"Nuclear shell structures in terms of classical periodic orbits"

K.A., Physica Scripta 91 (2016) 063002

Various properties of nuclear shapes and symmetries are clearly understood with the semiclassical periodic orbit theory

- Nuclear magic numbers and pseudospin symmetry
  special combination of surface diffuseness and spin-orbit coupling
  $\cdots$ bifurcation of circular PO
- Superdeformed/hyperdeformed shell structures
  remarkable shell effect at axis ratio $\approx$ 2:1 and 3:1
  $\cdots$ significance of the bridge-orbit bifurcations between two diameter POs
- Anomalous shell effect at large tetrahedral deformation
  Evolution of shell structure with 4 kinds of pure octupole deformations
  Anomalously strong shell effect at large Y_{32} (tetrahedral) deformation
  $\cdots$ crossings of the POs and emergences of the bridge orbits between them
- Prolate-oblate asymmetry and the prolate-shape dominance
  shell energy valleys $\cdots$ constant-action curves of the bridge orbit
  effect of the spin-orbit coupling on the bifurcations

PERIODIC ORBIT BIFURCATIONS AND LOCAL SYMMETRY RESTORATIONS IN EXOTIC SHAPE NUCLEAR MEAN FIELDS

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1. **Anomalous shell effect at large tetrahedral deformation**
   – Significance of tetrahedral shape degree of freedom
   – Evolution of shell structures with 4 kinds of pure octupole deformations
   – Convex tetrahedral deformation
2. **Semiclassical origin of the tetrahedral shell structure**
   – Trace formula – semiclassical formula for the level density
   – Periodic orbit bifurcations and local dynamical symmetries
   – Significant effect of the bridge-orbit bifurcations
3. **Summary**

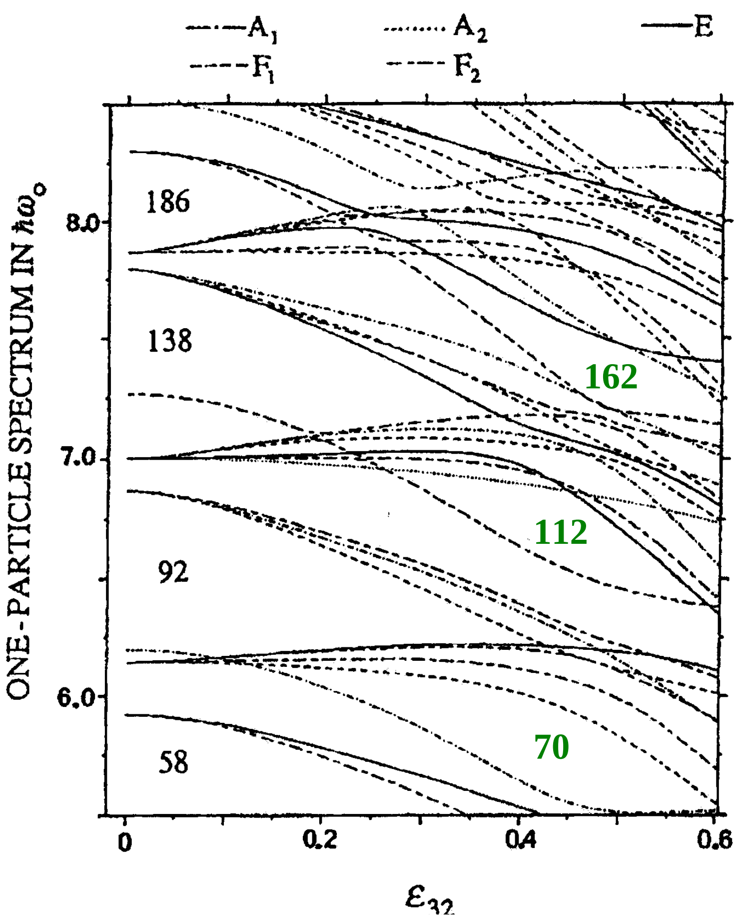
1. Anomalous shell effect at large tetrahedral deformation

Significance of tetrahedral shape degree of freedom

recent developments of experimental techniques

$\rightarrow$ neclei with various combination of (N, Z) have become available

possibility of exotic-shape nuclei in the region where the deformed shell effects for proton and neutron play constructive roles



