Stability of the wobbling motion in the tri-axially deformed odd-A nucleus

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= Plan of the Talk =

§1. Motivation and purposes

§2. Particle-rotor model

§3. Holstein-Primakoff bosons and top-on-top model

§4. Stability domains of wobbling modes

§5. Diagonal boson representations for I_y and j_x

§6. Taking account of the CAP effect in the moment of inertia

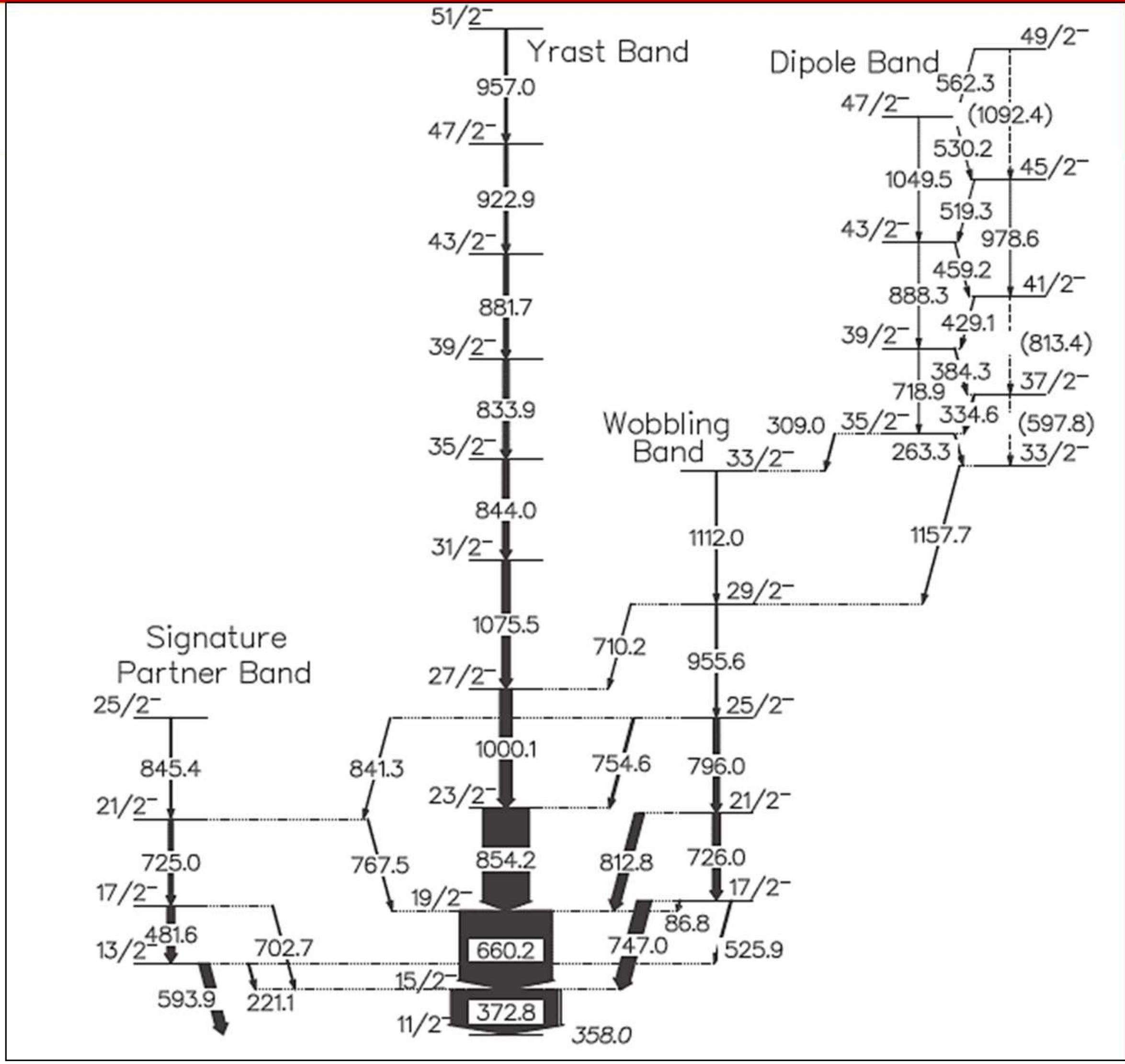
§7. Conclusion

§1. Motivation and purposes

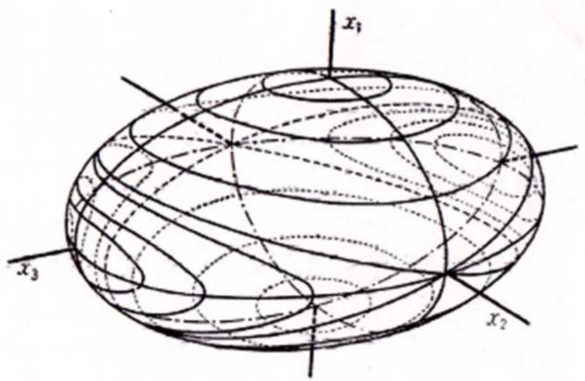
Experimental Results for 135Pr:
J.T.Matta *et al*,
PRL 114,082501 (2015)

(1) Is it transverse wobbling ?

(2) Is there any alternative possibility ?



Wobbling motion（Precession)



Landau and Lifschits (1958)： Goldstein(2002)

$$\mathcal{J}_1 < \mathcal{J}_2 < \mathcal{J}_3$$
$$2E = \frac{L_1^2}{\mathcal{J}_1} + \frac{L_2^2}{\mathcal{J}_2} + \frac{L_3^2}{\mathcal{J}_3} \text{ (ellipsoid)}$$
$$L^2 = L_1^2 + L_2^2 + L_3^2 \text{ (sphere)}$$
$$2E\mathcal{J}_1 < L^2 < 2E\mathcal{J}_3$$

The intersection between ellipsoid and sphere is the orbit for given *L* and *E*. In the plane perpendicular to x_1 or x_3 axis the orbit is ellipse, while to x_2 axis it becomes hyperbola. There is **no stable rotation around the x_2 axis with middle MoI**.

Bohr-Mottelson (1967)

$$\mathcal{J}_1 > \mathcal{J}_2 > \mathcal{J}_3 \qquad A_k = 1/(2\mathcal{J}_k)\ (k = 1,2,3 \text{ or } x,y,z)$$
$$I_+ = \sqrt{2I}c^+,\ I_- = \sqrt{2I}c,\ I_1 = I$$
$$H_{\mathrm{rot}} = A_1 I(I+1) + (n + \frac{1}{2})\hbar\omega$$
$$\hbar\omega = 2I\sqrt{(A_2 - A_1)(A_3 - A_1)}$$

There is no wobbling around 2-axis with middle MoI, because wobbling energy is imaginary.

§2. Particle-rotor model

- We attempt "Gedanken experiment" with particle-rotor model.
- Why particle-rotor model ?

$$H = H_{\rm rot} + H_{\rm sp}, \quad H_{\rm rot} = \sum_{k=x,y,z} A_k (I_k - j_k)^2, \quad H_{\rm sp} = \frac{V}{j(j+1)}\left[\cos\gamma(3j_z^2 - \vec{j}^2) - \sqrt{3}\sin\gamma(j_x^2 - j_y^2)\right]$$

(1) With either Rigid-body MoI, or Hydrodynamical MoI.

(2) For single-j case, use of Wigner-Eckart theorem directly relates H to Nilsson model. H is expressed ang. mom. vectors I and j.

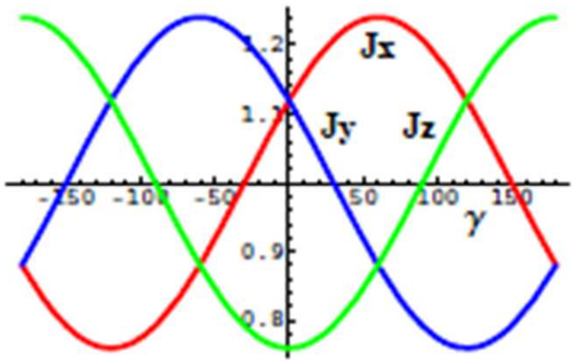
$$H_\delta = -\hbar\omega_0(\delta)\beta_2 r^2\left[\cos\gamma Y_{20} - \frac{1}{2}\sin\gamma(Y_{22} + Y_{2-2})\right]$$

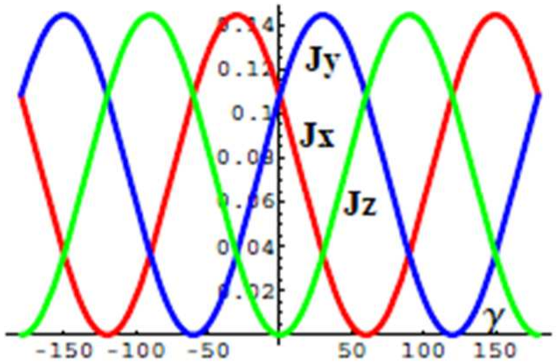
$$\langle jm|r^2\left[\cos\gamma Y_{20} - \frac{1}{2}\sin\gamma(Y_{22} + Y_{2-2})\right]|jm\rangle = -\frac{1}{8j(j+1)}\sqrt{\frac{5}{\pi}}\langle jm|r^2\left[\cos\gamma(3j_z^2 - \vec{j}^2) - \sqrt{3}\sin\gamma(j_x^2 - j_y^2)\right]|jm\rangle$$

(3) Comparison between Rig. MoI and Hyd. MoI with different nuclear radii, and periodicity in γ.

$$\mathcal{J}_k^{\mathrm{rig}} = \frac{\mathcal{J}_0}{1+(\frac{5}{16\pi})^{1/2}\beta_2}\left[1-\left(\frac{5}{4\pi}\right)^{1/2}\beta_2\cos\left(\gamma+\frac{2}{3}\pi k\right)\right]$$

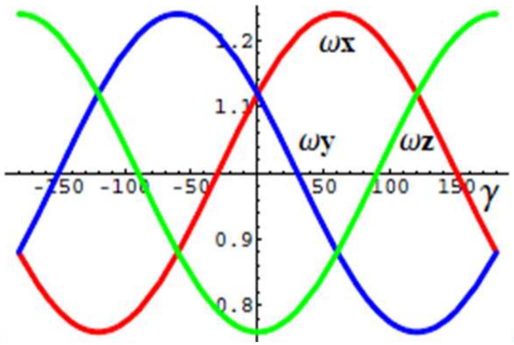
$$\mathcal{J}_k^{\mathrm{hyd}} = \frac{4}{3}\mathcal{J}_0\sin^2\left(\gamma+\frac{2}{3}\pi k\right)$$

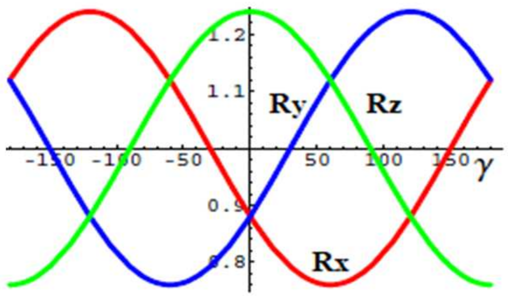




$$\omega_k = \omega_0\left[1-\sqrt{\frac{5}{4\pi}}\beta_2\cos(\gamma+\frac{2\pi k}{3})\beta_2\right]$$

$$R_k = R_0\left[1+\sqrt{\frac{5}{4\pi}}\beta_2\cos(\gamma+\frac{2\pi k}{3})\beta_2\right]$$





(4) We can compare the "exact solution" of the model obtained from diagonalization with the "approximate solution" from the Holstein-Primakoff bosonized Hamiltonian.

