

Shape changes and shape coexistence in rapidly rotating nuclei.

I.Ragnarsson

partly in collaboration with

A. Kardan, A.V. Afanasjev, B.G. Carlsson,
Hai-Liang Ma, E.S. Paul, C.M. Petrache,
M.A. Riley, J.S. Sharpey-Schafer and J. Simpson.



LUND INSTITUTE OF TECHNOLOGY
Lund University

Outline

- The Cranked Nilsson-Strutinsky (CNS) and Cranked Nilsson-Strutinsky Bogoliubov (CNSB) formalisms.
- Reinterpretation of ‘TSD’ bands in ^{164}Hf - terminating bands.
- Terminations at very high spin in ^{156}Dy - interplay between dg and sd pseudospin partner orbitals.
- How should we understand the TSD bands with wobbling excitations in $^{163,165,167}\text{Lu}$.

Cranked Nilsson-Strutinsky (CNS) -Bogoliubov (CNSB) models.

Modified oscillator or Nilsson potential:

$$V_{MO} = V(\varepsilon_2, \gamma, \varepsilon_4) - \kappa \hbar \omega_0^0 \left(2 \vec{l}_t \cdot \vec{s} + \mu (l_t^2 - \langle l_t^2 \rangle_N) \right).$$

The cranking Hamiltonian is diagonalized (CNS):

$$H_{CNS}^\omega = H_{MO} - \omega j_x; \quad H_{CNS}^\omega \chi_\nu^\omega = e_\nu^\omega \chi_\nu^\omega; \quad e_\nu = \langle \chi_\nu^\omega | H_{CNS} | \chi_\nu^\omega \rangle$$

CNSB: $H_{CNSB}^\omega = H_{MO} - \omega j_x - \Delta(P^\dagger + P) - \lambda \hat{N}$

Total quantities:

$$E_{tot} = E_{rld} + E_{shell} (+E_{pair}); \quad E_{shell} = \sum_{occ} e_\nu - \left\langle \sum_{occ} e_\nu \right\rangle \quad I = \sum_{occ} \langle j_x \rangle$$

Rotating liquid drop energy:

$$E_{rld}(Z, N, I, \varepsilon_i) = E_{ld}(Z, N, \varepsilon_i) + \frac{\hbar^2 I(I+1)}{2 \mathcal{J}_{\text{rig.}}(Z, N, \varepsilon_i)}$$

E_{ld} : Lublin-Strasbourg drop (LSD model) [or FRLDM]

$\mathcal{J}_{\text{rig.}}$: diffuse surface: $r_0 = 1.16$ fm, $a = 0.6$ fm

B.G. Carlsson and I.R., Phys. Rev. C74, 011302 (2006)

B.G. Carlsson *et al.* Phys. Rev. C78, 034316 (2008)

Hai-Liang Ma, B.G. Carlsson, I.R. and H. Ryde, Phys. Rev. C90, 014316 (2014)

Cranked Nilsson-Strutinsky (CNS) -Bogoliubov (CNSB) models.

Removal of virtual crossings - diabatic configurations.

CNS: Mesh in deformation space: $(\varepsilon_2, \gamma, \varepsilon_4)$.

Unique features to label orbitals and thus to fix conf's, e.g.:

- Diagonalization in rotating harmonic oscillator basis,
 N_{rot} -shells treated as pure
- Labelling of high- j shells and low- j shells, respectively

CNSB: Mesh in def. and pairing space: $(\varepsilon_2, \gamma, \varepsilon_4, \lambda_p, \lambda_n, \Delta_p, \Delta_n)$

- Minimization in $(\varepsilon_2, \gamma, \varepsilon_4, \lambda_p, \lambda_n, \Delta_p, \Delta_n)$ space
- Particle number projection.
- Removal of virtual interactions for fixed parameters

⇒ Possible to draw smooth PES's in the full (ε_2, γ) -plane.
Conf's: $(\pi, \alpha)_p(\pi, \alpha)_n$

Comparison CNS - CNSB, ^{161}Lu .

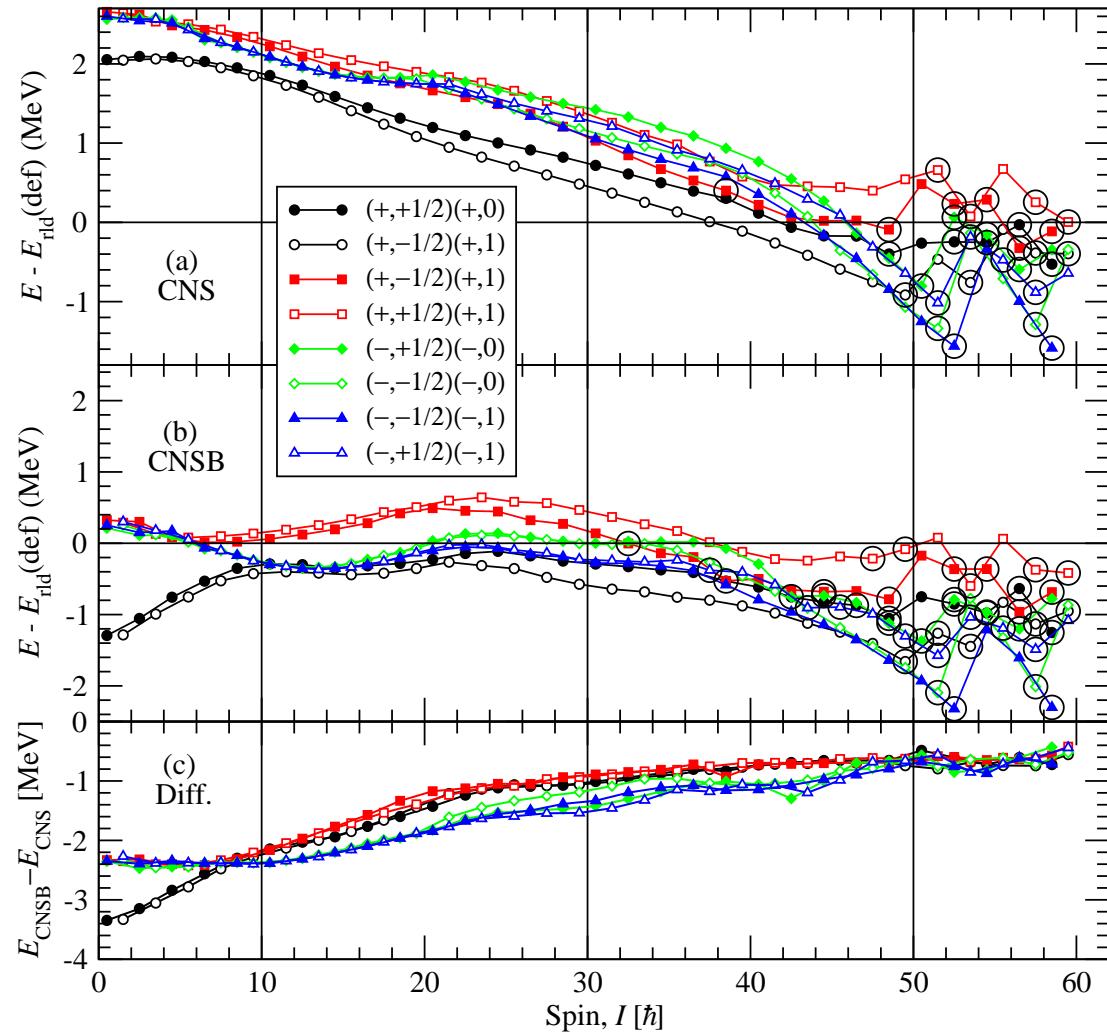
No pairing and pairing included.

Difference shows a rather smooth trend approaching zero at high spin.

⇒ Experiment down to rather low spin is well described by CNS if moment of inertia is renormalized.

No paired crossings at high spin

Trend towards more terminations (aligned non-collective states) with CNSB

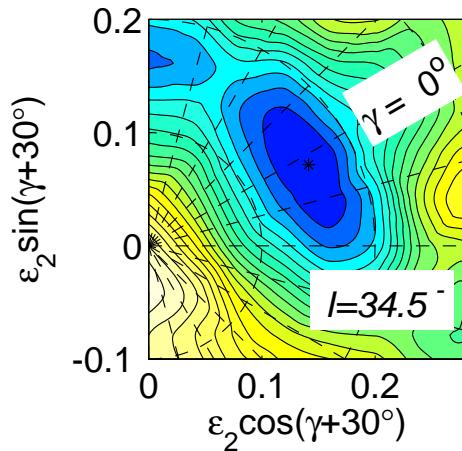


Hai-Liang Ma, B.G. Carlsson, I.R. and H. Ryde, PRC 90, 014316 (2014)

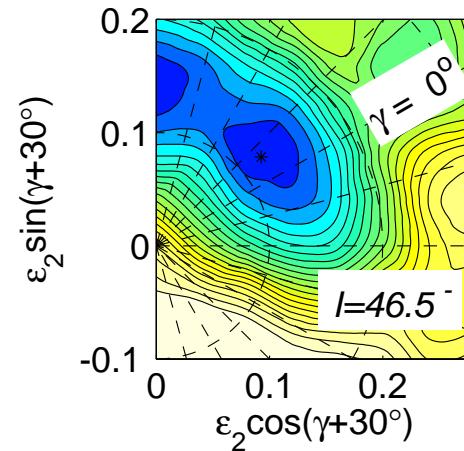
Total energy surfaces for ^{161}Lu .

$$(\pi_p, \alpha_p)(\pi_n, \alpha_n) = (+, -1/2)(-, 1); \quad \sim \pi(h_{11/2})^6 \nu(i_{13/2})^3, I_{max} = 52.5.$$

