

Isospin symmetry and its breaking in atomic nuclei

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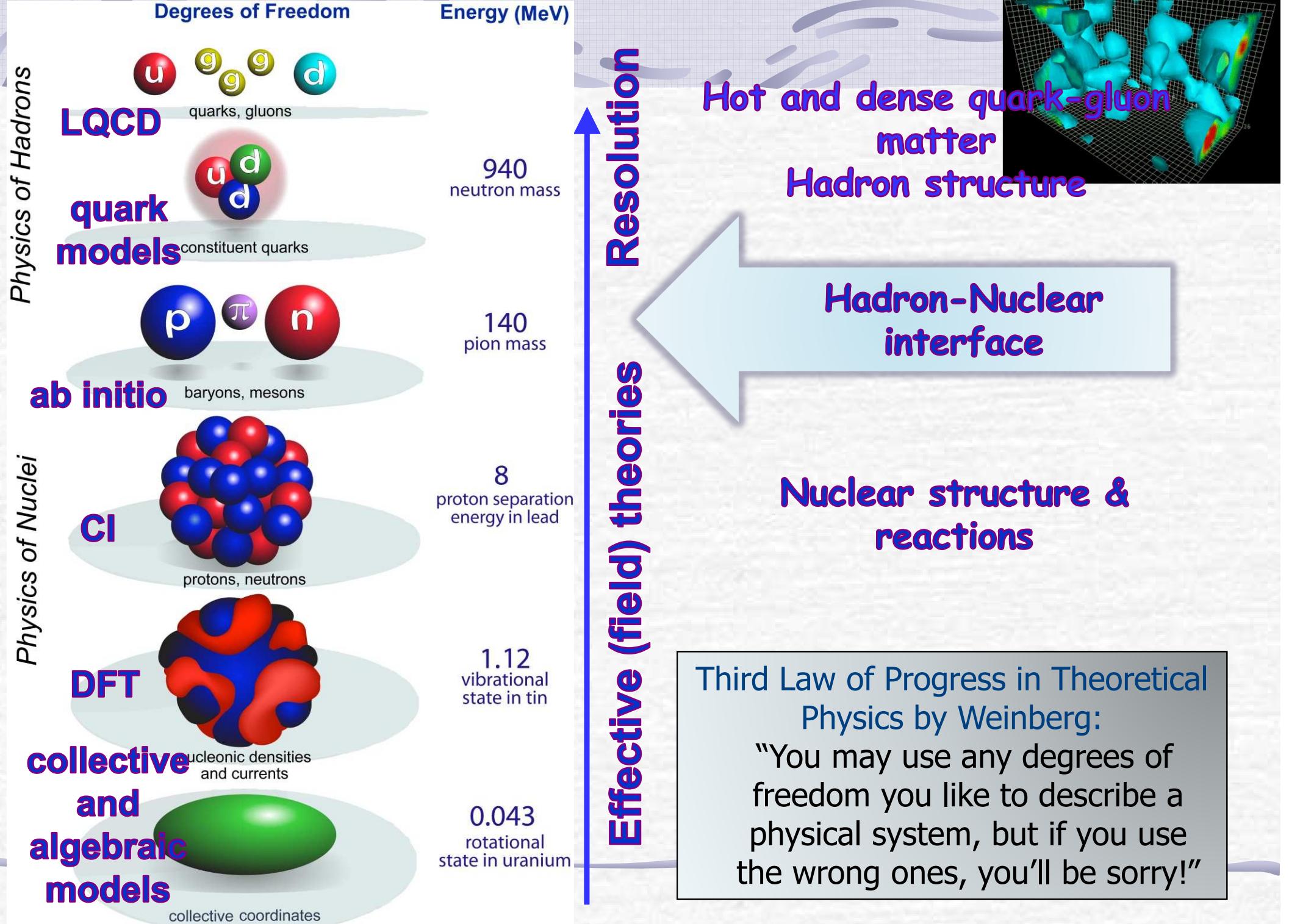
Isospin symmetry studies using SR-DFT and MR-DFT-rooted approaches:

- new developments:
 - DFT-rooted NCCI involving angular-momentum and isospin projections
 - pn-mixed SR functionals
 - charge-dependent functionals
- physics highlights
 - nuclear structure and beta decays
 - strong-force isospin symmetry breaking effects (TDE/MDE)

SSNET'16

International Workshop on
Shapes and Symmetries in Nuclei :
from Experiment to Theory

Gif sur Yvette, November 7th - 11th 2016



Effective or low-energy (low-resolution) theory explores separation of scales. Its formulation requires:

in coordinate space:

→ define R to separate short- and long-distance physics

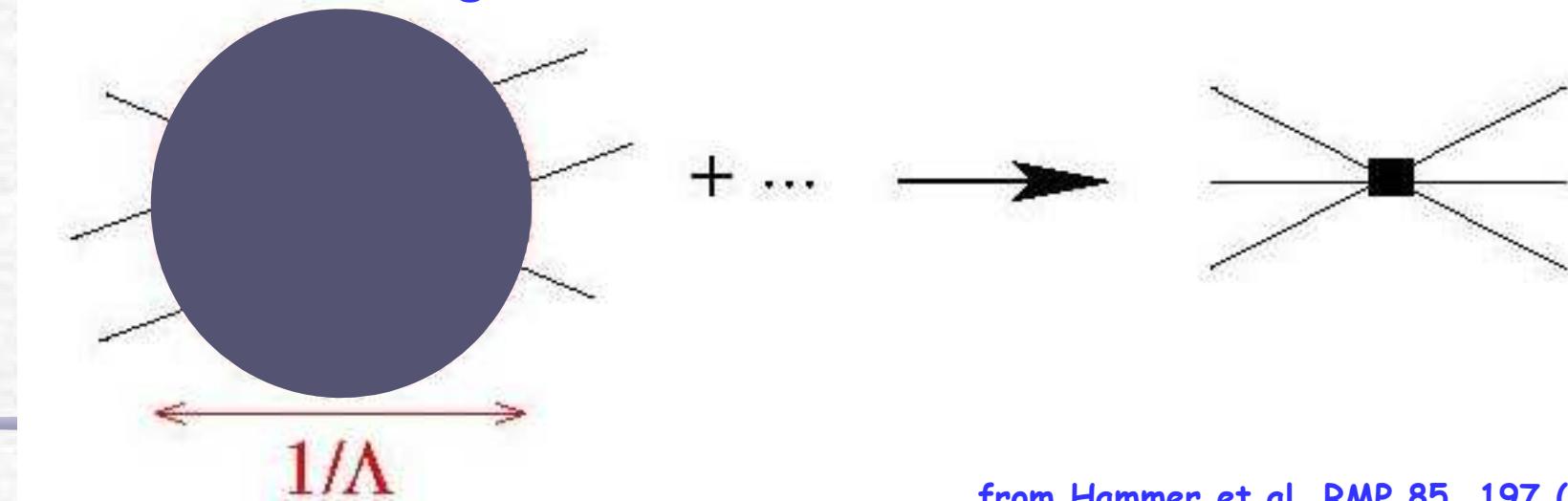
or, in momentum space:

→ define Λ ($1/R$) to separate low and high momenta

replace (complicated and, in nuclear physics, unknown) short distance (or high momentum) physics by a LCP (local correcting potential)

(there is a lot of freedom how this is done concerning both the scale and form but physics is (should be!) independent on the scheme!!!)

emergence of 3NF due to finite resolution



from Hammer et al. RMP 85, 197 (2013)

Nuclear effective theory for EDF (nuclear DFT)

is based on the same simple and very intuitive assumption that low-energy nuclear theory is independent on high-energy dynamics

ultraviolet →
cut-off Λ

$$v_S(q^2) \approx v_S(0) + v_S^{(1)}(0)q^2 + v_S^{(2)}(0)q^4 \dots ,$$

Coulomb
hierarchy of scales: $v_{eff}(\mathbf{r}) \approx v_{long}(\mathbf{r})$

$$\frac{2r_o A^{1/3}}{r_o} \sim 2A^{1/3}$$

protons

~ 10

$$+ ca^2 \delta_a(\mathbf{r}) \\ + d_1 a^4 \nabla^2 \delta_a(\mathbf{r}) + d_2 a^4 \nabla \delta_a(\mathbf{r}) \nabla \\ + \dots \\ + g_1 a^{n+2} \nabla^n \delta_a(\mathbf{r}) + \dots ,$$

Long-range part of the NN interaction
(must be treated exactly!!!)

$$\delta_a(\mathbf{r}) \equiv \frac{e^{-\mathbf{r}^2/2a^2}}{(2\pi)^{3/2} a^3} .$$

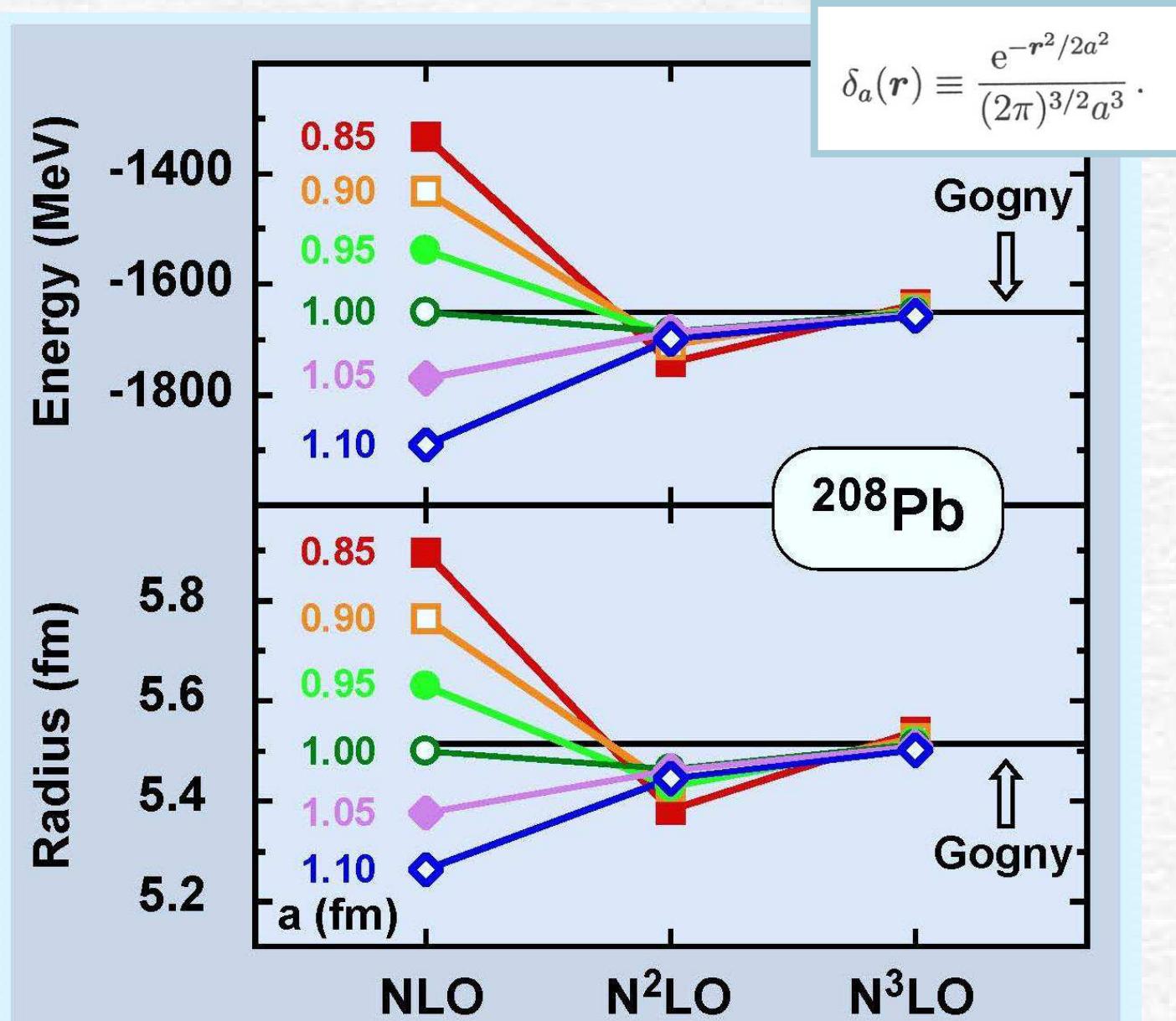
Gaussian regulator

where $\delta_a(\mathbf{r})$ denotes an arbitrary Dirac-delta model

There exist an „infinite” number
of equivalent realizations
of effective theories

Fourier regularization
local correcting potential

Proof of principle of the regularization range (scale) independence for the gaussian-regularized density-independent EDFs



Skyrme interaction - specific (local) realization of the nuclear effective interaction: $\lim_{a \rightarrow 0} \delta_a$

