

# Theoretical description of E0, E1, E2 and M1 giant resonances

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# Microscopic description of the nuclear states

nucleus Hamiltonian

$$\hat{H} = \hat{T}_{kin} + \hat{V}_{eff}$$

semi-phenomenological  
Skyrme, Gogny,

relativistic mean field

derived from the vacuum  
n-n interaction (Brueckner,  
chiral, UCOM, ...)

Hamiltonian eigenstates

$$\hat{H} |\nu\rangle = E_\nu |\nu\rangle$$

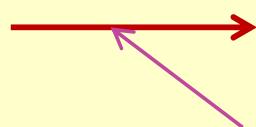
most used approaches:

**BCS (HFB) + RPA approach**

**large scale shell models**

**Knowing the structure and energies of the Hamiltonian eigen-states one can determine excitation probabilities  $B(X\lambda\mu ; |0\rangle \rightarrow |\nu\rangle)$  and strength function of given transition operator  $\hat{T}_{\lambda\mu}^{(X)}$  for all types  $X = el, mag, tor, com$  and multipolarities  $\lambda$**

$$S_k(X\lambda\mu ; E) = \sum_{\nu} (E_{\nu})^k |\langle \nu | \hat{T}_{\lambda\mu}^{(X)} | RPA \rangle|^2 \delta(E - E_{\nu})$$



$$S_k(X\lambda\mu ; E) = \sum_{\nu} (E_{\nu})^k |\langle \nu | \hat{T}_{\lambda\mu}^{(X)} | RPA \rangle|^2 \xi_{\Delta}(E - E_{\nu})$$

*to simulate escape width  
and coupling to complex  
configurations*

**where  $\xi_{\Delta}(E - E_{\nu})$  is the Lorentz weight function**

$$\xi_{\Delta}(E - E_{\nu}) = \frac{1}{2\pi} \frac{\Delta}{(E - E_{\nu})^2 + \frac{\Delta^2}{4}}$$

**with corresponding moments of str. function**

$$m_k(X\lambda\mu) = \int S_k(X\lambda\mu ; E) dE = \sum_{\nu} (E_{\nu})^k |\langle \nu | \hat{T}_{\lambda\mu}^{(X)} | RPA \rangle|^2$$

## Electromagnetic transition operators

$$\hat{T}(el \ \lambda\mu; k) = \frac{(2\lambda+1)!!}{c k^{\lambda+1}} \sqrt{\frac{\lambda}{\lambda+1}} \int d^3r [j_\lambda(kr) \hat{\vec{Y}}_{\lambda,\lambda,\mu}(\hat{r})] \cdot [\vec{\nabla} \times \hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\hat{T}(mag \ \lambda\mu; k) = \frac{-i(2\lambda+1)!!}{ck^\lambda(\lambda+1)} [\lambda(\lambda+1)]^{\frac{1}{2}} \int dr^3 j_\lambda(kr) \hat{\vec{j}}_{nuc}(\vec{r}) \cdot \vec{Y}_{\lambda,\lambda,\mu}(\hat{r})$$

where

vector spherical harmonics

$$\vec{Y}_{l,\lambda,\mu}(\hat{r}) = \sum_{m=-l}^l C_{lm1\mu-m}^{\lambda\mu} Y_{lm}(\hat{r}) \ \vec{\xi}_\mu$$

$$\begin{aligned} \hat{S}^2 \ \vec{\xi}_\mu &= 2 \ \vec{\xi}_\mu \\ \hat{S}_3 \ \vec{\xi}_\mu &= \mu \ \vec{\xi}_\mu \quad \mu = 0, \pm 1 \end{aligned}$$

$\hbar k c = \hbar \omega$  is the energy of absorbed or emitted photon

## current density and density operators:

$$\hat{\vec{j}}_{nuc}(\vec{r}) = \sum_{i=1}^A \frac{e_i^{(eff)}}{m_i} \hat{\vec{p}}_i \delta(\vec{r} - \vec{r}_i) + \sum_{i=1}^A (\vec{\nabla}_i \times \vec{\mu}_i^{(s)}) \delta(\vec{r} - \vec{r}_i)$$

$$\hat{\rho}(\vec{r}) = \sum_{i=1}^A e_i^{(eff)} \delta(\vec{r} - \vec{r}_i) , \quad \vec{\mu}_i^{(s)} = g_i^{(s,eff)} \vec{s}_i \frac{e\hbar}{2m_i}$$

where in the **pure el.mg. processes** (like photoabsorption or (e,e')) :

$$e_p^{eff} = 1 \quad e_n^{eff} = 0 \quad g_p^{eff} = 5.58 \xi \quad g_n^{eff} = -3.82 \xi$$

with the quenching  $\xi = 0.7$

in the **isoscalar T=0 processes** (like ( $\alpha, \alpha'$ )):

$$e_p^{eff}(T=0) = e_n^{eff}(T=0) = 1 \quad g_p^{(s,eff)}(T=0) = g_n^{(s,eff)}(T=0) = \frac{g_p^{eff} + g_n^{eff}}{2}$$

in the **isovector T=1 processes** (like partly in (p,p')):

$$e_p^{eff}(T=1) = -e_n^{eff}(T=1) = 1 \quad g_p^{(s,eff)}(T=1) = -g_n^{(s,eff)}(T=1) = \frac{g_p^{eff} - g_n^{eff}}{2}$$

$kr \ll kR \ll 1 \quad \rightarrow$

in nuclear physics (similarly as in atomic physics) one can use long wave decomposition:

$$kr \ll kR \ll 1$$



$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!} \left[ 1 + \frac{(-\frac{k^2 r^2}{2})}{2\lambda+3} + \dots \right]$$

$$\hat{T}(el \ \lambda\mu; k) = \hat{T}_{\lambda\mu}^{(el)} + k \hat{T}_{\lambda\mu}^{(tor)} + \dots$$

$$\hat{T}(mag \ \lambda\mu; k) = \hat{T}_{\lambda\mu}^{(mag)} + \dots$$

toroidal multipole operator appears  
as a second order term in long-wave  
decomposition of standard  $E^\lambda$  operator

where long-wave limits of the transition operators,  $\hat{T}_{\lambda\mu}^{(el)}$ ,  $\hat{T}_{\lambda\mu}^{(tor)}$  and  $\hat{T}_{\lambda\mu}^{(mag)}$  are

$$\hat{T}_{\lambda\mu}^{(el)} = \int d^3r \ \hat{\rho}(\vec{r}) \ r^\lambda Y_{\lambda\mu}(\hat{r})$$

$$\hat{T}_{\lambda\mu}^{(tor)} = -\frac{1}{2c(2\lambda+3)} \sqrt{\frac{\lambda}{\lambda+1}} \int d^3r \ r^{\lambda+2} \hat{\vec{Y}}_{\lambda\mu}(\hat{r}) \cdot [\vec{\nabla} \times \hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\hat{T}_{\lambda\mu}^{(mag)} = \frac{e\hbar}{2mc} \int d^3r \sum_{i=1}^A \left\{ \frac{2}{\lambda+1} g_i^{(l,eff)} \hat{\vec{l}}_i + g_i^{(s,eff)} \hat{\vec{s}}_i \right\} \delta(\vec{r} - \vec{r}_i) \cdot \vec{\nabla}_i r_i^\lambda Y_{\lambda\mu}(\hat{r})$$

There is another expression for the  $E\lambda$  transition operator

$$\hat{T}(E\lambda\mu; k) :$$

$$\hat{T}(E\lambda\mu; k) = \hat{T}_V(\lambda\mu; k) + \hat{T}_S(\lambda\mu; k)$$

where

$$\hat{T}_V(\lambda\mu; k) = \frac{2\lambda+1}{ck^{\lambda+1}(\lambda+1)} \sqrt{\lambda(\lambda+1)} \int d^3r j_\lambda(kr) Y_{\lambda\mu}(\hat{r}) \left[ \left( \vec{\nabla} \times \hat{\vec{j}}_{nuc}(\vec{r}) \right) - \left( \vec{\nabla} \hat{\rho}(\vec{r}) \times \vec{v} \right) \right]$$

$$\hat{T}_S(\lambda\mu; k) = -i \frac{(2\lambda+1)}{ck^{\lambda+1}} \int d^3r j_{\lambda\mu}(kr) Y_{\lambda\mu}(\hat{r}) \left[ \vec{\nabla} \cdot \hat{\vec{j}}(\vec{r}) \right]$$

with corresponding long-wave decompositions:

$$\hat{T}_S(\lambda\mu; k) = \hat{T}_{\lambda\mu}^{(el)} - k \hat{T}_{\lambda\mu}^{(com)} + \dots$$

$$\hat{T}_V(\lambda\mu; k) = k \hat{T}_{\lambda\mu}^{(vor)} + \dots$$

$$\hat{T}_{\lambda\mu}^{(com)} = -\frac{i}{2c(2\lambda+3)} \int d^3r r^{\lambda+2} Y_{\lambda\mu}(\hat{r}) \left[ \vec{\nabla} \cdot \hat{\vec{j}}_{nuc}(\vec{r}) \right]$$

$$\hat{T}_{\lambda\mu}^{(vor)} = -\frac{i\sqrt{2\lambda+1}}{c(2\lambda+3)\sqrt{\lambda+1}} \int d^3r r^{\lambda+1} \hat{\vec{j}}_{nuc}(\vec{r}) \vec{Y}_{\lambda\lambda+1\mu}(\hat{r})$$

$$\hat{T}_{\lambda\mu}^{(vor)} = \hat{T}_{\lambda\mu}^{(el)} + \hat{T}_{\lambda\mu}^{(com)}$$

and  $\hat{T}_{\lambda\mu}^{(com)} = -k \hat{T}_{\lambda\mu}^{(com)},$

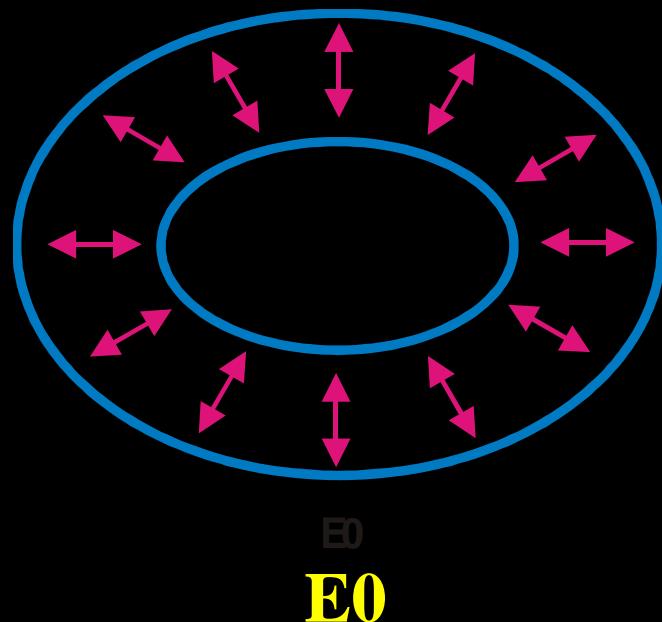
with the long-wave compression operator:

$$\hat{T}_{\lambda\mu}^{(com)}, = \frac{1}{2(2\lambda+1)} \int d^3r \hat{\rho}(\vec{r}) r^{\lambda+2} Y_{\lambda\mu}(\hat{r})$$



long-wave limit toroidal  $\hat{T}_{\lambda\mu}^{(tor)}$  and compression  $\hat{T}_{\lambda\mu}^{(com)}$  operators with the given  $\lambda$  are related each other and they are immediately connected with the second order term in the  $E\lambda$  transition operator  $\hat{T}(E\lambda\mu; k)$  decomposition in the linear momentum  $k$

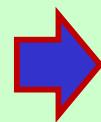
# giant monopole resonance



Giant Monopole Resonance (GMR) centroid  $E_{GMR}$  is connected with finite – nucleus incompressibility  $K_A$  by (see e.g. J.Blaizot, Phys. Rep. 64, 171 (1980))

$$E_{GMR} = \sqrt{\frac{\hbar^2 K_A}{m \langle r^2 \rangle}}$$

The incompressibility ( together with the nucleus mass and radius) belongs to the bulk properties used for the determination of the energy functional (n-n effective interaction ) parameters



GMR is the subject of intensive investigation from 60-s up to now

From the point of view of theory the position of  $E_{GMR}$  is usually obtained by means of moments of energy weighted E0 strength functions

$$E_{GMR} = \frac{m_1}{m_0} \quad \left| \begin{array}{l} m_k = \int dE S_k(E0; E) \\ S_k(E0; E) = \sum_{\nu} E^k | \langle \nu | \hat{M}_{\lambda=\mu=0}^{(IS)}(el) | 0 \rangle |^2 \delta(E - E_{\nu}) \end{array} \right.$$

where  $\hat{T}_{\lambda=\mu=0}^{(el)}(T=0) = \sum_{i=1}^A (r^2 Y_{00})_i$  is the isoscalar E0 transition operator

There are two main groups in the world providing the data on E0 resonance, namely:

Texas A&M University (TAMU):

D.H. Youngblood, et al., PRC69, 034315 (2004) -  $^{116}\text{Sn}$ ,  $^{208}\text{Pb}$ ,  $^{144}\text{Sm}$ ,  $^{154}\text{Sm}$

D.H. Youngblood, et al., PRC69, 054312 (2004) -  $^{90}\text{Zr}$

D.H. Youngblood, et al., PRC88, 021301(R) (2004) -  $^{92}\text{Zr}$ ,  $^{92}\text{Mo}$ ,  $^{90}\text{Zr}$ ,  $^{96}\text{Mo}$ ,  $^{96}\text{Mo}$ ,  $^{98}\text{Mo}$ ,  $^{100}\text{Mo}$

Research Center for Nuclear Physics (RCNP) at Osaka University

M.Uchida, et al., PRC69, 051301 (2004) -  $^{90}\text{Zr}$ ,  $^{116}\text{Sn}$ ,  $^{208}\text{Pb}$

M.Itoh, et al., PRC68, 064602 (2003) -  $^{144}\text{Sm}$ ,  $^{148}\text{Sm}$ ,  $^{150}\text{Sm}$ ,  $^{152}\text{Sm}$ ,  $^{154}\text{Sm}$

T.Li, et al., PRC99, 162503 (2007) -  $^{112-124}\text{Sn}$

All these papers give not only GMR centroids but also shapes of the GMR and both experimental groups used  $(\alpha, \alpha')$  reaction for the determination of E0 strength functions. However, in the case when both groups measured E0 strength function for the same nucleus ( $^{90}\text{Zr}$ ,  $^{144,154}\text{Sm}$ ,  $^{208}\text{Pb}$ ) one can see substantial differences in the E0 strength functions between both groups (mentioned already in P.Avogadro, et al., PRC88, 044319 (2013)).

From theoretical point of view in the most of papers only centroids of the E0 strength function were compared with experimental values (  $E_{GMR} = \frac{m_1}{m_0}$  ) but recently also shapes of the GMR are compared:

**Yoshida K., Nakatsukasa T., PRC 88, 034309 (2013)**

**Li-Gang Cao, Sagawa H., Colo G., PRC 86, 054313 (2012)**

**Vesely P., Toivanen J., Carlsson B.G., Dobaczewski J., Michel N., Pastore A., PRC 86, 024303 (2012)**

**Kvasil J., Nesterenko V.O., Repko A., Kleinig W., Reihard P.-G., arXiv: 1608.01483 v1[ucl-th] 4 Aug 2016**

**P.Avogadro, Nakatsukasa T., PRC 88, 044319 (2013)**

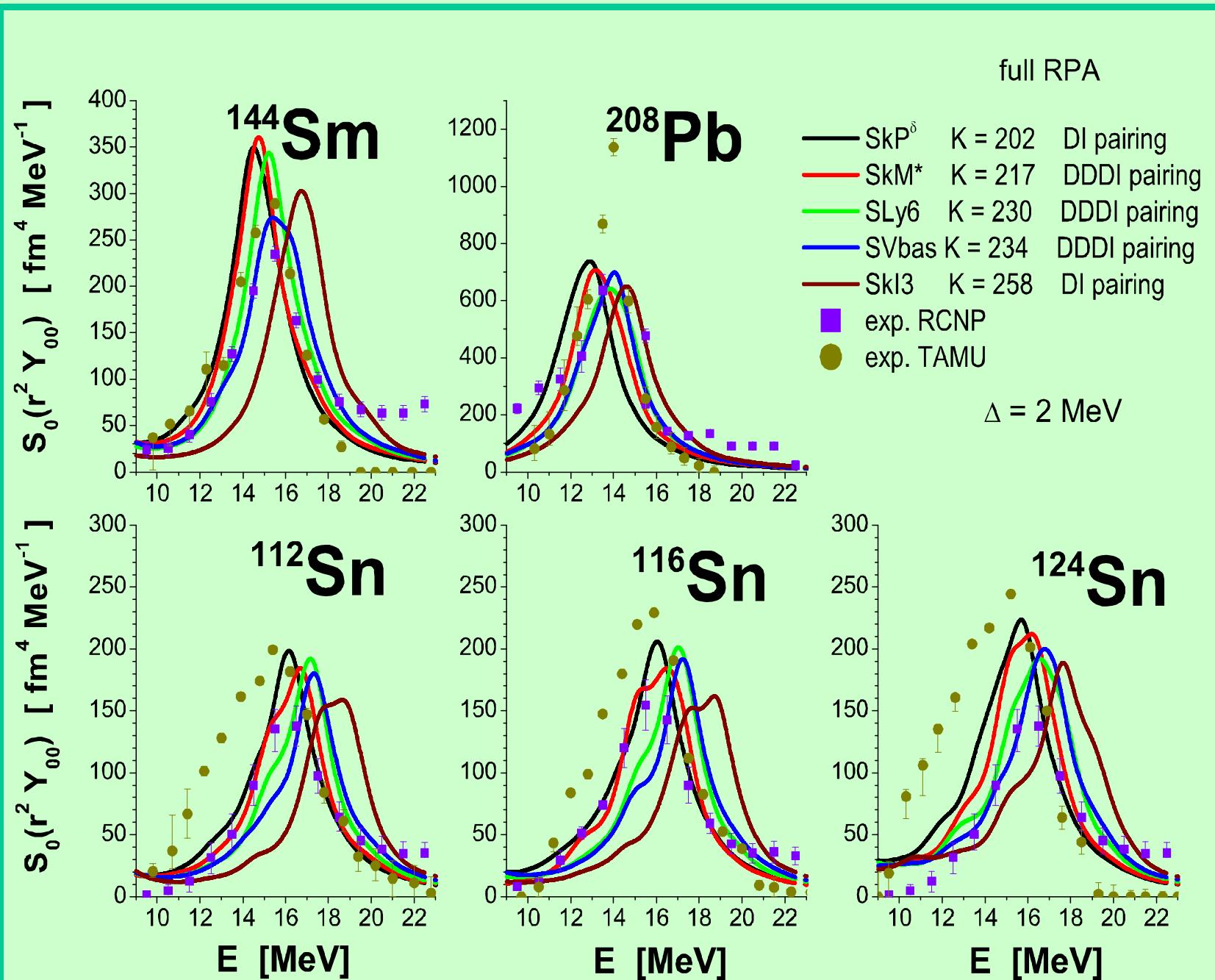
# Dependence of E0 strength function in the spherical nuclei (heavier $^{208}\text{Pb}, ^{144}\text{Sm}$ ; lighter $^{112}, ^{116}, ^{124}\text{Sn}$ ) on Skyrme parametrizations with different K