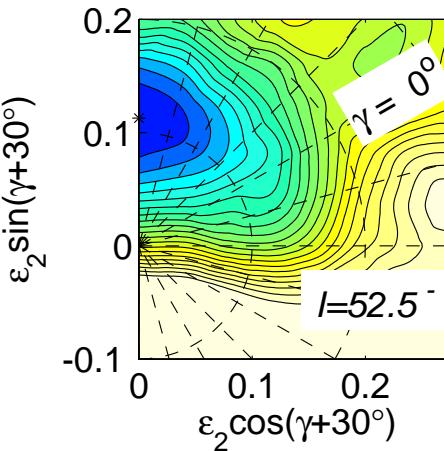
CNS: $I = 34.5$



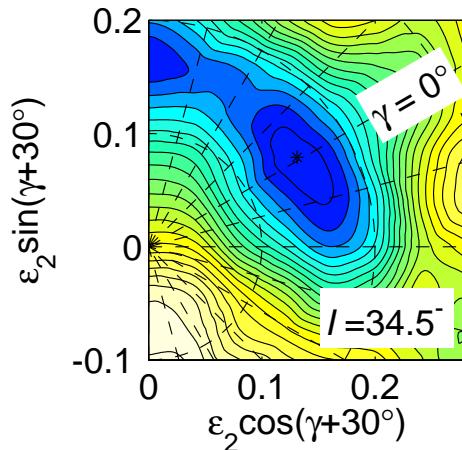
$I = 46.5$



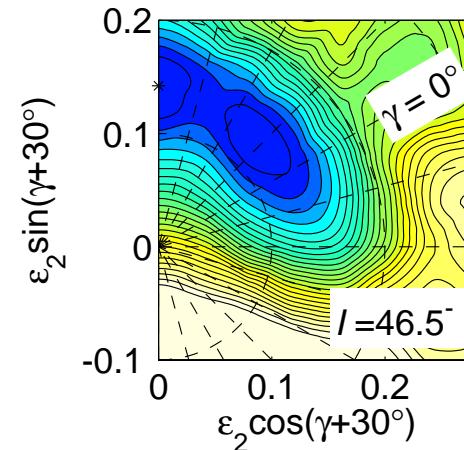
$I = 52.5$



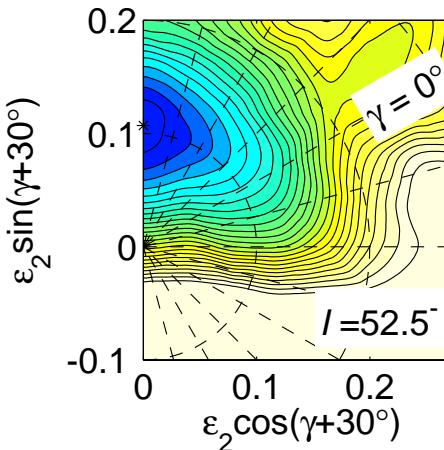
CNSB: $I = 34.5$



$I = 46.5$

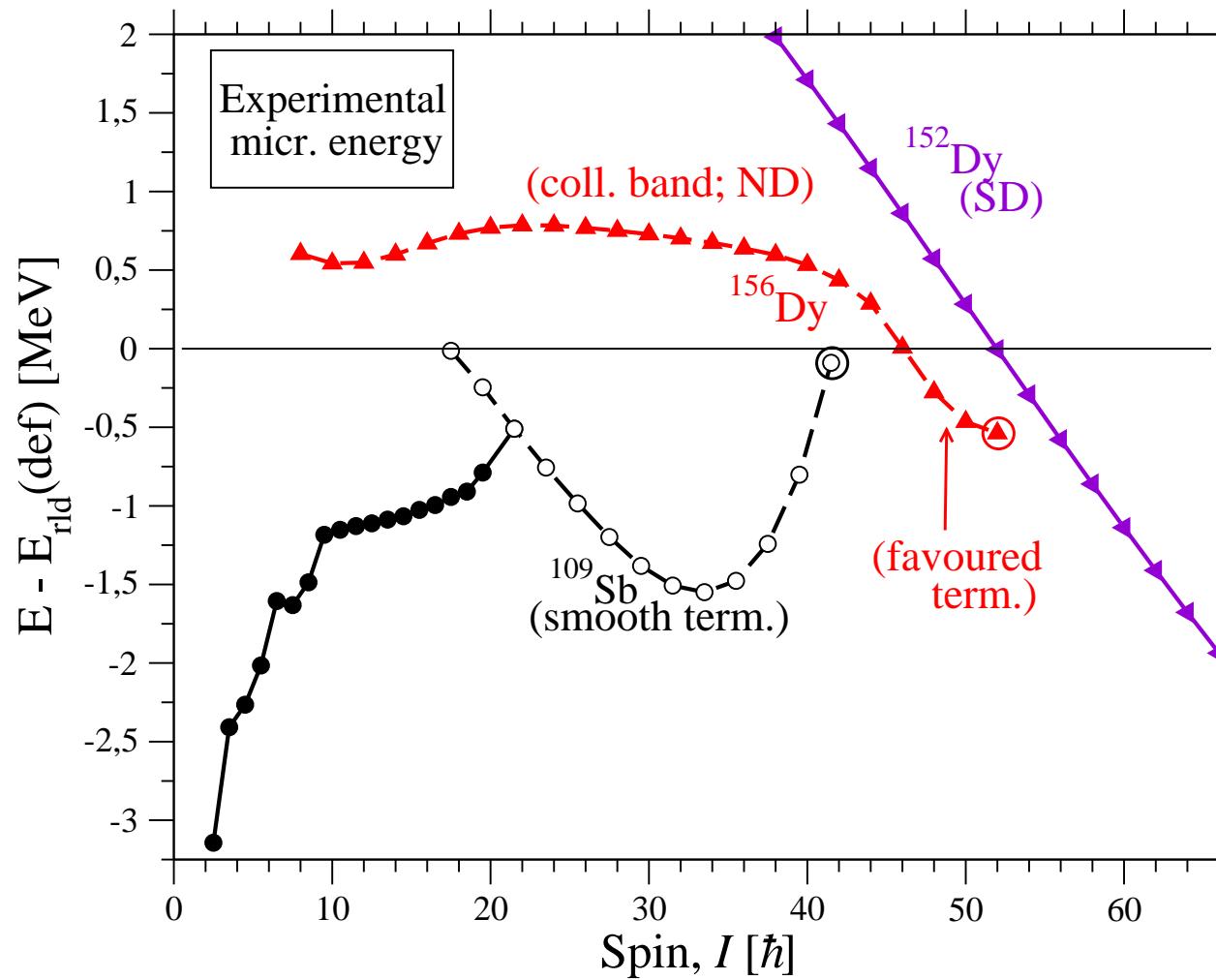


$I = 52.5$



Typical features of $E - E_{rld}$ plots

Different types of observed bands:



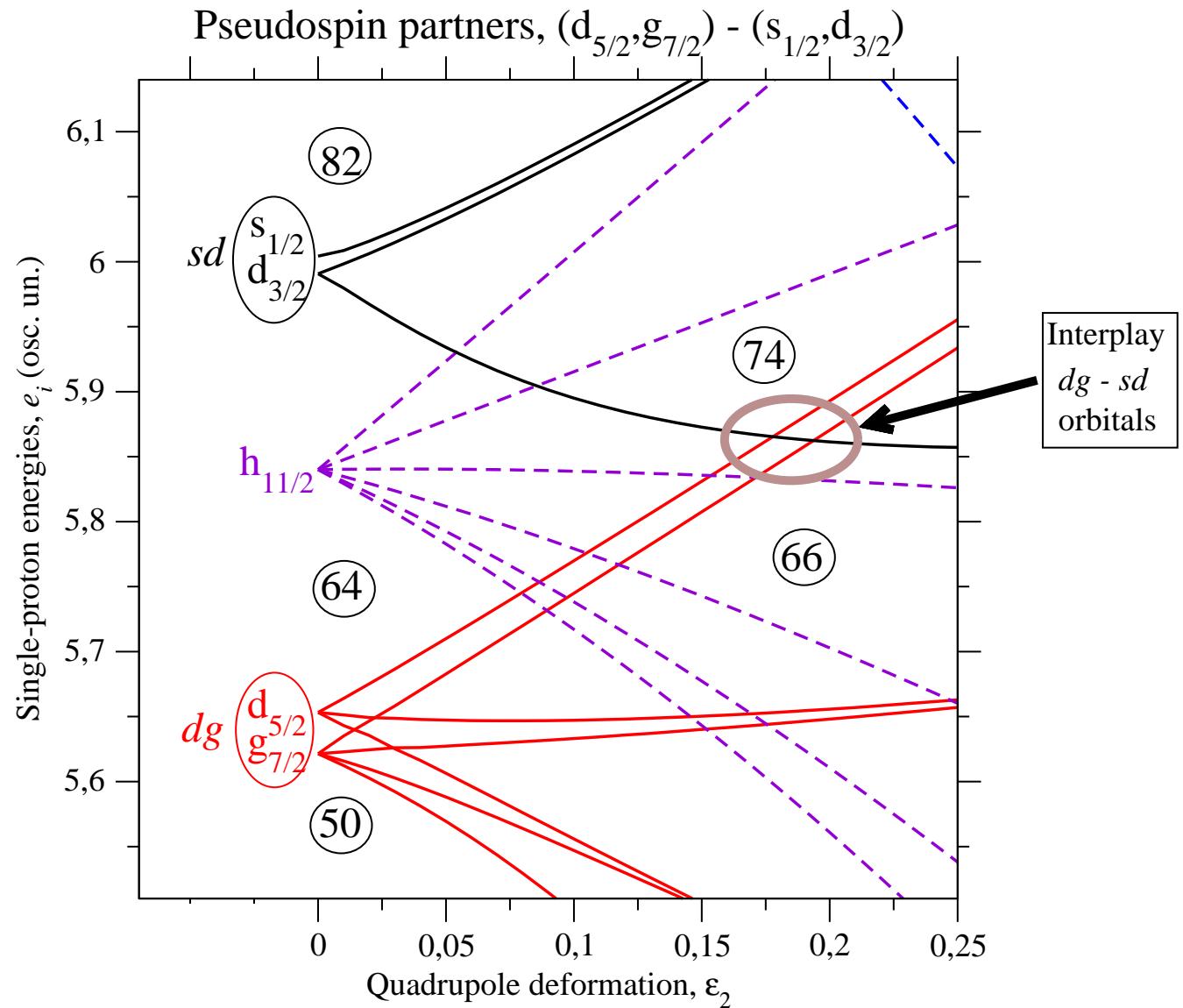
Interplay between dg and sd orbitals.

