Descriptions of triaxial band structures in ^{133}La

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Reference:

Triaxial-band structures, chirality and magnetic rotation in ^{133}La

C. M. Petrache,[1] Q. B. Chen,[2] S. Guo,[1,*] A. D. Ayangeakaa,[3,†] U. Garg,[3] J. T. Matta,[3,‡] B. K. Nayak,[3,§] D. Patel,[3,¶] J. Meng,[2] M. P. Carpenter,[4] C. J. Chiara,[4,5,**] R. V. F. Janssens,[4] F. G. Kondev,[6] T. Lauritsen,[4] D. Seweryniak,[4] S. Zhu,[4] S. S. Ghugre,[7] and R. Palit[8,9]

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Outline

- Introduction
- Theoretical framework
- Results and discussion
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Chirality

- **The investigation of chirality in atomic nuclei is one of the hottest topics in nuclear physics.**

ELSEVIER

Nuclear Physics A 617 (1997) 131–147

NUCLEAR PHYSICS A

Tilted rotation of triaxial nuclei

S. Frauendorf, Jie Meng [1]

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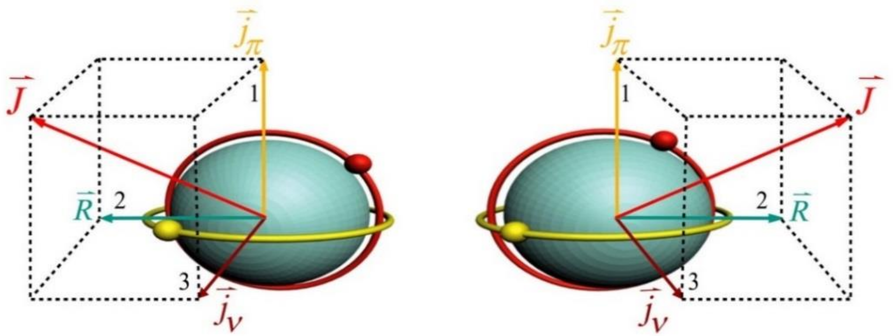
Received 14 November 1996

Originally suggested in 1997 chiral doublet bands



VOLUME 86, NUMBER 6 PHYSICAL REVIEW LETTERS 5 FEBRUARY 2001

Chiral Doublet Structures in Odd-Odd $N = 75$ Isotones: Chiral Vibrations

K. Starosta,[1,*] T. Koike,[1] C. J. Chiara,[1] D. B. Fossan,[1] D. R. LaFosse,[1] A. A. Hecht,[2] C. W. Beausang,[2] M. A. Caprio,[2] J. R. Cooper,[2] R. Krücken,[2] J. R. Novak,[2] N. V. Zamfir,[2,†] K. E. Zyromski,[2] D. J. Hartley,[3] D. L. Balabanski,[3,‡] Jing-ye Zhang,[3] S. Frauendorf,[4] and V. I. Dimitrov[4,‡]

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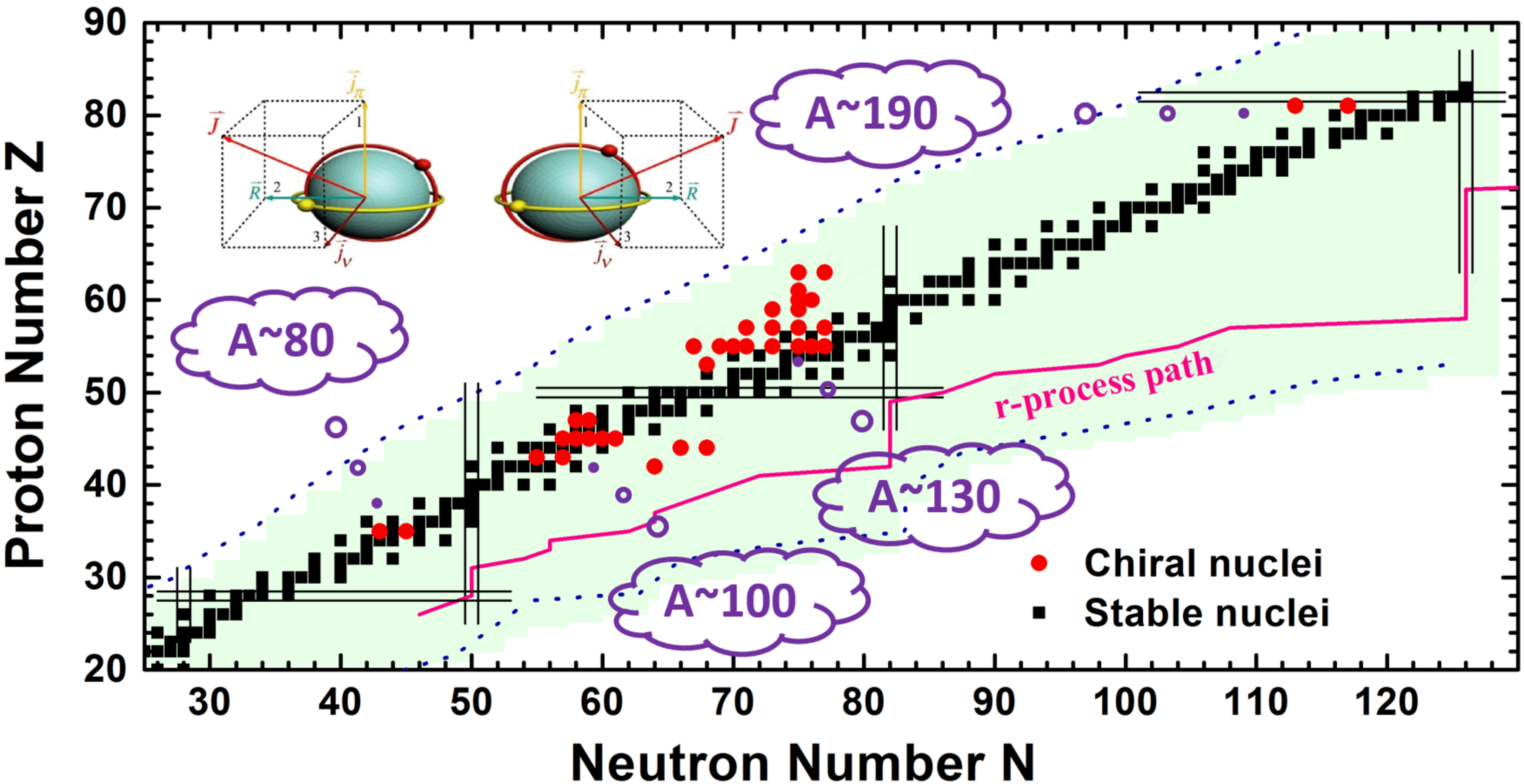
[4]Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556 and Institute for Nuclear and Hadronic Physics, Research Center Rossendorf, 01314 Dresden, Germany

(Received 24 July 2000)

Identified in 2001

Experimental progress

- **So far, more than 40 chiral doublet bands have been found in the A~80, 100, 130, and 190 mass regions.** *Meng&Zhang, JPG 37, 064025 (2010); Meng&Chen&Zhang, IJMPE 23, 1430016 (2014); Meng&Zhao, PS 91, 053008 (2016)*



