Nuclear Isomers: What we learn from Tin to Tin

Ashok Kumar Jain

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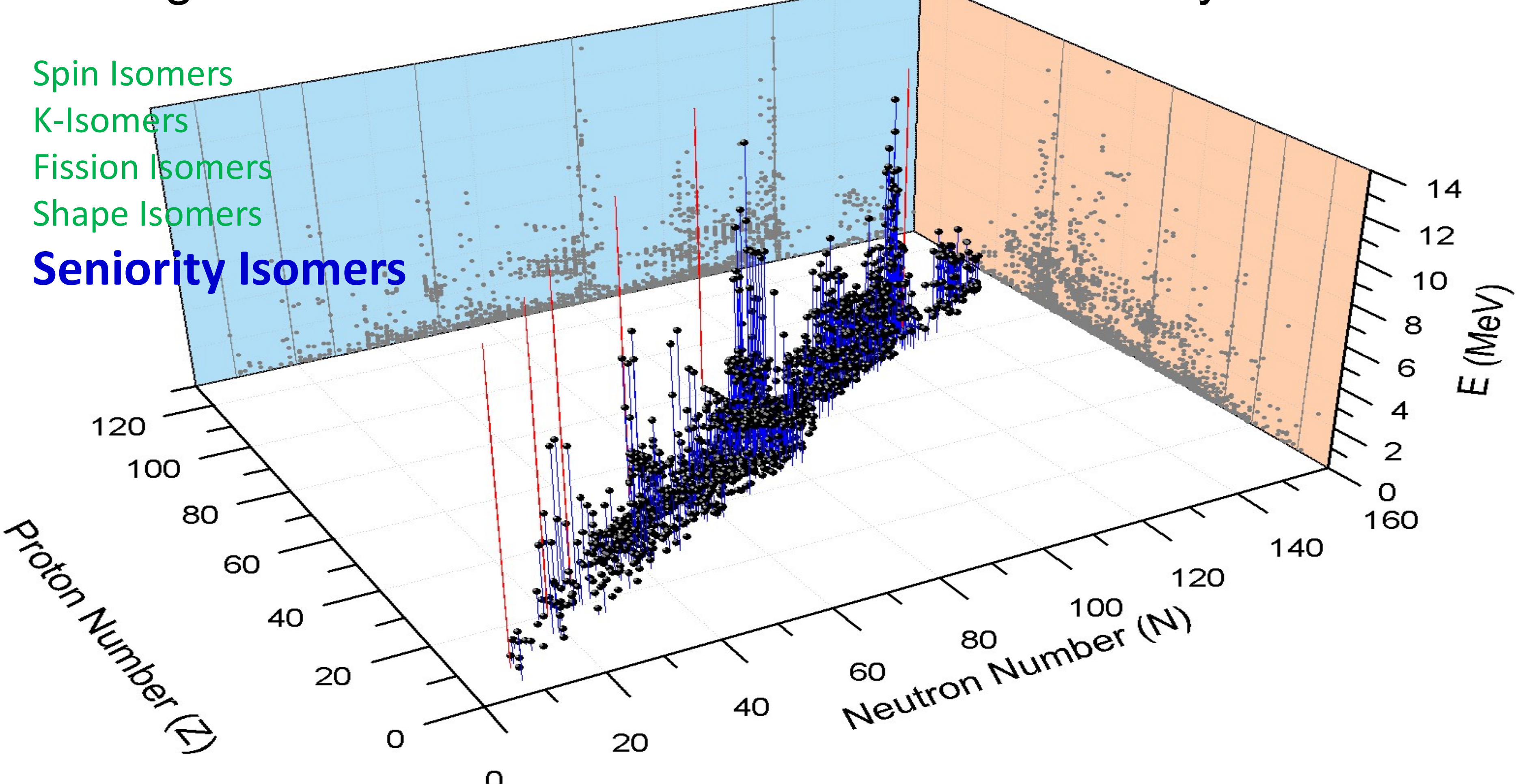
Outline

- Nuclear Isomers
- The longest chain of Sn isotopes from 100Sn to 138Sn including two doubly magic nuclei
- Beautiful concepts of Seniority and Generalized Seniority
- A new kind of odd-tensor Seniority Isomers, and a new set of selection rules
- Application to medium mass ^{100}Sn to heavy mass ^{132}Sn
- Application to Sn isotopes beyond ^{132}Sn
- What we learn and conclude from Sn to Sn
- Future possibilities

Chart of Nuclear Isomers

- Longer lived excited states due to hindered decays

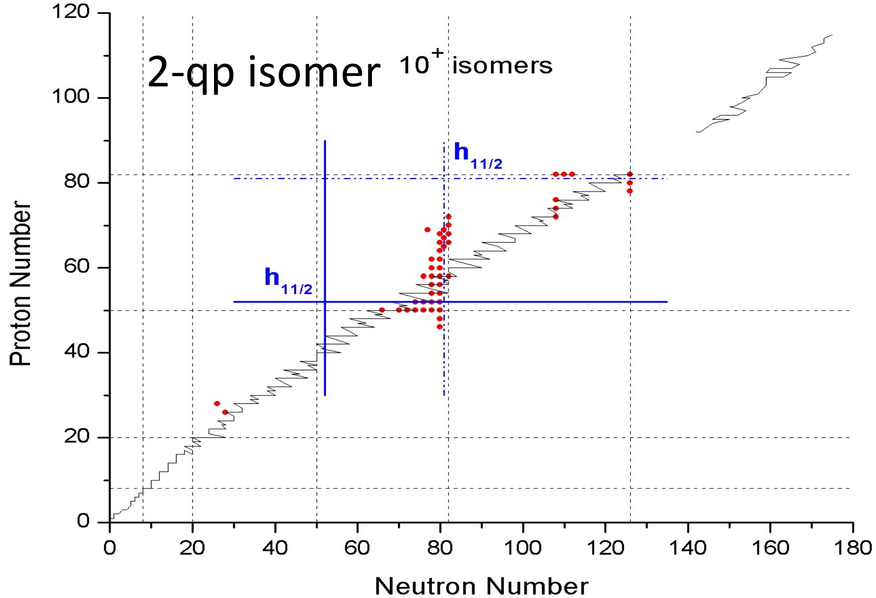
Spin Isomers
K-Isomers
Fission Isomers
Shape Isomers
Seniority Isomers

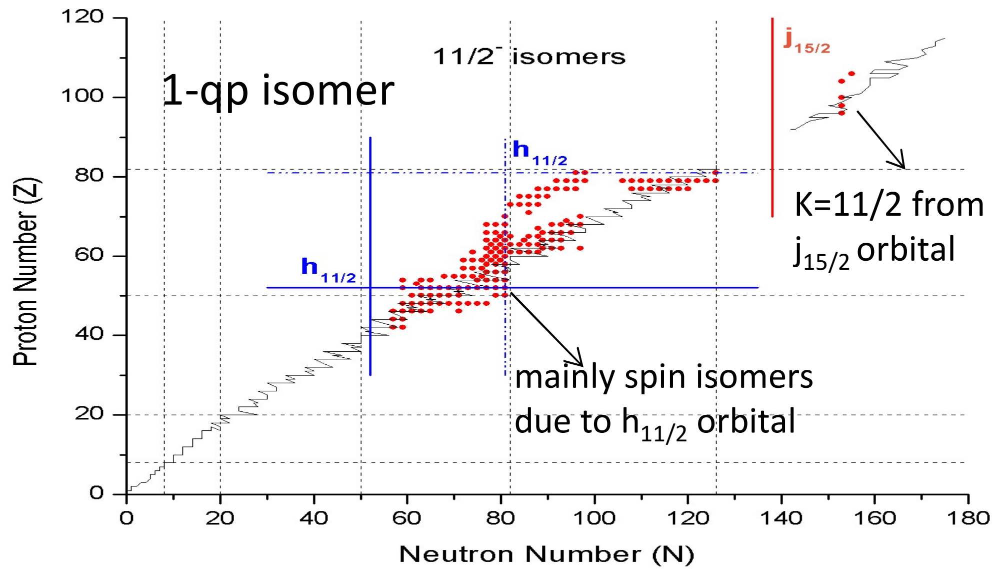


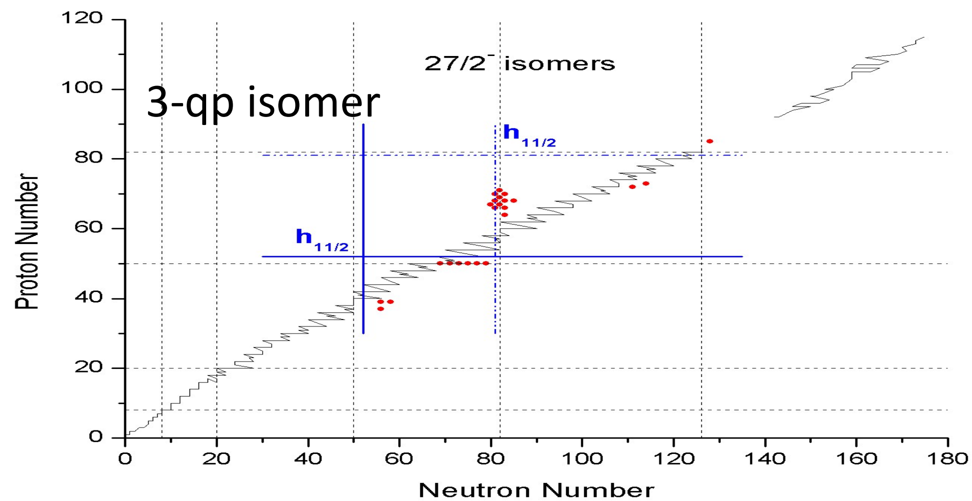
"Atlas of Nuclear Isomers", A. K. Jain, B. Maheshwari, S. Garg, M. Patial and B. Singh, Nuclear Data Sheets, 128 (2015) 1-130.

$h_{11/2}$: $11/2^{-}$, 10^{+} and $27/2^{-}$ isomers

Are they seniority isomers ??







A. K. Jain and B. Maheshwari, https://www-nds.iaea.org/publications/indc/indc-ind-0048/

Why Sn isotopes?

- The longest chain of isotopes in the nuclear chart
- From ^{100}Sn to ^{132}Sn, and beyond up to ^{138}Sn
- To explore the complex nuclear structure near the doubly magic numbers
- A useful set of data to study various systematics, like B(E2) and B(E3) values.
- Emergence of new type of seniority isomers which decay by E1, E3 transitions
- To test the realistic effective interactions
- Estimation of single particle states in the neutron-rich region

What is Seniority ?

- Seniority quantum number (v)= No. of unpaired nucleons
- Leads to a particle number independent energy across a set of isotopes/isotones for a given state
- Leads to a constant energy gap for Δv=0 or, 2 set of states
- Already proved that any interaction between identical fermions in single-*j* shell conserves seniority if $j \leq 7/2$.