Note pseudo-spin partners:

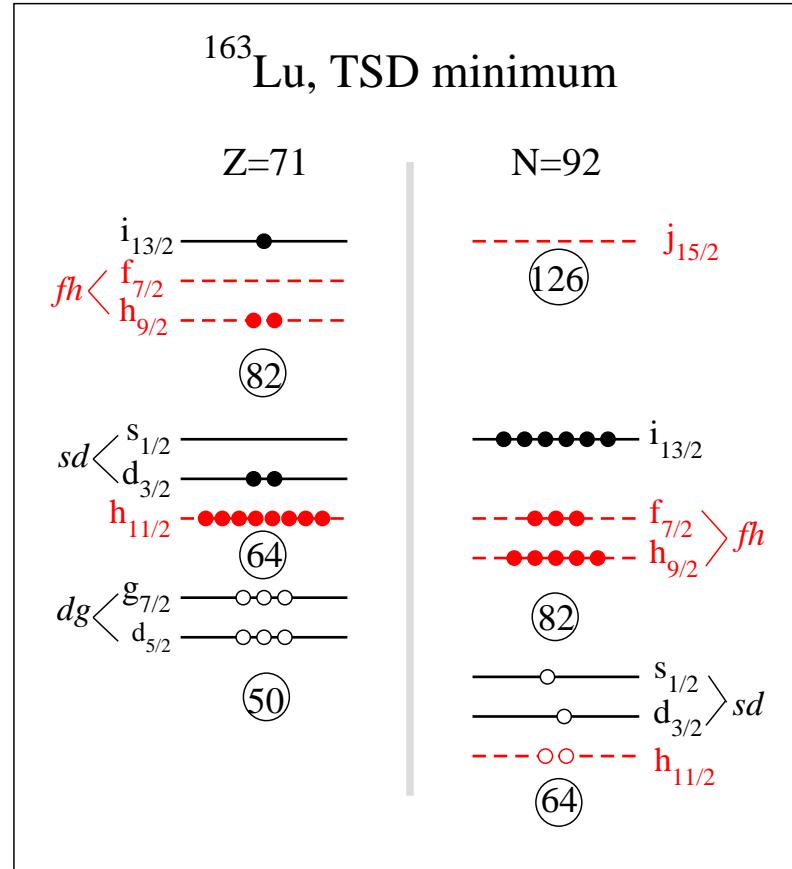
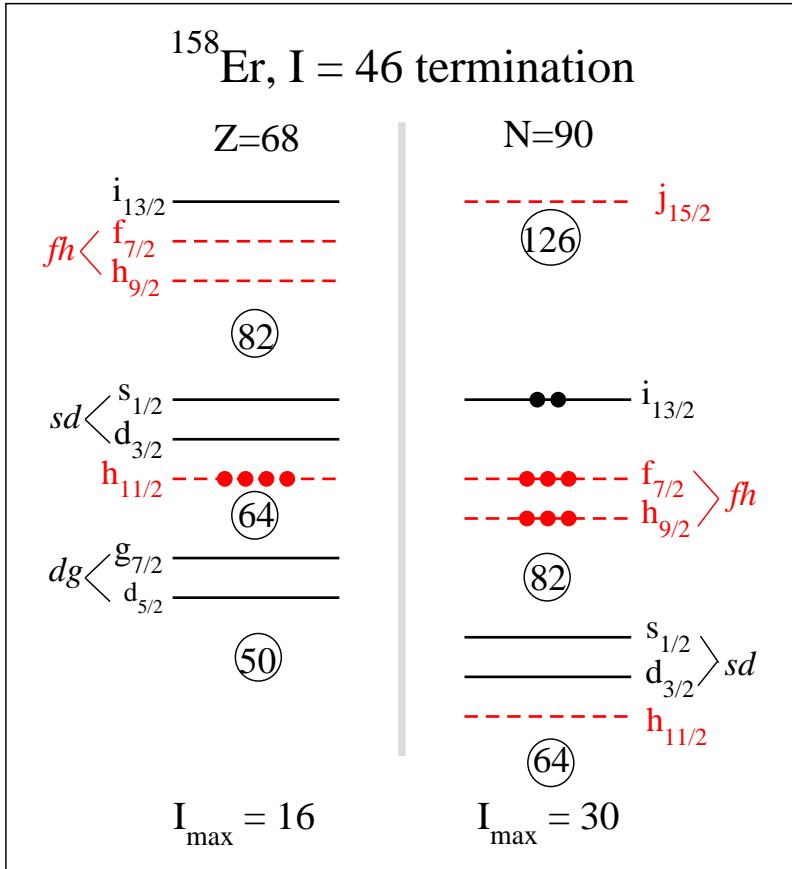
dg : $d_{5/2}g_{7/2}$

sd : $d_{3/2}s_{1/2}$

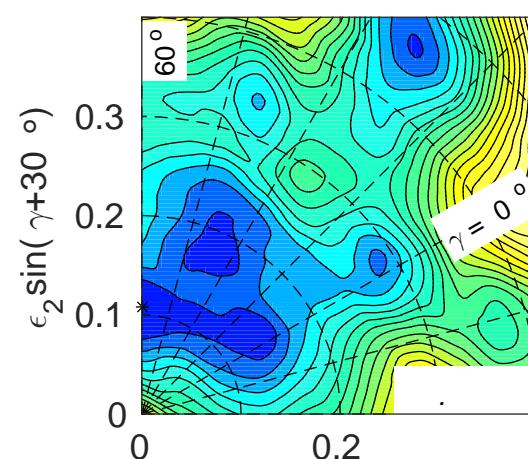
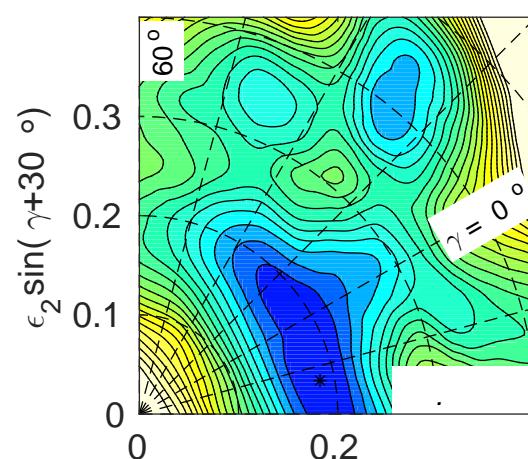
Creation of unpaired crossings at intermediate and high spin.



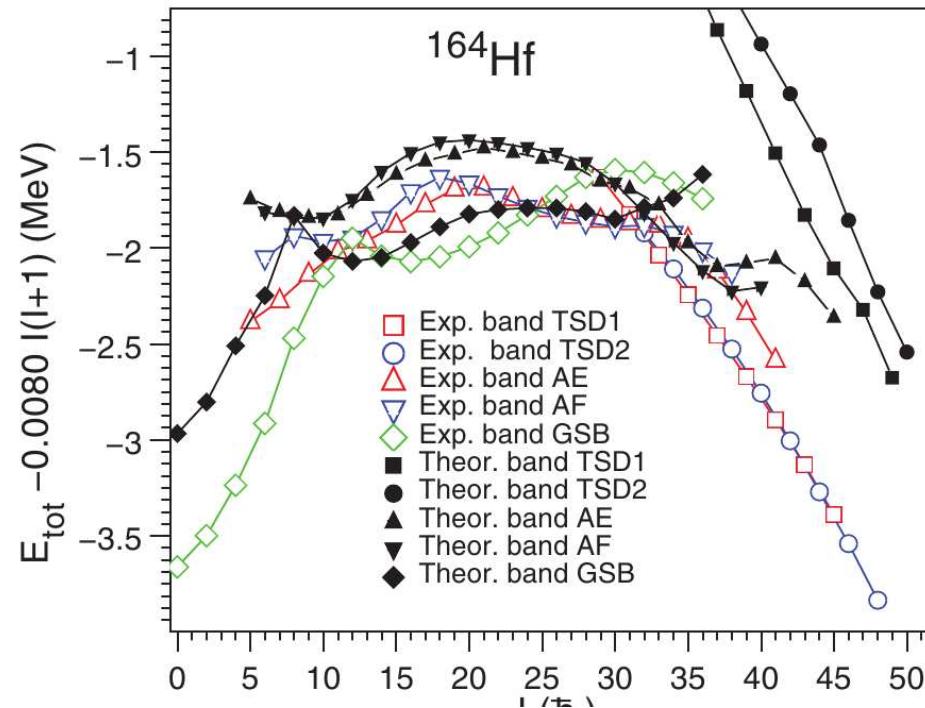
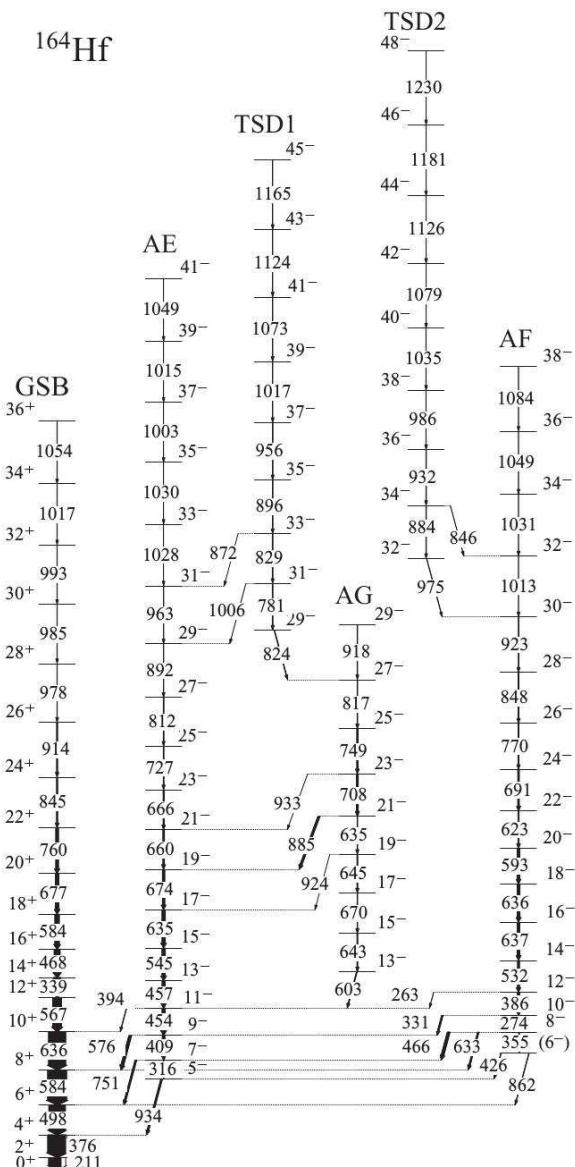
Typical configurations (relative to ^{146}Gd core).