Experimental progress

- **In the past three years, 6 papers were published at PRL!**

PRL **110,** 172504 (2013) PHYSICAL REVIEW LETTERS week ending 26 APRIL 2013

Evidence for Multiple Chiral Doublet Bands in ^{133}Ce

A. D. Ayangeakaa,[1] U. Garg,[1] M. D. Anthony,[1] S. Frauendorf,[1] J. T. Matta,[1] B. K. Nayak,[1,*] D. Patel,[1] Q. B. Chen (陈启博),[2] S. Q. Zhang (张双全),[2] P. W. Zhao (赵鹏巍),[2] B. Qi (亓斌),[3] J. Meng (孟杰),[2,4,5] R. V. F. Janssens,[6] M. P. Carpenter,[6] C. J. Chiara,[6,7] F. G. Kondev,[8] T. Lauritsen,[6] D. Seweryniak,[6] S. Zhu,[6] S. S. Ghugre,[9] and R. Palit[10,11]

2013-04-24

PRL **112,** 052501 (2014) PHYSICAL REVIEW LETTERS week ending 7 FEBRUARY 2014

Candidates for Twin Chiral Bands in ^{102}Rh

D. Tonev,[1] M. S. Yavahchova,[1] N. Goutev,[1] G. de Angelis,[2] P. Petkov,[1] R. K. Bhowmik,[3] R. P. Singh,[3] S. Muralithar,[3] N. Madhavan,[3] R. Kumar,[3] M. Kumar Raju,[4] J. Kaur,[5] G. Mohanto,[3] A. Singh,[5] N. Kaur,[5] R. Garg,[6] A. Shukla,[7] Ts. K. Marinov,[1] and S. Brant[8]

2014-02-03

PRL **113,** 032501 (2014) PHYSICAL REVIEW LETTERS week ending 18 JULY 2014

Multiple Chiral Doublet Bands of Identical Configuration in ^{103}Rh

I. Kuti,[1] Q. B. Chen,[2] J. Timár,[1] D. Sohler,[1] S. Q. Zhang,[2] Z. H. Zhang,[2] P. W. Zhao,[2] J. Meng,[2] K. Starosta,[3] T. Koike,[4] E. S. Paul,[5] D. B. Fossan,[6] and C. Vaman[6]

2014-07-14

PRL **112,** 202502 (2014) PHYSICAL REVIEW LETTERS week ending 23 MAY 2014

Resolution of Chiral Conundrum in ^{106}Ag: Doppler-Shift Lifetime Investigation

E. O. Lieder,[1,2] R. M. Lieder,[1,*] R. A. Bark,[1] Q. B. Chen,[3] S. Q. Zhang,[3] J. Meng,[3,4,5] E. A. Lawrie,[1] J. J. Lawrie,[1] S. P. Bvumbi,[1] N. Y. Kheswa,[1] S. S. Ntshangase,[1] T. E. Madiba,[1] P. L. Masiteng,[1] S. M. Mullins,[1] S. Murray,[1] P. Papka,[1] D. G. Roux,[6] O. Shirinda,[1] Z. H. Zhang,[3] P. W. Zhao,[3] Z. P. Li,[7] J. Peng,[8] B. Qi,[9] S. Y. Wang,[9] Z. G. Xiao,[10,11] and C. Xu[3]

2014-05-20

PRL **116,** 112501 (2016) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2016

Evidence for Octupole Correlations in Multiple Chiral Doublet Bands

C. Liu (刘晨),[1] S. Y. Wang (王守宇),[1,†] R. A. Bark,[2] S. Q. Zhang (张双全),[3,‡] J. Meng (孟杰),[3,4,5,§] B. Qi (亓斌),[1] P. Jones,[2] S. M. Wyngaardt,[5] J. Zhao (赵杰),[6,7] C. Xu (徐川),[3] S.-G. Zhou (周善贵),[6] S. Wang (王硕),[1] D. P. Sun (孙大鹏),[1] L. Liu (刘雷),[1] Z. Q. Li (李志泉),[1] N. B. Zhang (张乃波),[1] H. Jia (贾慧),[1] X. Q. Li (李湘庆),[3] H. Hua (华辉),[3] Q. B. Chen (陈启博),[3] Z. G. Xiao (肖志刚),[8,9] H. J. Li (李红洁),[8] L. H. Zhu (竺礼华),[4] T. D. Bucher,[2,5] T. Dinoko,[2,10] J. Easton,[2,10] K. Juhász,[11,*] A. Kamblawe,[2,5] E. Khaleel,[2,5] N. Khumalo,[2,10,12] E. A. Lawrie,[2] J. J. Lawrie,[2] S. N. T. Majola,[2,13] S. M. Mullins,[2] S. Murray,[2] J. Ndayishimye,[2,5] D. Negi,[2] S. P. Noncolela,[2,10] S. S. Ntshangase,[12] B. M. Nyakó,[14] J. N. Orce,[10] P. Papka,[2,5] J. F. Sharpey-Schafer,[2,10] O. Shirinda,[2] P. Sithole,[2,10] M. A. Stankiewicz,[2,13] and M. Wiedeking[2]

2016-03-14

PRL **112,** 202503 (2014) PHYSICAL REVIEW LETTERS week ending 23 MAY 2014

Exploring the Origin of Nearly Degenerate Doublet Bands in ^{106}Ag

N. Rather,[1] P. Datta,[2,*] S. Chattopadhyay,[1] S. Rajbanshi,[1] A. Goswami,[1] G. H. Bhat,[3] J. A. Sheikh,[3] S. Roy,[4] R. Palit,[4] S. Pal,[4] S. Saha,[4] J. Sethi,[4] S. Biswas,[4] P. Singh,[4] and H. C. Jain[4]

2014-05-20

- **Searching for more chiral candidates and novel phenomena are still hot topics.**

In this talk, new experimental results on ^{133}La will be reported.

Theoretical progress

- **Tilted axis cranking (TAC)**

See Professor Meng's talk for details

  - **Single-j model** *Frauendorf_Meng1997NPA;*
  - **Hybird Woods-Saxon and Nilsson model** *Dimitrov2000PRL*
  - **Skyrme Hartree-Fock model** *Olbratorwski2004PRL, 2006PRC*
  - **Covariant density functional theory (CDFT)** *Madokoro2000PRC*

    - TAC + RPA *Mukhopadhyay2007PRL; Almehed2011PRC*
    - TAC + Collective Hamiltonian *Chen2013PRC, 2016PRC*

- **Triaxial PRM**

  - **One-particle-one-hole PRM** *Frauendorf_Meng 1997NPA; Koike2004PRL; Peng2003PRC; Qi2009PRC, CPL2010; Chen2010PRC; Zhang2016CPC*
  - **Two quasiparticles PRM** *Starosta2002PRC; Koike2003PRC; Zhang2007PRC; Wang2007PRC, 2008PRC, 2010PRC; Qi2011CPL; Lawrie 2008PRC, 2010PLB;*
  - **n-particle-n-hole PRM** *Qi2009PLB, 2011PRC;*

    - PRM + CDFT *Ayangeakaa2013PRL; Lieder2014PRL; Kuti2014PRL; Liu2016PRL*

In this talk, the PRM+CDFT will be applied to describe the newly observed triaxial band structure in ^{133}La.