$v(1, 2) = t_0(1 + x_0 \hat{P}_\sigma) \delta(\mathbf{r}_{12})$
 $+ \frac{1}{2} t_1(1 + x_1 \hat{P}_\sigma) (\hat{k}'^2 \delta(\mathbf{r}_{12}) + \delta(\mathbf{r}_{12}) \hat{k}^2)$
 $+ [t_2(1 + x_2 \hat{P}_\sigma) \hat{k}' \delta(\mathbf{r}_{12}) \hat{k}$
 $+ \frac{1}{6} t_3(1 + x_3 \hat{P}_\sigma) \rho_0^\gamma(\mathbf{R}) \delta(\mathbf{r}_{12})$
 $+ iW_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) (\hat{k}' \times \delta(\mathbf{r}_{12}) \hat{k}) ,$

$\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2; \mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2; \hat{\mathbf{k}} = \frac{1}{2i}(\nabla_1 - \nabla_2)$ relative momenta
 $\hat{\mathbf{k}}' = -\frac{1}{2i}(\nabla_1 - \nabla_2)$
 $\hat{P}_\sigma = \frac{1}{2}(1 + \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2)$ spin exchange

Having defined the generator, the nuclear EDF is built using mean-field (HF or Kohn-Sham) methodology

$$E[\rho(\vec{r}_1, \vec{r}_2)] = \iint d\vec{r}_1 d\vec{r}_2 \mathcal{H}(\rho(\vec{r}_1, \vec{r}_2))$$

$$\mathcal{H}(\rho(\vec{r}_1, \vec{r}_2)) = V(\vec{r}_1 - \vec{r}_2) [\rho(\vec{r}_1)\rho(\vec{r}_2) - \rho(\vec{r}_1, \vec{r}_2)\rho(\vec{r}_2, \vec{r}_1)]$$

direct term exchange term

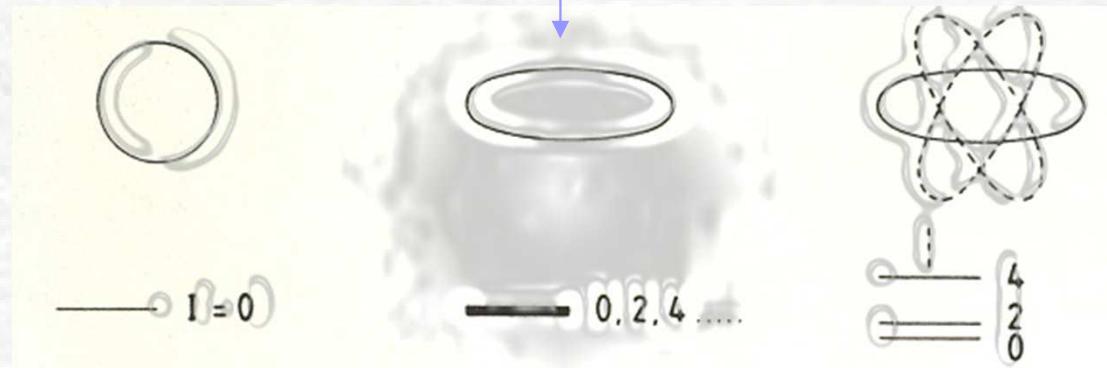
Skyrme local energy density functional

Skyrme (hadronic) interaction conserves such symmetries like:

- rotational (spherical) symmetry
- isospin symmetry: $V_{nn}^{\text{LS}} = V_{pp}^{\text{LS}} = V_{np}^{\text{LS}}$ (in reality approximate)
- particle number, parity...

• Self-consistent solutions (Slater's) break these symmetries (are deformed) spontaneously

$$\hat{R}(Q)|\varphi(Q_0)\rangle = |\varphi(Q')\rangle$$
$$\langle \varphi | \hat{H} | \varphi \rangle = \langle \varphi | \hat{R}^\dagger(Q) \hat{H} \hat{R}(Q) | \varphi \rangle$$



advantages:

built in correlations into single Slater determinant

disadvantages:

symmetry must be restored to compare theory to data



Isospin symmetry restoration

There are two sources of the isospin symmetry breaking:

- **unphysical**, caused solely by the HF approximation

→ Engelbrecht & Lemmer,
PRL24, (1970) 607

- **physical**, caused mostly by Coulomb interaction

(also, but to much lesser extent, by the strong force isospin non-invariance)

- Find self-consistent HF solution (including Coulomb) → deformed Slater determinant $|\text{HF}\rangle$:

$$|\text{HF}\rangle = \sum_{T \geq |T_z|} b_{T,T_z} |\alpha; T, T_z\rangle$$

See: Caurier, Poves & Zucker,
PL 96B, (1980) 11; 15

- Apply the isospin projector:

$$\hat{P}_{T_z T_z}^T = \frac{2T+1}{2} \int_0^\pi d\beta \sin \beta d_{T_z T_z}^{T*}(\beta) \hat{R}(\beta)$$

in order to create good isospin „basis”:

$$|\alpha; T, T_z\rangle = \frac{1}{b_{T,T_z}} \hat{P}_{T_z T_z}^T |\text{HF}\rangle$$

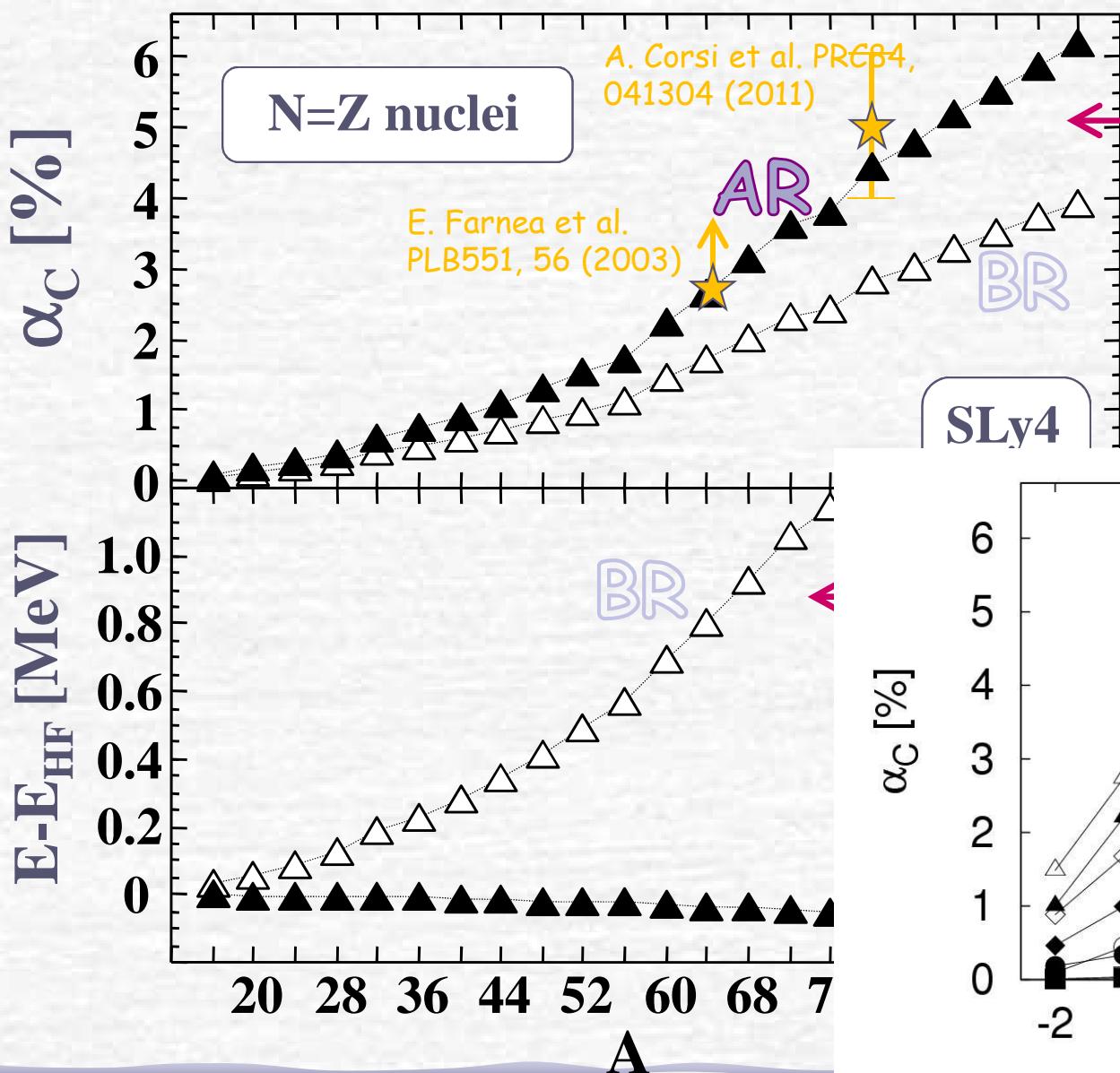
- Diagonalize total Hamiltonian in „good isospin basis” $|\alpha, T, T_z\rangle$
→ takes physical isospin mixing

$$\sum_{T' \geq |T_z|} \langle \alpha; T, T_z | \hat{H} | \alpha; T', T_z \rangle a_{T', T_z}^n = E_{n, T_z}^{\text{AR}} a_{T, T_z}^n$$

$$|\alpha; n, T_z\rangle = \sum_{T \geq |T_z|} a_{T, T_z}^n |\alpha; T, T_z\rangle,$$

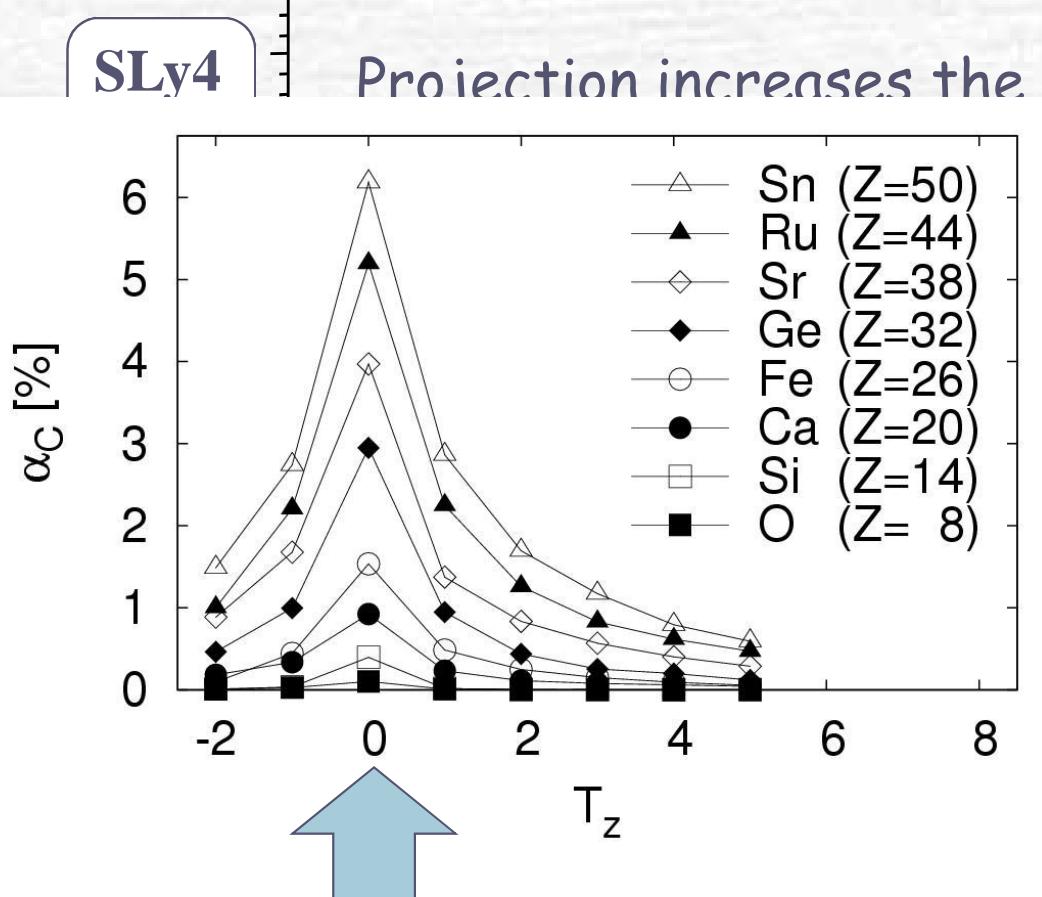
$$\alpha_C^{\text{AR}} = 1 - |a_{T=T_z}^{\mathbf{n=1}}|^2$$

Isospin mixing & energy in the ground states of e-e N=Z nuclei:



This is not a single Slo
There are no constraints on

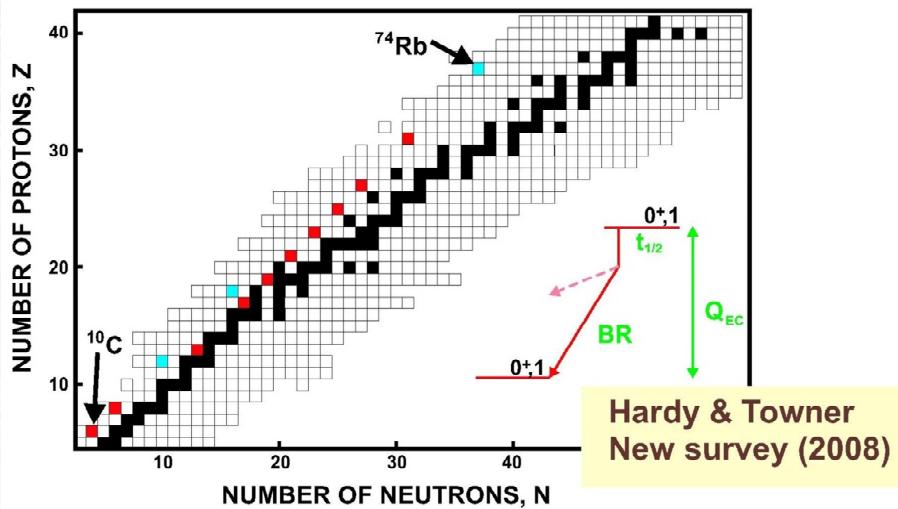
HF tries to reduce the isospin mixing by:
 $\Delta\alpha_C \sim 30\%$
 in order to minimize the total energy



Testing the fundamental symmetries of nature

Superallowed $0^+ > 0^+$ Fermi beta decays

adopted from J. Hardy's, ENAM'08 presentation



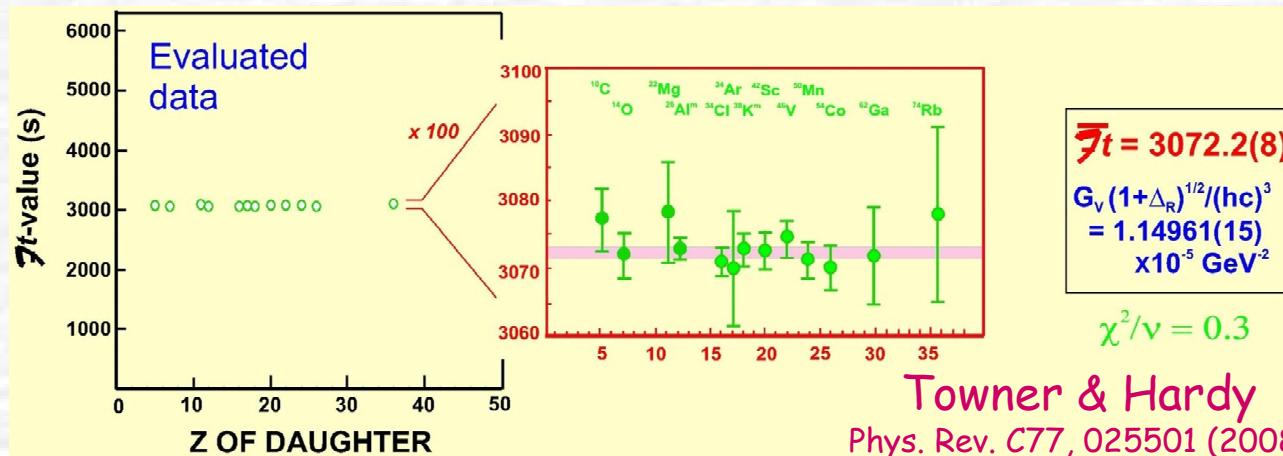
10 cases measured with accuracy $\overline{ft} \sim 0.1\%$
3 cases measured with accuracy $\overline{ft} \sim 0.3\%$

→ test of the CVC hypothesis
(Conserved Vector Current)

INCLUDING RADIATIVE CORRECTIONS

$$\overline{ft} = ft (1 + \delta'_R) [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2 (1 + \Delta_R)}$$

1.5% 0.3%
- 1.5% ~2.4%



Towner & Hardy
Phys. Rev. C77, 025501 (2008)

$$\overline{ft} = 3072.2(8)$$

$$G_V (1 + \Delta_R)^{1/2} / (hc)^3$$

$$= 1.14961(15) \times 10^{-5} \text{ GeV}^{-2}$$

$$\chi^2/\nu = 0.3$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

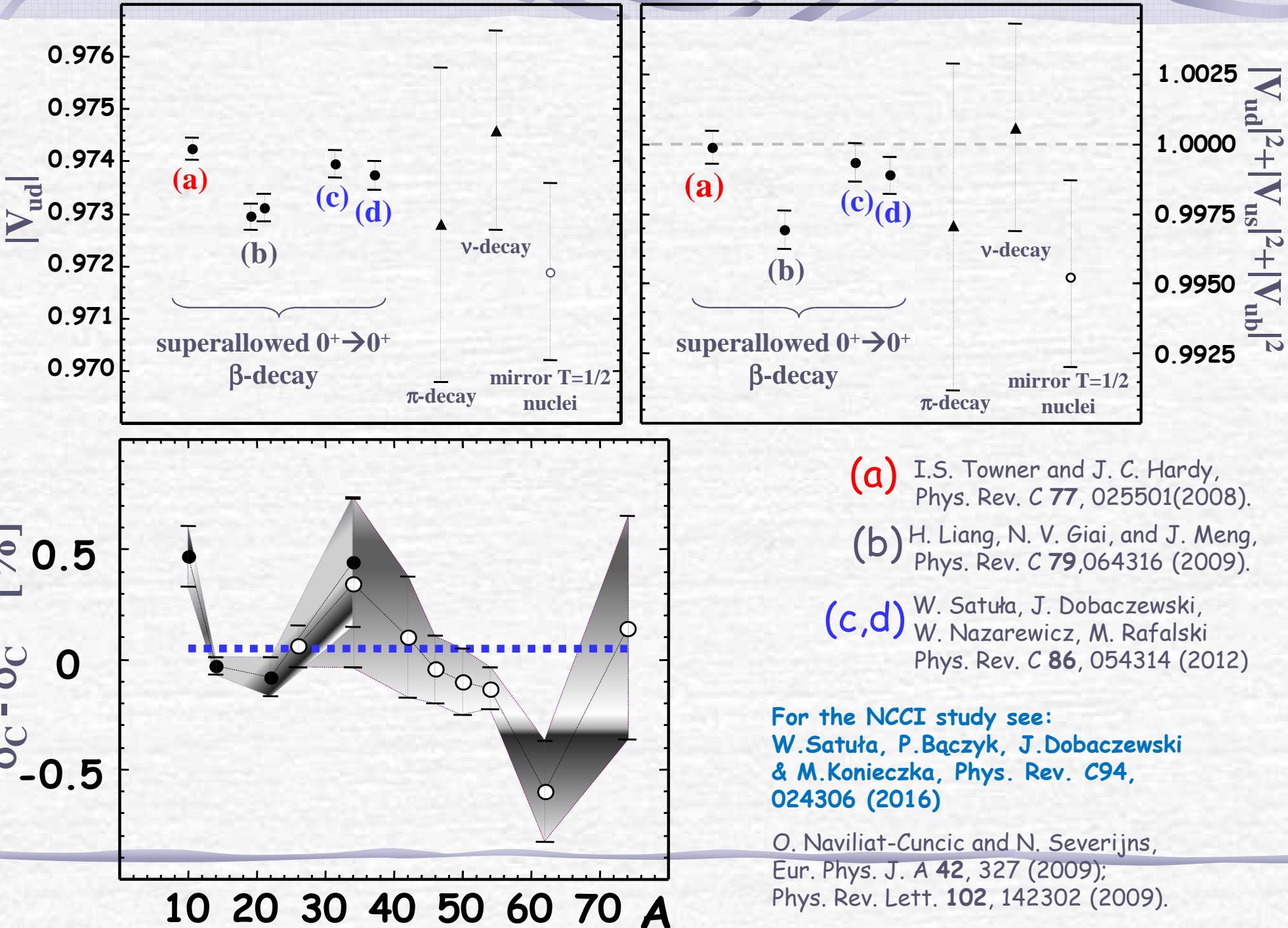
weak eigenstates CKM mass eigenstates
Cabibbo-Kobayashi-Maskawa

$|V_{ud}| = 0.97418 \pm 0.00026$
→ test of unitarity of the CKM matrix

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9997(6)$$

$0.9490(4)$ $0.0507(4)$ <0.0001

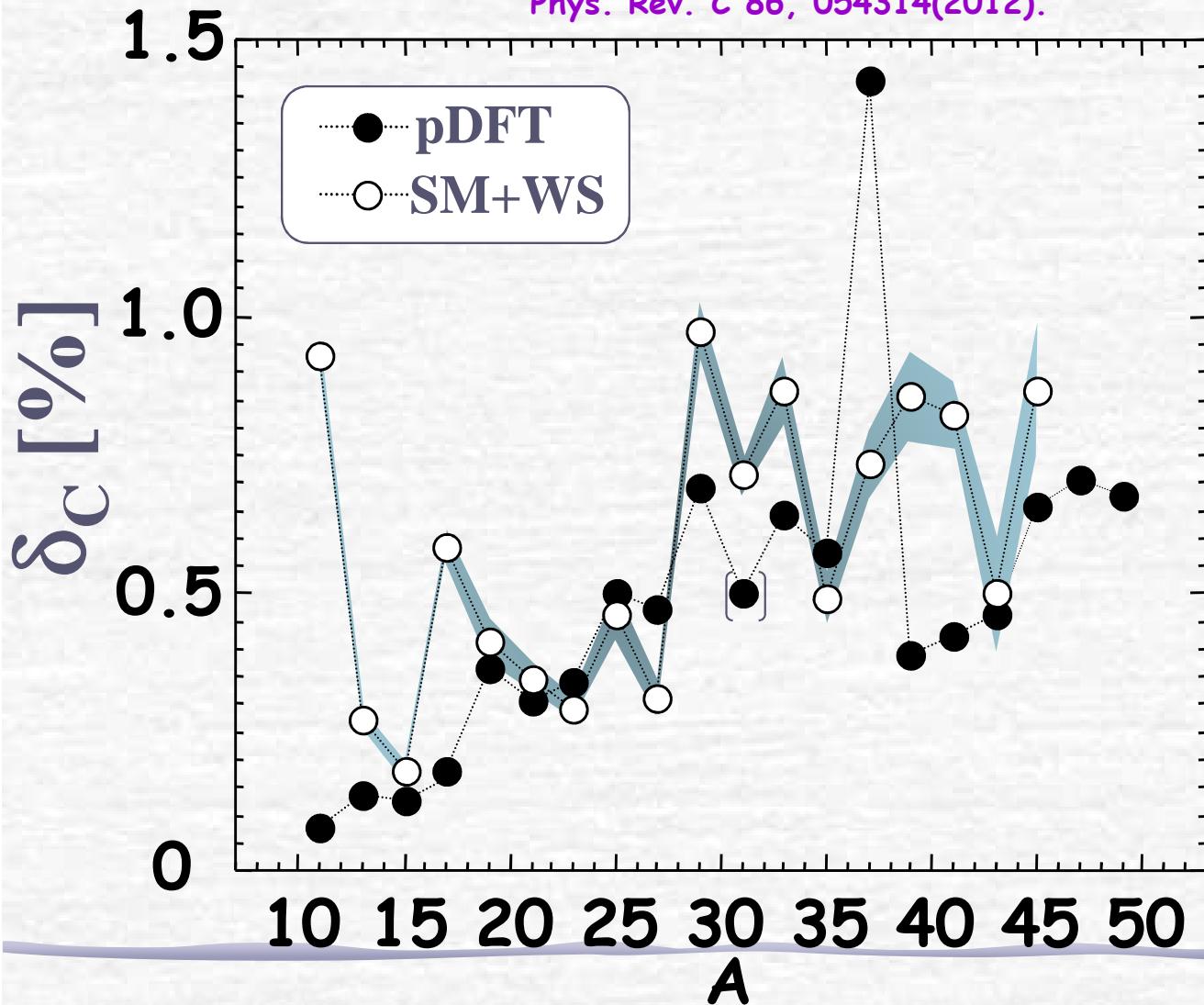
$|V_{ud}|$ & the CKM unitarity - world survey



Testing the fundamental symmetries of nature

Fermi beta decays in $T=1/2$ mirrors

W. Satuła, J. Dobaczewski, W. Nazarewicz, M. Rafalski
Phys. Rev. C 86, 054314(2012).



See also the NCCI study:

M. Konieczka, P. Baczyk, W. Satuła,
Phys. Rev. C 93, 042501(R) (2016).

SM+WS results from:
N. Severijns, M. Tandecki,
T. Phalet, and I. S. Towner,
Phys. Rev. C 78, 055501 (2008).

