Shape Transitions and Coexistence:

self-consistent mean-field and beyond

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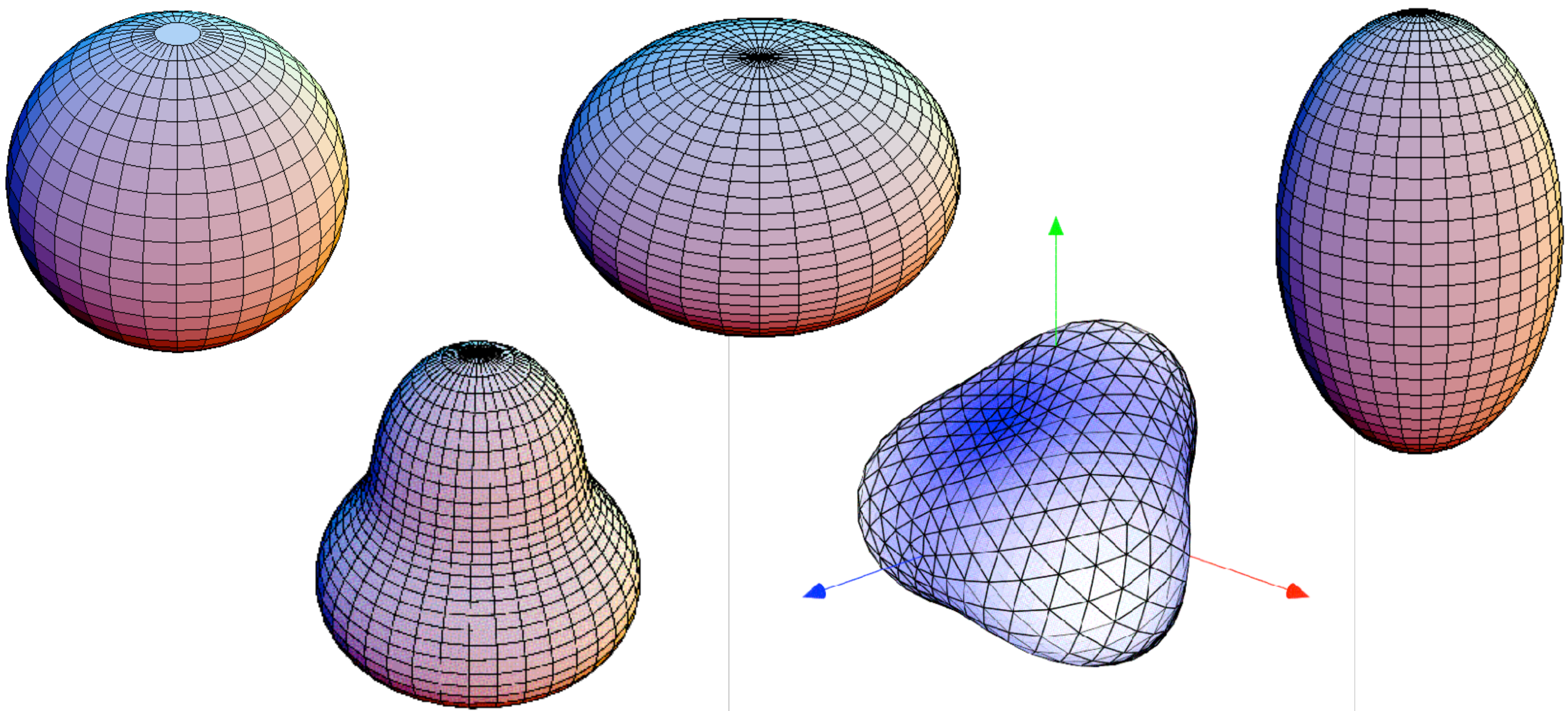
How can nuclear matter support a diversity of shapes?

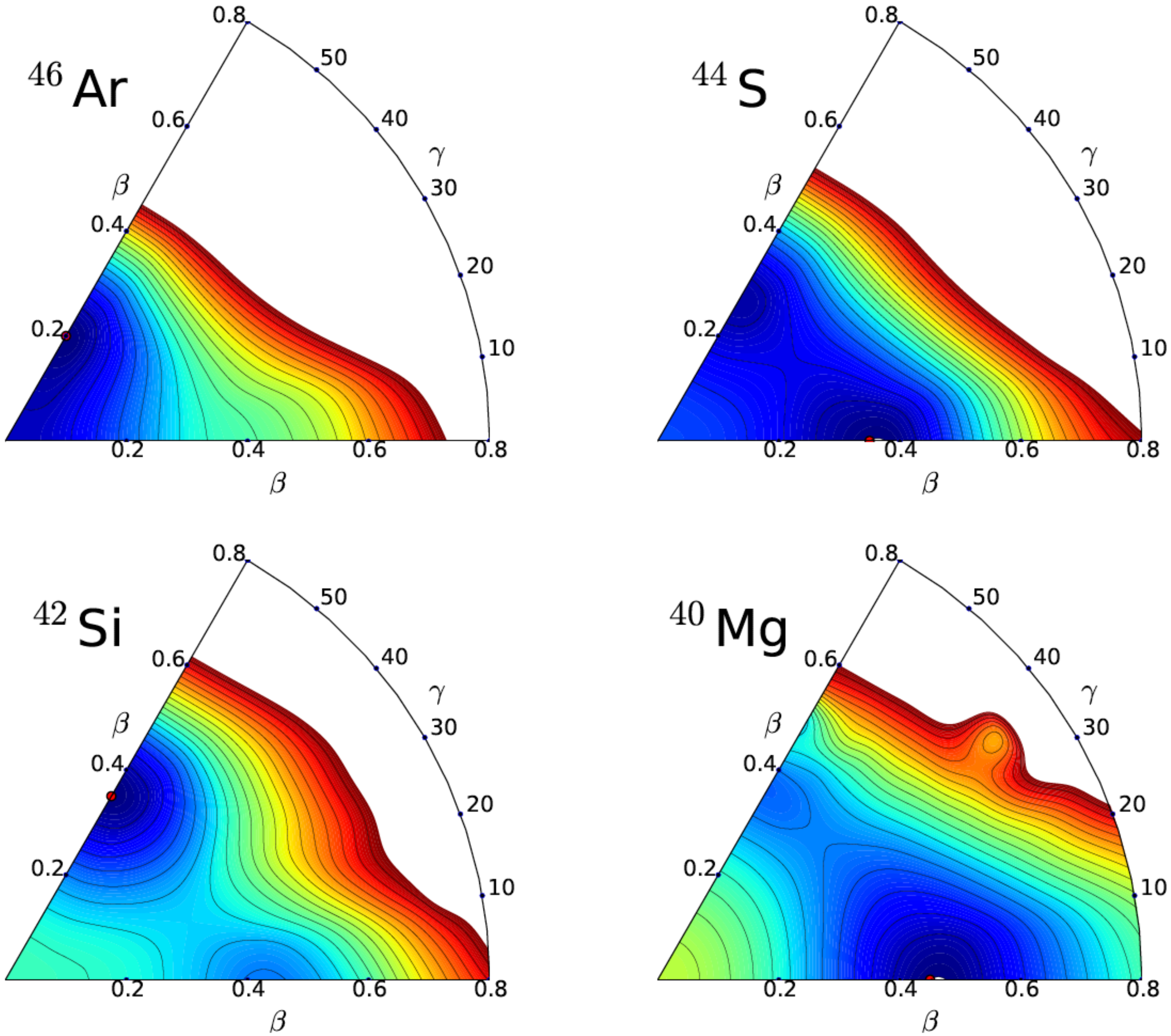


Evolution of shell structure with nucleon number and/or angular momentum.

Shape transitions within a single nucleus (shape coexistence) or as a function of nucleon number (shape evolution) → universal phenomena that occur in light, medium-heavy, heavy and superheavy nuclei.

Universal theory framework: Nuclear Energy Density Functionals

Basic implementation: the self-consistent mean field method → produces semi-classical energy surfaces as functions of intrinsic deformation parameters.

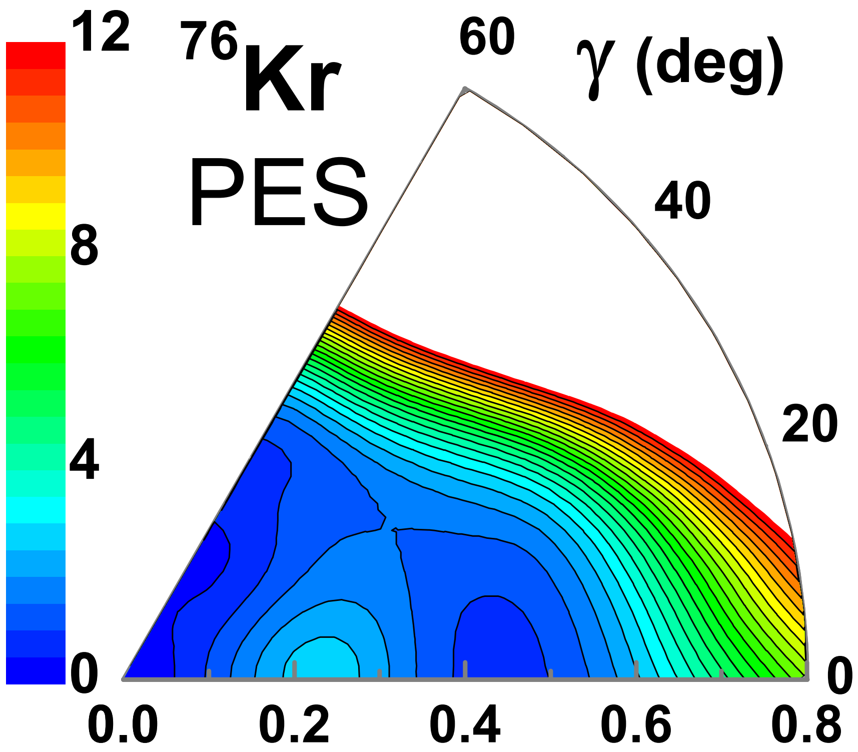


→ include static correlations: deformations & pairing

→ do not include dynamic (collective) correlations that arise from symmetry restoration and quantum fluctuations around mean-field minima

Collective Hamiltonian

Prog. Part. Nucl. Phys. **66**, 519 (2011).
Phys. Rev. C **79**, 034303 (2009).



... nuclear excitations determined by quadrupole vibrational and rotational degrees of freedom:

$$H_{\text{coll}} = \mathcal{T}_{\text{vib}}(\beta,\gamma) + \mathcal{T}_{\text{rot}}(\beta,\gamma,\Omega) + \mathcal{V}_{\text{coll}}(\beta,\gamma)$$

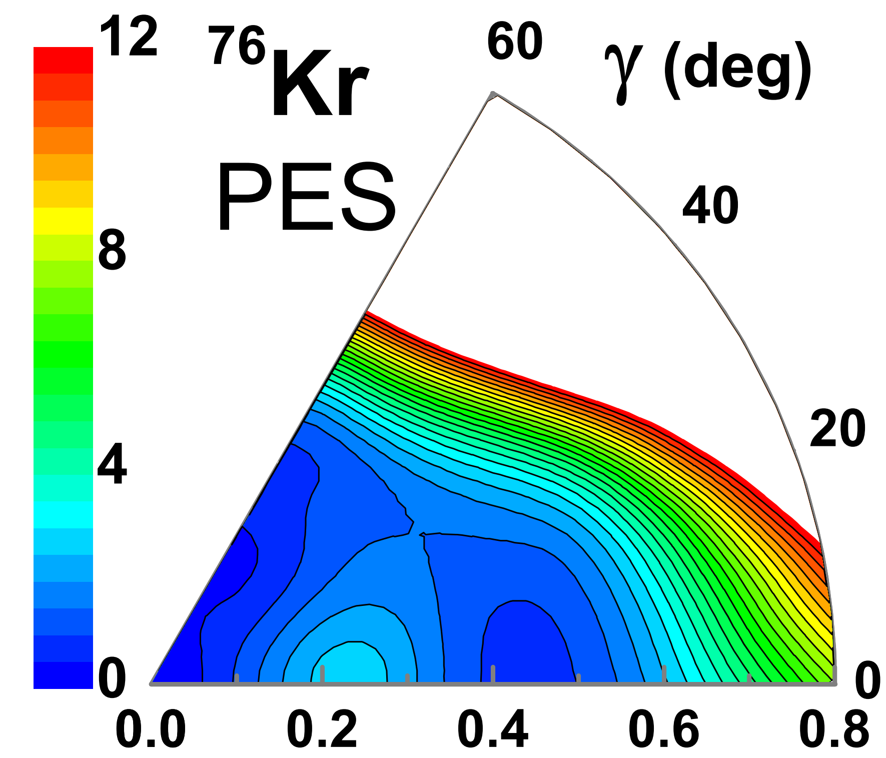
$$\mathcal{T}_{\text{vib}} = \frac{1}{2}B_{\beta\beta}\dot{\beta}^2 + \beta B_{\beta\gamma}\dot{\beta}\dot{\gamma} + \frac{1}{2}\beta^2 B_{\gamma\gamma}\dot{\gamma}^2$$

$$\mathcal{T}_{\text{rot}} = \frac{1}{2}\sum_{k=1}^{3}\mathcal{I}_k\omega_k^2$$

The dynamics of the collective Hamiltonian is determined by: the self-consistent collective potential, the three mass parameters: $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$, and the three moments of inertia I_k, functions of the intrinsic deformations β and γ.