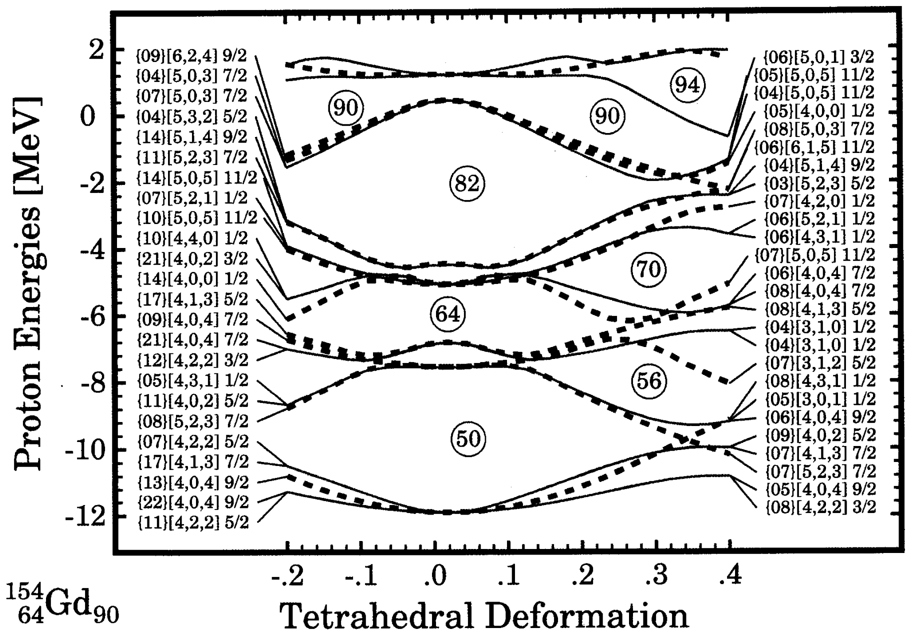
Hamamoto et al., Z. Phys. D21, 163 (1991)

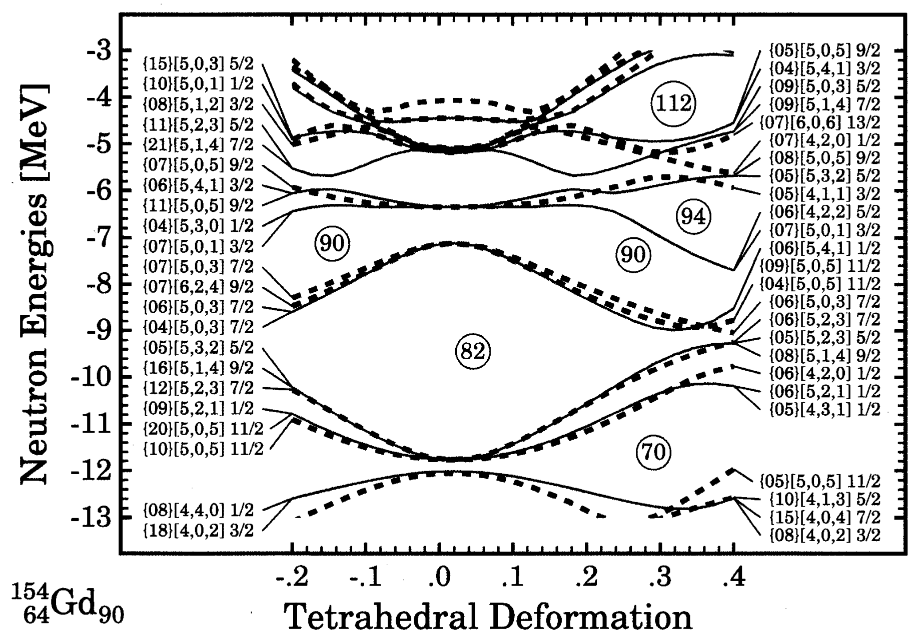
$\star$ modified oscillator model

$$V(\boldsymbol{r}) = \frac{1}{2} m\omega_0^2 r^2 \left\{ 1 + \frac{\varepsilon_{32}}{\sqrt{2}} (Y_{32} + Y_{32}^*) \right\} + v_{ll} \hbar\omega_0 (\boldsymbol{l}^2 - \langle \boldsymbol{l}^2 \rangle_N)$$

$\star$ remarkable shell gaps at

$$\beta_{32} = \frac{1}{2} \varepsilon_{32} \sim 0.25$$





Dudek et al., Int. J. Mod. Phys. E **16**, 533 (2007)

- ⋆ Woods-Saxon model (including spin-orbit coupling and Coulomb int.)

$$R(\theta,\varphi) = R_0\Big[1 + t_3\{Y_{32}(\theta,\varphi) + Y_{3-2}(\theta,\varphi)\}\Big]$$

- ⋆ remarkable shell effect at finite Y_{32} deformation ($\beta_{32} = \sqrt{2}t_3 \approx 0.4$)

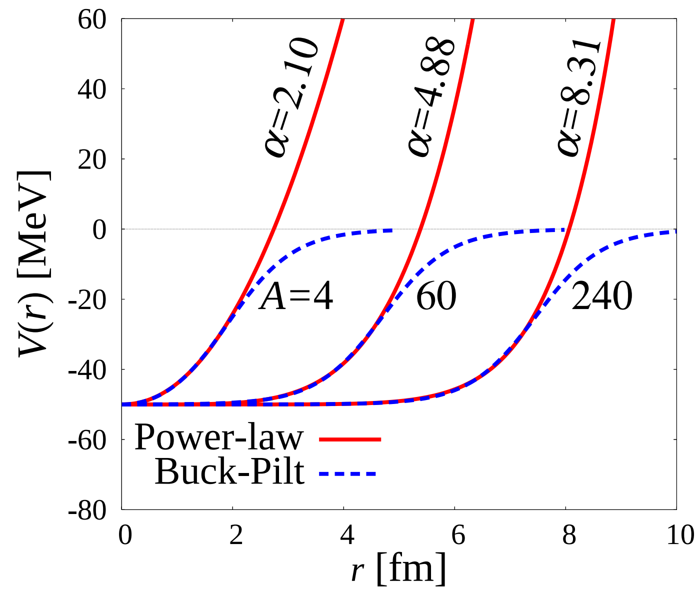
Evolution of shell structure with pure octupole deformations