Our NCCI scheme:

MEAN FIELD
compute a set of n self-consistent Slater determinants corresponding to low-lying p-h excitations

$$\varphi_1 \quad \varphi_2 \quad \varphi_3 \quad \cdots \quad \varphi_n$$

PROJECTION

compute the I-,K- and T-projected states

$$\Psi_{TIK}^{(1)} \quad \Psi_{TIK}^{(2)} \quad \Psi_{TIK}^{(3)} \quad \cdots \quad \Psi_{TIK}^{(n)}$$

K- AND T-MIXING

compute the K-mixing of Coulomb T-mixed states

$$\Psi_{\tilde{T}I\alpha}^{(1)} \quad \Psi_{\tilde{T}I\alpha}^{(2)} \quad \Psi_{\tilde{T}I\alpha}^{(3)} \quad \cdots \quad \Psi_{\tilde{T}I\alpha}^{(n)}$$

CONFIGURATION INTERACTION

solve the Hill-Wheeler equation

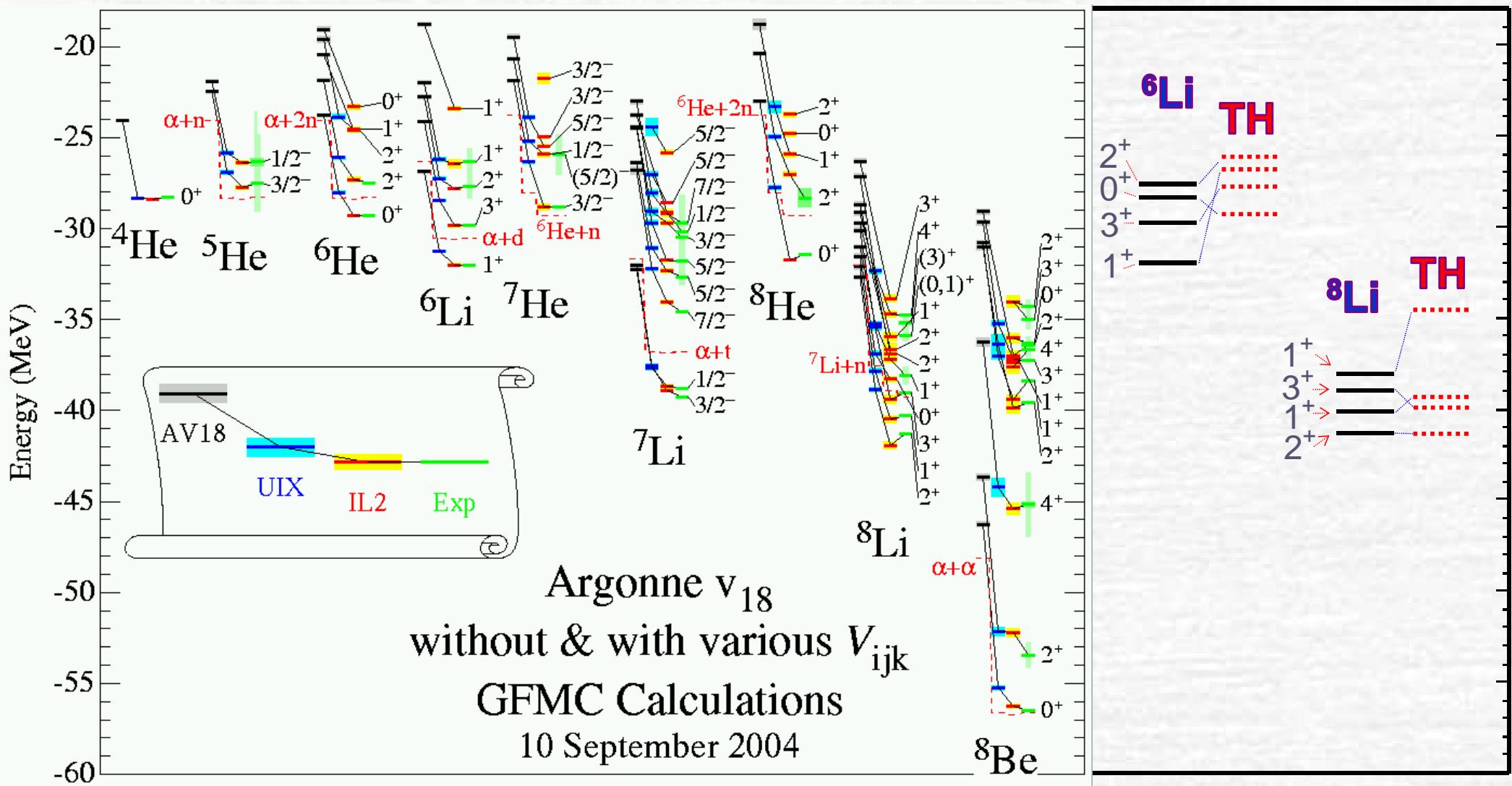
$$E_k, \quad | I_k \rangle$$

**Skyrme SV
(density independent)
is used at this stage**

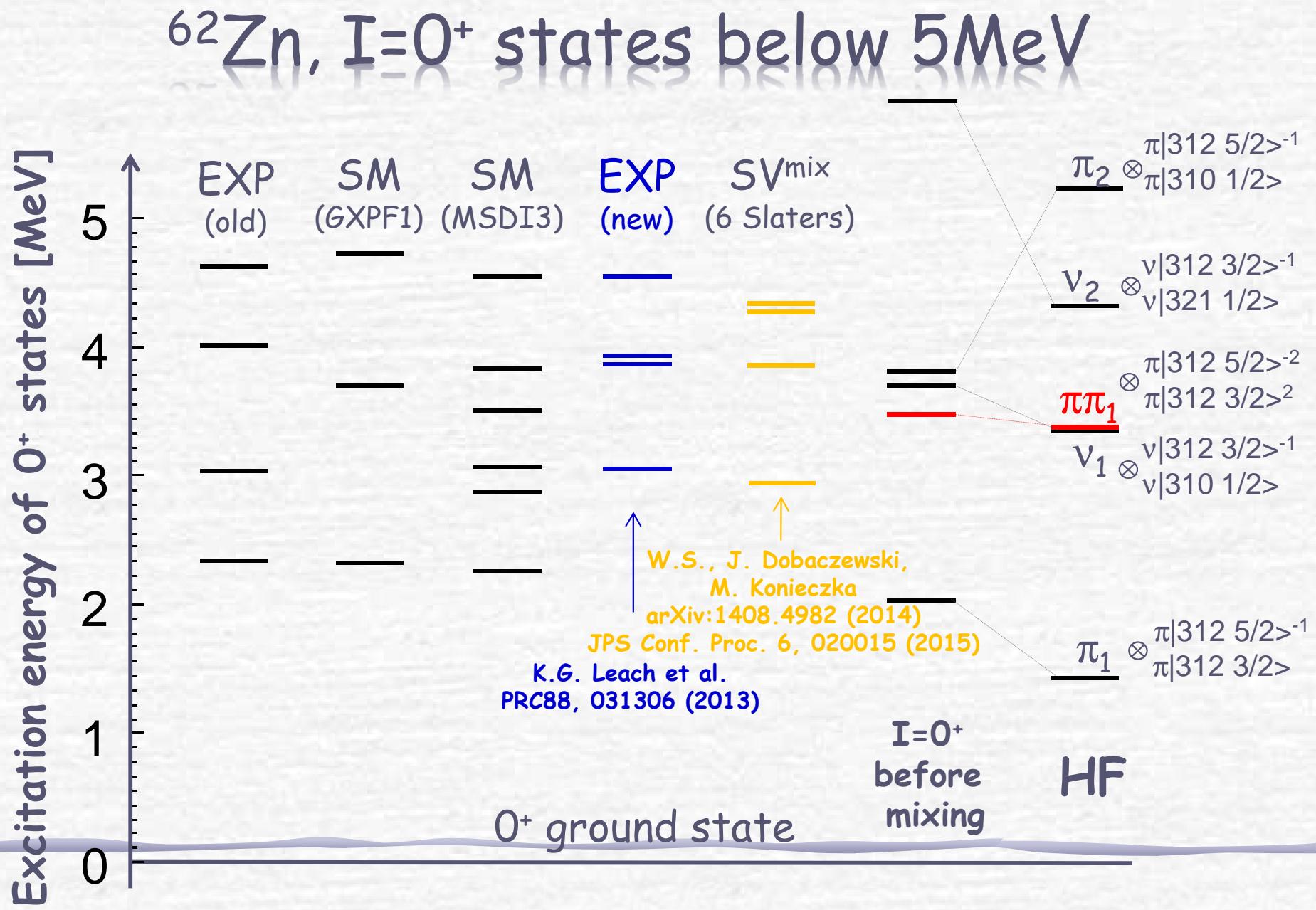
**Skyrme SV
(density independent)
is used at this stage**

mixing of states projected from just three-four p-h configurations

For details see: W.Satuła, P.Bączyk, J.Dobaczewski & M.Konieczka, Phys. Rev. C94, 024306 (2016)



No-core configuration-interaction formalism based on the isospin and angular momentum projected DFT

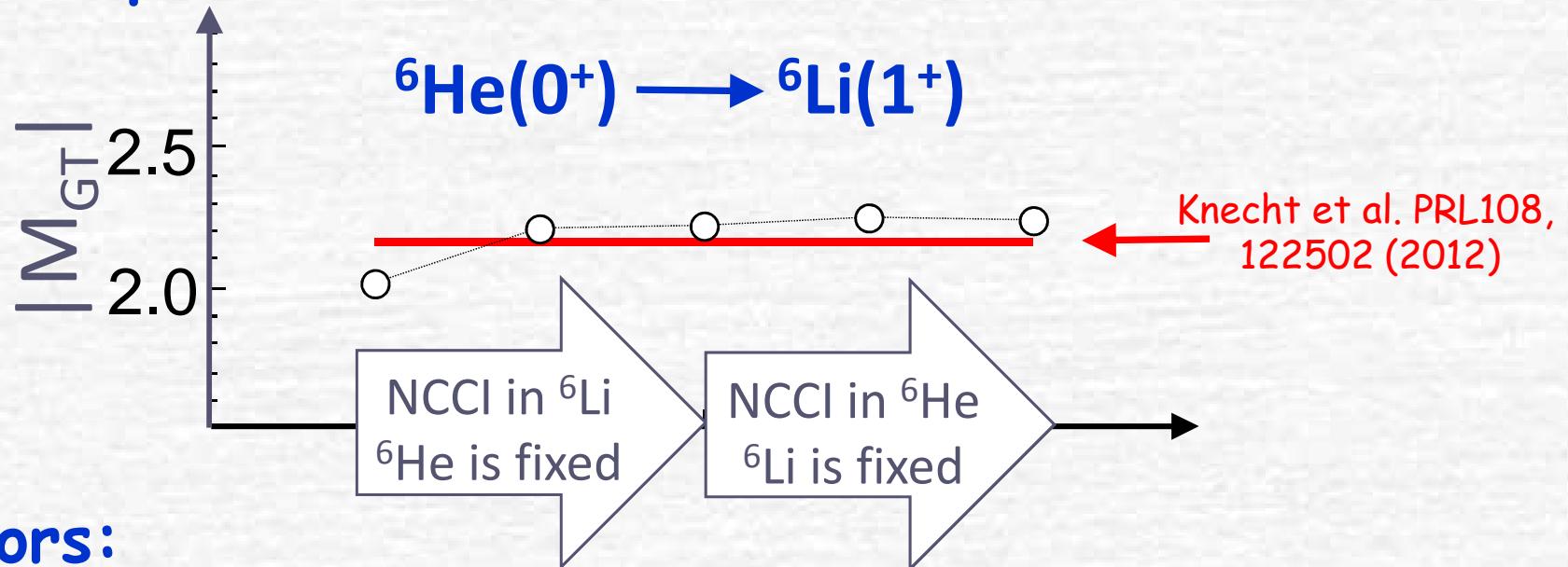


Gamow-Teller and Fermi matrix elements in T=1/2 sd- and ft- mirrors.