Outline

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Particle rotor model

- **PRM Hamiltonian:**

$$\hat{H}_{\mathrm{PRM}} = \hat{H}_{\mathrm{coll}} + \hat{H}_{\mathrm{intr}}$$

  - **Collective part:**

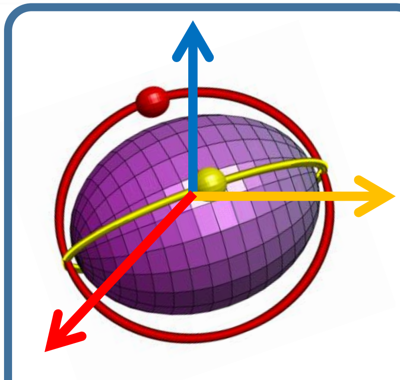
$$\hat{H}_{\mathrm{coll}} = \sum_i \frac{\hat{R}_i^2}{2\mathcal{J}_i} = \sum_i \frac{(\hat{I}_i - \hat{j}_i)^2}{2\mathcal{J}_i}$$

  - **Intrinsic part:**

$$\hat{H}_{\mathrm{intr}}^{p(n)} = \sum_\nu \varepsilon_{p,\nu} a_{p,\nu}^\dagger a_{p,\nu} + \sum_{\nu'} \varepsilon_{n,\nu'} a_{n,\nu'}^\dagger a_{n,\nu'}$$

$$h_{\mathrm{sp}} = \pm \frac{1}{2} C \left\{ \cos\gamma \left(\hat{j}_3^2 - \frac{j(j+1)}{3}\right) + \frac{\sin\gamma}{2\sqrt{3}} (\hat{j}_+^2 + \hat{j}_-^2) \right\}$$

(β, γ) by CDFT



- **EM transitions:**

$$B(\sigma\lambda, I \to I') = \sum_{\mu M'} \left| \langle I'M' | \mathcal{M}_{\lambda\mu}^\sigma | IM \rangle \right|^2$$

  - **E2 operator:**

$$\mathcal{M}(E2, \mu) = \sqrt{5/16\pi}\, \hat{Q}_{2\mu}$$

  - **M1 operator:**

$$\mathcal{M}(M1, \mu) = \frac{3}{4\pi} \frac{e\hbar}{2Mc} [(g_p - g_R)\hat{j}_{p\mu} + (g_n - g_R)\hat{j}_{n\mu}]$$

- **Angular momentum:**

$$R_k = \sqrt{\langle \hat{R}_k^2 \rangle}, \quad J_{pk} = \sqrt{\langle \hat{j}_{pk}^2 \rangle}, \quad J_{nk} = \sqrt{\langle \hat{j}_{nk}^2 \rangle}$$

See also Professor Starosta's talk

Covariant density functional theory

- **Energy density functional (*point-coupling*):** See also Professor Vretenar's talk

Dirac eq. $[i\gamma_\mu(i\partial^\mu - V^\mu) - (m+S)]\psi_k = 0$

$$E_{\rm DF} = \int d^3r\,\mathcal{E}(\boldsymbol{r})$$

$$= \int d^3r \sum_k v_k^2 \psi_k^\dagger(\boldsymbol{r})(-i\boldsymbol{\alpha}\cdot\boldsymbol{p} + m)\psi_k(\boldsymbol{r}) + \int d^3r \Big(\frac{\alpha_S}{2}\rho_S^2 + \frac{\beta_S}{3}\rho_S^3 + \frac{\gamma_S}{4}\rho_S^4 + \frac{\delta_S}{2}\rho_S\Delta\rho_S + \frac{\alpha_V}{2} j_\mu j^\mu$$

$$+ \frac{\gamma_V}{4}(j_\mu j^\mu)^2 + \frac{\delta_V}{2} j_\mu \Delta j^\mu + \frac{\alpha_{TV}}{2}\vec{j}^{\mu}_{TV}(\vec{j}_{TV})_\mu + \frac{\delta_{TV}}{2}\vec{j}^{\mu}_{TV}\Delta(\vec{j}_{TV})_\mu + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - F^{0\mu}\partial_0 A_\mu + e A_\mu j_p^\mu \Big)$$

- **Local densities (ρ) & currents (j); *S*: scalar; *V*: vector; *T*: isovector**
- ***α*, *β*, *γ*, *δ* denote ~11 parameters, like: PC-F1, DD-PC1, PC-PK1, ……**

Burvenich2002PRC, Niksic2008PRC, Zhao2010PRC

- **Constraint calculation on quadrupole moment:**

- **(β, γ) constraint:** $\delta\Big[\langle \hat{H}\rangle + \sum_{\mu=0,2} C_{2\mu}(\langle \hat{Q}_{2\mu}\rangle - q_{2\mu})^2\Big] = 0$
- **β^2 constraint:** $\delta\Big[\langle \hat{H}\rangle + C((\langle \hat{Q}_{20}\rangle)^2 + 2\langle \hat{Q}_{22}\rangle^2 - \mu)^2\Big] = 0$

$$q_{20} = \sqrt{5/16\pi}\langle \hat{Q}_{20}\rangle$$
$$q_{22} = \sqrt{15/32\pi}\langle \hat{Q}_{22}\rangle$$
$$\hat{Q}_{20} = 2z^2 - x^2 - y^2$$
$$\hat{Q}_{22} = x^2 - y^2$$

$$\beta = \frac{4\pi}{3AR_0^2}\sqrt{q_{20}^2 + 2q_{22}^2}, \quad \gamma = \arctan\left(\frac{\sqrt{2}q_{22}}{q_{20}}\right)$$

