Exotic shapes and exotic symmetries

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I. Deformation

II. Quarteting

III. Clustering

IV. Connection

V. Conclusion

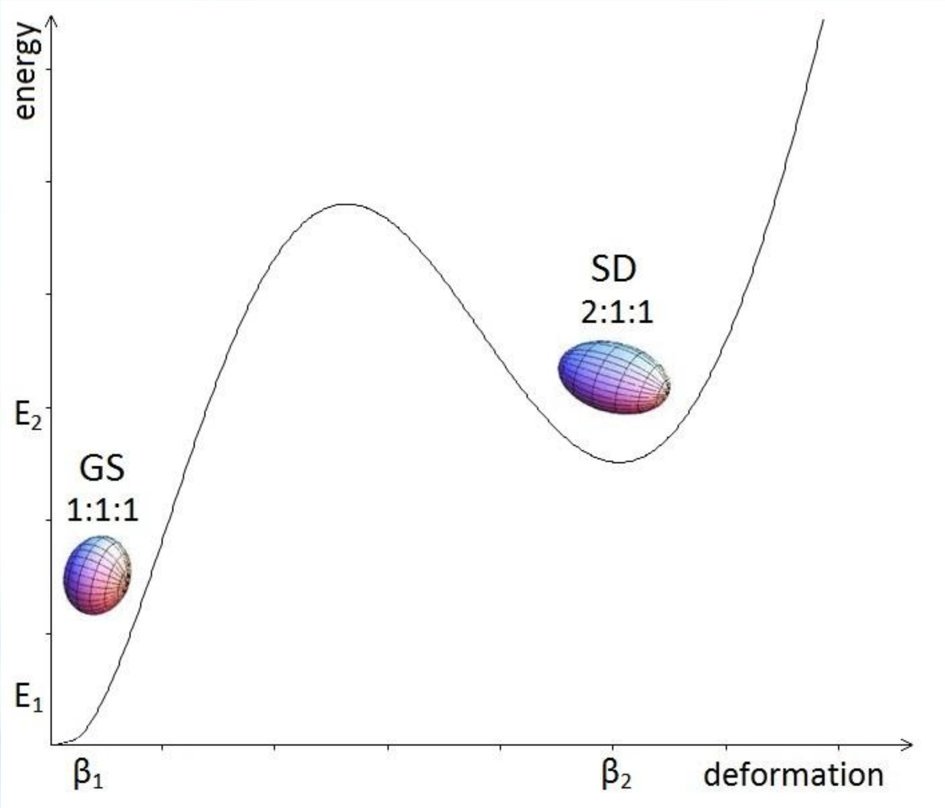
I. Deformation

Spherical, superdeformed, hyperdeformed,…
shapes with ratios of major axes

1:1:1 , 2:1:1 , 3:1:1, …

turn out to be exceptianally stable.

One way of seeing it is to investigate the energy-surface as a function of the quadrupole deformation.


energy
E2
E1
GS
1:1:1
SD
2:1:1
β1
β2
deformation

A non-typical way is the investigation of the stability of the quadrupole deformation, or the SU(3) symmetry. (The SU(3) quantum numbers and the parametrs of the quadrupole deformation are uniquely related to each other.)

The validity of the SU(3) symmetry is rather limited. But its generalization, the quasi-dynamical SU(3) is widely applicable.

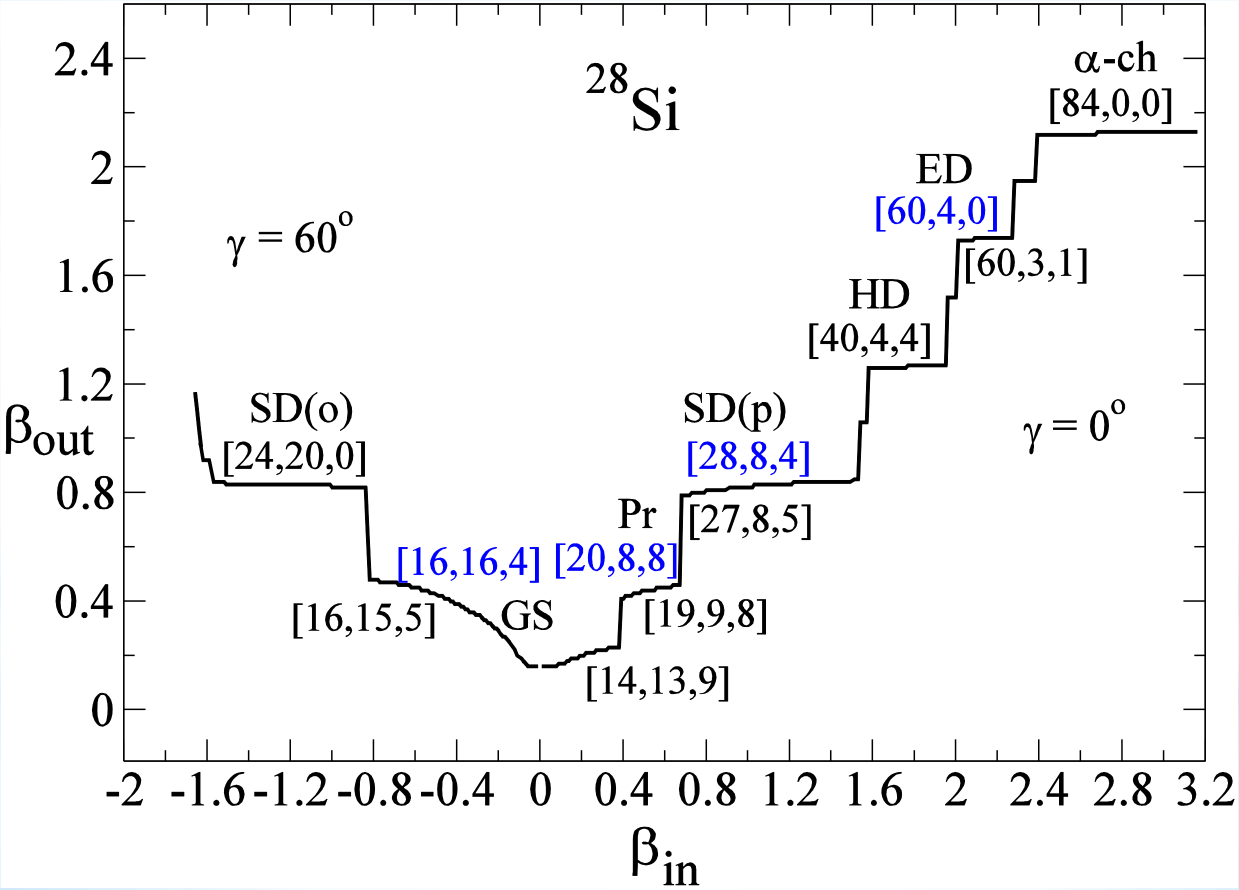
Scenario:

quadrupole deformation > Nilsson-calculation >

quasi-dynamical SU(3) > quadrupole deformation

Self-consistency + stability.