PHYSICAL REVIEW VOLUME 63, NUMBERS 9 AND 10 MAY 1 AND 15, 1943

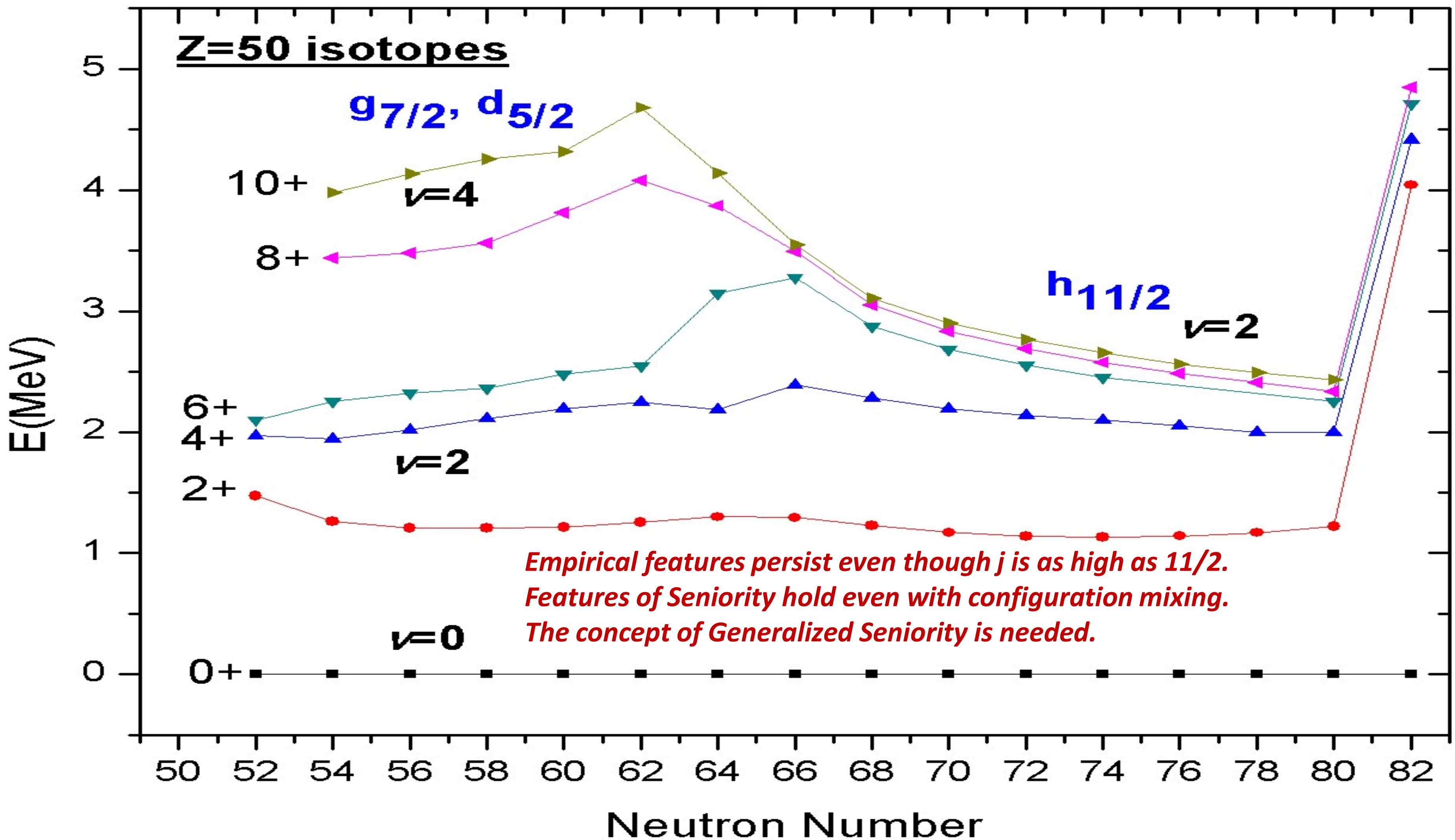
Theory of Complex Spectra. III

GIULIO RACAH
The Hebrew University, Jerusalem, Palestine

(Received February 8, 1943)

The consideration of the phases of the fractional-parentage coefficients allows the extension of the matrix methods to configurations with more than two equivalent electrons. Tables are given for the parentages of the terms of p^n and d^n. Applications are made to the spin-orbit interaction of the d^n terms and to the electrostatic interaction between the configurations d^n, $d^{n-1}s$, and $d^{n-2}s^2$. Errata in Part II are indicated.

Empirical evidences of good seniority



Single-j: Quasi-spin scheme (The theoretical framework)

- The pair creation operator

$$S_j^+ = \frac{1}{2}\sum(-1)^{j-m} a_{jm}^+ a_{j,-m}^+$$

- Therefore, the pair annihilation operator

$$S_j^- = \frac{1}{2}\sum(-1)^{j-m} a_{jm} a_{j,-m}$$

- Also, we define

$$S_j^0 = \frac{1}{2}\sum a_{jm}^+ a_{jm} - \frac{2j+1}{4}$$

- E_{pair}= 2s(s+1)- ½ (Ω-n) (Ω+2-n) = ½ (n-v) (2Ω+2-n-v)
  where s = ½ (Ω-v), S_j^0 = ½ (n- Ω), and Ω= ½ (2j+1)

Decay properties in Seniority scheme (Single-j)

The reduced transition probability

$$B(EL) = \frac{1}{2J_i + 1}\left|\left(J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i \right\| J_i \right)\right|^2$$

For L even, κ=0 component of quasi-spin vector

For Δν=0, seniority conserving transitions

$$\left\langle j^n v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i) \right\| j^n v l' J_i \right\rangle = \left(\frac{\Omega - n}{\Omega - v} \right) \left\langle j^v v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i) \right\| j^v v l' J_i \right\rangle$$

For Δν=2, seniority changing transitions

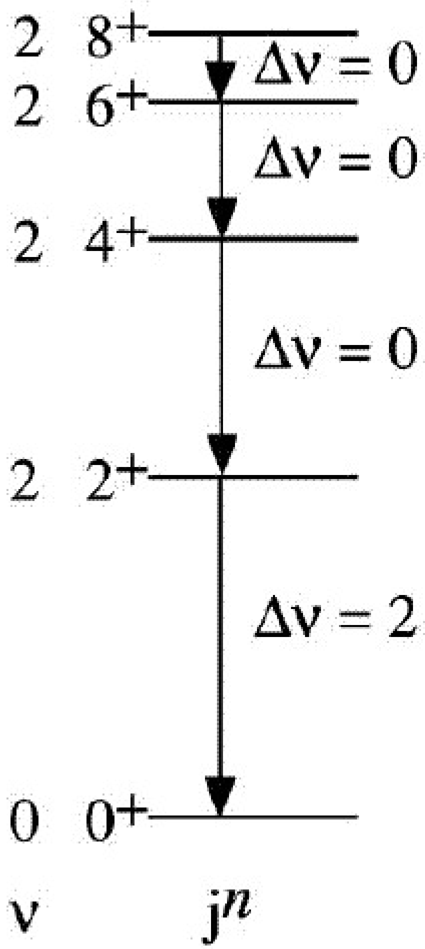
$$\left\langle j^n v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i) \right\| j^n, v \mp 2, l' J_i \right\rangle = \sqrt{\frac{(n - v + 2)(2\Omega + 2 - n - v)}{2(2\Omega + 2 - 2v)}} \left\langle j^v v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i) \right\| j^v, v \mp 2, l' J_i \right\rangle$$

For L odd, quasi-spin scalar, only Δν=0

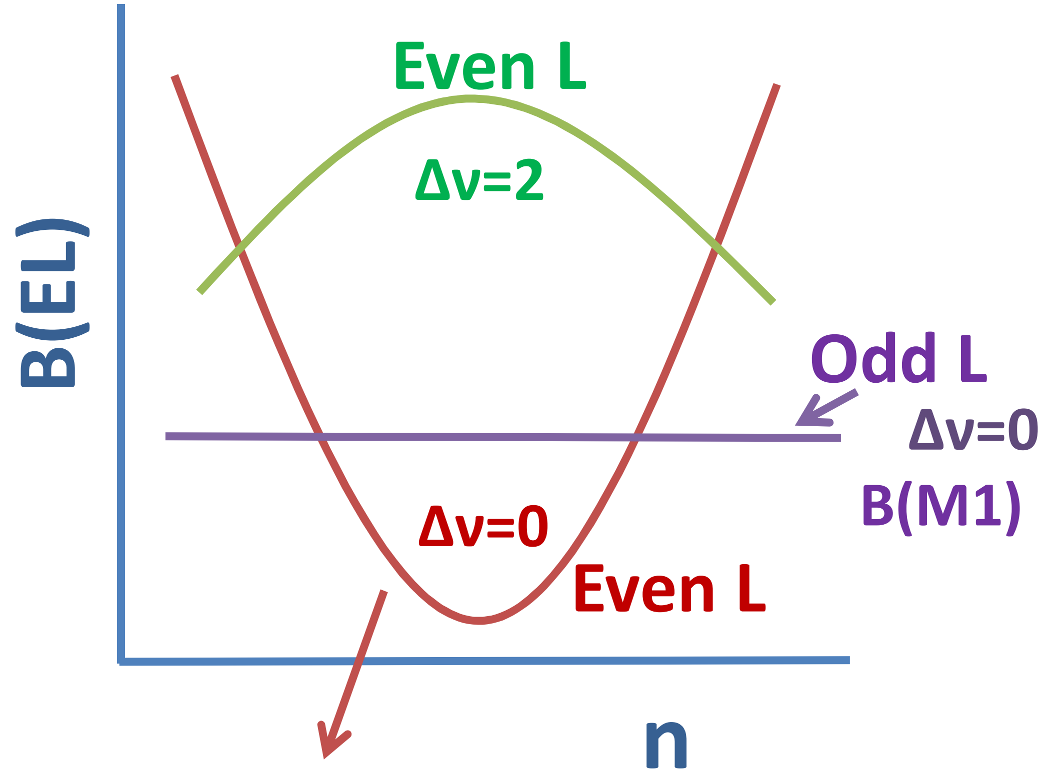
$$\left\langle j^n v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i) \right\| j^n v' l' J_i \right\rangle = \left\langle j^v v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i) \right\| j^v v' l' J_i \right\rangle \delta_{v,v'}$$

Here pair degeneracy $\Omega = \frac{1}{2}(2j + 1)$

Decay properties in single-j seniority scheme



j=9/2



Origin of single-j Seniority isomers

As expected: E2, seniority(v) isomers

- 10^+, v=2 ($h_{11/2}$) in even-even Sn-isotopes
- $27/2^-$, v=3 ($h_{11/2}$) in even-odd Sn-isotopes
- Qualitatively understood in terms of seniority isomers, as expected from the single-j scheme. **However, no quantitative results were available so far.**

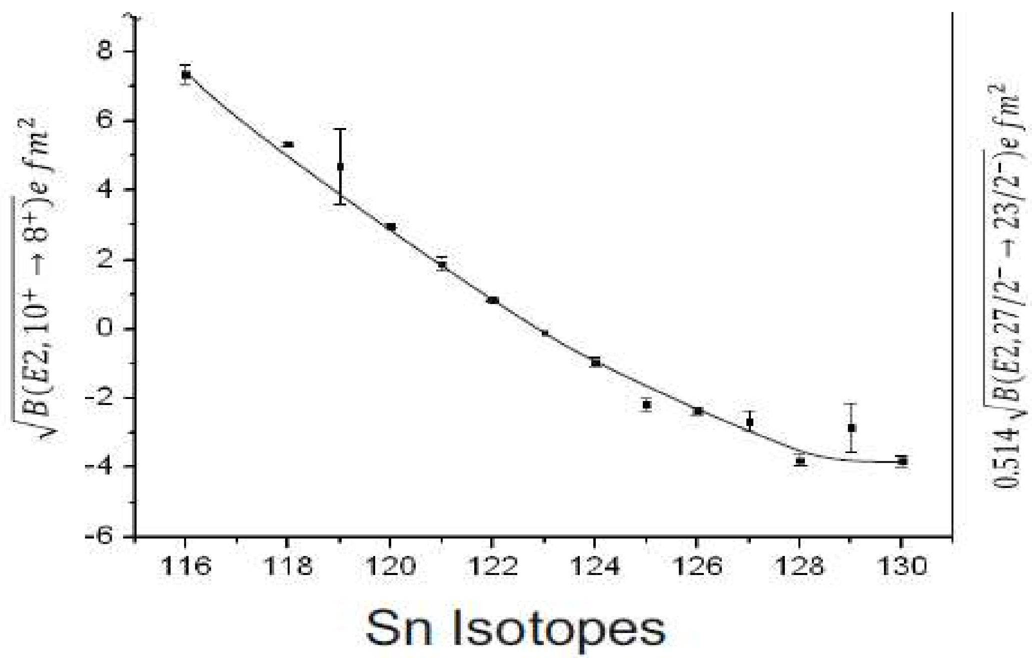


Fig. taken from: Iskra *et al.*, Acta Physica Polonica B, 46 (2015) 651.