This is a basic strategy of present Gedanken Experiment

§3. Holstein-Primakoff bosons and top-on-top model

$$[I_k, j_l] = 0 \qquad [I_i, I_j] = -\mathrm{i}I_{i\times j}, \ [j_i, j_j] = \mathrm{i}j_{i\times j}.$$

Holstein-Primakoff boson transformation:

$$\begin{aligned}
I_+ &= I_-^\dagger = I_y + iI_z = -\hat{a}^\dagger\sqrt{2I - \hat{n}_a}, \\
I_x &= I - \hat{n}_a \qquad \text{with} \quad \hat{n}_a = \hat{a}^\dagger\hat{a}; \\
j_+ &= j_-^\dagger = j_y + ij_z = \sqrt{2j - \hat{n}_b}\,\hat{b}, \\
j_x &= j - \hat{n}_b \qquad \text{with} \quad \hat{n}_b = \hat{b}^\dagger\hat{b}
\end{aligned}$$

$$\sqrt{2I - \hat{n}_a} \simeq \sqrt{2I}\left(1 - \frac{\hat{n}_a}{4I}\right),$$

$$\sqrt{2j - \hat{n}_b} \simeq \sqrt{2j}\left(1 - \frac{\hat{n}_b}{4j}\right).$$

Bosonized approximate Hamiltonian:

$$H_\mathrm{B} \simeq H_0 + H_2 + H_4,$$

Bosonized approximate Hamiltonian :

$$H_0 = A_x^{\rm rig}(I-j)^2 + \frac{1}{2}(A_y^{\rm rig} + A_z^{\rm rig} - 2A_x^{\rm rig}) \quad -2V\cos(\gamma - \pi/3)\left(1 - \frac{3}{4j(j+1)}\right),$$

$$H_2 = \begin{pmatrix} \hat{a} & \hat{b} & \hat{a}^\dagger & \hat{b}^\dagger \end{pmatrix} \begin{pmatrix} A & G & B & F \\ G & C & F & D \\ B & F & A & G \\ F & D & G & C \end{pmatrix} \begin{pmatrix} \hat{a}^\dagger \\ \hat{b}^\dagger \\ \hat{a} \\ \hat{b} \end{pmatrix},$$

$$A = \frac{1}{2}(I - \frac{1}{2})A_{yzx} + jA_x^{\rm rig}, \quad B = \frac{1}{2}(I - \frac{1}{4})A_{yz}, \quad A_{yzx} = A_y^{\rm rig} + A_z^{\rm rig} - 2A_x^{\rm rig}, \; A_{yz} = A_y^{\rm rig} - A_z^{\rm rig},$$

$$C = \frac{1}{2}(j - \frac{1}{2})a_{yzx} + IA_x^{\rm rig}, \quad D = \frac{1}{2}(j - \frac{1}{4})a_{yz}, \quad a_{yz} = A_{yz} + \frac{2\sqrt{3}V}{j(j+1)}\sin(\gamma - \pi/3),$$

$$F = \frac{1}{2}\sqrt{Ij}(A_y^{\rm rig} + A_z^{\rm rig}), \quad G = \frac{1}{2}\sqrt{Ij}A_{yz}, \quad a_{yzx} = A_{yzx} + \frac{6V}{j(j+1)}\cos(\gamma - \pi/3).$$

NOTE:

We call the approximation employed in the previous slide "the next-to-leading order approximation", which collect all the terms up to the 4^{th} order in boson operators.

In contrast , "the leading order approximation" includes all the contributions up to the second order in boson operators.

We assume that all products of HP boson operators are rearranged to be in normal order forms.

Note that, by this way, a part of the anharmonicity effect arising from the higher order terms is taken into account by the lading-order approximation.

Diagonalization is attained by solving eigenvalue equation algebraically.

Quasiboson operators are introduced by boson Bogoliubov transformation:

$$\begin{pmatrix} \hat{a} \\ \hat{b} \\ \hat{a}^\dagger \\ \hat{b}^\dagger \end{pmatrix} = \begin{pmatrix} u_+ & w_+ & u_-^* & w_-^* \\ v_+ & t_+ & v_-^* & t_-^* \\ u_- & w_- & u_+^* & w_+^* \\ v_- & t_- & v_+^* & t_+^* \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \alpha^\dagger \\ \beta^\dagger \end{pmatrix}$$

Eigenvalue equation:

$$\begin{pmatrix} A & G & B & F \\ G & C & F & D \\ -B & -F & -A & -G \\ -F & -D & -G & -C \end{pmatrix} \begin{pmatrix} u_+ & w_+ & u_-^* & w_-^* \\ v_+ & t_+ & v_-^* & t_-^* \\ u_- & w_- & u_+^* & w_+^* \\ v_- & t_- & v_+^* & t_+^* \end{pmatrix} = \begin{pmatrix} u_+ & w_+ & u_-^* & w_-^* \\ v_+ & t_+ & v_-^* & t_-^* \\ u_- & w_- & u_+^* & w_+^* \\ v_- & t_- & v_+^* & t_+^* \end{pmatrix} \begin{pmatrix} \omega_\alpha & 0 & 0 & 0 \\ 0 & \omega_\beta & 0 & 0 \\ 0 & 0 & -\omega_\alpha & 0 \\ 0 & 0 & 0 & -\omega_\beta \end{pmatrix}$$

The leading order approximation gives :

$$H_2 \simeq 2\omega_\alpha(\hat{n}_\alpha + 1/2) + 2\omega_\beta(\hat{n}_\beta + 1/2), \qquad \boxed{\hat{n}_\alpha = \alpha^\dagger\alpha \quad \text{and} \quad \hat{n}_\beta = \beta^\dagger\beta.}$$

The next-to-leading order approximation :

$$\boxed{\begin{aligned} H_{\mathrm{B}} \simeq\ & H_0 + \omega_\alpha + \omega_\beta + C_0 + (2\omega_\alpha + C_\alpha)\hat{n}_\alpha \\ & + (2\omega_\beta + C_\beta)\hat{n}_\beta + C_{\alpha\alpha}\hat{n}_\alpha^2 + C_{\beta\beta}\hat{n}_\beta^2 + C_{\alpha\beta}\hat{n}_\alpha\hat{n}_\beta, \end{aligned}}$$

where six constants C’s are additional contributions from higher order terms describing anharmonicity effect.

In order to clarify the meaning of two quantum number

$$\hat{n}_\alpha = \alpha^\dagger \alpha \quad \text{and} \quad \hat{n}_\beta = \beta^\dagger \beta.$$

we consider the case of V=0 (i.e., pure rotor vase), and with I_x, j_x in diagonal forms, then we have

$$E_{\rm rot} \simeq A_x^{\rm rig} R(R+1) - \frac{p+q}{2} n_\alpha^2 \ + \left(2R\sqrt{pq} + \sqrt{pq} - \frac{p+q}{2}\right)\left(n_\alpha + \frac{1}{2}\right), \quad R = |I - j| + n_\beta$$

which gives energies of wobbling and precession $\omega_\alpha \sim (I-j)\sqrt{pq}, \quad \omega_\beta \sim (I-j)A_x^{\rm rig}.$