## Skyrme BCS+RPA

parametrizations with  $K \sim 230$  MeV fits the experimental values for heavier nuclei (Sm, Pb)

experimental values for lighter Sn isotopes require parametrizations with the lower values of  $K$  ( $K \sim 200$  MeV like  $\text{SkP}^\delta$ )

differences in RCNP and TAMU data



# GMR in the isotopes $^{144-154}\text{Sm}$

comparison of SRPA results with TAMU and RCNP exp. data

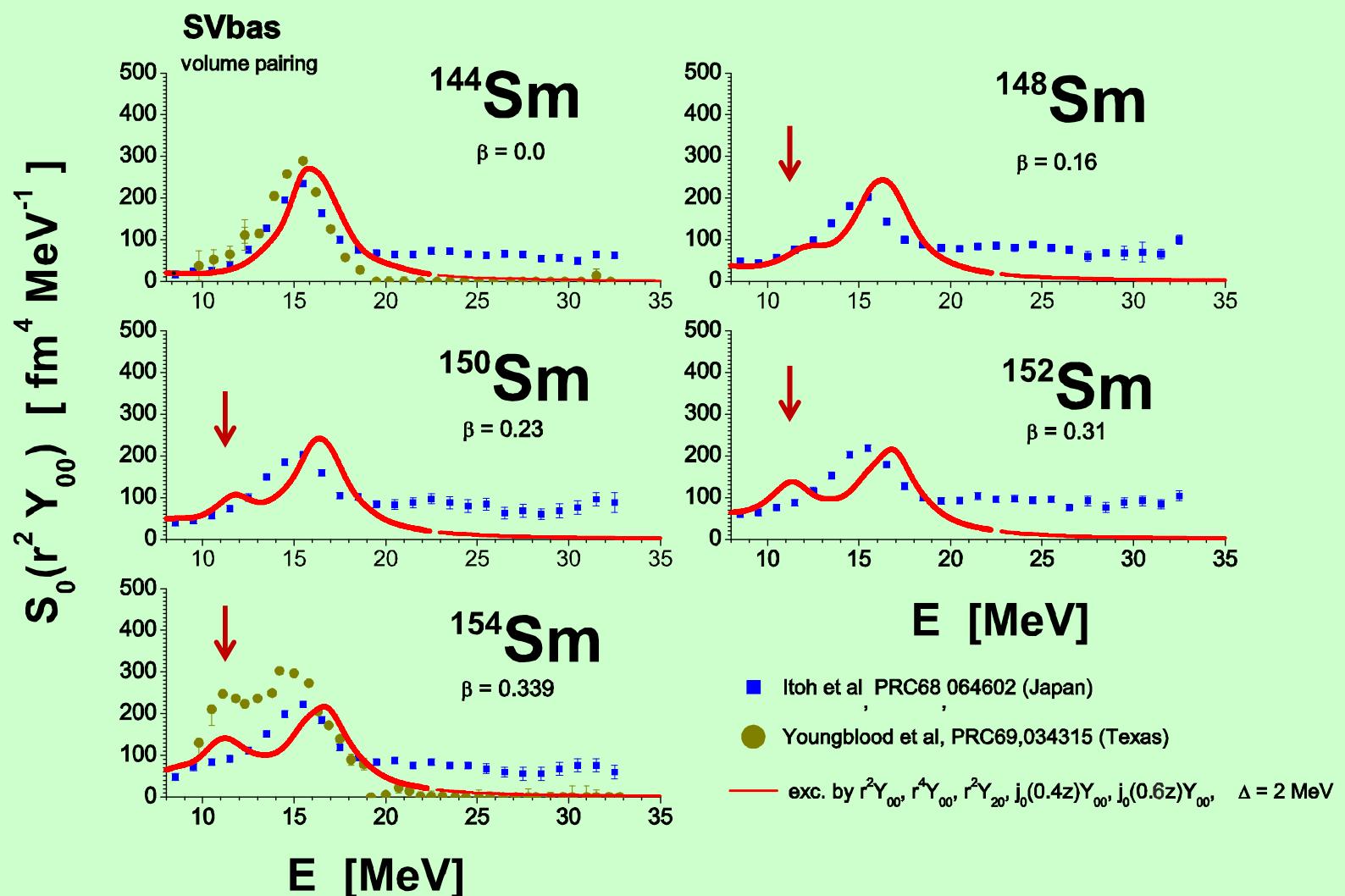
TAMU data renormalized for absolute units:

$$S_0(r^2Y_{00}; fm^4 MeV^{-1}) = S_1(r^2Y_{00}; fraction EWSR MeV^{-1}) \times EWSR_{cl} \times E^{-1}$$

different shapes of exp. GMR for TAMU and RCNP data:  
much bigger deformation effect in TAMU data

E0-E2 coupling is switched on

↓ position of the E2  $\mu=0$  resonance peak

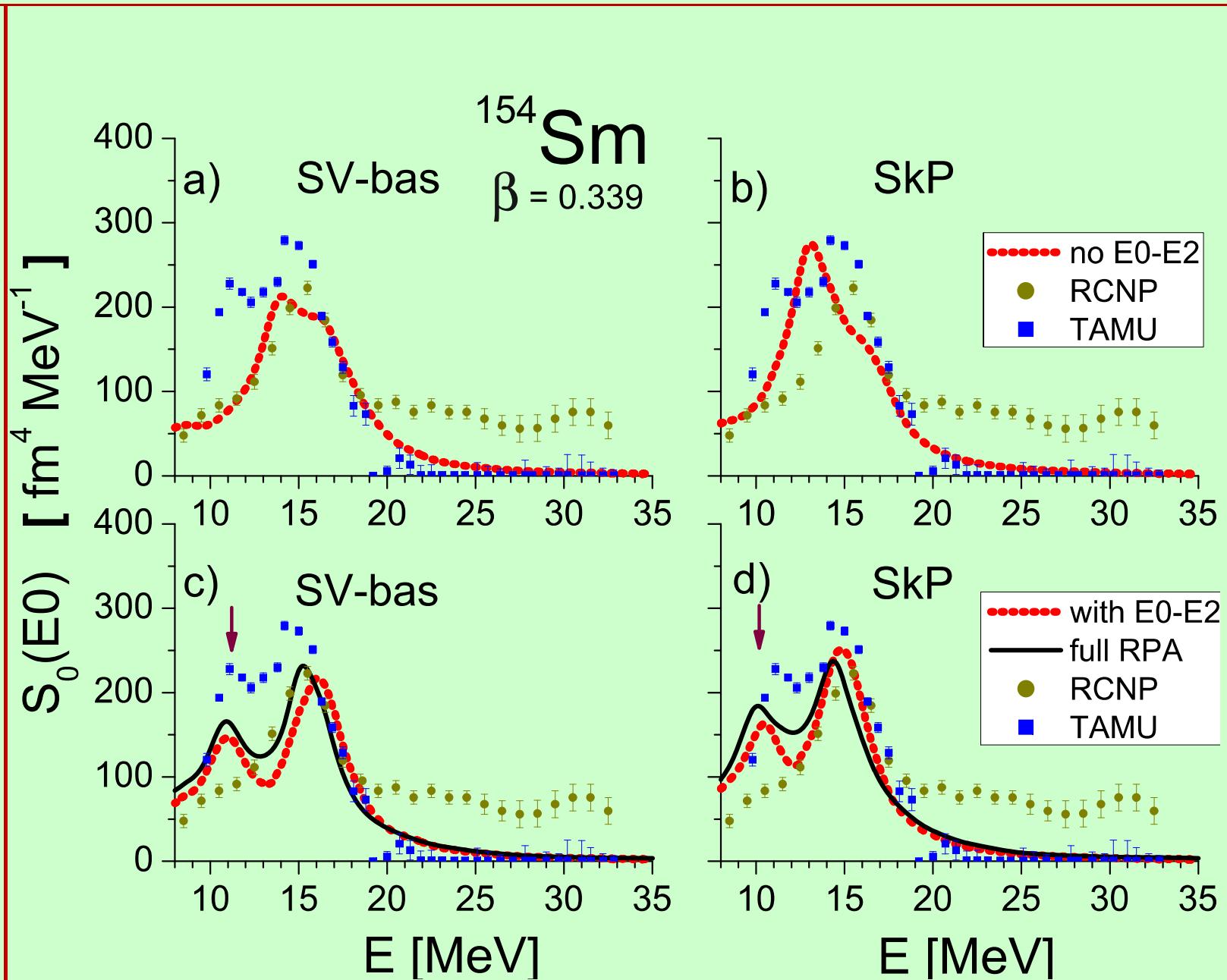


# Comparison of the GMR in $^{154}\text{Sm}$ calculated with SV-bas and SkP $^\delta$ parametrizations in the framework of sRPA (without and with E0-E2 mixing) and fRPA with corresponding experimental data

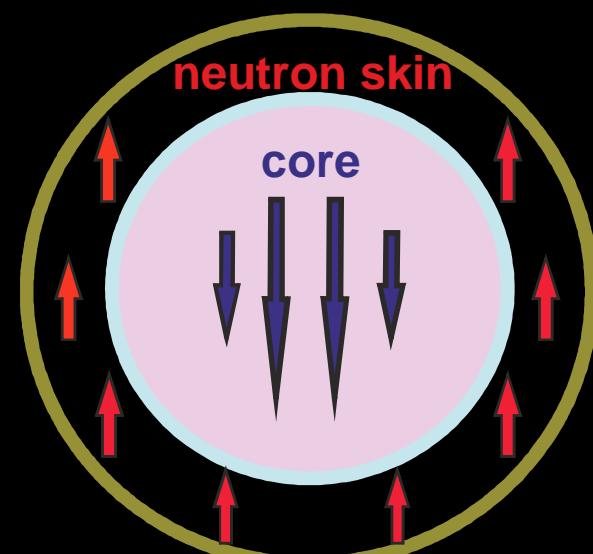
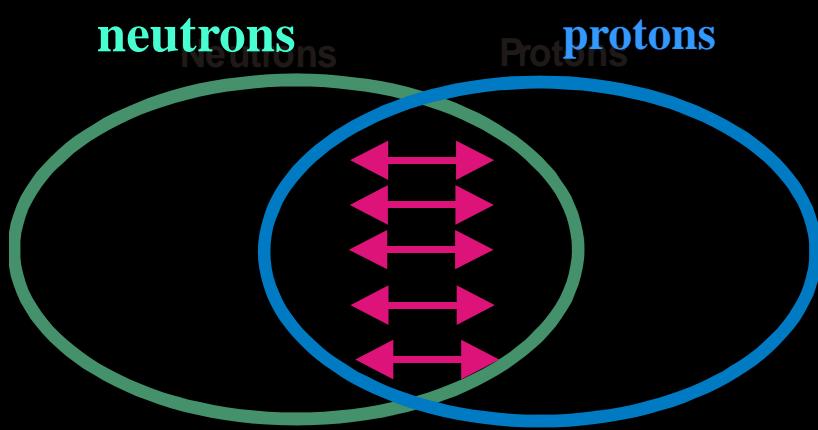
small discrepancies  
between sRPA and  
fRPA

the SV-bas Skyrme  
interaction gives  
better agreement with  
experiment

↓ position of the  
E2  $\mu=0$  resonance  
peak



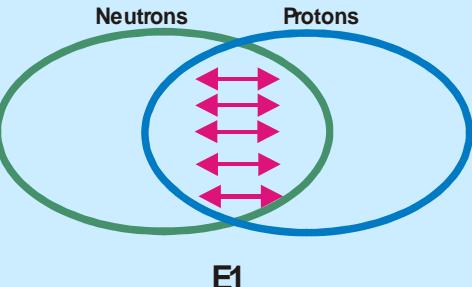
# giant dipole resonance (pygmy resonance)



# Photoabsorption cross section for Sm isotopes

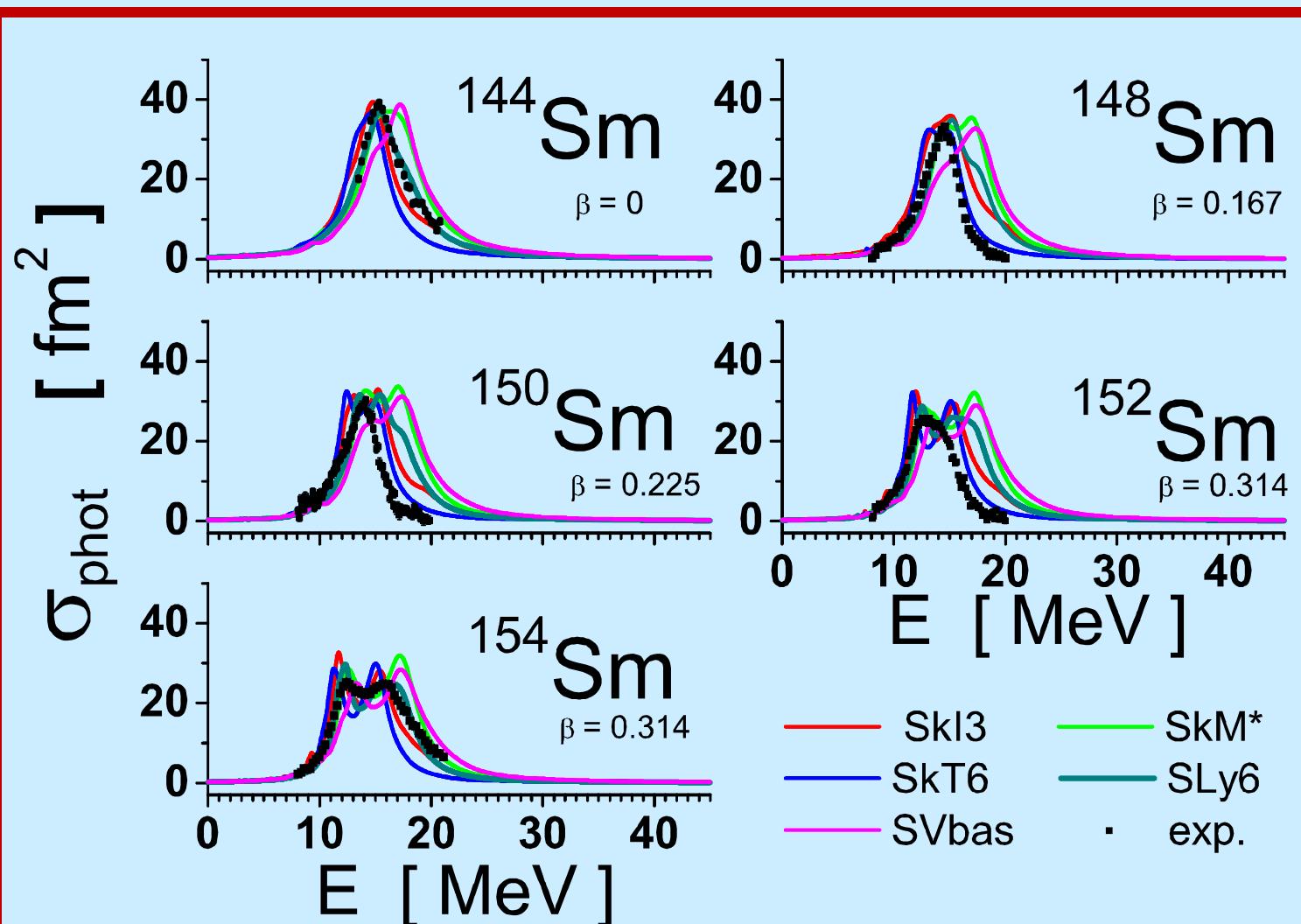
$$\sigma_{phot}(E) = \frac{16\pi^3 \alpha_{FS}}{9e^2} E S_0(E1; E)$$

$$S_0(E1; E) = \sum_{\mu=0,\pm 1} \sum_{\nu} |\langle \nu | \hat{T}_{1\mu}^{(el)}(T=1) | RPA \rangle|^2 \xi_{\Delta}(E - E_{\nu})$$



$$\begin{aligned} \hat{T}_{1\mu}^{(el)}(T=1) &= \\ &= \int d^3r \hat{\rho}_{T=1}(\vec{r}) r Y_{1\mu}(\hat{r}) \\ &= \sum_{i=1}^A e_i^{(eff)}(T=1) r_i Y_{1\mu}(\hat{r}_i) \end{aligned}$$