Multi-j: quasi-spin scheme

I. Talmi, Simple Models of Complex Nuclei. (Harwood, 1993).

- For multi-j shell, we define $S^+ = \sum S_j^+$
- The commutation between pair creation operator S^+ and single-nucleon Hermitian tensor $T_\kappa^{(k)}$, can be written as

$$[S^+, T_\kappa^{(k)}] = \frac{1}{\sqrt{2k+1}} \sum_{j<j'} \left[1 + (-1)^k\right] \left(j \left\| T^{(k)} \right\| j'\right) \left(a_j^+ \times a_{j'}^+\right)_\kappa^{(k)}$$

- κ is the component of tensor with rank k.
- a_j^+ is the creation operator for particle in j-orbit.
- Therefore,
  - For k odd, the tensor → quasi-spin scalar.
  - For k even, the tensor → κ =0 component of the quasi-spin vector.

For many orbitals with different j

- The pair creation operator now becomes,

$$S_1^+ = \sum_j (-1)^{l_j} S_j^+$$

where l_j denotes the orbital angular momentum of the j-orbit.

Green and Moszkowski, Phys. Rev. 139, 790 (1965).

- **The new selection rules:**
  - **The phase factor $(-1)^k$ modifies as $(-1)^{l+l'+k}$, therefore,**
    - **For $l+l'+k$ odd, $T_\kappa^{(k)}$ tensor is a quasi-spin scalar.**
    - **For $l+l'+k$ even, $T_\kappa^{(k)}$ tensor is a κ=0 component of the quasi-spin vector.**

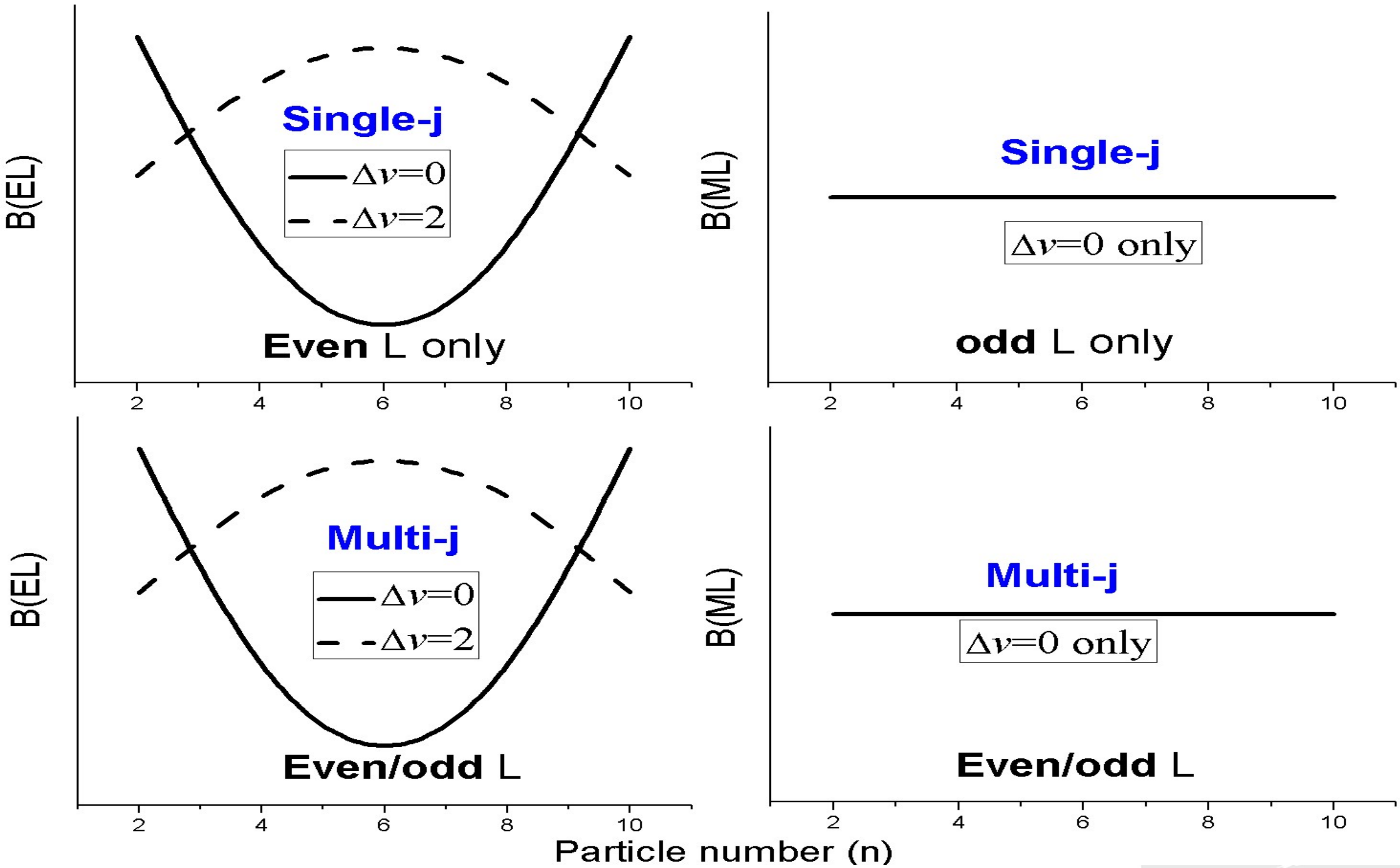
Electric/ magnetic multipole transitions

- $l+l'+L$ always remains even, for any L value (odd or even) in electric transitions.
  - l and l' control the parity of the initial and final states of the transition.
  - L specifies the nature of the transition.
  - For L odd, $l+l'$ = odd.
  - For L even, $l+l'$ = even.

- Hence, we conclude that Electric transitions, for both the odd and even L, now behave as the κ=0 component of the quasi-spin vector.

- The magnetic transitions, on the other hand, for both even and odd L, behave as quasi-spin scalar due to $l+l'+L$ =odd.

Comparison b/w single-j and multi-j shell



Seniority isomerism in single-j and multi-j shell

- Single-j : Only **even tensor** electric transitions between the same seniority states can have a minimum in the middle of their transition probabilities.→ Since parity change is not possible in single-j case, therefore, only E2 transitions correspond to seniority isomerism.

- Multi-j : Electric transition probabilities now have a dip in the middle irrespective of the nature of the involved tensor (**even or odd**) between the same seniority states. → One can expect seniority isomers in all E1, E2, E3,…. transitions depending upon the initial and final states.

A new kind of seniority isomers

- We show for the first time the occurrence of seniority isomers, which decay by odd-tensor transitions to the same seniority states.
- In our recent paper, we have presented an evidence of odd-tensor E1 decaying generalized seniority isomers.

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Odd tensor electric transitions in high-spin Sn-isomers and generalized seniority

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ABSTRACT

The similar behavior of the $B(E1)$ values of the recently observed 13^- odd tensor $E1$ isomers and the $B(E2)$ values of the 10^+ and 15^- even tensor $E2$ isomers in the Sn-isotopes has been understood in terms of the generalized seniority for multi-j orbits by using the quasi-spin scheme. This simple approach proves to be quite successful in explaining the measured transition probabilities and the corresponding half-lives in the high-spin isomers of the semi-magic Sn-isotopes. Hence, we show for the first time the occurrence of seniority isomers in the 13^- Sn-isomers, which decay by odd-tensor $E1$ transitions to the same seniority states.

Odd-tensor seniority isomers in $^{120\text{-}126}$Sn

- Odd tensor seniority isomers have been discovered for the very first time. Here we show the 13- isomers.
- Take ^{114}Sn as a core by freezing $g_{7/2}$ and $d_{5/2}$ orbits.
- The remaining active orbits:
  - $h_{11/2}$, $d_{3/2}$ and $s_{1/2}$.
  - Possible mixed configurations

50-82 valence space
$g_{7/2}$, $d_{5/2}$, $h_{11/2}$, $d_{3/2}$ and $s_{1/2}$ orbits

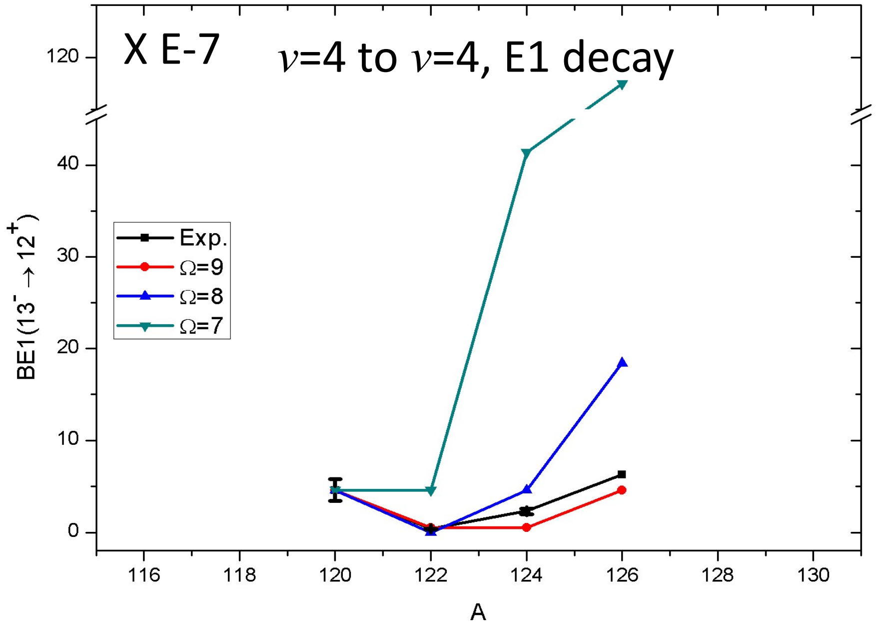
$$\tilde{j} = h_{11/2} \otimes s_{1/2} \quad \Omega = 7$$

