

# TESTING THEORIES

or

## The terrible dangers of $\chi^2$ tests

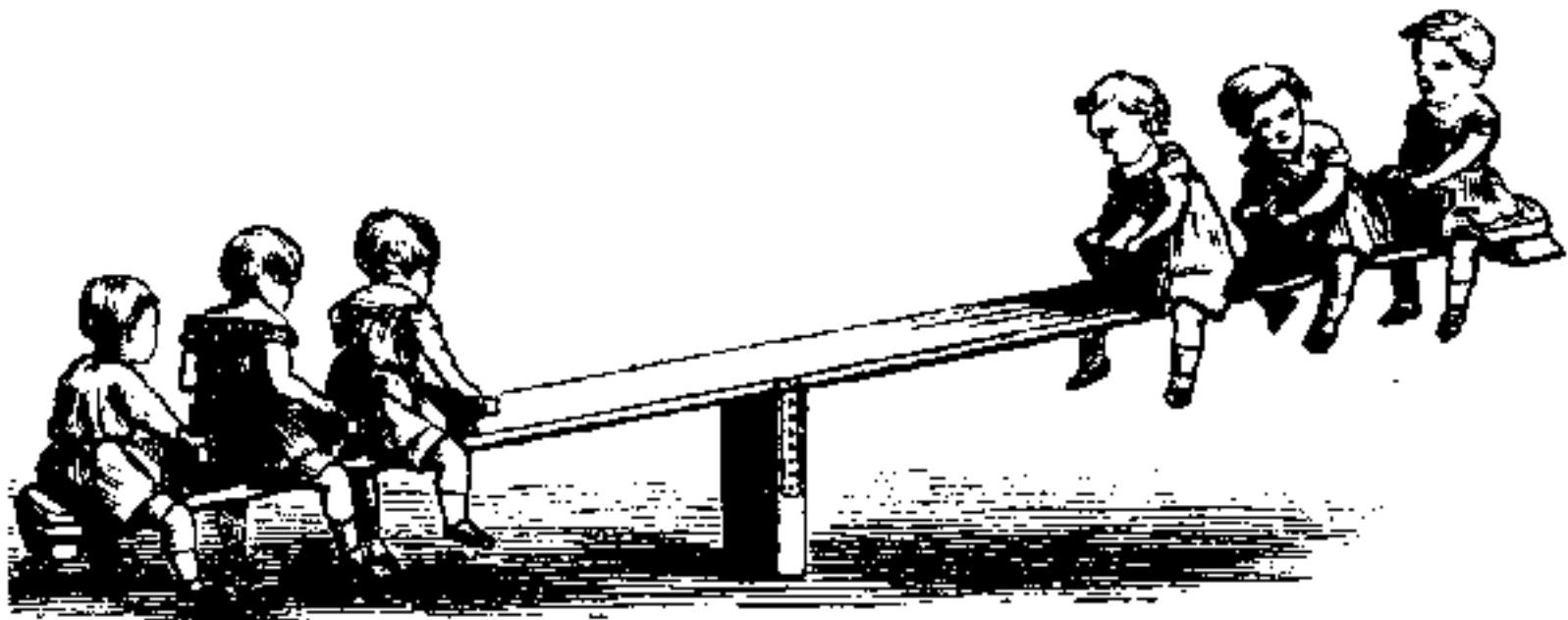
R. F. Casten  
Yale, MSU-FRIB

SSNET, Nov. 2016

“ $\chi^2$ ” here is a placeholder name for many kinds of blind statistical tests of goodness of fit

# **Scary thought:**

**At this time, the voting is starting in  
the US !**



# TESTING THEORIES

Which theory better?

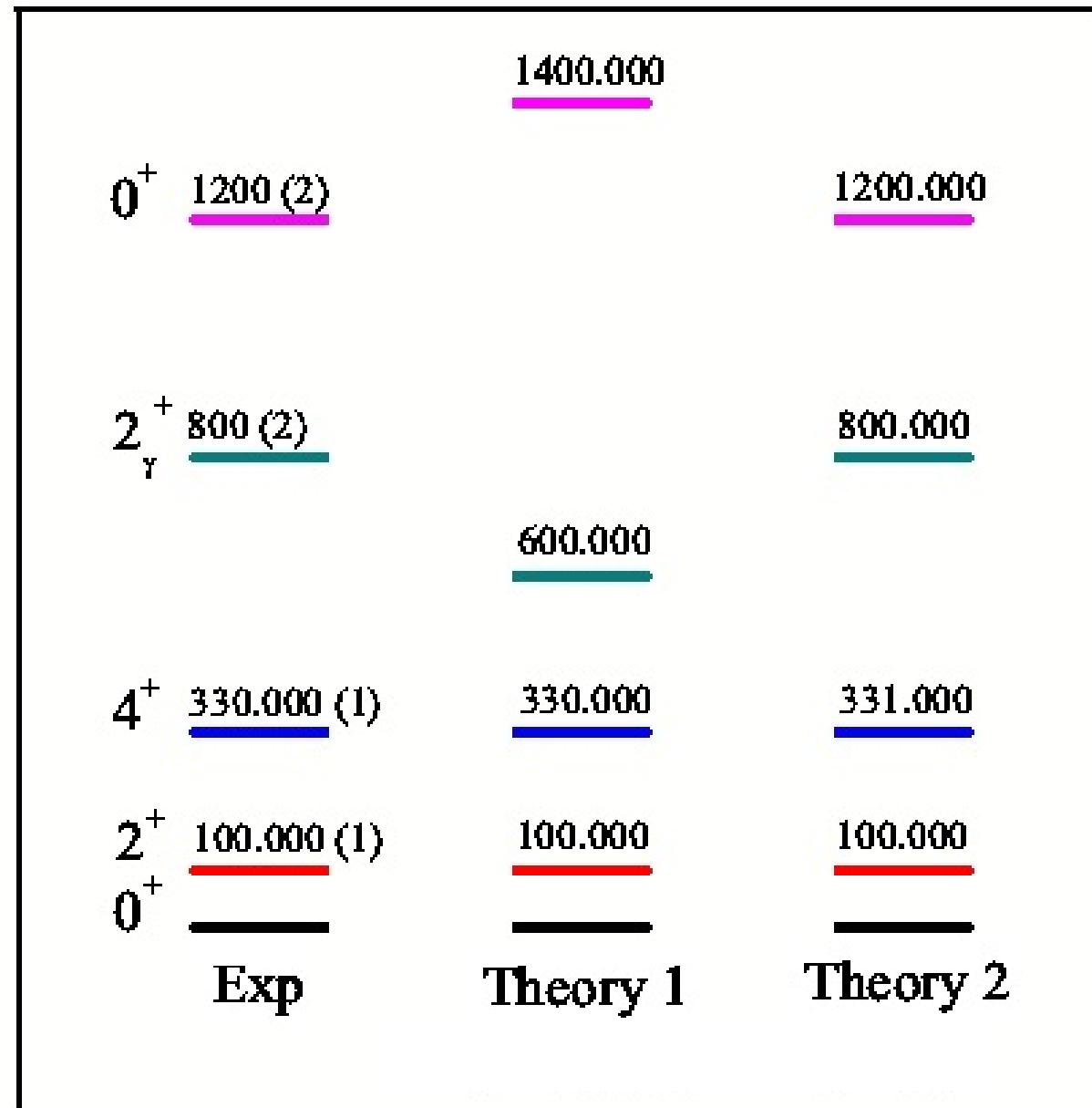
Now look at  $\chi^2$

$$\chi^2 = \sum_{i=1}^k \left( \frac{O_i(\text{exp}) - O_i(\text{theor})}{\sigma_i} \right)^2$$