- best agreement for Sly6
- the most of Skyrme parametrizations describes experimental deformation effects

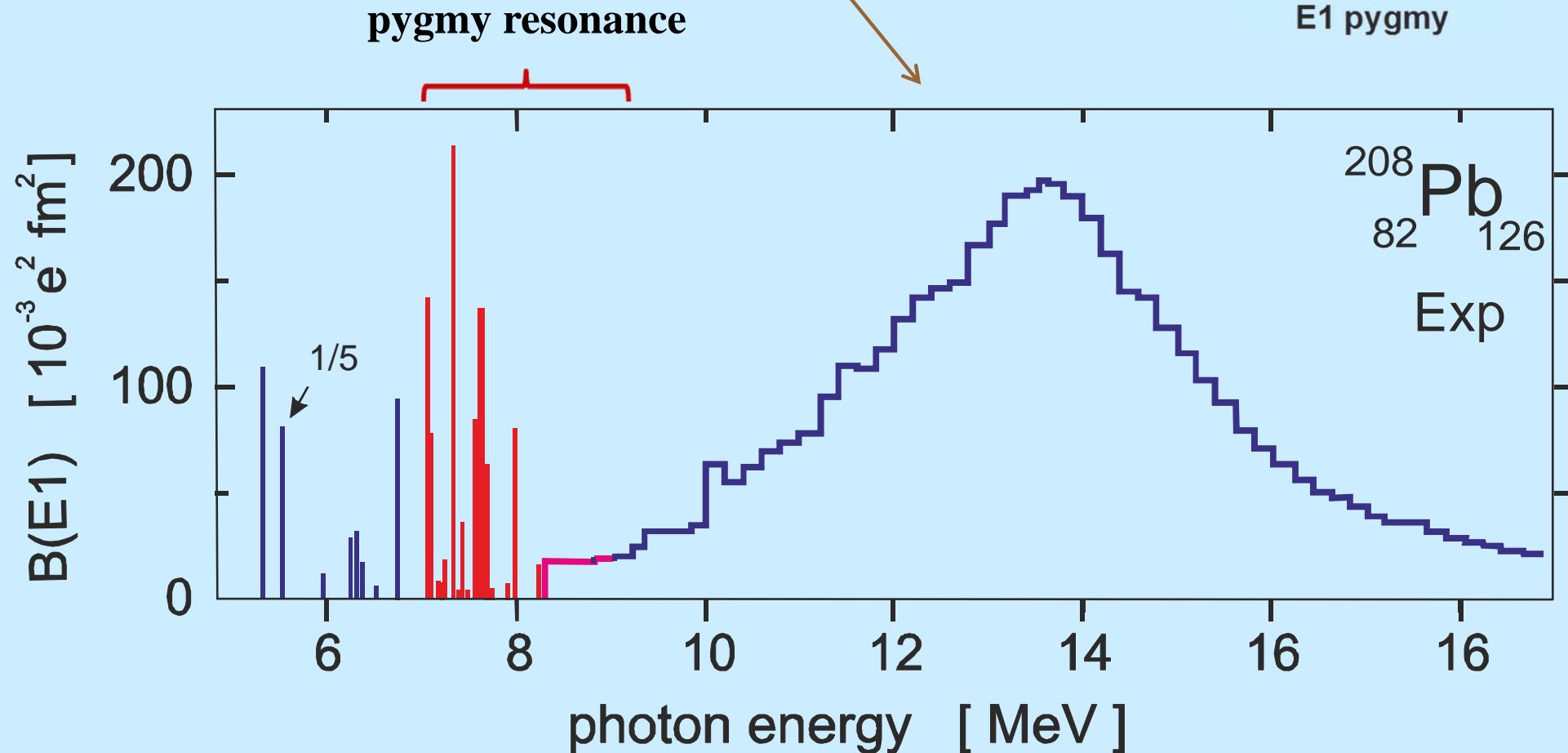
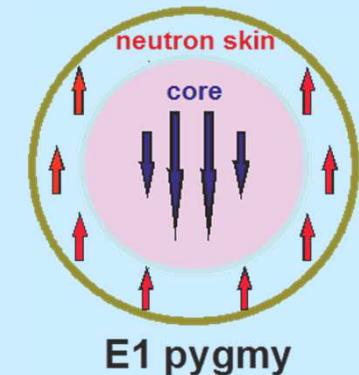


low energy part of the E1 strength function demonstrates a bunch of states called **pygmy resonance** – see e.g.:

Kneissl U, et al., J. Phys. G 32, R217 (2006)

Krasznahorkay A., et al. Phys.Rev. A64, 067302 (2001)

Ryezayeva N., et al., Phys.Rev. Lett. 89, 272502 (2002)



# TR and CR constitute low- and high-energy ISGDR branches

D.Y. Youngblood et al, 1977  
 H.P. Morsch et al, 1980  
 G.S. Adams et al, 1986  
 B.A. Devis et al, 1997  
 H.L. Clark et al, 2001  
 D.Y. Youngblood et al, 2004  
 M.Uchida et al, PLB 557, 12 (2003),  
 PRC 69, 051301(R) (2004)

$(\alpha, \alpha')$   
 experiment →

There are also the ISGDR data for

$^{56}\text{Fe}$ ,  $^{58,60}\text{Ni}$ ,  $^{90}\text{Zr}$ ,  $^{116}\text{Sn}$ ,  $^{144}\text{Sm}$ , ...

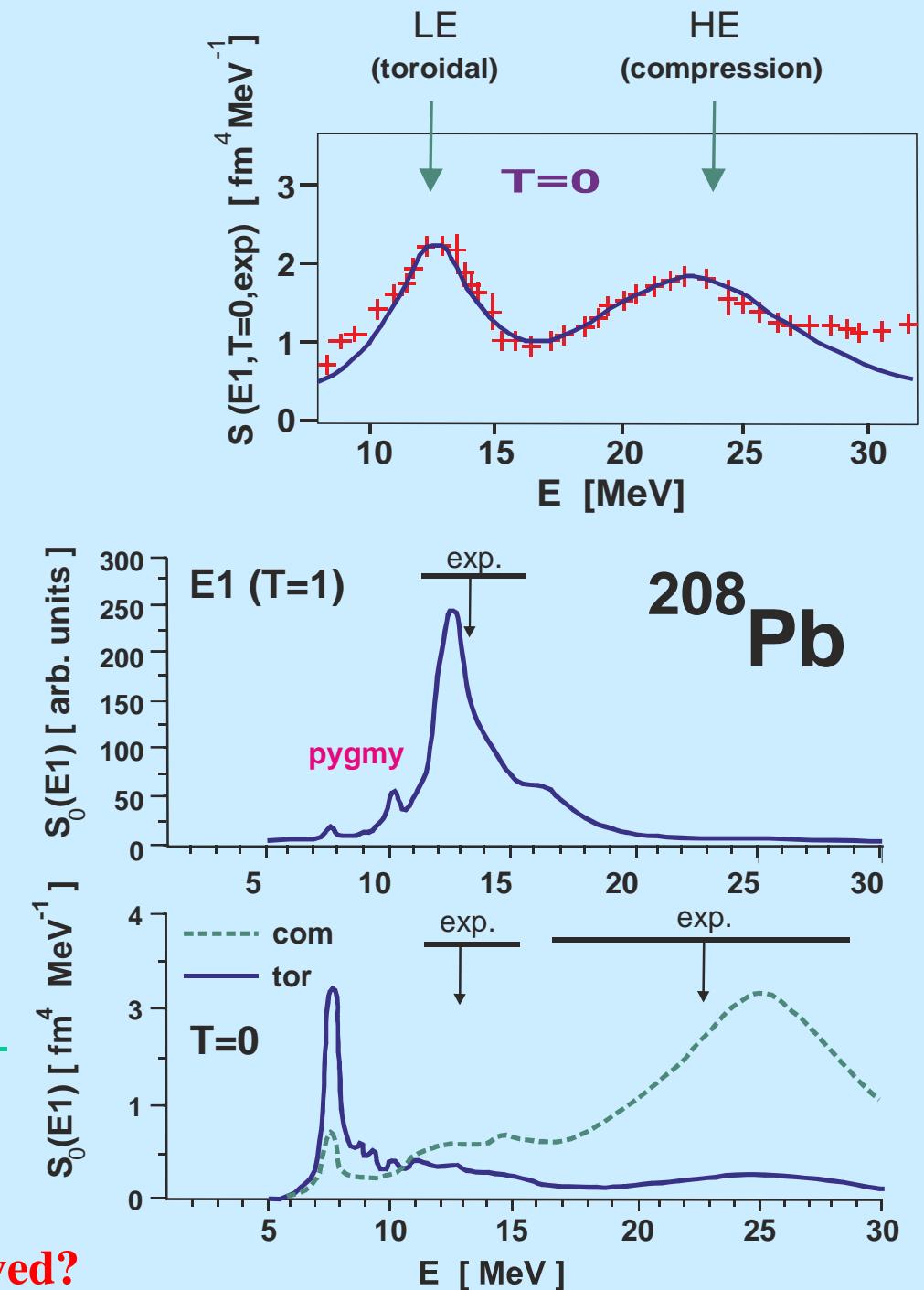
$$\hat{T}_{1\mu}^{(tor)} = -\frac{1}{10c} \sqrt{\frac{1}{2}} \int d^3r \left[ r^3 + \frac{5}{3}r <r^2> \right] \vec{Y}_{11\mu}(\hat{r}) \cdot [\vec{\nabla} \times \hat{j}_{nuc}(\vec{r})]$$

$$\hat{T}_{1\mu}^{(com)} = \frac{1}{10} \int d^3r \hat{\rho}_{nuc}(\vec{r}) \left[ r^3 - \frac{5}{3}r <r^2> \right] Y_{1\mu}(\hat{r})$$

A. Repko, P.-G. Reinhard,  
 V.O. Nesterenko, J. Kvasil,  
 PRC 87, 024305 (2013). Skyrme RPA, SLy6

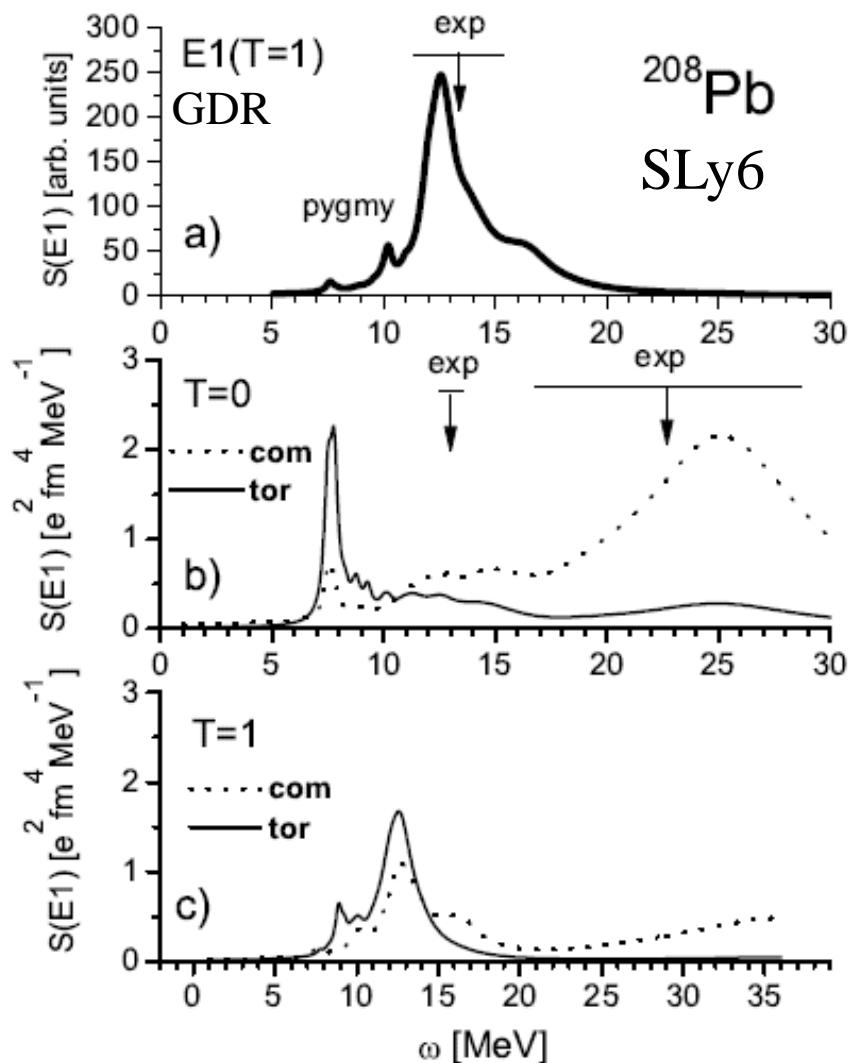
theory →

- discrepancy between theory and experiment for TR
- perhaps, Uchida observed not TR but low-energy CR fraction. Then TR is not still observed?

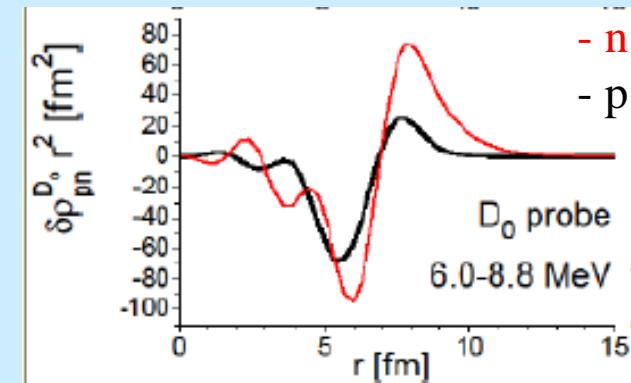


Isoscalar E1 modes (toroidal and compression)  
play an important role for states with energies  
in the pygmy resonance region → new  
point of view for the interpretation of pygmy  
resonance

A. Repko, P.G. Reinhard, V.O.Nesterenko,  
J. Kvasil, PRC, 87, 024305 (2013)

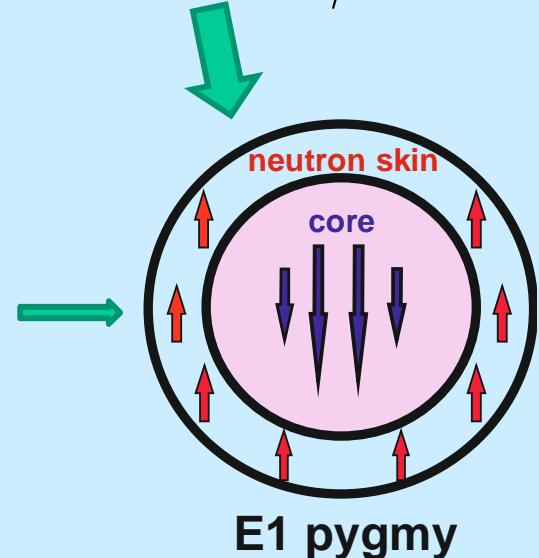


Typical PDR transition density:

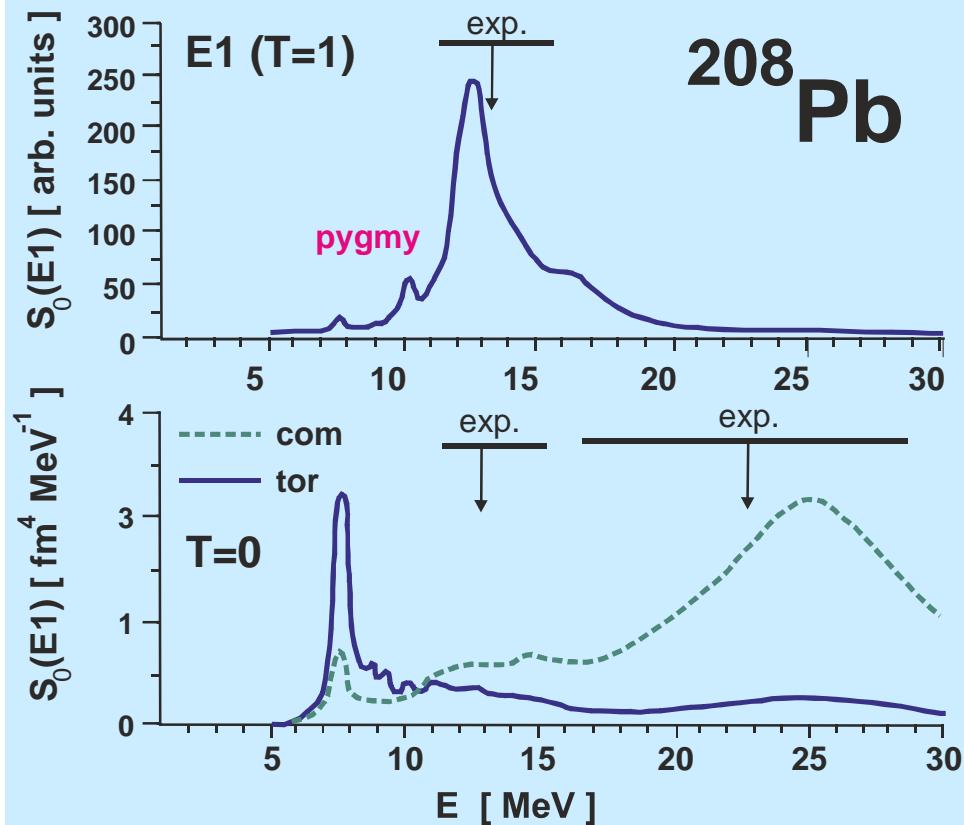


$$\begin{aligned} \delta\rho_\nu(\vec{r}) &= \langle \nu | \hat{\rho}(\vec{r}) | RPA \rangle = \\ &= \left\langle \nu \left| \sum_{i=1}^A e_i^{(eff)} \delta(\vec{r} - \vec{r}_i) \right| RPA \right\rangle \end{aligned}$$

typical interpretation  
of pygmy resonance:  
oscillation of neutron  
skin with respect to  
 $N=Z$  core