The NCCI study

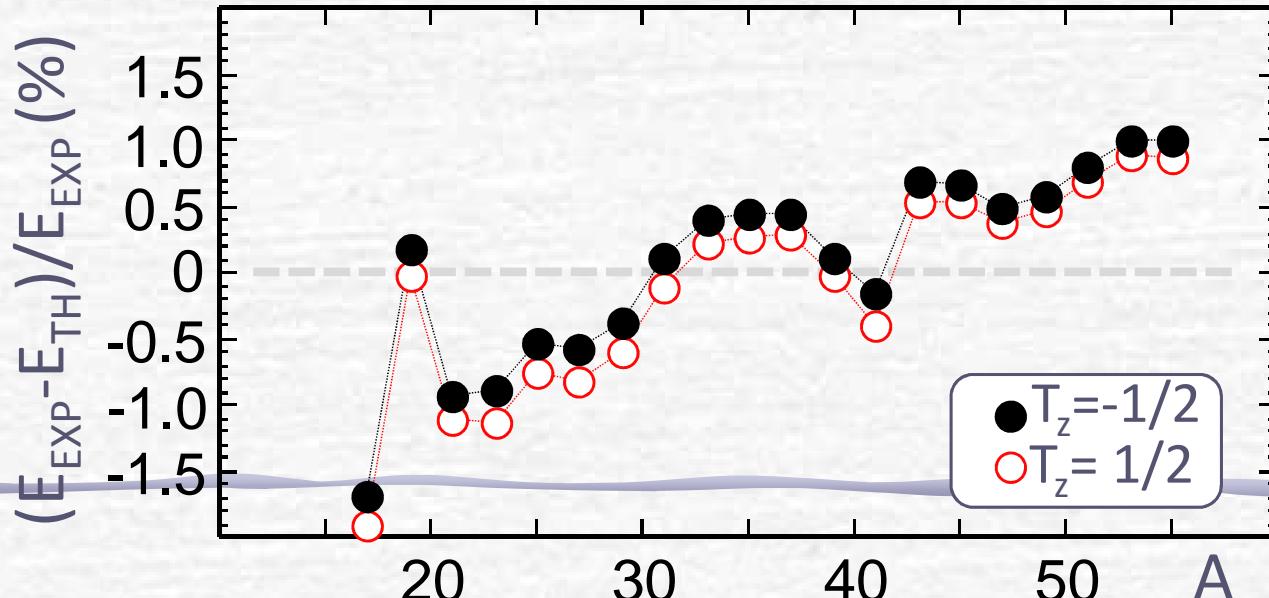
M.Konieczka, P.Bączyk, W.Satuła, Phys. Rev. C93, 042501(R) (2016); [arXiv:1509.04480](https://arxiv.org/abs/1509.04480)

Proof-of-principle calculation:



T=1/2 mirrors:

○ masses:



Shell-model:

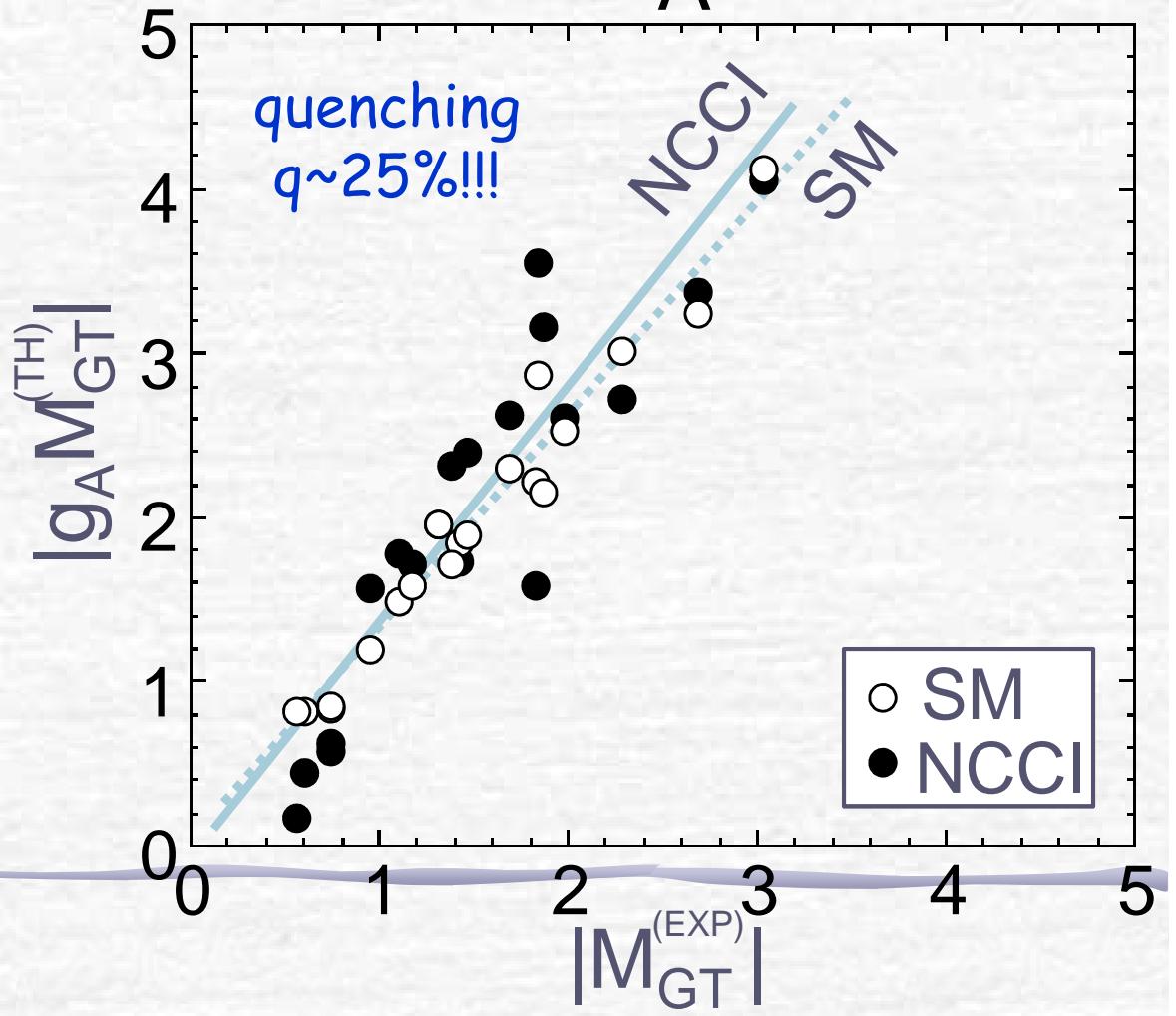
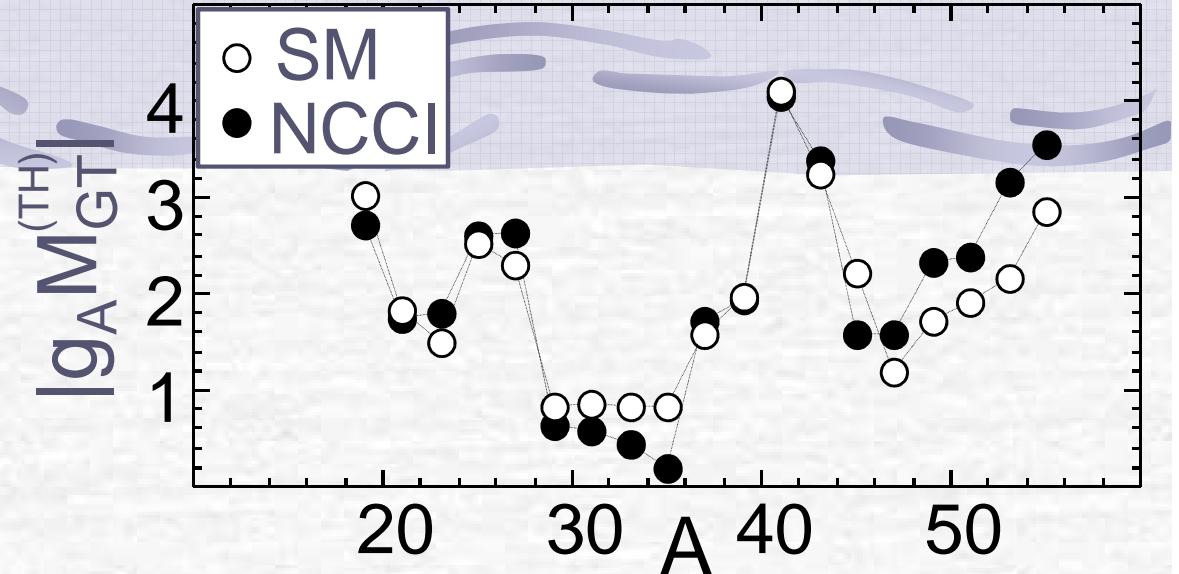
B. A. Brown and B. H. Wildenthal,
 Atomic Data and Nuclear
 Data Tables **33**, 347 (1985).

G. Martinez-Pinedo *et al.*,
 Phys. Rev. C **53**, R2602 (1996).

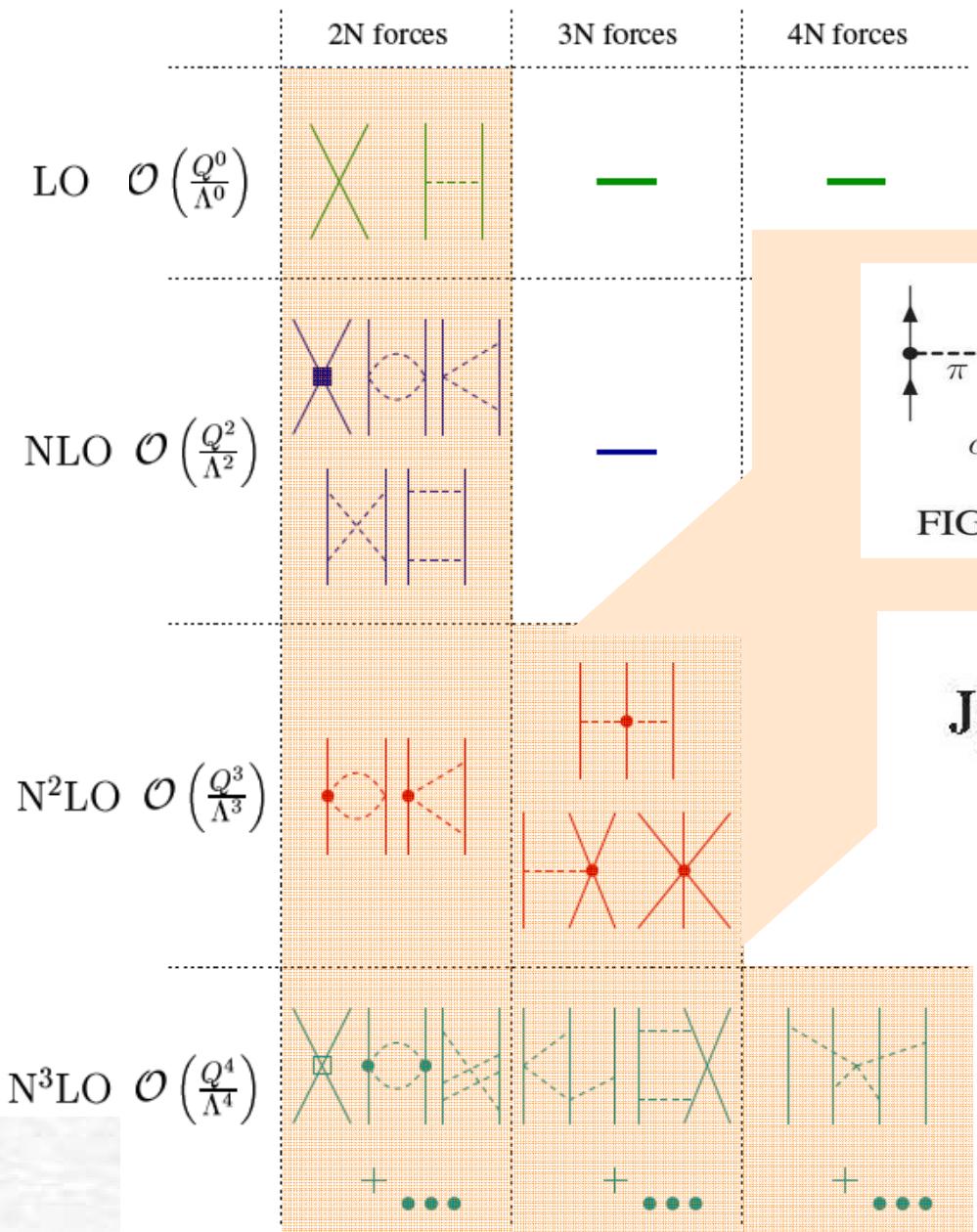
T. Sekine *et al.*,
 Nucl. Phys. A **467**, (1987).