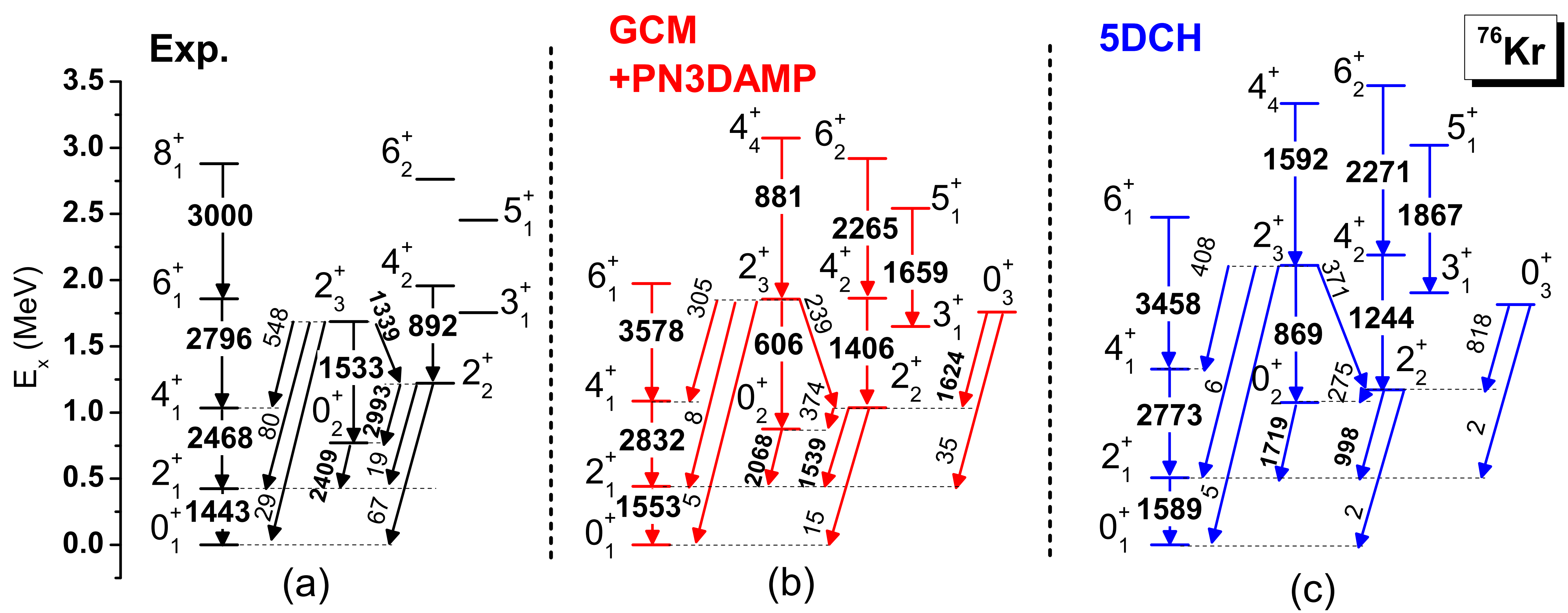
... collective eigenfunction:

$$\Psi_\alpha^{IM}(\beta,\gamma,\Omega) = \sum_{K\in\Delta I}\psi_{\alpha K}^{I}(\beta,\gamma)\Phi_{MK}^{I}(\Omega)$$



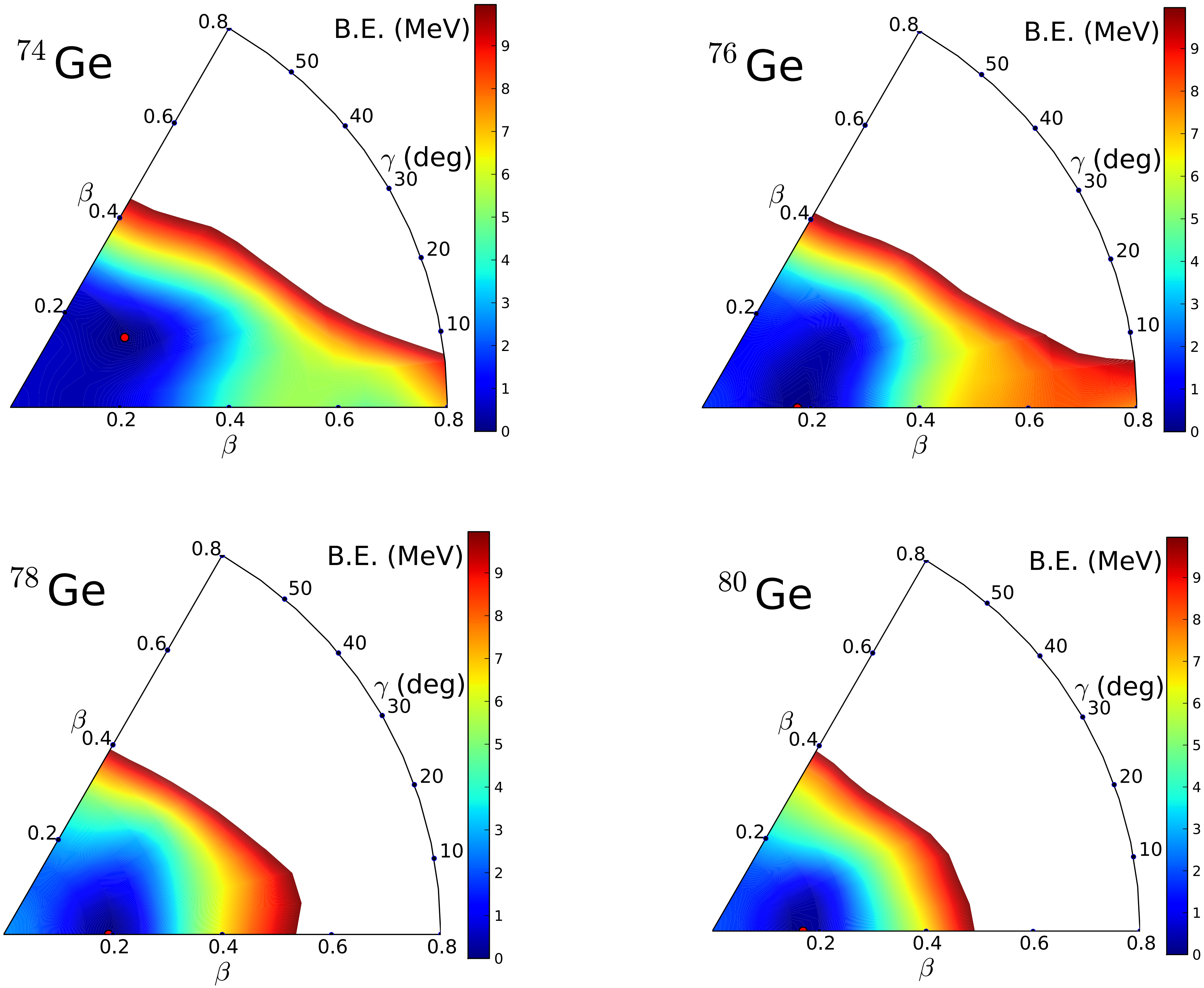
✔ an intuitive interpretation of mean-field results in terms of ***intrinsic shapes*** and ***single-particle states***

✔ the ***full model space*** of occupied states can be used; no distinction between core and valence nucleons, ***no need for effective charges!***

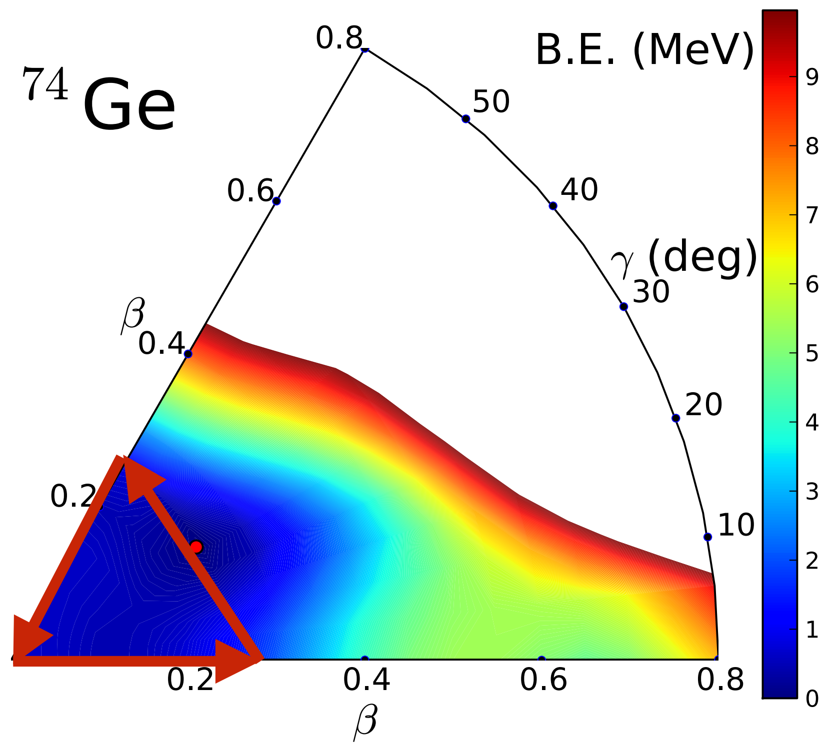


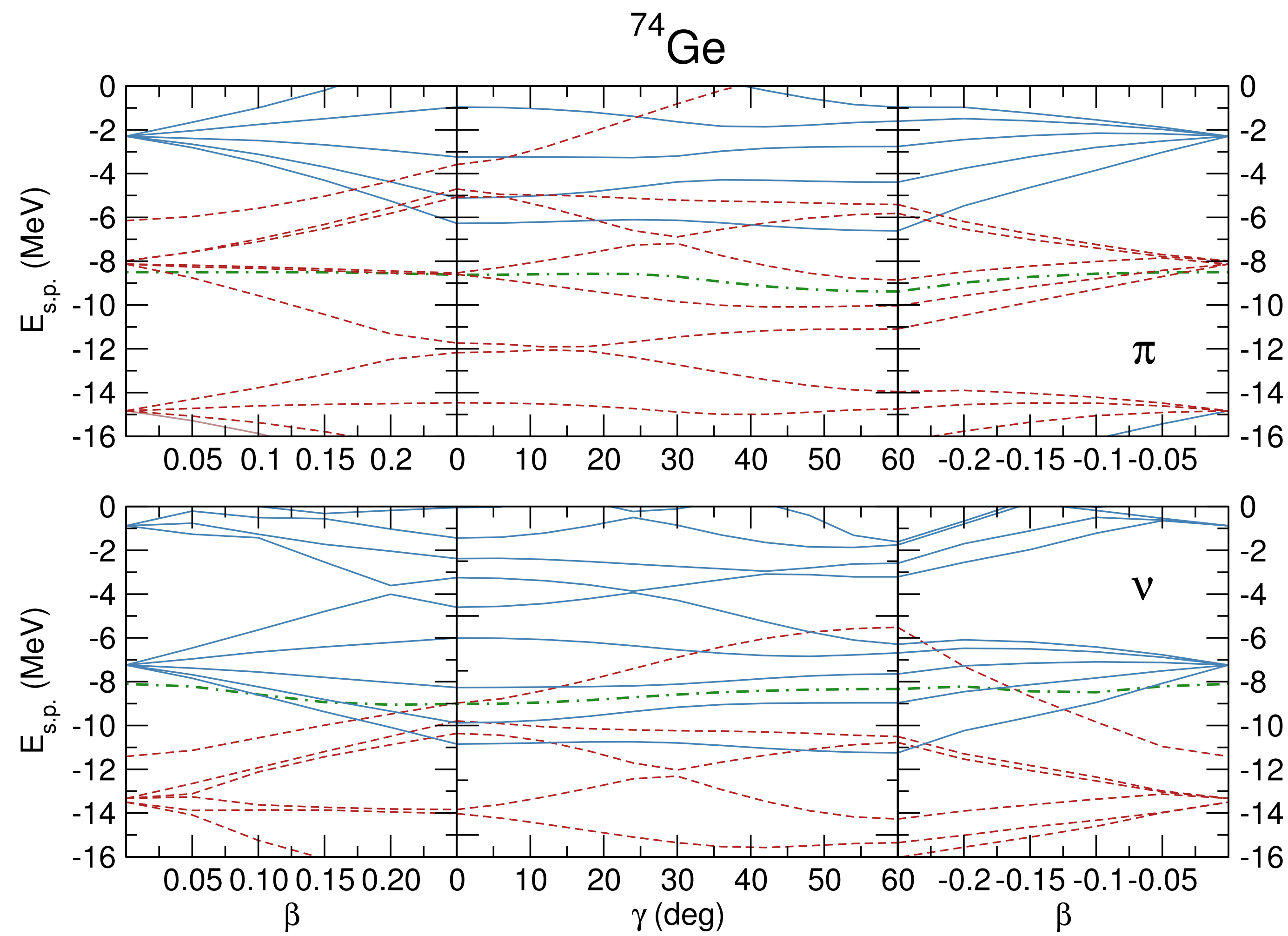
Shape evolution and triaxiality in germanium isotopes