$$\tilde{j} = h_{11/2} \otimes d_{3/2} \quad \Omega = 8$$

$$\tilde{j} = h_{11/2} \otimes d_{3/2} \otimes s_{1/2} \quad \Omega = 9$$

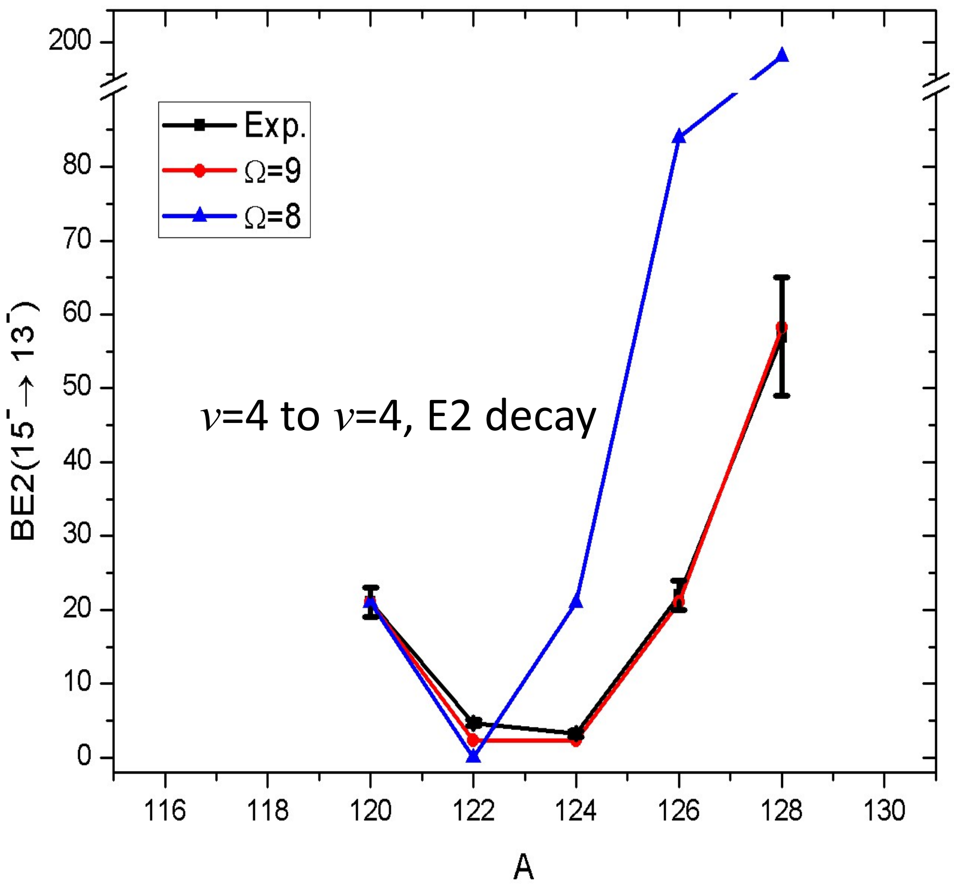
$$B(EL) \propto \left(\frac{\Omega - n}{\Omega - v} \right)^2, \Delta v = 0$$

$$B(EL) \propto \frac{(n - v + 2)(2\Omega + 2 - n - v)}{2(2\Omega + 2 - 2v)}, \Delta v = 2$$

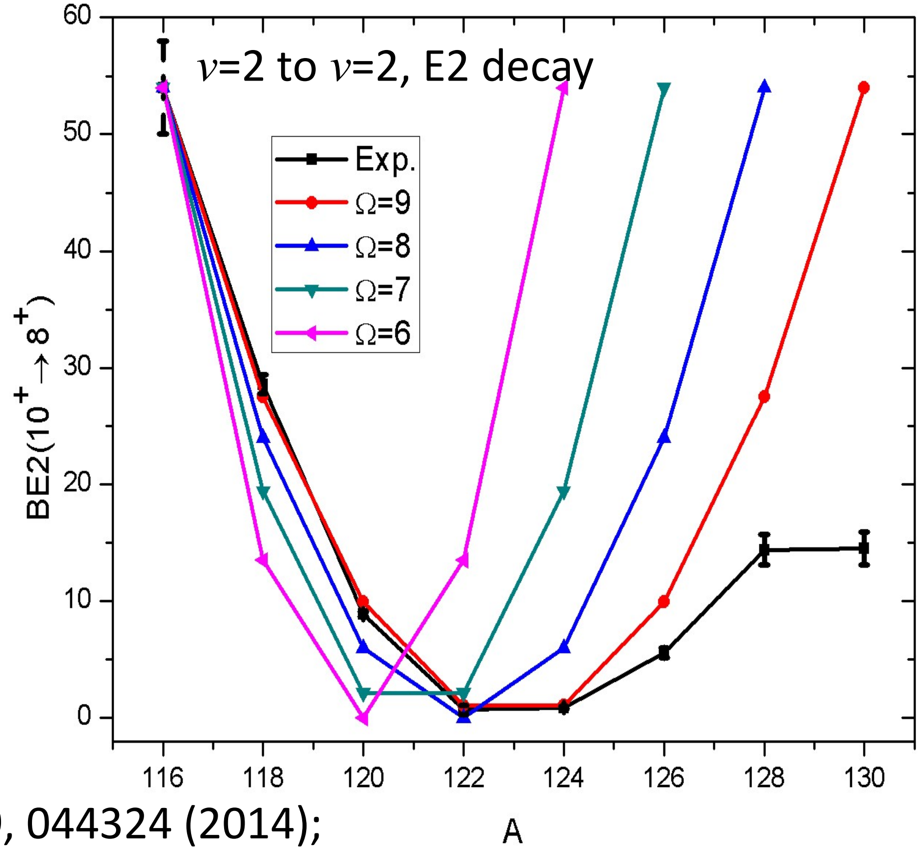


Only Ω=9 explains the B(E1) behavior.

The role of configuration mixing in Sn-isomers



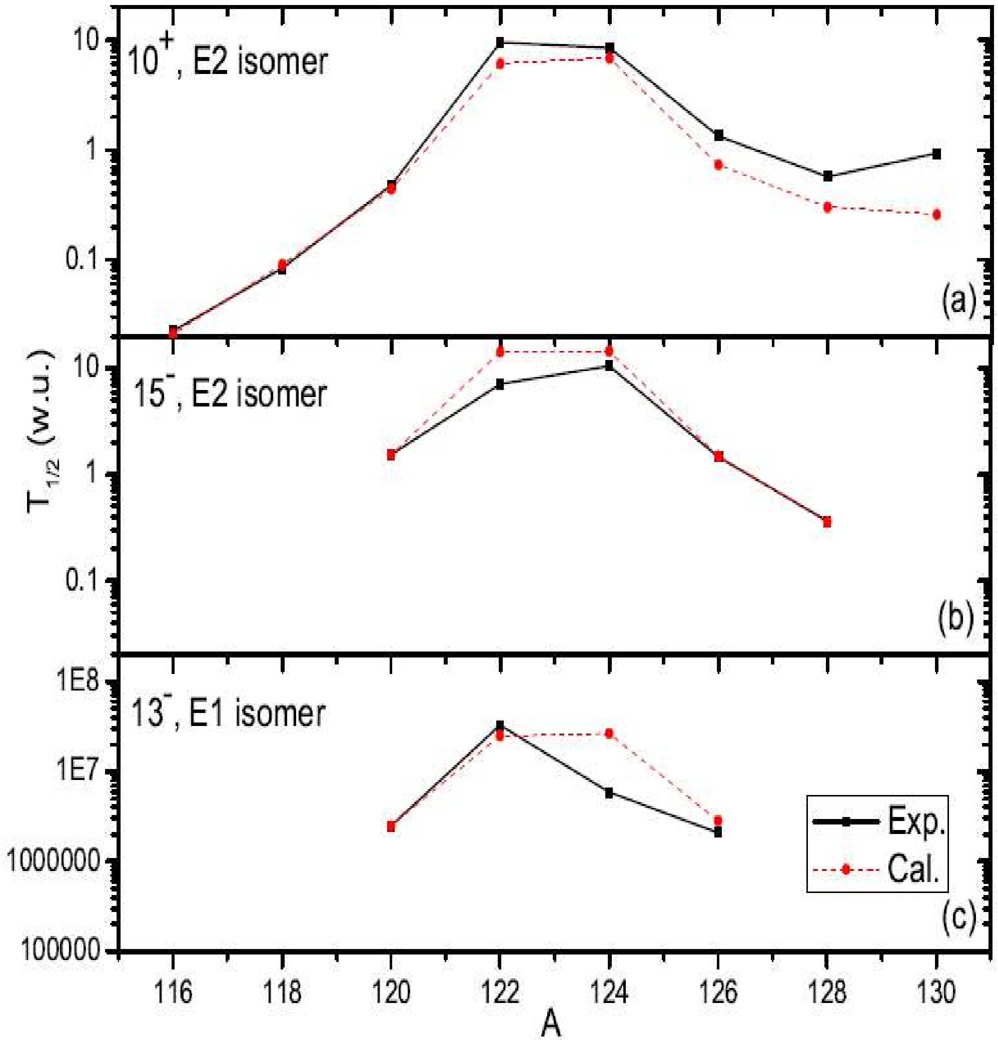
10^+ isomers: so far qualitatively explained as $h_{11/2}{}^2$ config.
We explain them as generalized seniority Isomers, involving config mixing.



E2 and E1 isomers behave similar to each other.
B(E2)s are in e^2fm^4 , and B(E1)s are in e^2fm^2.

Exp. Data for 13- and 15- isomers: Phys. Rev. C 89, 044324 (2014);
for 10+ isomers: Phys. Rev. Lett. 68, 11 (1992), Phys. Rev. C 62, 057305 (2000).

Half-life calculations for Sn isomers



Total transition probabilities

$$T(E1) = 1.587 \times 10^{15} {E_{\gamma i}}^3 \times B(E1) \times (1+\alpha_i)$$

$$T(E2) = 1.223 \times 10^{9} {E_{\gamma i}}^5 \times B(E2) \times (1+\alpha_i)$$

Where B(EL) are in units of e^2fm^L,
L: multipolarity,
E_γ are in MeV,
and T(EL) are in second^{-1}

$$T_{\frac{1}{2}}^{\gamma_i} = 0.693/T(EL)$$ in seconds

Total half-life

$$T_{\frac{1}{2}} = T_{\frac{1}{2}}^{\gamma_i} \times B.R._i$$

$$B.R._i = \frac{I_{\gamma i}(1+\alpha_i)}{\sum_i I_{\gamma i}(1+\alpha_i)}$$

Twin parabolas in the B(E2) variation of Sn isotopes

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Asymmetric behavior of the $B(E2\uparrow; 0^+ \rightarrow 2^+)$ values in $^{104-130}$Sn and generalized seniority

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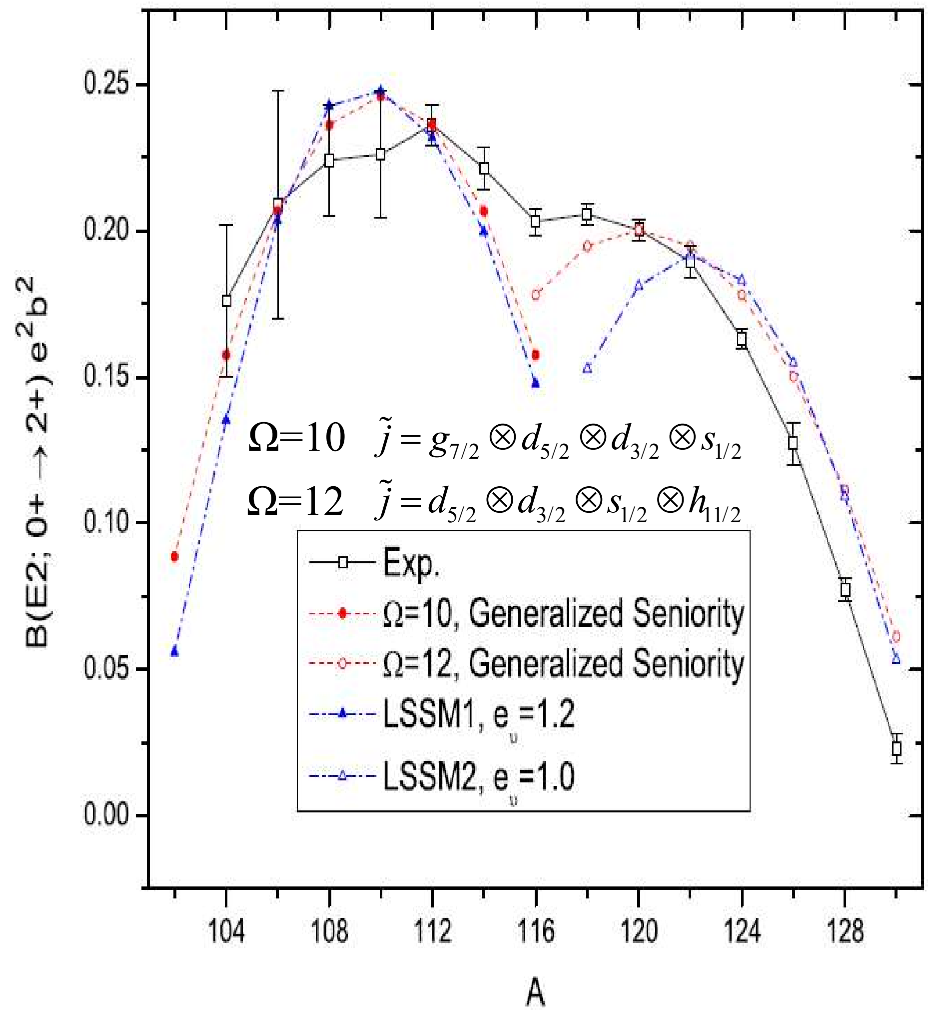
Abstract

We present freshly evaluated $B(E2\uparrow; 0^+ \rightarrow 2^+)$ values across the even–even Sn-isotopes which confirm the presence of an asymmetric behavior as well as a dip in the middle of the full valence space. We explain these features by using the concept of generalized seniority. The dip in the $B(E2)$ values near ^{116}Sn is understood in terms of a change in the dominant orbits before and after the mid shell, which also explains the presence of asymmetric peaks in the $B(E2)$ values. This approach helps in deciding the most active valence spaces for a given set of isotopes, and single out the most useful truncation scheme for Large Scale Shell Model (LSSM) calculations. The LSSM calculations so guided by generalized seniority are also able to reproduce the experimental data on $B(E2)\uparrow$ values quite well.