^{163}Lu , $I = 30.5$:
 $I = 54.5$:

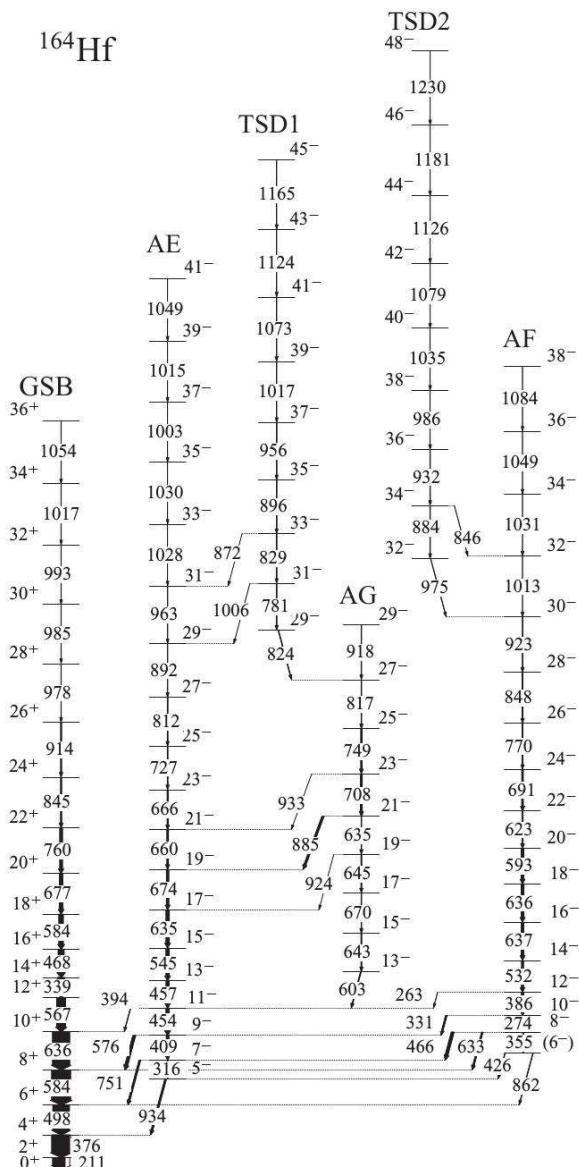


Observed bands in ^{164}Hf , TSD interpretation.



Marsh *et al.*, PRC 88, 041306 (2013):
Two triaxial superdeformed (TSD)
bands observed!
Mainly based on moment of inertia
(similar down-slope in figure above)

^{164}Hf , problems with TSD interpretation.



TSD band in ^{164}Hf predicted and searched for since long.

Problems with TSD interpretation:

- ‘Easy’ connection to normal deformed bands
- Observed too low in energy
- Signature degeneracy not explained

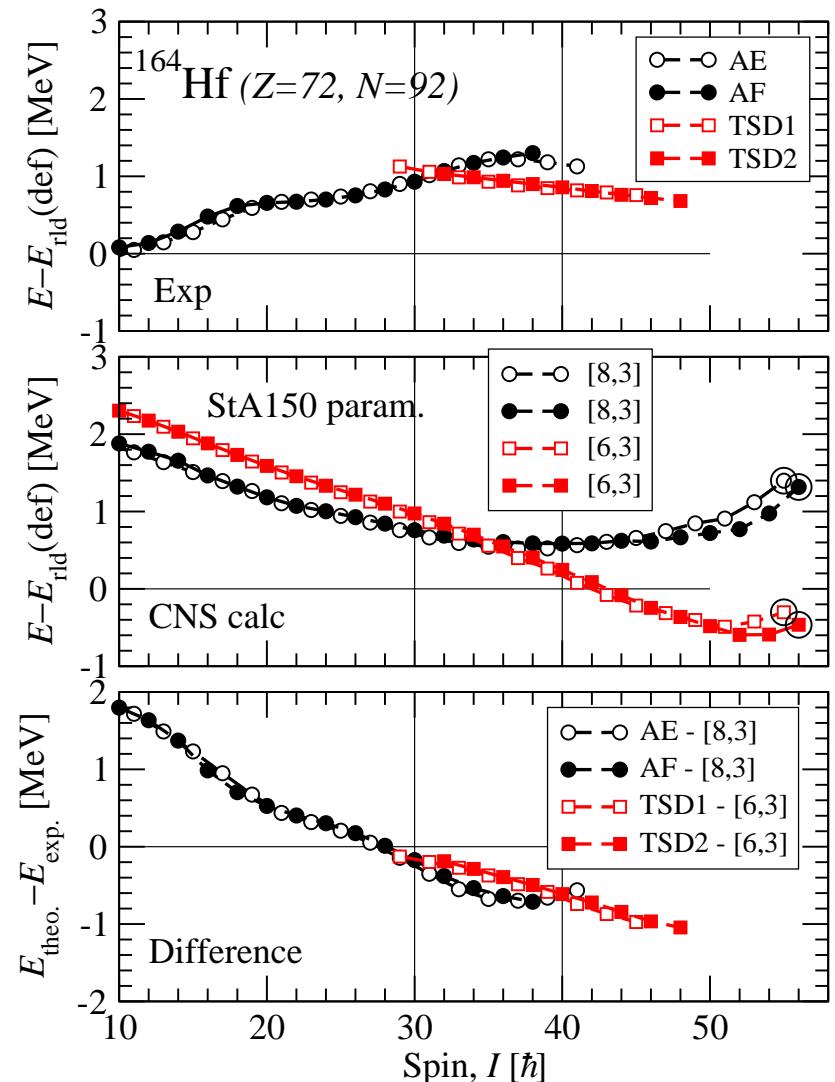
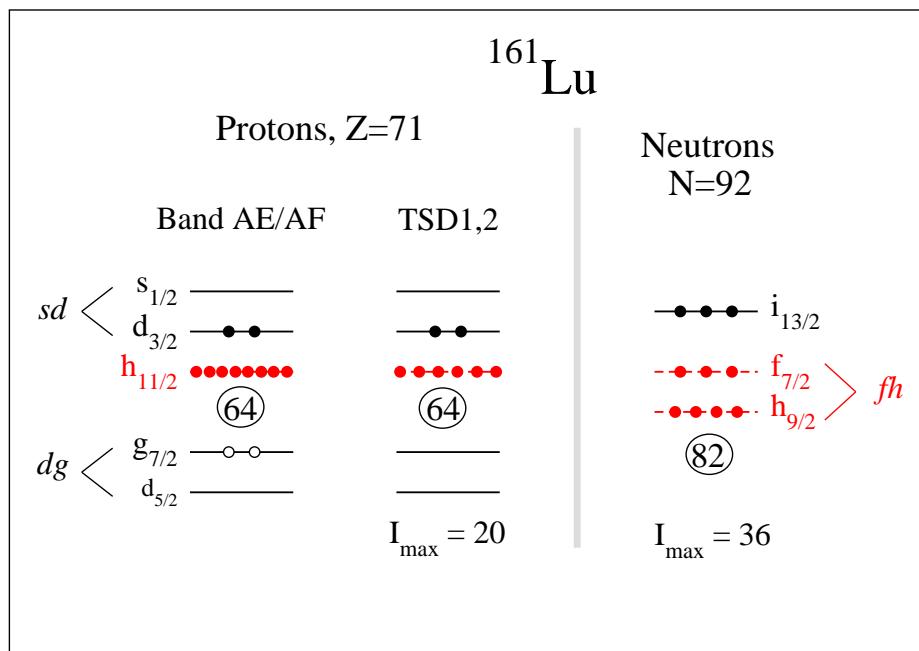
Alternative interpretation?
More interesting!

Exp. compared with CNS calc., ^{164}Hf .

Unpaired conf's for AE and AF bands:

$$\pi(h_{11/2})^6 \text{ or } \pi(h_{11/2})^8$$

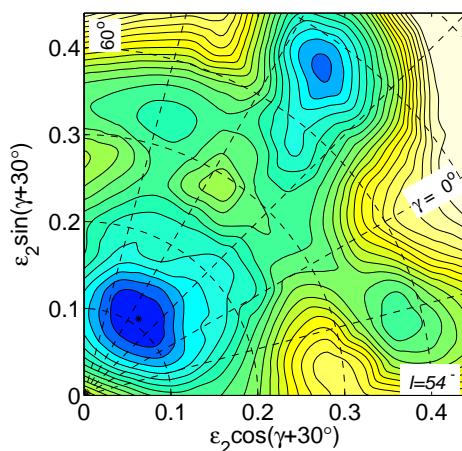
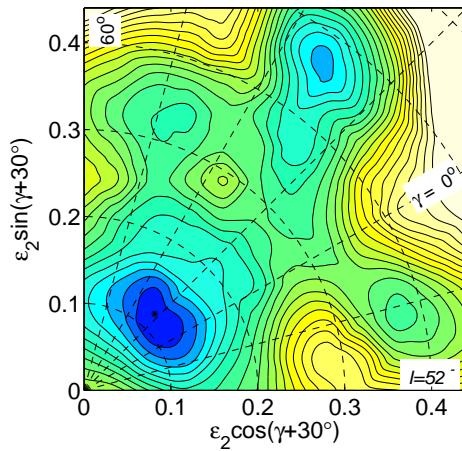
$$\nu(h_{9/2}f_{7/2})^7(i_{13/2})^3$$



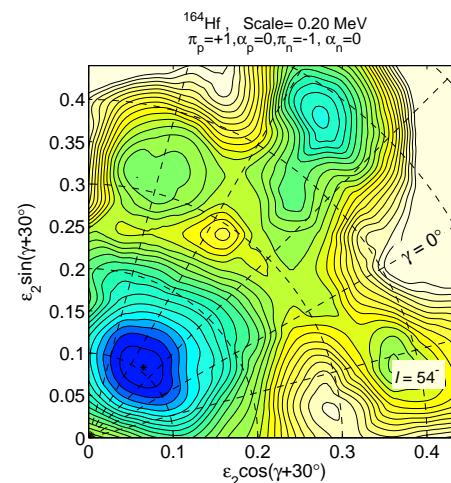
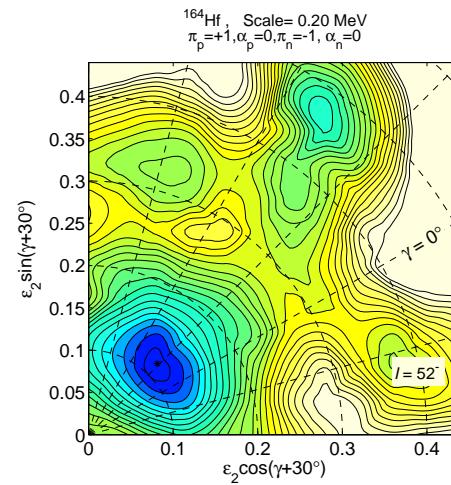
Total energy surfaces, ^{164}Hf .