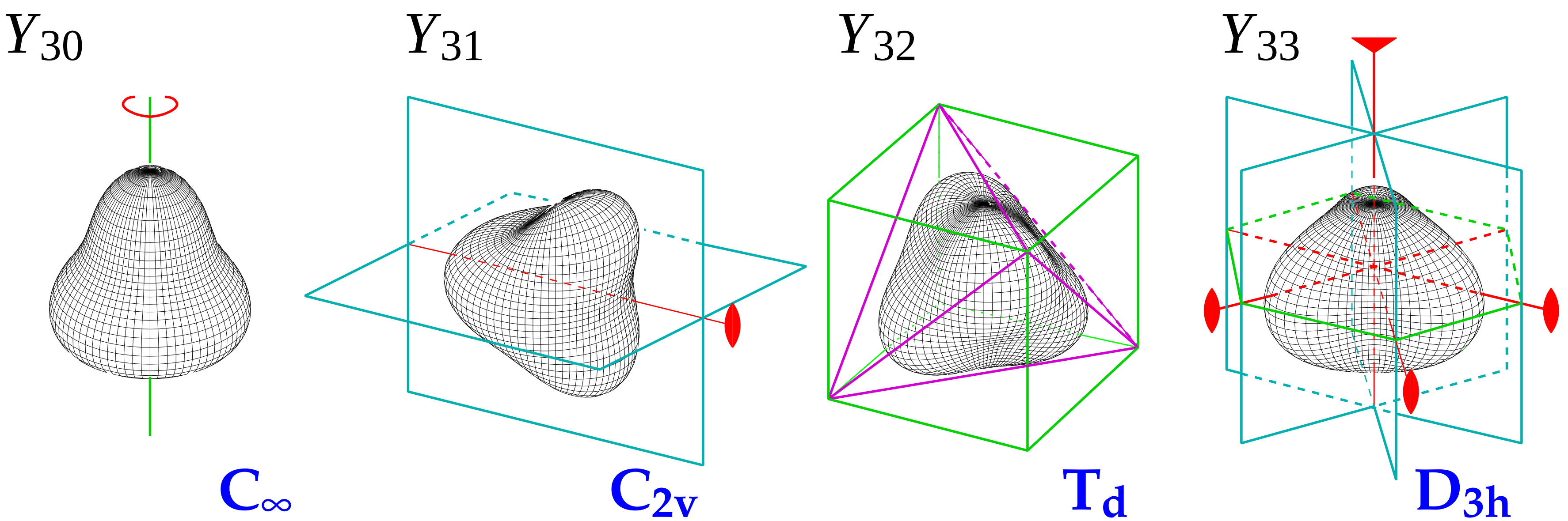
Radial power-law potential
with pure octupole deformation

$$V(\boldsymbol{r};\beta_{3K}) = U_0\left(\frac{r}{R_0 f(\Omega;\beta_{3K})}\right)^{\alpha}$$

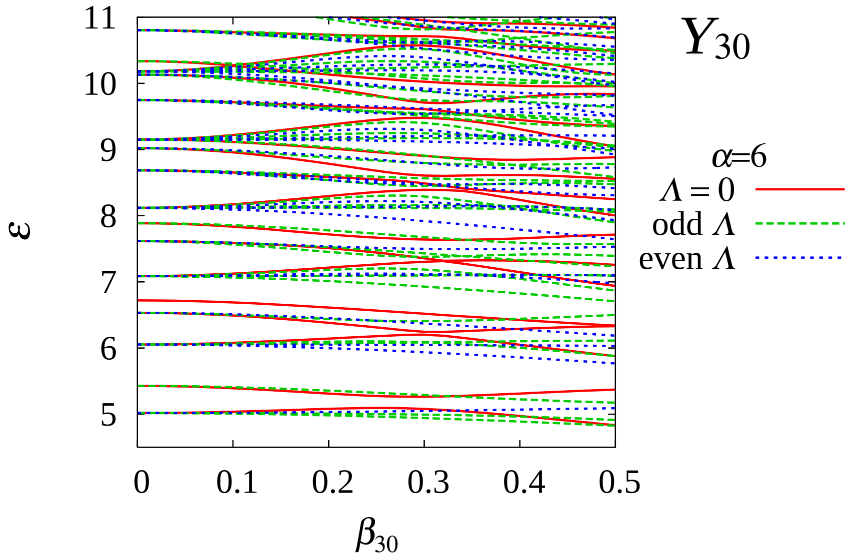
$$f(\Omega;\beta_{3K}) = \exp\left[\beta_{3K}\tilde{Y}_{3K}(\Omega)\right],$$