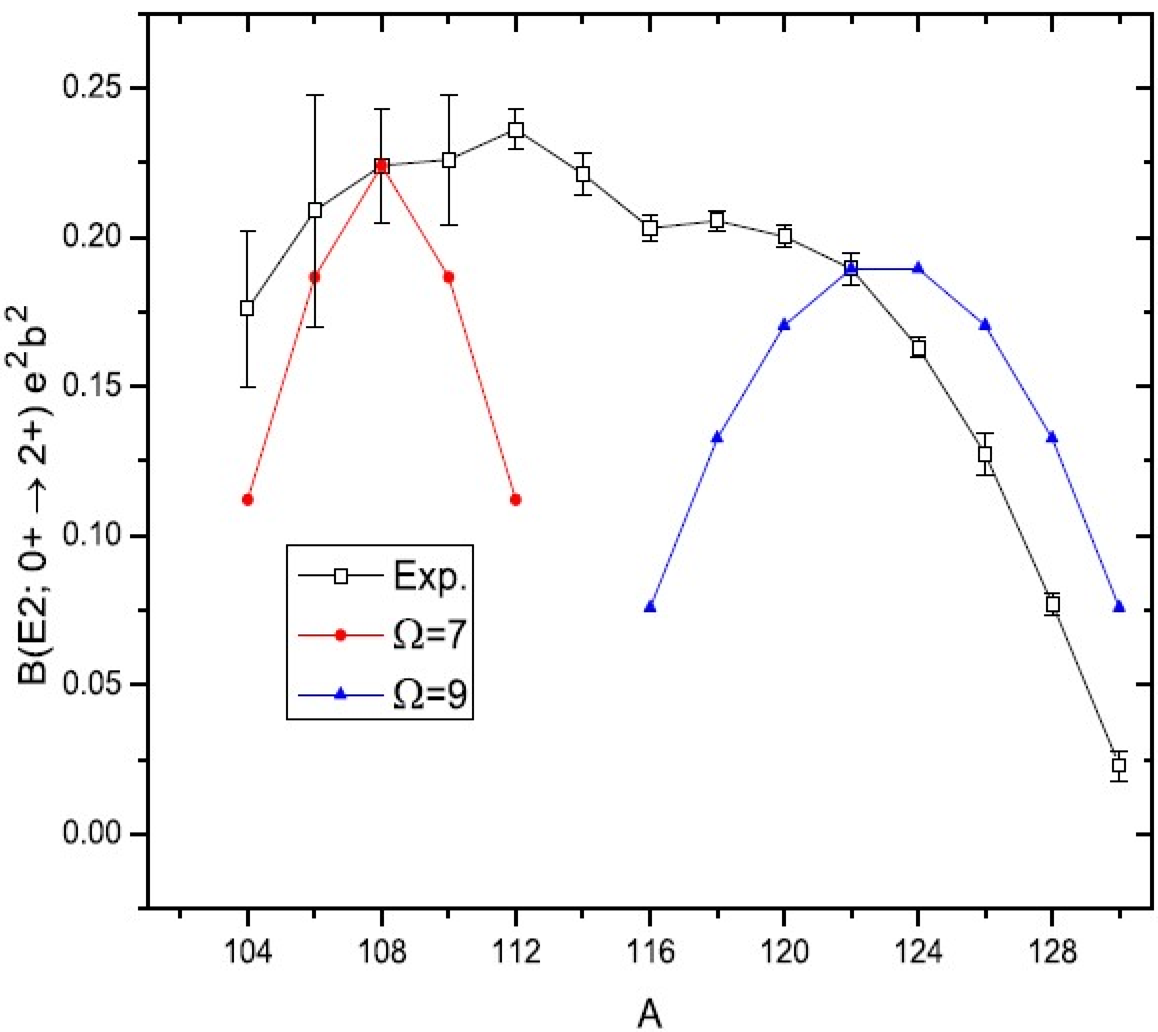
The first excited 2^+ states in Sn-isotopes

- **Expected:** a single parabolic trend coming from generalized seniority using full valence space.
- **Observed:** A minima in the middle of the valence space from recent measurements and our evaluation of the data.
- **Reason:** The different rates of filling of the orbits involved, in agreement with Morales *et al.* [**Phys. Lett. B 703, 606-608 (2011)**]
- Our calculations based on Generalized seniority, before and after the mid-shell, explain the data very well.
- LSSM calc, guided by generalized seniority assigned by us, also explains the trend very well.



B. Maheshwari, A. K. Jain and B. Singh, Nucl. Phys. A **952**, 62 (2016).

Other configurations do not explain!