$I = 52, 54$

CNS

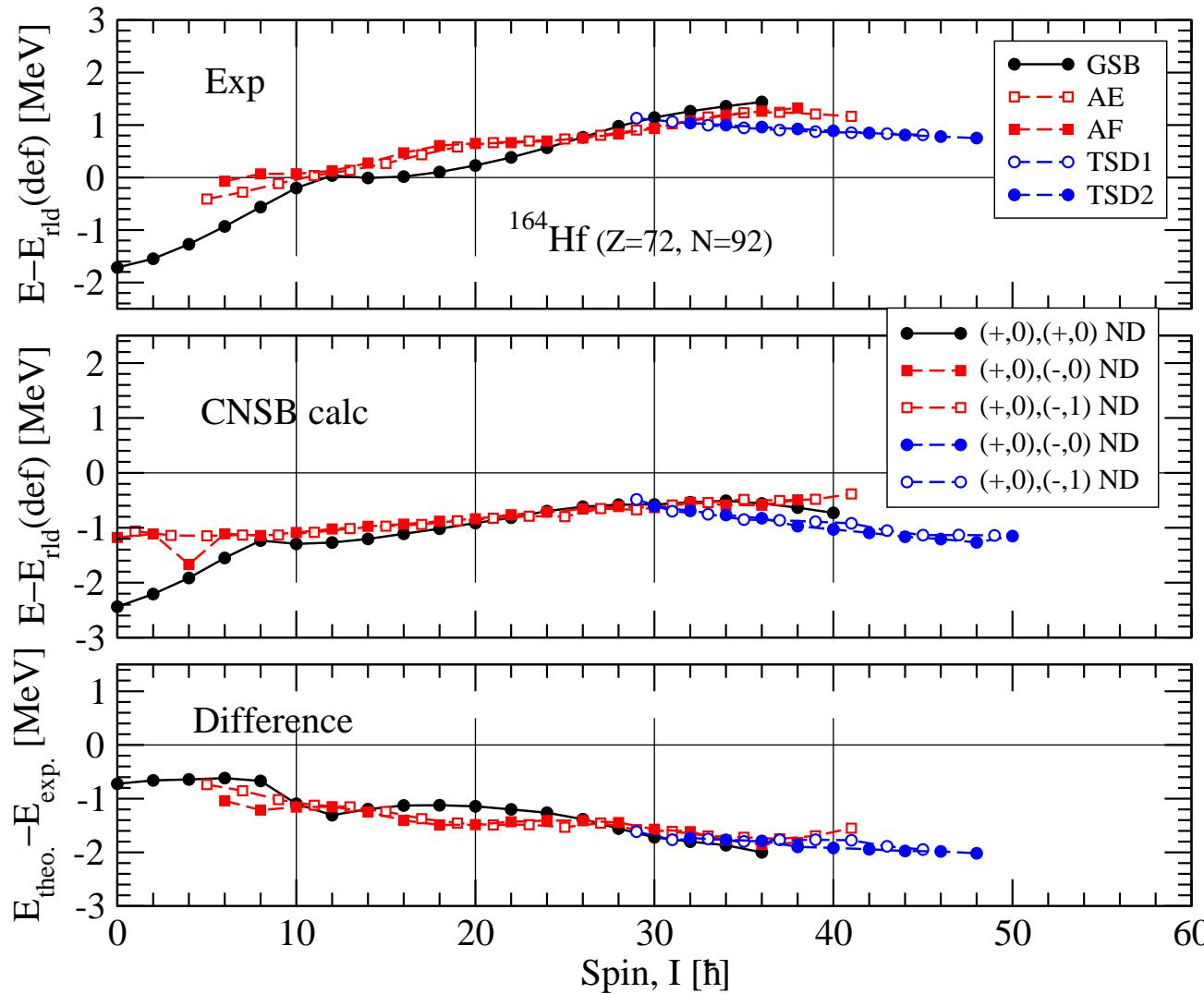


CNSB



Very similar
PES's in CNS
and CNSB.

Exp. compared with CNSB calc., ^{164}Hf .



CNSB calculation,
intermediate pa-
rameters.

Lowest and 2nd
lowest $(+, 0)(-, 0)$
and $(+, 0)(-, 1)$
bands

Summary, ^{164}Hf .

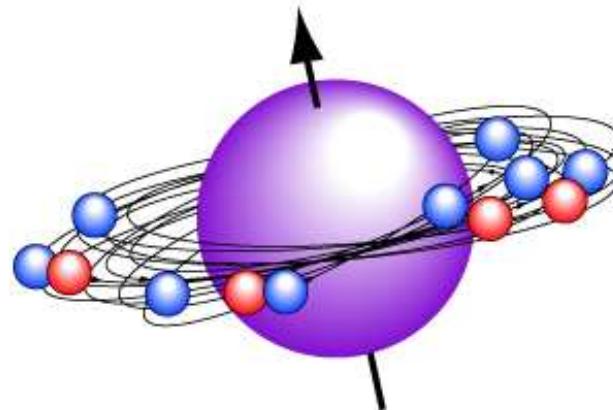
TSD1,TSD2 \rightarrow TB1, TB2

TB: terminating band

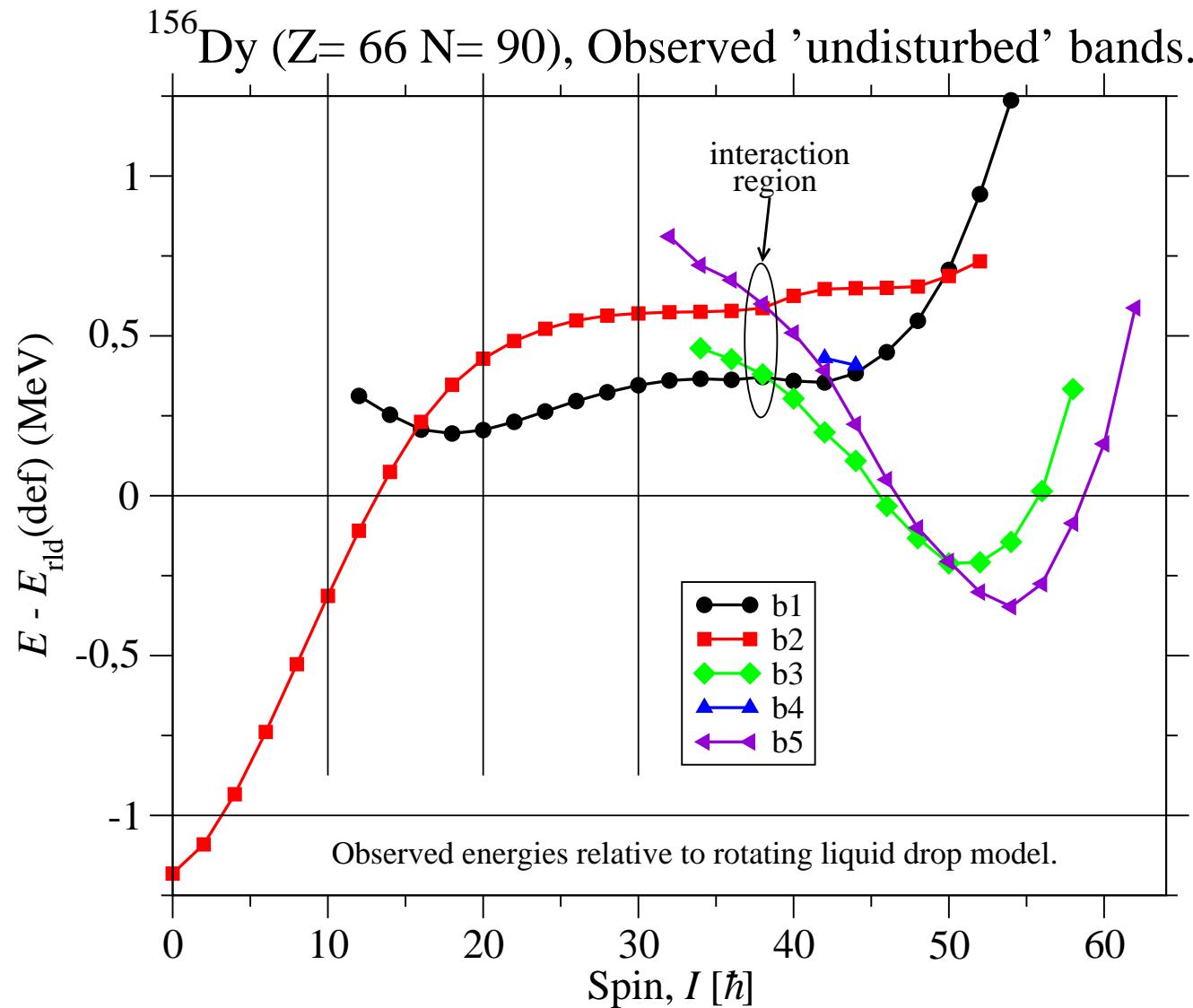
^{146}Gd core + 18 valence particles

Should be possible to observe these bands in an extensive spin range from $I \approx 30$ to full alignment at $I = 55, 56$.

(cf. ^{156}Dy with a tentative fully aligned $I = 62$ state with 18 particles + holes.)