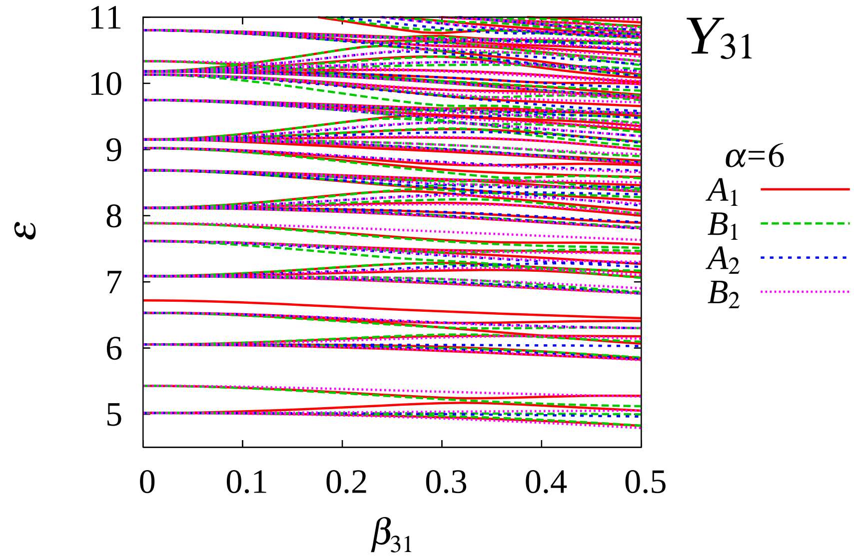
$$\tilde{Y}_{3K} = Y_{30},\ \frac{1}{\sqrt{2}}(Y_{3K}+Y^*_{3K})$$

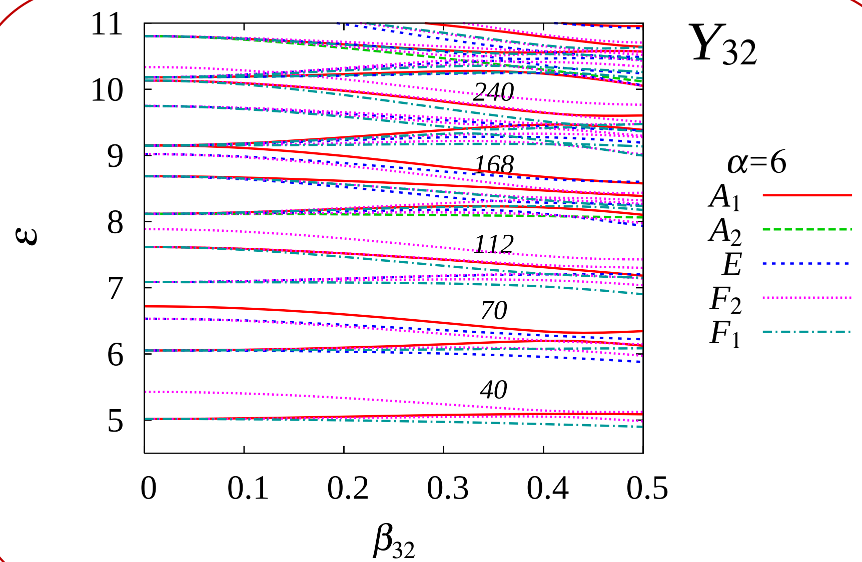


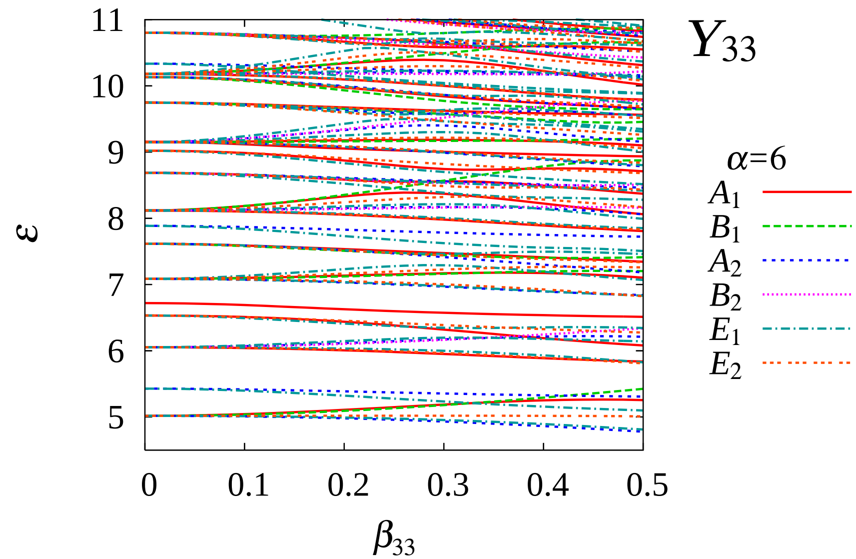


Single-particle level diagrams









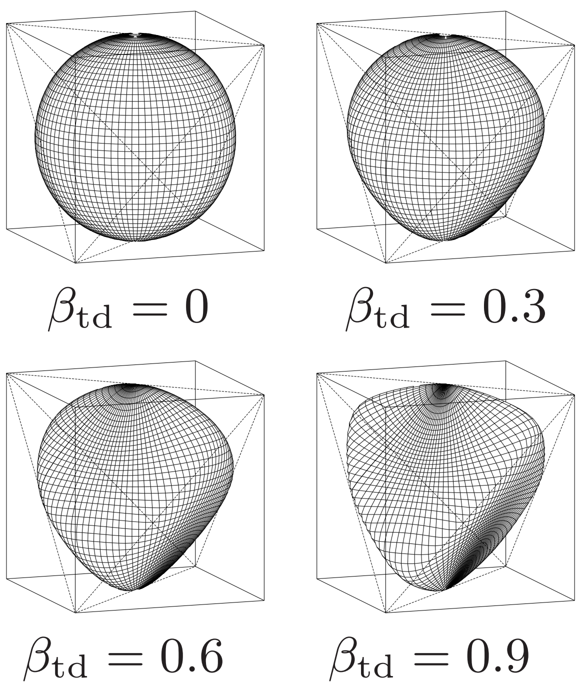
remarkable enhancement of shell effect for large Y_{32} deformation