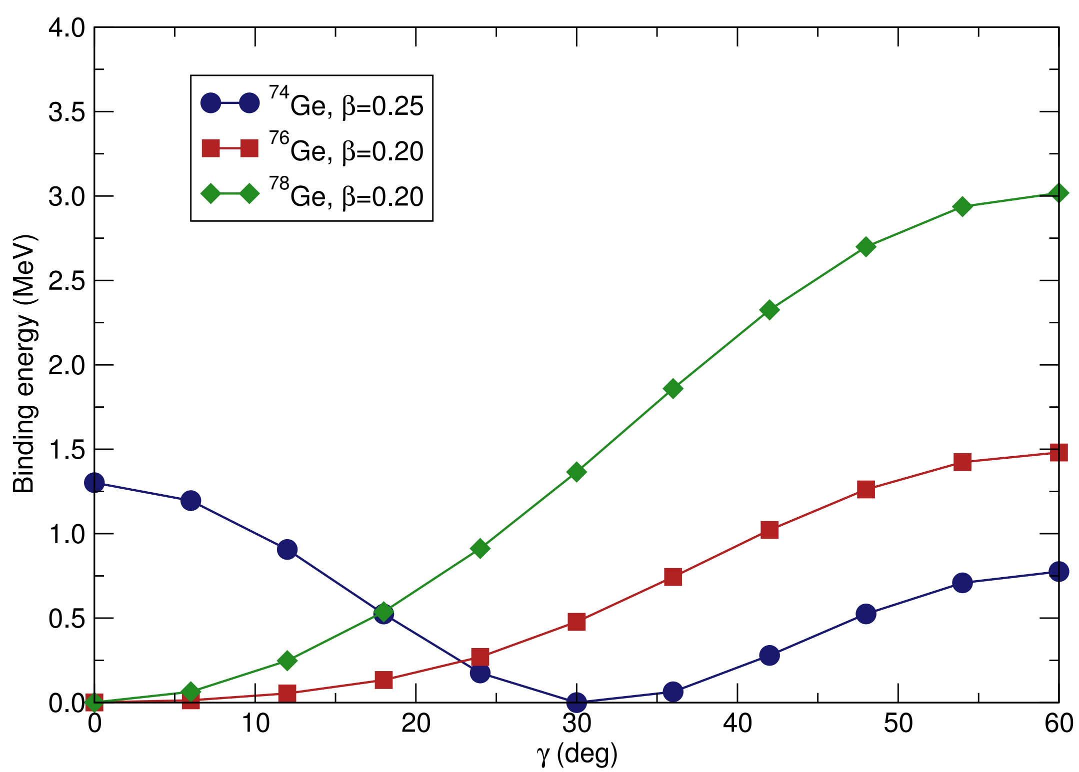
Phys. Rev. C 89, 044325 (2014).



… formation of deformed minima → regions of low single-particle level density around the Fermi surface.