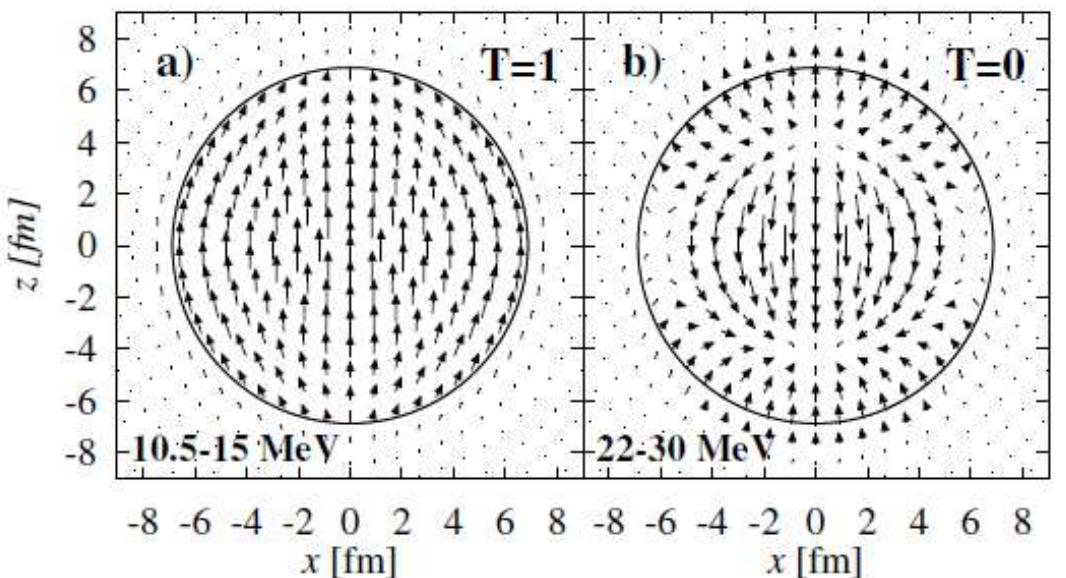


Two peaks at 7.5 and 10.3 MeV are in agreement  
to RMF calculations:  
(D. Vretenar, N. Paar, P. Ring, PRC, 63, 047301 (2001))



GDR

compression

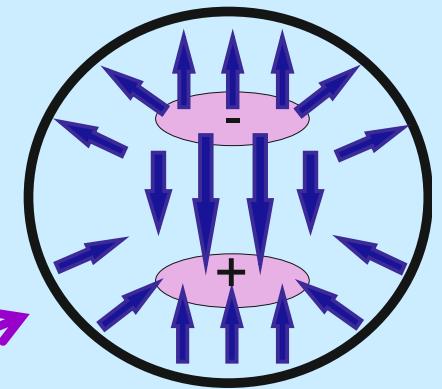


transition current fields for isovector GDR ( $T=1$ ) and isoscalar ( $T=0$ ) compression modes

$$\hat{j}(\vec{r}) = -i \sum_{q=n,p} e_q^{(eff)} \sum_{i \in q} \left( \delta(\vec{r} - \vec{r}_i) \vec{\nabla}_i - \vec{\nabla} \delta(\vec{r} - \vec{r}_i) \right)$$

$$\delta j_\nu(\vec{r}) = \langle \nu | \hat{j}(\vec{r}) | RPA \rangle$$

transition density of the convection current for the RPA state  $|\nu\rangle$



E1 compression

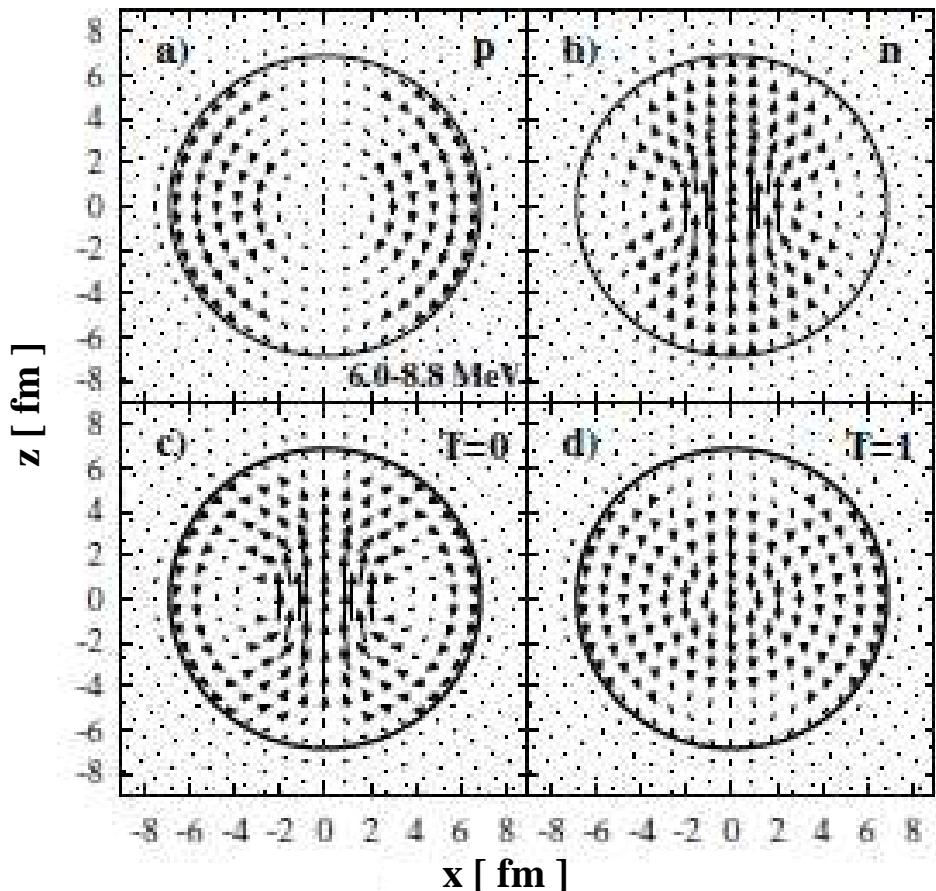
|        |   |
|--------|---|
| $T=0:$ | $e_{eff}^n = e_{eff}^p = 1$                         |
| $T=1:$ | $e_{eff}^p = \frac{N}{A}, e_{eff}^n = -\frac{Z}{A}$ |
| $p:$   | $e_{eff}^p = 1, e_{eff}^n = 0$                      |
| $n$    | $e_{eff}^p = 0, e_{eff}^n = 1$                      |

# transition current in HF and RPA level

$$\hat{j}(\vec{r}) = -i \sum_{q=n,p} e_q^{(eff)} \sum_{i \in q} (\delta(\vec{r} - \vec{r}_i) \vec{\nabla}_i - \vec{\nabla} \delta(\vec{r} - \vec{r}_i))$$

$$\delta \vec{j}_v(\vec{r}) = \langle v | \hat{j}(\vec{r}) | RPA \rangle$$

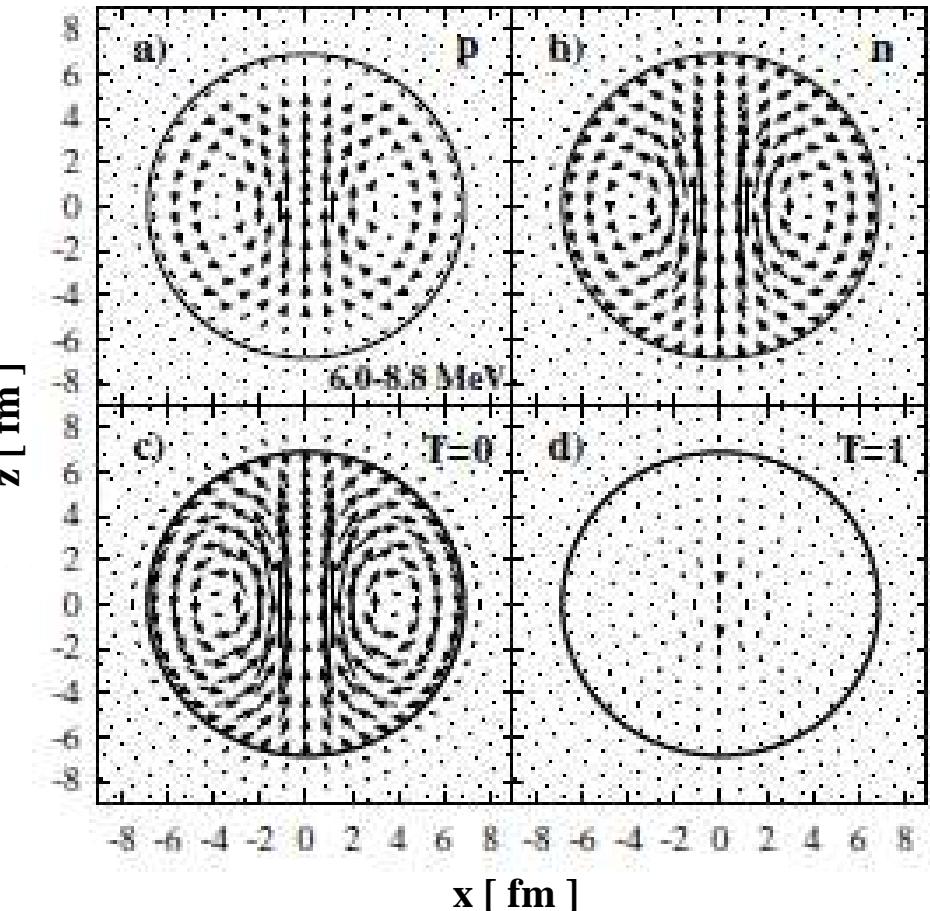
HF



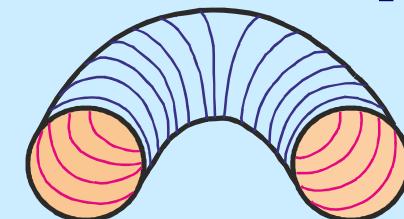
- both isoscalar and isovector
- toroidal flow mainly from neutrons

So the toroidal flow is basically formed already by the mean-field, but residual interaction makes it collective and more impressive.

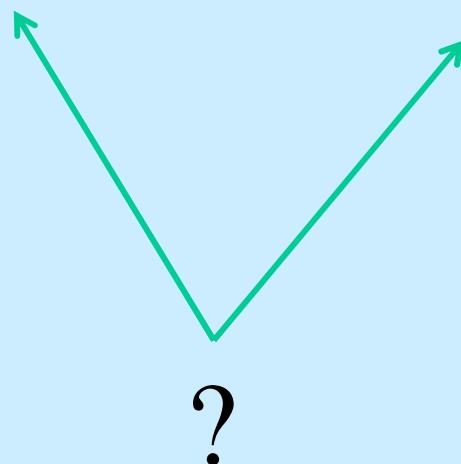
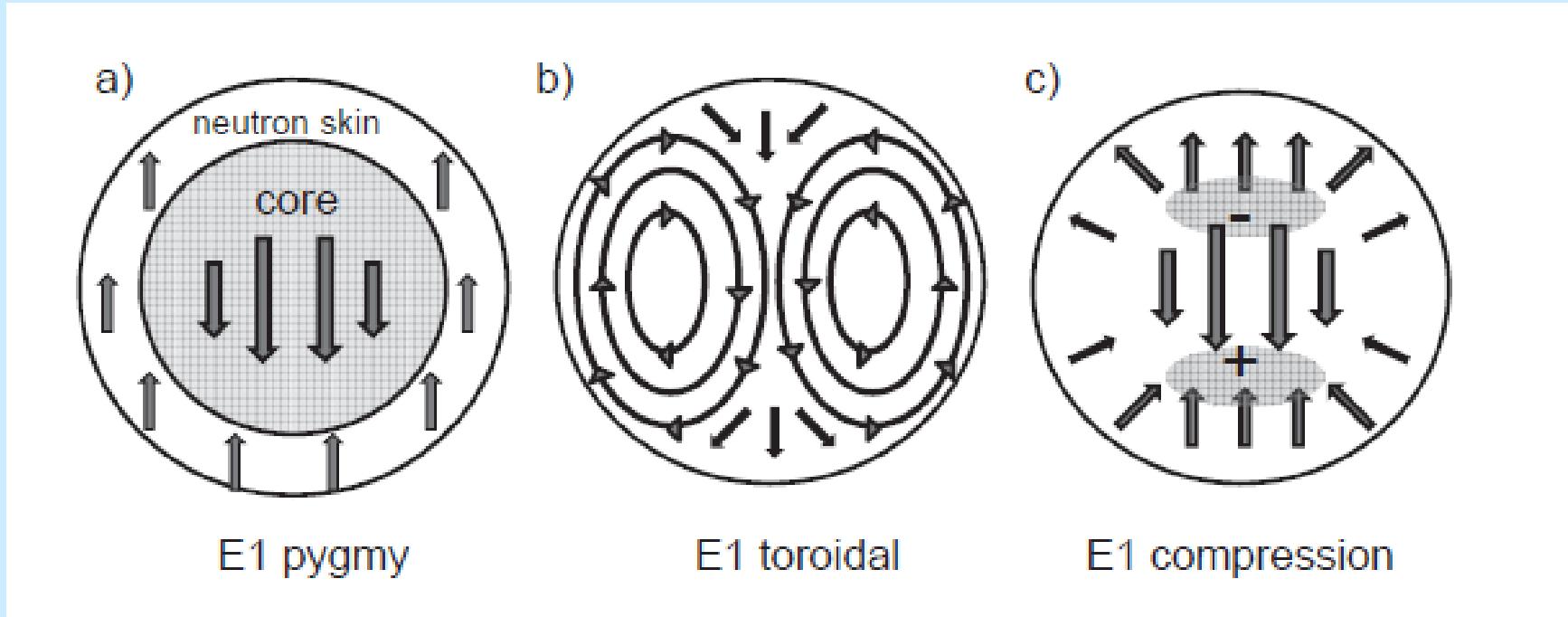
RPA



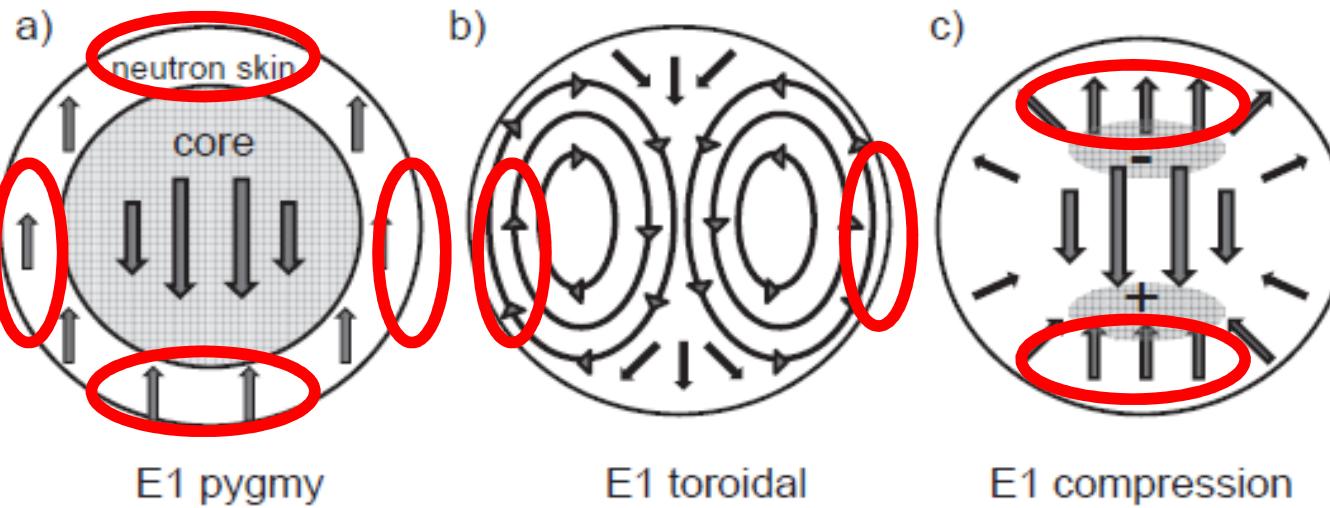
- mainly isoscalar
- toroidal flow from both n/p