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Results and discussion

Experimental information

The experiment was performed by our experimental collaborators

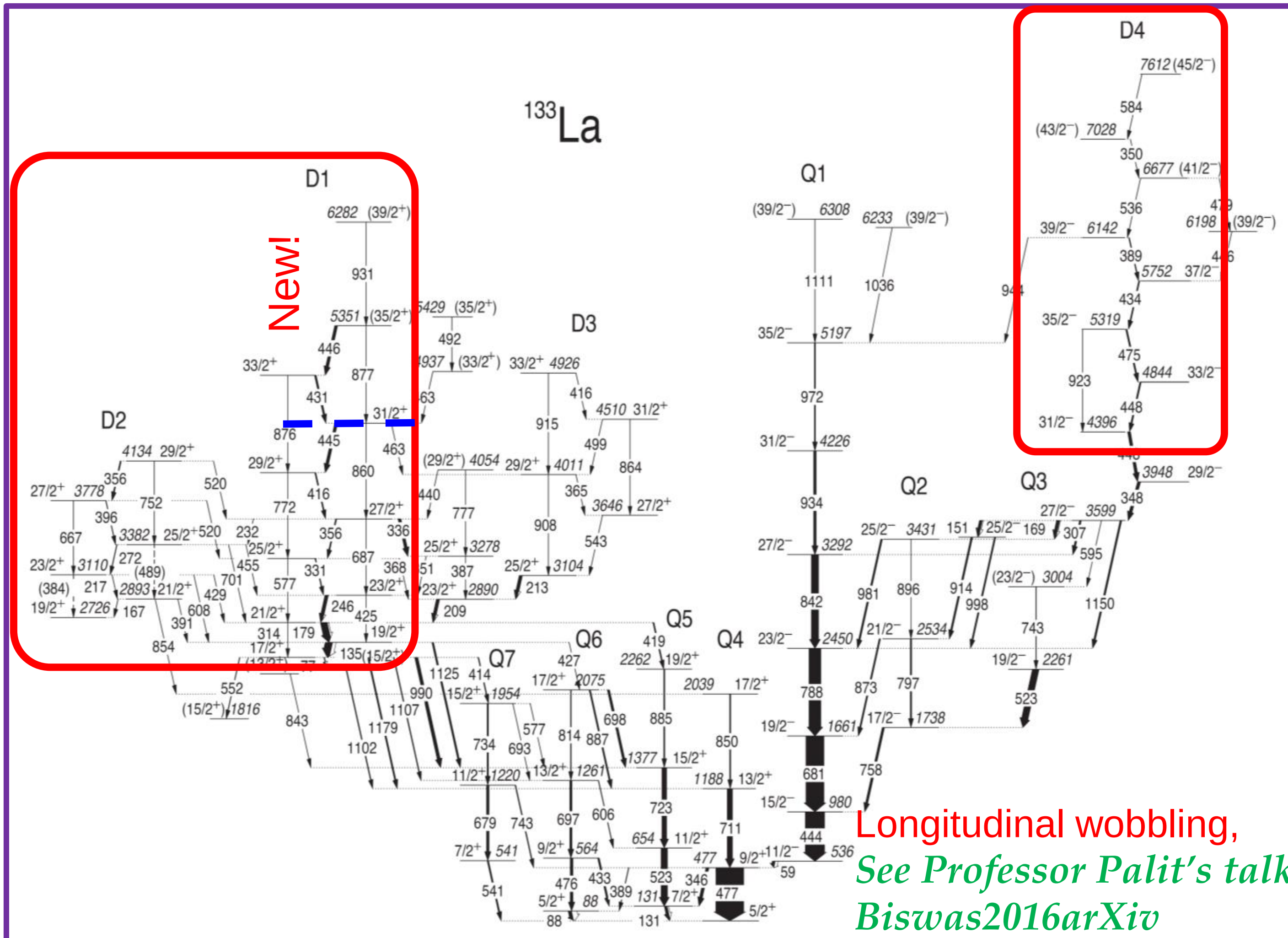
- **Facility:** Two separate experiments were performed using the ATLAS superconducting linear accelerator facility at Argonne National Laboratory (Nov. 2008, Aug. 2011).
- **Reaction:** ^{116}Cd(^{22}Ne, p4n) with a bombarding energy of 112 MeV.
- **Target:** (1) a foil of isotopically enriched ^{116}Cd, sandwiched between a 50 μg/cm^2 thick front layer of Al and a 150 μg/cm^2 Au backing; (2) a target of the same enrichment and thickness but evaporated onto a 55 μg/cm^2 thick Au foil.
- **Detector:** Gammasphere array, 101 (88) HPGe detectors.
- **Event:** A combined total of approximately 4×10^9 four- and higher-fold coincidence events were accumulated during the two experiments.

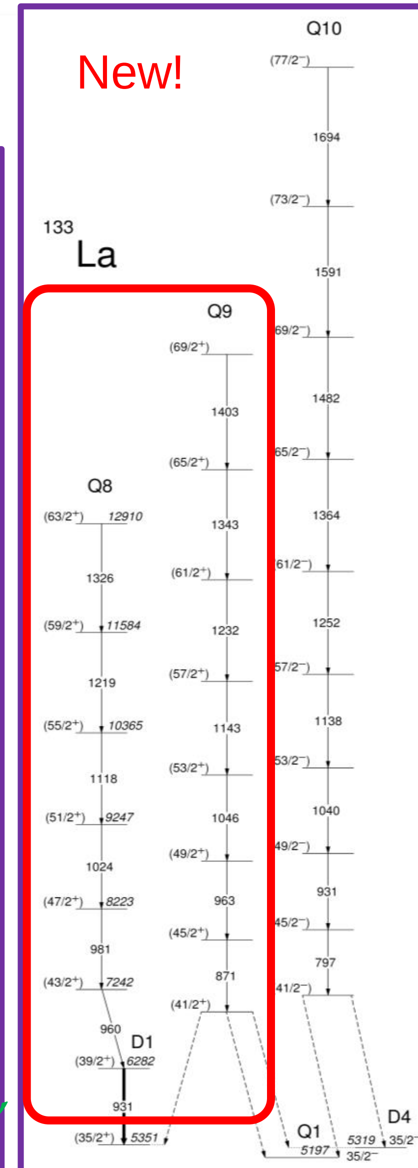
See details also at

Ayangeakaa2013PRL, 2016PRC (^{133}Ce), *Petrache2016PRC* (^{134}Ce)