Convex tetrahedral shape parametrization

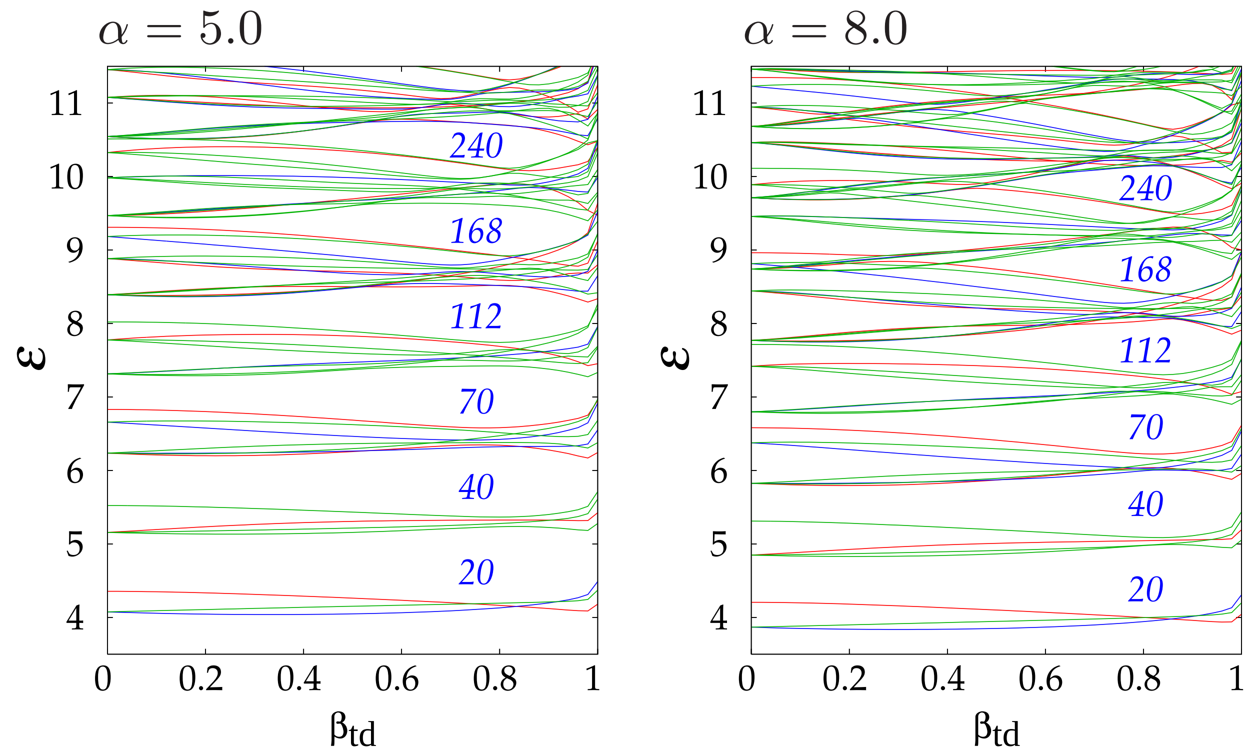
K. A. and Y. Mukumoto, Phys. Rev. C **89** (2014), 054308

$$f^2 + \beta_{\rm td}\{\tfrac{1}{2} + (\tfrac{2}{15}P_{32}\sin 3\varphi)f^3 - (\tfrac{1}{10} + \tfrac{2}{5}P_{40} + \tfrac{1}{420}P_{44}\cos 4\varphi)f^4\} = 1$$

$\beta_{\rm td} = 0$ … sphere ($f = 1$) $\longrightarrow$ $\beta_{\rm td} = 1$ … tetrahedron