# Does the toroidal flow contradicts the familiar PDR picture?

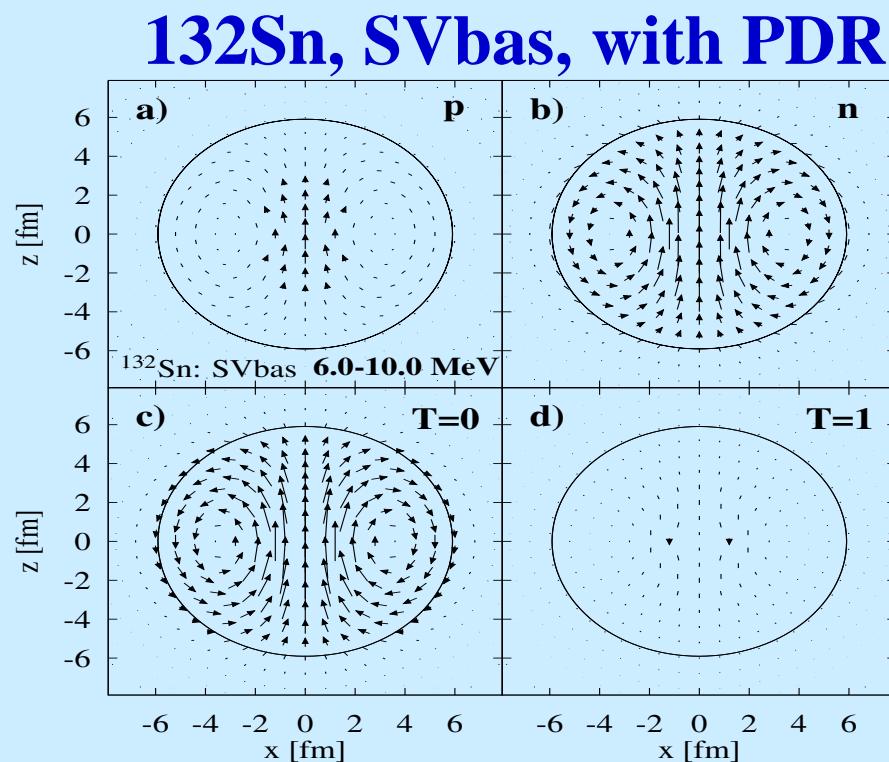
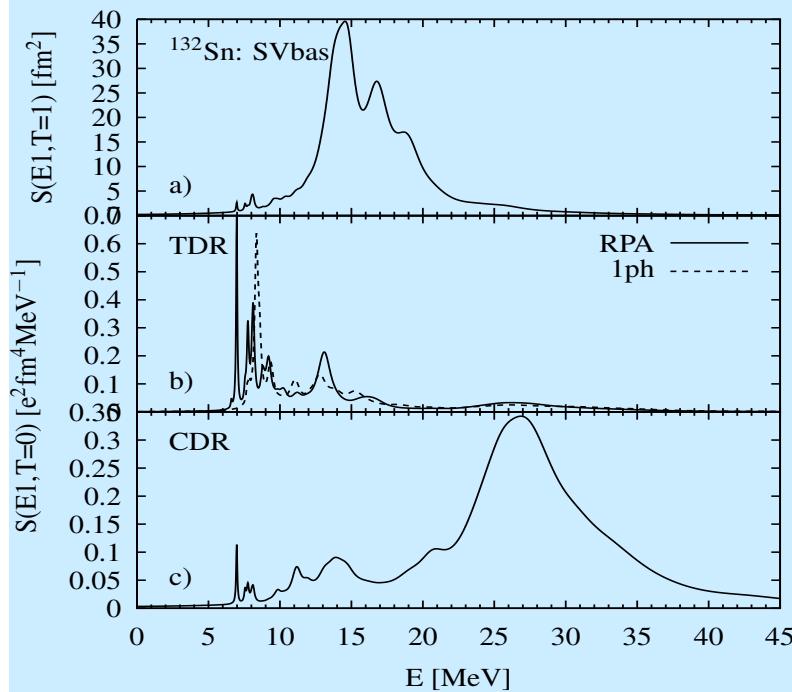


**The answer is no**

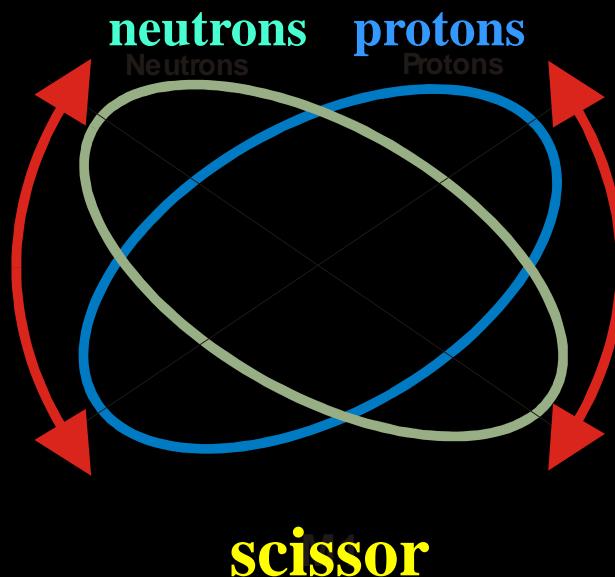


V.O. Nesterenko, A. Repko,  
P.-G. Reinhard, and J. Kvasil,  
"Relation of E1 pygmy and toroidal  
resonances",  
arXiv:1410.5634[nucl-th],

- PDR can be viewed as a local peripheral part of TR and CR
- our calculations demonstrate the TR motion in PDR energy region for other nuclei: Ni, Zr, Sn, ...

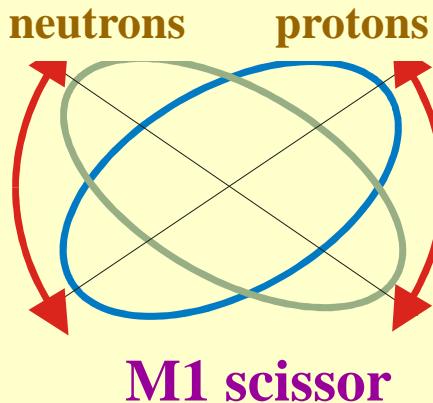


# Giant M1 resonance (scissor, spin-flip resonance)



# Strength function $S(M1, E) = \sum_{\mu=0,\pm 1} S(M1\mu, E)$ in $^{142-152}\text{Nd}$ isotopes

$$g_i^{(s,eff)} = 0.7 g_i^{(free)}$$



orbital (scissor) part contributes only for deformed systems:

$$B_{scissor}(M1, 0^+ \rightarrow K^\pi = 1^+) \approx$$

$$\approx 0.004 E_{sc} A^{5/3} (0.95\beta)^2 (2\frac{Z}{A})^2 \mu_N^2$$

two-rotor model:

N. Lo Iudice, A. Richter, Phys. Lett. B304, 193 (1993)

time-odd densities and currents should be involved in the Skyrme functional

so called high-energy scissor mode - see:

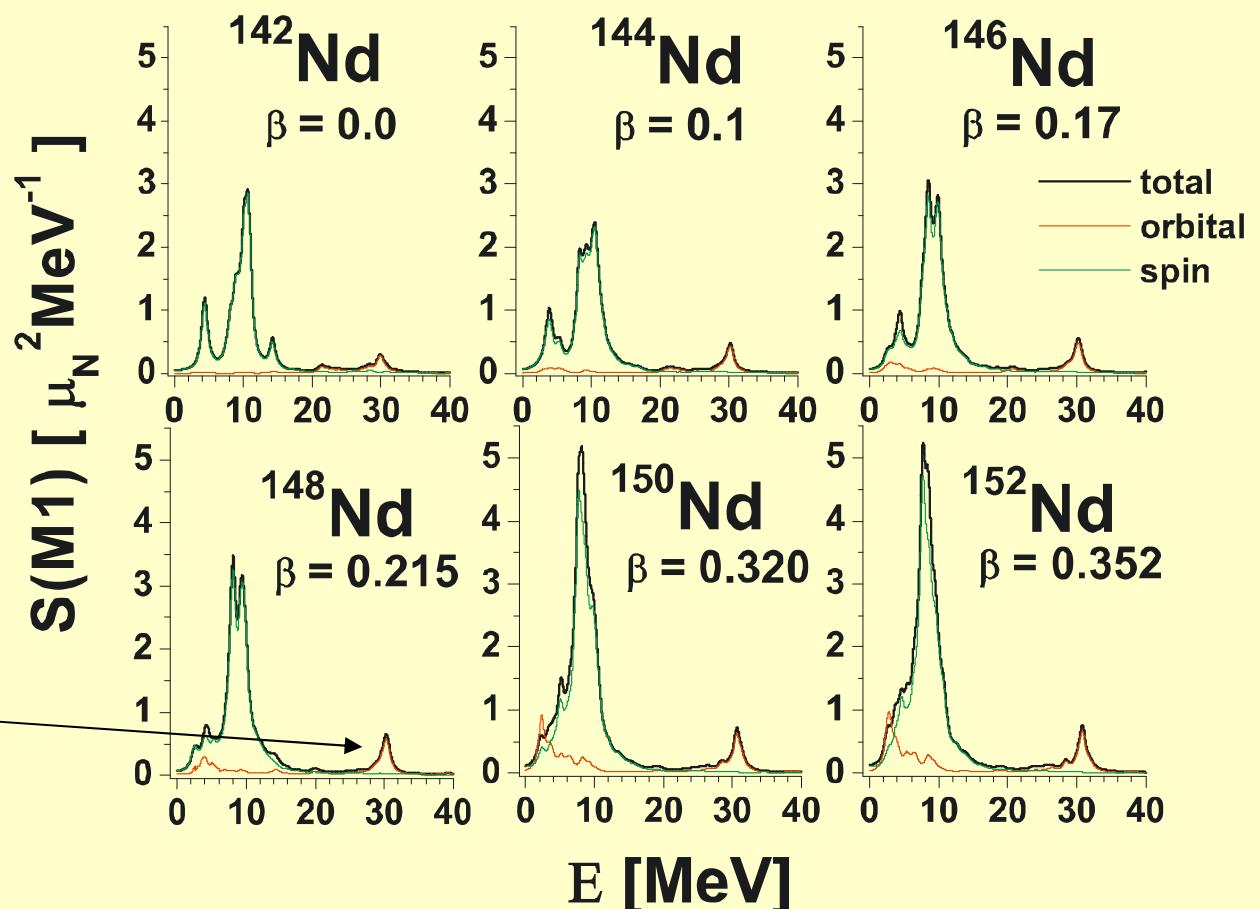
I.Hamamoto, W.Nazarewicz, Phys.Lett. B297, 25 (1992);  
J.Kvasil, N.Lo Iudice, F.Andreozzi, F.Knapp, A.Porrino, Phys.Rev. C73, 034502 (2006)

$$\hat{T}_{1\mu}^{(mag)} = \mu_N \sqrt{\frac{3}{4\pi}} \sum_{i=1}^A \left( g_i^{(l,eff)} l_\mu^{(i)} + g_i^{(s,eff)} s_\mu^{(i)} \right)$$

↑  
scissor      ↑  
spin-flip

$$g_i^{(l,eff)} = \begin{cases} 0 & \text{neutrons} \\ 1 & \text{protons} \end{cases}$$

exciting operators:  
 $\sum s_{1\mu}, \sum r^2 s_{1\mu}, \sum r^2 Y_{2\mu}$



experimental values of energies and  $B(M1)$  values for the first  $1^+$  state which is, according to experimentalists, interpreted as the scissor state

two-rotor model estimation ( N. Lo Iudice, A. Richter, Phys. Lett. B304, 193 (1993) ) :

$$E_{sc} = 66 (0.95\beta) A^{-1/3} \text{ MeV}$$

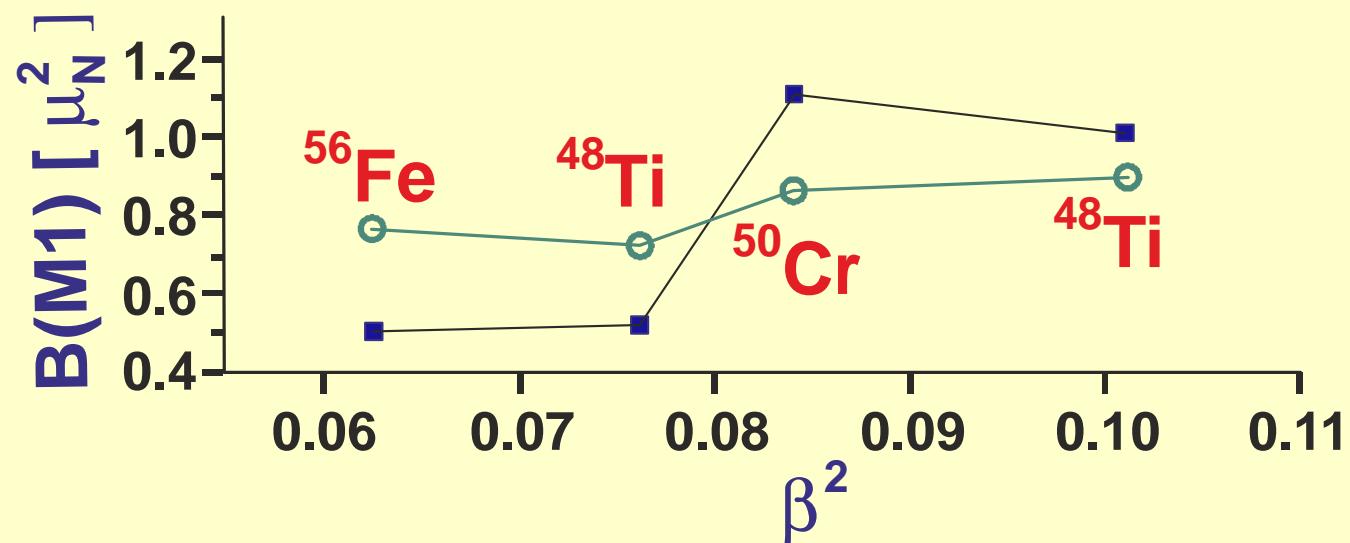
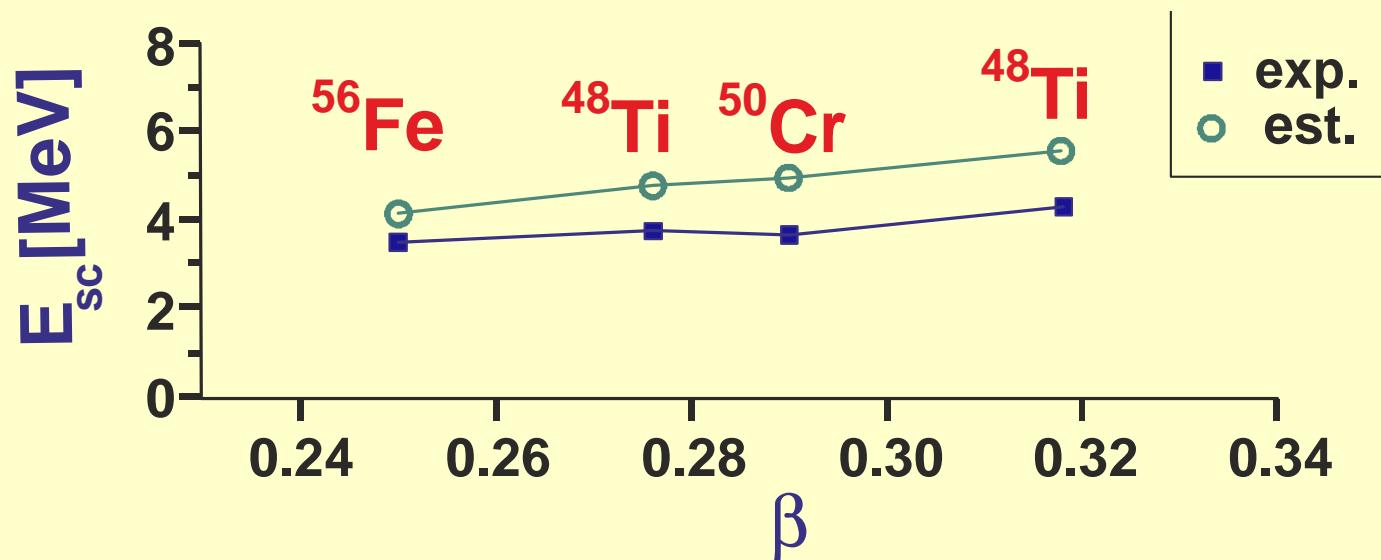
$$B_{scissor}(M1, 0^+ \rightarrow K^\pi = 1^+) \approx 0.004 E_{sc} A^{5/3} (0.95\beta)^2 (2 \frac{Z}{A})^2 \mu_N^2$$

exp. data:

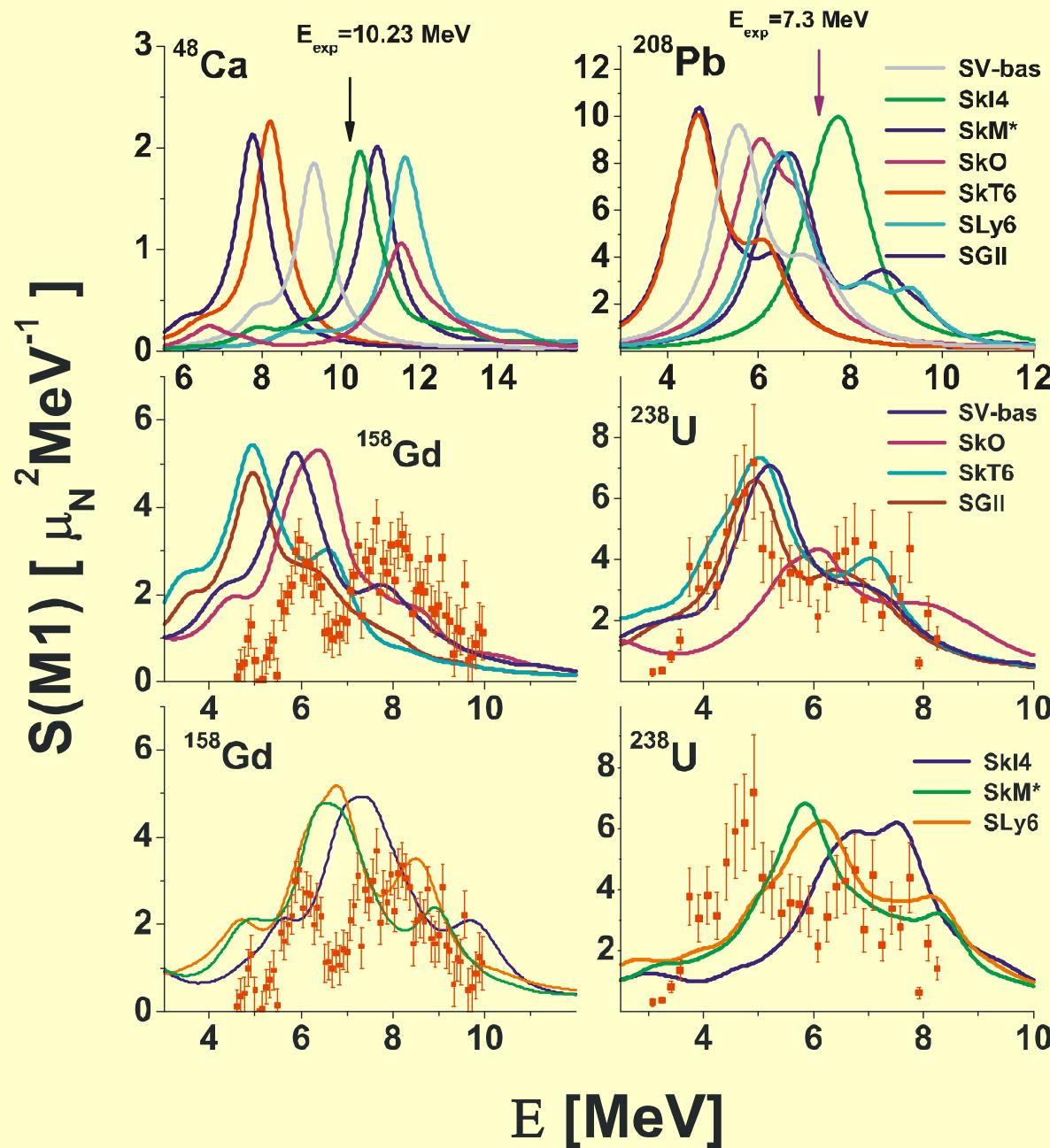
P. Von Brentano, et al., Prog. Part. Nucl. Phys. 46, 197 (2001)

K. Heyde, P. von Neumann Cosel,  
A. Richter, Rev. Mod. Phys. 82,  
2365 (2010)

R.W. Fearick, et al., Nucl. Phys. A 727, 41 (2003)



# Comparison of M1 strength functions calculated with different Skyrme parametrizations with experimental values.



experimental values from:

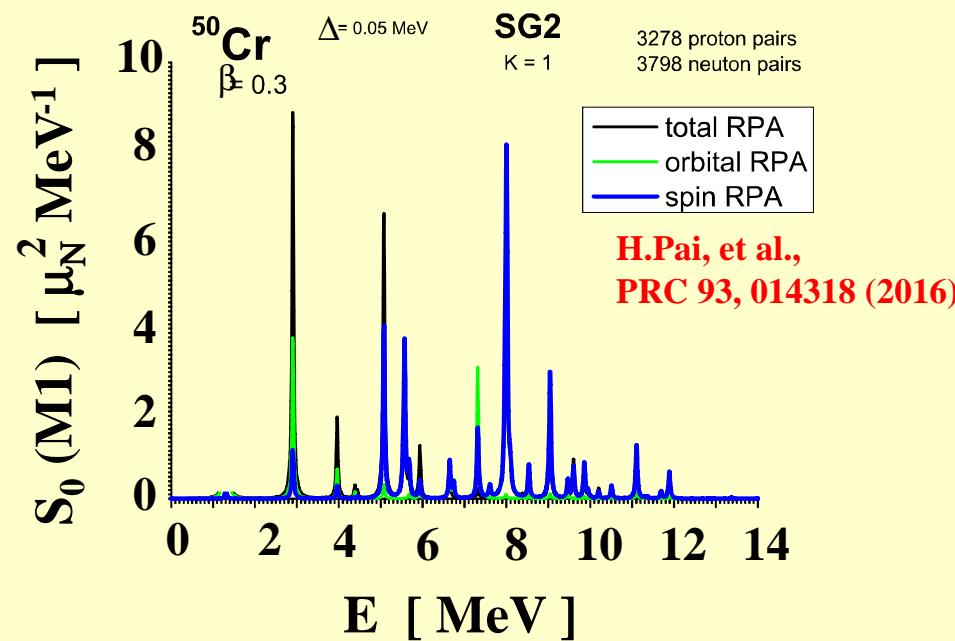
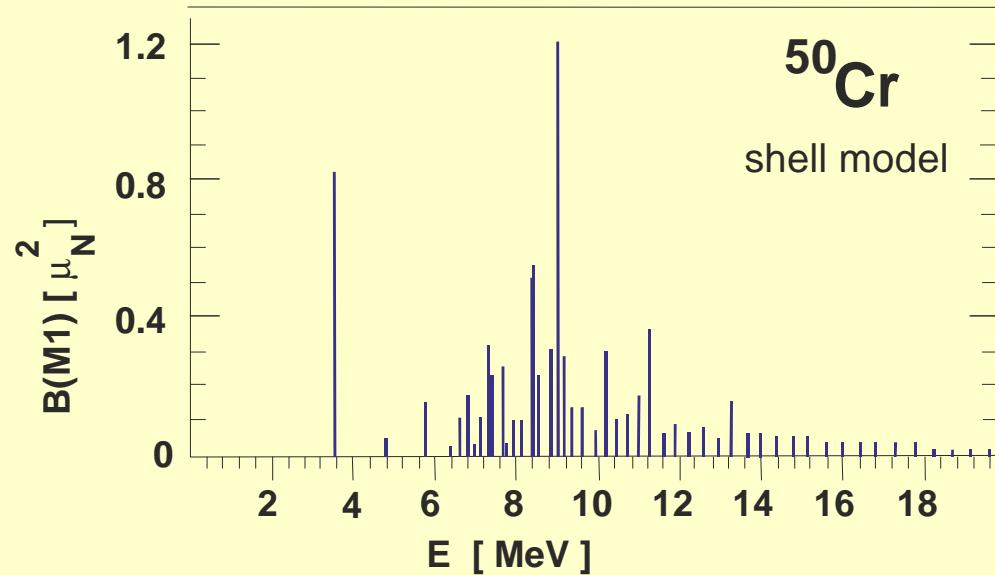
**Ca - S.K.Nanda, et al.,**  
PRC 29, 660 (1984)

**Pb – R.M.Laszewski, et al.,**  
PRL 61, 1710 (1988)

**Gd+U – P.Sarriguren, et al.,**  
PRC 54, 690 (1996)

none of used parametrizations  
describes M1 strength for all  
investigated nuclei

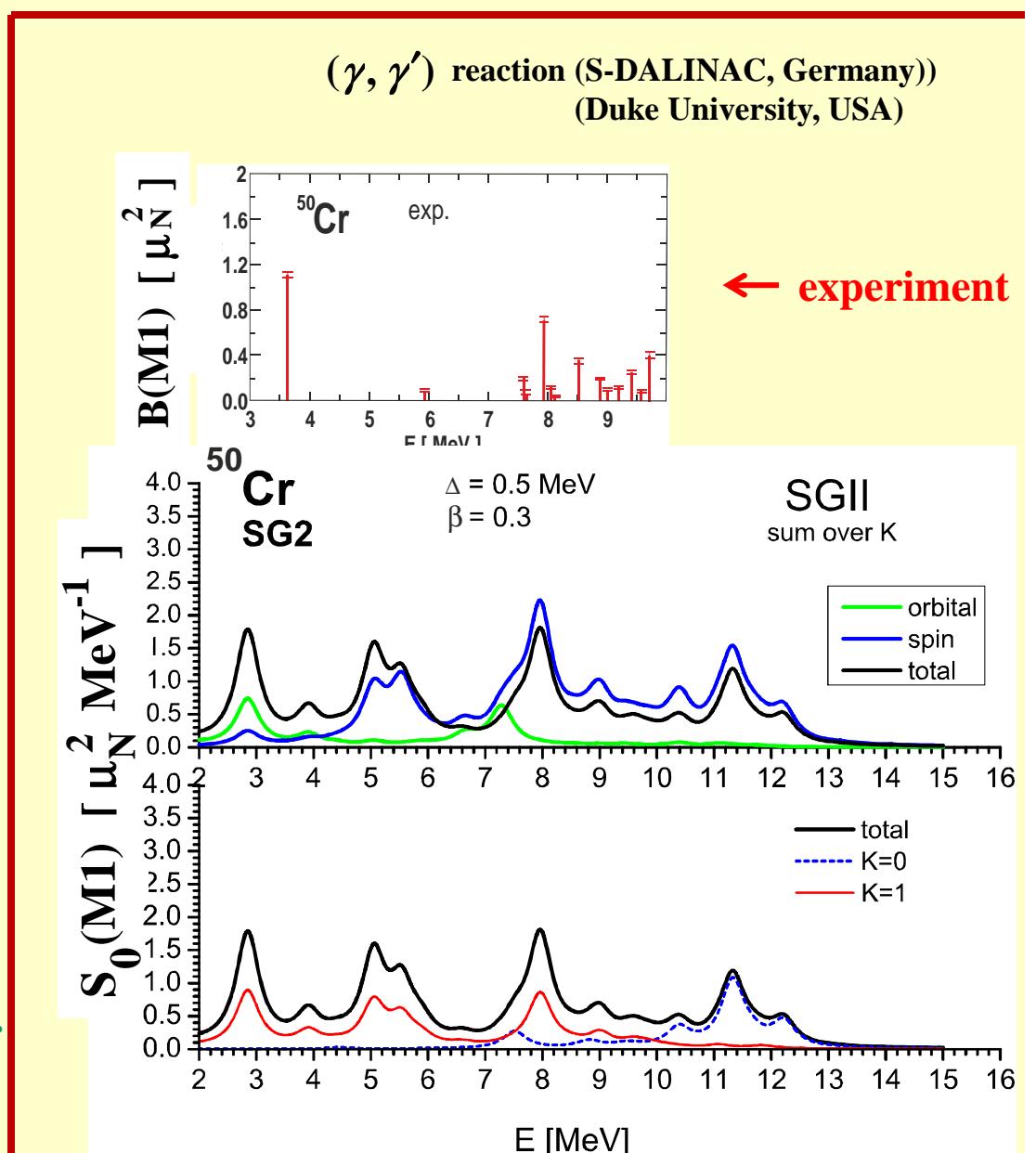
V.O. Nesterenko, J.Kvasil, P.Vesely, W. Kleinig,  
P.-G. Reinhard, Yu. Pononarev, J. Phys. G:  
Nucl. Part. Phys. 37, 064034 (2010)



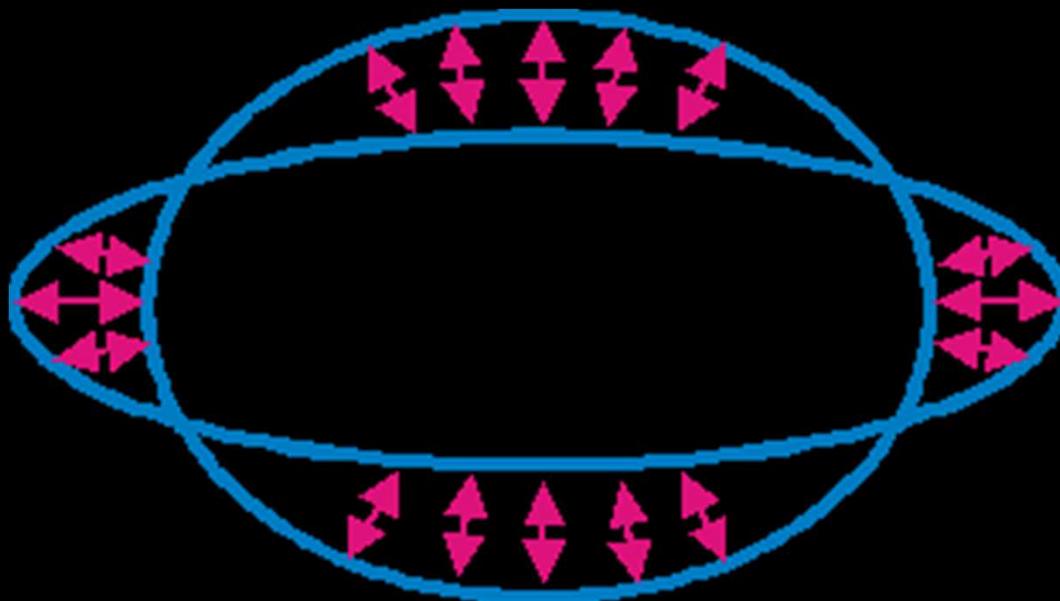
the peak at  $E=2.72$  MeV (Skyrme QRPA) and  $E=3.3$  MeV (shell model) corresponds to the exp. peak at  $E=3.6$  MeV and corresponding  $1^+$  state has dominant orbital part (scissor character)

large-scale shell model calculation  
A.Poves et al., Nucl.Phys. A 694, 157 (2001)

the complex multi-phonon components are not involved in the Skyrme QRPA approach while they are contained in the shell model



# giant quadrupole resonance



E2

# giant quadrupole resonance

## electric quadrupole transition operators

$$\hat{T}_{2\mu}^{(el)}(T=0) = \sum_{i=1}^A \mathbf{r}_i^2 Y_{2\mu}(\hat{\mathbf{r}}_i)$$

$$\hat{T}_{2\mu}^{(el)}(T=1) = \sum_{i=1}^A \mathbf{r}_i^2 Y_{2\mu}(\hat{\mathbf{r}}_i) \mathbf{2}(t_z)_i \quad \mu = 0, \pm 1, \pm 2$$

$$\hat{T}_{2\mu}^{(el)}(T=p) = \sum_{i=1}^Z \mathbf{r}_i^2 Y_{2\mu}(\hat{\mathbf{r}}_i)$$

opposite to  $\lambda = 1$  case GQR is experimentally observed also for isoscalar case

