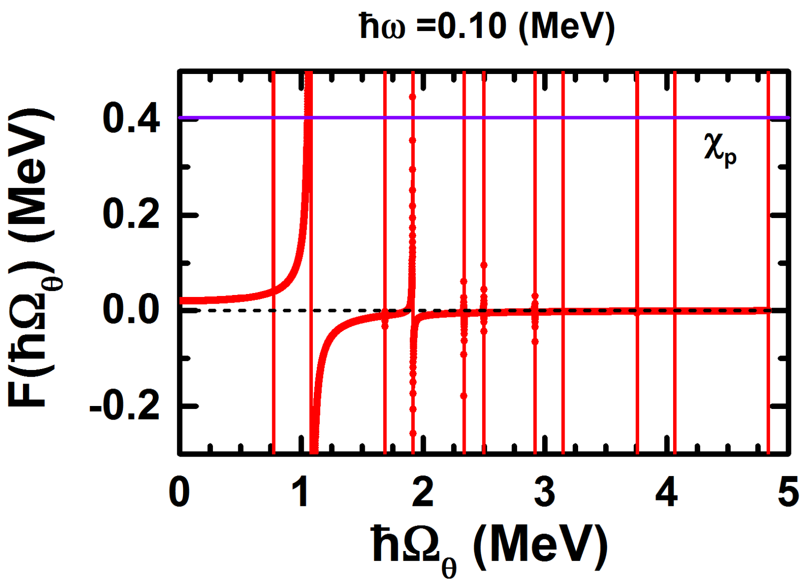
$$\Rightarrow\left\{\begin{array}{l} B_{\theta\theta}=2\sum_{l\neq 0}\frac{\varepsilon_l\left|\langle l|\frac{\partial\boldsymbol{\omega}}{\partial\theta}\cdot\hat{\boldsymbol{j}}|0\rangle\right|^2}{[\varepsilon_l^2-\hbar^2\Omega_\theta^2]^2}, \quad B_{\varphi\varphi}=2\sum_{l\neq 0}\frac{\varepsilon_l\left|\langle l|\frac{\partial\boldsymbol{\omega}}{\partial\varphi}\cdot\hat{\boldsymbol{j}}|0\rangle\right|^2}{[\varepsilon_l^2-\hbar^2\Omega_\varphi^2]^2}, \\ B_{\theta\varphi}=B_{\varphi\theta}=2\sum_{l\neq 0}\frac{\varepsilon_l\langle l|\frac{\partial\boldsymbol{\omega}}{\partial\theta}\cdot\hat{\boldsymbol{j}}|0\rangle\langle 0|\frac{\partial\boldsymbol{\omega}}{\partial\varphi}\cdot\hat{\boldsymbol{j}}|l\rangle}{[\varepsilon_l^2-\hbar^2\Omega_\theta^2][\varepsilon_l^2-\hbar^2\Omega_\varphi^2]}. \end{array}\right.$$

Vibrational frequencies

- **How do we calculate the vibrational frequencies?**



$$2\sum_{l\neq 0}\frac{\left|\omega\sin\theta\sin\varphi\langle l|\hat{j}_1|0\rangle-\omega\sin\theta\cos\varphi\langle l|\hat{j}_2|0\rangle\right|^2\varepsilon_l}{\varepsilon_l^2-(\hbar\Omega_\varphi)^2}$$
$$=\left[\omega\sin\theta\cos\varphi\langle 0|\hat{j}_1|0\rangle+\omega\sin\theta\sin\varphi\langle 0|\hat{j}_2|0\rangle\right].$$