J. Darai, J. Cseh, D.G. Jenkins , Phys. Rev. C86 (2012) 064309.



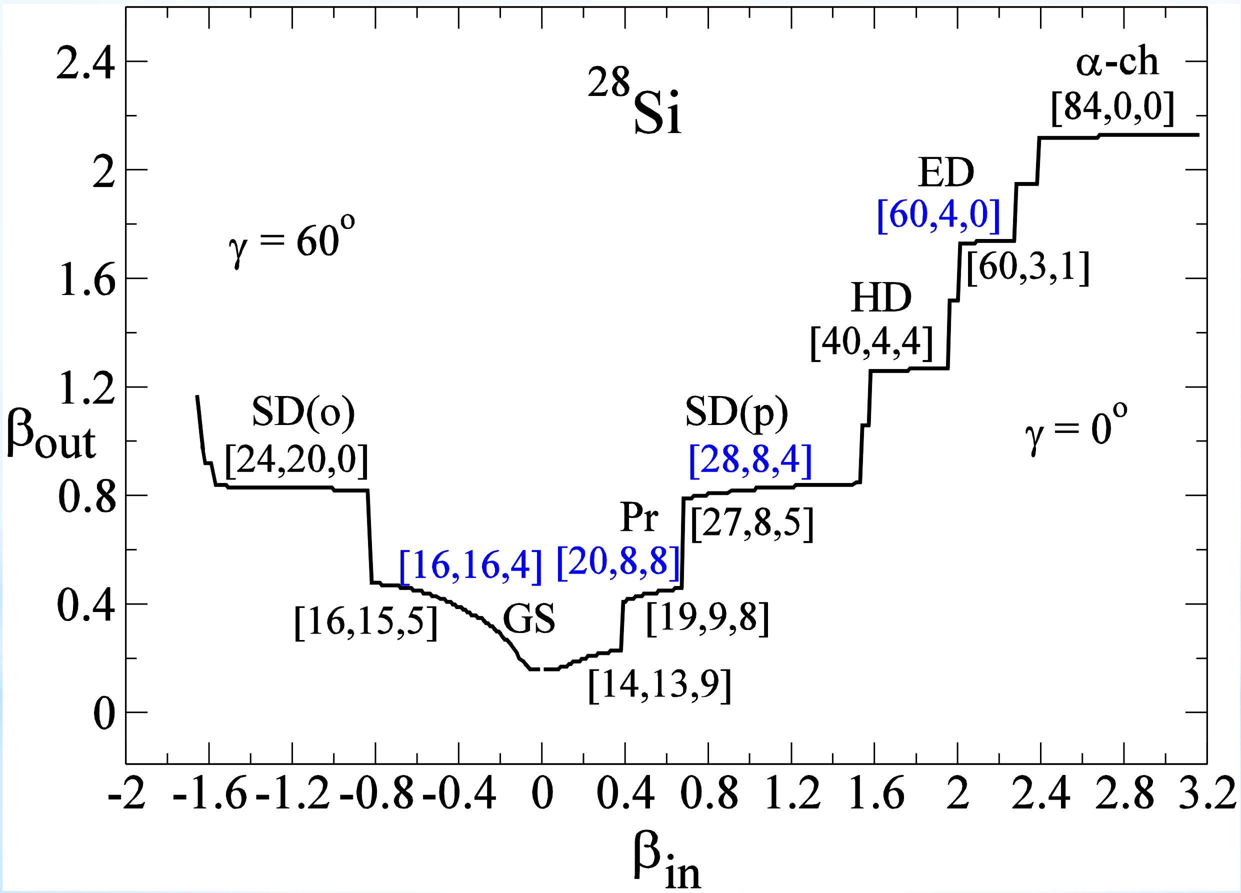
Energy-minima and stabil deformation (symmetry) are in good agreement. (Nice to see that different methods give the same results.)

Symmetry: selection rule, connection to cluster-configuration, connection to reaction channels.

Connection between shape (structure) and reactions: clustering (the appearance of moleclule-like configuration).

A cluster-configuration is defined by a reaction channel.

J. Darai, J. Cseh, D.G. Jenkins , Phys. Rev. C86 (2012) 064309.



Experimental varification:
moment of inertia, reaction channels.

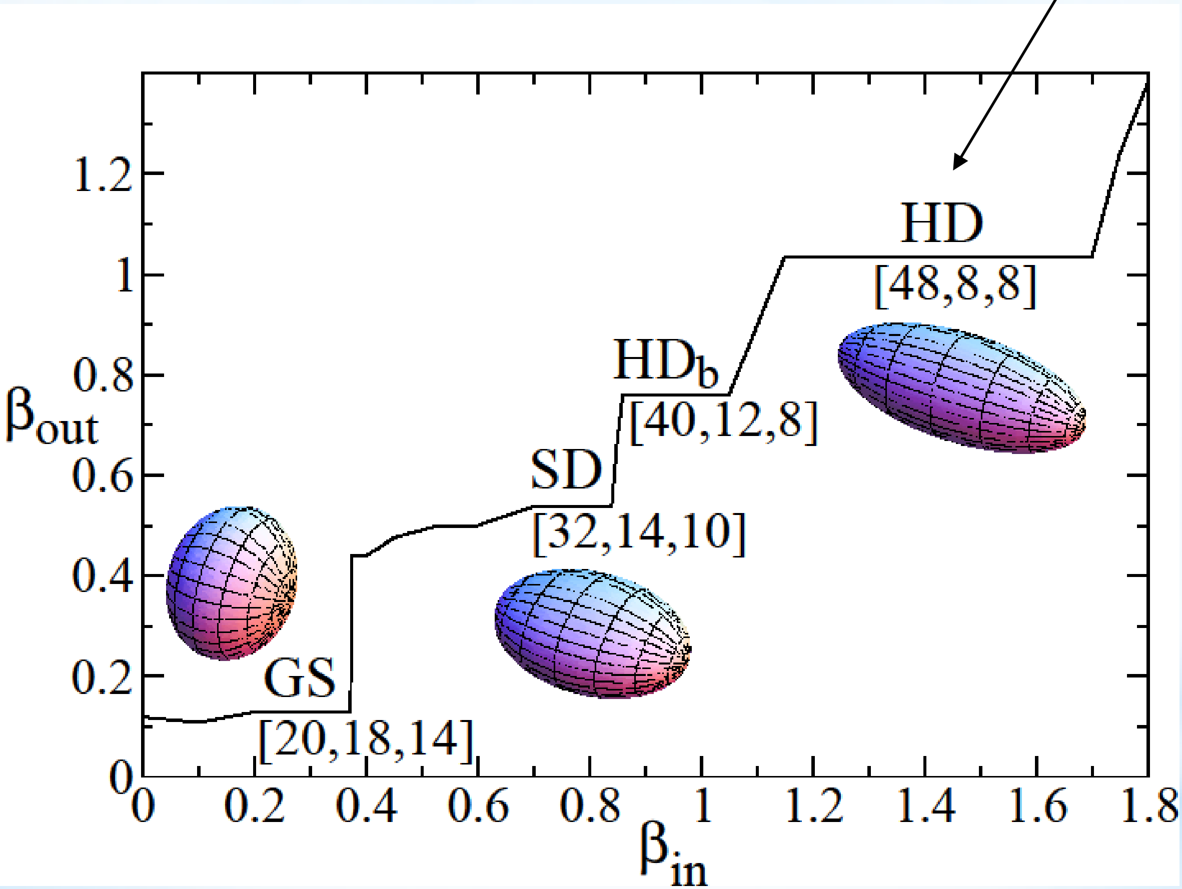
D.G. Jenkins et al, Phys. Rev. C86 (2012) 064308.

Shape isomers > clusterization > reaction channel:

Hyperdeformed state of ^{36}Ar

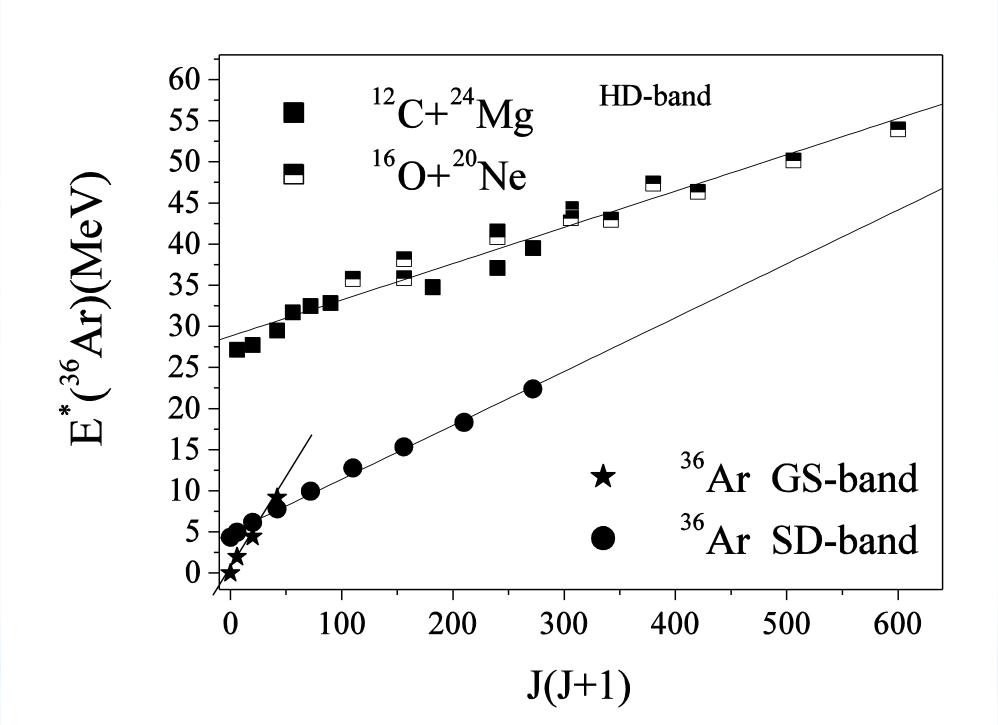
W.D.M. Rae, A. Merchant, Phys. Lett. B 279 (1992) 207.
J. Cseh et al, Phys. Rev. C70 (2004) 034311.
W. Sciani et al, Phys. Rev. C80 (2009) 034319.
J. Cseh et al, Phys. Rev. C80 (2009) 034320.