## empirical centroids of ISGQR and IVGQR:

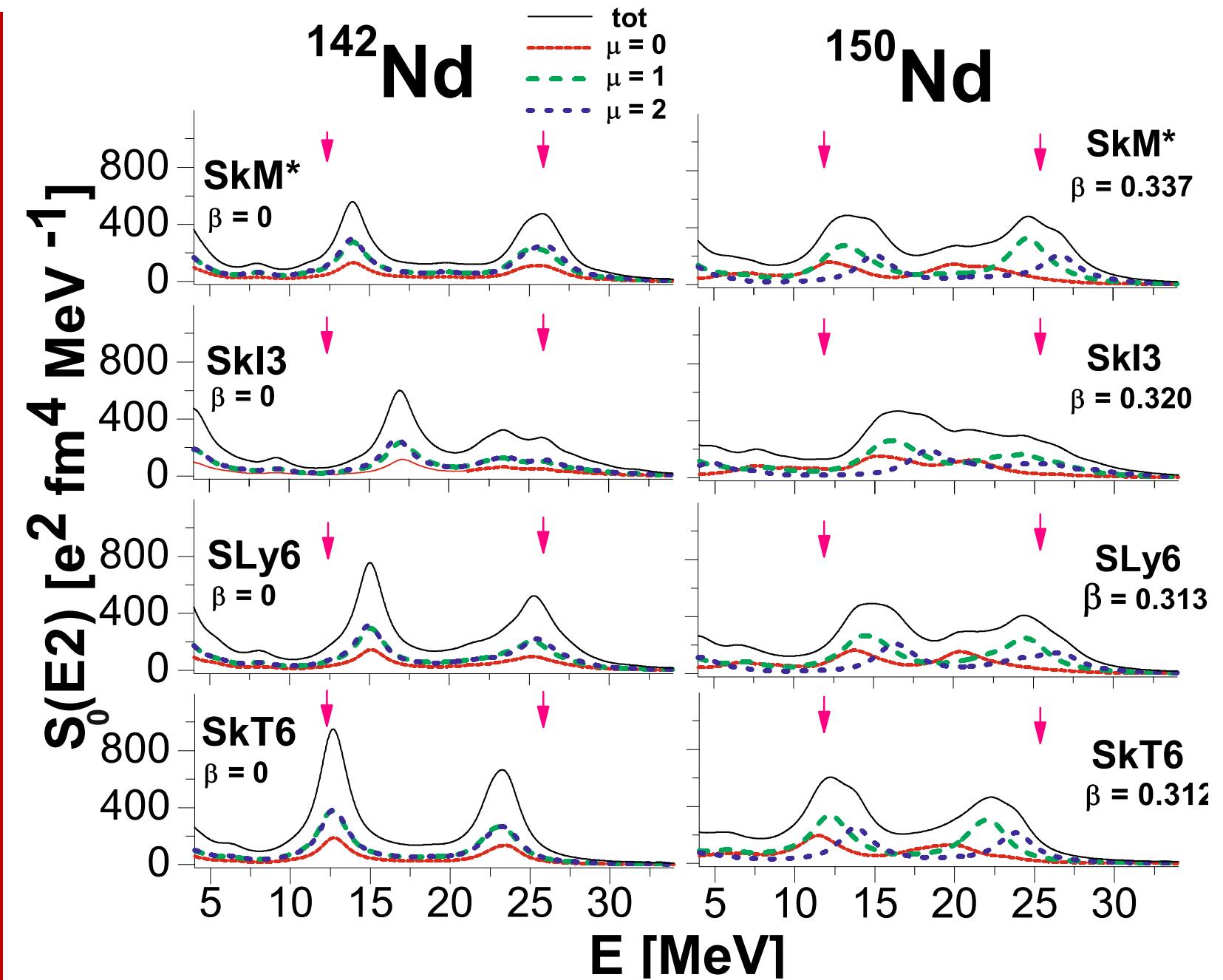
$$E_{ISGQR}^{(\text{exp-emp})} = (61.0 \pm 1.7) A^{-1/3} \text{ MeV}$$

$$E_{IVGQR}^{(\text{exp-emp})} = (59.2 \pm 2.6) A^{-1/3} \text{ MeV}$$

# proton ( $T = p$ ) E2 strength function for spherical $^{142}\text{Nd}$ and deformed $^{150}\text{Nd}$

deformation splitting  
of GQR

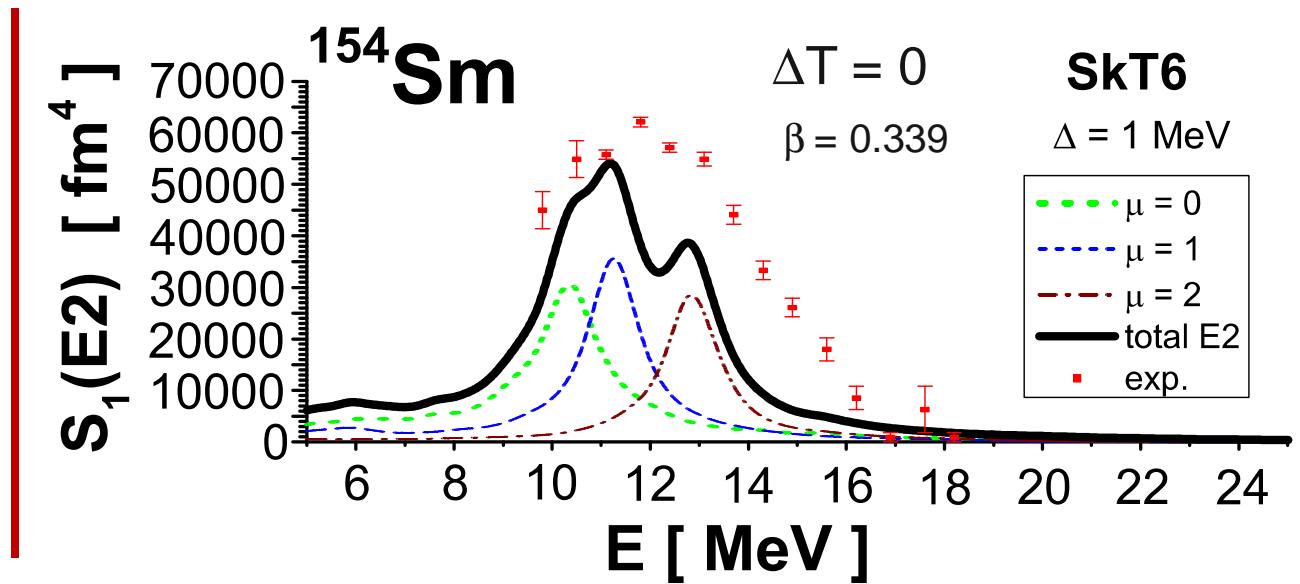
only some Skyrme  
parametrizations  
correspond to  
experiment



# energy weighted isoscalar E2 strength function for deformed $^{154}\text{Sm}$ – comparison with experiment

deformation splitting

missing strength probably caused by the neglecting of the complex multi-phonon configurations

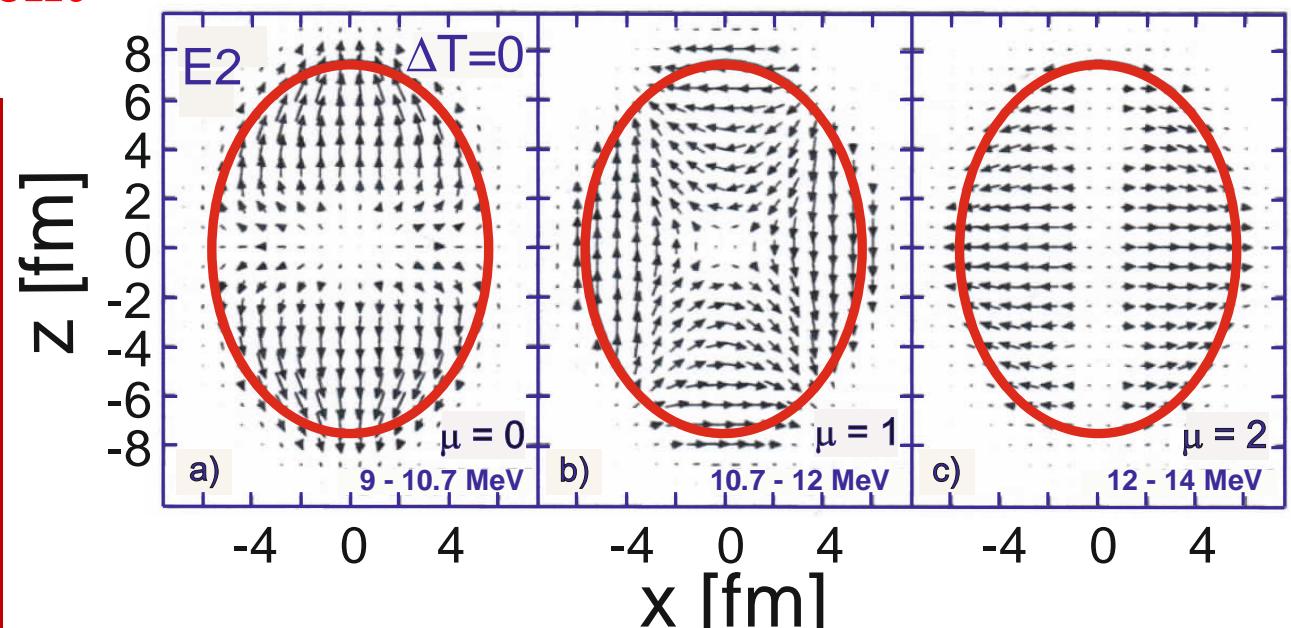


## transition isoscalar current for E2

$$\delta \vec{j}_{T=0}^{(D)}(\vec{r}) = \sum_{\nu \in (\omega_1, \omega_2)} \langle RPA | \hat{D}_\mu | \nu \rangle \langle \nu | \hat{j}_T(\vec{r}) | RPA \rangle$$

with

$$\hat{D}_\mu = \sum_{i=1}^A r^2 Y_{2\mu}(\hat{r})$$



# conclusions and perspectives

# concluding remarks

- standard Skyrme BCS+RPA was used for the demonstration of the quality of the description of Giant resonances; the complex configuration approach was demonstrated by the EMPM method used for GDR in the light  $^{16}\text{O}$
- comparison with corresponding exp. data is qualitatively good; better agreement requires to go beyond the RPA approach – generator coordinate method or shell model approach
- in order to test different theoretical approaches to a nuclear structure the more experimental data on level spectra, giant resonances, transitional probabilities and other properties are necessary
- since the extraction of exp. data about nuclear structure from concrete experiments using different processes (nuclear reactions) is dependent on theoretical mechanism of given process ( extraction of nuclear structure data from experiments is usually model dependent ), it is necessary to improve theoretical description of these processes

# perspectives

## theoretical models of nuclear structure:

- **mean field approaches:** HFB+RPA+beyond RPA, based on the effective nucleon - nucleon interaction or energy functionals (Skyrme interaction, Gogny interaction, relativistic interaction), beyond RPA – generator coordinate method:

**T.R. Rodrigues, J.L. Egido, Phys. Rev. C81, 064323 (2010)**

*Triaxial angular momentum projection and configuration mixing calculations with the Gogny force*

- **large scale shell model**

- **no core shell models:**

ab initio approaches – starting with bare nucleon-nucleon interaction (with nowadays computers only light nuclei can be analyzed):

**B.R. Barrett, P. Navratil, J.P. Vary, Progress in Particle and nuclear physics 69, 131 (2013)**  
*Ab initio no core shell model*

**Dytrych T., Sviratcheva K.D., Bahri K.D., Draayer J.R., Phys. Rev. Lett. 98, 1620503 (2005)**

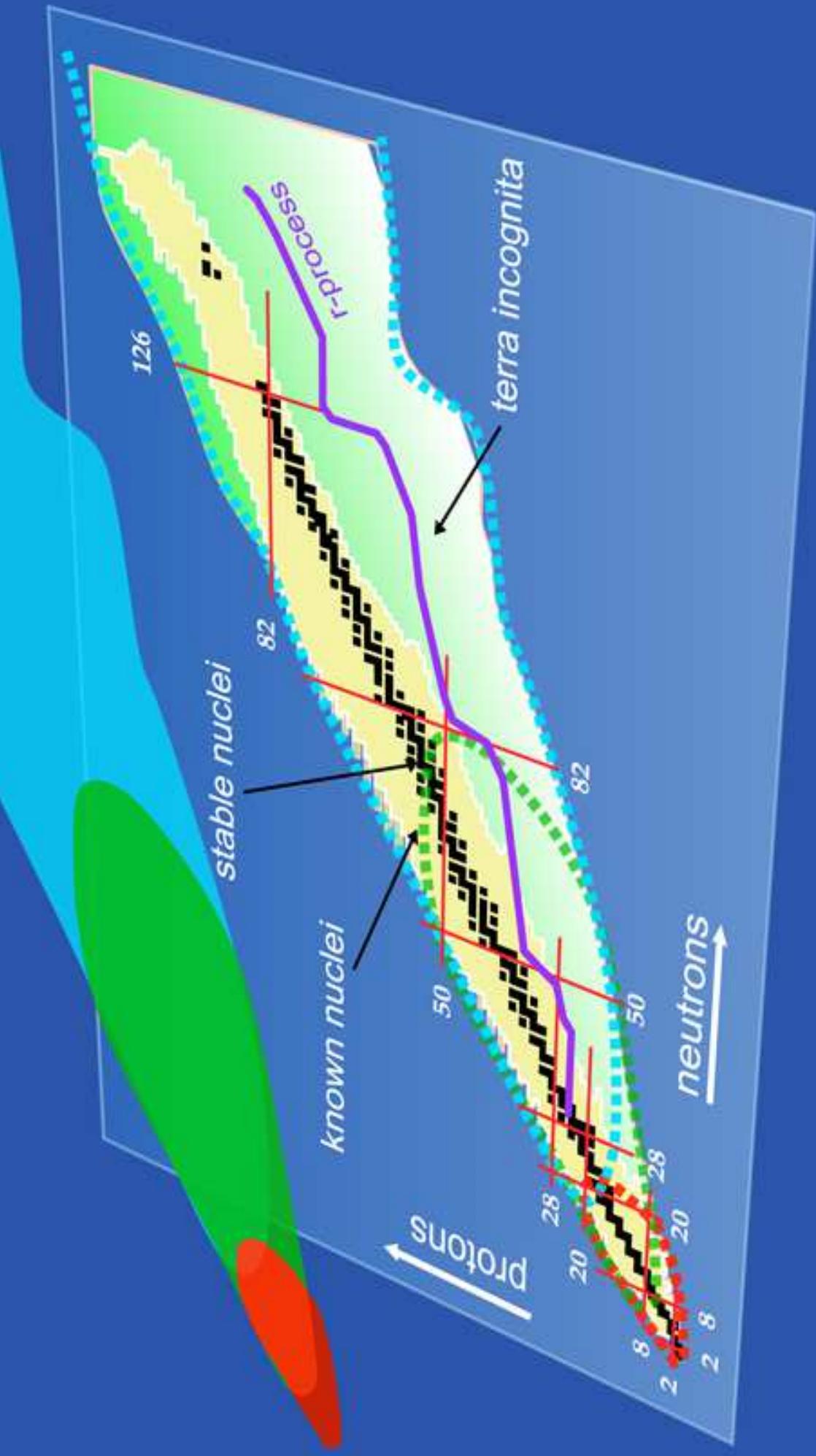
- core shell models : inert even-even core (reduction of the configuration space to valence nucleons)

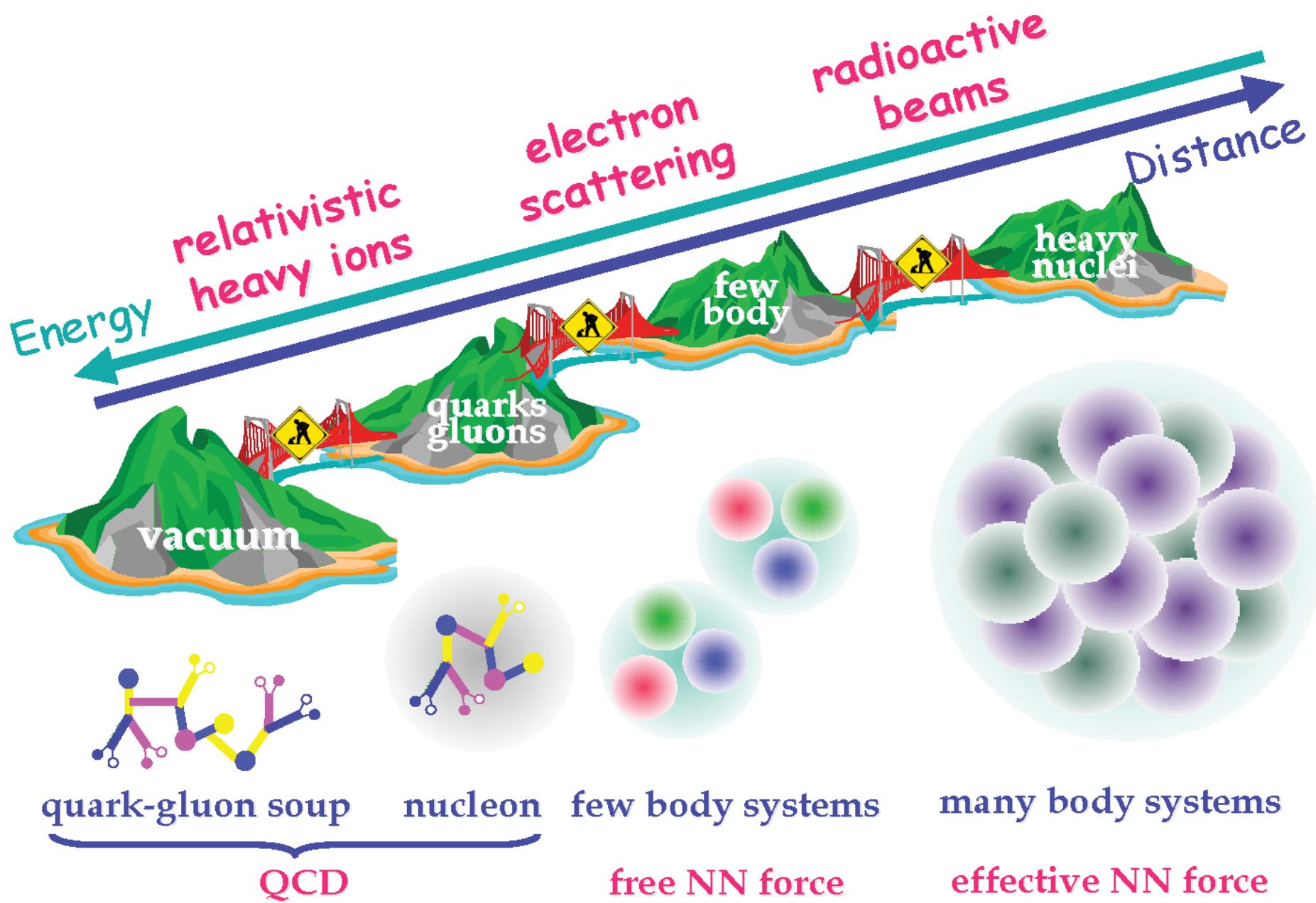
Tsunoda Y., Otsuka T., Shimizu N., Honna M., Utsuno Y.,  
Phys. Rev. C89, 032 (2014)

Caurier A.P., Martines-Pinedo G., Nowacki F., Poves  
A., Zuker A.P., Rev. Mod. Phys. 77, 427 (2005)