Super-precise  
data can mislead

Need Theoretical  
Uncertainties

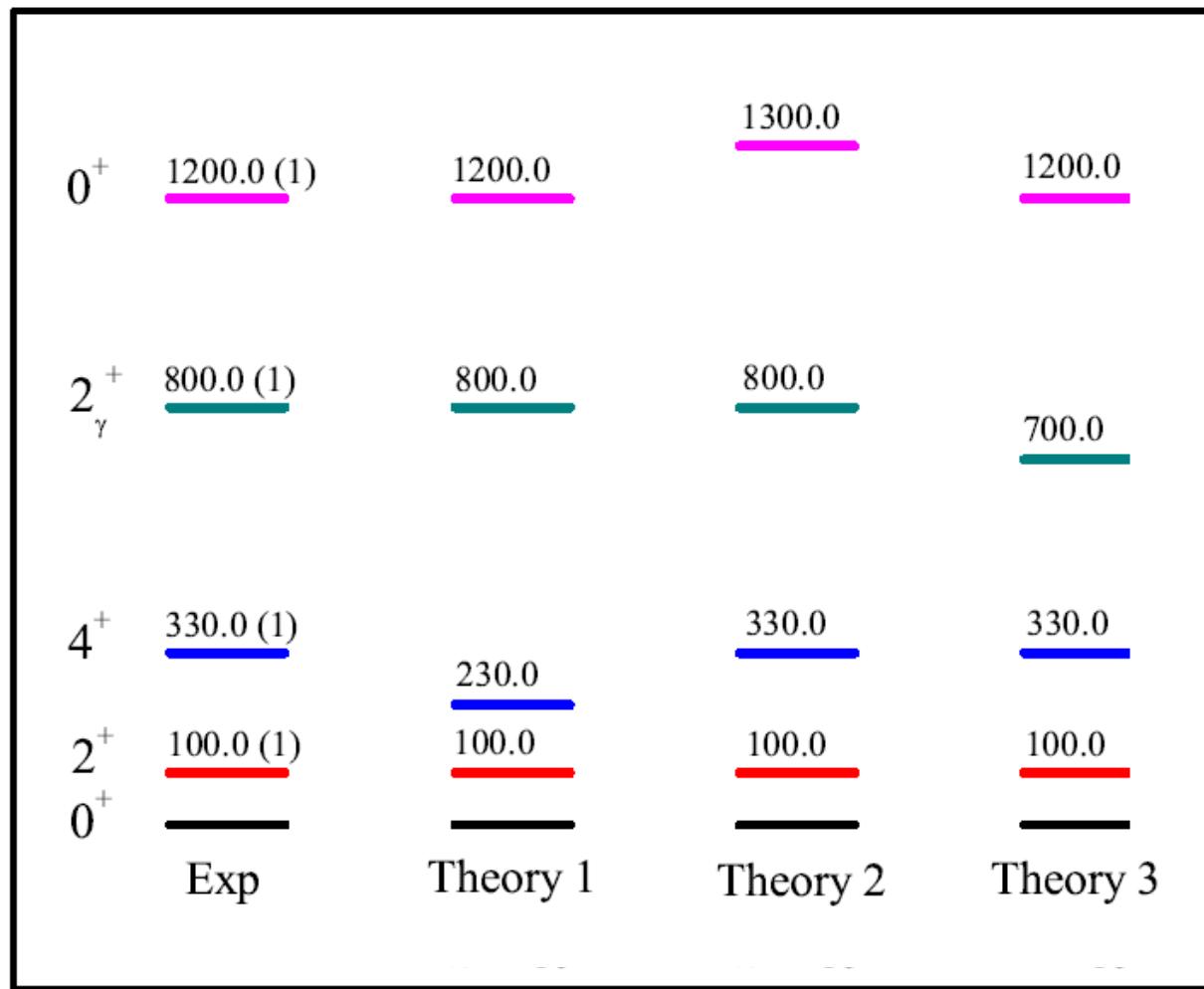
Assume Theor Uncert = 10 keV



800      0.01

# Evaluating theories with equal discrepancies

Theoretical  
uncertainties  
are  
observable-  
dependent



Theoretical uncertainties: Yrast: Few keV ; Vibrations ~ 100 keV

$$\chi^2 \sim 1000 \quad 1$$

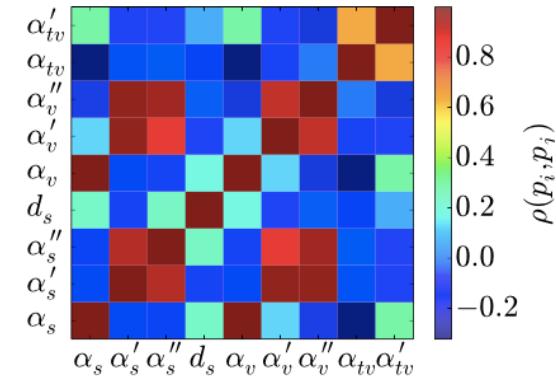
# How to estimate theoretical uncertainties

**Micro models (interacting SM, RPA, DFT, Ab initio,...):**

**Model ----- Space ----- Interaction ----- computation**

Fine for comprehensive studies, but a challenge for single calculations (many!!)

J. Phys. G: Nucl. Part. Phys. **42**, 030301 (2015).



**Phenomenological models:**

$$H = aOp_1 + bOp_2 + cOp_3 \dots$$

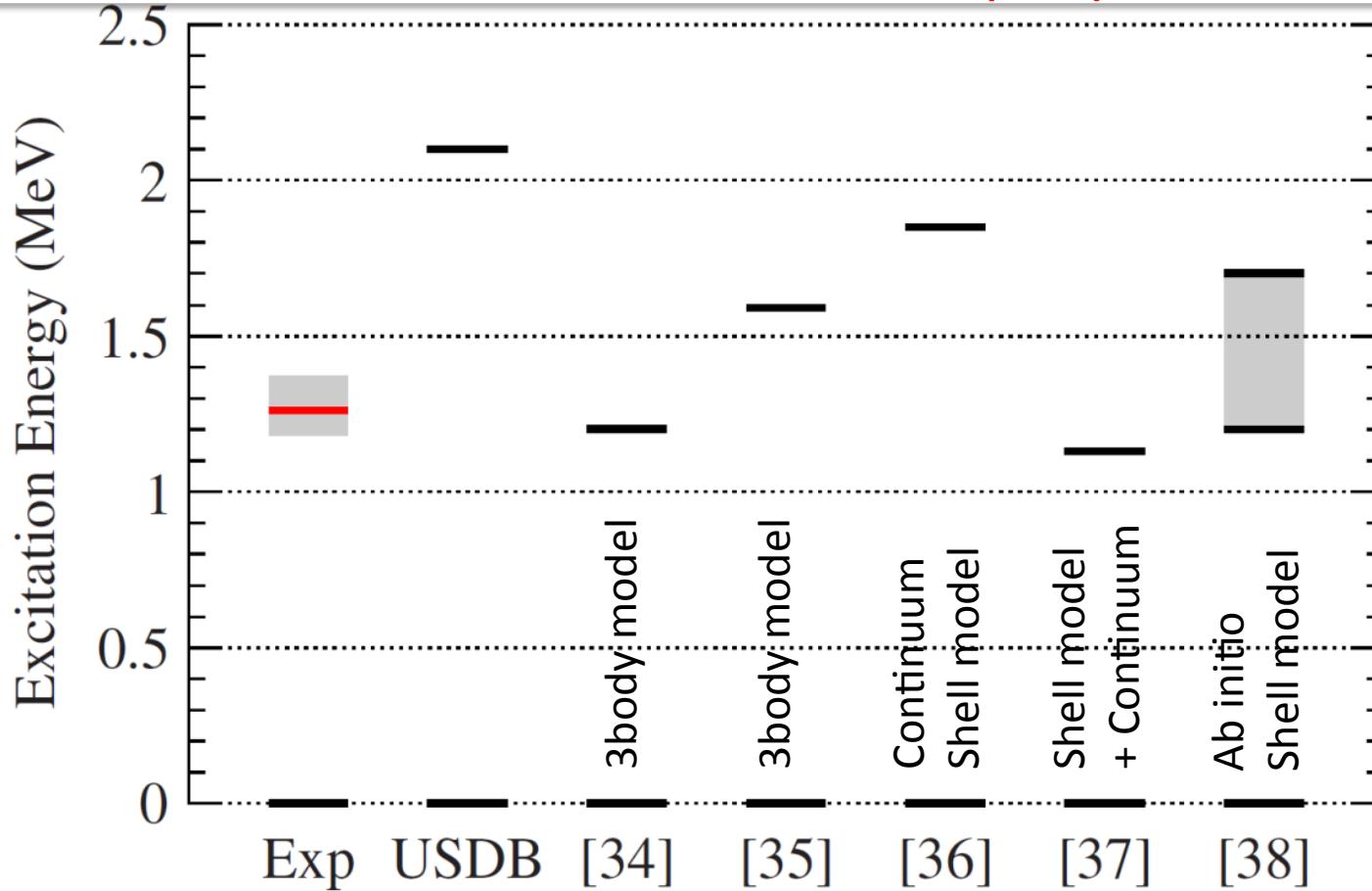
Fit to data. Ambiguities in fitting: choices of observables

**Analytic models (Alaga rules, rotational energies, AHV, ...):**

Difficult: At least go by range of historical agreement

# $^{26}\text{O}$ measurement of first $2^+$ state energy – Comparison with theories

Kondo et al, PRL 116, 102503 (2016)



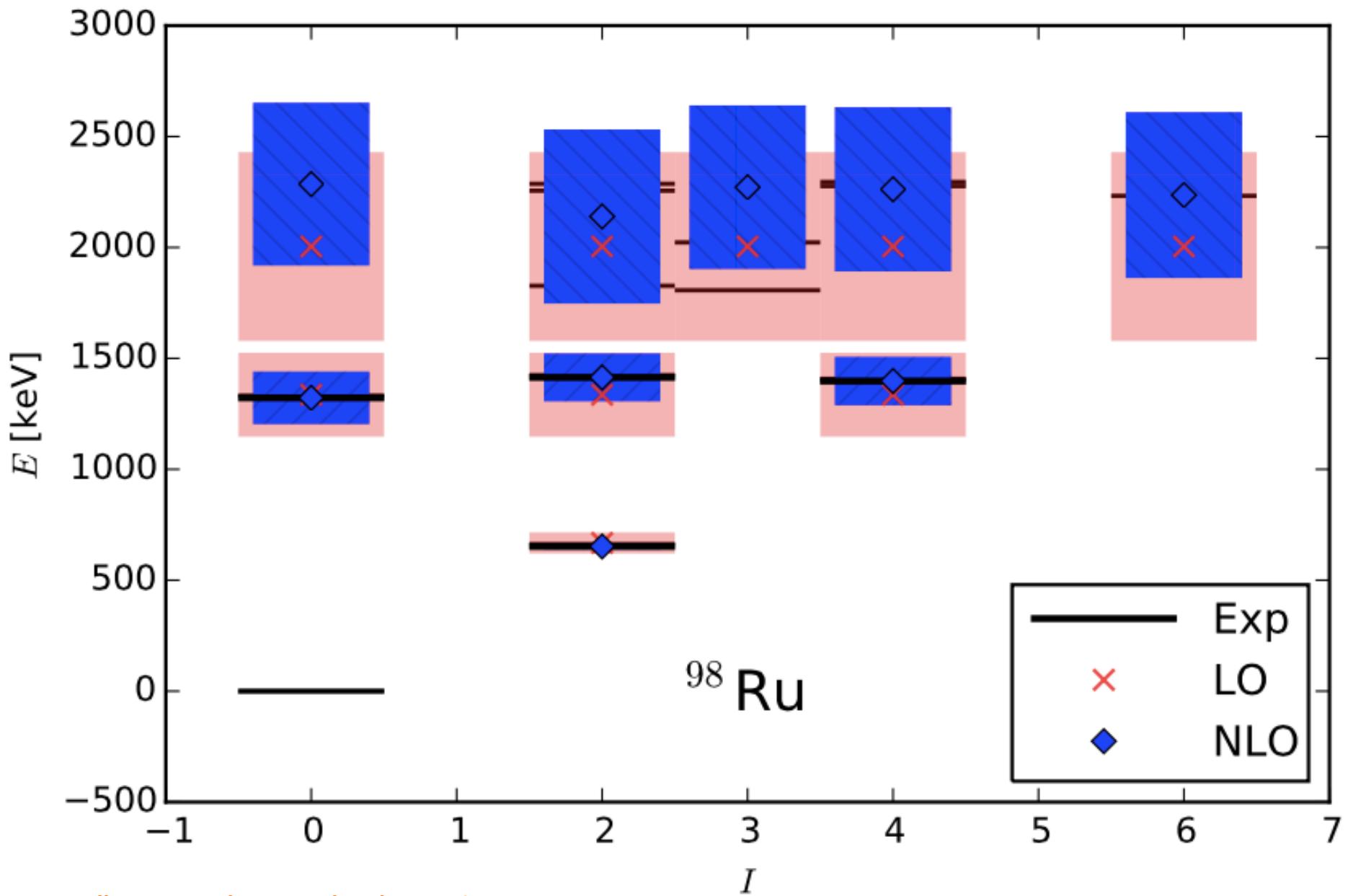
[34] K. Hagino et al., PRC90, 027303, (2014)

[35] L.V. Grigorenko et al., PRC91, 064617, (2015)

[36] A. Volya et al., PRC74, 064314, (2006)

[37] K. Tsukiyama et al., PTEP2015, 093D01 (2015)

[38] S.K. Bogner et al., PRL113, 142501, (2014)