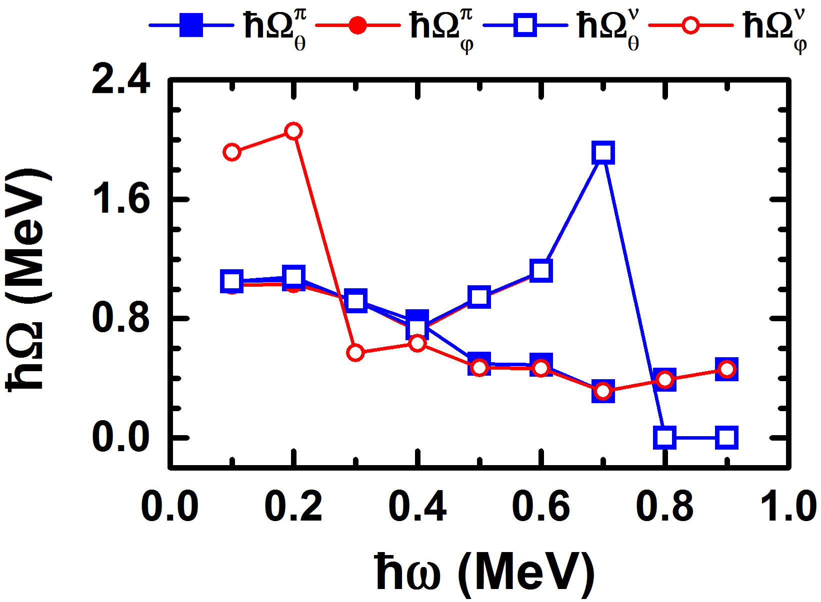


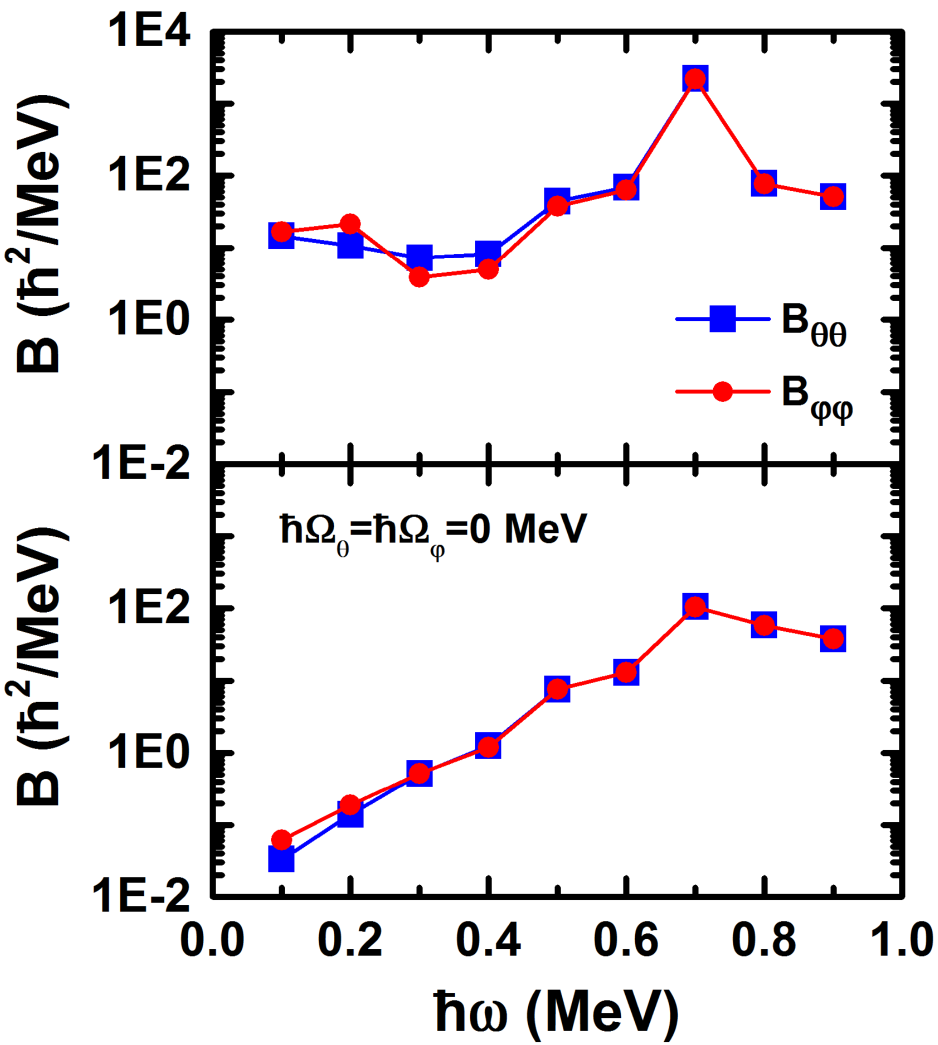
$$2\sum_{l\neq 0}\frac{\left|-\omega\cos\theta\cos\varphi\langle l|\hat{j}_1|0\rangle-\omega\cos\theta\sin\varphi\langle l|\hat{j}_2|0\rangle+\omega\sin\theta\langle l|\hat{j}_3|0\rangle\right|^2\varepsilon_l}{\varepsilon_l^2-(\hbar\Omega_\theta)^2}$$
$$=\left[\omega\sin\theta\cos\varphi\langle 0|\hat{j}_1|0\rangle+\omega\sin\theta\sin\varphi\langle 0|\hat{j}_2|0\rangle+\omega\cos\theta\langle 0|\hat{j}_3|0\rangle\right].$$