HD state of ^{36}Ar



HD state of ^{36}Ar

Conclusion of ^{24}Mg + ^{12}C scattering + previous ^{20}Ne + ^{16}O results



Moment of inertia: is the same as the theoretically predicted moment of inertia.

Reaction channels coincide with our preferred clusterizations.

Reaction:

Extension of the statistical pre-equilibrium (exciton) model for considering also the heavy ion collisions.

E. Betak, J. Cseh, in Proc. 14th Int. Conf. On Nucl. Reaction Mechanisms, Varenna 2015, CERN 2015.

Relation between the shape and the cluster configurations: the shell connection.

Shape isomers from Nilsson-calculation: deformed shell model.

Magic numbers of the deformed shell model and cluster configurations
W.D.M: Rae, Int. J. Mod. Phys. A 3 (1988) 1343.

For the spherical shell model the cluster-connection is better understood.

A natural step in between: quarteting.

II. Quarteting

Important in different branches of culture.

Quartet

Importance of quarteting in nuclear structure:

- short-range attractive nucl.-nucl. force: occupy the same single-particle orbital.

- Pauli-principle: 2p+2n.

Semimicroscopic Algebraic Quartet Model (SAQM)

J. Cseh, Phys. Lett. B 743 (2015) 213.

Quartet-concept: 2p+2n, shell-like

A. Arima, V. Gillet, J. Ginocchio,Phys. Rev. Lett. 25 (1970) 1043.

Quartet-symmetry: permut. [4], $U^{ST}(4)$ [1,1,1,1]

M. Harvey, Nucl. Phys. A202 (1973) 191.

2p+2n may sit in different shells;

any number of major shell excitation.

Algebraic description: detailed spectrum

(like the IBM-type quartet models

J. Dukelsky et al, Phys. Lett. B115 (1982) 359.

F. Iachello, A.D. Jackson, Phys. Lett. B108 (1982) 151.)

Formalism: SU(3)

J.P. Elliott, Proc. Roy. Soc.245 (1958) 128; 562,

E.P. Wigner, Phys. Rev. 51 (1937) 106.

Algebra-chain:

$$
\begin{array}{llllllll}
U(3) & \supset & SU(3) & \supset & SO(3) & \supset & SO(2) \\
\big|[n_1, n_2, n_3], & & (\lambda, \mu), K, & & L, & & M\big\rangle
\end{array}
$$

1. Complete set of basis states
2. Dynamical symmetry:

   H in terms of invariant operators

   Eigenvalue-problem: analytical solution

Physical operators: in terms of U(3) generators

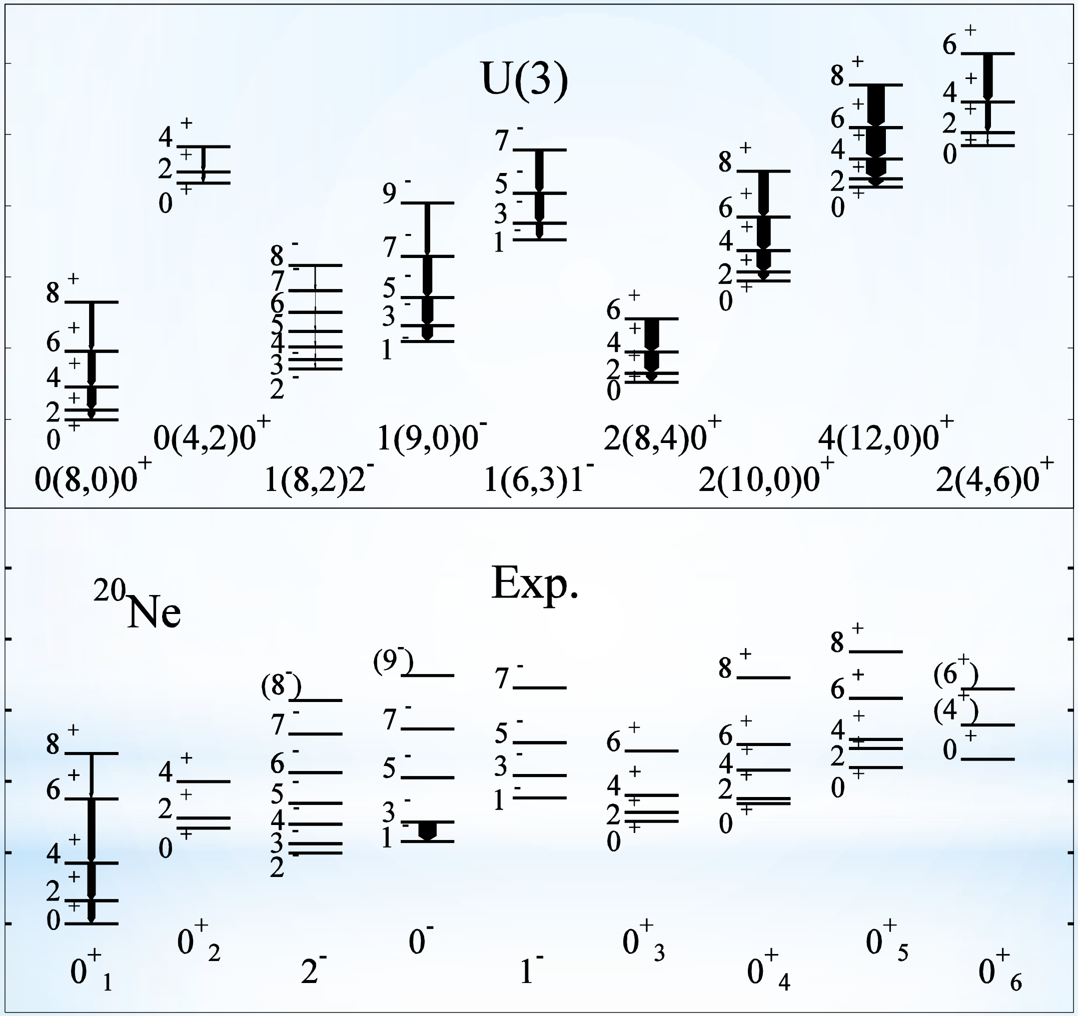
$$\hat{T}^{(E0)} = e^{(0)}\,\hat{n}, \quad \hat{T}^{(E2)} = e^{(2)}\hat{Q}^{(2)}_m, \quad \hat{T}^{(M1)} = m^{(1)}\hat{L}^{(1)}_m,$$

Hamiltonian: spherical scalar, e.g.:

$$\hat{n}, \qquad \hat{Q}^{(2)}\cdot\hat{Q}^{(2)}, \qquad \hat{L}^{(1)}\cdot\hat{L}^{(1)},$$

$$E = (h\omega)n + a(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu) + d\,\frac{1}{2\theta}\hat{L}^2,$$

$$B(E2, I_i \to I_f) = \frac{2I_f + 1}{2I_i + 1}\alpha^2 \left|\left\langle (\lambda,\mu)KI_i, (1,1)2 \parallel (\lambda,\mu)KI_f \right\rangle\right| C^{(2)}(\lambda,\mu)$$



U(3)
0(8,0)0$^+$
0(4,2)0$^+$
1(8,2)2$^-$
1(9,0)0$^-$
1(6,3)1$^-$
2(8,4)0$^+$
2(10,0)0$^+$
4(12,0)0$^+$
2(4,6)0$^+$
Exp.
^{20}Ne
0^+_1
0^+_2
2^-
0^-
1^-
0^+_3
0^+_4
0^+_5
0^+_6

SAQM

Model space: symmetry-governed truncation of the no-core SU(3) shell model.