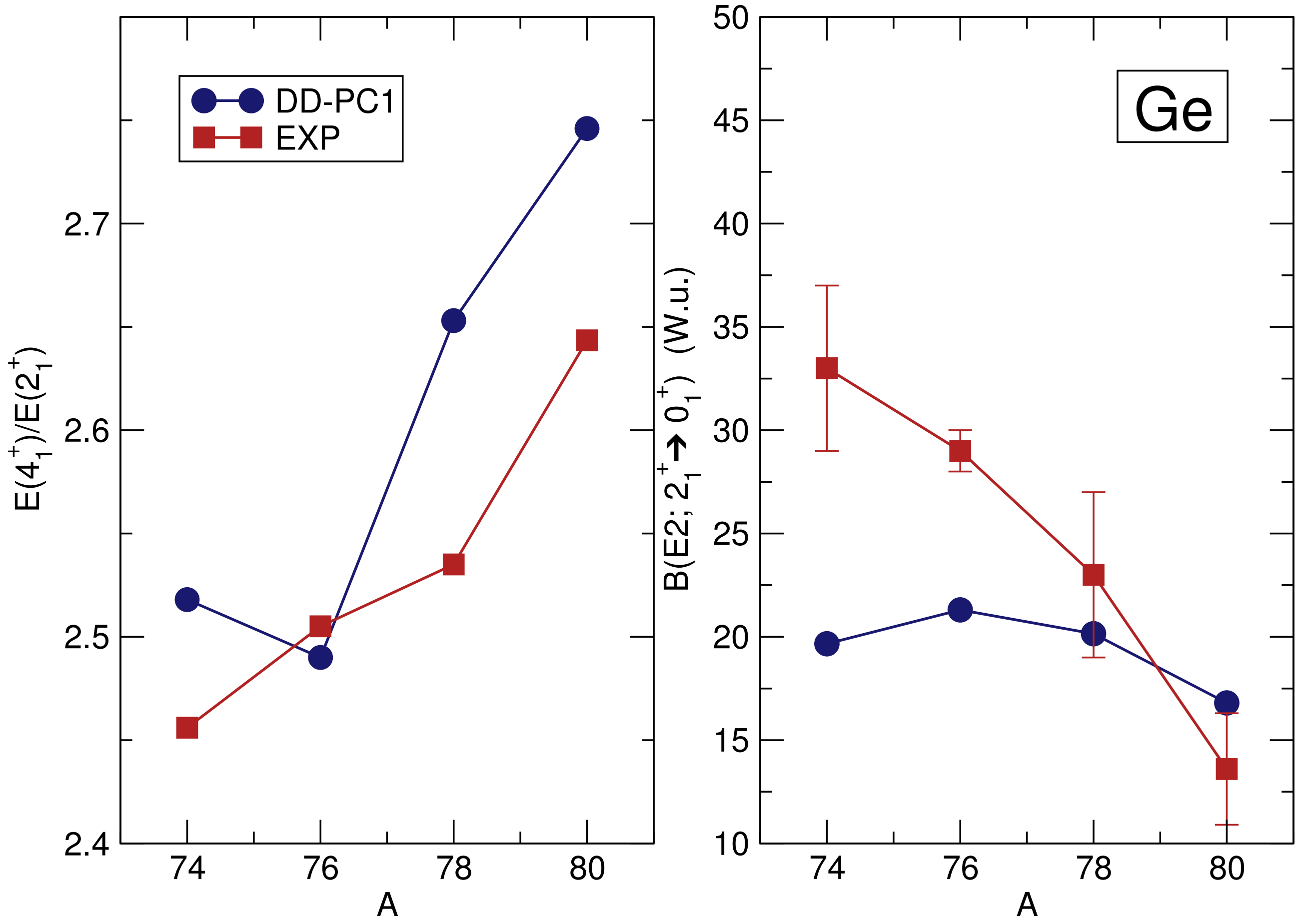


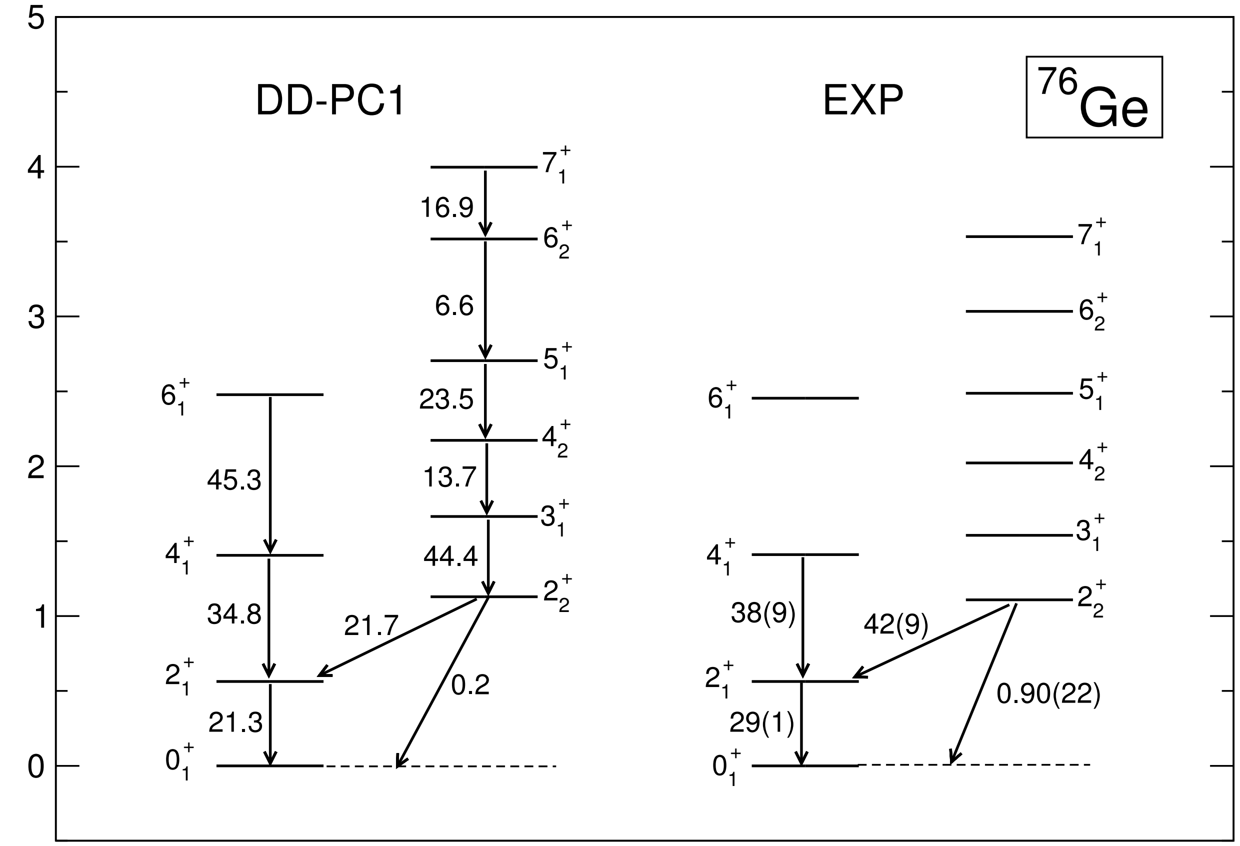


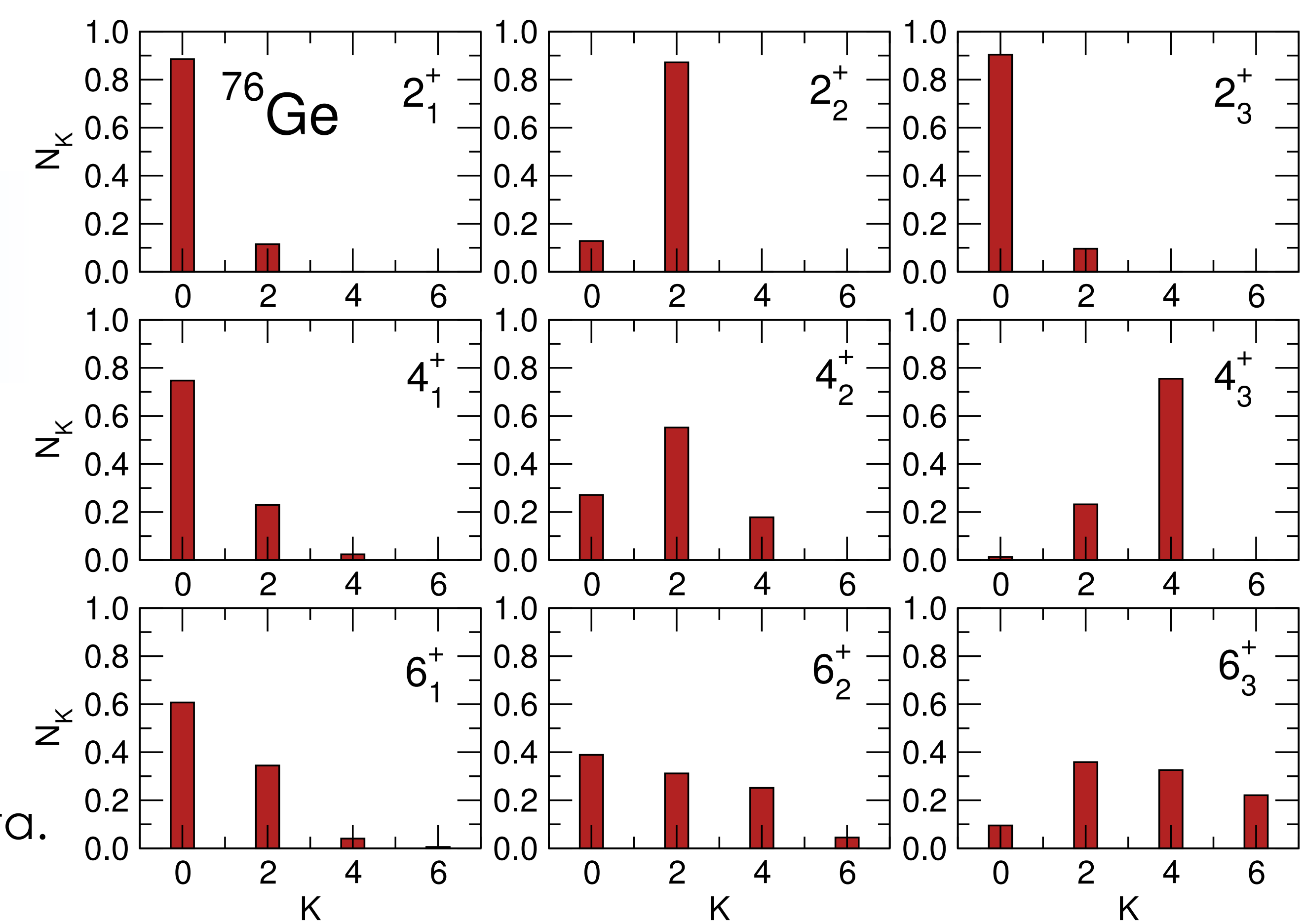
…shallow triaxial minimum at γ = 30°

→ important role of dynamical effects related to restoration of broken symmetries and fluctuations in collective coordinates.

Quadrupole collective Hamiltonian based on the functional DD-PC1







Distribution of *K* components (projection of the angular momentum on the body-fixed symmetry axis) in the collective wave functions of the nucleus ^{76}Ge.

$$N_K = 6\int_0^{\pi/3}\int_0^{\infty} \left|\psi^J_{\alpha,K}(\beta,\gamma)\right|^2 \beta^4 |\sin 3\gamma| d\beta d\gamma .$$

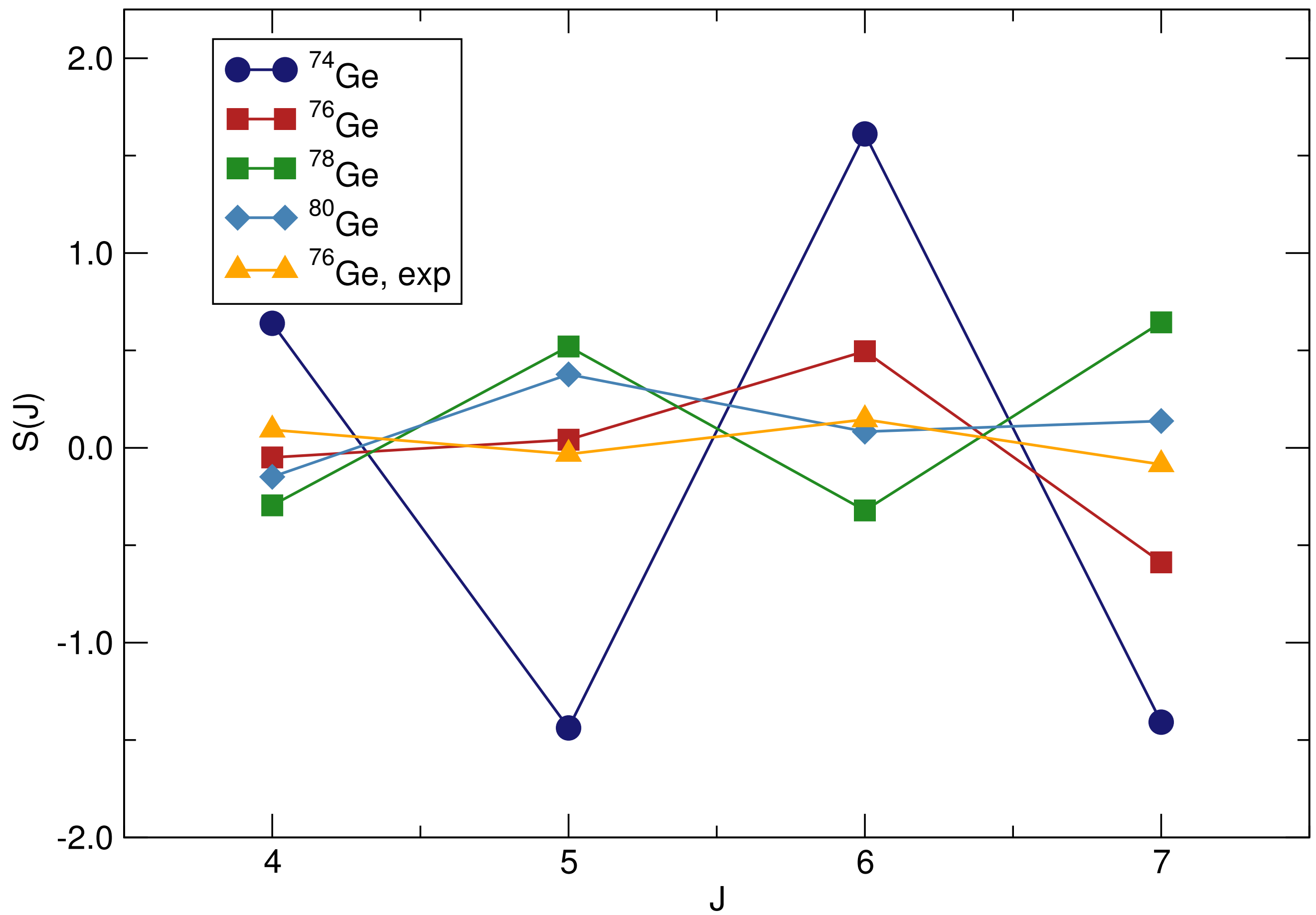
→ more K-mixing for states with higher angular momenta.

The level of K-mixing is reflected in the staggering in energy between odd- and even-spin states in the γ band:

$$S(J) = \frac{E[J_\gamma^+] - 2E[(J-1)_\gamma^+] + E[(J-2)_\gamma^+]}{E[2_1^+]}$$

Deformed γ-soft potential $\Rightarrow$ S(J) oscillates between negative values for even-spin states and positive values for odd-spin states.

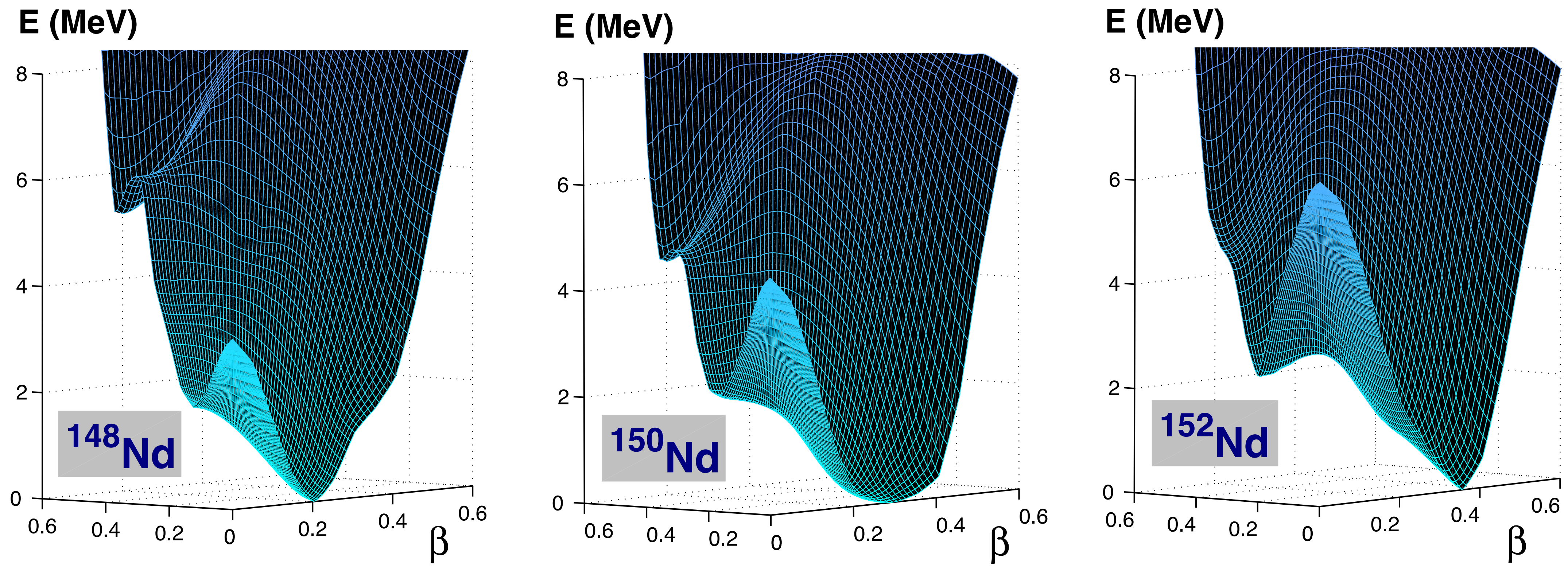
γ-rigid triaxial potential $\Rightarrow$ S(J) oscillates between positive values for even-spin states and negative values for odd-spin states.



The mean-field potential of ^{76}Ge is γ soft. The inclusion of collective correlations (symmetry restoration and quantum fluctuations) drives the nucleus toward triaxiality, but not strong enough to stabilize a $\gamma \approx 30^\circ$ shape.

Phys. Rev. C 89, 044325 (2014).