- **The lowest solutions are taken as the vibrational frequency.**

Self-consistent mass parameter

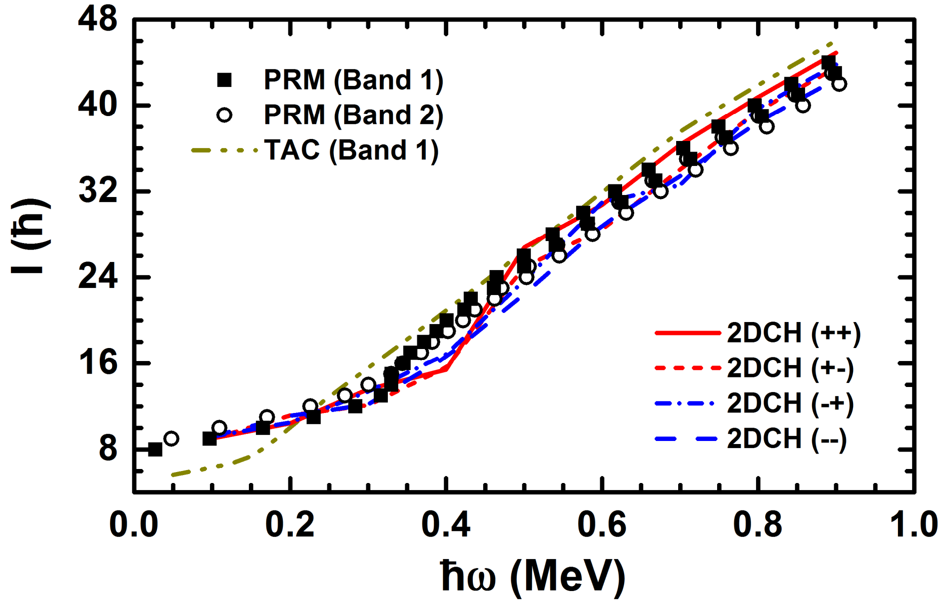
- The obtained vibrational frequencies and mass parameters at the minimum of potential energy surface.