NCCI vs shell-model:

- The NCCI takes into account a core and its polarization
- Completely different model spaces
- Different treatment of correlations
- Different interactions



Renormalization of axial-vector coupling constant by 2B-currents



c_i from πN and NN:

$$c_1 = -0.9^{+0.2}_{-0.5}, \quad c_3 = -4.7^{+1.2}_{-1.0}, \quad c_4 = 3.5^{+0.5}_{-0.2}$$

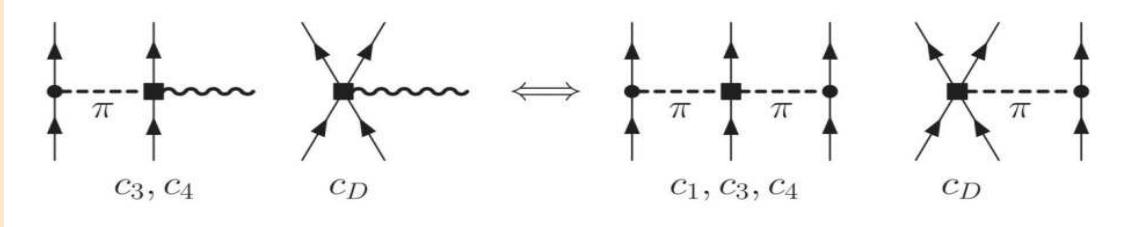


FIG. 1. Chiral 2b currents and 3N force contributions.
Menendez et al. PRL107, 062501 (2011)

$$\begin{aligned} \mathbf{J}_{i,2b}^{\text{eff}} = & -g_A \boldsymbol{\sigma}_i \tau_i^- \frac{\rho}{F_\pi^2} \left[\frac{c_D}{g_A \Lambda_\chi} + \frac{2}{3} c_3 \frac{\mathbf{p}^2}{4m_\pi^2 + \mathbf{p}^2} \right. \\ & \left. + I(\rho, P) \left(\frac{1}{3}(2c_4 - c_3) + \frac{1}{6m} \right) \right], \end{aligned}$$

β^- decays of ^{14}C and $^{22;24}O$
Ekstrom et al. PRL 113, 262504 (2014)
 $q^2 \sim 0.84-0.92$ (from Ikeda sum rule)

See also:

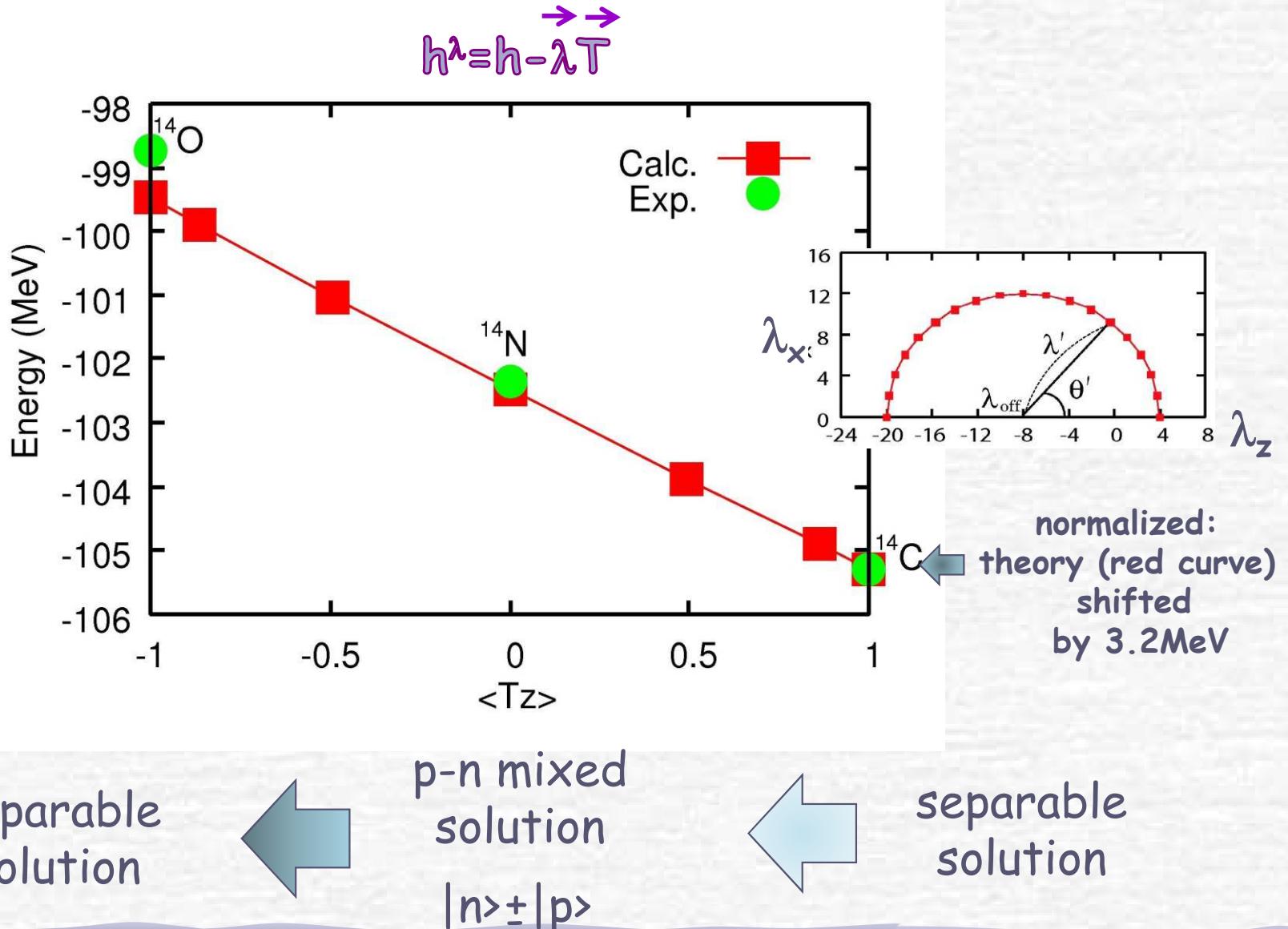
Klos et al. PRC89, 029901 (2013)

Engel et al. PRC89, 064308 (2013)

$q \sim 0.9$

$T=1, I=0^+$ isobaric analogue states
from self-consistent 3D-isocranked HF: $h^\lambda = h - \lambda T$

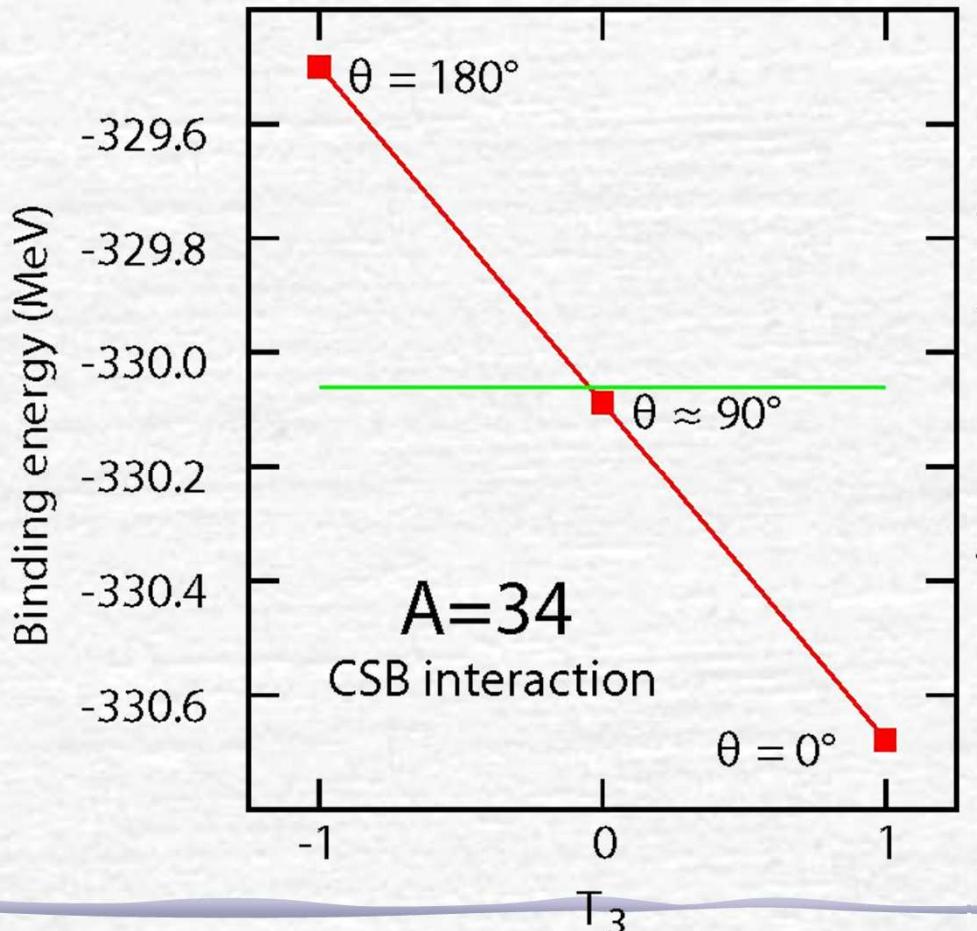
K. Sato, J. Dobaczewski, T. Nakatsukasa, and W. Satuła, Phys. Rev. C88 (2013), 061301



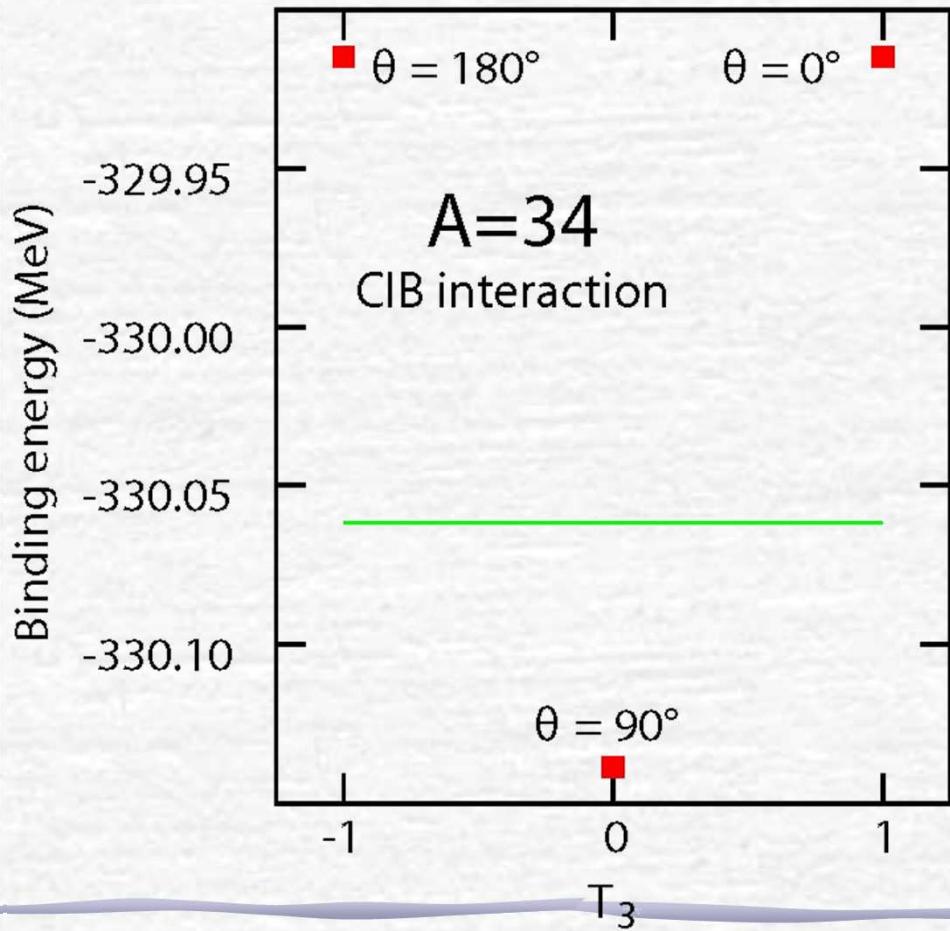
CD local (strong) corrections to the Skyrme force (class II (CIB) and III (CSB) Henley-Miller forces)

$$\hat{V}^{\text{II}}(i,j) = \frac{1}{2}t_0^{\text{II}} \delta(r_i - r_j) \left(1 - x_0^{\text{II}} \hat{P}_{ij}^\sigma\right) \left[3\hat{\tau}_3(i)\hat{\tau}_3(j) - \hat{\vec{\tau}}(i) \circ \hat{\vec{\tau}}(j)\right], \quad \mathcal{H}_{\text{II}} = \frac{1}{2}t_0^{\text{II}} (1 - x_0^{\text{II}}) (\rho_n^2 + \rho_p^2 - 2\rho_n\rho_p - 2\rho_{np}\rho_{pn} - s_n^2 - s_p^2 + 2s_n \cdot s_p + 2s_{np} \cdot s_{pn}),$$

$$\hat{V}^{\text{III}}(i,j) = \frac{1}{2}t_0^{\text{III}} \delta(r_i - r_j) \left(1 - x_0^{\text{III}} \hat{P}_{ij}^\sigma\right) [\hat{\tau}_3(i) + \hat{\tau}_3(j)], \quad \mathcal{H}_{\text{III}} = \frac{1}{2}t_0^{\text{III}} (1 - x_0^{\text{III}}) (\rho_n^2 - \rho_p^2 - s_n^2 + s_p^2),$$