T. Dytrych et al, J. Phys. G 35 (2008) 123101.

Effective model:

lowast-grade U(3), renormalization,

which describes the deformation and shape coexistence within the spherical shell model.

D.J. Rowe et al, Phys. Rev. Lett. 97 (2006) 202501.

1) ^{16}O: positive-parity spectrum: self-consistency argument, no-parameter model.
2) SAQM
3) Symmetry-adapted no-core shell model

   (SA-NCSM): work is in progress to test how far the effective model is in line with the microscopic content of the ab initio model.

III. Clustering

Semimicroscopic Algebraic Cluster Model

J. Cseh, Phys. Lett. B281 (1992) 173;

J. Cseh, G. Lévai, Ann. Phys. (NY) (1994) 165.

Internal structure of clusters:

Elliott-model with $U^{ST}_{C_i}(4) \otimes U_{c_i}(3)$ algebraic structure.

J.P. Elliott, Proc. Roy. Soc. 245 (1958) 128; 562.

Relative motion:

(truncated) vibron model with $U_R(4)$ algebraic structure

F. Iachello, Phys. Rev. C23 (1981) 2778;

F. Iachello, R.D. Levine, J. Chem. Phys. 77 (1982) 3046.

Binary clusterization:

$$U_{C_1}^{ST}(4)\otimes U_{c_1}(3)\otimes U_{C_2}^{ST}(4)\otimes U_{c_2}(3)\otimes U_R(4).$$

Operators: group generators, matrix elements, algebraic.

Model space: only Pauli-allowed states,

as in the Microscopic Cluster Model with U(3) basis.

(H. Horiuchi, T. Hecht, Y. Suzuki, K. Kato,…)

IV. Connection

Strong $(U^{ST}(4) \otimes U(3))$ coupled basis;

intersection with the quartet (shell model).

Even more: identical parts of their spectra:

Multichannel Dynamical Symmetry (MUSY).

Multichannel Dynamical Symmetry (MUSY)

J. Cseh, Phys. Rev. C50 (1994) 2240.

Unified description of different cluster configurations
(including the shell or quartet limit).

Channel: reaction channel, which defines the clusterization.

Simplest case: two-channel symmetry.

Composite symmetry:

- Simple SU(3) dynamical symmetry in both configurations.
- A further symmetry connects them.

  The latter one is a symmetry of the pseudo-space of the particle-indeces.

Similar logical structure like SUSY in nuclear spectroscopy.

Simplest case

Two-channel dynamical symm. of binary configurations

J. Cseh, K. Kato, Phys. Rev. C87 (2013) 067301.

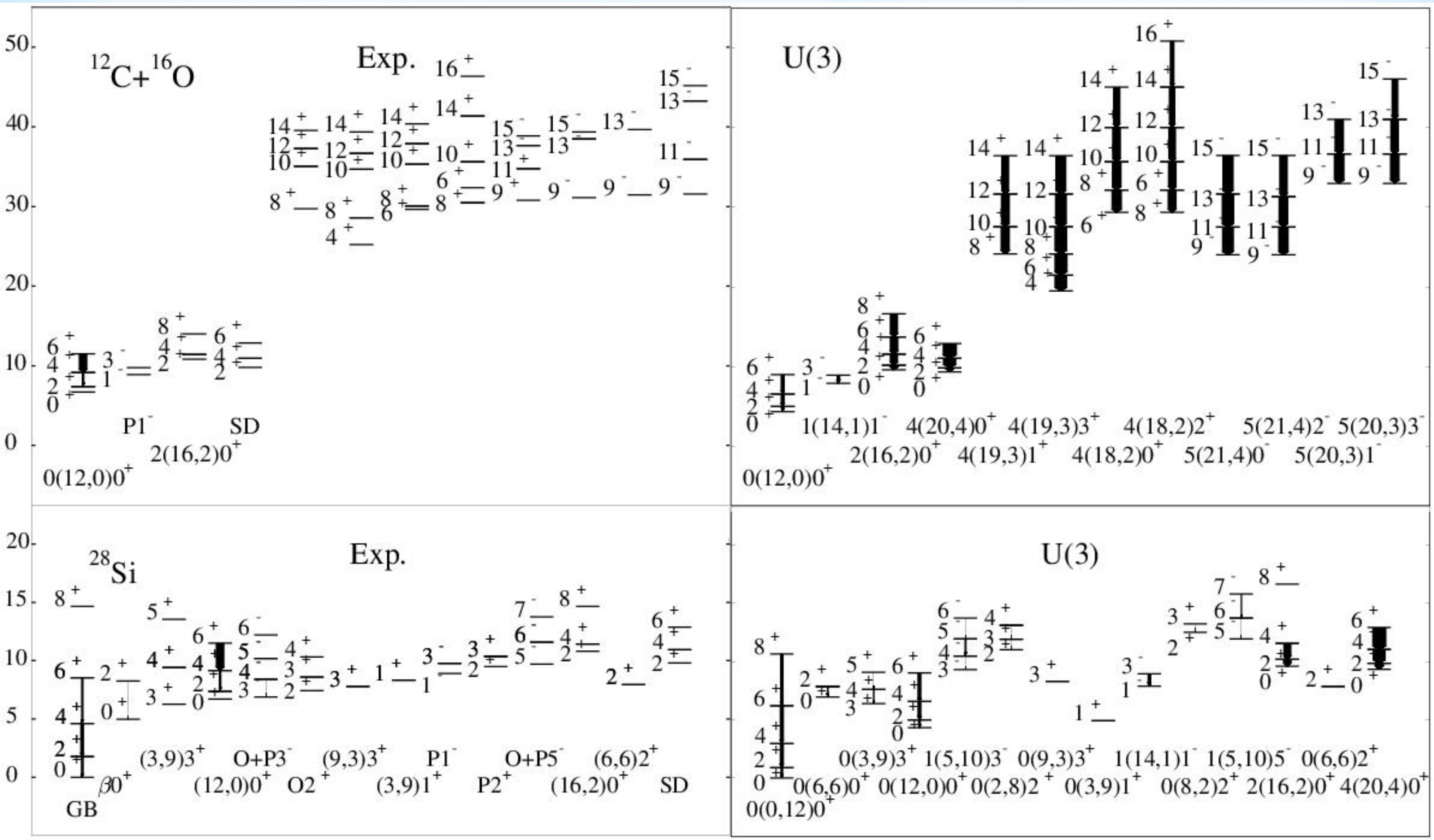
Geometrically: a symmetry with respect to the Talmi-Moshinsky transformations that connects the Jacobi coordinate systems of the two configurations.

The two-channel symmetry is a consequence of a usual dynamical symmetry of an underlying three-cluster configuration.