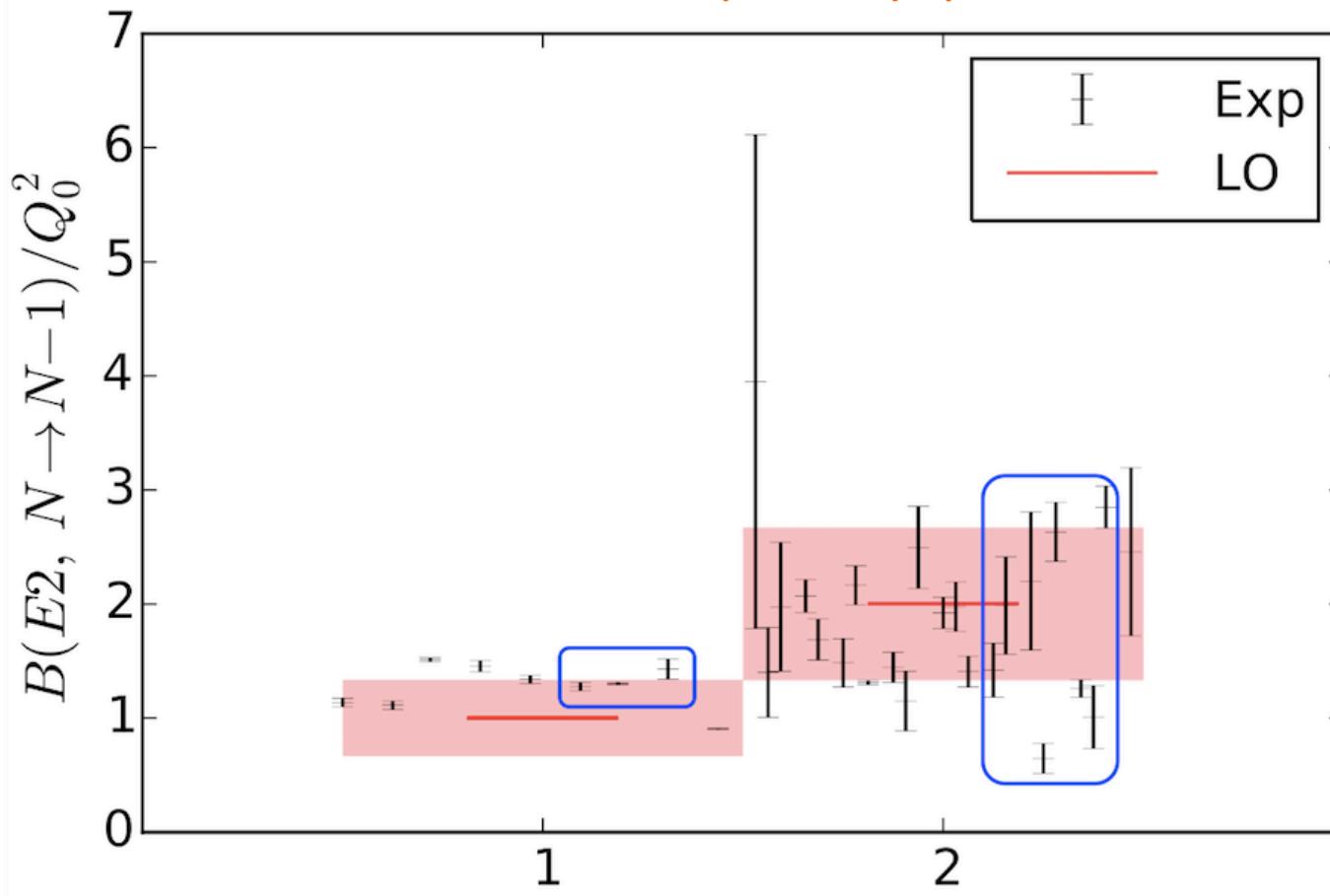


# Phonon-changing B(E2) values in vibrational nuclei Ni to Te

## (Cd values in blue rectangular boxes)

Effective field theory for nuclear vibrations with quantified uncertainties

E. A. Coello Perez and T. Papenbrock, preprint, 2015

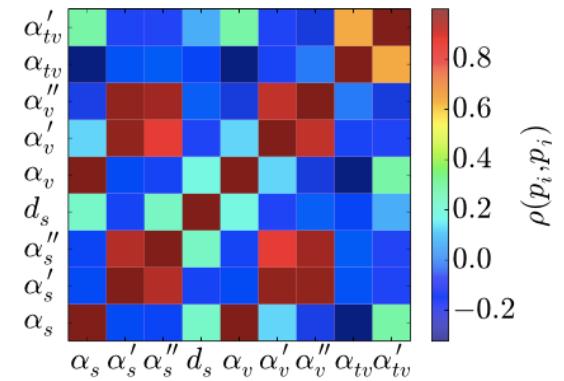


“Thus, the claim that traditional vibrational models do not describe the existing data is hard to quantify in the absence of theoretical uncertainties.”

# How to estimate theoretical uncertainties

**Micro models (interacting SM, RPA, DFT, Ab initio,...):**

**Model ----- Space ----- Interaction ----- computation**



**Phenomenological models:**

$$H = aOp_1 + bOp_2 + cOp_3 \dots$$

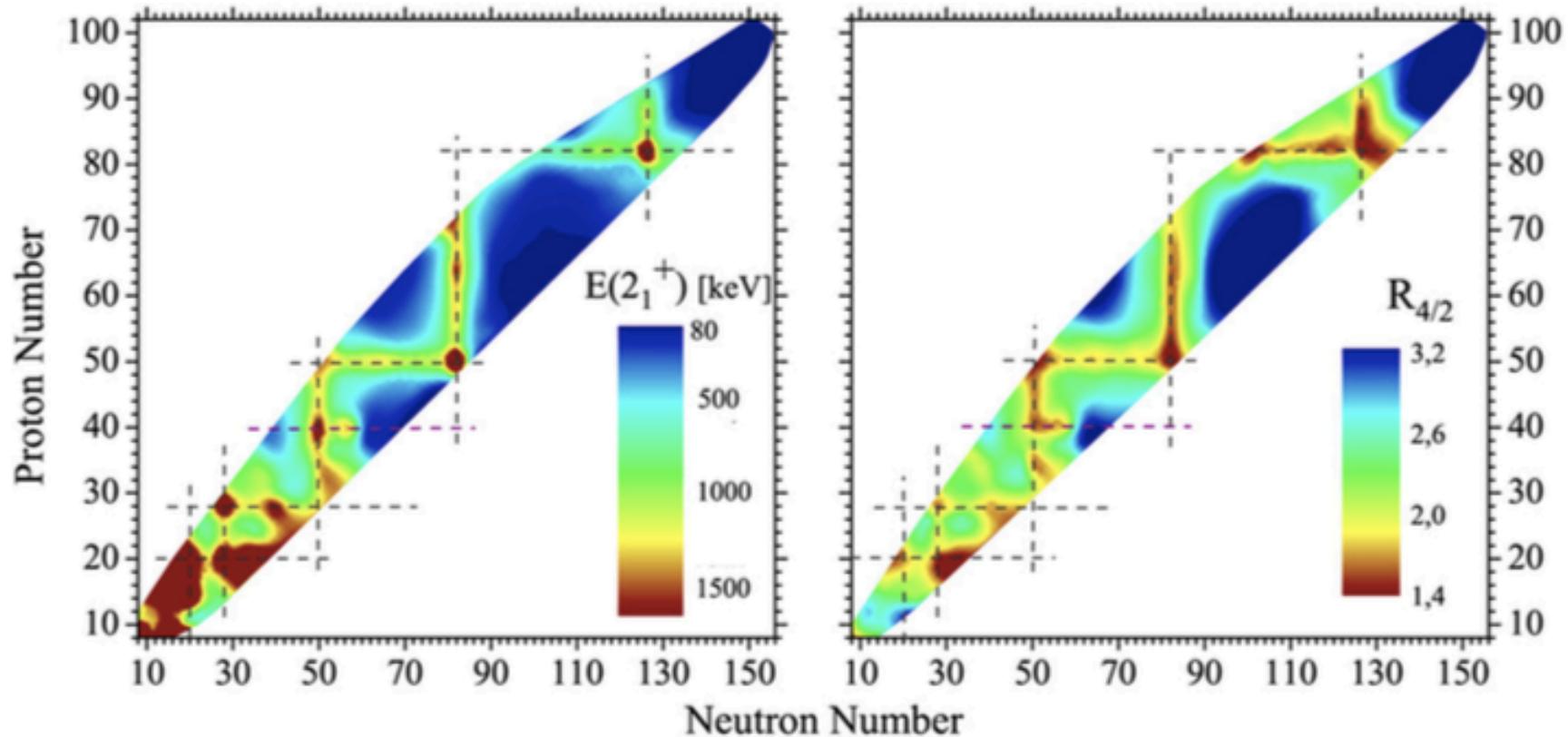
**Fit to data. Ambiguities in fitting: choices of observables**