Shape phase transitions in medium-heavy and heavy nuclei

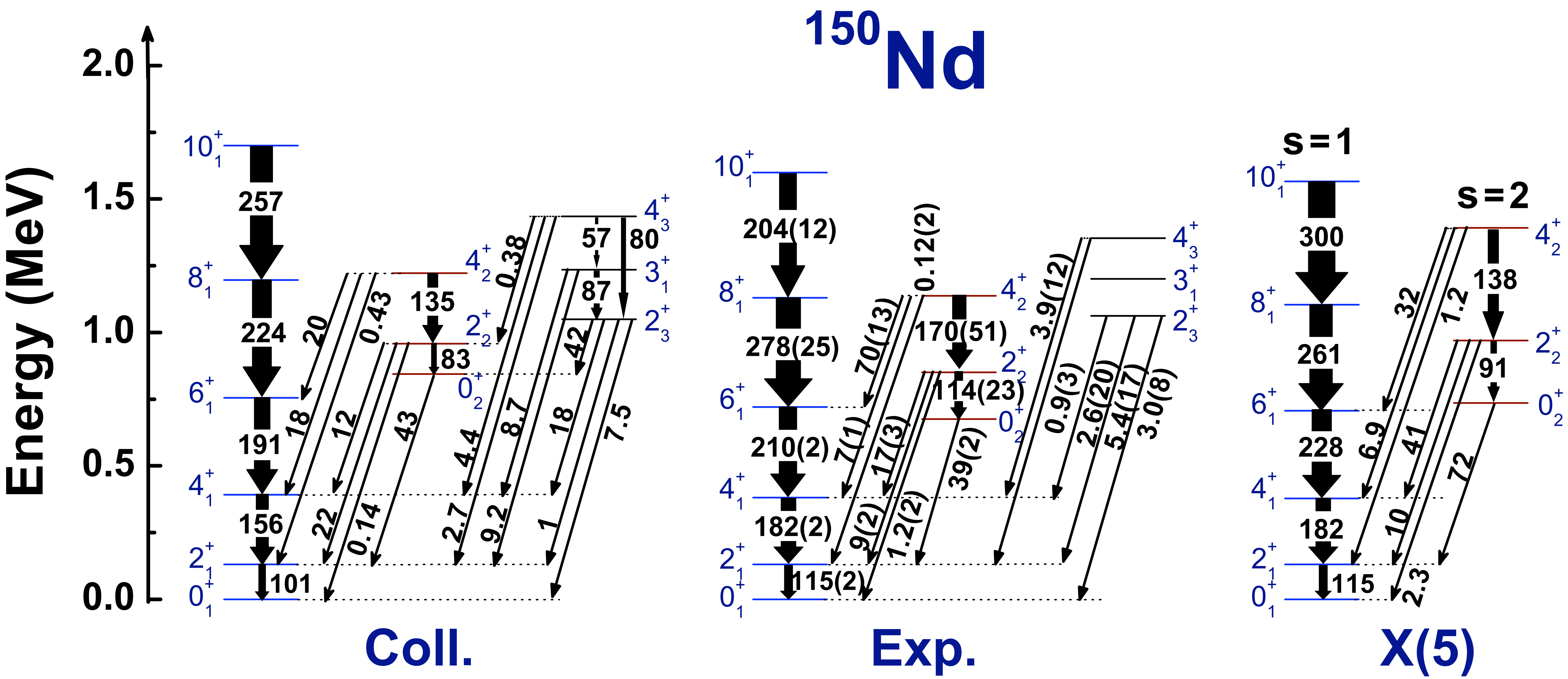


N ≈ 90 rare-earth nuclei → one of the best examples of shape evolution and shape phase transition.

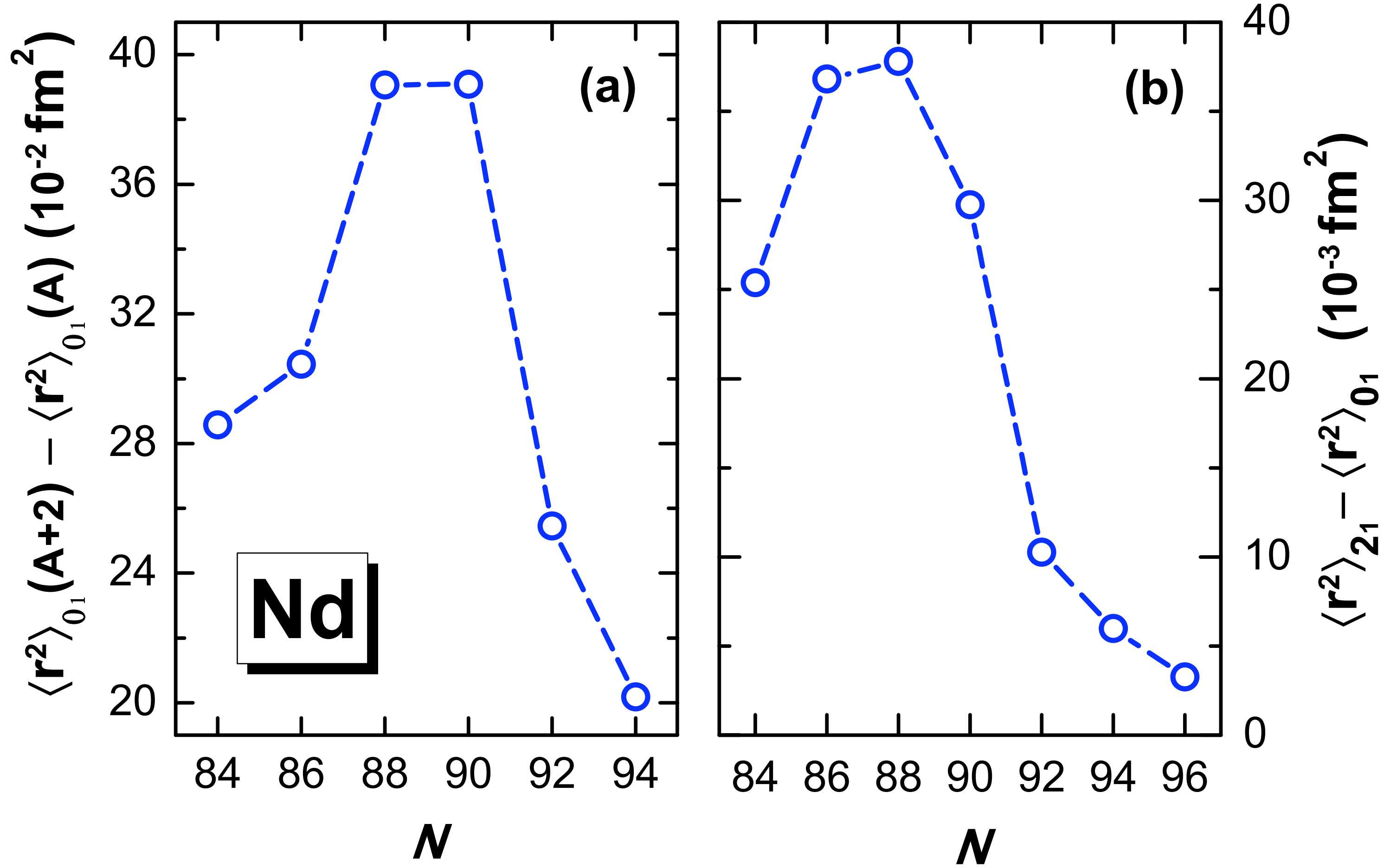
Ground-state transitions:

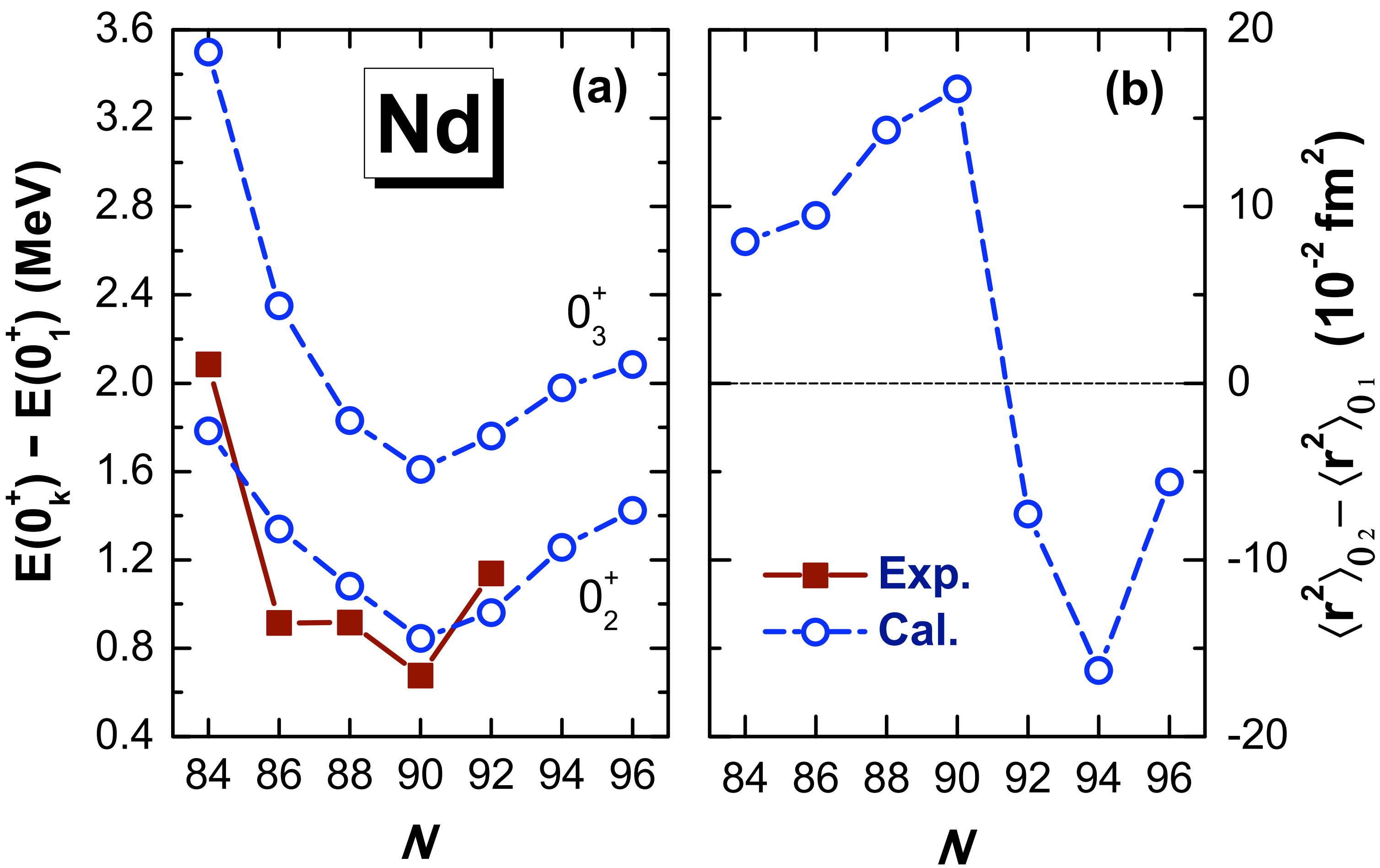
⇒ the physical control parameter - nucleon number - discrete integer values!
⇒ order parameters - expectation values of operators that as observables characterize the state of a nuclear system.

Experimental evidence for a first-order shape phase transition at N≈90 → associated with the X(5) critical symmetry.

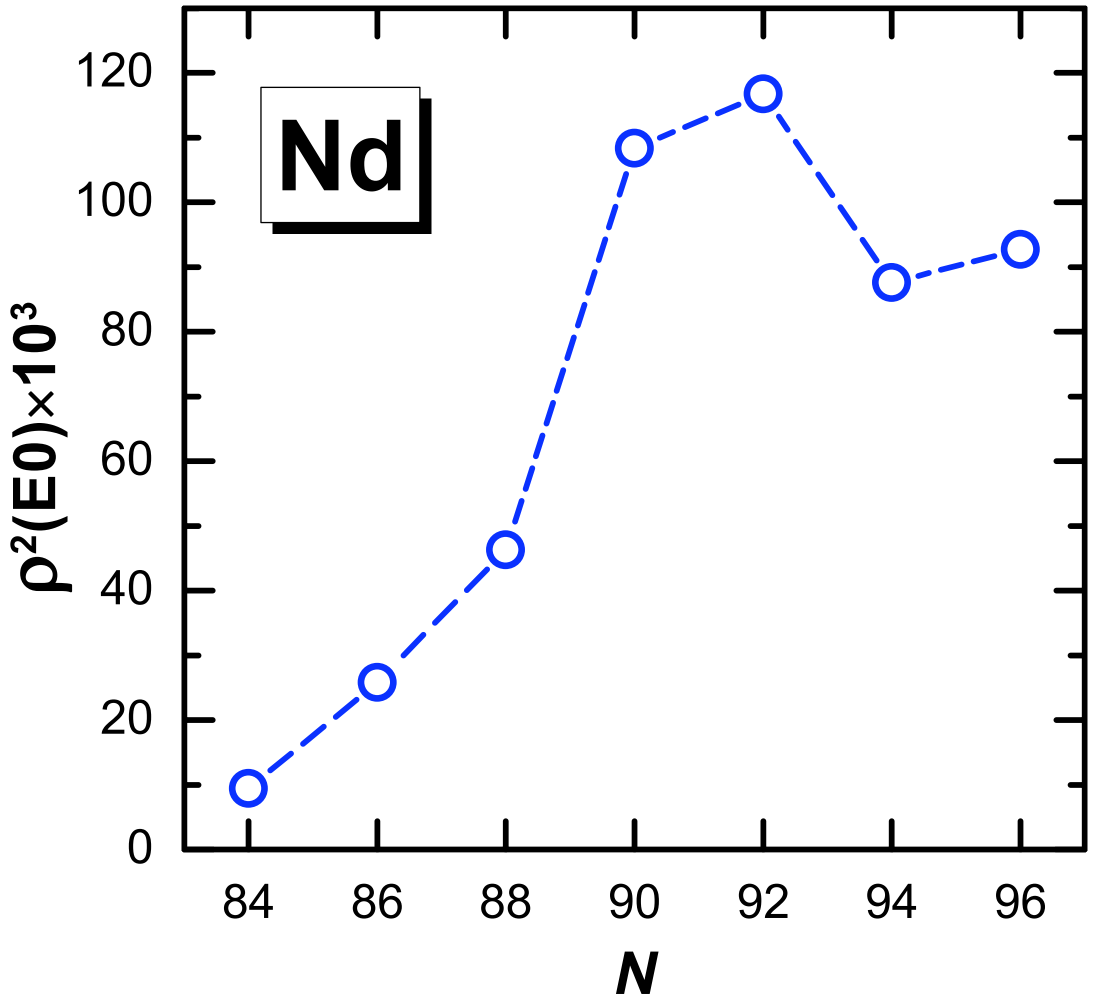


... using collective wave functions obtained by diagonalization of the five-dimensional Hamiltonian ...