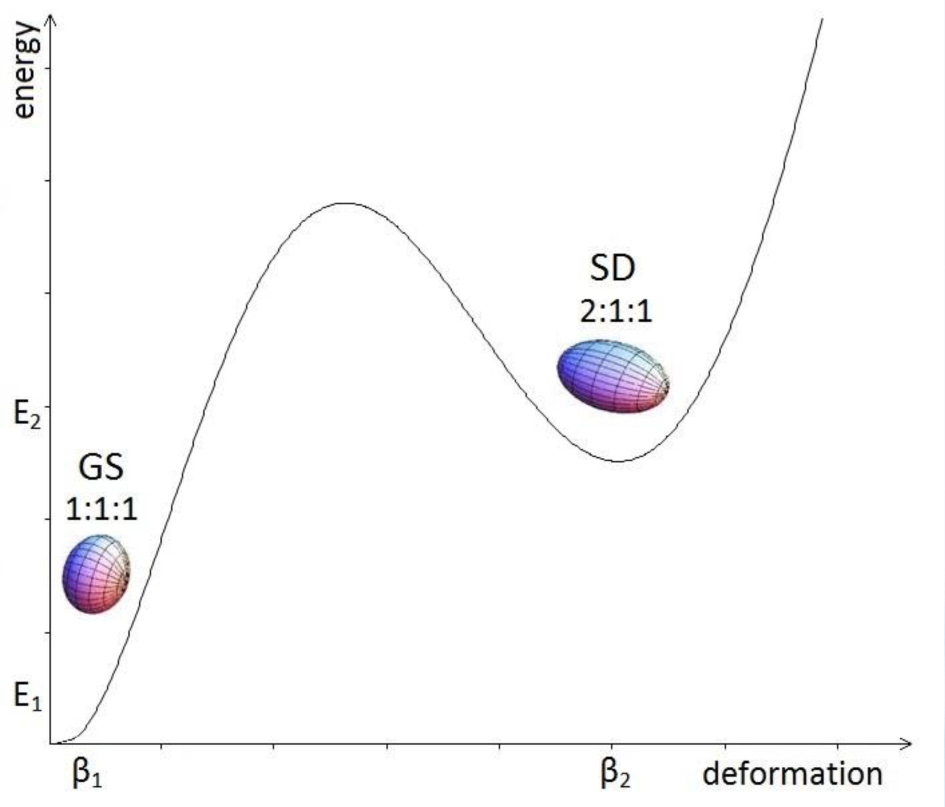
The quartet (shell) model limit is a special cluster configuration.

Quartet spectrum > MUSY >   cluster spectra

J. Cseh, G. Riczu, Phys. Lett. B 757 (2016) 312.



$^{12}C+^{16}O$
Exp.
U(3)
0(12,0)0+
P1-
2(16,2)0+
SD
1(14,1)1-
4(20,4)0+
4(19,3)3+
4(18,2)2+
5(21,4)2-
5(20,3)3-
2(16,2)0+
4(19,3)1+
4(18,2)0+
5(21,4)0-
5(20,3)1-
0(12,0)0+
^{28}Si
Exp.
U(3)
GB
β0+
(3,9)3+
(12,0)0+
O+P3-
O2+
(9,3)3+
(3,9)1+
P1-
P2+
O+P5-
(16,2)0+
(6,6)2+
SD
0(0,12)0+
0(6,6)0+
0(3,9)3+
0(12,0)0+
1(5,10)3-
0(2,8)2+
0(9,3)3+
0(3,9)1+
1(14,1)1-
0(8,2)2+
1(5,10)5-
2(16,2)0+
0(6,6)2+
4(20,4)0+



energy
E2
E1
GS
1:1:1
SD
2:1:1
β1
β2
deformation

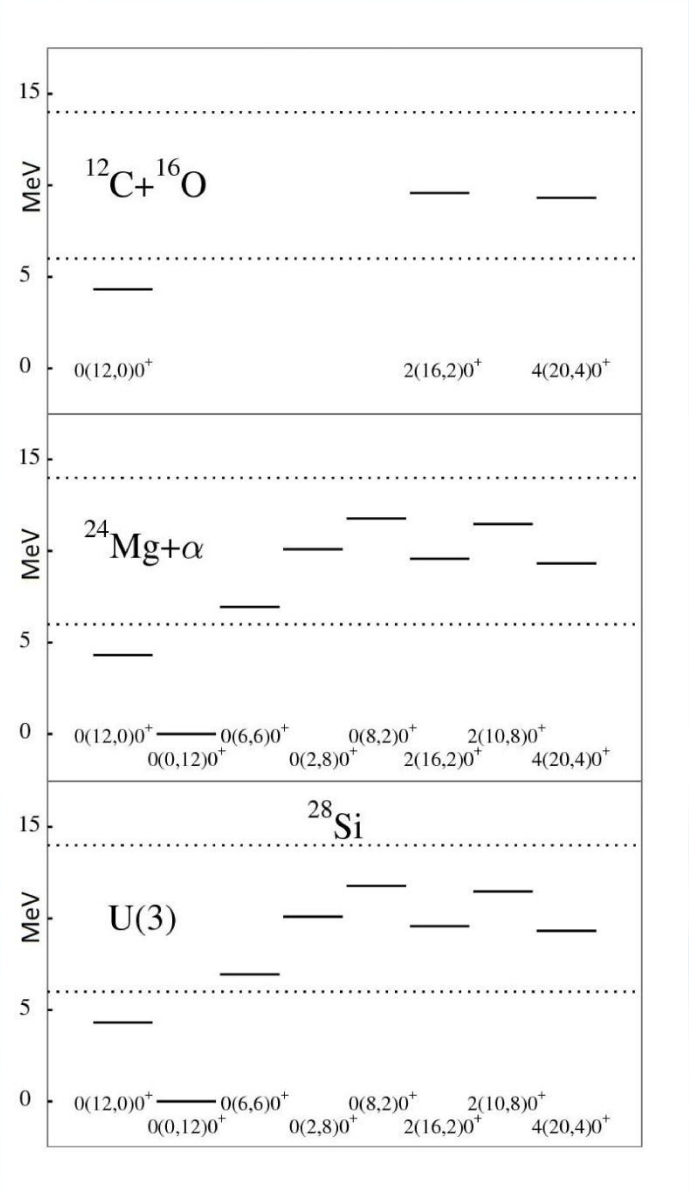
0^+ states in ^{28}Si

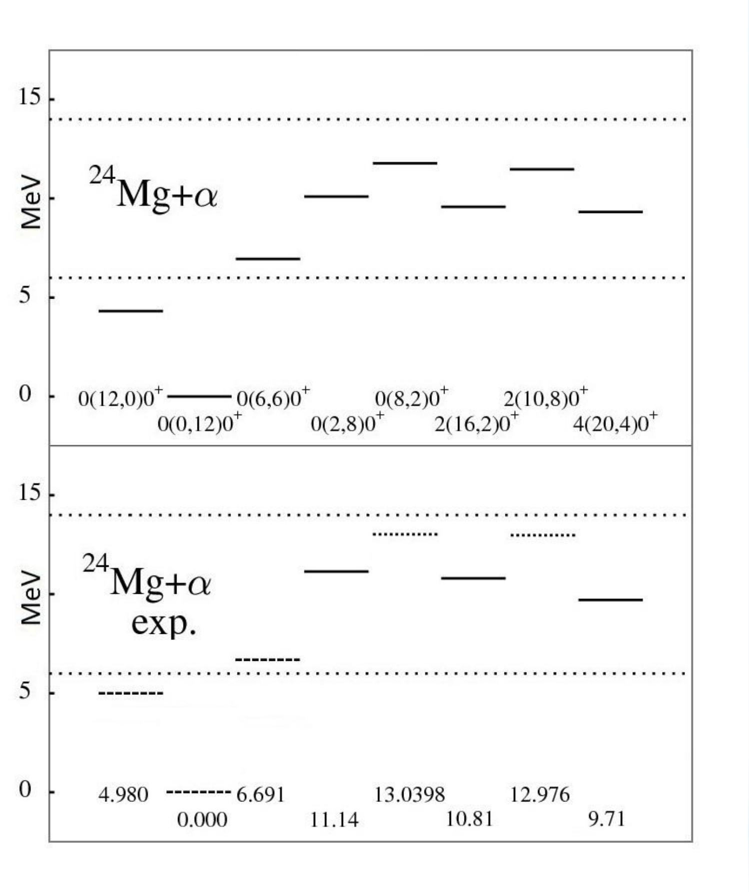
P Adsley et al, in preparation

Inelastic alpha-scattering at very forward angles

E_{exct}: 6 – 14 MeV

4 (+2) 0^+ states.