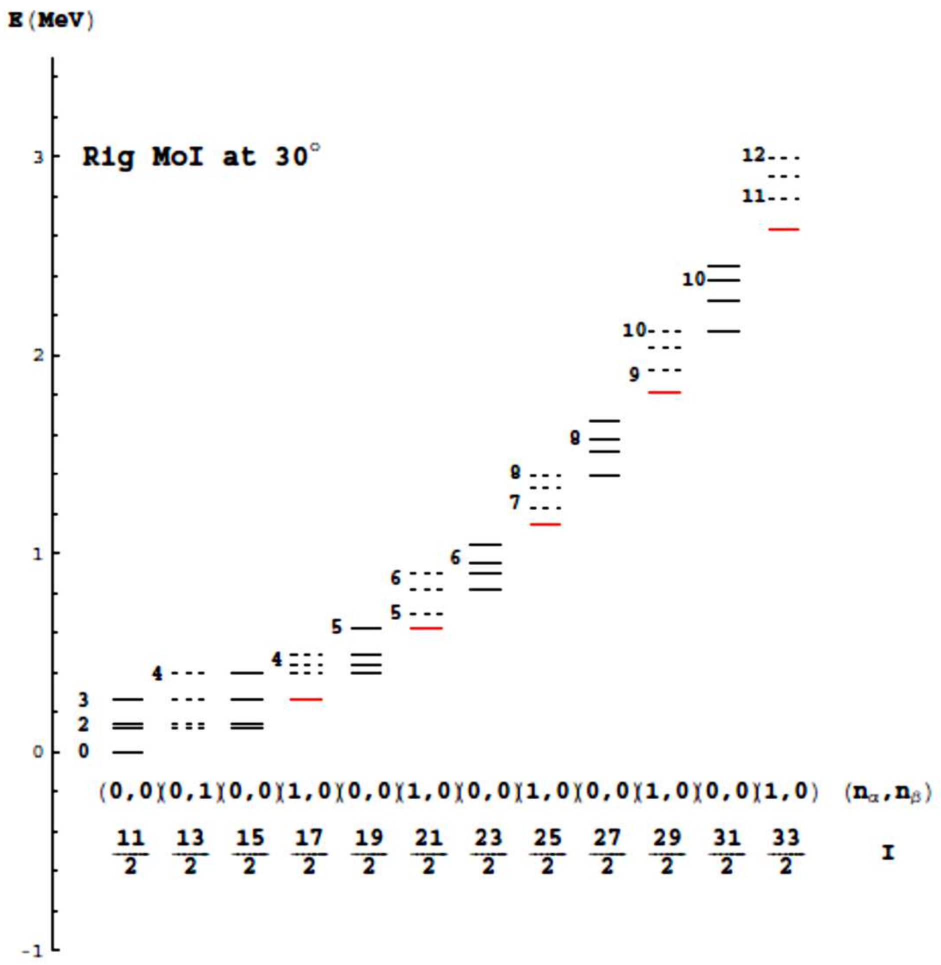
This solution gives exact result

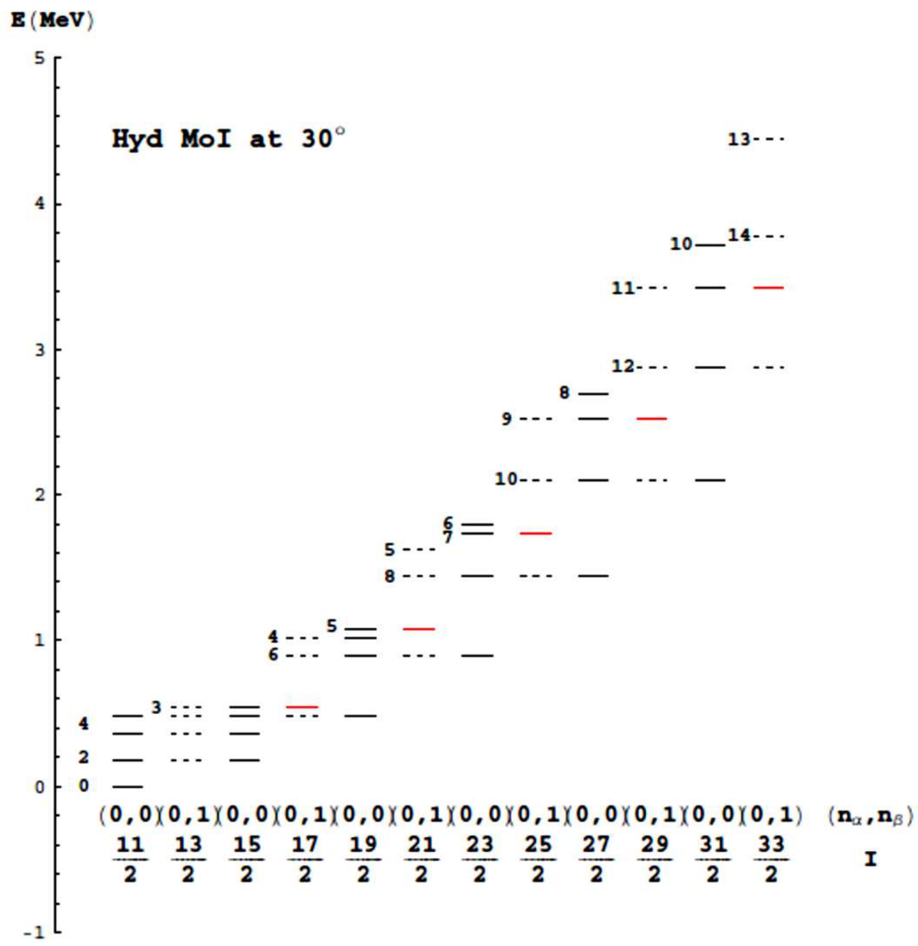
$$E_{\rm rot} = A_z^{\rm rig} R(R+1) - (A_z^{\rm rig} - A_x^{\rm rig})(R - n_\alpha)^2.$$

in the symmetric limit $A_y^{\rm rig} = A_z^{\rm rig}$ ($\gamma = 60^\circ$)

Thus, the integral values coincide with the numbering of the D_2-invariant independent bases.

Calculated energy levels



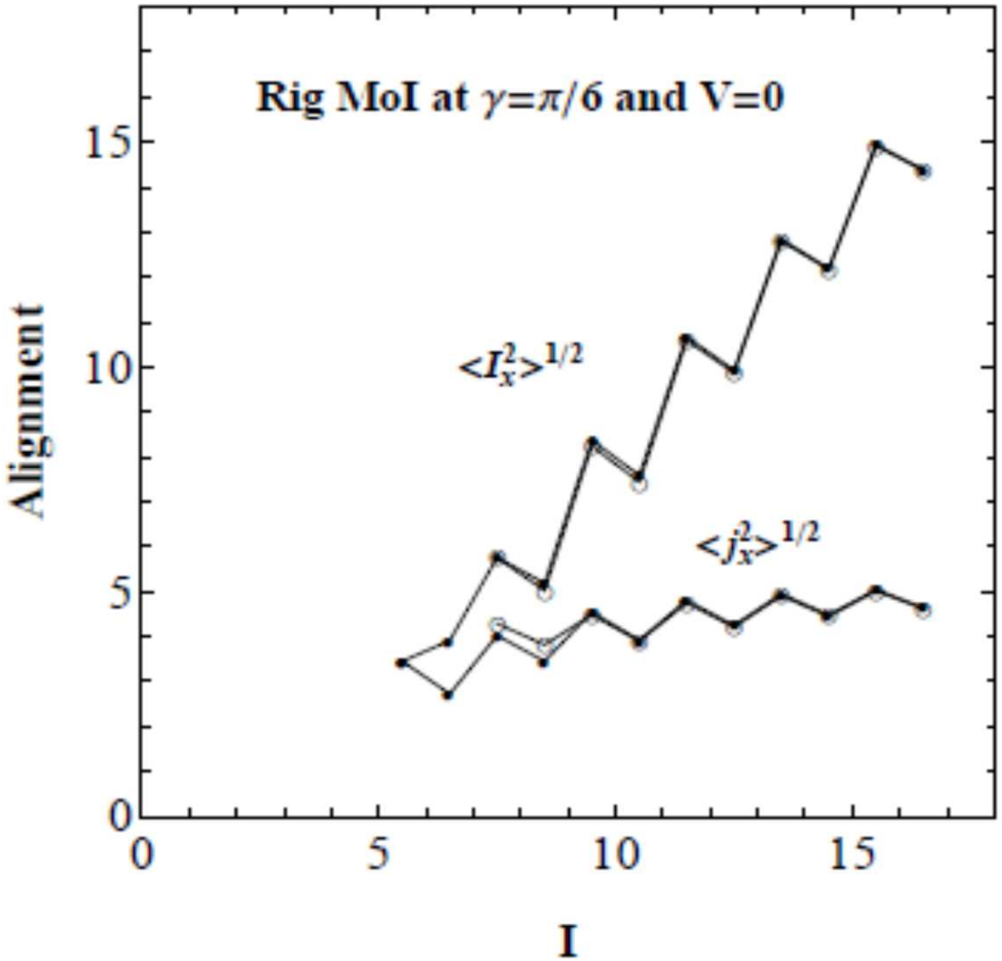


(1) Comparison with exact result Rig. MoI (Solid circles)

<u>Calculated alignment:</u>

$$\mathcal{J}_x^{\text{rig}} \geq \mathcal{J}_y^{\text{rig}} \geq \mathcal{J}_z^{\text{rig}}$$

I_x and j_x are in diagonal representation

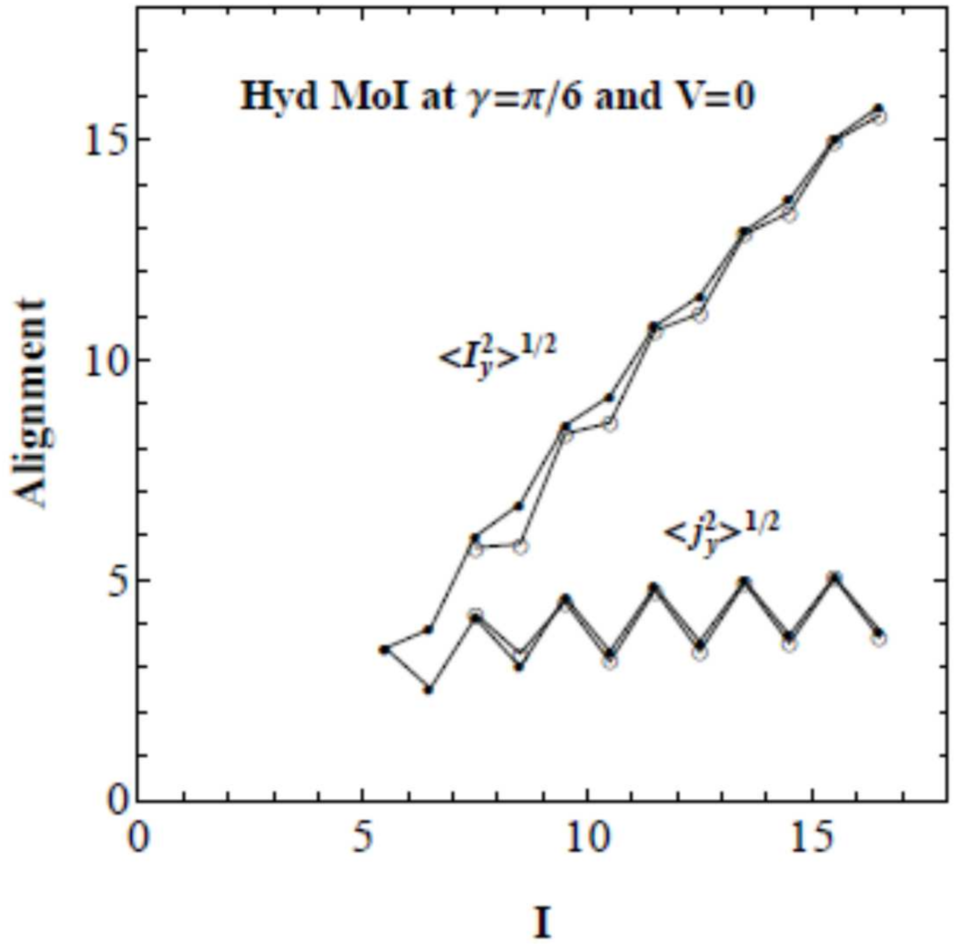


(2) Comparison with exact result with hyd. MoI (Solid circles)