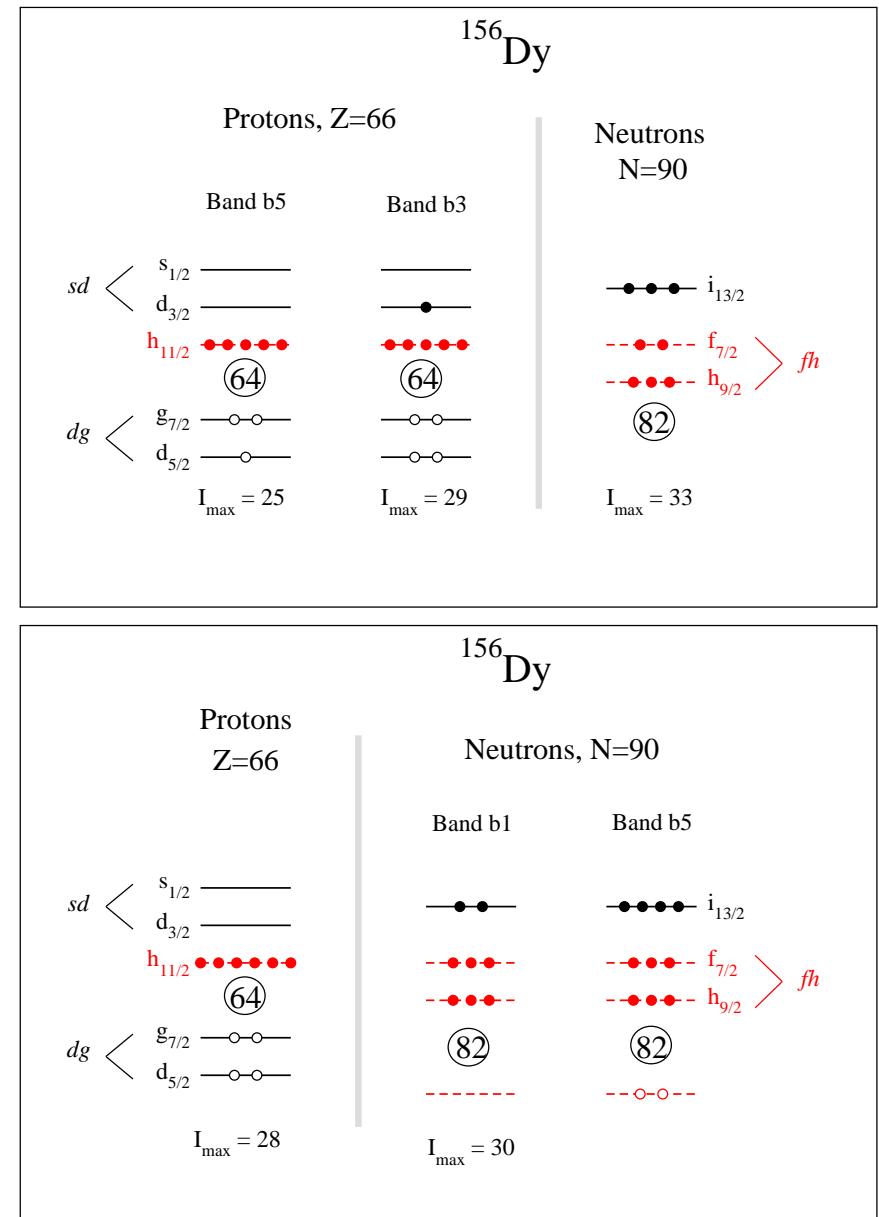
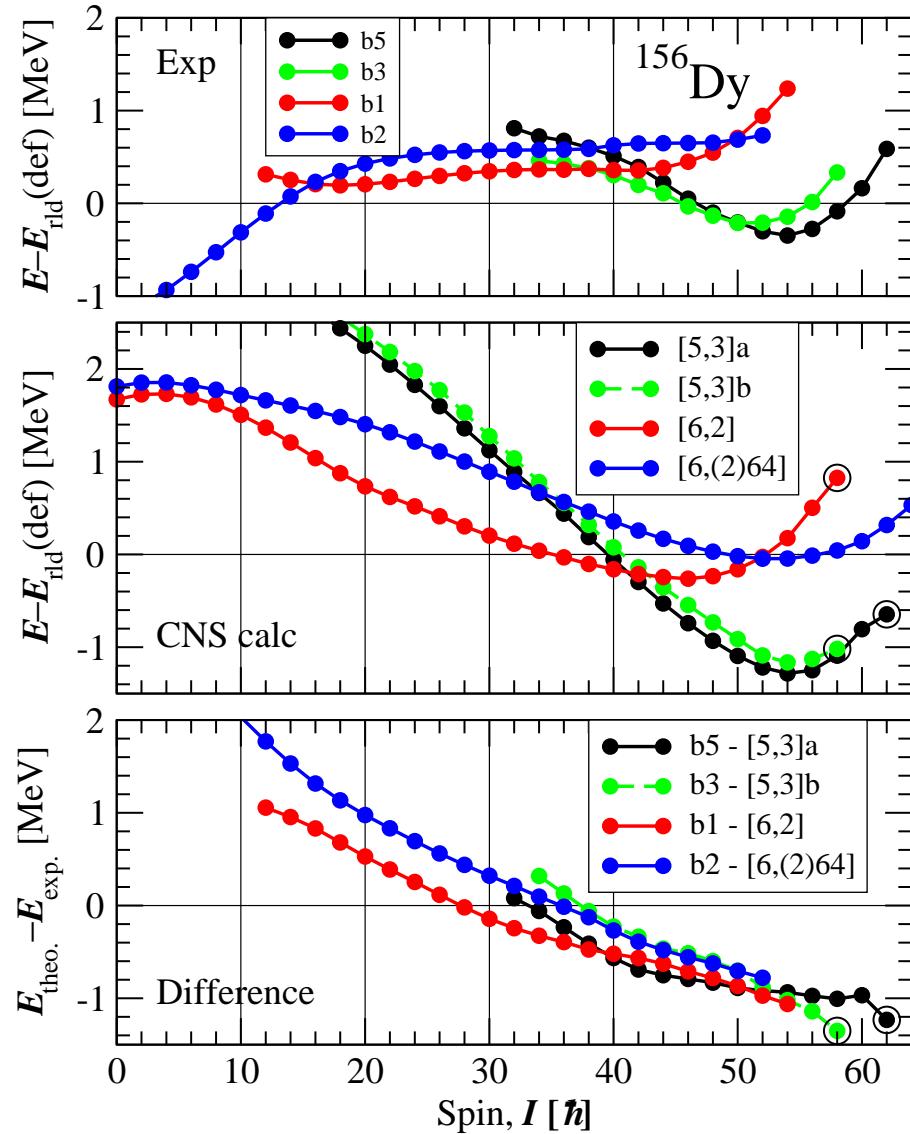


Observed energies in ^{156}Dy relative rotating liq. drop.

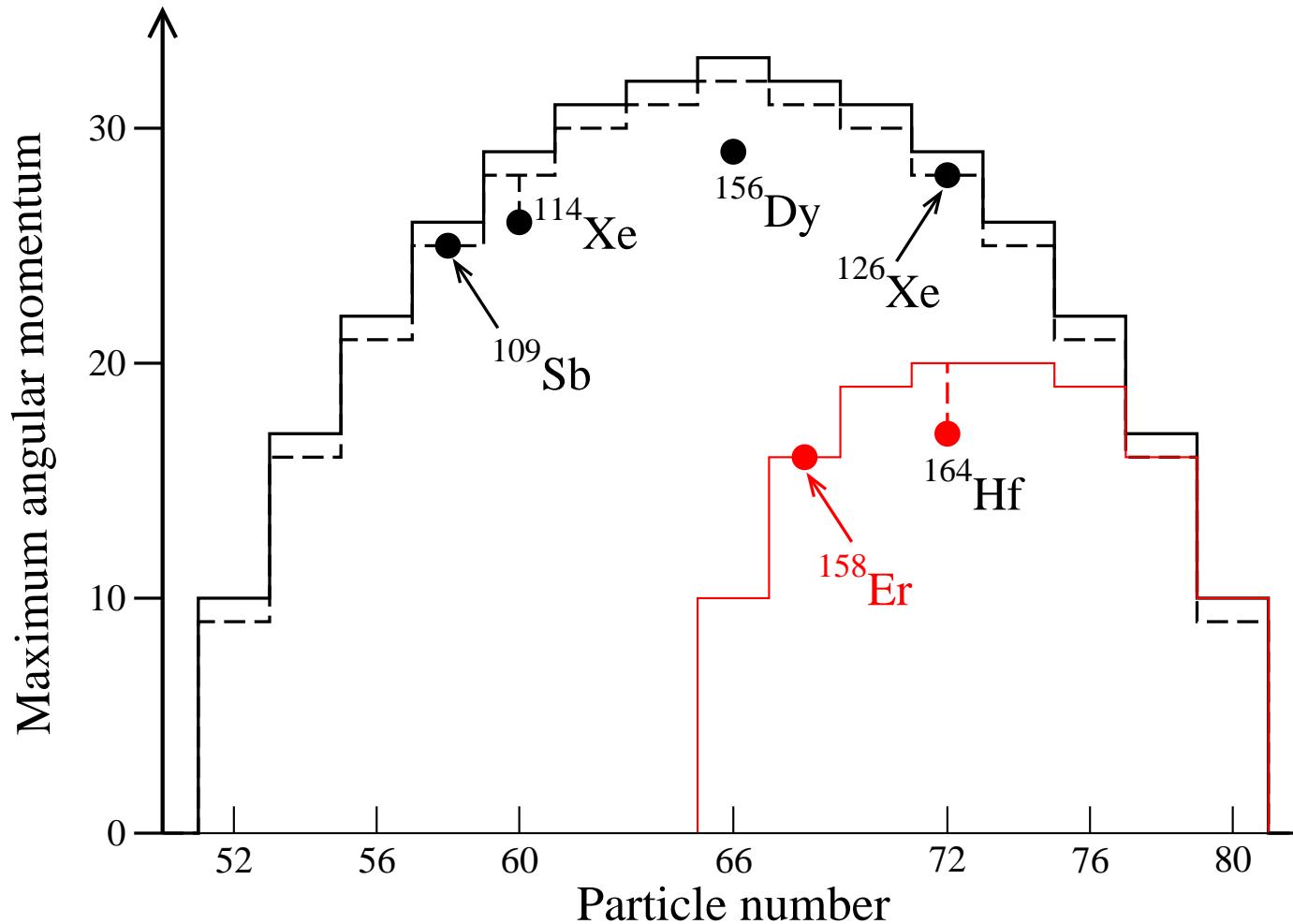


F.G. Kondev *et al.*, Phys. Lett. B437, 35 (1998).

Configuration assignments for ^{156}Dy (+, 0) bands.



Maximum spin in major shells.



Interpretation of ‘TSD’ bands in $^{163,165,167}\text{Lu}$?