**Analytic models (Alaga rules, rotational energies, AHV, ...):**

**Difficult: At least go by range of historical agreement**

# The Evolution of Structure

Two key observables,  $E(2^+_1)$ ,  $R_{42}$

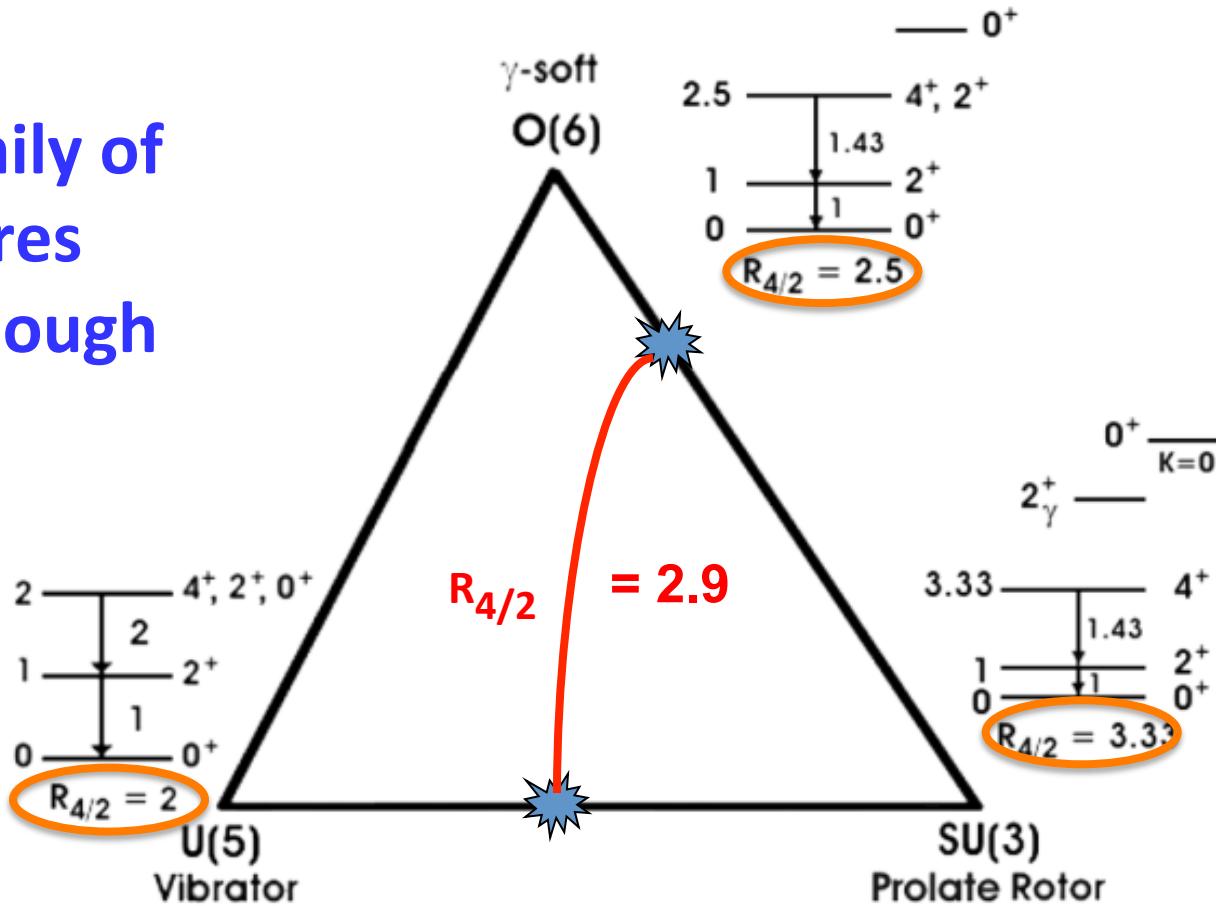


# Structural triangle for collective behavior

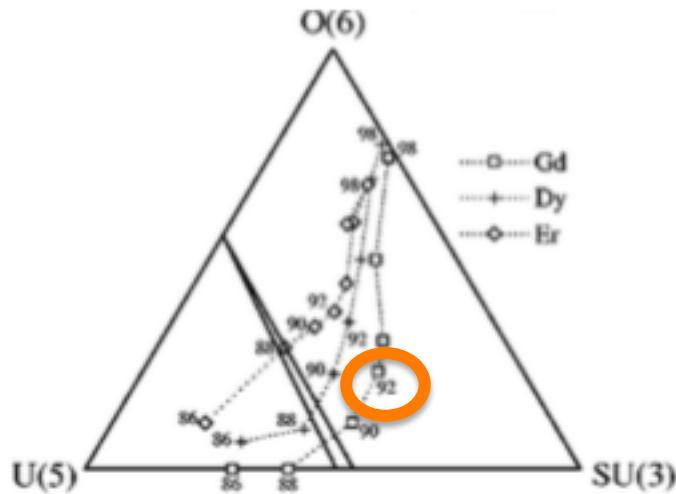
(based on the IBM but similar for GCM)

What does an observable tell us? Consider  $R_{4/2}$   
 $R_{4/2} = 2.0$  (harm. vib);  $= 3.33$  (Ax. Rotor);  $= 2.5$  (Gamma-soft rotor)

Entire family of structures  
 $R_{4/2}$  not enough

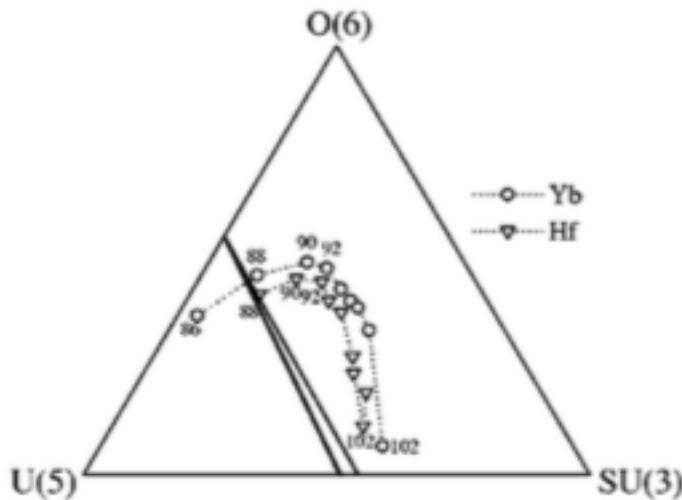


# The symmetry triangle of the IBM

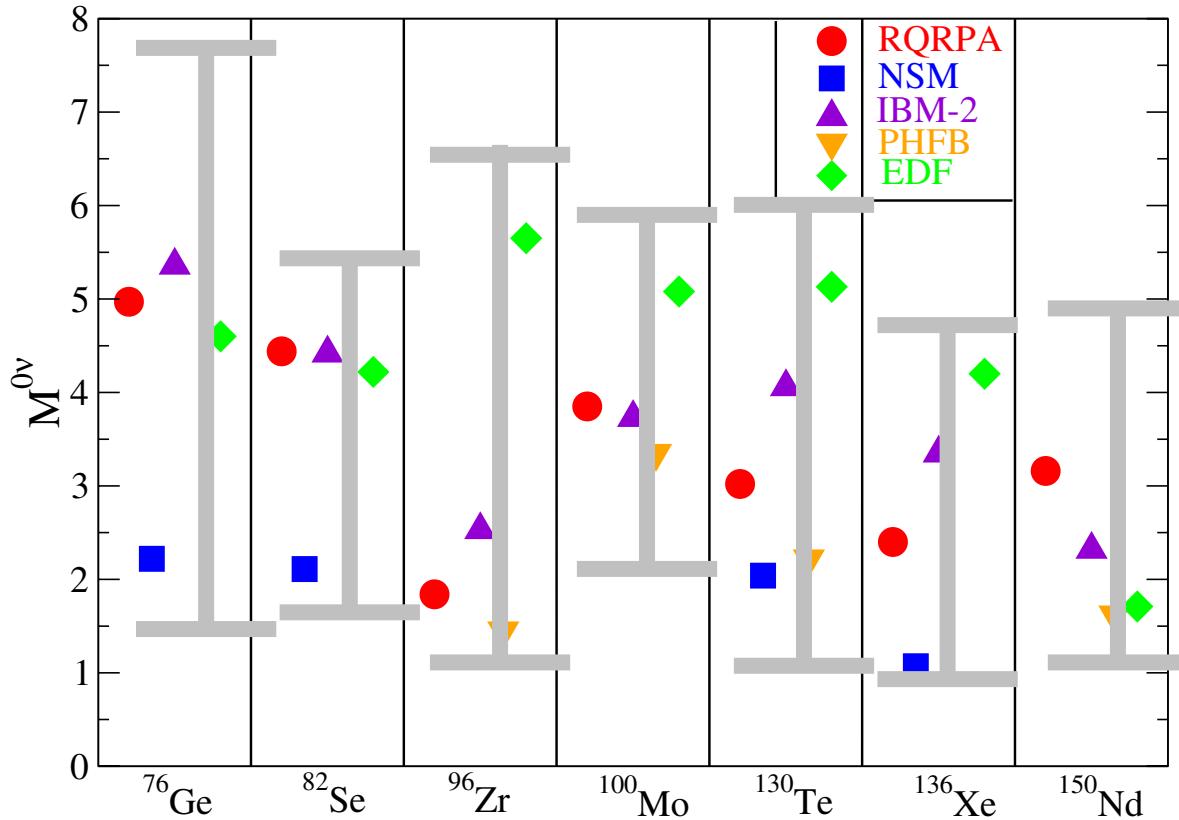


Uncertainties in  
phenomenological models.

Regions of parameter  
acceptability



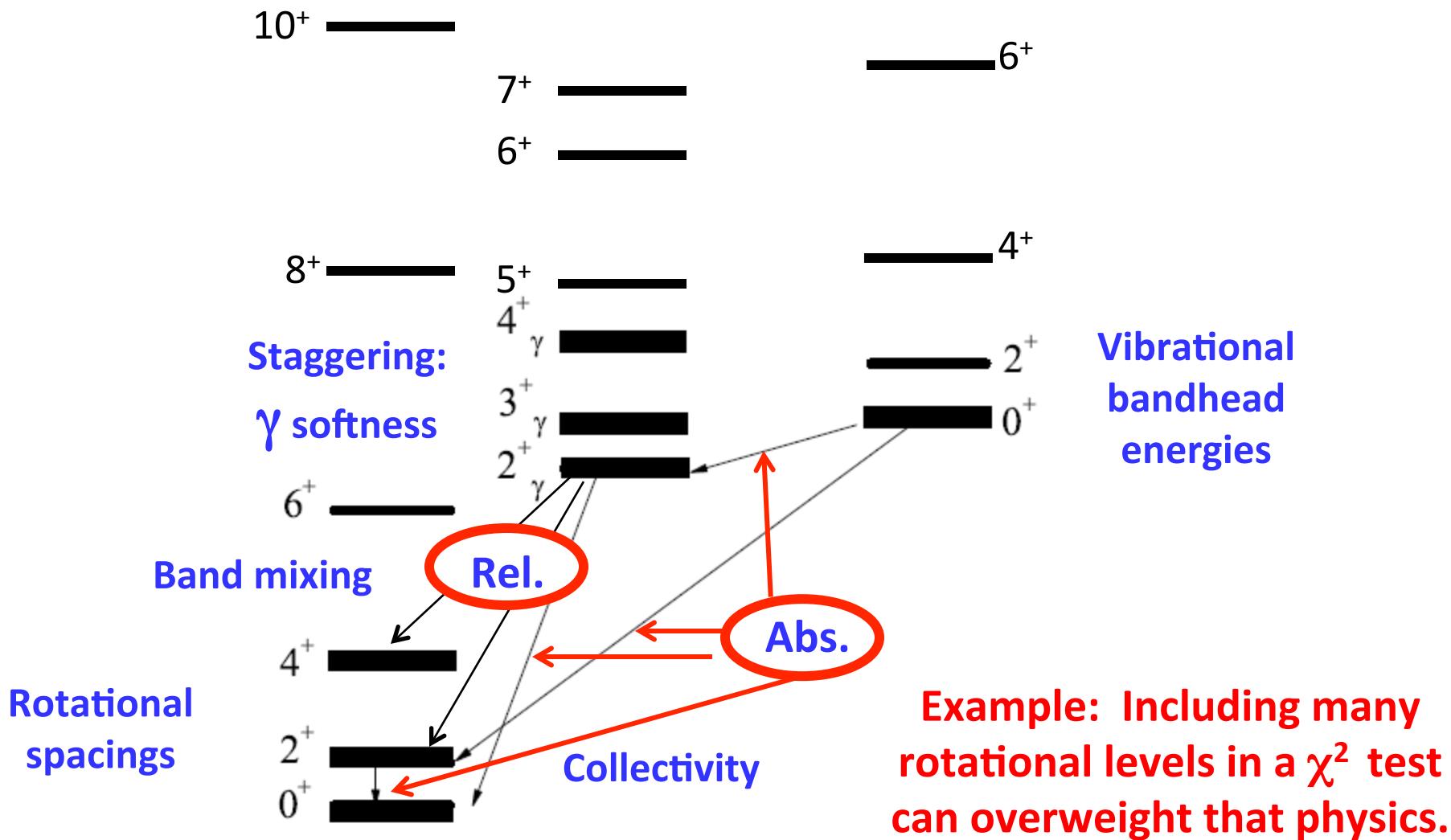
# Current $0\nu\beta\beta$ predictions



*"There is generally significant variation among different calculations of the nuclear matrix elements for a given isotope. For consideration of future experiments and their projected sensitivity it would be very desirable to reduce the uncertainty in these nuclear matrix elements."* (Neutrinoless Double Beta Decay NSAC Report 2014)