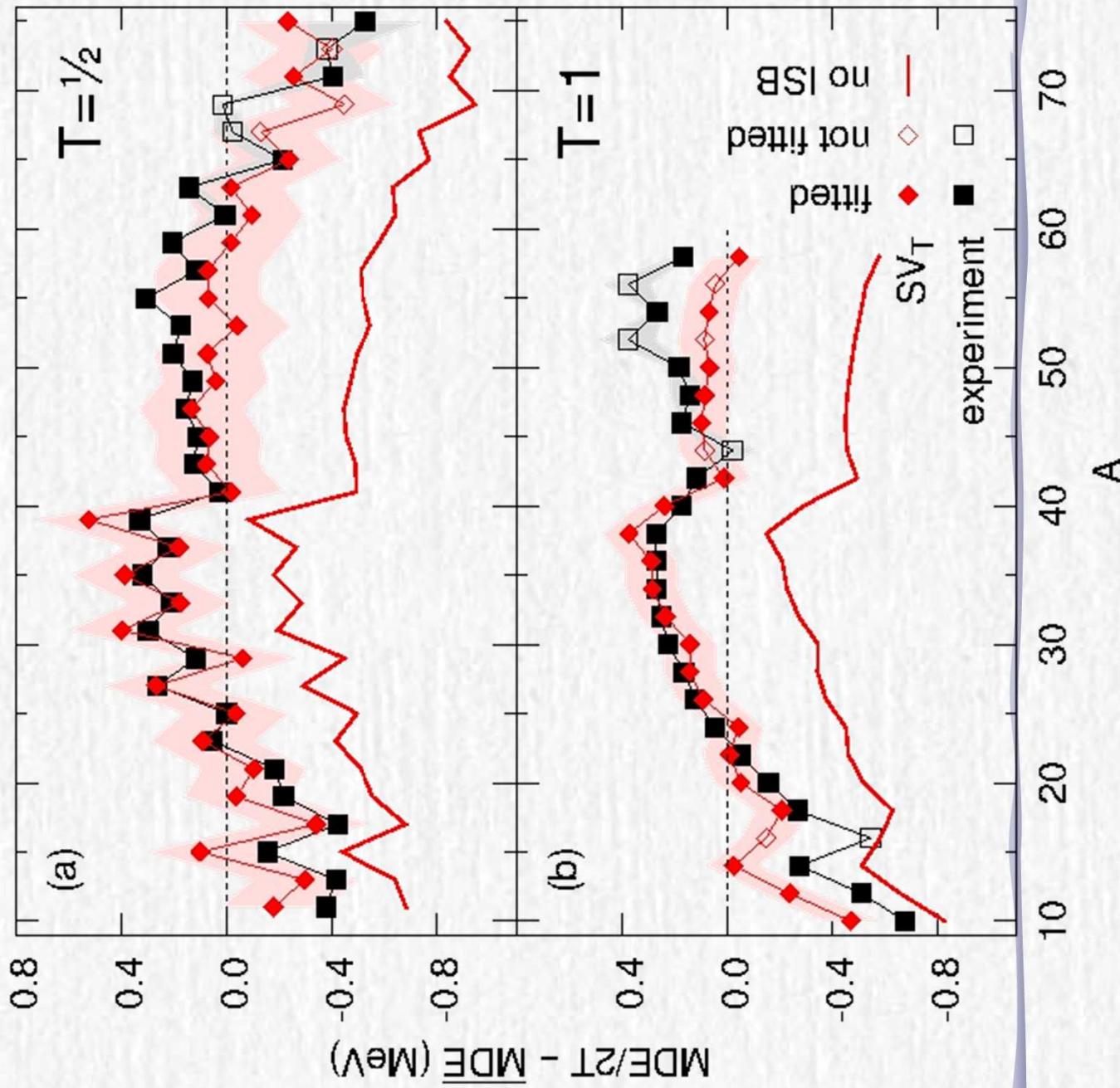


Class III corrects for MDE



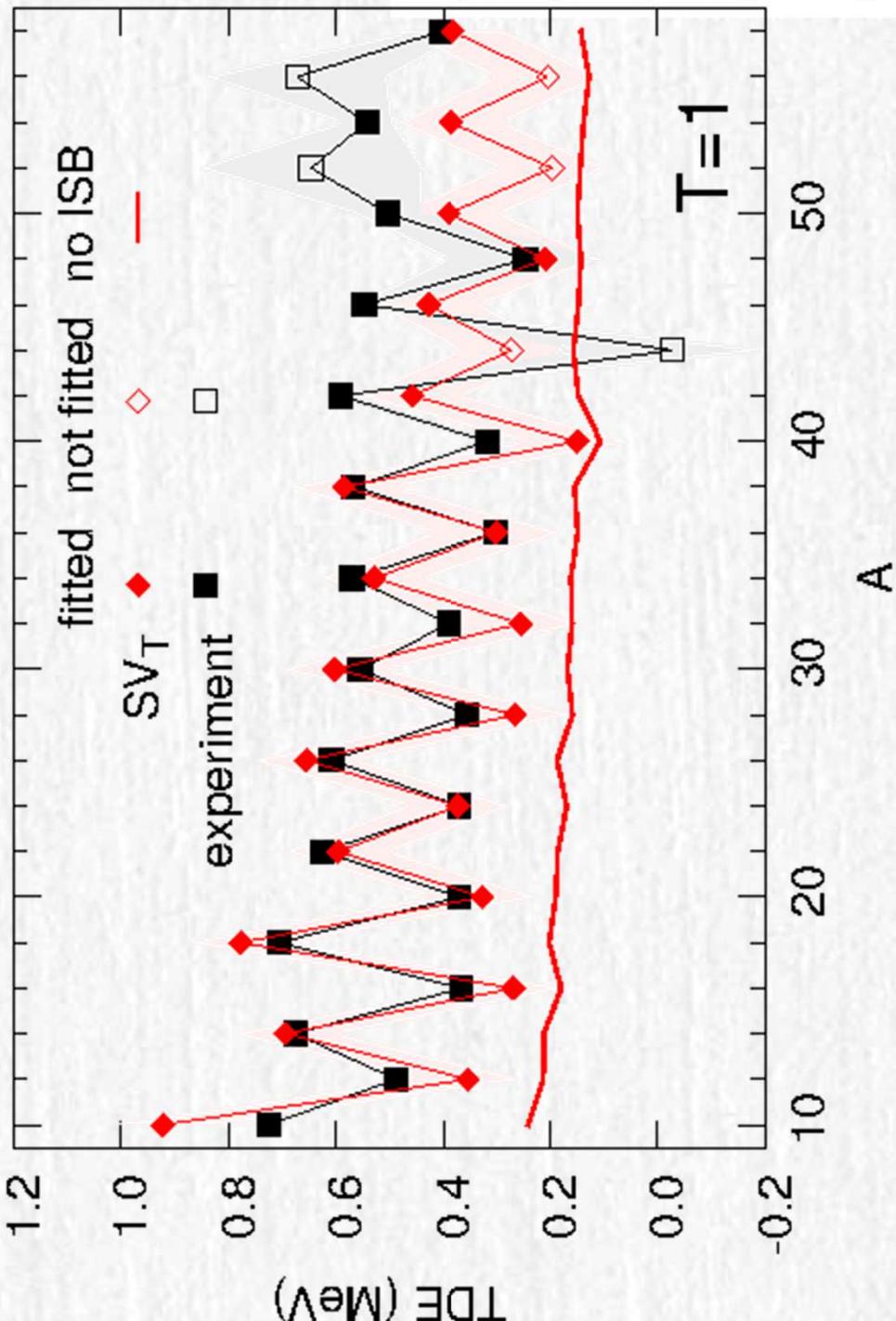
Class II corrects for TDE

Mirror displacement energies with class II and III local corrections to the Skyrme force



Triplet Displacement Energies (TDE) with class II and III local corrections to the Skyrme force

$$\begin{aligned}\hat{V}^{\text{II}}(i,j) &= \frac{1}{2}t_0^{\text{II}}\delta(\mathbf{r}_i - \mathbf{r}_j)\left(1 - x_0^{\text{II}}\hat{P}_{ij}^\sigma\right)\left[3\hat{\tau}_3(i)\hat{\tau}_3(j) - \hat{\tau}(i) \circ \hat{\tau}(j)\right], & \mathcal{H}_{\text{II}} &= \frac{1}{2}t_0^{\text{II}}\left(1 - x_0^{\text{II}}\right)\left(\rho_n^2 + \rho_p^2 - 2\rho_n\rho_p - 2\rho_{np}\rho_{pn}\right. \\ &\quad \left.- s_n^2 - s_p^2 + 2s_n \cdot s_p + 2s_{np} \cdot s_{pn}\right), \\ \hat{V}^{\text{III}}(i,j) &= \frac{1}{2}t_0^{\text{III}}\delta(\mathbf{r}_i - \mathbf{r}_j)\left(1 - x_0^{\text{III}}\hat{P}_{ij}^\sigma\right)\left[\hat{\tau}_3(i) + \hat{\tau}_3(j)\right], & \mathcal{H}_{\text{III}} &= \frac{1}{2}t_0^{\text{III}}\left(1 - x_0^{\text{III}}\right)\left(\rho_n^2 - \rho_p^2 - s_n^2 + s_p^2\right),\end{aligned}$$

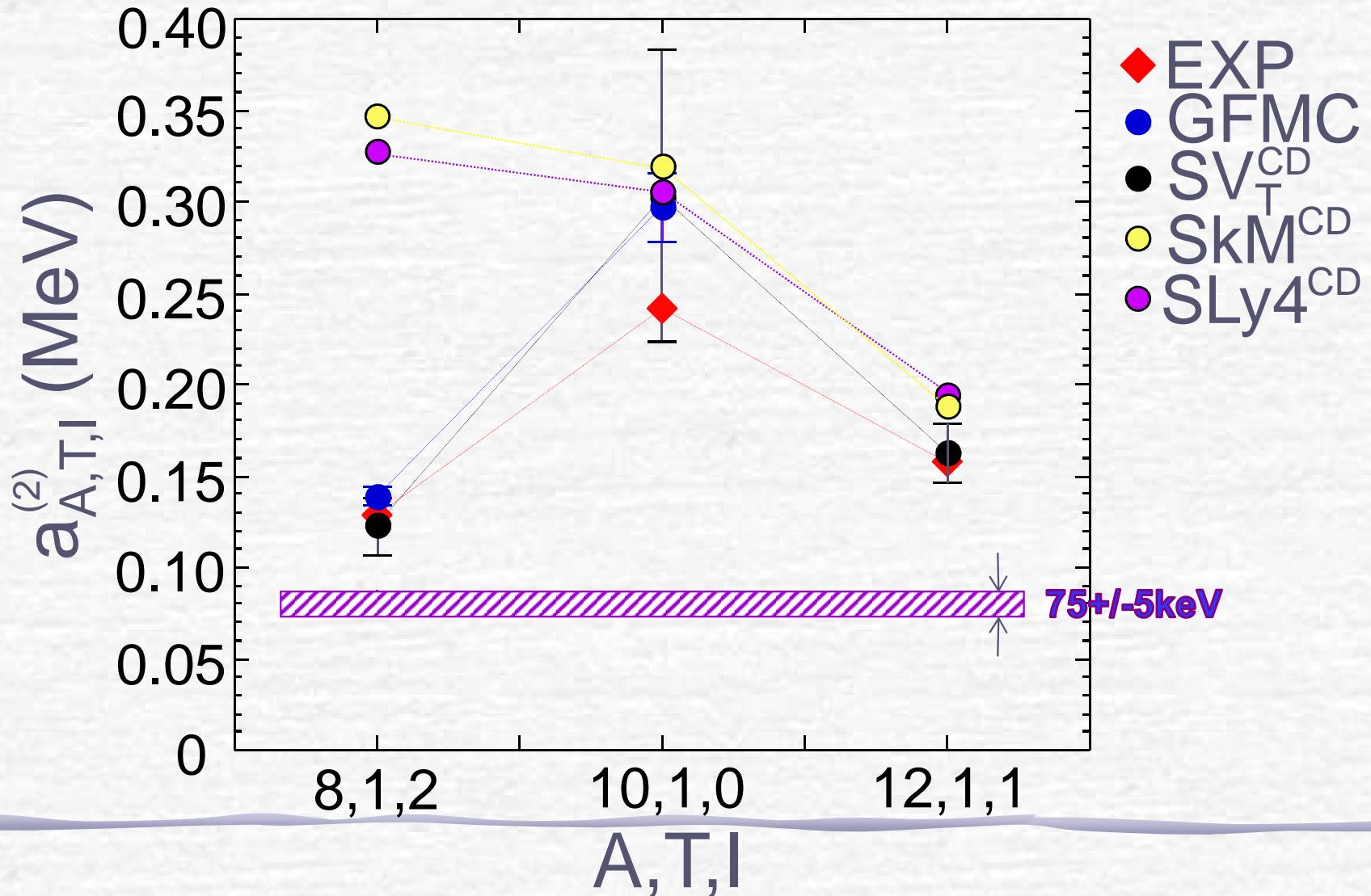


$$\begin{aligned} \text{TDE} = & BE(T=1, T_z=-1) + BE(T=1, T_z=+1), \\ & -2BE(T=1, T_z=0) \end{aligned}$$

