## Octupole deformation in the nuclear chart based on the 3D Skyrme Hartree－Fock plus BCS model

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## Introduction

How to investigate nuclear deformation? There are many aspects for the Deformation

## Spontaneous symmetry breaking

The spherical symmetry is broken by the correlation between individual particle motions and quantum fluctuations ("single-particle" vs. "collectivity").

## S.P. states in deformed mean field

The degeneracy of single-particle states is solved in deformed mean field.
$\leftarrow$ Neutron, Proton states in the vicinity of Fermi surface have a key role to define the nuclear shape

## Role of Pairing

Amplitude of pairing correlation which is sensitive to he level density has a important role for nuclear shape.

## Many kinds of deformation

Quadrupole(prolate, oblate, triaxial), Octupole(I=3,m), Hexadecapole( $\beta_{4}$ )

How do we regard the nuclear deformation?

- Rotational bands ■ Large quadrupole moment ■ Changes of S.P. states
- Isomer in nuclear fission Changes of reaction cross section(?)


## Method <br> S.E. et al., PRC82(2010) 034306, N.Tajima, et al., NPA603 (1996) 23.

## Procedure to calculate self-consistent HF+BCS state

Interaction (ph) : Skyrme (SkM*), ( $p p, h h$ ) : Constant (monopole)

$$
\Delta_{k}(t)=\sum_{l>0} G_{k l} \kappa_{l}(t) \quad G_{k l}=g f\left(\epsilon_{k}^{0}\right) f\left(\epsilon_{l}^{0}\right)
$$

$f(\epsilon)$ : cutoff function
1, HF calculation : Imaginary-time method
2, Unoccupied states are calculated up to the cutoff energy.
$\epsilon_{k}^{0}$ : s.p. energy at g.s.

3 , Occupation probabilities $v_{k}$ are evaluated by BCS gap equation.
$3^{\prime}$, Calculate density including $\left|v_{k}\right|^{2}$
4, HF+BCS calculation : Imaginary-time method
$f(\varepsilon)=\left(1+\exp \left[\frac{\varepsilon-\epsilon_{\mathrm{c}}}{0.5 \mathrm{MeV}}\right]\right)^{-1 / 2 \stackrel{\text { Cutoff Ene. }}{\downarrow} \theta\left(e_{\mathrm{c}}-\varepsilon\right)}$
$\epsilon_{\mathrm{c}}=\underset{\substack{\lambda}}{\tilde{\lambda}}+5.0 \mathrm{MeV} \quad e_{\mathrm{c}}=\epsilon_{\mathrm{c}}+2.3 \mathrm{MeV}$

Iterative calculation
up to the convergence

Ave. of LUMO \& HOMO


Method s.E. et al., PRC82(2010) 034306, N.Tajima, et al., NPA603 (1996) 23.

$$
\Delta_{k}(t)=\sum_{l>0} G_{k l} \kappa_{l}(t) \quad G_{k l}=g f\left(\epsilon_{k}^{0}\right) f\left(\epsilon_{l}^{0}\right)
$$

Pairing strength is constant in the Time-Evolution...

How to decide the pairing strength $\boldsymbol{g}$ at each nuclide?

$$
\tilde{\Delta}=\frac{g_{\tau}}{2} \tilde{\Delta} \int_{-\infty}^{\infty} d \varepsilon \frac{f^{2}(\varepsilon) \bar{D}_{\tau}(\varepsilon)}{\sqrt{\left(\varepsilon-\bar{\lambda}_{\tau}\right)^{2}}+f^{2}(\varepsilon) \tilde{\Delta}^{2}} \quad \tilde{\Delta}=12 A^{-1 / 2}
$$

Nucleon \#: $\quad N_{\tau}=\int_{-\infty}^{\infty} d \varepsilon \frac{\left(\varepsilon-\bar{\lambda}_{\tau}\right)^{2} \bar{D}_{\tau}(\varepsilon)}{\sqrt{\left(\varepsilon-\bar{\lambda}_{\tau}\right)^{2}}+f^{2}(\varepsilon) \tilde{\Delta}^{2}}$
Level Density : $\quad \bar{D}_{\tau}(\varepsilon)=\frac{1}{2 \pi^{2}} \int d r\left(\frac{2 m_{\tau}^{*}(r)}{\hbar^{2}}\right)^{3 / 2}\left(\varepsilon-V_{\tau}(r)\right)^{1 / 2} \Theta\left(\varepsilon-V_{\tau}\right)$