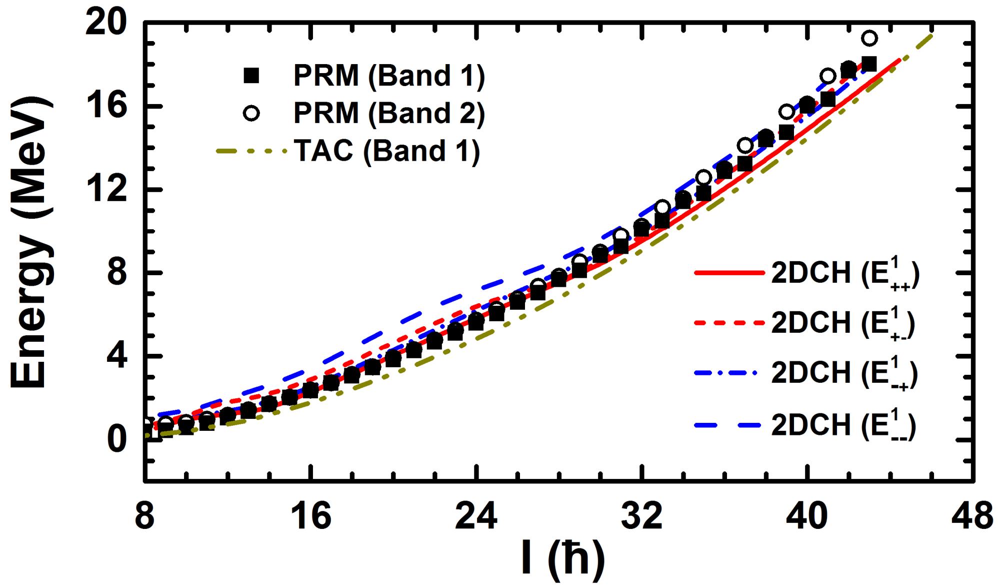




- With the vibrational frequency, the mass parameters are enhanced at the low rotational frequencies.