15
MeV
5
0
$^{12}C+^{16}O$
$0(12,0)0^+$
$2(16,2)0^+$
$4(20,4)0^+$
15
MeV
5
0
$^{24}Mg+\alpha$
$0(12,0)0^+$
$0(0,12)0^+$
$0(6,6)0^+$
$0(2,8)0^+$
$0(8,2)0^+$
$2(16,2)0^+$
$2(10,8)0^+$
$4(20,4)0^+$
^{28}Si
15
MeV
5
0
U(3)
$0(12,0)0^+$
$0(0,12)0^+$
$0(6,6)0^+$
$0(2,8)0^+$
$0(8,2)0^+$
$2(16,2)0^+$
$2(10,8)0^+$
$4(20,4)0^+$

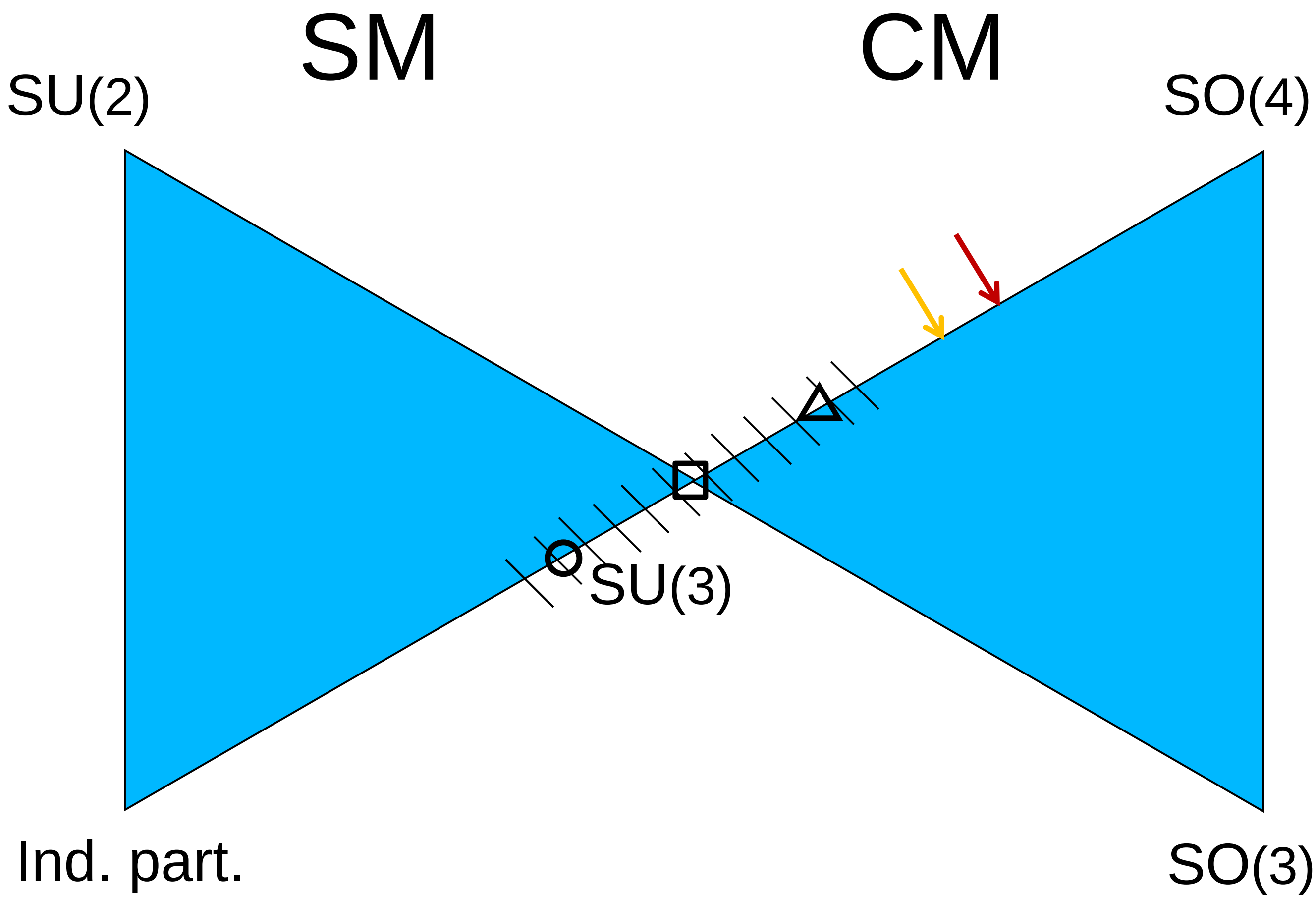


P. Adsley et al, arXiv 1609.00296 [nucl-ex]

In the language of the quantum phases:

The phase diagrams of the shell (quartet) and cluster models match each other at the SU(3) multichannel dynamical symmetry.

J. Cseh, J. Phys. Conf. Ser. 205 (2010) 012021 .


SM
CM
SU(2)
SO(4)
SU(3)
Ind. part.
SO(3)

VI. Summary and conclusions

The semimicroscopic algebraic approach to deformation, quarteting and clustering (SAQM & SACM) seems to be promising for light nuclei.

Their connecting symmetry (MUSY) has a great predictive power (^{28}Si: high-lying cluster spectra from low-lying quartet spectrum).

Some theoretical predictions seems to be experimentally verified: reactions to populate SD ^{28}Si, HD ^{36}Ar.

Predicted $^{12}C+^{16}O$ spectrum, unified description of the spectra of the first and second minima.

Phenomen.	Model	Method	Symm.	Ex.
Shape isom.	Nilsson	q-self-cons	QDS	SD, HD
Quarteting	SAQM	eff.mod.	DS	gs-sp.
Clustering	SACM	diff.conf.	MUSY	clus.pr.

Thank you for your attention!

Shell, collective and cluster models:

(sympl., contr.sp., semi- or fully algebr.)

SU(3) dynamical symmetry.

No-core shell models (NCSM), L-S coupling, SU(3)

from ab initio to semimicroscopic:

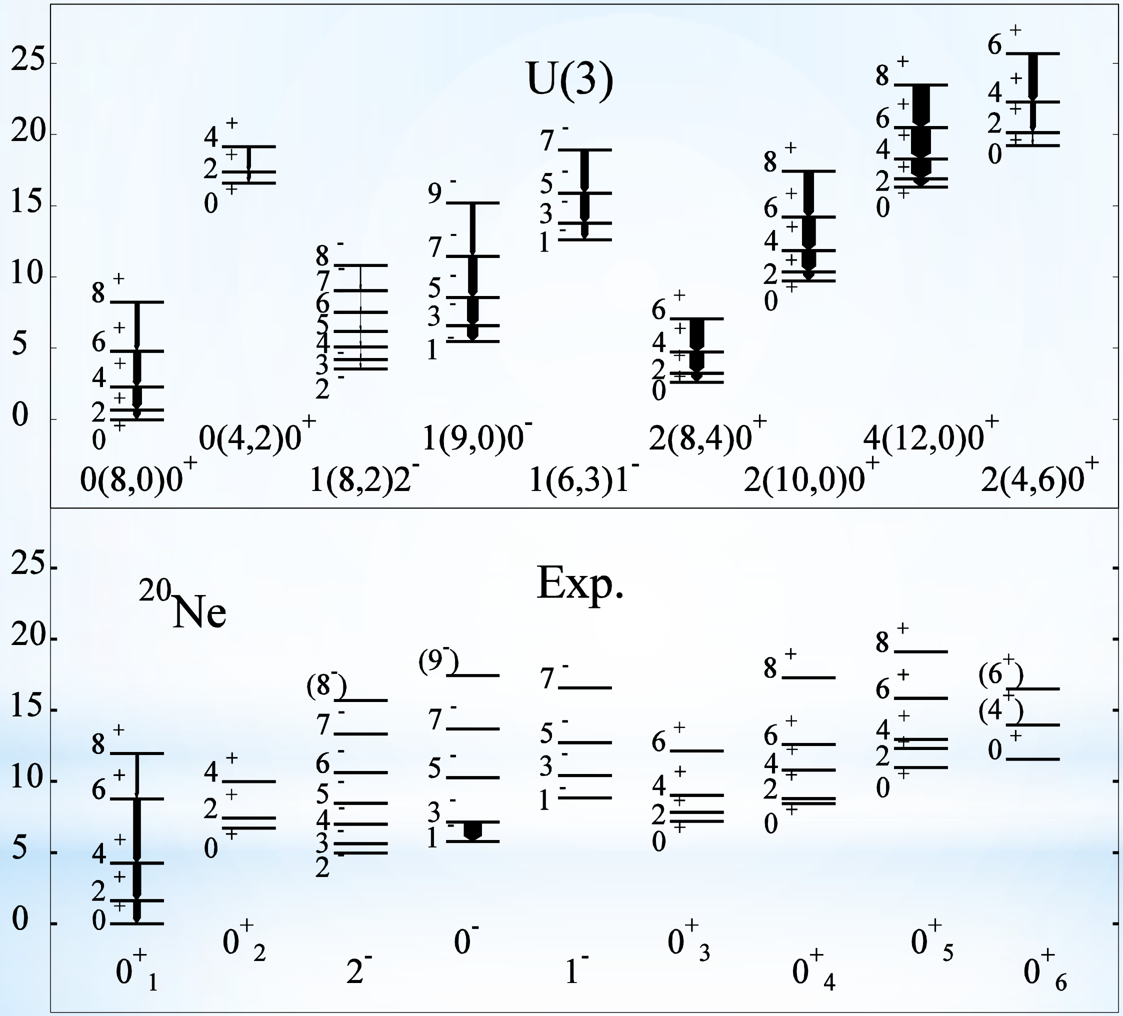
Symmetry-		Symplectic		SAQM	SACM
adapted	>	fully	>	semimicroscopic	
Ab initio		microscopic		quartet	cluster

Clustering:

The appearance of moleclule-like configuration.

A cluster-configuration is defined by a reaction channel.

Best-known example: alpha-clustering.



U(3)
0(8,0)0+
0(4,2)0+
1(8,2)2-
1(9,0)0-
1(6,3)1-
2(8,4)0+
2(10,0)0+
4(12,0)0+
2(4,6)0+
Exp.
20Ne
0+1
0+2
2-
0-
1-
0+3
0+4
0+5
0+6
0
5
10
15
20
25

$$\hat{H} = (h\omega)\hat{n} + a\hat{C}^{(2)}_{SU3} + b\hat{C}^{(3)}_{SU3} + d\,\frac{1}{2\theta}\hat{L}^2,$$

$$B(E2, I_i \to I_f) = \frac{2I_f + 1}{2I_i + 1}\alpha^2 \left| \left\langle (\lambda, \mu) K I_i, (1,1) 2 \,\|\, (\lambda, \mu) K I_f \right\rangle \right| C^{(2)}(\lambda, \mu)$$

Binary configurations: 3 dynamical symmetries

$$SU_{C1}(3)\otimes SU_{C2}(3)\otimes U_R(4)\supset SU_C(3)\otimes SU_R(3)\supset SU(3)\supset SO(3)$$

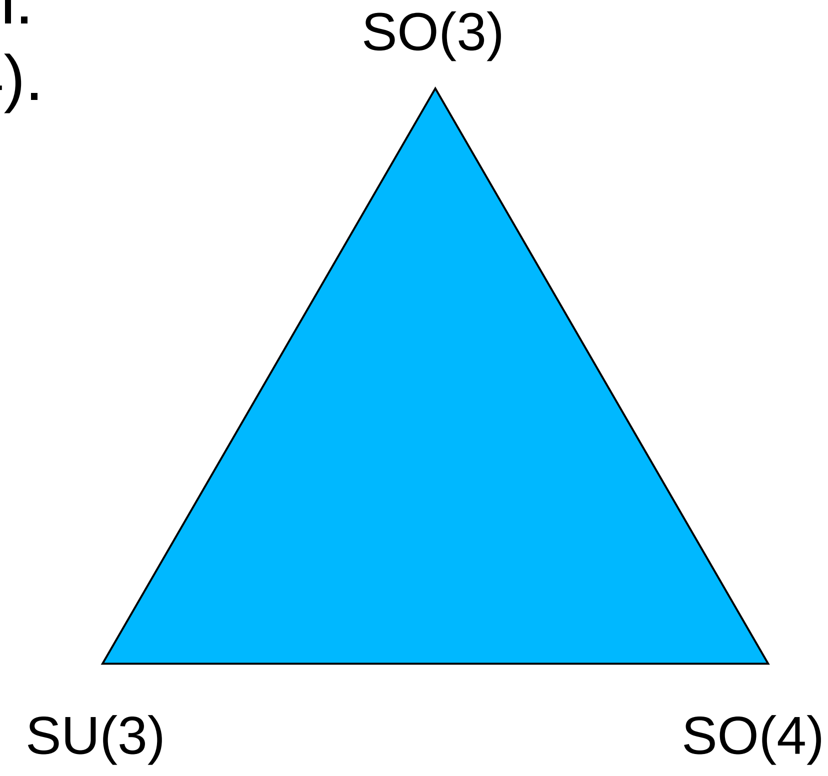
$$SU_{C1}(3)\otimes SU_{C2}(3)\otimes U_R(4)\supset SU_C(3)\otimes O_R(4)\supset SO_C(3)\otimes SO_R(3)\supset SO(3)$$

$$SU_{C1}(3)\otimes SU_{C2}(3)\otimes U_R(4)\supset SU_C(3)\otimes SU_R(3)\supset SO_C(3)\otimes SO_R(3)\supset SO(3)$$

Phases and clusters

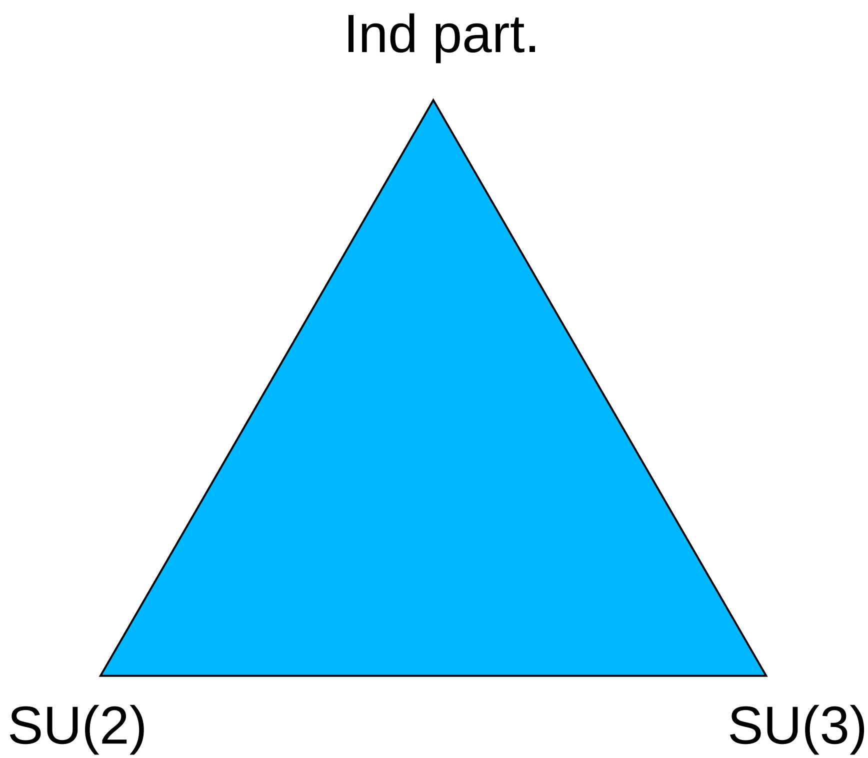
Rel. motion: vibron model.
Vibron model: U(3) – O(4).
Cluster model
- Coupling to int. d. f.
- Pauli-principle.