Single-particle level diagram



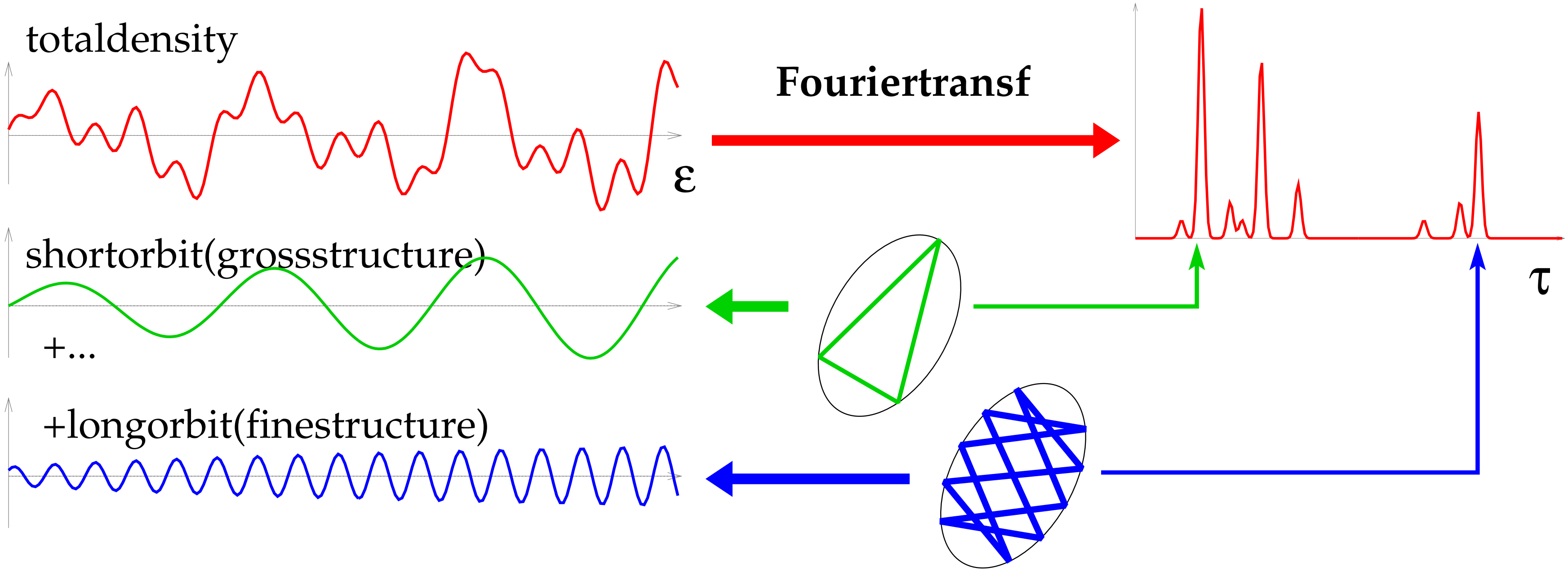
- tetrahedral shell effect much more pronounced than that of pure Y_{32}
- sharp surface + tetrahedral $\Rightarrow$ restoration of dynamical symmetry $\sim$ SU(3)?

2. Semiclassical origin of the tetrahedral shell structure

The Gutzwiller Trace Formula

Fluctuation part of the level density ... irregular oscillation ?

➡ sum of regular oscillations ⇐ contribution of **classical periodic orbits**



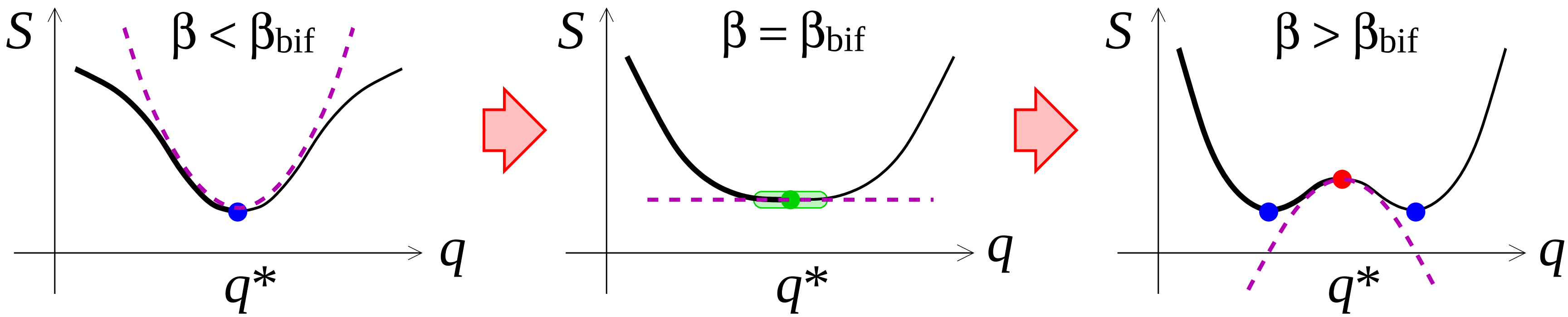
Trace Formula

$$\delta g(\varepsilon) \sim \sum_{\mathrm{po}} A_{\mathrm{po}}(\varepsilon) \cos\left[\frac{S_{\mathrm{po}}(\varepsilon)}{\hbar} - \nu_{\mathrm{po}}\right], \qquad S_{\mathrm{po}}(\varepsilon) = \oint_{\mathrm{po}} \boldsymbol{p} \cdot d\boldsymbol{r} = \hbar\varepsilon\tau_{\mathrm{po}}$$

☞ M. C. Gutzwiller, J. Math. Phys. **8** (1967), 1979; **12** (1971), 343.