Isobaric Multiplet Mass Equation (IMME)

$$E_{A,T}(T_z) = \sum_{n \leq 2T} a_n(A, T) Q_n(T, T_z)$$

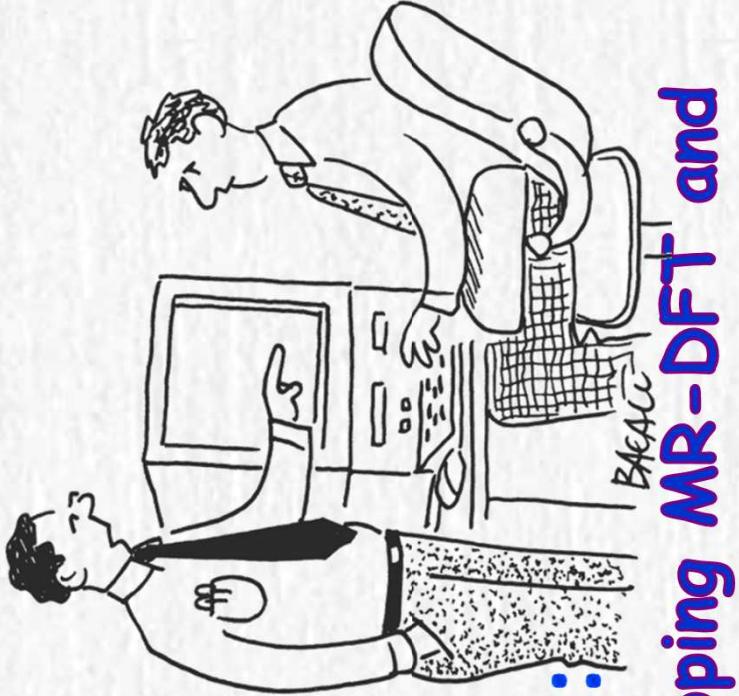
$$Q_0 = 1, Q_1 = T_z, \text{ and } Q_2 = \frac{1}{2}\{T_Z^2 - T(T+1)\}$$



MR-DFT-rooted methods are extremely attractive because:

"The purpose of computing is insight, not numbers"

(Richard Hamming, 1962)



"According to Einstein's theory, if we move the computer real fast, we can go back in time and recover the files you accidentally deleted."

Perspectives are good because:

- spectacular progress in developing **MR-DFT** and **DFT-rooted NCCI methods**
- opens new opportunities in studying new high-precision data, in particular, on short lived nuclei

Challenges for Low-Energy Nuclear Theory

- Perform proof-of-principle lattice QCD calculation for the lightest nuclei
- Develop first-principles framework for light, medium-mass nuclei, and nuclear matter from 0.1 to twice the saturation density
- Derive predictive nuclear energy density functional rooted in first-principles theory
- Carry out predictive and quantified calculations of nuclear matrix elements for fundamental symmetry tests.
- Unify the fields of nuclear structure and reactions.
- Develop predictive microscopic model of fusion and fission that will provide the missing data for astrophysics and nuclear energy research.
- Develop and utilize tools for quantification of theoretical uncertainties.
- Provide the microscopic explanation for observed, and new, (partial-) dynamical symmetries and simple patterns



Mazurian Lakes Conference on Physics

XXXV Mazurian Lakes Conference on Physics

Exotic nuclei – laboratories for fundamental laws of nature

Piaski, Poland, September 3 – 9, 2017

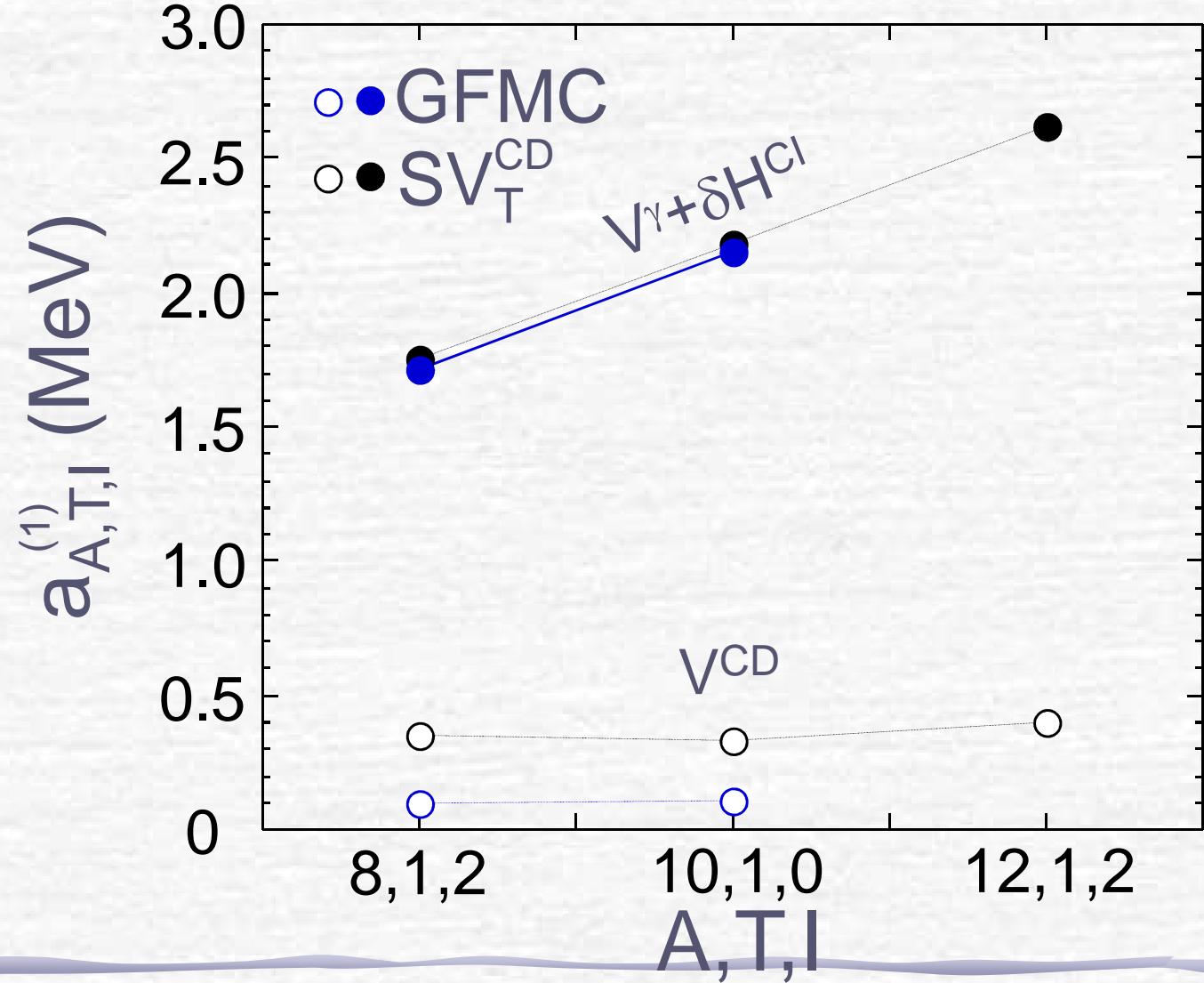
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- Exotic nuclei and fundamental symmetry tests
- Challenges in nuclear theory
- Nuclear structure and reactions
- Nuclear astrophysics and nucleosynthesis
- Nuclear fission and super-heavy elements
- Novel experimental techniques and facilities
- Interdisciplinary studies and societal applications

Organized by:

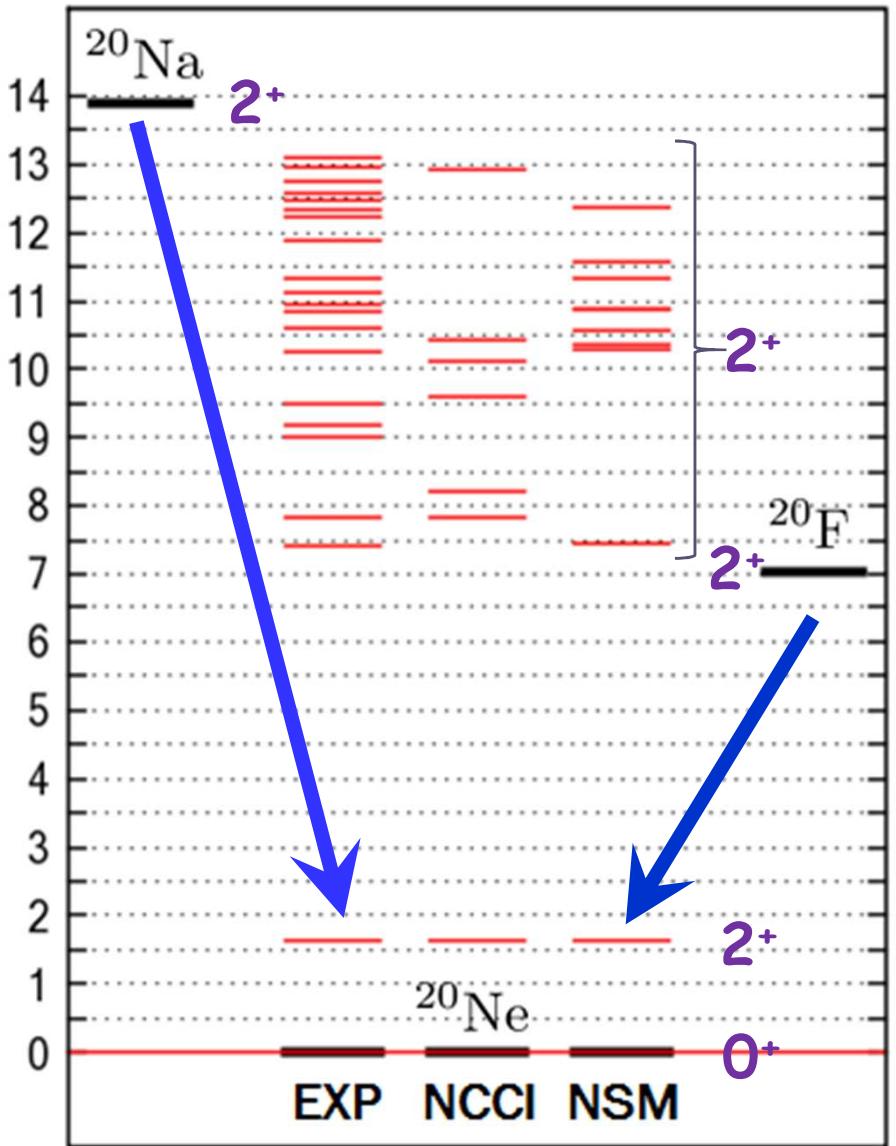




GT beta decay to excited states

(from Maciek Konieczka)

Beta decay to ^{20}Ne ; 6 SD in NCCI



Gamow-Teller matrix elements for $^{20}\text{Na} \rightarrow ^{20}\text{Ne}$

ITn	NCCI	NSM	EXP
2 0 1	0.326	0.551	0.450
2 1 1	0.752	0.774	0.532
2 0 2	2.343	2.171	—
2 1 2	0.198	0.181	—
2 0 3	0.980	1.322	1.133
2 1 3	0.531	0.384	—

Isospin symmetry breaking effects
in GT decays of T=1 nuclei

			$ M_{GT-} - M_{GT\pm} $
β^-	β^\pm	MR-DFT	EXP
$^{18}\text{Ne} \rightarrow ^{18}\text{F}$	$^{18}\text{F} \rightarrow ^{18}\text{O}$	0.012	0.008
$^{20}\text{Na} \rightarrow ^{20}\text{Ne}$	$^{20}\text{F} \rightarrow ^{20}\text{Ne}$	0.003	-0.023
$^{24}\text{Al} \rightarrow ^{24}\text{Mg}$	$^{24}\text{Na} \rightarrow ^{24}\text{Mg}$	0.002	0.006
$^{28}\text{P} \rightarrow ^{28}\text{Si}$	$^{28}\text{Al} \rightarrow ^{28}\text{Si}$	0.035	0.015
$^{30}\text{S} \rightarrow ^{30}\text{P}$	$^{30}\text{P} \rightarrow ^{30}\text{Si}$	-0.009	-0.017
$^{32}\text{P} \rightarrow ^{32}\text{S}$	$^{32}\text{Cl} \rightarrow ^{32}\text{S}$	0.000	0.034

VERY PRELIMINARY RESULTS !!!