Pairing strength is changed depending on the shell-structure (also on the interaction: effective mass $m^{*}$, centroid energy $V$ )

## Calculation space for the self-consistent HF+BCS states

Fully 3D-Spherical meshed box:
Our subject is Nuclear chart. We calculate the even-even nuclei with Z= 6-92.
For light nuclei ( $6<Z<20$ ),
we use the box has radius $\mathbf{1 2}[\mathrm{fm}]$ and meshed by $0.8[\mathrm{fm}]$.
For middle heavy nuclei ( $20<Z<82$ ),
we use the box has radius $\mathbf{1 5}[\mathrm{fm}]$ and meshed by $\mathbf{1 . 0}[\mathrm{fm}]$.
For heavy nuclei ( $82<Z<92$ ), we use the box has radius $\mathbf{2 0}[\mathrm{fm}]$ and meshed by $\mathbf{1 . 0}[\mathrm{fm}]$.


$$
\begin{aligned}
\phi_{l}(\vec{r}, \sigma ; t) & \rightarrow \phi_{l}(x, y, z, \sigma ; t) \\
& x+y+z \sim 15,000-32,000
\end{aligned}
$$

Each single-particle state has many lattice points to describe the wave function.

## Results

3D HF+BCS Cal. w/ SkM* From N=Z to N=2Z, Z=6-92 even-even (Total \# 1005)


## Comparison with m.v. Stoitsov, et. al. PRC68 (2003) 054312

Axial deformed $\mathrm{HFB}+\mathrm{THO}+\mathrm{LN}$ Cal. w/ SkM* $+\delta$-Vol. pairing


## Results (Deformed)

3D HF+BCS Cal. w/ SkM* Deformed nuclei: $\left|\beta_{2}\right|>0.05$


## Results (Deformed: prolate)

3D HF+BCS Cal. w/ SkM* Deformed nuclei: $\left|\beta_{2}\right|>0.05, \gamma<1.5^{\circ}$

Quadrupole Deformation : Prolate (\# 375 / 1005)


## Results (Deformed: oblate)

3D HF+BCS Cal. w/ SkM* Deformed nuclei: $\left|\beta_{2}\right|>0.05,58.5^{\circ}<\gamma<60^{\circ}$


## Results (Deformed: triaxial)

3D HF+BCS Cal. w/ SkM* Deformed nuclei: $\left|\beta_{2}\right|>0.05,1.5^{\circ}<\gamma<58.5^{\circ}$

Quadrupole Deformation : Triaxial (\# 101 / 1005)


## Results (octupole)

3D HF+BCS Cal. w/ SkM* can describe octupole moments in the ground state.
The octupole deformation parameter is defined using the axis of rotational symmetry. ( <Q22> = <xy-yx>=0 $\rightarrow \mathrm{z}$ is regarded as the axial symmetric axis.)
Octupole deformation parameter $\beta_{3}, \beta_{3 \mathrm{~m}}$

$$
\begin{aligned}
& \beta_{3}=\left(\sum_{m=-3}^{3} \alpha_{3 m}^{2}\right)^{1 / 2} \\
& \beta_{3 m}=\left(\alpha_{3 m}^{2}+\alpha_{3-m}^{2}\right)^{1 / 2}
\end{aligned}
$$

The mass-multipole moment

$$
\begin{array}{r}
\alpha_{l m} \equiv \frac{4 \pi}{3 A \bar{R}^{l}} \int r^{l} X_{l m}(\Omega) \rho(r) d \boldsymbol{r} \\
\bar{R}=\sqrt{5\left\langle\sum_{i=1}^{A} r_{i}^{2}\right\rangle / 3 A} \begin{array}{l}
X_{l 0}=Y_{l 0} \\
X_{l m}=\frac{1}{\sqrt{2}}\left(Y_{l-|m|}+Y_{l-|m|}^{*}\right) \\
X_{l-|m|}=\frac{-i}{\sqrt{2}}\left(Y_{l|m|}-Y_{l|m|}^{*}\right)
\end{array}
\end{array}
$$