Effect of PO bifurcations to the deformed shell structures

Typical bifurcation scenario (ex. “pitchfork” bifurcation)



Bifurcation $\cdots$ zero curvature $S''(q^*) = 0$

$\to$ continuous family of quasi-stationary points (local family of PO)

local dynamical symmetry (q being the “negligible” coordinate)

coherent contribution to the trace integral

$\to$ enhancement of $A_{\rm po}$ $\to$ growth of shell effect

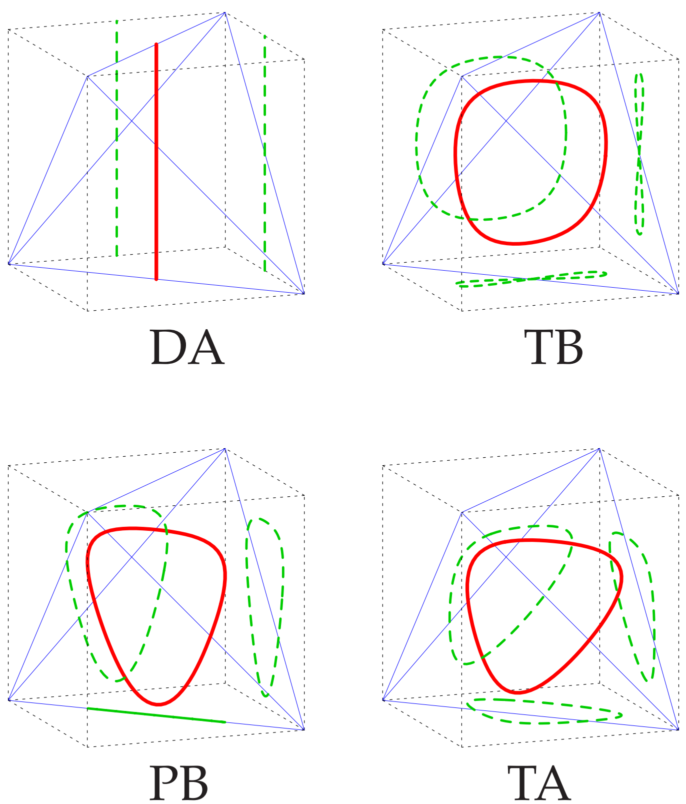
Enhancement of shell effect around a certain deformation

$\cdots$ Significant effect of the bifurcations in short POs

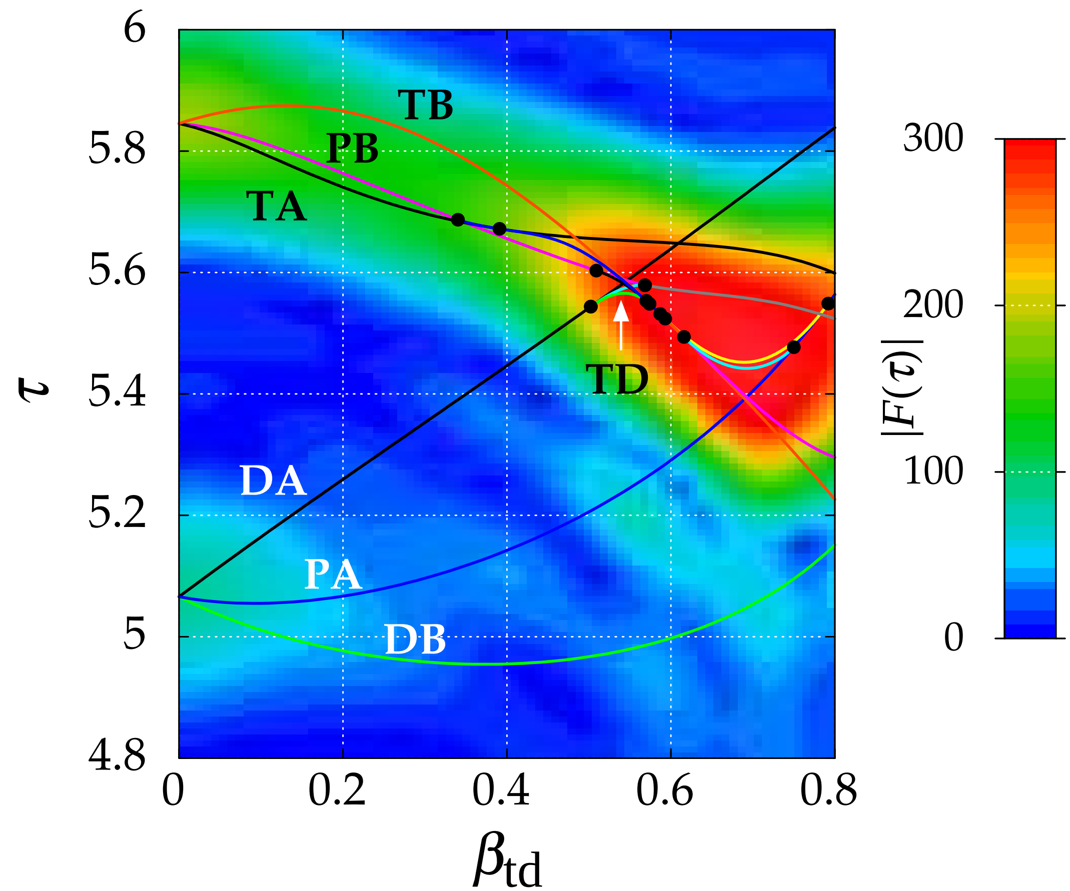
Quantum-classical correspondence in tetrahedral potential