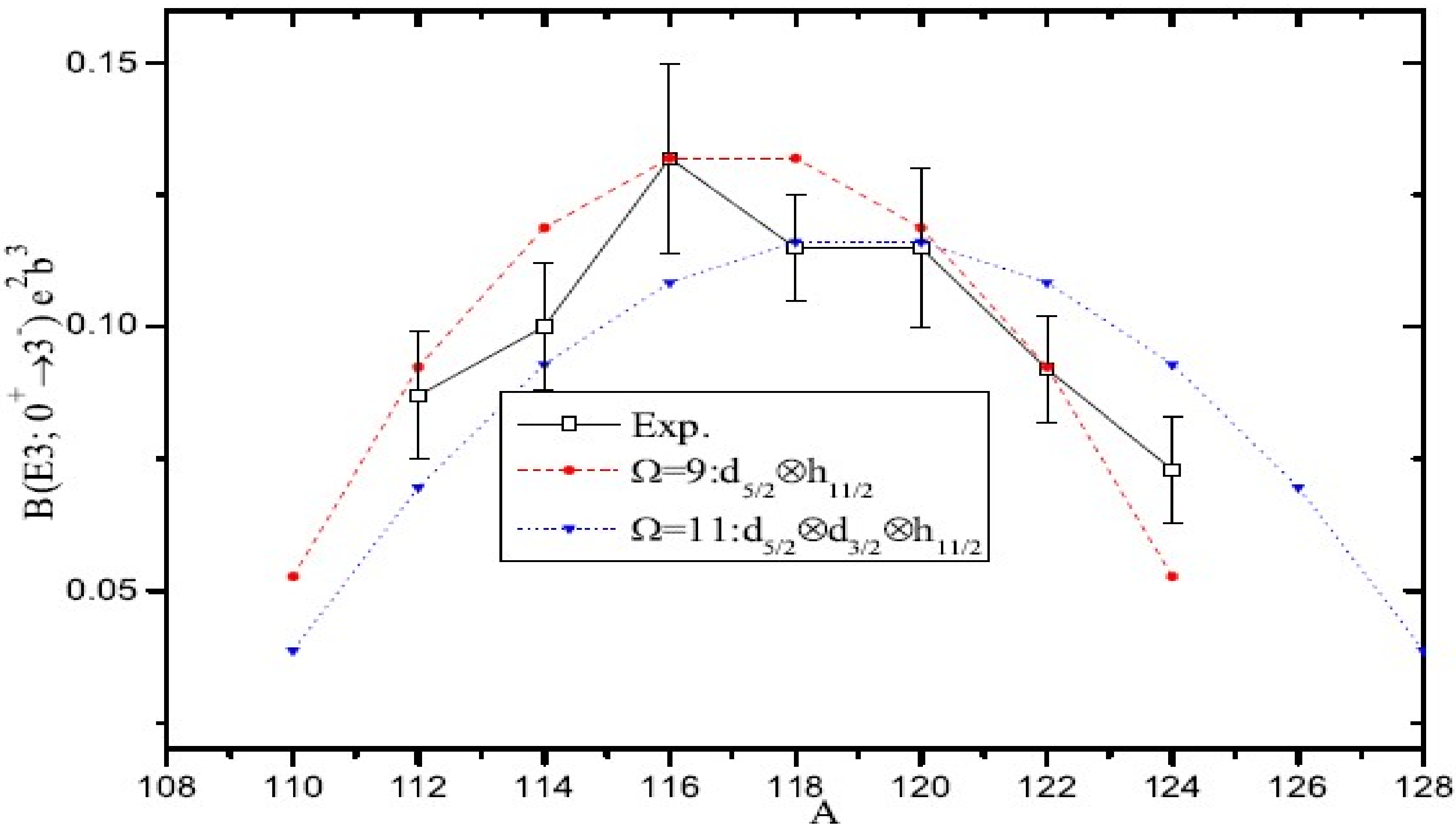


$\Omega=7,\ g_{7/2}*d_{5/2}$

$\Omega=9,\ d_{3/2}*s_{1/2}*h_{11/2}$

The first excited 3- states in Sn isotopes

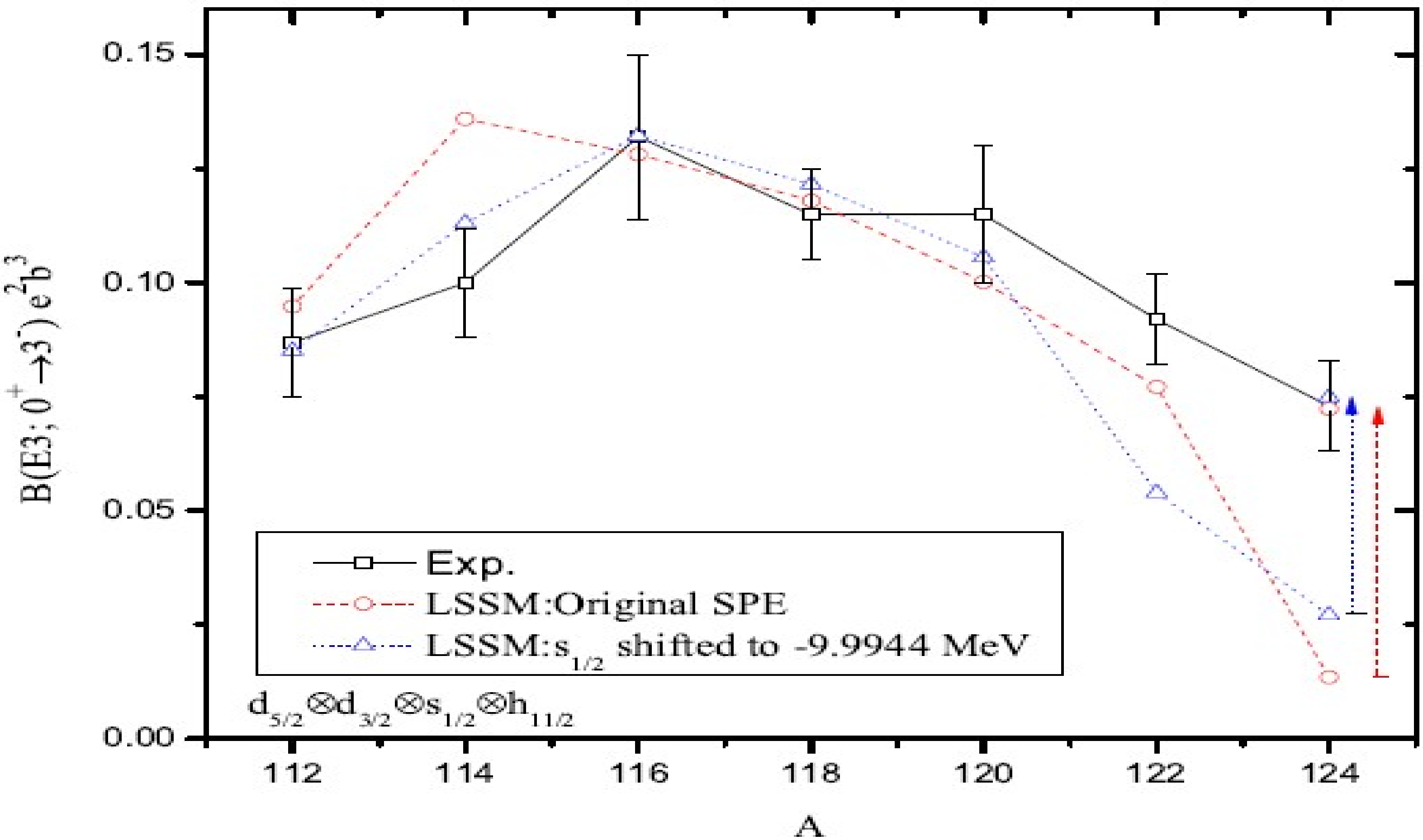
Active set of orbitals : $g_{7/2}, d_{5/2}, d_{3/2}, s_{1/2}, h_{11/2}$
Assumed core: N=58 ($g_{7/2}$ is full.)



Experimental data : Kibedi and Spear
Atomic Data and Nucl. Data Tables **80,** 35 (2002).

LSSM calculations

Original SPE	-10.6089,	-10.2893,	-8.7167,	-8.6944,	-8.8152 MeV
	0g7/2,	1d5/2,	1d3/2,	2s1/2 ,	0h11/2



SN100PN: B. A Brown, N. J. Stone, J. R. Stone, I. S. Towner, and M. Hjorth-Jensen, Phys. Rev. C 71 (2005) 044317.

Generalized Seniority in n-rich nuclei beyond ^{132}Sn

- We understand the B(E2) properties of n-rich Sn-isomers, and explain the anomaly at ^{136}Sn in terms of seniority mixing.

PHYSICAL REVIEW C **91**, 024321 (2015)

6^+ isomers in neutron-rich Sn isotopes beyond $N = 82$ and effective interactions

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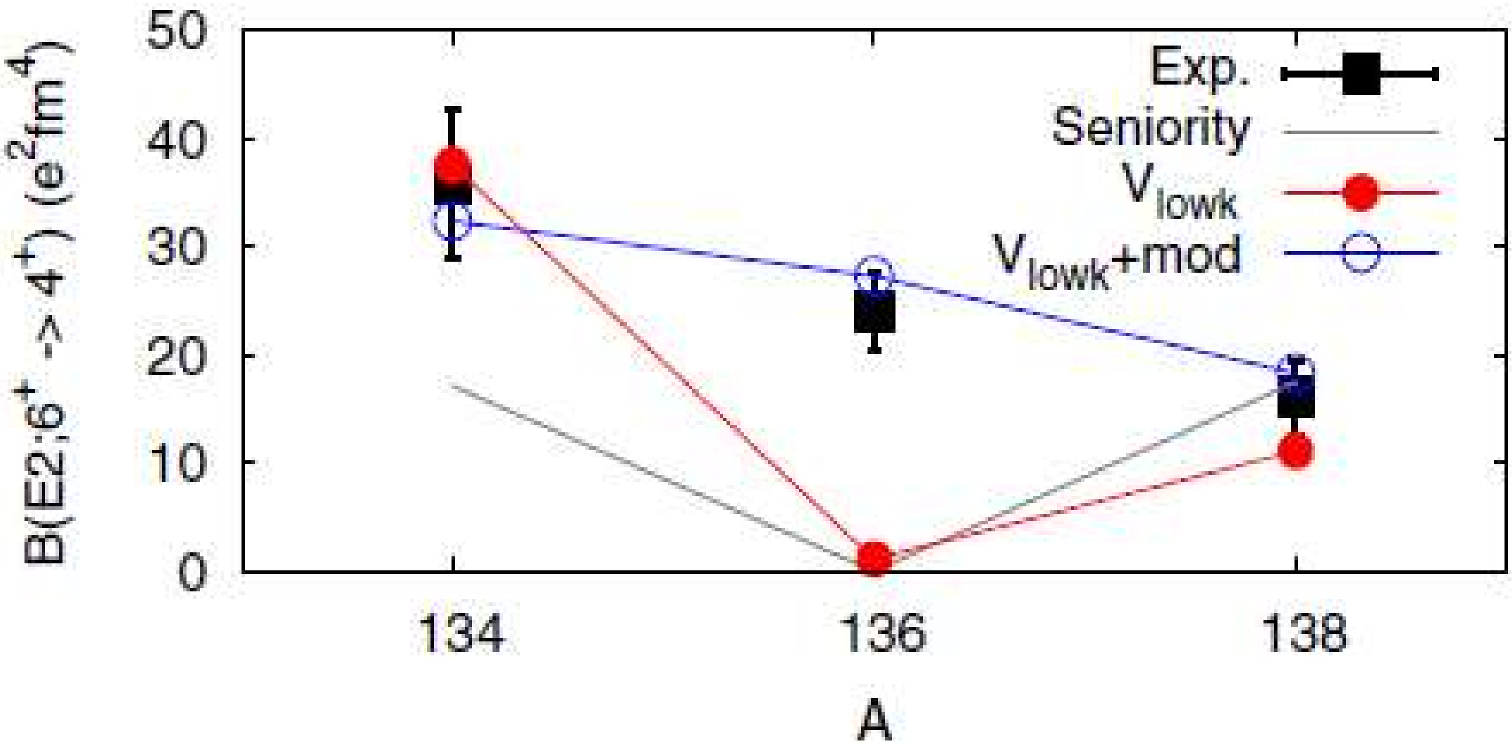
(Received 20 November 2014; revised manuscript received 5 January 2015; published 23 February 2015)

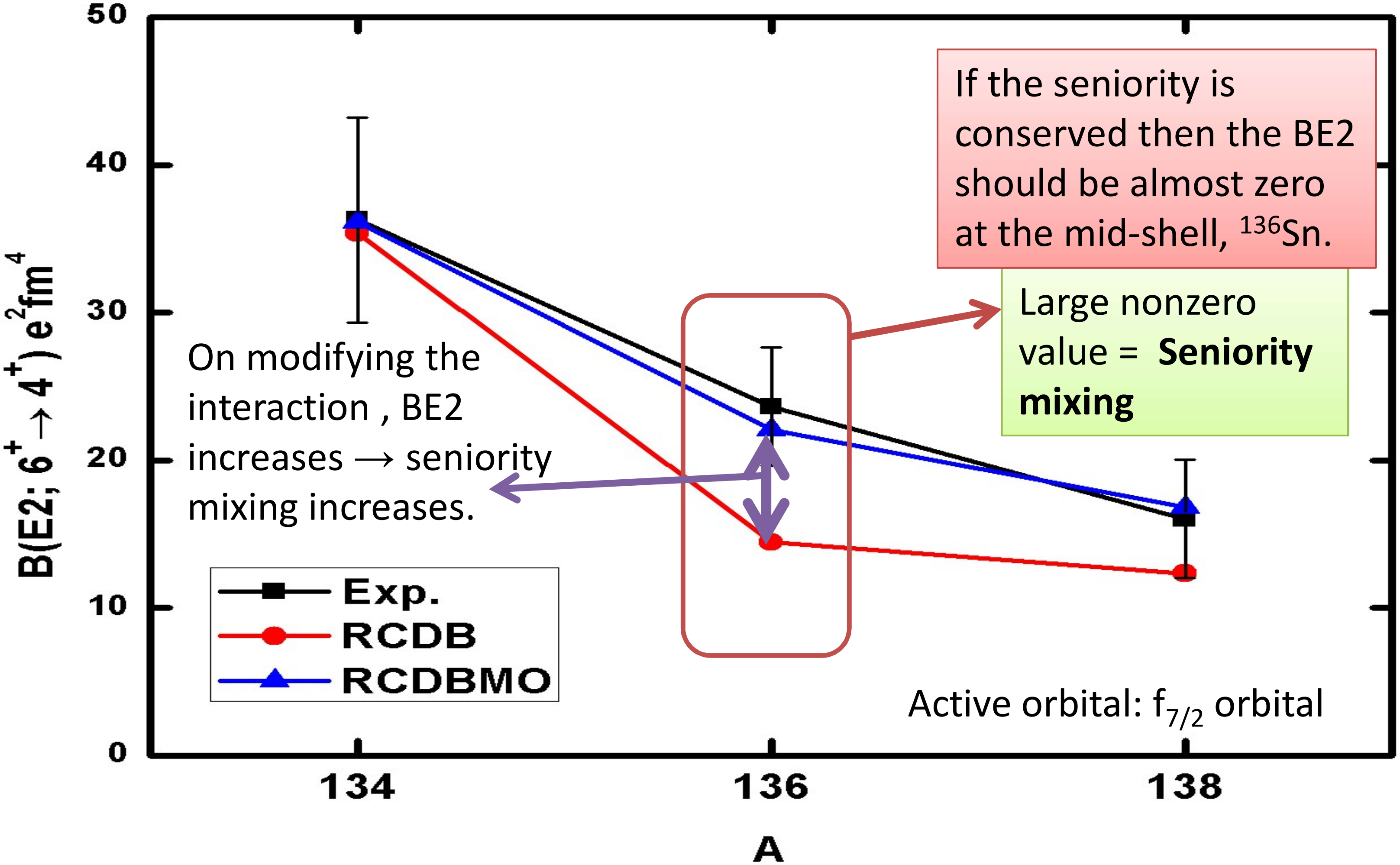
Recent observation of the 6^+ seniority isomers and measurements of the $B(E2)$ values in the $^{134-138}$Sn isotopes lying close to the neutron drip line have raised some questions about the validity of the currently used effective interactions in the neutron-rich region. Simpson *et al.* [Phys. Rev. Lett. **113**, 132502 (2014)] had to modify the diagonal and nondiagonal $\nu f_{7/2}^2$ two-body matrix elements of the $V l k$ interaction by ~150 keV in their shell model calculations in order to explain the data of ^{136}Sn. In contrast, we are able to explain the observed energy levels and the $B(E2)$ values after marginal reduction of the same set of matrix elements by 25 keV in the RCDB (renormalized CD-Bonn) interaction. The observed mismatch in reproducing the data of ^{136}Sn is due to the seniority mixing. Further, we do not find it necessary to consider the core excitations, and the RCDB interaction seems better suited to explain the data beyond $N = 82$ magic number.