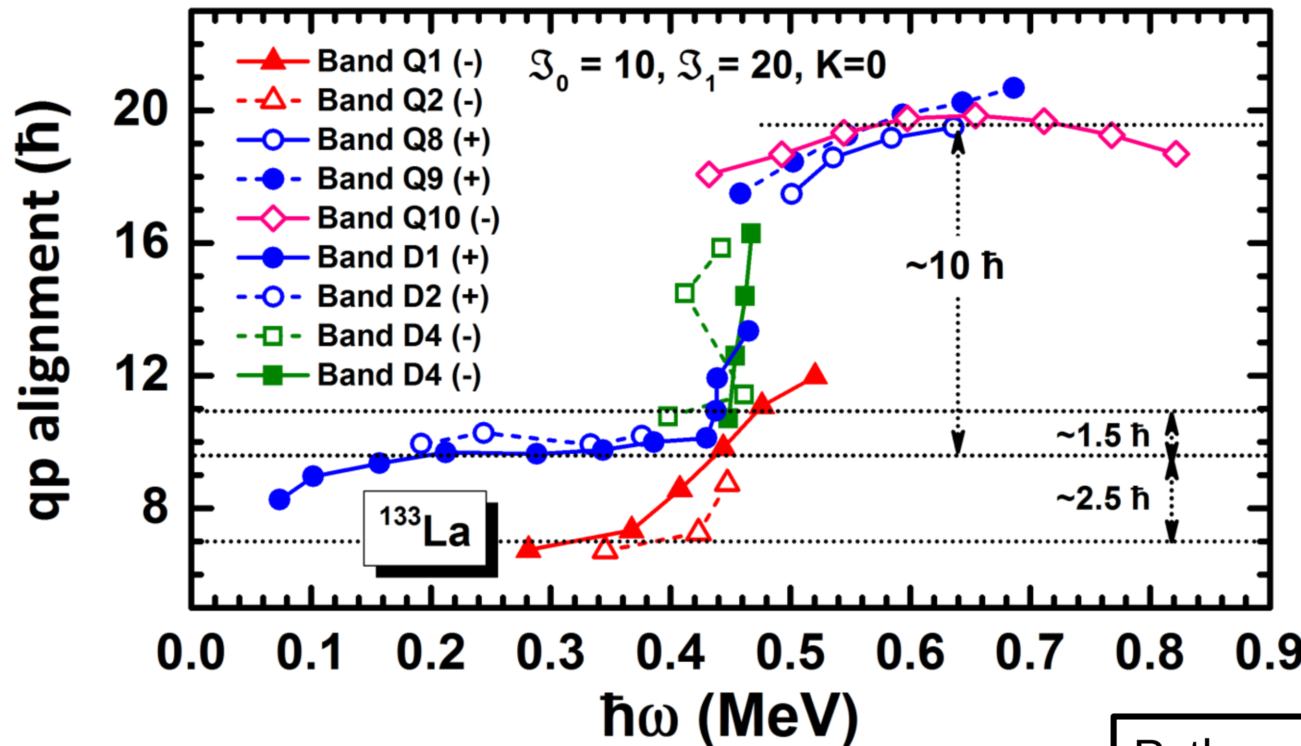
Level scheme

- **Level scheme of ^{133}La**





Quasiparticle alignments



Pythagorean theorem

- **Quasiparticle alignments**
  - **Q1, Q2: ~7ℏ,** $\pi(1h_{11/2})^1$ Longitudinal wobbling band, *Biswas2016arXiv*
  - **D1, D2: ~10ℏ,** $\pi(d_{5/2}, g_{7/2})^1 \otimes \nu(1h_{11/2})^{-2}$ $\sqrt{5/2^2 + (11/2 + 9/2)^2} \approx 10.3\hbar$
  - **Q8, Q9: ~20ℏ,** $\pi(d_{5/2}, g_{7/2})^1 (1h_{11/2})^2 \otimes \nu(1h_{11/2})^{-2}$ $5/2 + 10 + 10 \approx 22.5\hbar$
  - **D4: ~11ℏ,** $\pi(h_{11/2})^1 \otimes \nu(1h_{11/2})^{-2}$ Low $\sqrt{11/2^2 + 10^2} \approx 11.4\hbar$

    $\pi(h_{11/2})^3 \otimes \nu(1h_{11/2})^{-2}$ High $\sqrt{(11/2 + 9/2 + 7/2)^2 + 10^2} \approx 16.8\hbar$

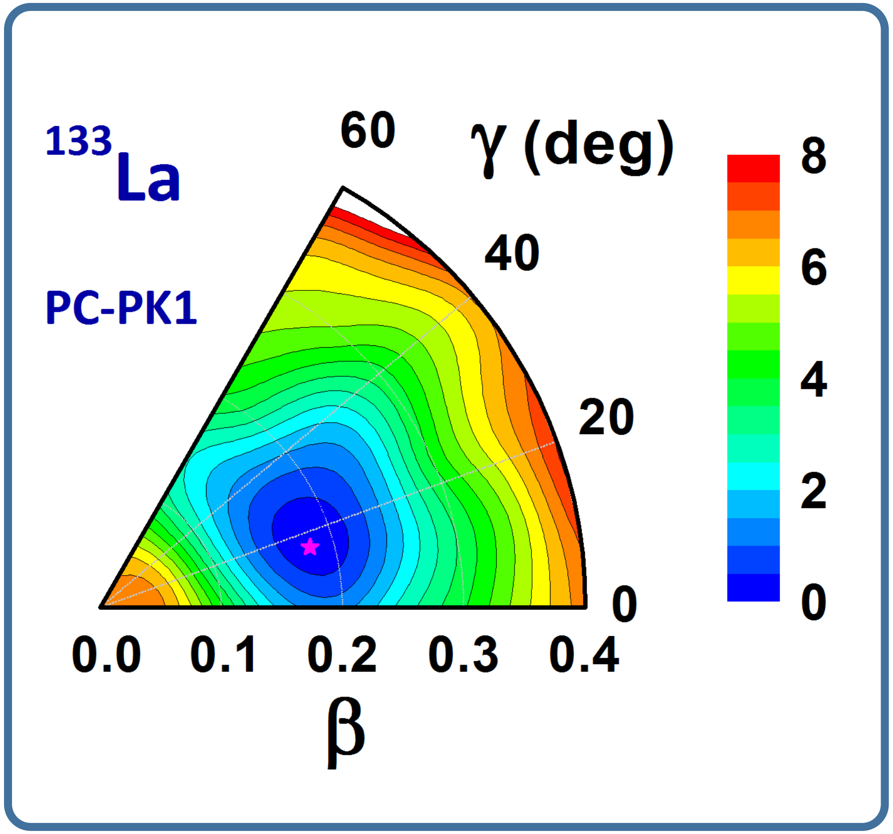
Results and discussion

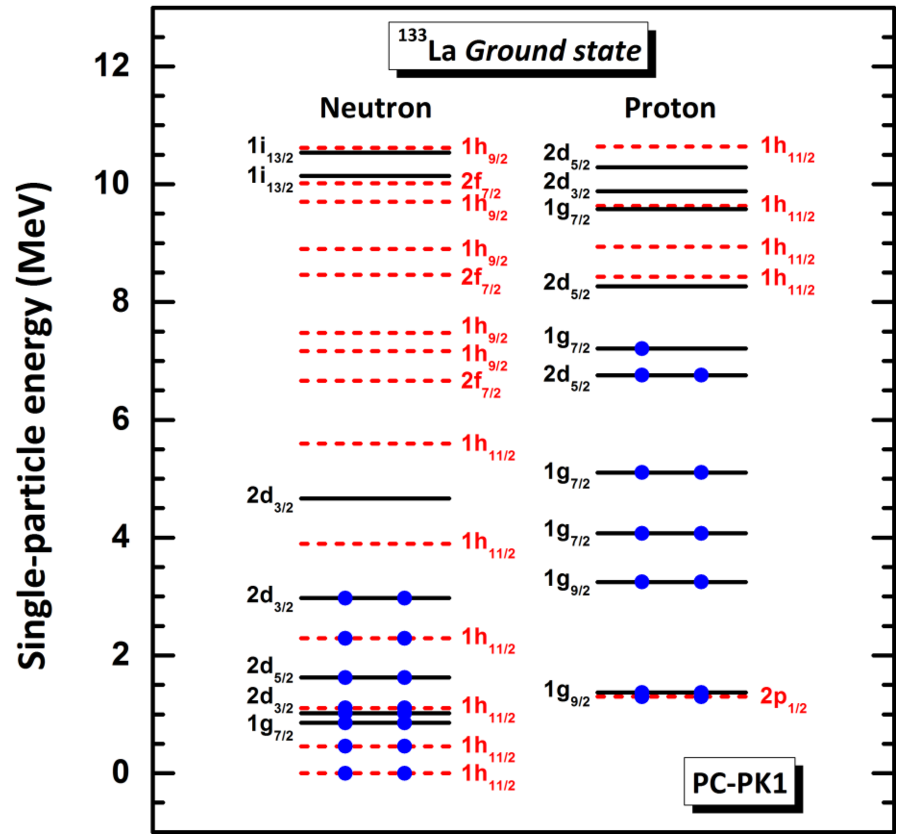
❑ Triaxial RMF calculations

- **The constrained triaxial relativistic mean field calculations have been performed to obtain the potential energy surface in the β-γ plane and potential energy curve along β-direction.** ***Meng2006PRC***