<u>Calculated alignment:</u>

$$\mathcal{J}_y^{\text{hyd}} > \mathcal{J}_x^{\text{hyd}} > \mathcal{J}_z^{\text{hyd}}$$

I_y and j_y are in diagonal representstion



§4. Stability domains of wobbling modes

Eigenvalue equation:

$$\begin{vmatrix} A-\omega & G & B & F \\ G & C-\omega & F & D \\ -B & -F & -A-\omega & -G \\ -F & -D & -G & -C-\omega \end{vmatrix} = 0.$$

This equation is reduced to $\omega^4 - b\omega^2 + c = 0,$

with $b \equiv A^2 - B^2 + C^2 - D^2 + 2(G^2 - F^2),$

$$c \equiv (A^2 - B^2)(C^2 - D^2) + (G^2 - F^2)^2 + 4FG(AD + BC) - 2(AC + BD)(F^2 + G^2).$$

We get real solutions

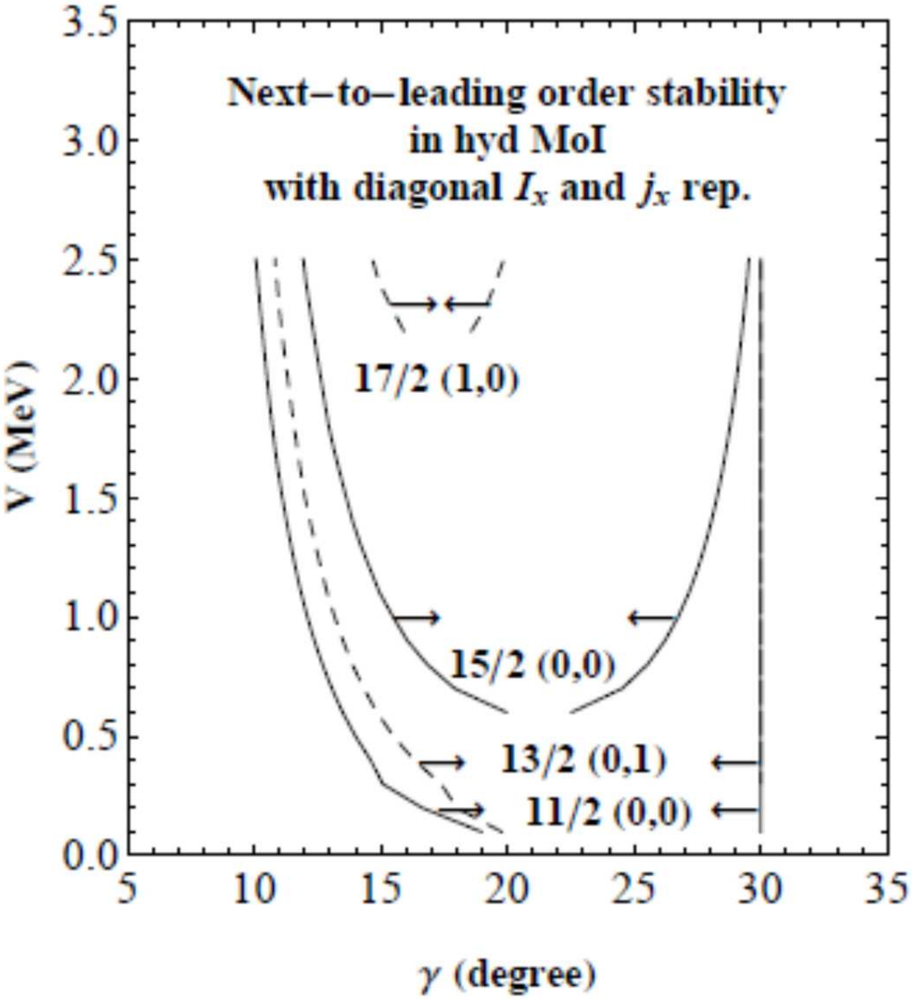
$$2\omega_{(\pm)}^2 = b \pm \sqrt{b^2 - 4c},$$

only when conditions $b^2 - 4c \geq 0, \quad c > 0, \quad b > 0.$ **are satisfied.**

(1) Stability domain in $\gamma - V$ plane for hyd. MoI:

I_x and j_x are in diagonal Representation.

In the region of $V > 2$MeV I=17/2(1,0) level appears. But, there does not appear any level with (1,0) .



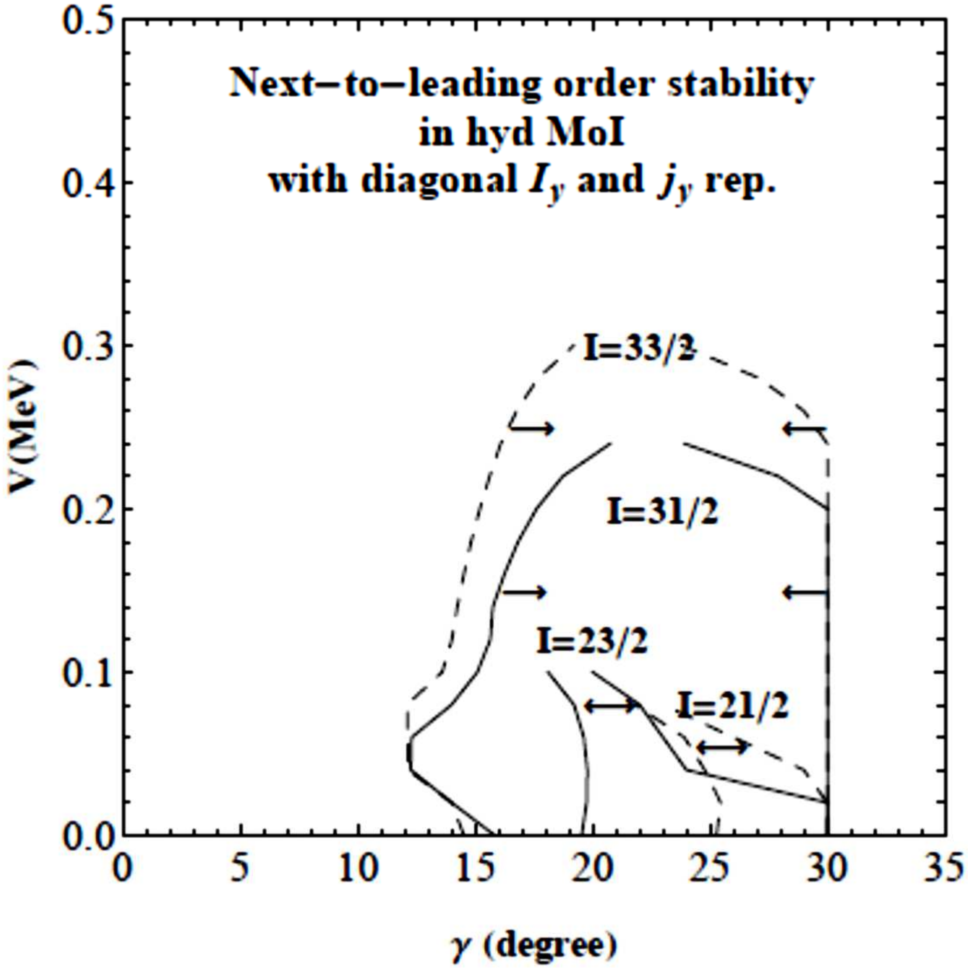
(2) Stability domain in $\gamma - V$ plane for hyd. MoI:

I_y and j_y are in diagonal representation.

Practically there does not exist any stability region.

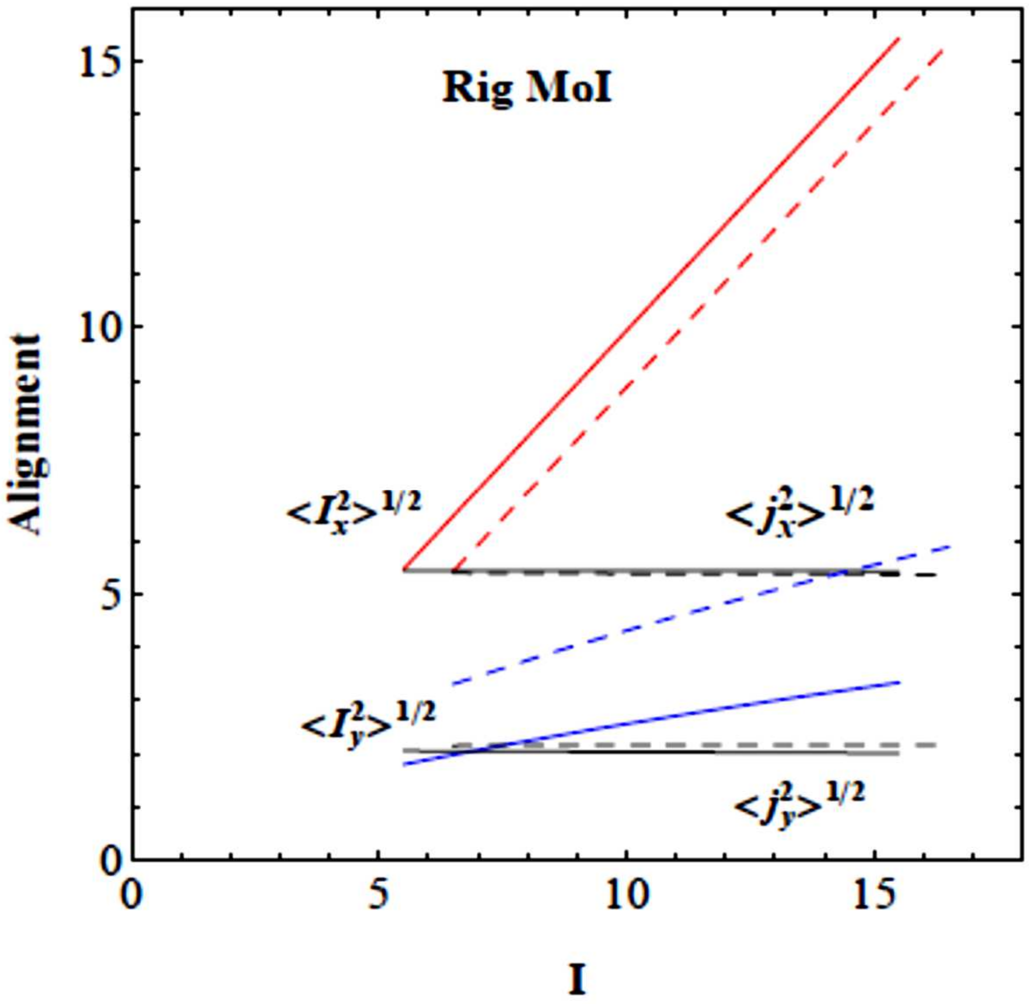
$$\mathcal{J}_y^{\text{hyd}} > \mathcal{J}_x^{\text{hyd}} \geq \mathcal{J}_z^{\text{hyd}} \text{ in } 5^\circ \leq \gamma \leq 30^\circ$$