... microscopic calculation of order parameters for a first-order nuclear QPT between spherical and axially deformed shapes.

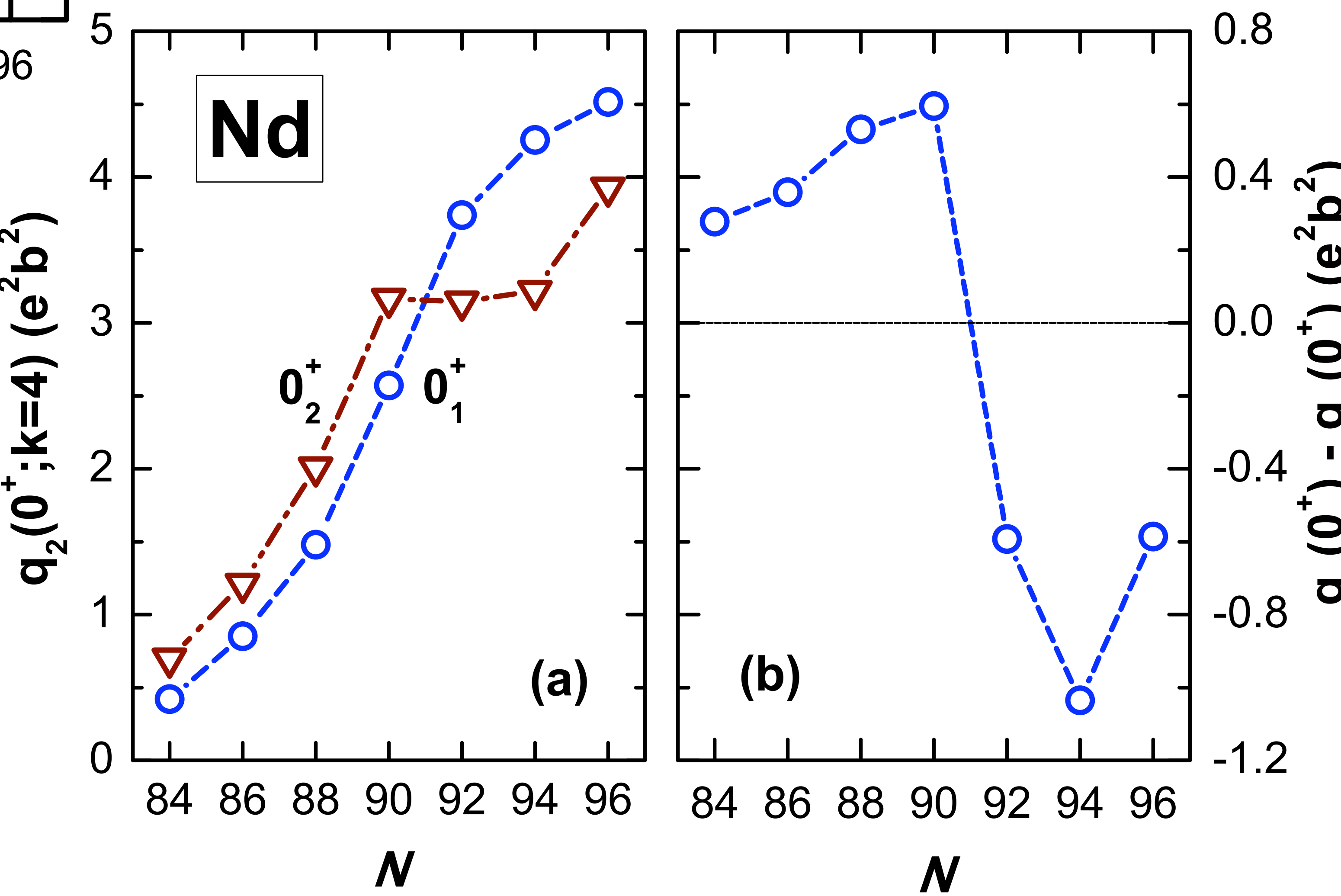


$$\hat{T}(E0) = \sum_k e_k r_k^2$$

$$\rho^2(E0; 0_2^+ \to 0_1^+) = \left| \frac{\left\langle 0_2^+ | \hat{T}(E0) | 0_1^+ \right\rangle}{eR^2} \right|^2$$

q shape invariants:

$$q_2(0_n^+; k) = \sum_{j=1}^{k} B(E2; 0_n^+ \to 2_j^+)$$



Lowest 0+ excitations in N ≈ 90 rare-earth nuclei

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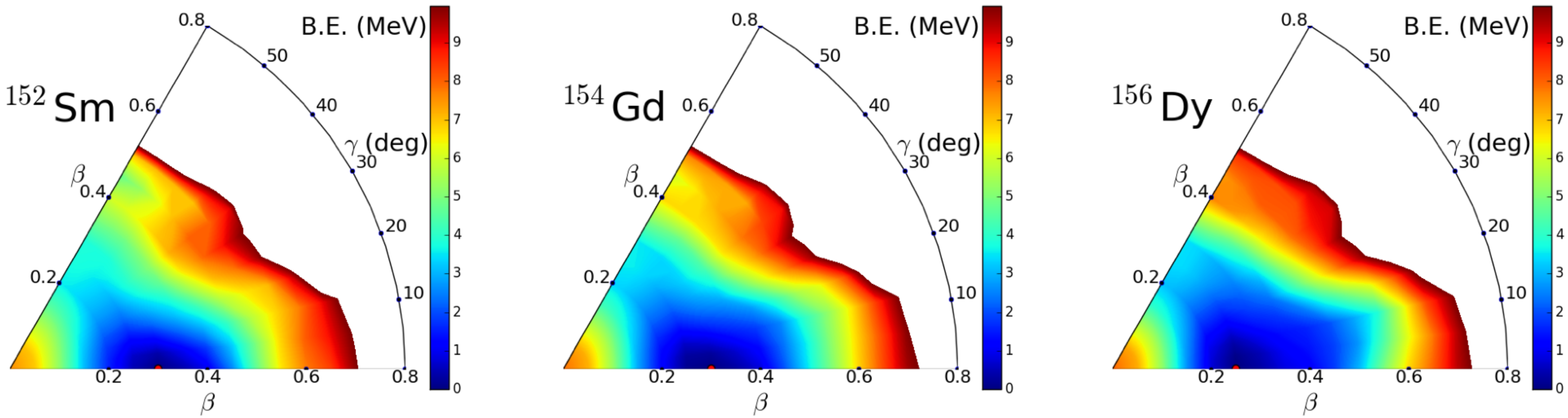
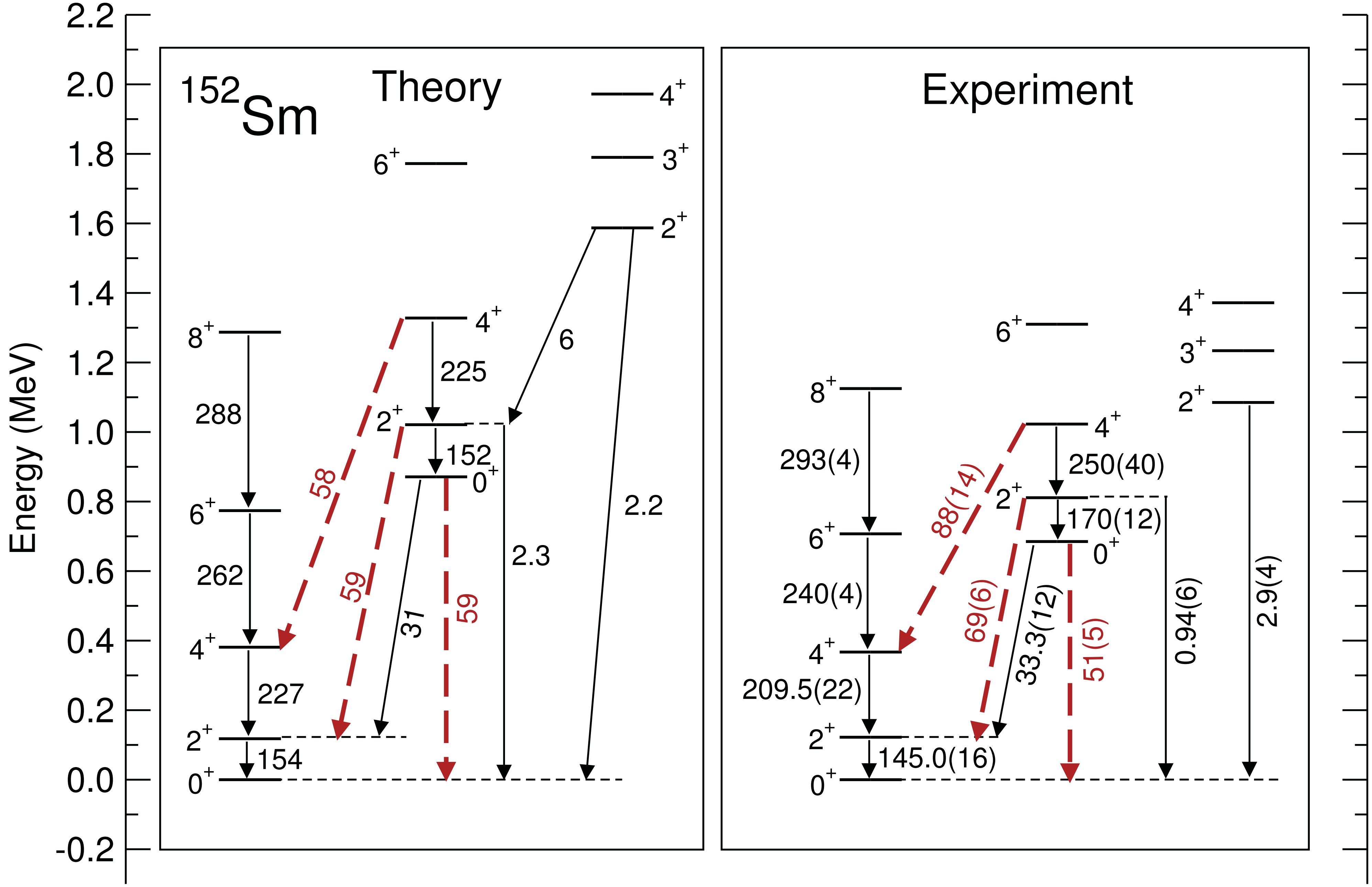


… eigenspectra of the 5D quadrupole collective Hamiltonian:

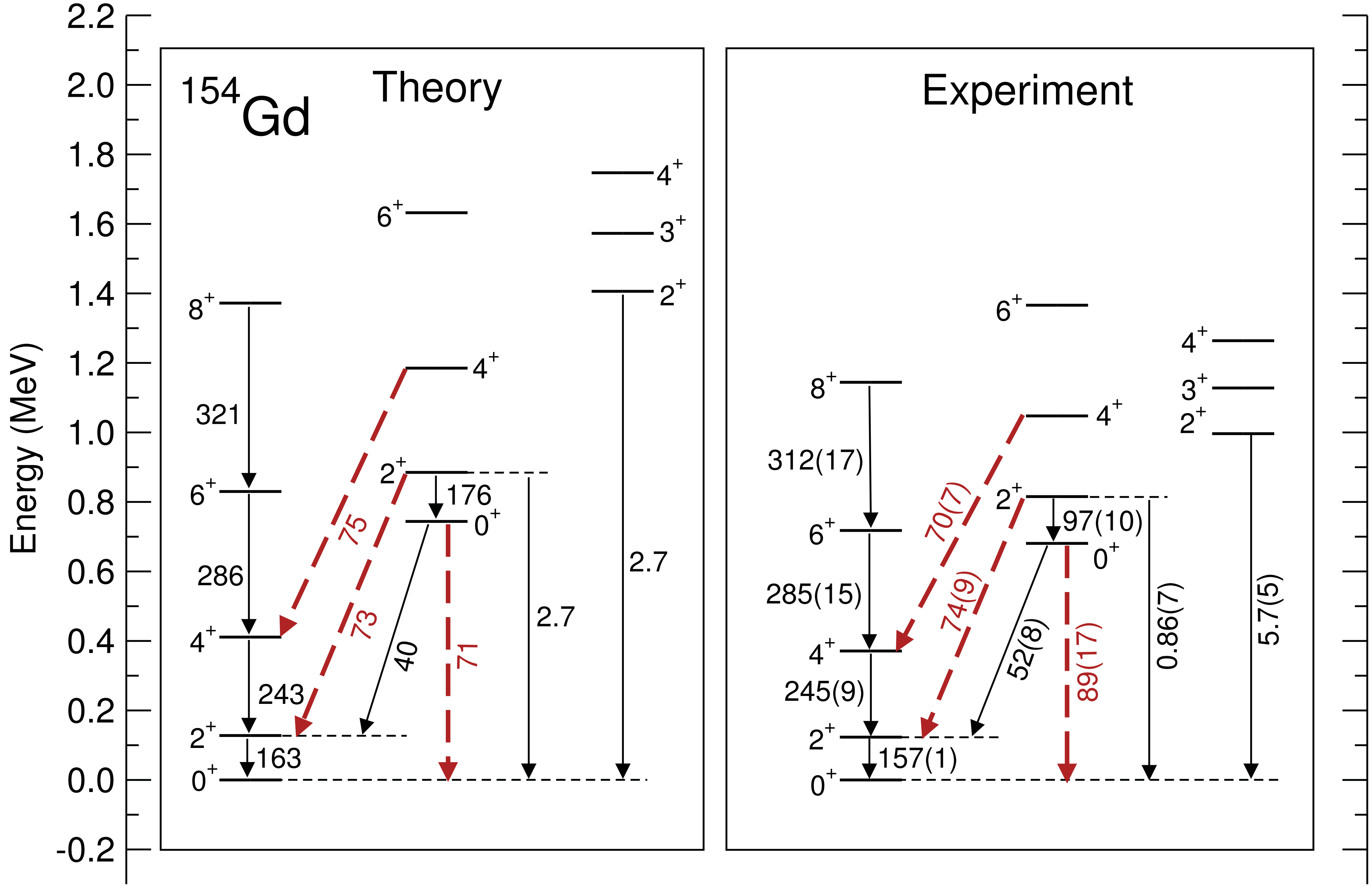


Criteria for the excited 0⁺ state to be labelled as a β-vibration:

$$B(E2; 0_\beta^+ \to 2_1^+) \approx 12 - 33\ W.u.$$

$$B(E2; 2_\beta^+ \to 0_1^+) \approx 2.5 - 6\ W.u.$$

$$\rho^2(E0; 0_2^+ \to 0_1^+) \approx (85 - 230) \times 10^{-3}$$



Criteria for the excited 0^+ state to be labelled as a β-vibration:

$$B(E2; 0_\beta^+ \to 2_1^+) \approx 12 - 33\ W.u.$$

$$B(E2; 2_\beta^+ \to 0_1^+) \approx 2.5 - 6\ W.u.$$

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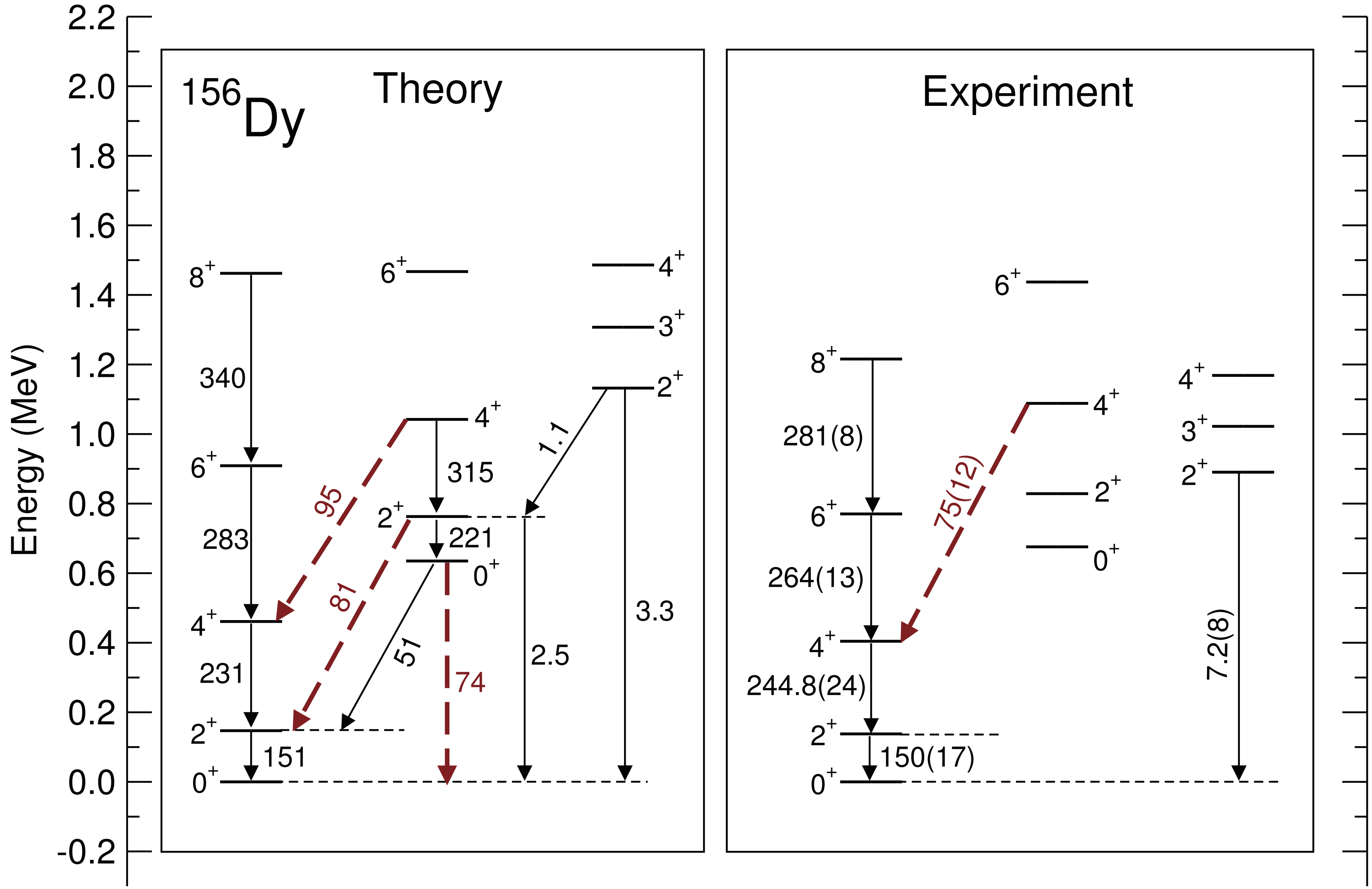

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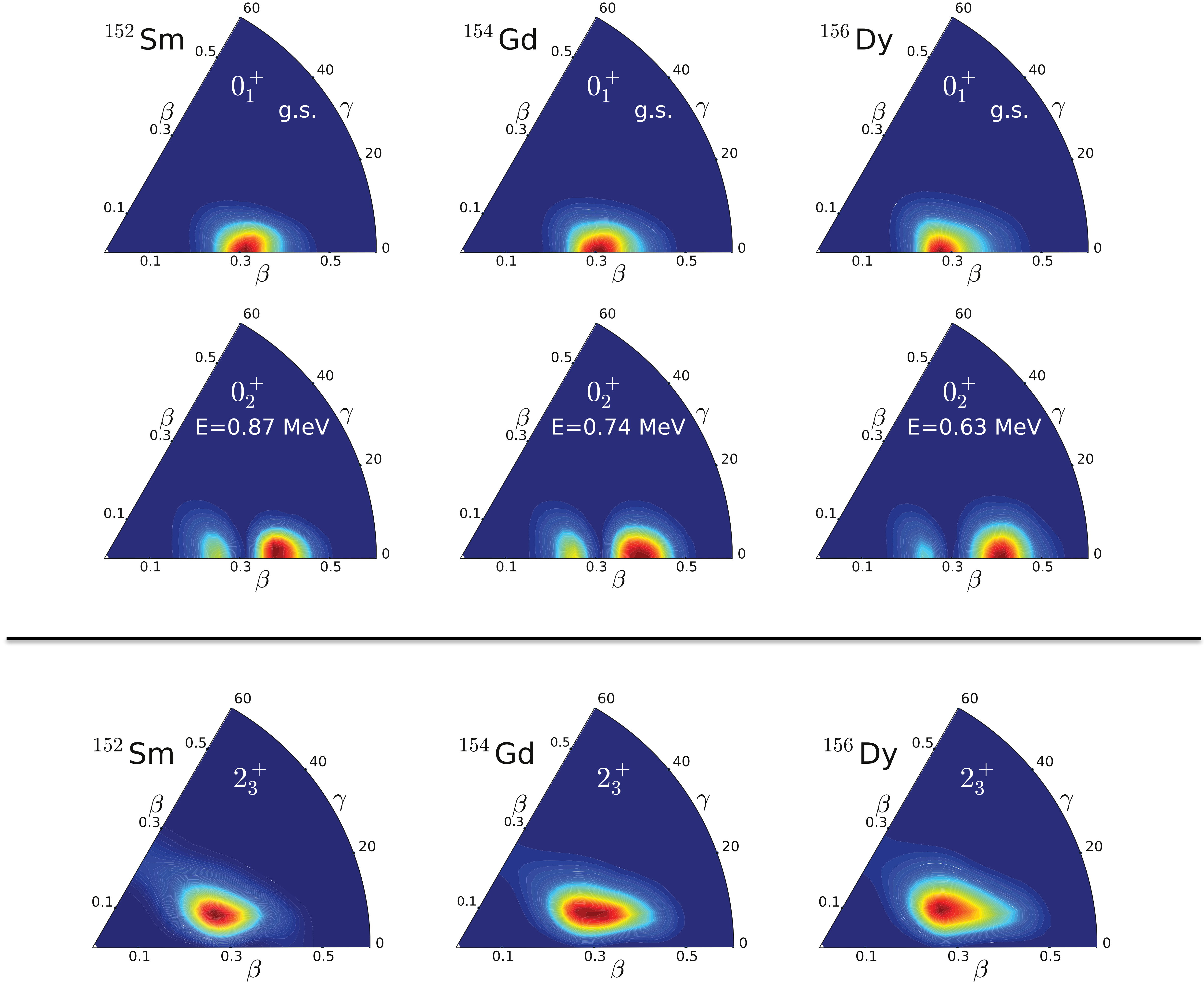
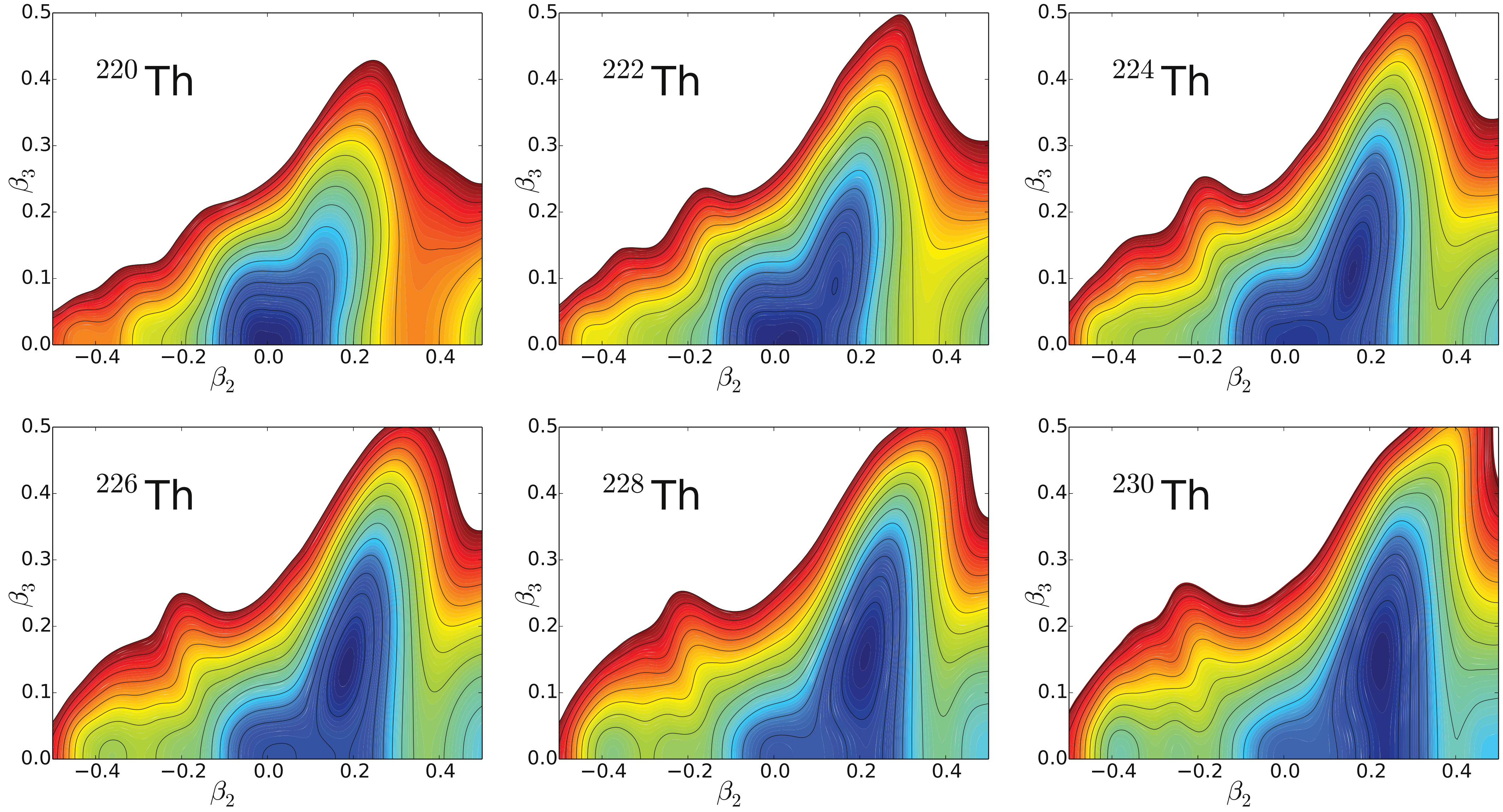
β-vibration or shape coexistence?