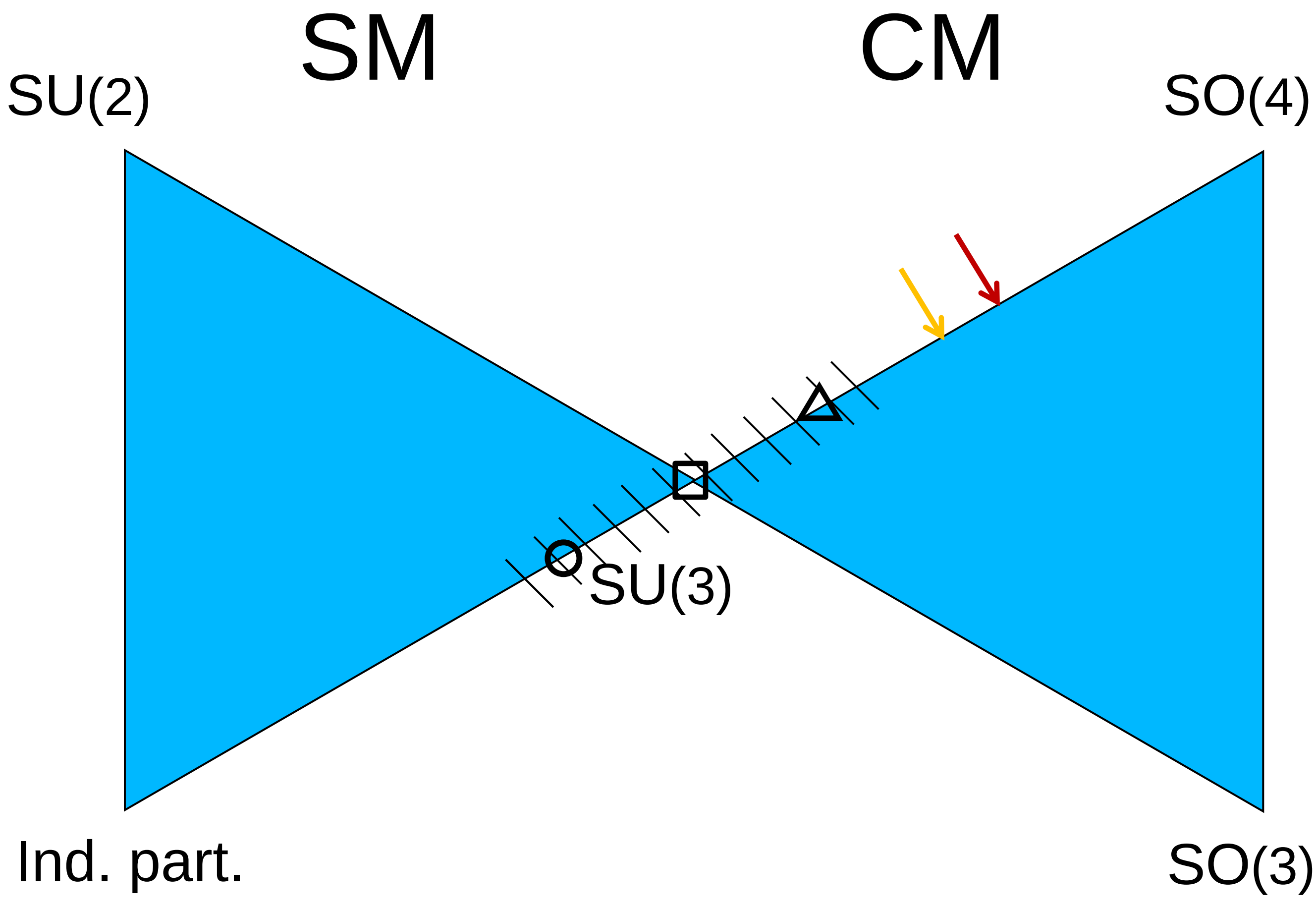
U(3) shell-like clusters,
O(4) rigid molecules.



J. Cseh, *J. Phys. Conf. Ser. 205 (2010) 012021.*

Shell model: P. Van Isacker, *Nucl. Phys. A. 704 (2002) 232C.*




SM
CM
SU(2)
SO(4)
SU(3)
Ind. part.
SO(3)

○ 4 nucleons full sd shell model

C. Vargas et al., *PRC 58 (1998) 1488*.

□ Symplectic no-core shell model

G.K. Tobin et al., *PRC 89 (2014) 034312.*

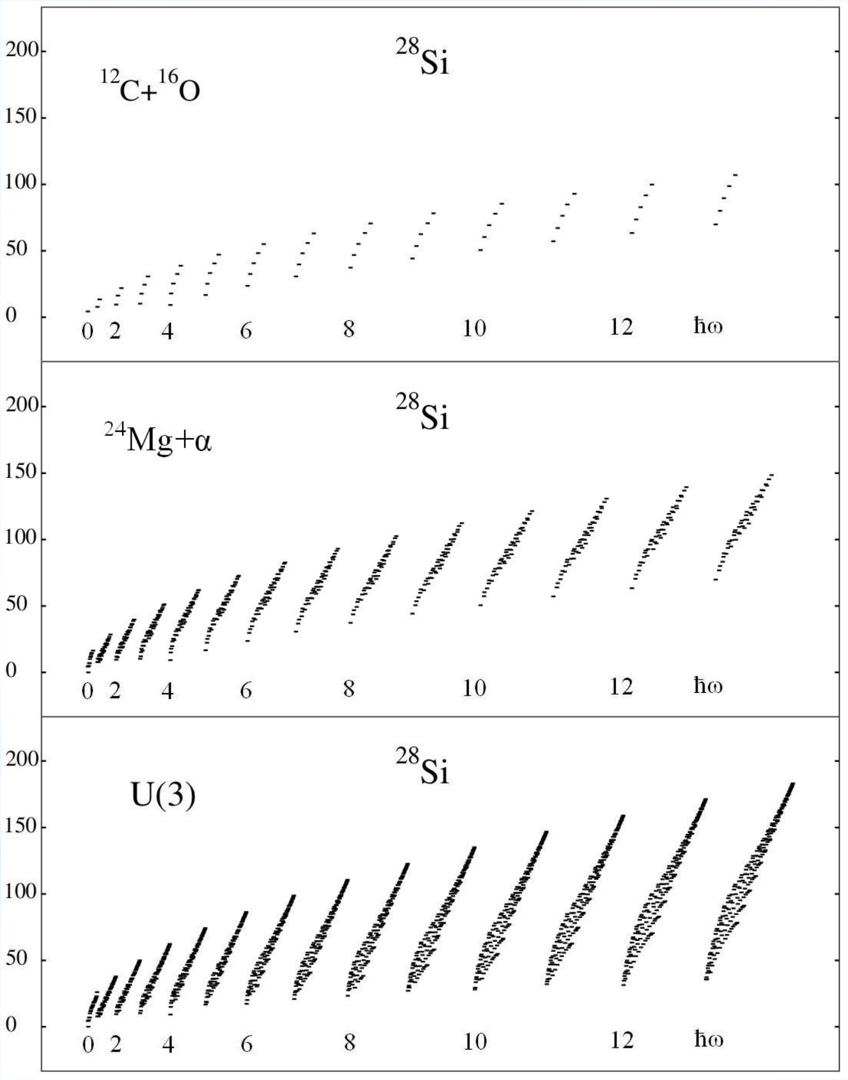
(G.s) overlap (cluster space) > 65%

△ SACM

H.Yepez et al., *PRC 86 (2012) 034309.*

//// Quasi-cluster: fr rigid mol. to strong LS

N. Itagaki et al., *PRC 83 (2011) 014302.*



28Si
12C+16O
24Mg+α
U(3)
200
150
100
50
0
0 2 4 6 8 10 12 ħω

$$E = 37.8\,n - 2.4879\,C_2(SU(3)) + 0.0454\,C_3(SU(3))$$
$$- 0.9837\,K^2 + 1.1661\,1/(2\Theta)L(L+1)$$