$$\mathcal{J}_0 = 25\text{MeV}^{-1}$$



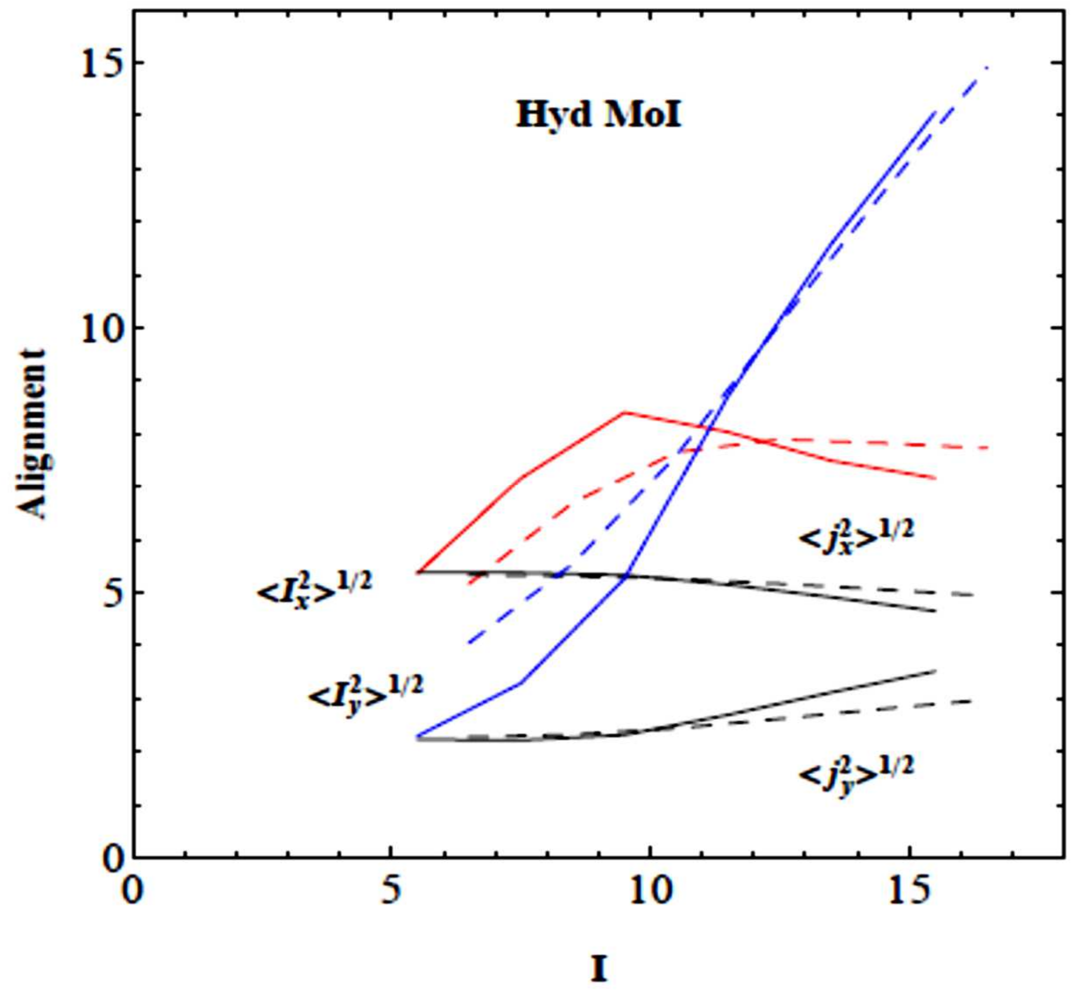
(3) Alignment calculated by exact diagonalization
with rig. MoI:

Clear evidence of
the alignment of I
along x-axis is seen.



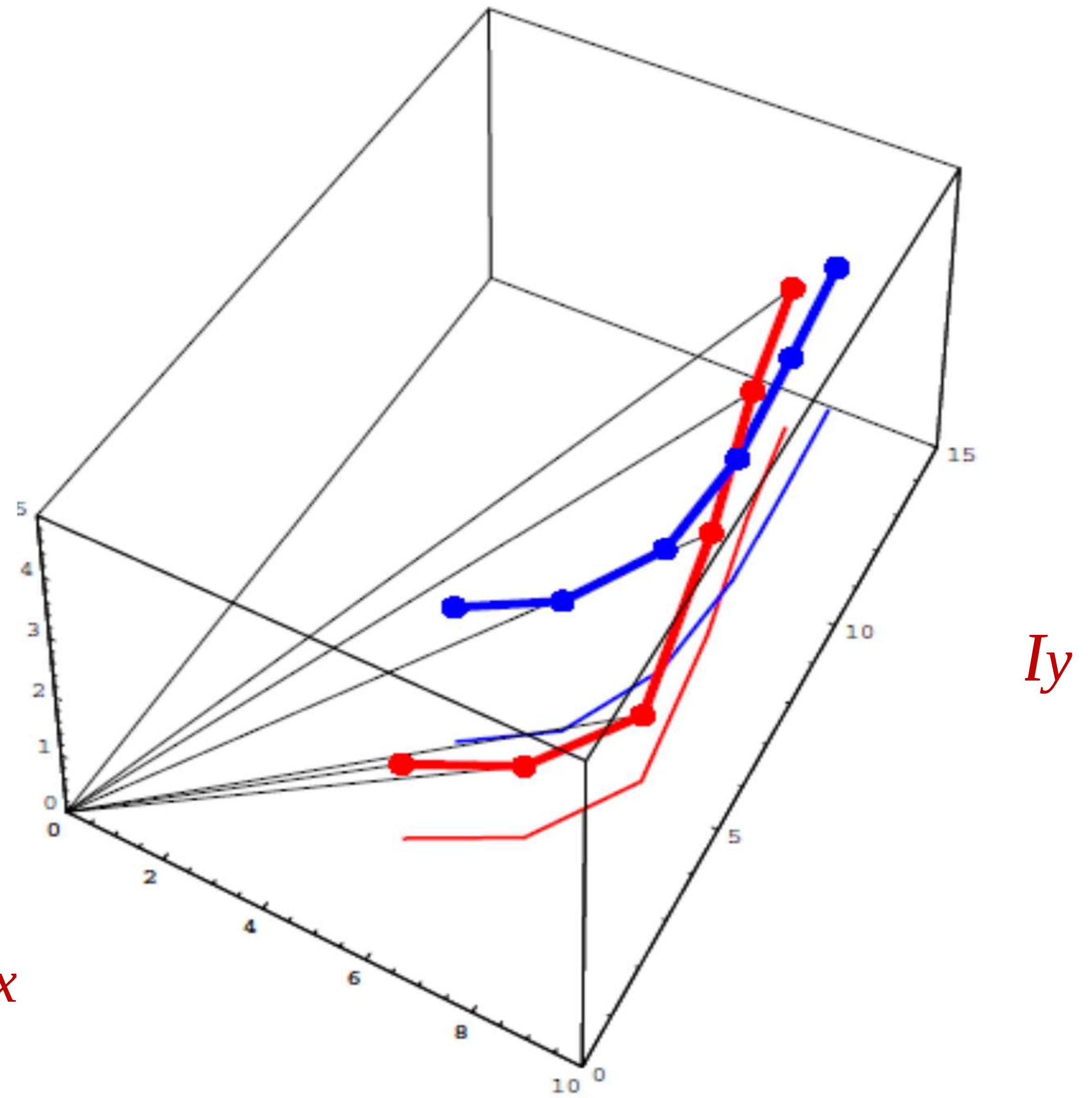
(4) Alignment calculated by exact diagonalization with hyd MoI:

Any clear evidence of alignment of I and j is not found.



Average direction of vector I :

Red curve corresponds to levels with I-j=even
blue curve to levels with I-j=odd.
(Thin lines on the I_x-I_y plane are the projections of the vector head trajectories.)



§5. Diagonal boson representations for I_y and j_x

HP Boson representation:

$$
\begin{aligned}
I_+ &= I_-^\dagger = I_z + \mathrm{i}I_x = -\hat{a}'^\dagger\sqrt{2I - \hat{n}_{a'}},\\
I_y &= I - \hat{n}_{a'} \qquad \text{with} \quad \hat{n}_{a'} = \hat{a}'^\dagger \hat{a}';\\
j_+ &= j_-^\dagger = j_y + \mathrm{i}j_z = \sqrt{2j - \hat{n}_{b'}}\,\hat{b}',\\
j_x &= j - \hat{n}_{b'} \qquad \text{with} \quad \hat{n}_{b'} = \hat{b}'^\dagger \hat{b}'.
\end{aligned}
$$

Unitary transformation of boson operators to eliminate liner terms in terms in *H*:

$$\hat{a}' = \hat{a} + p, \quad \hat{b}' = \hat{b} + q.$$

Then, p becomes purely imaginary, so we put $p =$ir and obtain

$$r = \sqrt{\frac{I}{2}}\frac{j\,A_x^{\mathrm{hyd}}}{A-B}, \qquad q = \sqrt{\frac{j}{2}}\frac{I\,A_y^{\mathrm{hyd}}}{C+D},$$