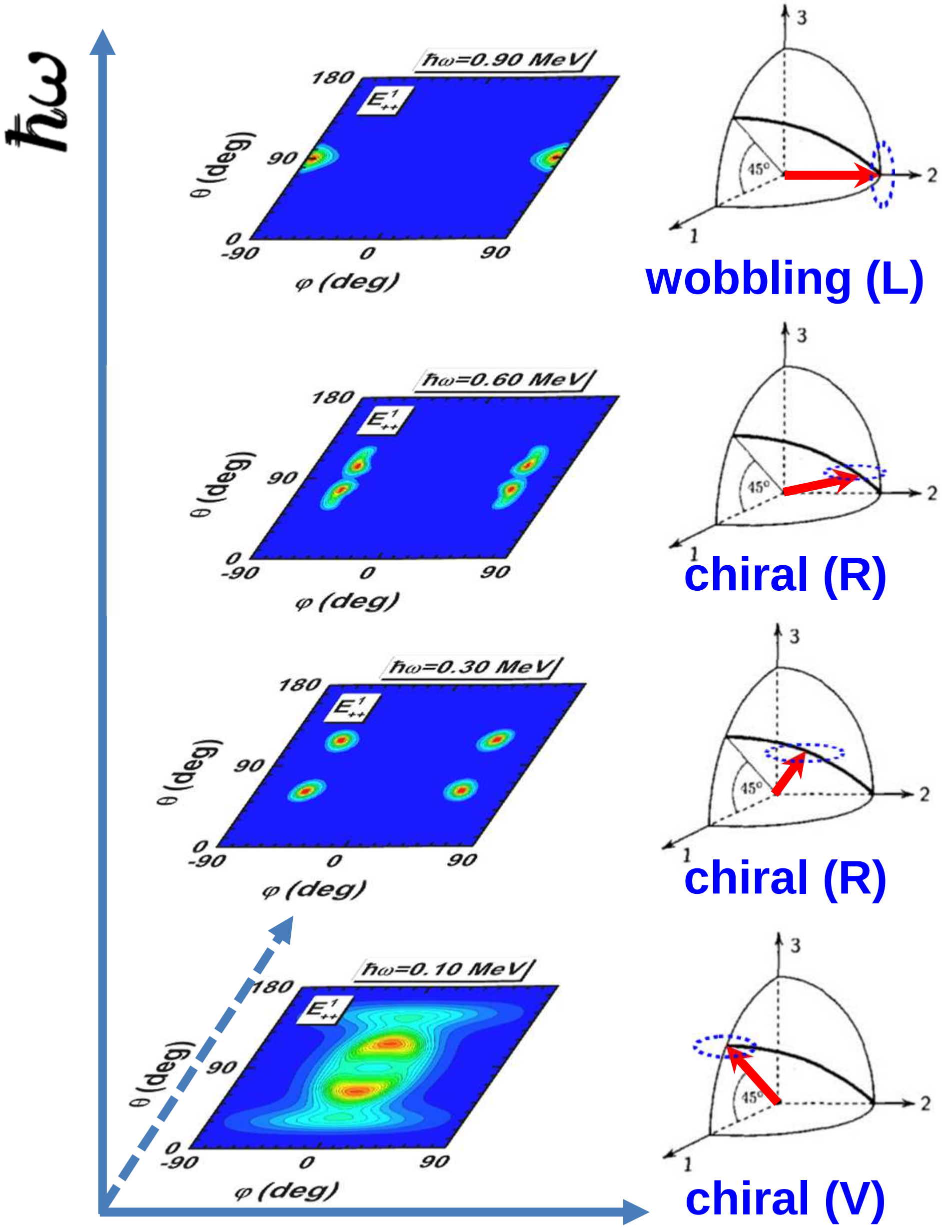
I-ω and energy

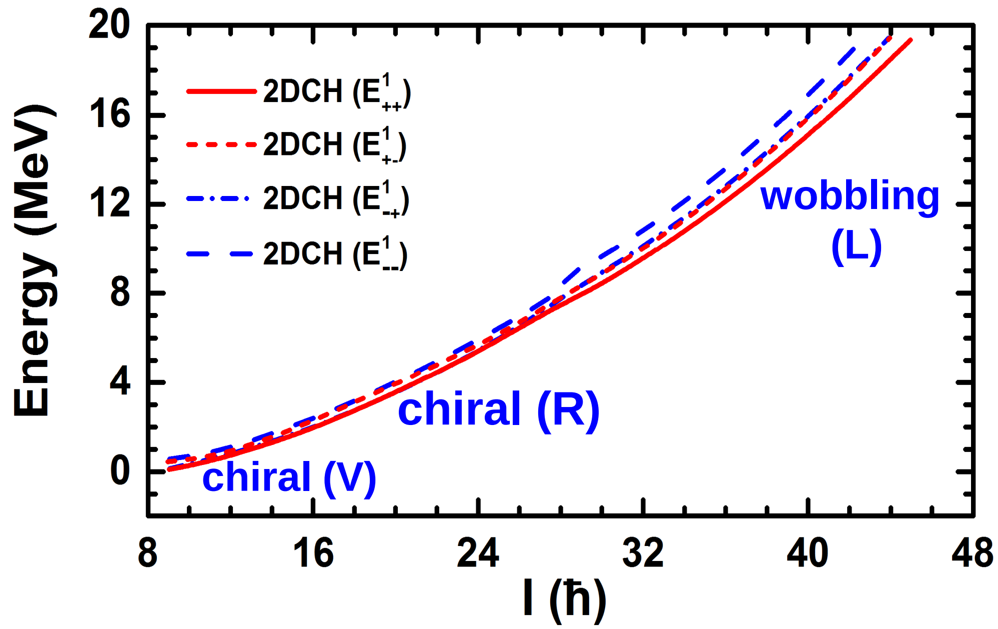




- After including the vibrational frequencies in the calculations of the mass parameter, the I-ω and energy spectra at low spins are reasonably reproduced.

A brief summary





- From chiral vibration (CV) to chiral rotation (CR) to longitudinal wobbling (LW).

Outline

- Introduction
- Collective Hamiltonian
- One-dimensional
- Two-dimensional
- Summary and perspective