## Results (Deformed: octupole)

3D HF+BCS Cal. w/ SkM* Octupole deformed nuclei: $\left|\beta_{3}\right|>0.01$


## Results (Deformed: octupole)

3D HF+BCS Cal. w/ SkM* Octupole deformed nuclei: $\left|\beta_{3}\right|>0.01$


## Results (Deformed: octupole $\beta_{30}$ )

3D HF+BCS Cal. w/ SkM* Octupole deformed nuclei: $\left|\beta_{3}\right|>0.01$


## Results (Deformed: octupole $\beta_{31}$ )

3D HF+BCS Cal. w/ SkM* Octupole deformed nuclei: $\left|\beta_{3}\right|>0.01$


Results (Deformed: octupole $\beta_{32}$ )
3D HF+BCS Cal. w/ SkM* Octupole deformed nuclei: $\left|\beta_{3}\right|>0.01$


Results (Deformed: octupole $\beta_{33}$ )
3D HF+BCS Cal. w/ SkM* Octupole deformed nuclei: $\left|\beta_{3}\right|>0.01$
$\beta_{33}$ for Total (\# 1005)


Results (Octupole correlations: ${ }^{144} \mathrm{Ba}$ ) Skyrme $\mathrm{HF}+\mathrm{BCS}+$ Constraints
W/ $Q_{30}$ constraints


Results (Octupole correlations: ${ }^{220} \mathrm{Rn}$ ) Skyrme $\mathrm{HF}+\mathrm{BCS}+$ Constraints
W/ $Q_{30}$ constraints


Results ( $\beta_{2}$ vs. $\beta_{3}:{ }^{144} \mathrm{Ba},{ }^{220} \mathrm{Rn}$ ) $w / Q_{2 m}$ constraints \& w/o $Q_{3 m}$ constraints



In both nuclide, $\beta_{31}, \beta_{32}, \beta_{33}$ do not appear under $\beta_{2}$ constraints (prolate).
Octupole deformation disappear under the large quadrupole deformation.

Results ( $\beta_{30}$ vs. $\beta_{31}$ vs. $\beta_{32}$ vs. $\left.\beta_{33}:{ }^{144} \mathrm{Ba},{ }^{220} \mathrm{Rn}\right)$ skyrme $\mathrm{HF}+\mathrm{BCS}+$ Constraints w/ $\mathrm{Q}_{3 \mathrm{~m}}$ constraints


In both, $\beta_{30}$ is lowest in the energy surface under the octupole constraints.
In this work, ${ }^{144} \mathrm{Ba}$ does not have local minimum on the $\beta_{31}, \beta_{32}$ and $\beta_{33}$ surfaces. Although we need more investigation, ${ }^{220} \mathrm{Rn}$ has some possibility of a local minimum on the $\beta_{31}$ energy surface.

## Summary

$\checkmark$ We investigate the ground states for even-even nuclei in the nuclear chart by 3D HF+BCS. We found about $54 \%$ deformed nuclide in the chart.
$\checkmark$ In the quadrupole deformed nuclei, the Prolate nuclei is $70 \%$, Oblate is $12 \%$, and Triaxial is $18 \%$. We found the Prolate dominance.
$\checkmark$ The nuclei with octupole deformation in their ground states are found (about 30 nuclei). They appear in the mass region with $Z=54-70,86-92$ and $N=\mathbf{8 6 - 9 0}, 130-138$, which is consistent to the region of $\Delta l=\mathbf{3}$ correlation. They might appear only with the correlations in both proton and neutron.
$\checkmark$ The octupole deformed nuclei might have only pear shape ( $\mathrm{Q}_{30}$ type), although this work are performed in the 3D coordinate.
$\checkmark$ We investigate octupole correlation in ${ }^{144} \mathrm{Ba}$ and ${ }^{220} \mathrm{Rn}$ using multi-constraints for $\mathrm{Q}_{30}, \mathrm{Q}_{31}, \mathrm{Q}_{32}$, and $\mathrm{Q}_{33}$.

## Remains to do ...

> Stable calculation for octupole deformed nuclei
$>$ Octupole correlation strength
> Relation between deformed nuclei region and pairing strength
> Space between single-particle orbits

## Results (Octupole Def. vs. ph-state)

*Taken from P. Ring \& P. Shock text book


Octupole Nuclide

$\Delta l=3$ pair ph-states
$Z: d_{5 / 2}-h_{11 / 2}$, $f_{7 / 2}-i_{13 / 2}$
$N: f_{7 / 2}-i_{13 / 2}$, $g_{9 / 2}-j_{15 / 2}$
*Taken from P. Ring \& P. Shock text book


Figure 2.21c. Same as Fig. 2.21a for protons in heavy nuclei