Expressions in the next-to-leading order approximation:

$$H_0 = A_x^{\rm hyd} j(j+1) + A_y^{\rm hyd} I(I+1) - 2V\cos\left(\gamma - \frac{\pi}{3}\right) - \frac{I(jA_z^{\rm hyd})^2}{A-B} - \frac{j(IA_y^{\rm hyd})^2}{C+D} - \frac{1}{4}(A_x^{\rm hyd} + A_y^{\rm hyd} - 2A_z^{\rm hyd})$$

$$+ \frac{3V}{2j(j+1)}\cos\left(\gamma - \frac{\pi}{3}\right).$$

$$H_2 = \begin{pmatrix} \hat{a} & \hat{b} & \hat{a}^\dagger & \hat{b}^\dagger \end{pmatrix} \begin{pmatrix} A & \mathrm{i}F & B & -\mathrm{i}F \\ -\mathrm{i}F & C & -\mathrm{i}F & D \\ B & \mathrm{i}F & A & -\mathrm{i}F \\ \mathrm{i}F & D & \mathrm{i}F & C \end{pmatrix} \begin{pmatrix} \hat{a}^\dagger \\ \hat{b}^\dagger \\ \hat{a} \\ \hat{b} \end{pmatrix},$$

$$A = \frac{I}{2}(A_z^{\rm hyd} + A_x^{\rm hyd} - 2A_y^{\rm hyd})\left(1 - \frac{1}{2I}\right),$$

$$B = \frac{I}{2}(A_z^{\rm hyd} - A_x^{\rm hyd})\left(1 - \frac{1}{4I}\right),$$

$$C = \frac{j}{2}\left[A_y^{\rm hyd} + A_z^{\rm hyd} - 2A_x^{\rm hyd} + \frac{6V}{j(j+1)}\cos\left(\gamma - \frac{\pi}{3}\right)\right] \times \left(1 - \frac{1}{2j}\right),$$

$$D = \frac{j}{2}\left[A_y^{\rm hyd} - A_z^{\rm hyd} + \frac{2\sqrt{3}V}{j(j+1)}\sin\left(\gamma - \frac{\pi}{3}\right)\right]\left(1 - \frac{1}{4j}\right),$$

$$F = \frac{\sqrt{Ij}}{2}A_z^{\rm hyd}.$$

(H_4 is not shown here.)

Eigenvalue equation :

$$\begin{pmatrix} A & iF & B & -iF \\ -iF & C & -iF & D \\ B & iF & A & -iF \\ iF & D & iF & C \end{pmatrix}\begin{pmatrix} u_+ & w_+ & u_-^* & w_-^* \\ v_+ & t_+ & v_-^* & t_-^* \\ u_- & w_- & u_+^* & w_+^* \\ v_- & t_- & v_+^* & t_+^* \end{pmatrix} = \begin{pmatrix} u_+ & w_+ & u_-^* & w_-^* \\ v_+ & t_+ & v_-^* & t_-^* \\ u_- & w_- & u_+^* & w_+^* \\ v_- & t_- & v_+^* & t_+^* \end{pmatrix}\begin{pmatrix} \omega_\alpha & 0 & 0 & 0 \\ 0 & \omega_\beta & 0 & 0 \\ 0 & 0 & -\omega_\alpha & 0 \\ 0 & 0 & 0 & -\omega_\beta \end{pmatrix}$$

$$\begin{aligned} u_\pm &= (A - B \pm \omega_{(+)})\left[(C+D)(2F^2 - BC + BD)\right. \\ &\quad \left. + B\omega_{(+)}^2\right] N_+, \\ v_\pm &= \pm iF(C + D \pm \omega_{(+)})\left[(A-B)^2 - \omega_{(+)}^2\right] N_+, \\ w_\pm &= (A - B \pm \omega_{(-)})\left[(C+D)(2F^2 - BC + BD)\right. \\ &\quad \left. + B\omega_{(-)}^2\right] N_-, \\ t_\pm &= \pm iF(C + D \pm \omega_{(-)})\left[(A-B)^2 - \omega_{(-)}^2\right] N_-, \end{aligned}$$

$$N_\pm^{-2} = 4\omega_\pm \Big[(A-B)\left[(C+D)(2F^2 - BC + BD)\right. \\ \left. + B\omega_\pm^2\right]^2 + F^2(C+D)\left((A-B)^2 - \omega_\pm^2\right)^2 \Big].$$

Eigenvalue equation is solved to determine two positive eigenvalues :

$$2\omega^2_{(\pm)} = b \pm \sqrt{b^2 - 4c}$$

$$b \equiv A^2 - B^2 + C^2 - D^2,$$

$$c \equiv (A^2 - B^2)(C^2 - D^2) - 4F^2(A - B)(C + D).$$

Within the leading-order-approximation:

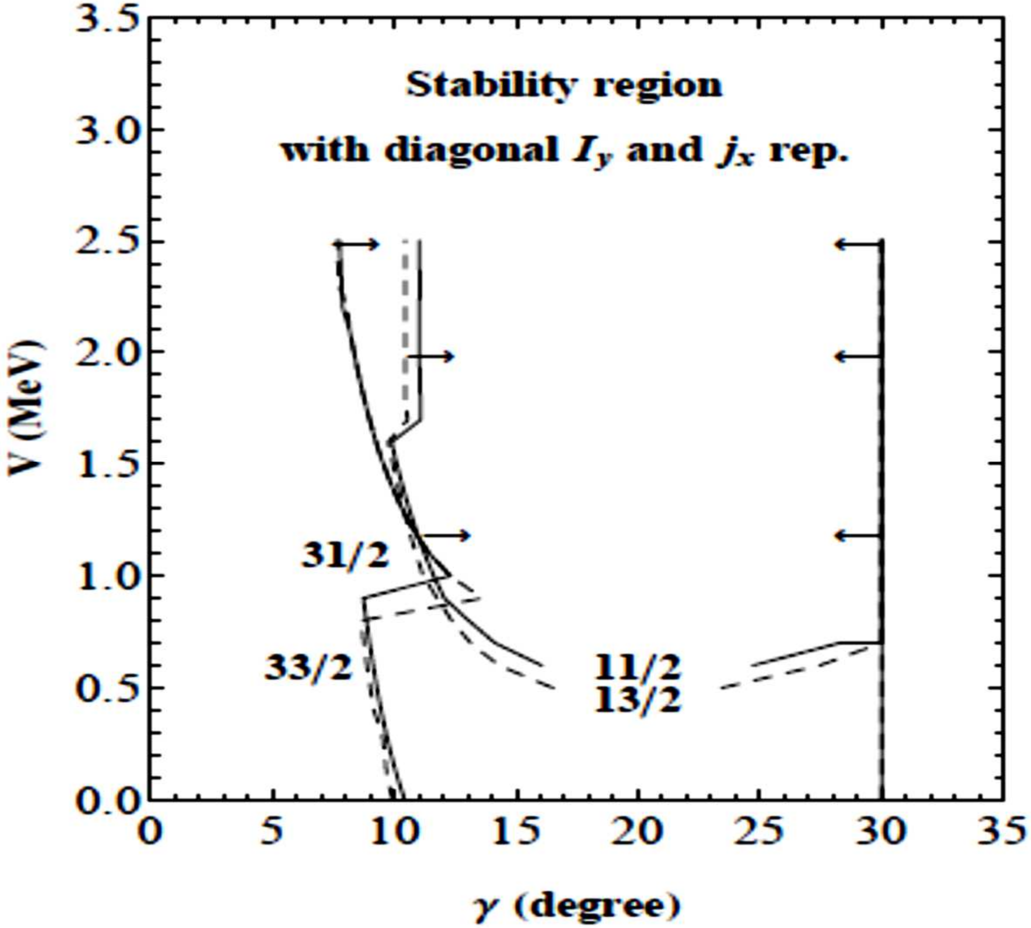
$$V_1 = \frac{2\sqrt{3}V \sin\gamma}{j(j+1)(A_x^{\rm hyd} - A_y^{\rm hyd})} - 1,$$

$$V_2 = \frac{2\sqrt{3}V \sin(\gamma + \frac{\pi}{3})}{j(j+1)(A_z^{\rm hyd} - A_y^{\rm hyd})} + \frac{A_z^{\rm hyd} - A_x^{\rm hyd}}{A_z^{\rm hyd} - A_y^{\rm hyd}}.$$

$$b = I^2(A_x^{\rm hyd} - A_y^{\rm hyd})(A_z^{\rm hyd} - A_y^{\rm hyd})\left[1 + V_1 V_2 (\frac{j}{I})^2\right],$$

$$c = [Ij(A_x^{\rm hyd} - A_y^{\rm hyd})(A_z^{\rm hyd} - A_y^{\rm hyd})]^2\, V_1 \left[V_2 - (\frac{A_z^{\rm hyd}}{A_z^{\rm hyd} - A_y^{\rm hyd}})^2\right].$$

Stability region in the $\gamma - V$ plane for hyd MoI and I_y and j_x Represented in diagonal forms



In the asymptotic region of large ang. mom. $(I \geq 2j)$

$$\omega_{(-)}^2 \simeq j^2(A_x^{\rm hyd} - A_y^{\rm hyd})(A_z^{\rm hyd} - A_y^{\rm hyd})\, V_1 \left[V_2 - \left(\frac{A_z^{\rm hyd}}{A_z^{\rm hyd} - A_y^{\rm hyd}}\right)^2 \right]$$

This expression suggest that corresponding mode is the precession of j , because it vanishes when j=0.

On the other hand,

$$\omega_{(+)}^2 \simeq I^2(A_x^{\rm hyd} - A_y^{\rm hyd})(A_z^{\rm hyd} - A_y^{\rm hyd})$$

which gives Bohr-Mottelson's wobbling formula.

To discuss low ang. mom. region, we need further details.