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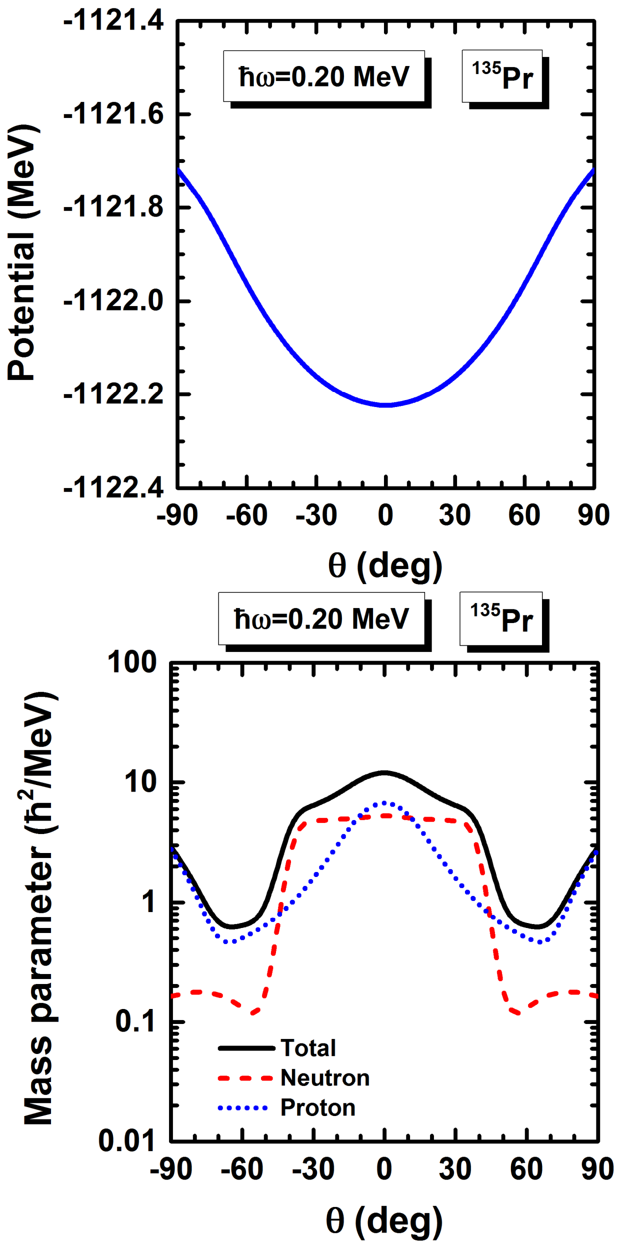
Summary and perspective

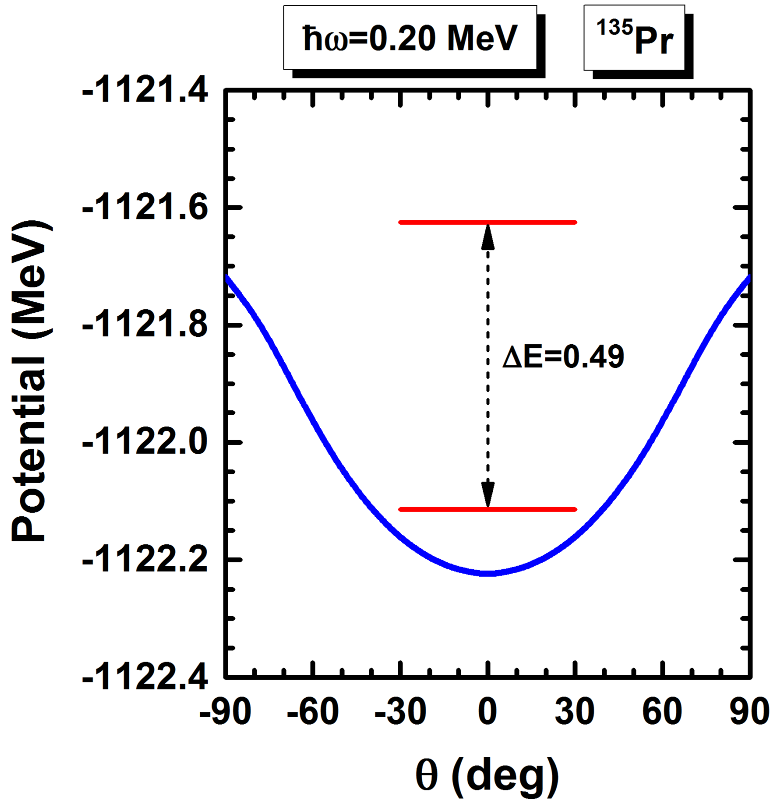
Collective Hamiltonian based on TAC model with particle-hole configuration has been developed to investigate the chiral rotation and wobbling motion.

- **1-D Collective H. :** *Chen, Zhang, Zhao, Jolos, and Meng, PRC 87, 024314 (2013)*
- **2-D Collective H. :** *Chen, Zhang, Zhao, Jolos, and Meng, PRC 94, 044301 (2016).*
- **Door is open to develop Collective Hamiltonian based on TAC density functional.**

Thank you!

Preliminary results by TAC-RHB





- The collective potential and mass parameter are calculated by two-dimensional TAC-RHB.
- The energy difference is agreement with the experimental results. *Matta2015PRL*