Simpson et al. PRL 113, 132502 (2014).



mod: Reduced TBME of $f_{7/2}$ by 150 keV

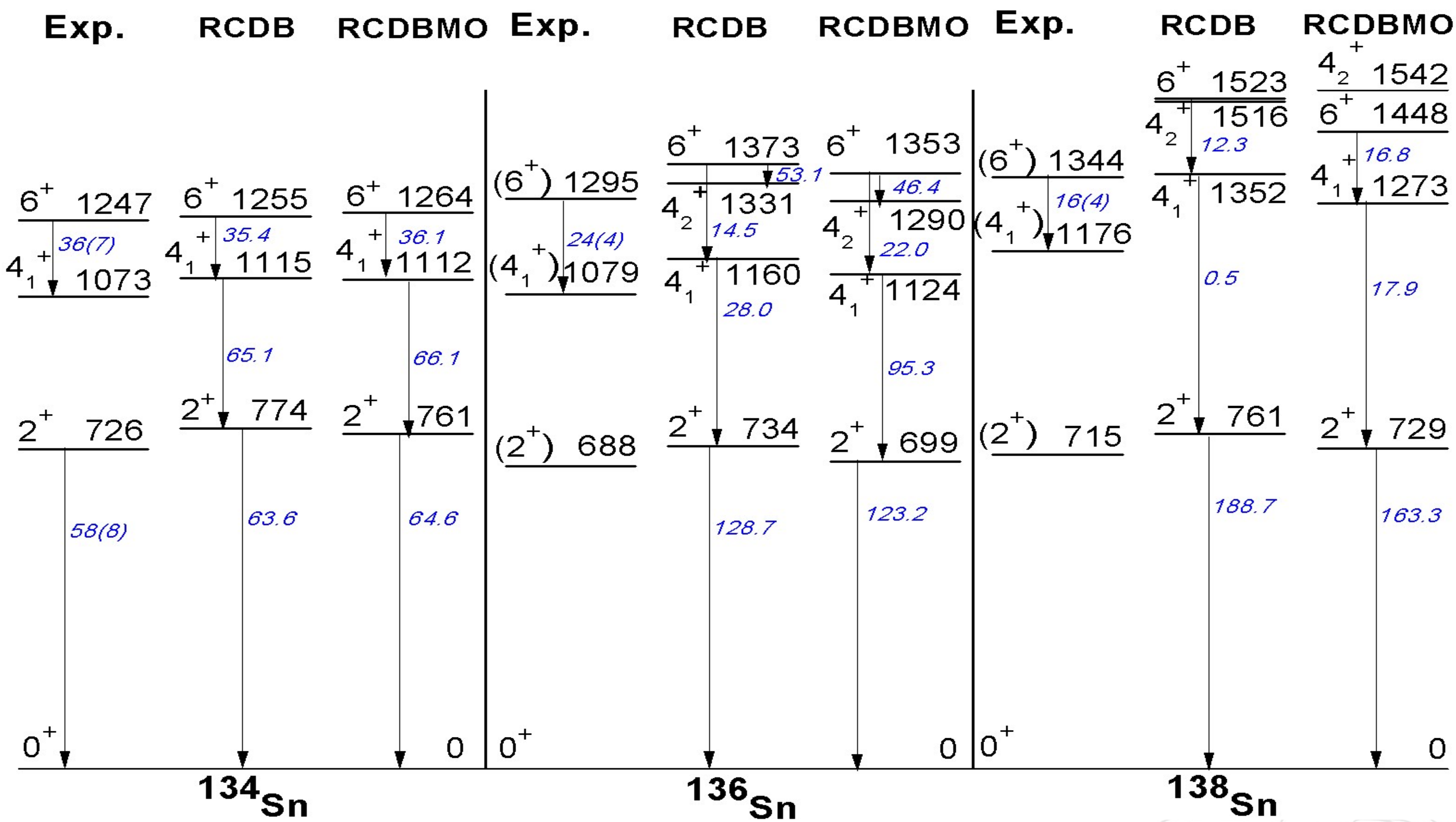
A small change in TBME & seniority mixing



RCDBMO: modified RCDB by reducing the diagonal and non-diagonal $\nu f_{7/2}{}^2$ TBME by 25 keV.

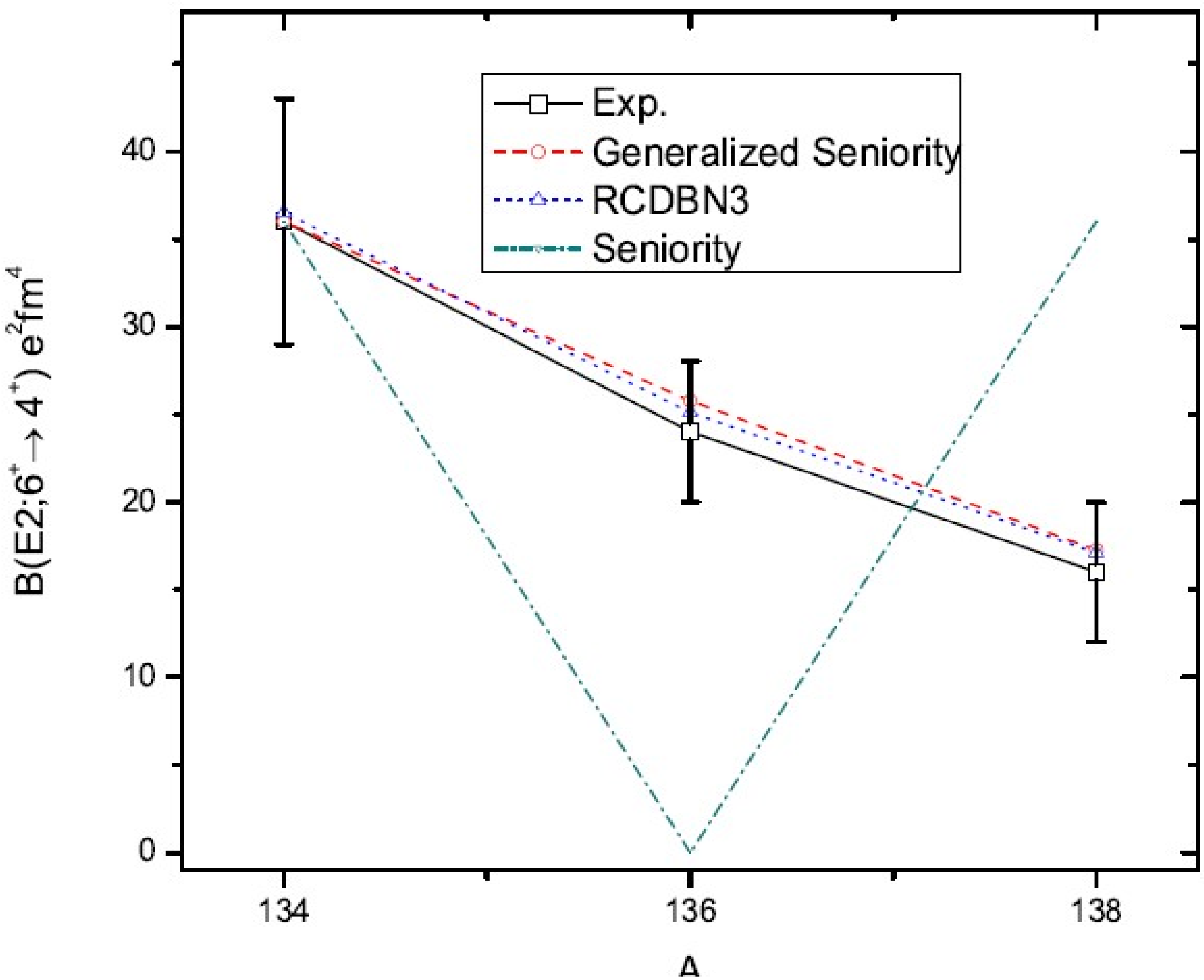
6⁺ seniority isomers beyond ¹³²Sn

B. Maheshwari, A. K. Jain and P. C. Srivastava, Phys. Rev. C **91**, 024321 (2015)



Exp. Data: Simpson *et al.* Phys. Rev. Lett. 113, 132502 (2014)

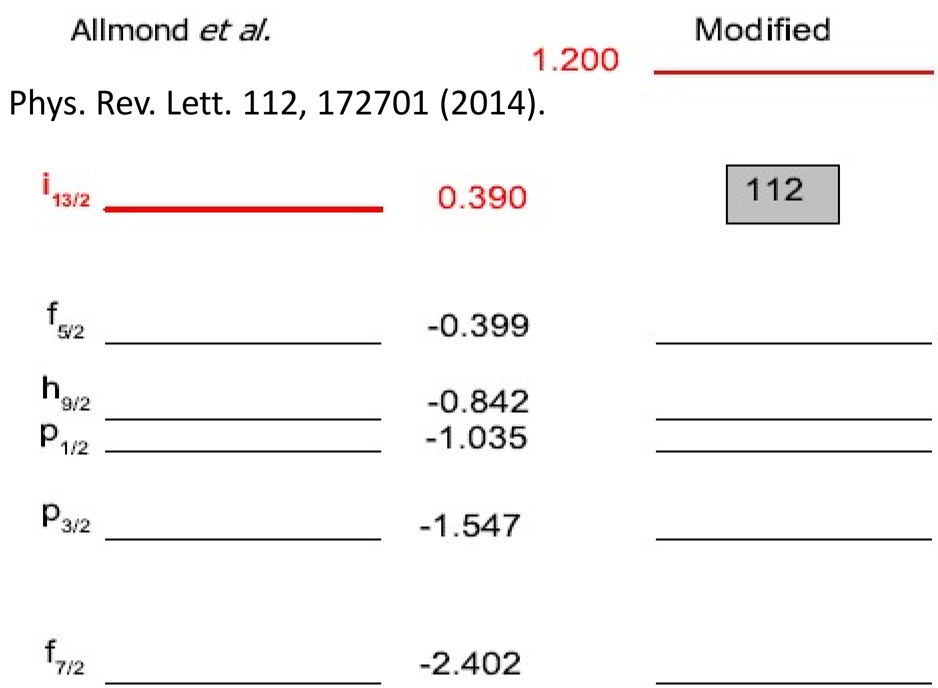
Generalized Seniority



We take the mixing of all the active orbitals($h_{9/2}$, $f_{7/2}$, $p_{3/2}$, $f_{5/2}$, $p_{1/2}$) except $i_{13/2}$ for the generalized seniority calculations.