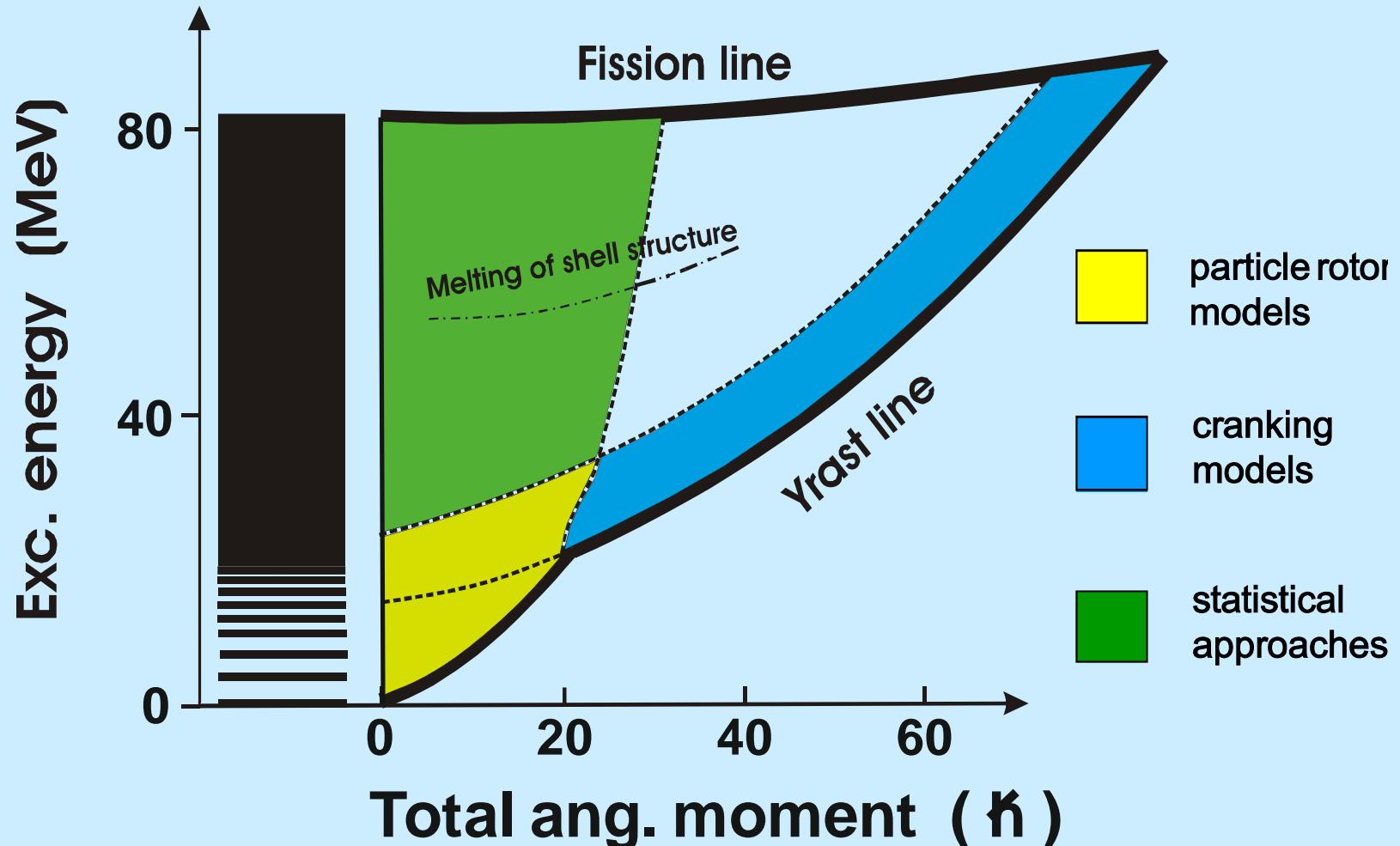
# Nuclear Landscape

- Ab initio
- Configuration Interaction
- Density Functional Theory

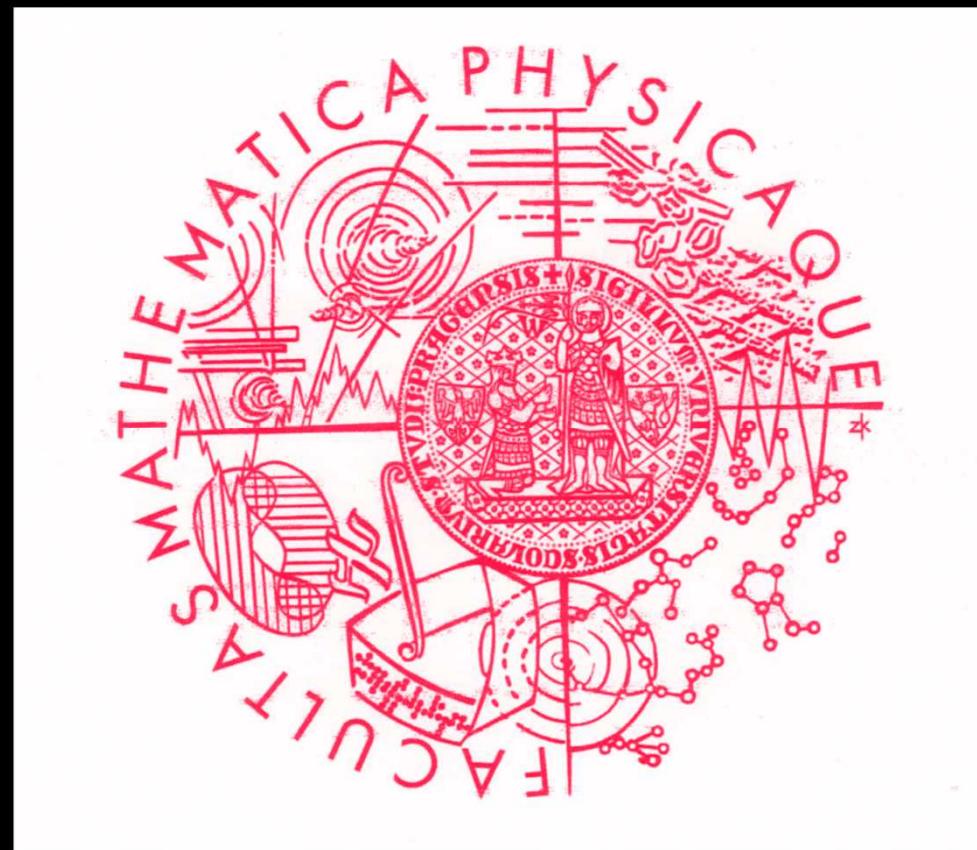




# possible nuclear states in the plane (ang. momentum , energy)



# Thank you for your attention



**beyond RPA**

As an example of the beyond RPA description of giant resonances –  
**Equation of motion multi-phonon method**

**De Gregorio G., Knapp F., Lo Iudice N., Vesely P., PRC 93, 044314 (2016)**  
**Andreozzi F., Knapp F., Lo Iudice N., Porrino A., Kvasil J., PRC 78, 054308 (2008)**

$$\hat{H} = \sum_i \epsilon_i a_i^+ a_i + \frac{1}{4} \sum_{ijkl} V_{ijkl} a_i^+ a_j^+ a_l a_k \quad | \quad V_{ijkl} - \text{chiral } NNLO_{opt}$$

**1-st step:** starting HF basis from the given  $V_{ijkl}$  is created

**2-nd step:** Tamm-Dancoff phonons (TDP),  $Q_\lambda^+ = \sum_{ph} c_{ph}^{(\lambda)} a_p^+ a_h$  are obtained by  
 the diagonalization of the 1p-1h block:

$$\sum_{p'h'} A_{ph p'h'} c_{p'h'}^{(\lambda)} = E_\lambda^{(TDA)} c_{ph}^{(\lambda)}$$

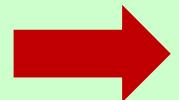
$$A_{ph p'h'} = \delta_{hh'} \delta_{pp'} (\epsilon_p - \epsilon_h) + V_{p'h' \bar{h}p}$$

**3-d step:** n – TDP basis is obtained by the action of TDA phonons

$$\text{on all } n-1 \text{ phonon states: } |n; \beta\rangle = \sum_{\lambda\alpha} C_{\lambda\alpha}^{(\beta)} Q_\lambda^+ |n-1; \alpha\rangle$$

amplitudes  $C_{\lambda\alpha}^{(\beta)}$  are obtained by the diagonalization of the block spanned by basis states  $Q_\lambda^+ |n-1; \alpha\rangle$

**4-th step:** construction and diagonalization of the total nucleus Hamiltonian matrix

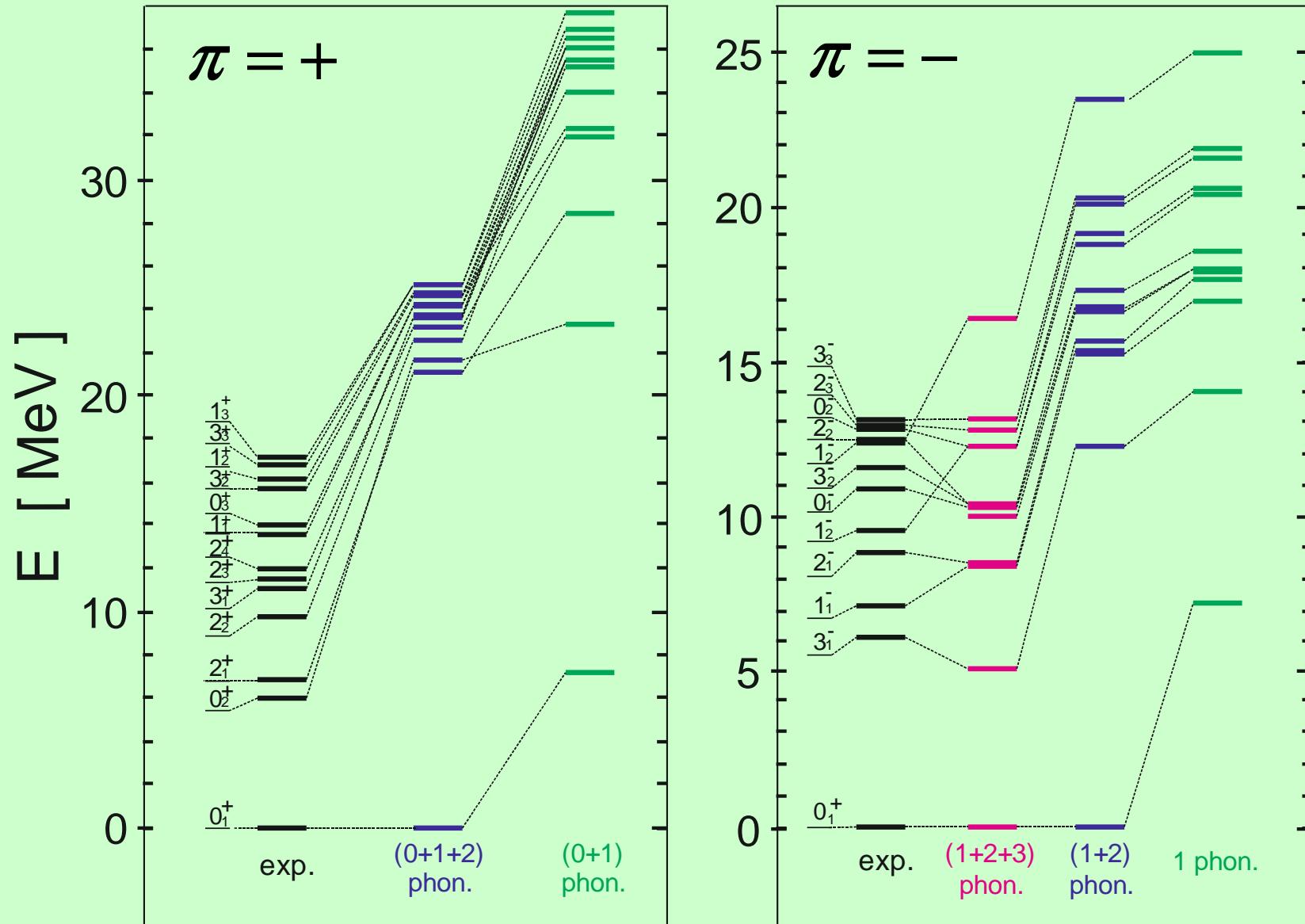


final spectrum:

$$|\Psi_\nu\rangle = \sum_{n=0}^3 \sum_{\alpha} d_{n\alpha}^{(\nu)} |n; \alpha\rangle \leftrightarrow E_\nu$$

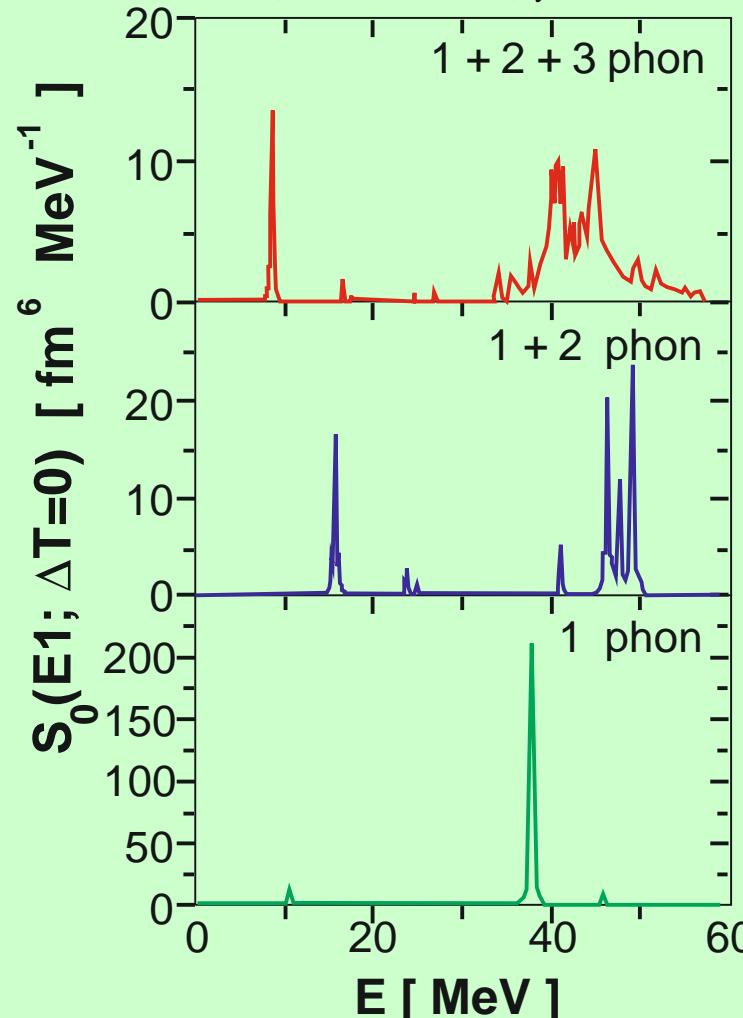
|                 | $ 0\rangle$                     | $ n=1\rangle$                     | $ n=2\rangle$                   | $ n=3\rangle$                 |
|-----------------|---------------------------------|-----------------------------------|---------------------------------|-------------------------------|
| $\langle 0 $    | $E^{(0)}$                       | $\langle 0   H   n=1 \rangle = 0$ | $\langle 0   H   n=2 \rangle$   | $\langle 0   H   n=3 \rangle$ |
| $\langle n=1  $ | $E_1^{(1)}$                     | $E_2^{(1)}$                       | $\dots$                         | $0$                           |
|                 | $0$                             | $E_\beta^{(1)}$                   | $\dots$                         | $\dots$                       |
| $\langle n=2  $ | $\langle n=2   H   n=1 \rangle$ | $E_1^{(2)}$                       | $E_2^{(2)}$                     | $\dots$                       |
|                 |                                 | $0$                               | $\dots$                         | $0$                           |
|                 |                                 | $E_\beta^{(2)}$                   | $\dots$                         | $\dots$                       |
| $\langle n=3  $ | $\langle n=3   H   0 \rangle$   | $\langle n=3   H   n=1 \rangle$   | $\langle n=3   H   n=2 \rangle$ | $E_1^{(3)}$                   |
|                 |                                 |                                   |                                 | $E_2^{(3)}$                   |
|                 |                                 |                                   |                                 | $0$                           |
|                 |                                 |                                   |                                 | $E_\beta^{(3)}$               |
|                 |                                 |                                   |                                 | $\dots$                       |

Experimental and calculated (within the EMPM) spectrum of low-lying states in  $^{16}\text{O}$ . The EMPM calculations with the 1- phonon, (1+2)- phonon, and (1+2+3)- phonon subspaces in the configuration space are mutually compared. In the case of the positive parity states the mean field ground state is added to the configuration space.



**Isoscalar (left panel) and isovector (right panel) E1 strength function in  $^{16}\text{O}$  calculated in the framework of the EMPM with 1-, (1+2)-, and (1+2+3)- TDA phonon subspaces in the configuration space. Width of the averaging Lorentzian  $\Delta = 0.5 \text{ MeV}$**

$$T_{1\mu}^{(com)} = \sum_{i=1}^A \left( r^3 - \frac{5}{3} \langle r^2 \rangle r \right)_i Y_{1\mu}(\hat{r})$$



$$\hat{T}_{1\mu}^{(el)} = \frac{N}{A} e \sum_{i=1}^Z r_i Y_{1\mu}(\hat{r}_i) - \frac{Z}{A} e \sum_{i=1}^N r_i Y_{1\mu}(\hat{r}_i)$$