- **Numerical details:**
  - **Interaction parameter: PC-PK1;** ***Zhao2010PRC***
  - **Harmonic oscillator shells: $N_f = 12$;**
  - **Paring correlations are neglected.**

PES and SPE



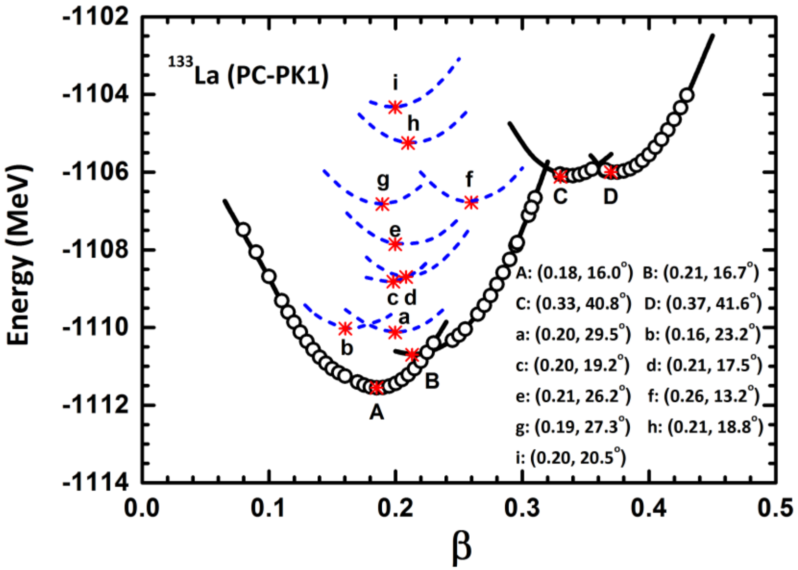


- **Potential energy surface (PES)**
  - Minimum: ($\beta = 0.18, \gamma = 16^o$).
  - A moderate γ softness.
- **Single particle energy level (SPE)**
  - Ground state: $\pi(1g_{7/2})^{-1}$.

Potential energy curve

- **Potential energy curve and configuration information**

Adiabatic cal. (circles) & configuration-fixed cal. (lines)



The deformation parameters will be input to the PRM.

State	E_x(MeV)	(β_2, γ)	Configuration	π	Band
A	0.00	$(0.18, 16.0°)$	$\pi g_{7/2}^{-1}$	+	$Q4$
B	0.85	$(0.21, 16.7°)$	$\pi h_{11/2}^{1}$	−	$Q1$, $Q2$
C	5.44	$(0.33, 40.8°)$	$\pi p_{3/2}^{-1}$	−	
D	5.55	$(0.37, 41.6°)$	$\pi h_{11/2}^{1}$	−	

State	E_x(MeV)	(β_2, γ)	Active nucleon configuration	π	Band
a	1.43	$(0.20, 29.5°)$	$\pi h_{11/2}^{1} \otimes \nu(s_{1/2}, d_{3/2})^{-1} h_{11/2}^{-1}$	+	
b	1.52	$(0.16, 23.2°)$	$\pi d_{5/2}^{1} \otimes \nu h_{11/2}^{-2}$	+	$D1$, $D2$
c	2.74	$(0.20, 19.2°)$	$\pi h_{11/2}^{1} \otimes \nu h_{11/2}^{-2}$	−	$D4$-low
d	2.86	$(0.21, 17.4°)$	$\pi d_{5/2}^{1} h_{11/2}^{2}$	+	
e	3.70	$(0.21, 26.2°)$	$\pi d_{5/2}^{1} h_{11/2}^{2} \otimes \nu(s_{1/2}, d_{3/2})^{-1} h_{11/2}^{-1}$	−	
f	4.67	$(0.26, 13.2°)$	$\pi d_{5/2}^{1} h_{11/2}^{2} \otimes \nu(s_{1/2}, d_{3/2})^{-1} h_{11/2}^{-2} (f_{7/2}, h_{9/2})^{1}$	−	$Q10$
g	4.71	$(0.19, 27.3°)$	$\pi d_{5/2}^{1} h_{11/2}^{2} \otimes \nu h_{11/2}^{-2}$	+	$Q8$, $Q9$
h	6.30	$(0.21, 18.8°)$	$\pi h_{11/2}^{3} \otimes \nu(s_{1/2}, d_{3/2})^{-1} h_{11/2}^{-1}$	+	
i	7.23	$(0.20, 20.5°)$	$\pi h_{11/2}^{3} \otimes \nu h_{11/2}^{-2}$	−	$D4$-high

Results and discussion

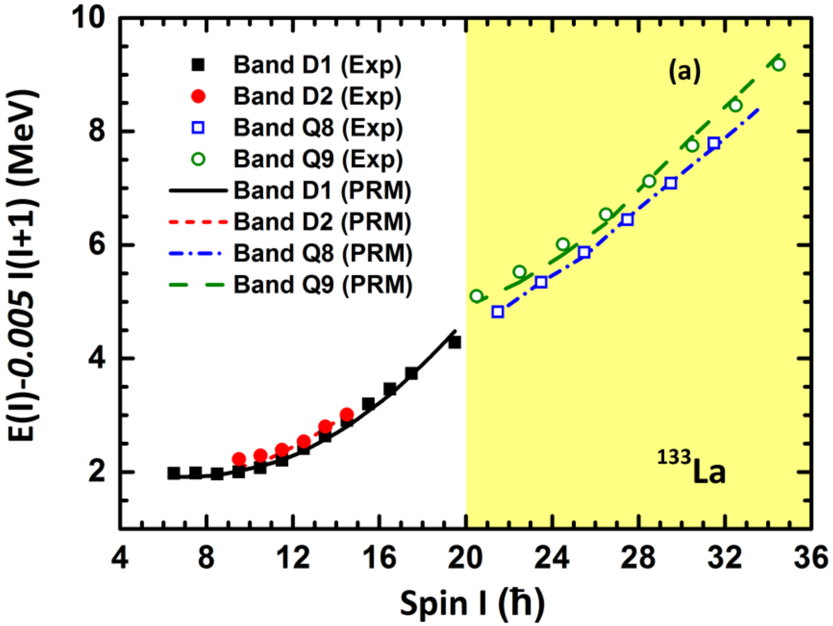
Triaxial PRM calculations

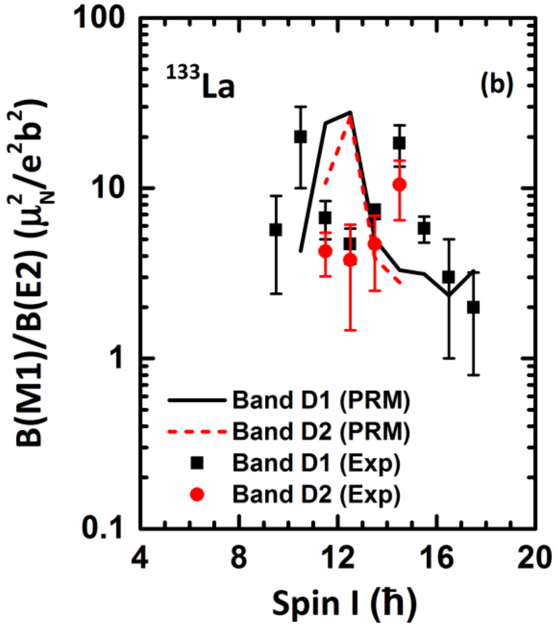
- **Numerical details of bands D1, D2 and Q8, Q9**
  - Deformation parameters: taken from CDFT.
  - Irr. MOI: $\mathcal{J}_0 = 19\ \hbar^2/\mathrm{MeV}$ and $\mathcal{J}_0 = 41\ \hbar^2/\mathrm{MeV}$.
  - Coriolis attenuation factor: ξ=0.96 and 0.94.