$N = 82 - 126$ for these isotopes consists of $h_{9/2}$, $i_{13/2}$, $f_{7/2}$, $p_{3/2}$, $f_{5/2}$ and $p_{1/2}$ orbitals

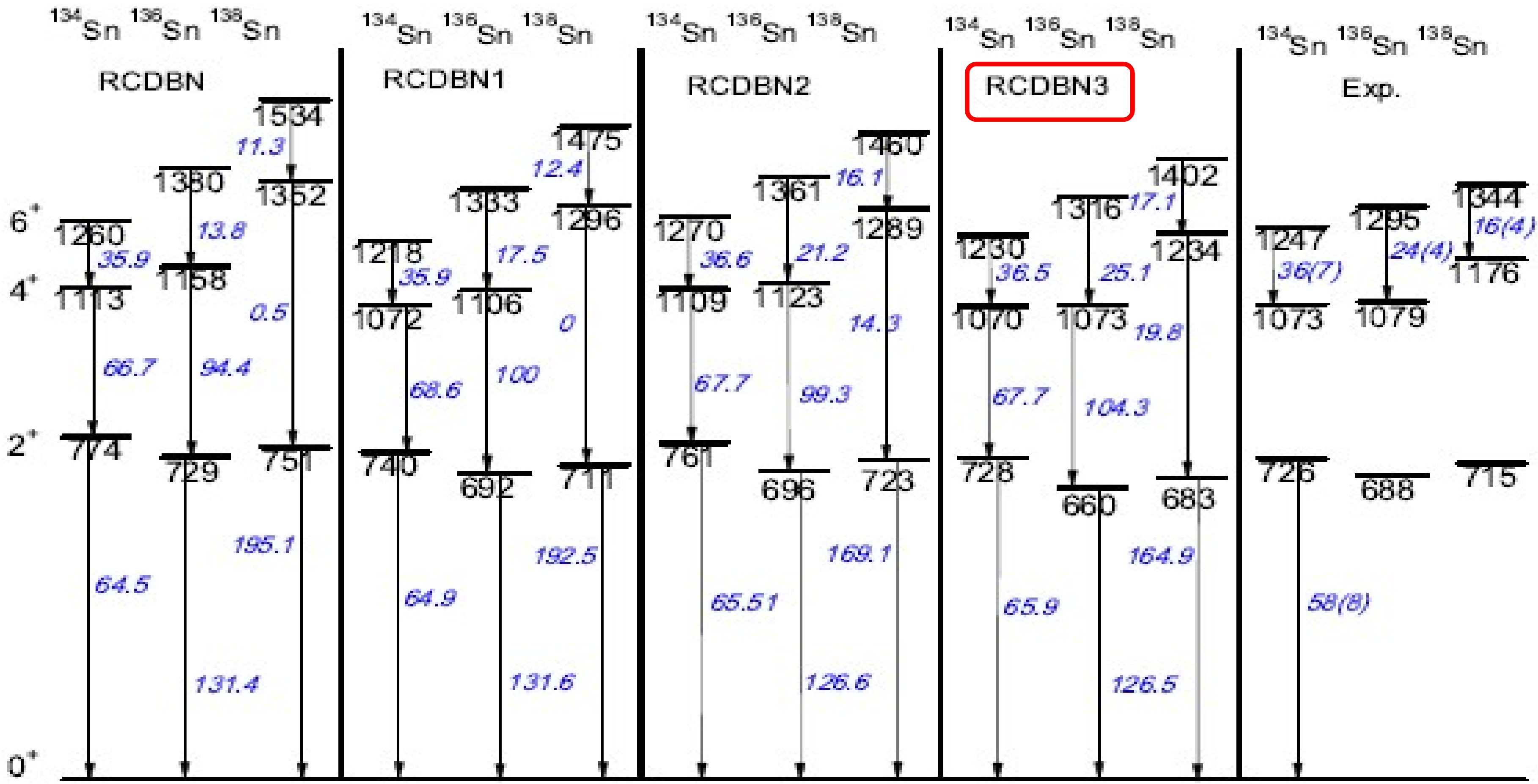
LSSM calculations with new set of SPE



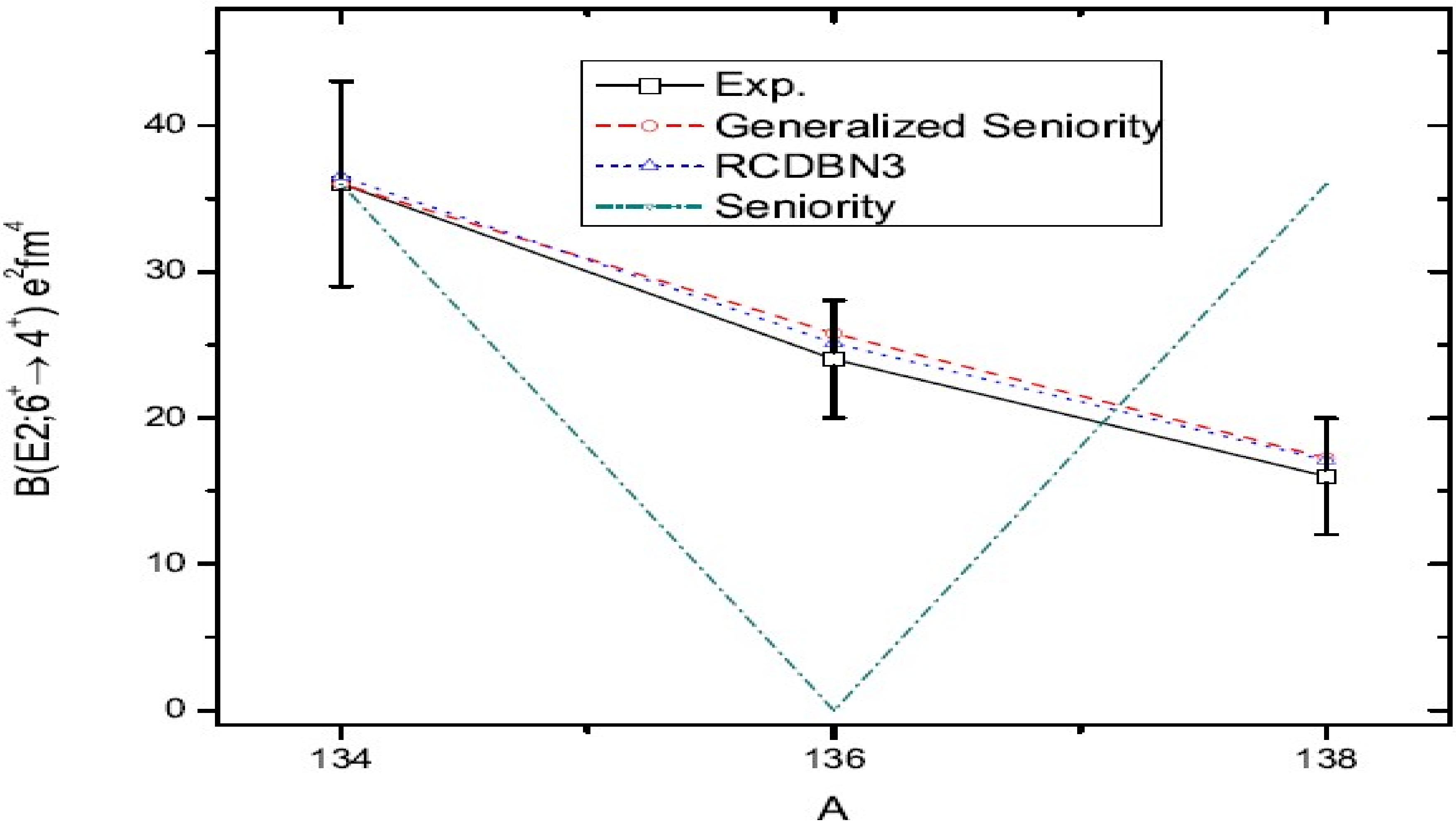
- RCDBN : new set of energies
- RCDBN1 : $i_{13/2}$ shifted as presented.
- RCDBN2: 25 keV reduction from $f_{7/2}^2$ TBME.
- RCDBN3: $i_{13/2}$ shifted + 25 keV reduction.

Level schemes from LSSM results and Exp. data

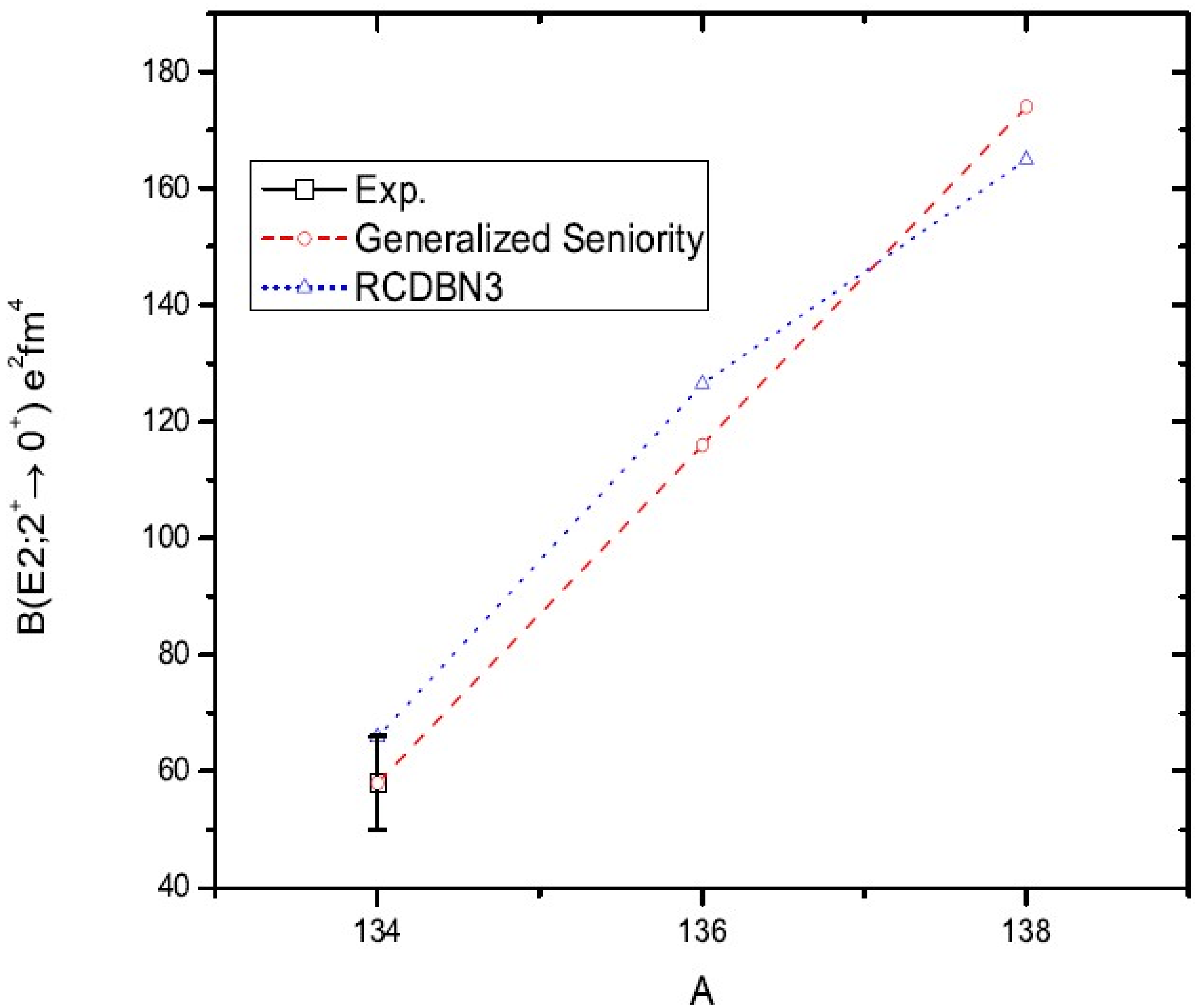
Both level energies and B(E2) values are consistent to the measured values



RCDBN3 results and Generalized Seniority



B(E2)s for the 2_1^+ states in $^{134-138}Sn$



What we learn and conclude from Sn to Sn

- The concept of Seniority has been extended to the Generalized Seniority for multi-j shell in a very simple way. Isomers in the Z=50 isotopic chain have been addressed.
- We find for the first time **emergence of the odd tensor E1, seniority isomers in multi-j** configurations.
- We show the **role of generalized seniority and configuration mixing** in explaining the E2 seniority isomers, which were only qualitatively understood as pure-seniority isomers before.
- We understand the **B(E2) and B(E3) systematics in Sn-isotopes** in terms of generalized seniority for seniority changing transitions and support the interpretation using LSSM calculations, though these LSSM calculations are not perfect in terms of used truncations (our computational limitation).
- We explore the recently measured **6^+ seniority isomers in $^{134-138}$Sn**, and explained the so-called anomaly at ^{136}Sn by using the generalized seniority and SM calculations. It also suggests a modified location for the i13/2 neutron orbital.
- The generalized seniority scheme has also been applied to the isomers in **odd-A Sn** isotopes (not shown here).
- **To conclude**, ***Generalized seniority scheme behaves as a nearly good quantum number*** in understanding various properties in the semi-magic chains. Predictions can be made for the gaps in the measurements.
- **The present work could be extended for the non-degenerate orbitals and both type of nucleons in future**. This will open up new directions.

Merci

Thank You