Wobbling excitation observed for the TSD bands in $^{163,165,167}\text{Lu}$.

⇒ These bands are triaxial!

but

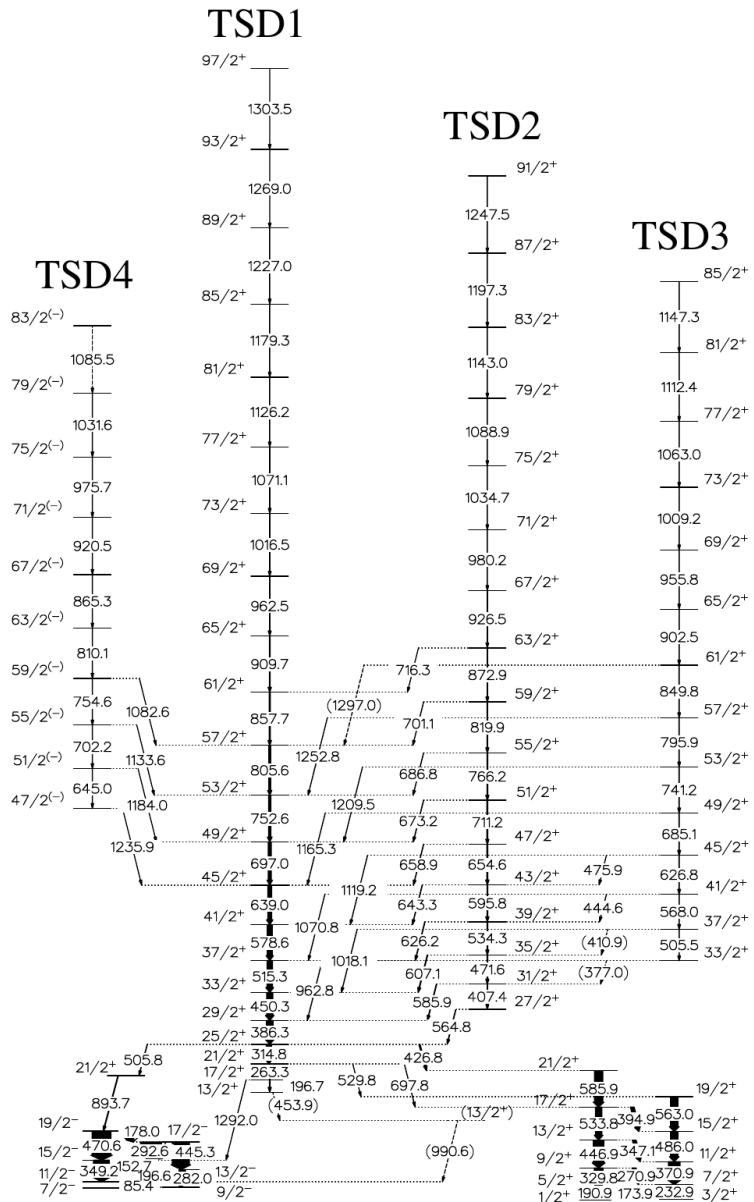
what about arguments for ‘strong deformation’ or ‘superdeformation’?
(formed in a secondary minimum at larger deformation).

Arguments for ‘superdeformation’:

- Not linked or very difficult to find linking transitions.
- Large transitional quadrupole moment Q_t .
- Large moment of inertia

????

Linking of TSD bands in ^{163}Lu .



Easy linking of 'TSD' bands!!
Also in $^{165,167}\text{Lu}$!

Transition quadrupole moments - ND

TABLE II. Quadrupole moments of ND bands based on the [523]7/2⁻ configuration in Tm, Ho, and Lu nuclei. The last column provides the reference and identifies the method used to measure the moments by the following symbols: FT-DSAM $F(\tau)$; LS-DSAM line shape; RD-recoil distance; LRIMS-laser resonance ionization; KaX-kaonic x ray; PiX-pionic x ray; MuX-muonic x ray. The error bars are statistical only and do not include the systematic uncertainty in the stopping powers. Note that for some entries, a range of values is given. The reader is referred to the cited reference for further details.

Nuclide	Band	$Q_t(e \text{ b})$	Method [Ref.]
¹⁶³ Tm	1	$6.40^{+0.57}_{-0.33}$	FT [present work]
¹⁶³ Tm	2	$6.39^{+0.33}_{-0.31}$	FT [present work]
¹⁶³ Ho	ND	6.78 ± 1.13	LRIMS [31]
¹⁶⁵ Ho	ND1	$6.42 \pm 0.15, 6.78 \pm 0.04$	KaX, PiX [32]
		6.74 ± 0.04	PiX [33]
		6.57 ± 0.06	MuX [34]
¹⁶⁵ Ho	ND2	5.76 ± 0.07	MuX [34]
¹⁶³ Lu	ND1	$4.88^{+1.36}_{-0.68} - 6.78^{+2.66}_{-1.39}$	LS + RD [30]
¹⁶³ Lu	ND2	$2.13^{+0.62}_{-0.43} - 6.72^{+0.77}_{-0.40}$	LS + RD [30]

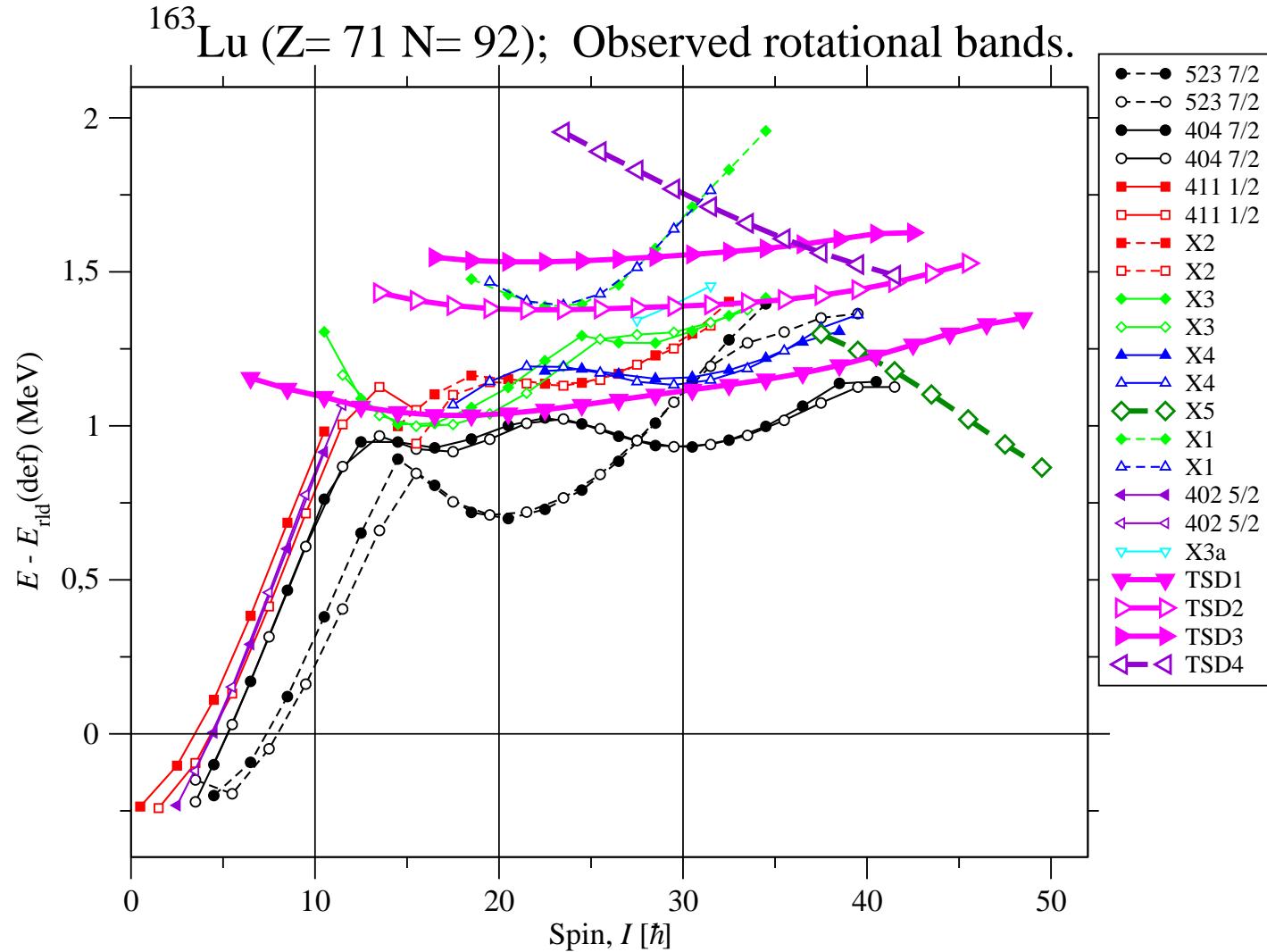
Transition quadrupole moments - SD

TABLE III. Quadrupole moments of TSD bands in Tm, Lu, and Hf nuclei. The last column provides the reference and identifies the method used to measure the moments by the following symbols: FT-DSAM $F(\tau)$; LS-DSAM line shape. The error bars are statistical only and do not include the systematic uncertainty in the stopping powers. Note that for some entries, a range of values is given. The reader is referred to the cited reference for further details.