^{152}Sm
0_1^+
g.s.
^{154}Gd
0_1^+
g.s.
^{156}Dy
0_1^+
g.s.
0_2^+
E=0.87 MeV
0_2^+
E=0.74 MeV
0_2^+
E=0.63 MeV
^{152}Sm
2_3^+
^{154}Gd
2_3^+
^{156}Dy
2_3^+
β
γ

Quadrupole and octupole shape transition in thorium

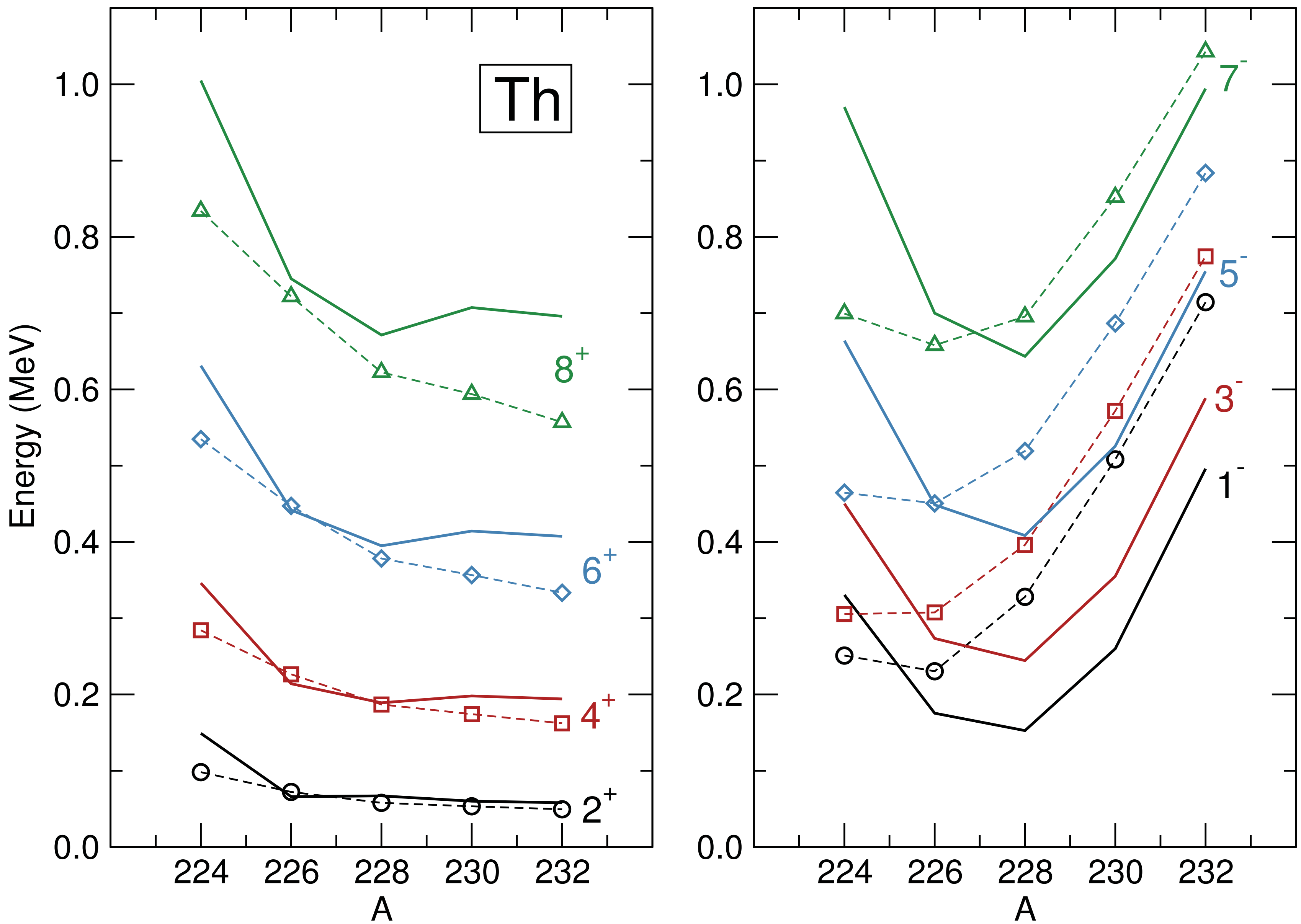
J. Phys. G: Nucl. Part. Phys. 43 (2016) 024005

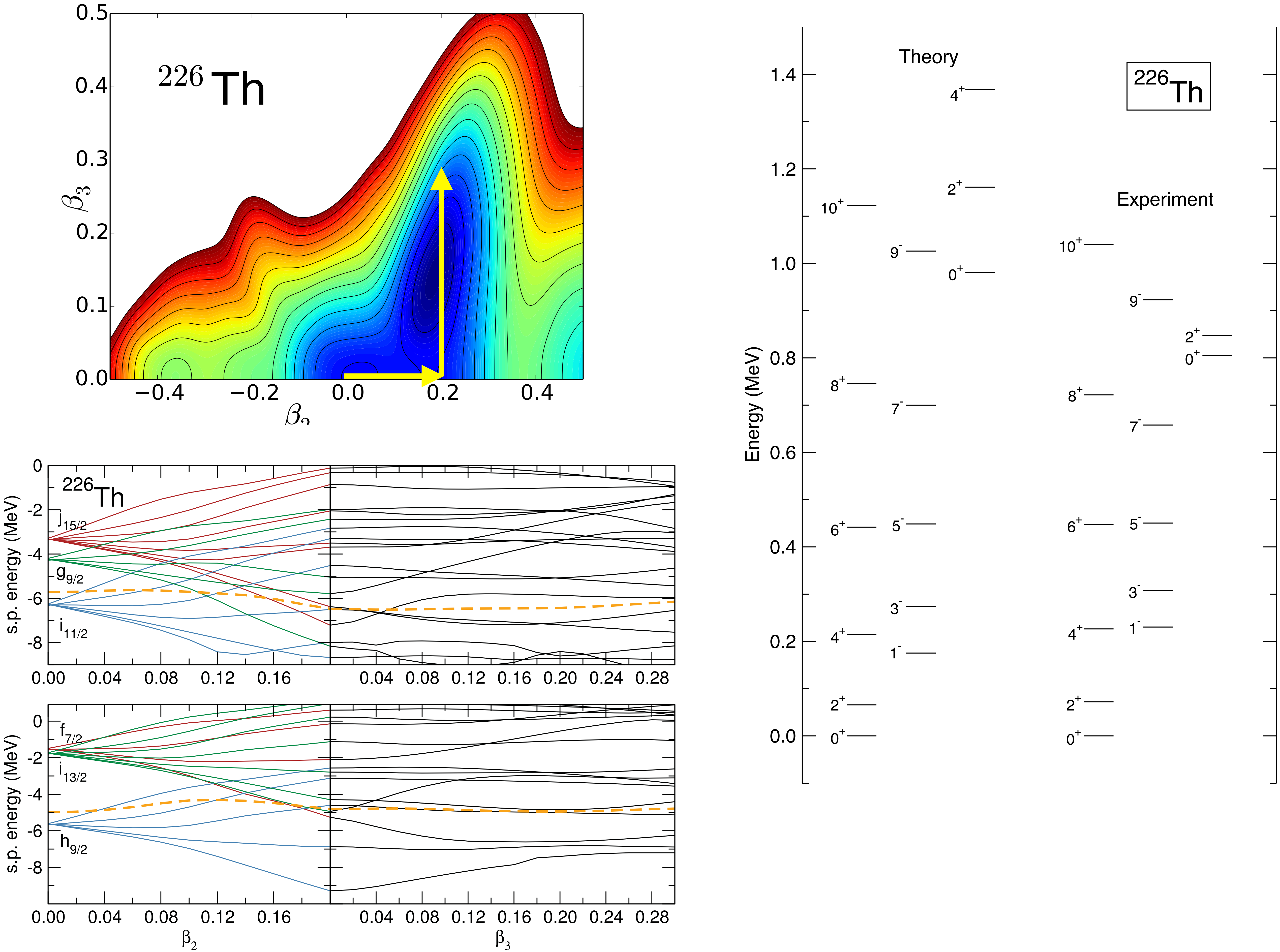


…quadrupole-octupole collective Hamiltonian:

$$\hat{H}_{\mathrm{coll}} = -\frac{\hbar^2}{2\sqrt{w\mathcal{I}}}\left[\frac{\partial}{\partial\beta_2}\sqrt{\frac{\mathcal{I}}{w}}B_{33}\frac{\partial}{\partial\beta_2} - \frac{\partial}{\partial\beta_2}\sqrt{\frac{\mathcal{I}}{w}}B_{23}\frac{\partial}{\partial\beta_3} - \frac{\partial}{\partial\beta_3}\sqrt{\frac{\mathcal{I}}{w}}B_{23}\frac{\partial}{\partial\beta_2} + \frac{\partial}{\partial\beta_3}\sqrt{\frac{\mathcal{I}}{w}}B_{22}\frac{\partial}{\partial\beta_3}\right. + \frac{\hat{J}^2}{2\mathcal{I}} + V(\beta_2,\beta_3)$$

… systematics of energy spectra of the positive-parity ground- state band ($K^{\pi} = 0^{+}$) and the lowest negative-parity ($K^{\pi} = 0^{-}$) sequences in $^{224-232}$Th.





226Th
β3
β2
226Th
j15/2
g9/2
i11/2
s.p. energy (MeV)
f7/2
i13/2
h9/2
s.p. energy (MeV)
β2
β3
Theory
226Th
Experiment
Energy (MeV)

Nuclear Energy Density Functional Framework

✔ …description of universal collective phenomena that reflect the organisation of nucleonic matter in finite nuclei → universal theory framework that can be applied to different mass regions.

✔ NEDFs provide an economic, global and accurate microscopic approach to nuclear structure that can be extended from relatively light systems to superheavy nuclei, and from the valley of β-stability to the particle drip-lines.

✔ NEDF-based structure models that take into account collective correlations → microscopic description of low-energy observables: excitation spectra, transition rates, changes in masses, isotope and isomer shifts, related to shell evolution with nuclear deformation, angular momentum, and number of nucleons.