MOI are adjusted to the energy spectra

- **Numerical details of band D4**
  - Deformation parameters: taken from CDFT.
  - Irr. MOI: $\mathcal{J}_0 = 25\ \hbar^2/\mathrm{MeV}$ and $\mathcal{J}_0 = 20\ \hbar^2/\mathrm{MeV}$.

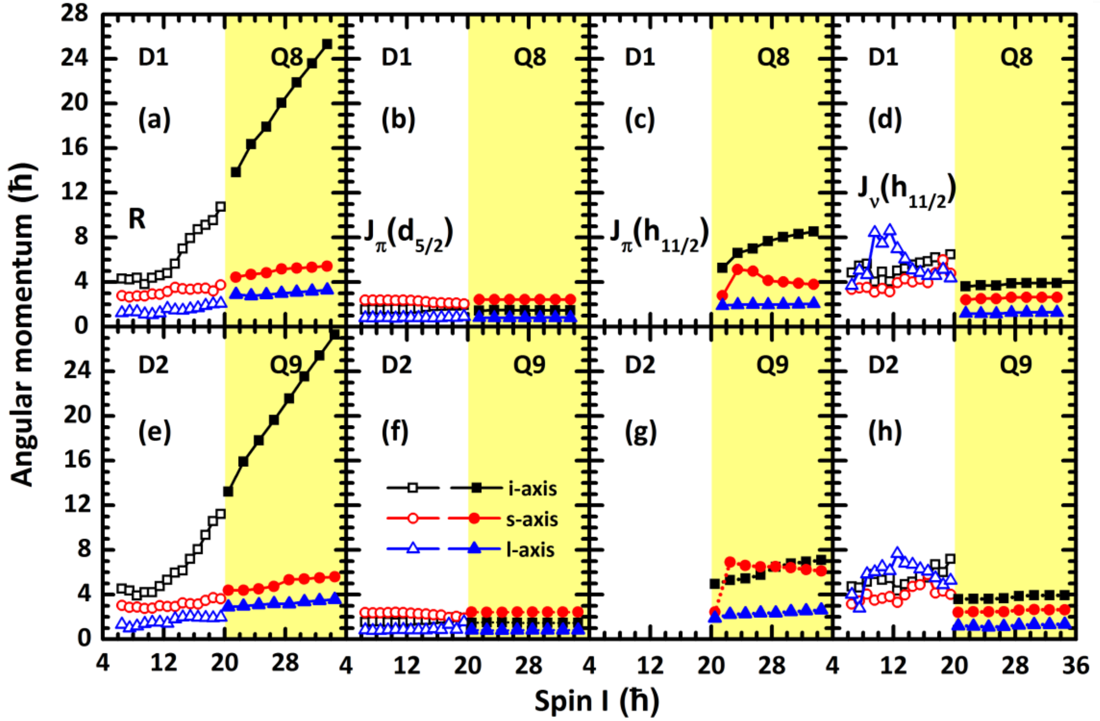
Bands D1, D2 and Q8, Q9





- **Bands D1, D2**
  - The experimental energy spectra and B(M1)/B(E2) of bands D1, D2 are well reproduced.
  - The energy differences between the doublets are small. Their B(M1)/B(E2) values are similar. → **Chiral doublet bands**
- **Bands Q8, Q9**
  - They are quadrupole bands. The experimental energy spectra is well reproduced.