Figure courtesy of Witek Nazarewicz

# Not all observables equally important; many multiply-test same physics



What are the key observables for testing collective models?

# Parameters

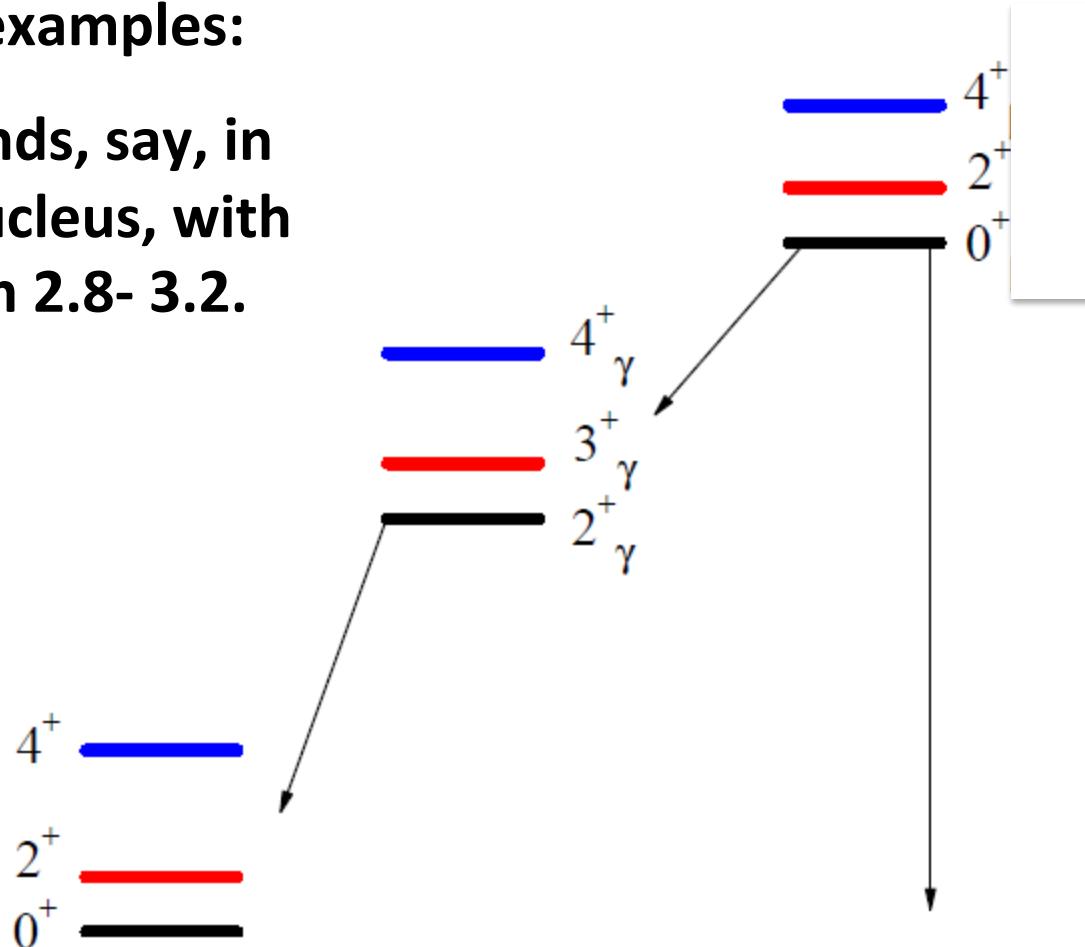
Interpreting this level scheme with bandmixing

**How many parameters?**

One of many examples:

Three quasi bands, say, in  
a transitional nucleus, with

$R_{42}$  values from 2.8- 3.2.



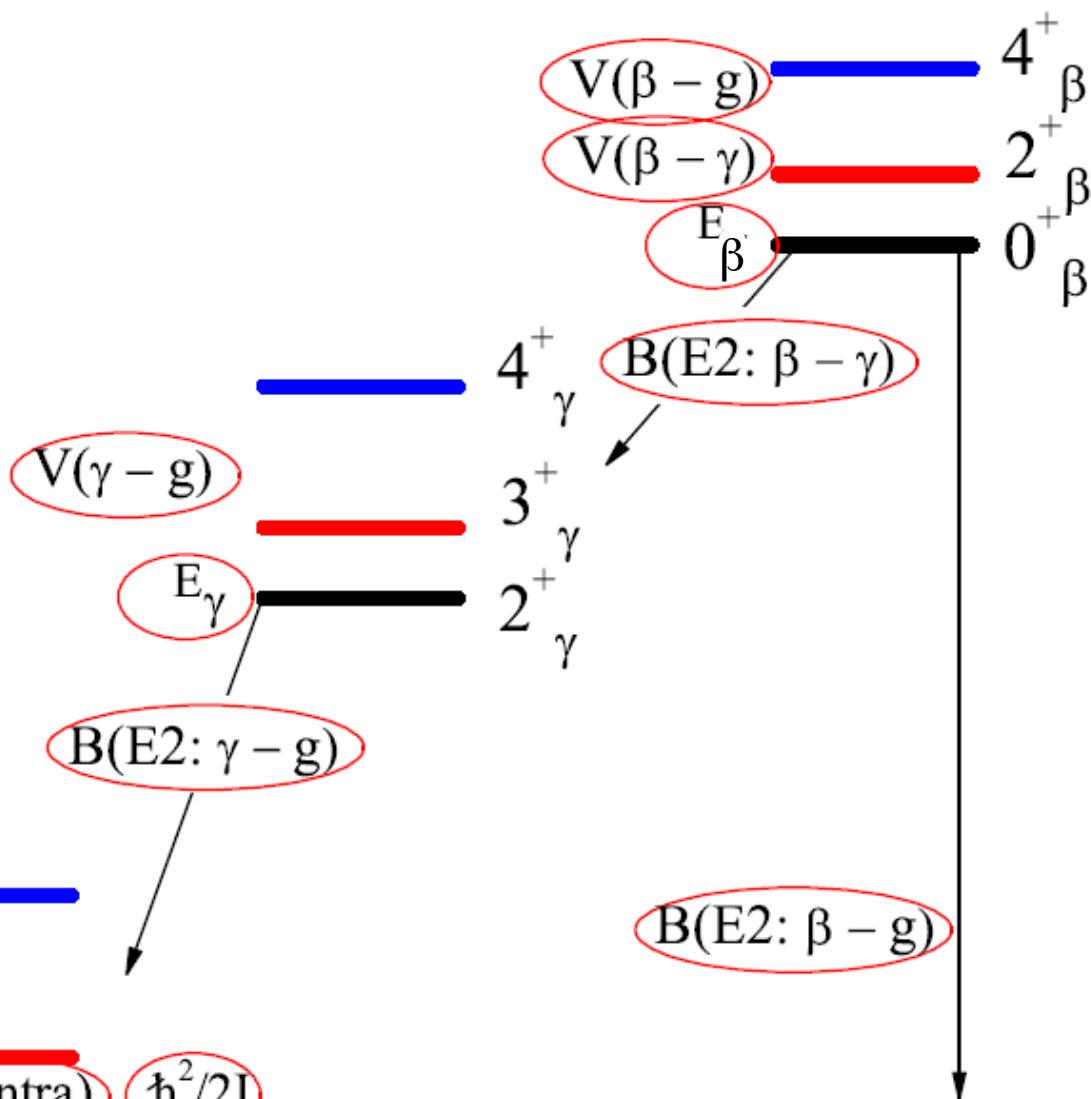
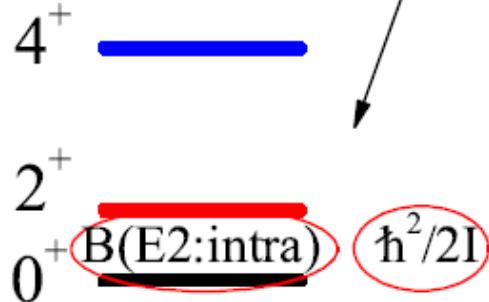
# Beware of multi - (often hidden) parameter predictions

To fit quasi-bands,  
need to start with  
quasi bands, with  
energies adjusted  
to fit final states:

$4_{\text{gr}}, 3_{\gamma}, 4_{\gamma}, 2_3, 4_3$

**15**

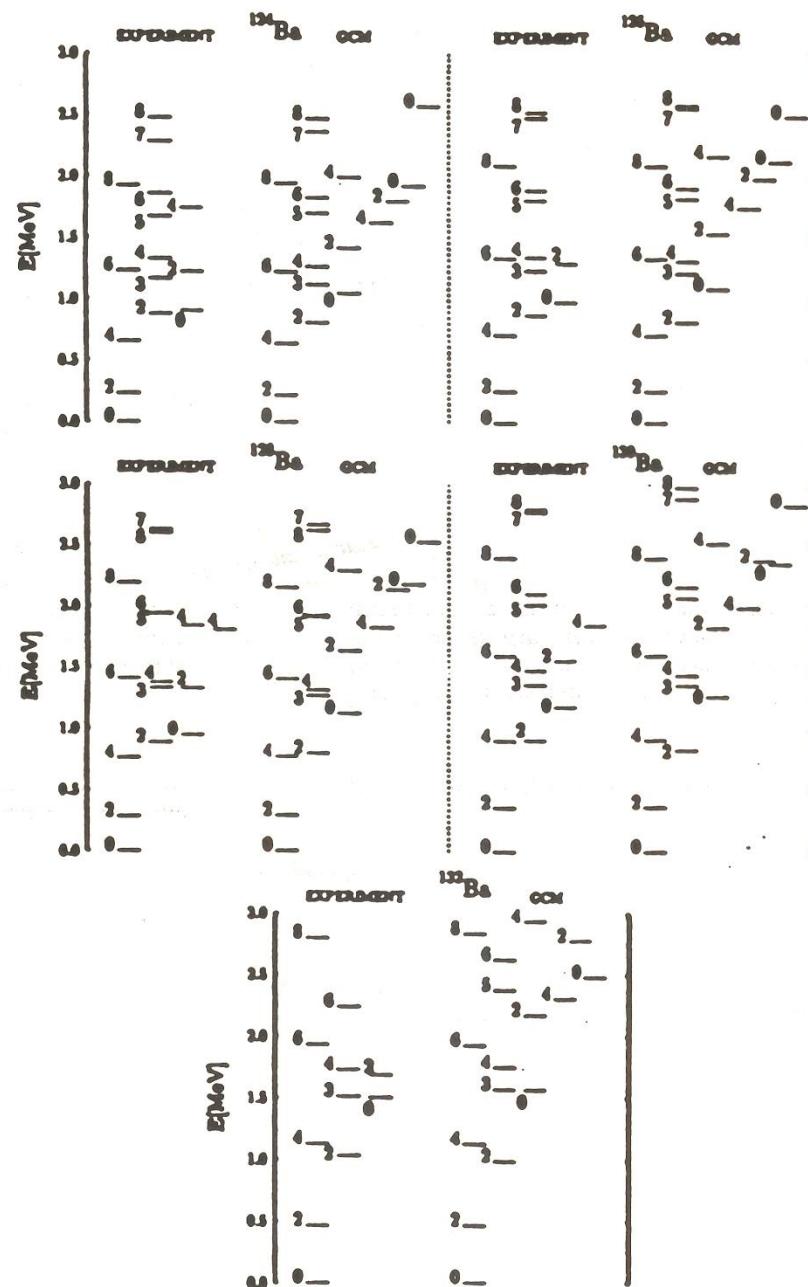
**in all**

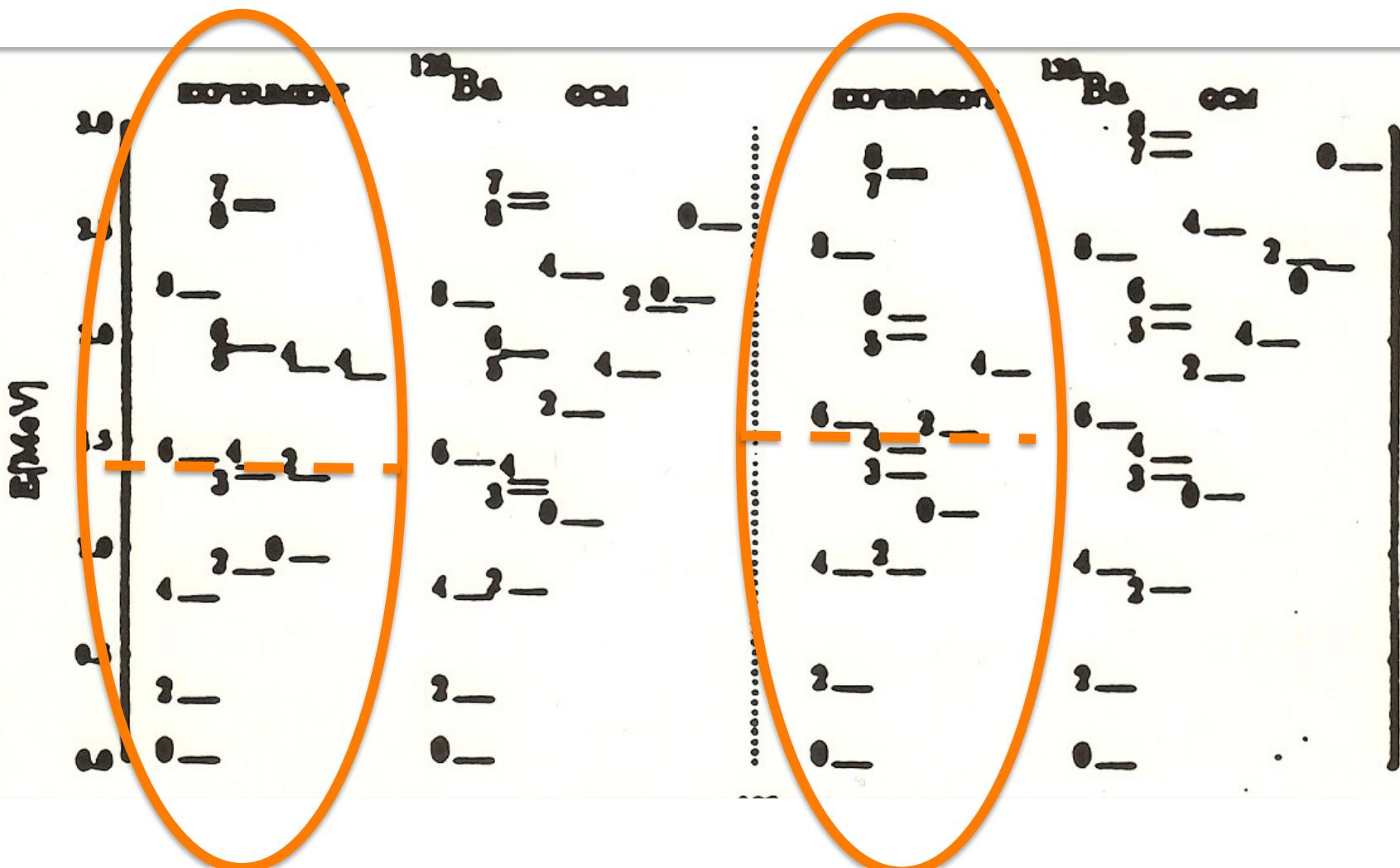


# Dealing with multi-parameter models

Set of very similar Ba level schemes

Fit with Geometric Collective model with parametrized Hamiltonian





# Geometric Collective Model Calculations in the Ba isotopes

	$^{124}\text{Ba}$	$^{126}\text{Ba}$	$^{128}\text{Ba}$	$^{130}\text{Ba}$	$^{132}\text{Ba}$
$B_2 (10^{-42} \text{ MeV s}^2)$	46.91602	53.8611	56.42193	62.70904	57.933
$P_3 (10^{43} \text{ MeV}^{-1} \text{s}^{-2})$	-0.13	-0.118	-0.107141	-0.107602	-0.125873
$C_2$	-209.0632	-252.7933	-263.6945	-352.6598	-314.2383
$C_3$	135.8335	220.6479	248.7190	362.6218	442.1007
$C_4$	1708.728	3449.529	4323.475	8848.212	9517.914
$C_5$	-229.18361	-946.7563	-1371.759	-86.12527	8525.92
$C_6$	-36499.49	-50421.41	-41041.91	-15557.86	74370.15
$D_6$	36499.49	41910.09	32030.03	10191.77	-546.2191
$V_{\perp}$	-5.7				

# Which model is better?

## $^{168}\text{Er}$ relative $\gamma$ to ground band B(E2)s

$J_i^\pi \rightarrow J_f^\pi$	$^{168}\text{Er}$	ALAGA	$Z_\gamma = 0.035$	PDS
$2_\gamma^+ \rightarrow 0^+$	56.2(11)	70	56.9	64.3
$2_\gamma^+ \rightarrow 2^+$	100	100	100	100
$2_\gamma^+ \rightarrow 4^+$	7.3(4)	5	7.6	6.3
$3_\gamma^+ \rightarrow 2^+$	100	100	100	100
$3_\gamma^+ \rightarrow 4^+$	62.6(14)	40	62.9	49.3
$4_\gamma^+ \rightarrow 2^+$	19.3(4)	34	20.2	28.1
$4_\gamma^+ \rightarrow 4^+$	100	100	100	100
$4_\gamma^+ \rightarrow 6^+$	13.1(12)	8.64	16.0	12.5
$5_\gamma^+ \rightarrow 4^+$	100	100	100	100
$5_\gamma^+ \rightarrow 6^+$	123(14)	57.1	117	79.6
$6_\gamma^+ \rightarrow 4^+$	11.2(10)	26.9	11.0	20.3
$6_\gamma^+ \rightarrow 6^+$	100	100	100	100
$6_\gamma^+ \rightarrow 8^+$	37.6(72)	10.6	23.6	18.0

Parameters: 0 1 0

Alaga works surprisingly well.

PDS (Ami's talk: takes into account finite valence nucleon number) always closer to data than Alaga. But doesn't go far enough.

Bandmixing accounts extremely well for the data.

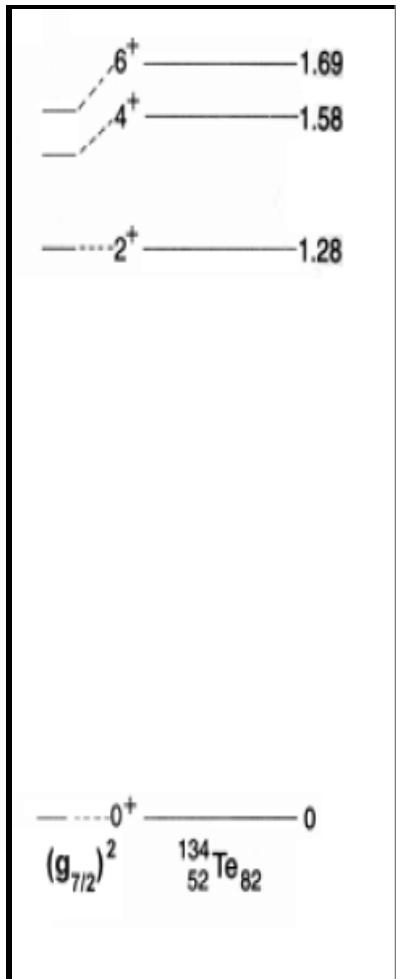
# Main points

- Statistical tests can be extremely useful but also dangerous if conducted blindly
- Don't inadvertently overweight super-precise data
- Include theoretical uncertainties
- Focus on key observables
- Don't overweight multiple observables testing the same physics
- Count parameters (including hidden ones)