Nuclide	Band	Q_t (e b)	Q_{sf} (e b)	Method [Ref.]
^{163}Tm	TSD1	$7.42^{+0.44}_{-0.37}$	$10.23^{+1.79}_{-1.34}$	FT [present work]
^{163}Tm	TSD2	$7.70^{+1.04}_{-0.57}$	$9.65^{+2.85}_{-2.25}$	FT [present work]
^{163}Lu	TSD1	$7.4^{+0.7}_{-0.4}, 7.7^{+2.3}_{-1.3}$ $7.63^{+1.46}_{-0.88} - 9.93^{+1.14}_{-0.99}$	$6.7^{+0.7}_{-0.7}, 7.0^{+0.7}_{-0.7}$	FT [39] LS [40]
^{163}Lu	TSD2	$6.68^{+1.70}_{-1.02} - 8.51^{+0.95}_{-0.73}$		LS [40]
^{164}Lu	TSD1	$7.4^{+2.5}_{-1.3}$	$6.7^{+0.7}_{-0.7}$	FT [39]
^{165}Lu	TSD1	$6.0^{+1.2}_{-0.2}, 6.4^{+1.9}_{-0.7}$	$5.4^{+0.5}_{-0.5}, 5.8^{+0.6}_{-0.6}$	FT [39]
^{167}Lu	TSD1	$6.9^{+0.3}_{-0.3}$	$4.4^{+0.4}_{-0.2}$	FT [41]
^{168}Hf	TSD1	$11.4^{+1.1}_{-1.2}$	$10.5^{+1.7}_{-1.6}$	FT [14]
^{174}Hf	TSD1	$13.8^{+0.3}_{-0.4}$	$8.4^{+0.3}_{-0.3}$	FT [17]
^{174}Hf	TSD2	$13.5^{+0.2}_{-0.3}$	$8.0^{+0.3}_{-0.2}$	FT [17]
^{174}Hf	TSD3	$13.0^{+0.8}_{-0.4}$	$10.3^{+0.6}_{-0.8}$	FT [17]
^{174}Hf	TSD4	$12.6^{+0.8}_{-0.8}$	$10.2^{+1.6}_{-1.3}$	FT [17]

Typical values: $A = 163 - 167$: 7 eb
 $A = 168 - 175$: 13 eb

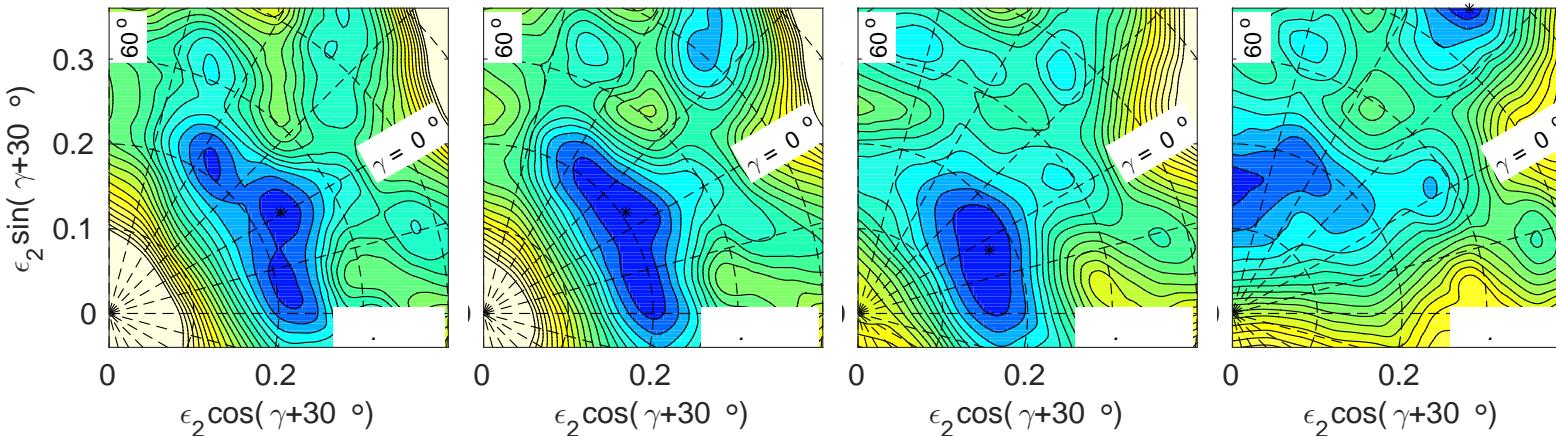
Energy systematics for observed bands in ^{163}Lu .



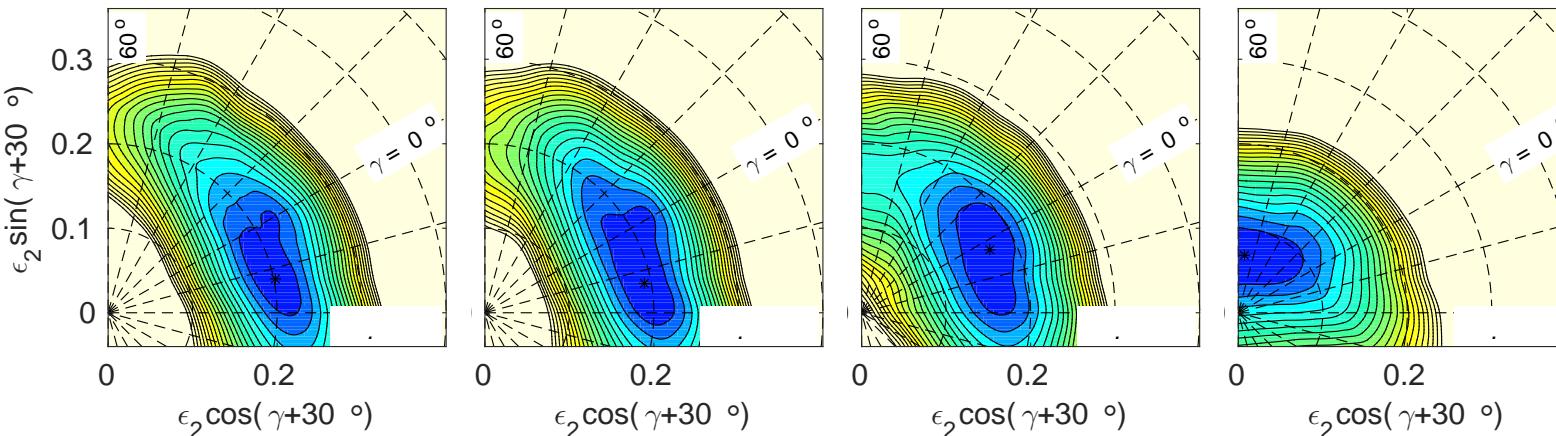
Similar moments of inertia for ND bands and TSD bands.

Total energy surfaces for ^{163}Lu .

Scan calc., $(\pi, \alpha) = (+, 1/2)$



Fixed conf. $\pi(h_{11/2})^7\nu(i_{13/2})^3$:



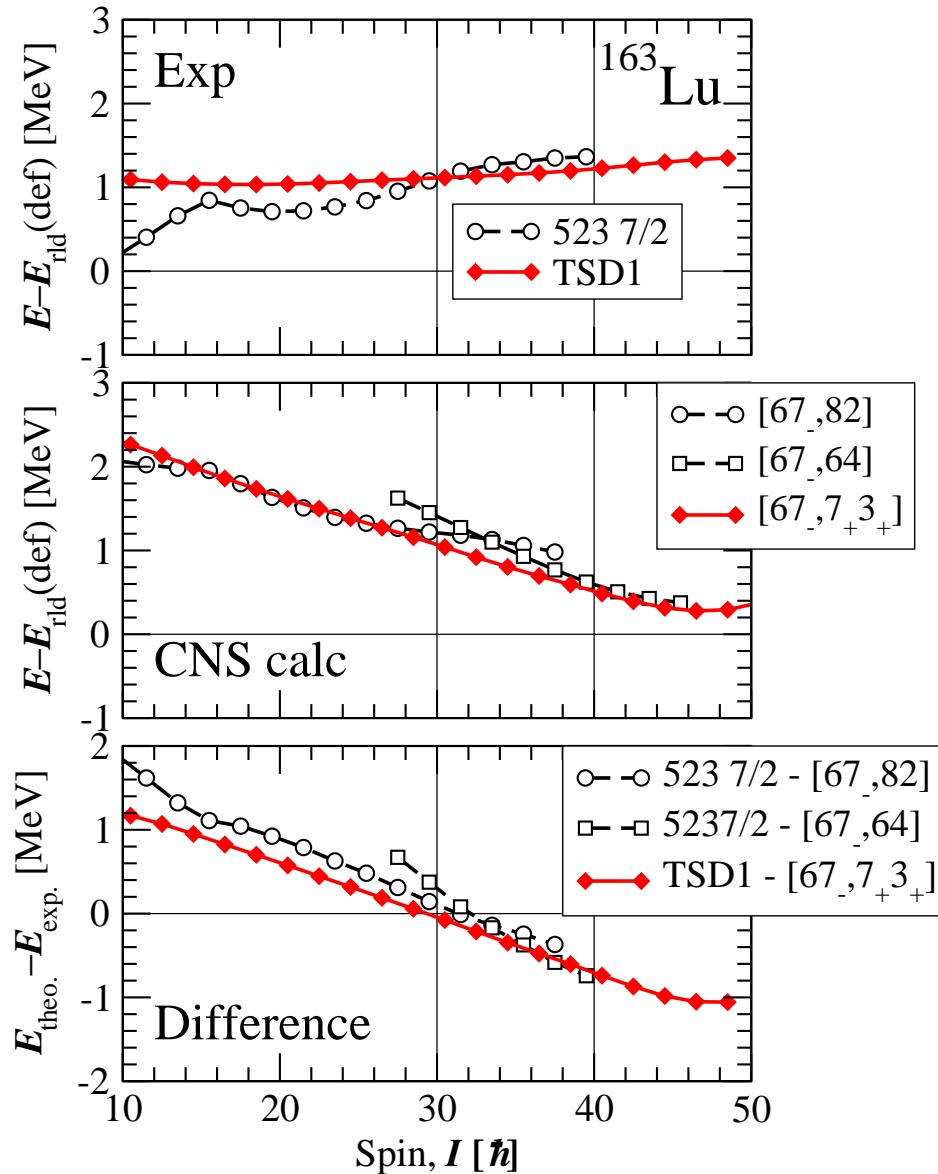
$I = 6.5$

24.5

42.5

60.5

Comparison, exp -calc; ^{163}Lu .



Differences well collected with typical slope in CNS calc's.

Summary

- CNSB - diabatic orbitals for fixed values of $(\varepsilon_2, \gamma, \varepsilon_4, \lambda_p, \lambda_n, \Delta_p, \Delta_n) \Rightarrow$ possible to get smooth energy surfaces for *fixed* spin values I in full (ε_2, γ) -plane.
- Minimization of total energy in these variables.
- Combination of CNS and CNSB powerful method to analyze high-spin data.
- TSD bands reinterpreted as TB's in ^{161}Lu and ^{164}Hf .
- Experiments are coming close to the maximum spin which can be formed from the particle in the $Z, N = 50 - 82$ shell.
- The 'TSD' bands in $^{163,165,167}\text{Lu}$ appear not to be strongly deformed - possible reinterpretation.