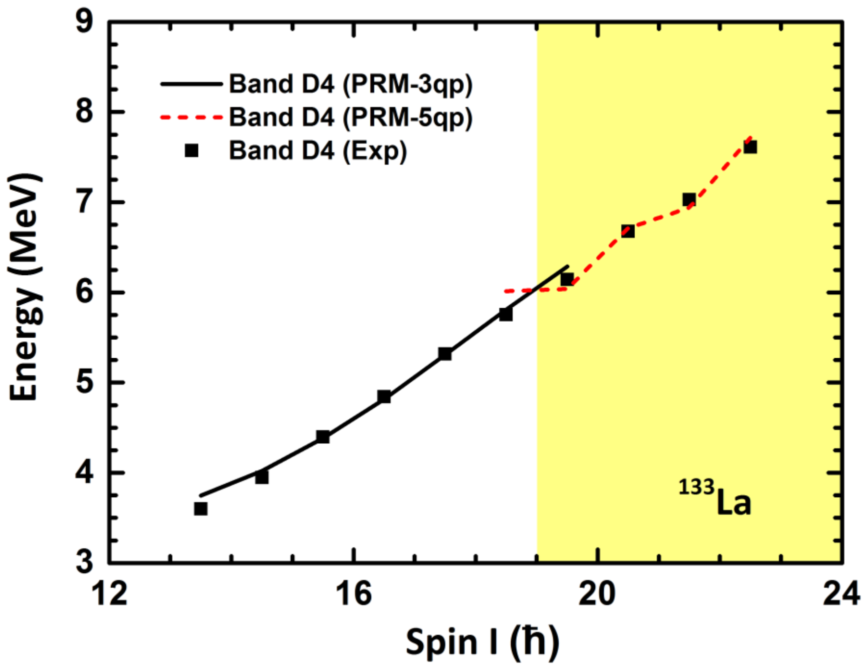
Bands D1, D2 and Q8, Q9

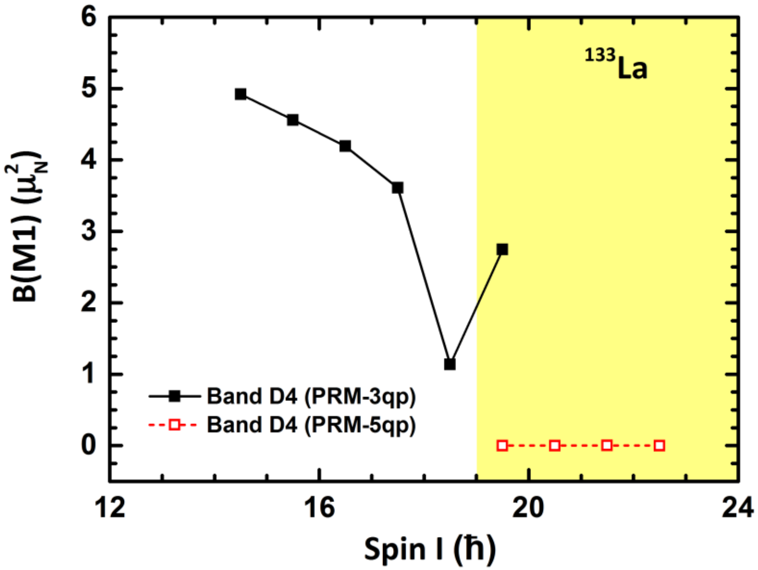


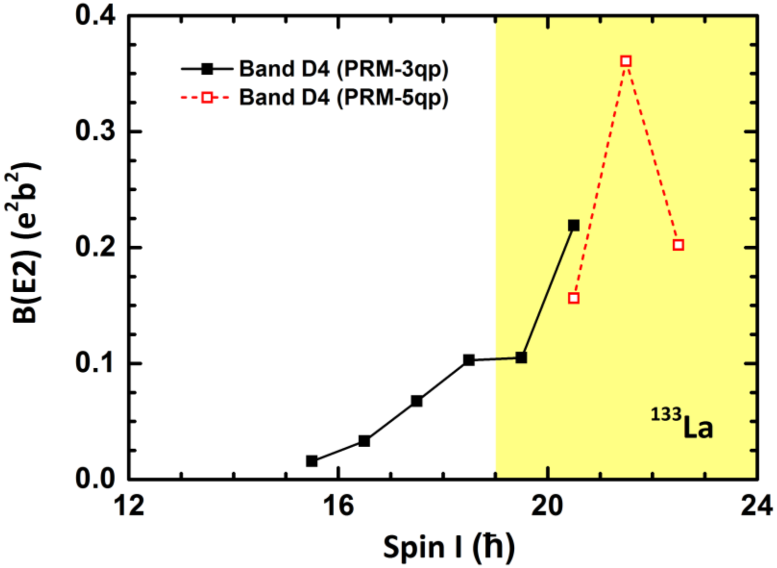
- **Bands D1, D2**
  - The angular momenta of bands D1 and D2 are similar. The rotor and proton mainly align along the i- and s-axes. The neutron holes are significant the l-axis and s-axis. This leads to the 3D chiral geometry.
- **Bands Q8, Q9**
  - The angular momenta mainly along the i-axis.

➡ **A transition occurs from an aplanar rotation to a principal axis one.**

Band D4



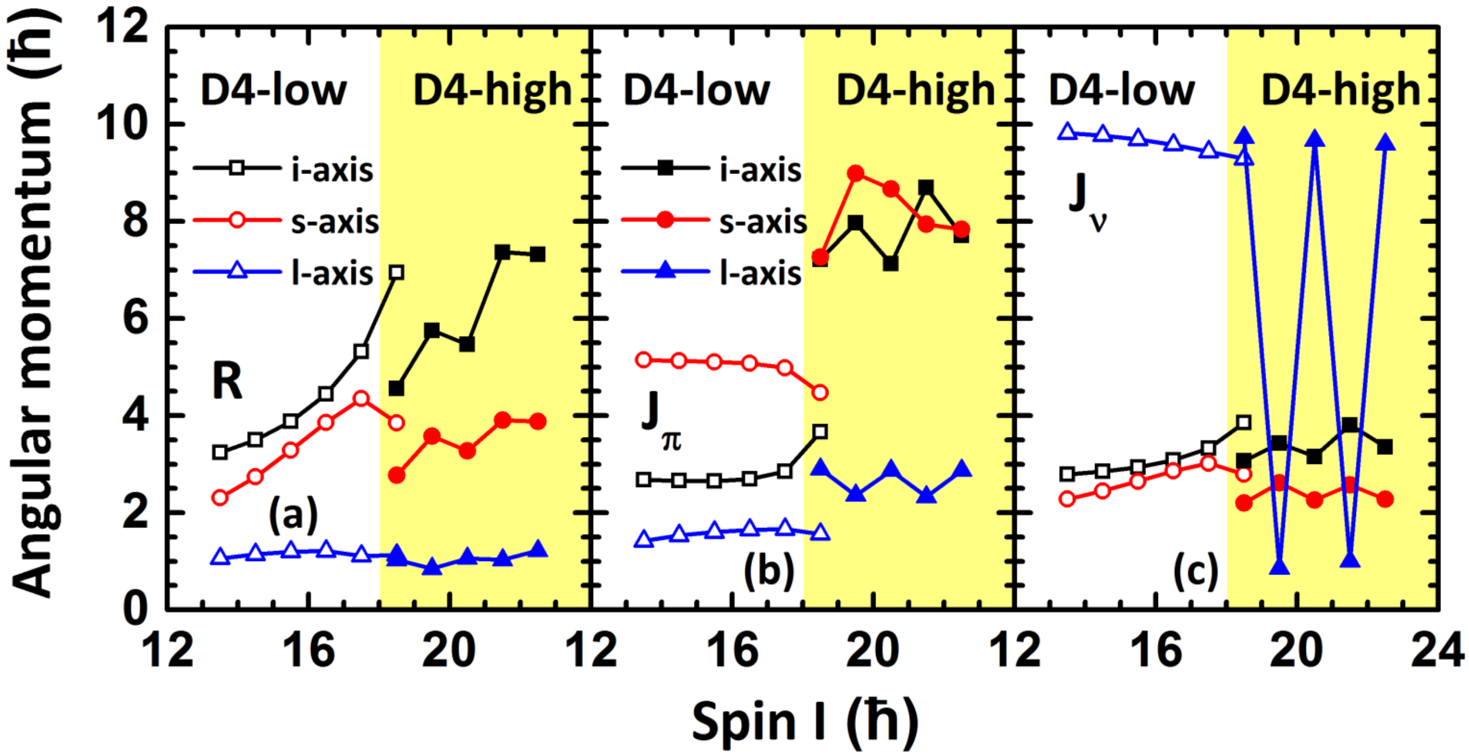




- **Band D4**
  - The experimental energy spectra of band D4 is reproduced respectively by 3-qp and 5-qp configuration.
  - B(M1) decreases while B(E2) increases at low spin part. B(M1) is large while B(E2) is small.

➡ **Magnetic rotational band: lifetime**

Band D4



- **Band D4**
  - Low: the rotor lies in the i-s plane. The proton, and the neutron mainly align along the s-, and l-axis. This leads an aplanar rotation but very close to l-s plane.
  - High: obvious odd-even staggering, corresponding to the signature splitting of energy spectra.

➡ **A transition occurs from a planar rotation to a principal axis one.**

Outline

- Introduction
- Theoretical framework
- Results and discussion
- Summary

Summary

- **With the experimental results of ^{133}La by our collaborators, detailed calculations using constrained CDFT and PRM have been performed to assign the configurations for the observed bands, and understand their structures.**

- **The nearly degenerated bands D1 and D2 is interpreted as chiral doublet bands. The dipole band D4 is interpreted as magnetic rotational band. At their high spin part, transitions from tilted axis rotation to principal axis rotation are found.**

- **In the furfure, the lifetime measurement is highly expected for these bands to reach a definitive conclusion.**

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