“Behavior of eigen-amplitudes”

Unitary transformation of shifts:

$$\begin{aligned} \hat{a} &= \exp[ir(\hat{a}'^{\dagger} + \hat{a}')]\,\hat{a}'\,\exp[-ir(\hat{a}'^{\dagger} + \hat{a}')] \\ &= \hat{a}' - ir \\ \hat{b} &= \exp[q(\hat{b}'^{\dagger} - \hat{b}')]\,\hat{b}'\,\exp[-(\hat{b}'^{\dagger} - \hat{b}')] \\ &= \hat{b}' - q \end{aligned}$$

Unitary transformation to quasiparticle operators.

$$\begin{pmatrix} \hat{a} \\ \hat{b} \\ \hat{a}^{\dagger} \\ \hat{b}^{\dagger} \end{pmatrix} = \begin{pmatrix} u_{+} & w_{+} & u_{-}^{*} & w_{-}^{*} \\ v_{+} & t_{+} & v_{-}^{*} & t_{-}^{*} \\ u_{-} & w_{-} & u_{+}^{*} & w_{+}^{*} \\ v_{-} & t_{-} & v_{+}^{*} & t_{+}^{*} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \alpha^{\dagger} \\ \beta^{\dagger} \end{pmatrix}$$

Eigenvalue equation :

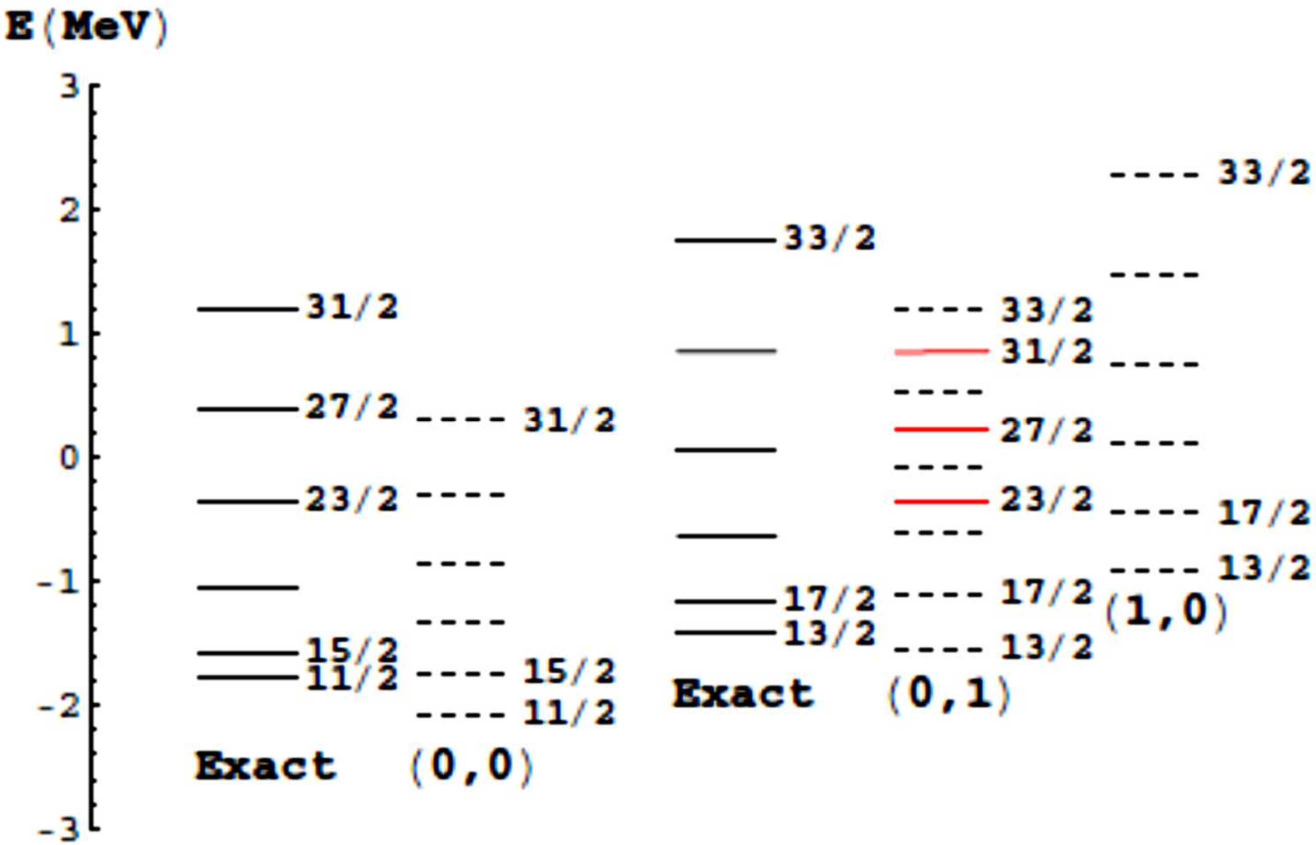
$$\begin{pmatrix} A & iF & B & -iF \\ -iF & C & -iF & D \\ B & iF & A & -iF \\ iF & D & iF & C \end{pmatrix} \begin{pmatrix} u_+ & w_+ & u_-^* & w_-^* \\ v_+ & t_+ & v_-^* & t_-^* \\ u_- & w_- & u_+^* & w_+^* \\ v_- & t_- & v_+^* & t_+^* \end{pmatrix} = \begin{pmatrix} u_+ & w_+ & u_-^* & w_-^* \\ v_+ & t_+ & v_-^* & t_-^* \\ u_- & w_- & u_+^* & w_+^* \\ v_- & t_- & v_+^* & t_+^* \end{pmatrix} \begin{pmatrix} \omega_\alpha & 0 & 0 & 0 \\ 0 & \omega_\beta & 0 & 0 \\ 0 & 0 & -\omega_\alpha & 0 \\ 0 & 0 & 0 & -\omega_\beta \end{pmatrix}$$

$$\begin{aligned} u_\pm &= (A - B \pm \omega_{(+)})\left[(C+D)(2F^2 - BC + BD)\right. \\ &\quad \left. + B\omega_{(+)}^2\right] N_+, \\ v_\pm &= \pm iF(C + D \pm \omega_{(+)})\left[(A-B)^2 - \omega_{(+)}^2\right] N_+, \\ w_\pm &= (A - B \pm \omega_{(-)})\left[(C+D)(2F^2 - BC + BD)\right. \\ &\quad \left. + B\omega_{(-)}^2\right] N_-, \\ t_\pm &= \pm iF(C + D \pm \omega_{(-)})\left[(A-B)^2 - \omega_{(-)}^2\right] N_-, \end{aligned}$$

$$N_\pm^{-2} = 4\omega_\pm \Big[(A-B)\left[(C+D)(2F^2 - BC + BD)\right. \\ \left. + B\omega_\pm^2\right]^2 + F^2(C+D)\left((A-B)^2 - \omega_\pm^2\right)^2 \Big].$$

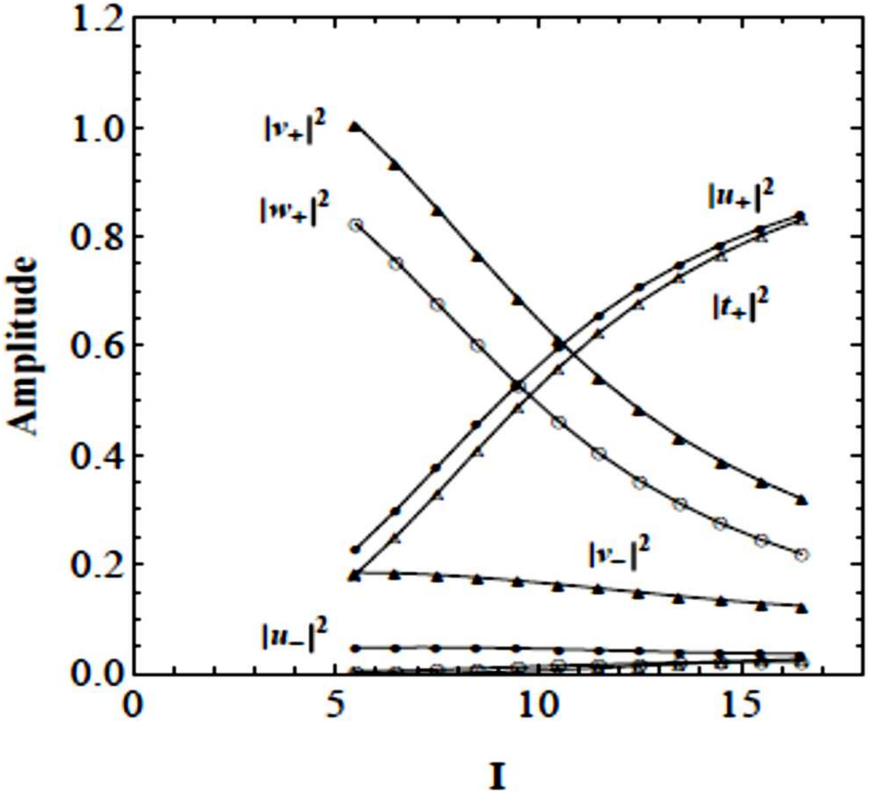
Behavior of calculated energy levels and eigenamplitudes

Approximate energy levels compared with exact ones

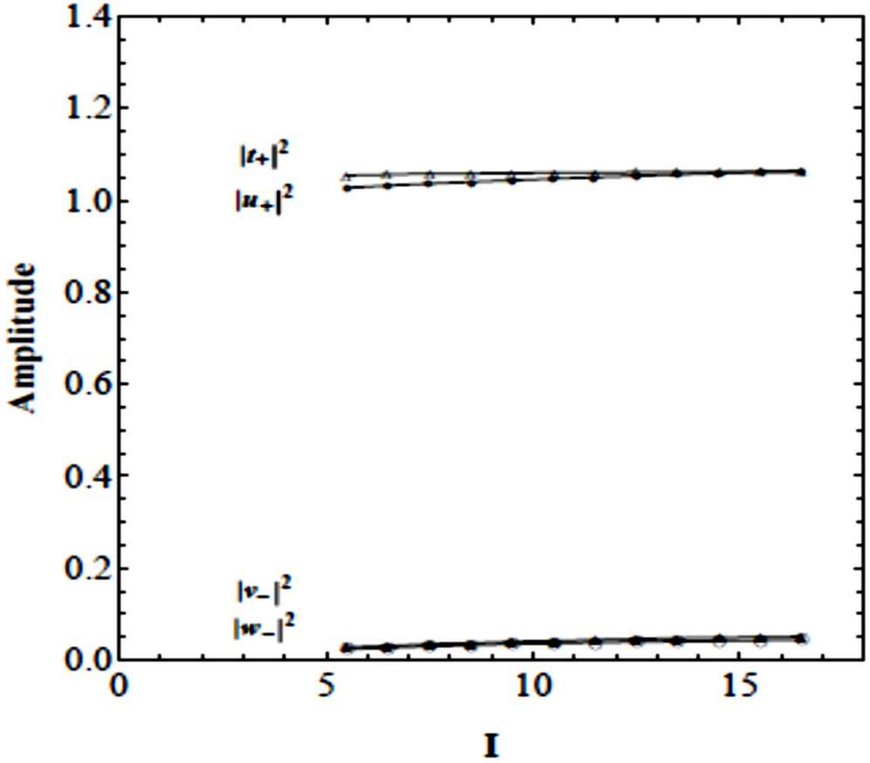


Behavior of eigen-amplitudes:

Hyd. MoI



Rig. MoI



Conclusion based on the behavior of eigen-amplitudes:

Inverse transformation:

$$\begin{pmatrix} \alpha \\ \beta \\ \alpha^\dagger \\ \beta^\dagger \end{pmatrix} = \begin{pmatrix} u_+^* & v_+^* & -u_-^* & -v_-^* \\ w_+^* & t_+^* & -w_-^* & -t_-^* \\ -u_- & -v_- & u_+ & v_+ \\ -w_- & -t_- & w_+ & t_+ \end{pmatrix} \begin{pmatrix} \hat{a}' - ir \\ \hat{b}' - q \\ \hat{a}'^\dagger + ir \\ \hat{b}'^\dagger - q \end{pmatrix}.$$

Inverse transformation:

$$\begin{pmatrix} \alpha \\ \beta \\ \alpha^\dagger \\ \beta^\dagger \end{pmatrix} = \begin{pmatrix} u_+^* & v_+^* & -u_-^* & -v_-^* \\ w_+^* & t_+^* & -w_-^* & -t_-^* \\ -u_- & -v_- & u_+ & v_+ \\ -w_- & -t_- & w_+ & t_+ \end{pmatrix} \begin{pmatrix} \hat{a}' - ir \\ \hat{b}' - q \\ \hat{a}'^\dagger + ir \\ \hat{b}'^\dagger - q \end{pmatrix}$$

$$I_y = I - \hat{a}'^\dagger \hat{a}'$$
$$j_x = j - \hat{b}'^\dagger \hat{b}'$$

In low spin region ($I \ll 2j$), amplitudes v_+ and w_+ dominate:

For ω_-, $\beta \cong w_+^*(\hat{a}' - ir)$,
indicating wobbling about y-axis

For ω_+, $\alpha \cong v_+^*(\hat{b}' - q)$,
indicating precession of $\vec{j}$ about x-axis.

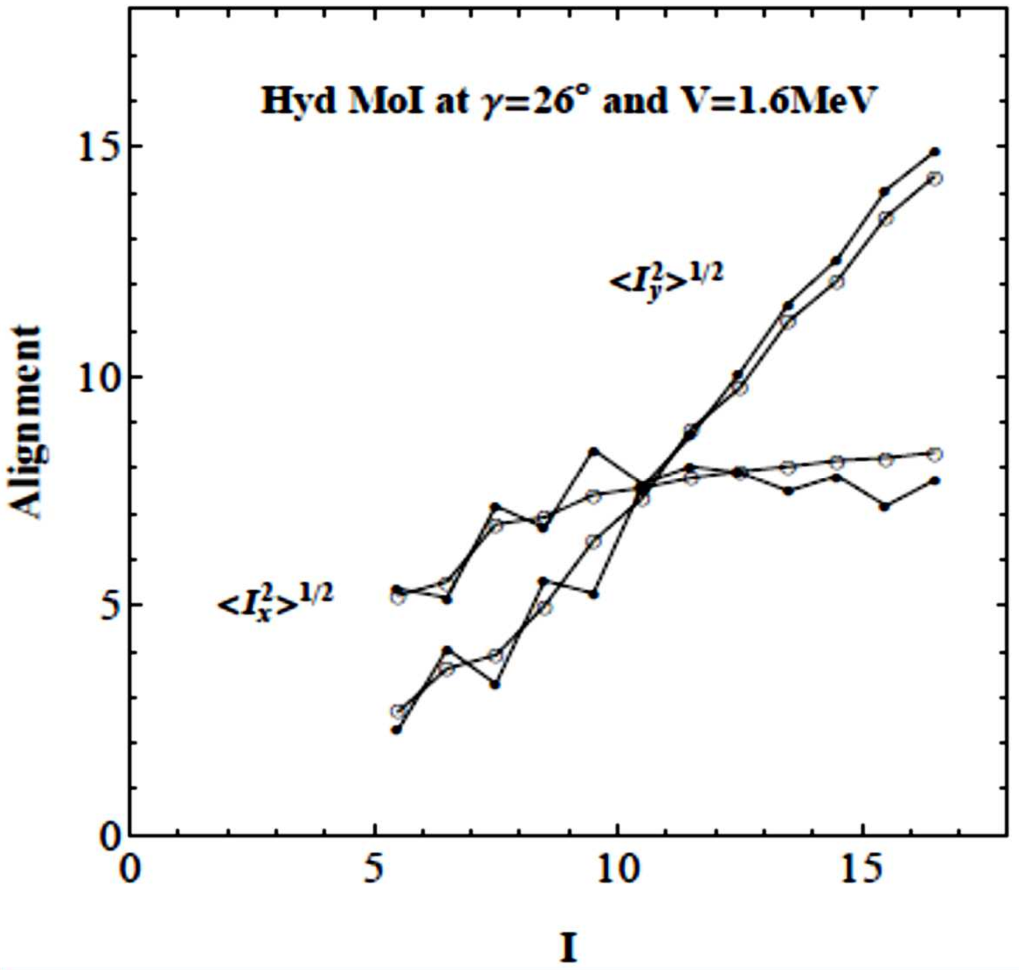
In high spin region ($I \gg 2j$), amplitudes u_+ and t_+ dominate:

For ω_-, $\beta \cong t_+^*(\hat{b}' - q)$,
indicating precession of $\vec{j}$ about x-axis

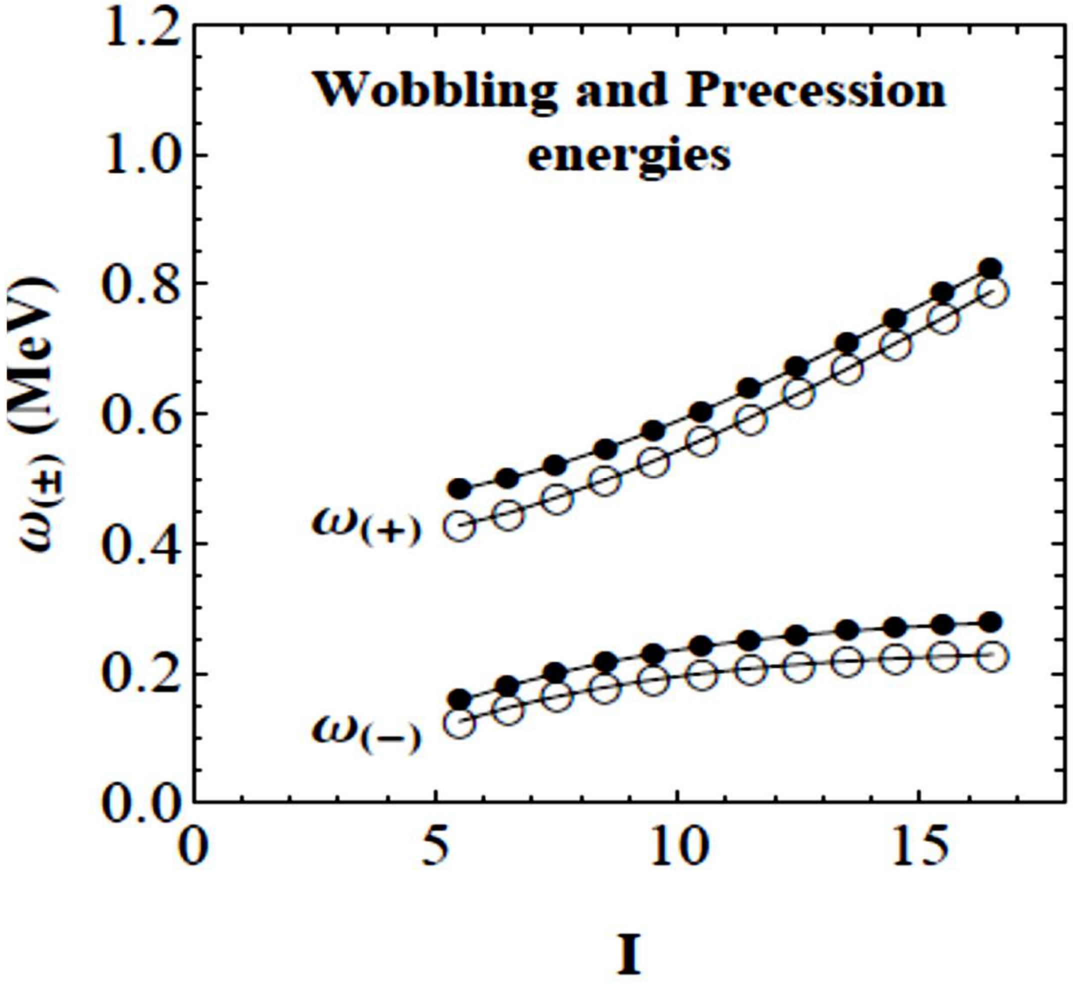
For ω_+, $\alpha \cong u_+^*(\hat{a}' - ir)$,
indicating wobbling about y-axis.

Comparison of alignment in leading order approximation with exact result

(solid circles)



Excitation energies of harmonic excitation as functions of I



§6. Taking account of the CAP effect in the moment of inertia

We have revisited Constrained Hartree-Fock-Bogoliubov (CHFB) theory to investigate change of moment of inertia with increasing angular momentum I due to Coriolis anti-pairing (CAP) effect.

Regarding the cranking term as perturbation, we start from BCS basis and take into account the CAP effect in the second order perturbation.

Due to finiteness of nuclear system, super-normal phase transition takes place gradually. In our final result, we derived analytic relations among angular momentum *I*, gap parameter and moment-of-inertia.

For details, K.T. and K. Sugawara-Tanabe, PRC, 91, 034328(2015).

Conclusion

1. The transverse wobbling is not described within a framework of particle-rotor model with hyd MoI and reasonable potential strength.
2. The reason is as follows. The single-particle spin *j*, which is pinned along *x*-axis, is expected to pull the total ang. mom. *I* toward the same direction to decrease $(I_x - j_x)^2$. However , this expectation is not realistic, because 1/ $\mathcal{J}_y^{\rm hyd}$ is much smaller than 1/ $\mathcal{J}_x^{\rm hyd}$ so that the alignment of *I* toward *y*-axis is not prevented.

3. The hyd MoI does not harmonize with the microscopic formalism. Such a typical example is that, there is no way to relate the hyd MoI to the CAP effect. From such a view point, the rig MoI is preferable.

Reproduction of 135Pr data will be presented by Sugawara-Tanabe in Thursday morning.