# **BACKUPS**

Doubly magic  
plus 2 nucleons

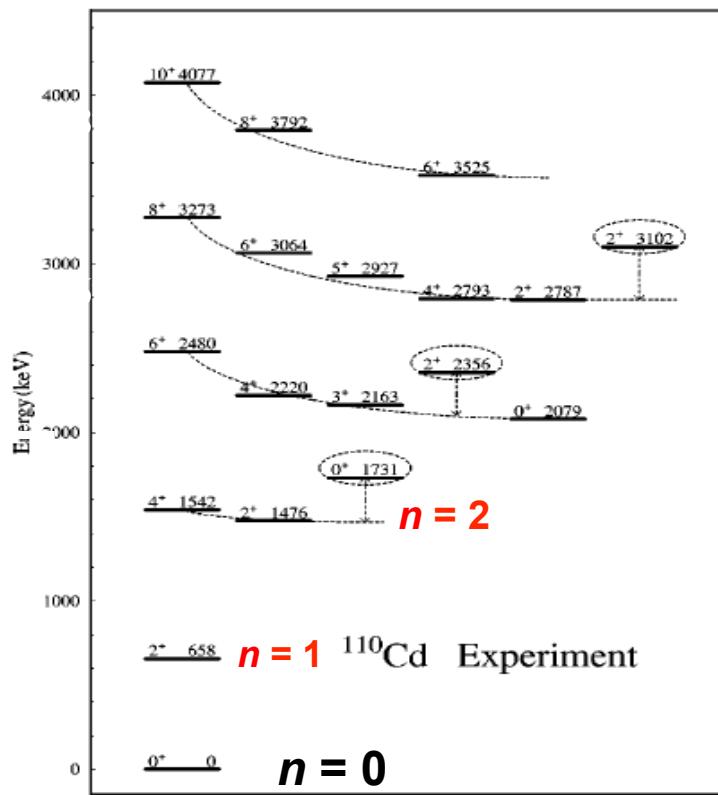
$$R_{4/2} < 2.0$$



Vibrator (H.O.)

$$E(J) = n (\hbar \omega_0)$$

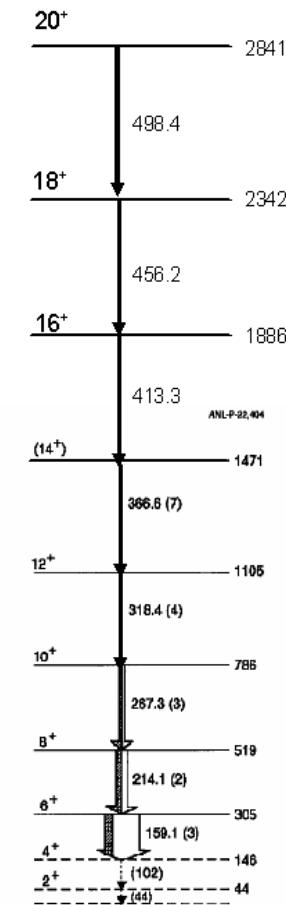
$$R_{4/2} = 2.0$$



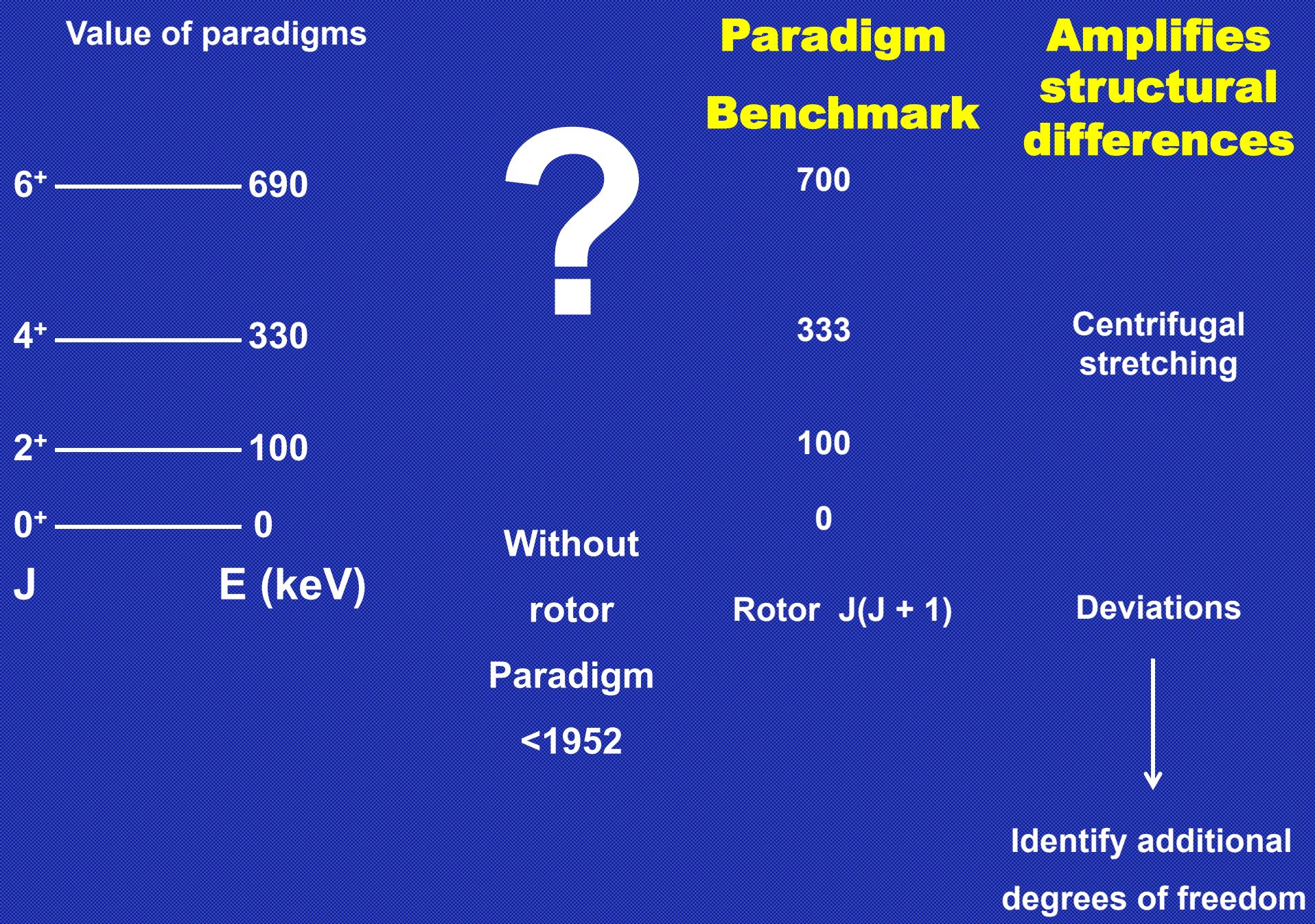
Rotor

$$E(J) \propto (\hbar^2/2I)J(J+1)$$

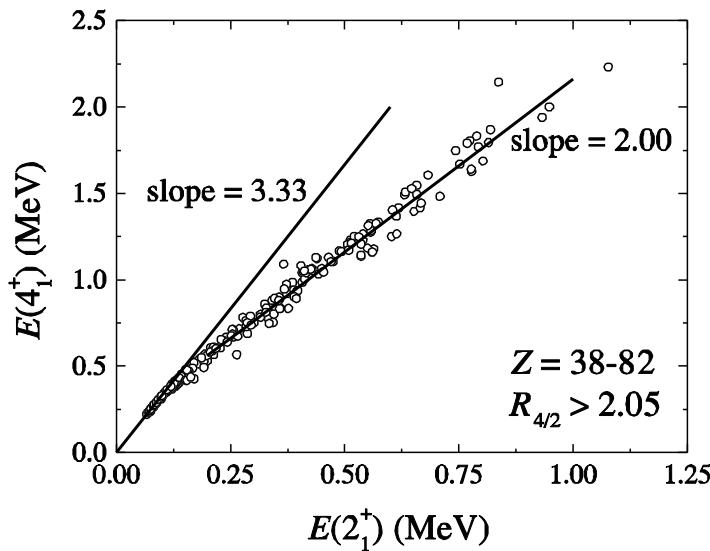
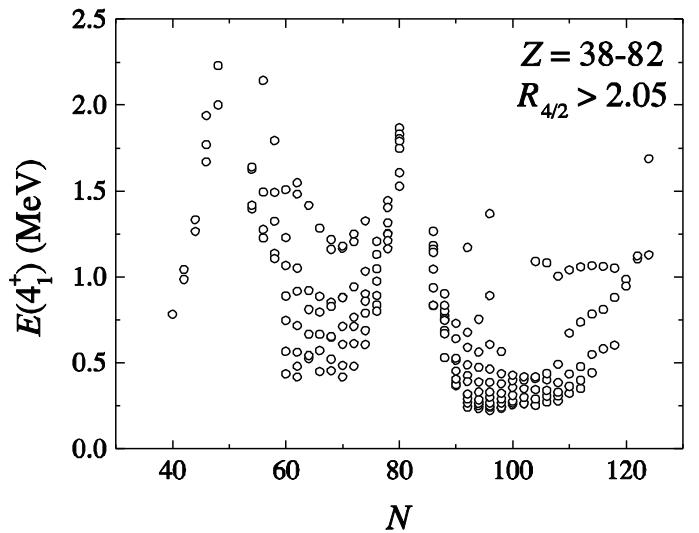
$$R_{4/2} = 3.33$$



$^{254}\text{No}_{102}$



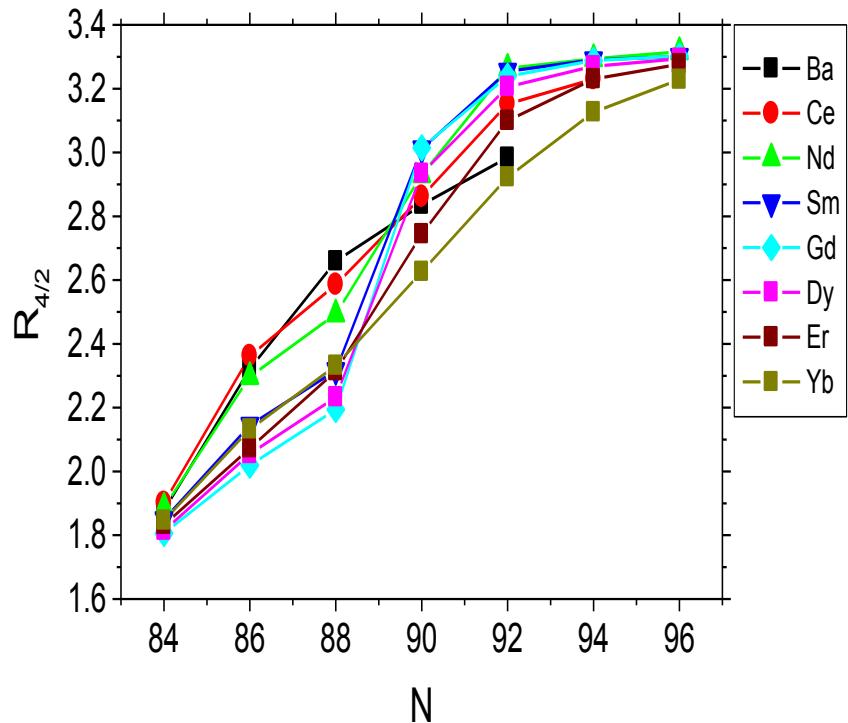
# Structural evolution: Look at data from different perspectives