Classical periodic orbits

$(\alpha = 5.0,\ \beta_{\rm td} = 0.3)$



Fourier amplitude $|F|$ and periods $\tau_{\rm PO}$



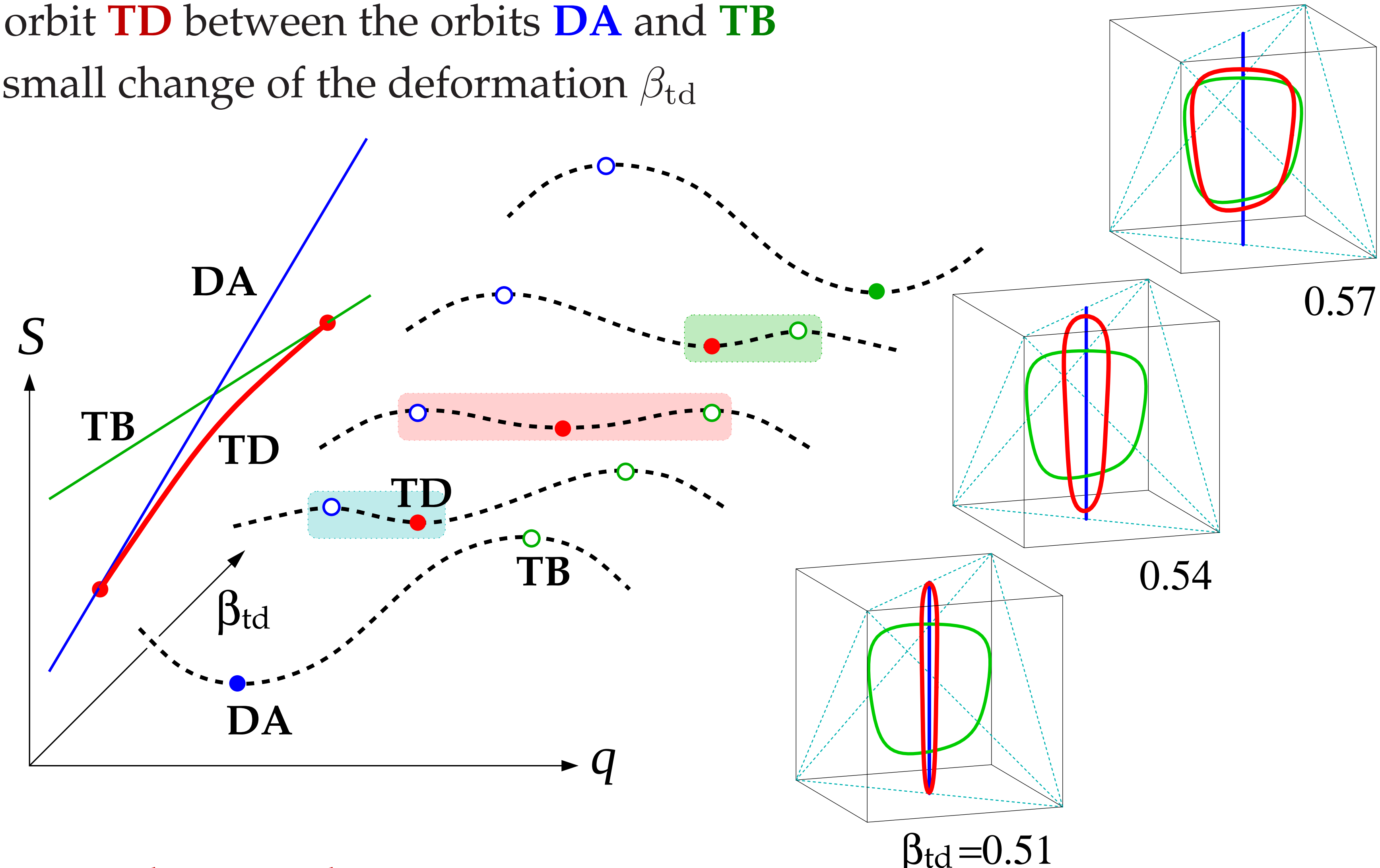
Simultaneous bifurcations of many PO's having almost the same periods $\tau_{\rm po}$

Emergences of **bridge orbits** between different orbits

... dynamical symmetry restored in wide region of the phase space

Bridge orbit bifurcations

Bridge orbit **TD** between the orbits **DA** and **TB**
within small change of the deformation β_{td}



Approximate dynamical symmetry
widely-distributed quasi-stationary points (quasi-periodic orbits)

$\Rightarrow$ coherent contribution to the level density

3. Summary

Semiclassical analysis of nuclear shell structures with the radial power-law potential model

- ❑ Periodic orbit bifurcations $\cdots$ indication of dynamical symmetries
  significant roles in emergence of strong shell effect at a certain deformation
- ❑ Bridge orbit bifurcation $\cdots$ dynamical symmetry between two orbits
  especially strong effect to the quantum shell effect
  $\Rightarrow$ anomalous shell effect at large tetrahedral deformation

Future subjects

- ❑ Effect of the fragment shell structures to the fission deformation
  $\Rightarrow$ puzzles in asymmetric fission (e.g., neutron deficient Hg isotopes)
- ❑ Shell evolutions in unstable nuclei