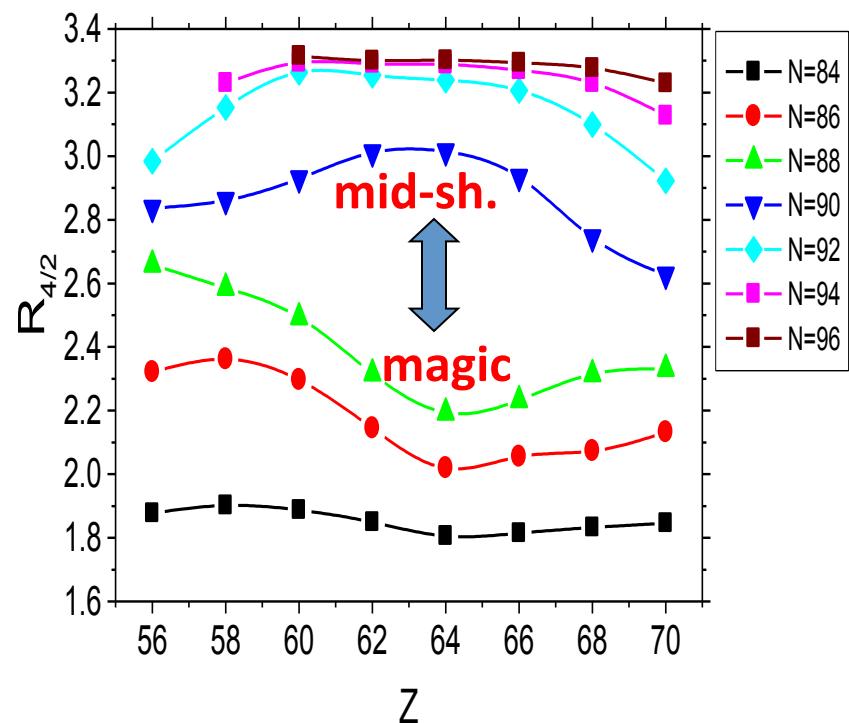
Time out – warning:  
**BEWARE OF FALSE CORRELATIONS!**



# An important example: different perspectives!

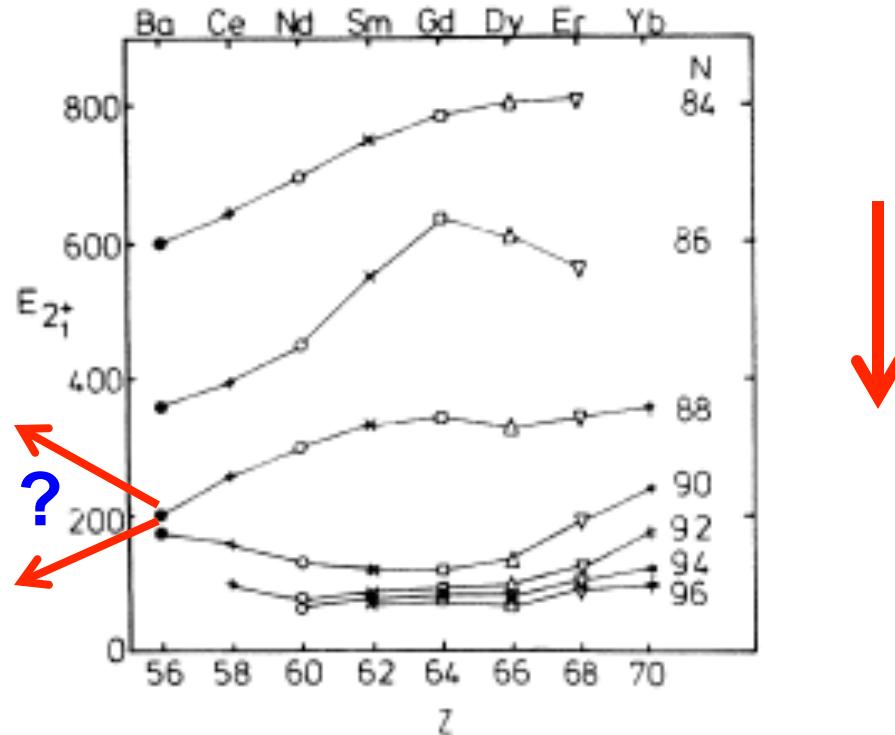


Onset of deformation



Onset of deformation  
as a  
quantum phase transition  
*mediated by a*  
*change in proton shell structure*  
*driven by*  
*the p-n interaction*

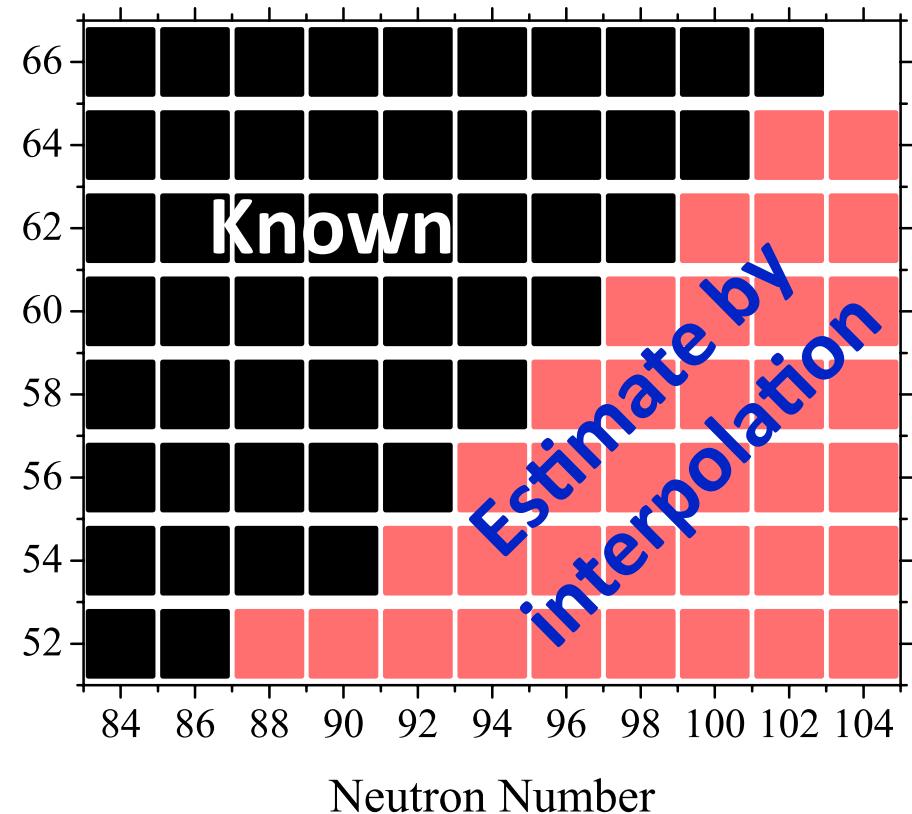
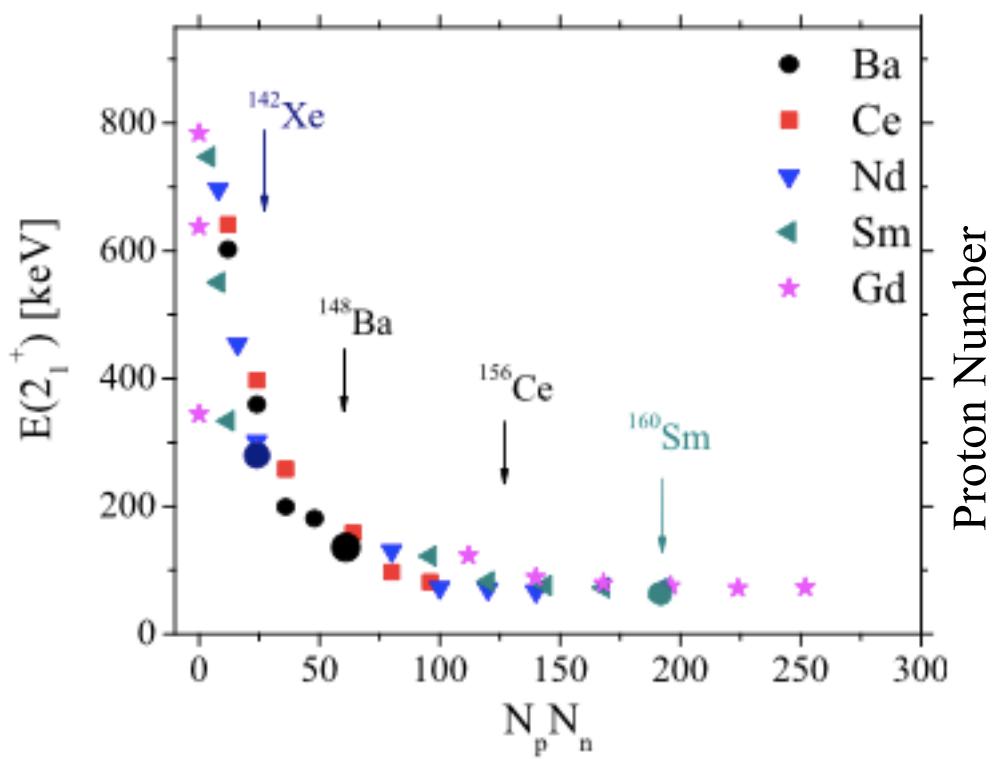
# The $N_p N_n$ scheme: Interpolation vs. Extrapolation



$$N_p N_n = 4 \times 6 = 24$$

- $^{156}\text{Te}$  (52, 104)       $^{156}\text{Gd}$  (64, 92)       $^{184}\text{Pt}$  (78, 106).
- $N_p N_n : 2 \times 22 = 44$        $14 \times 10 = 140$        $4 \times 20 = 80$

# $N_p N_n$ Plots



Estimation by **interpolation**