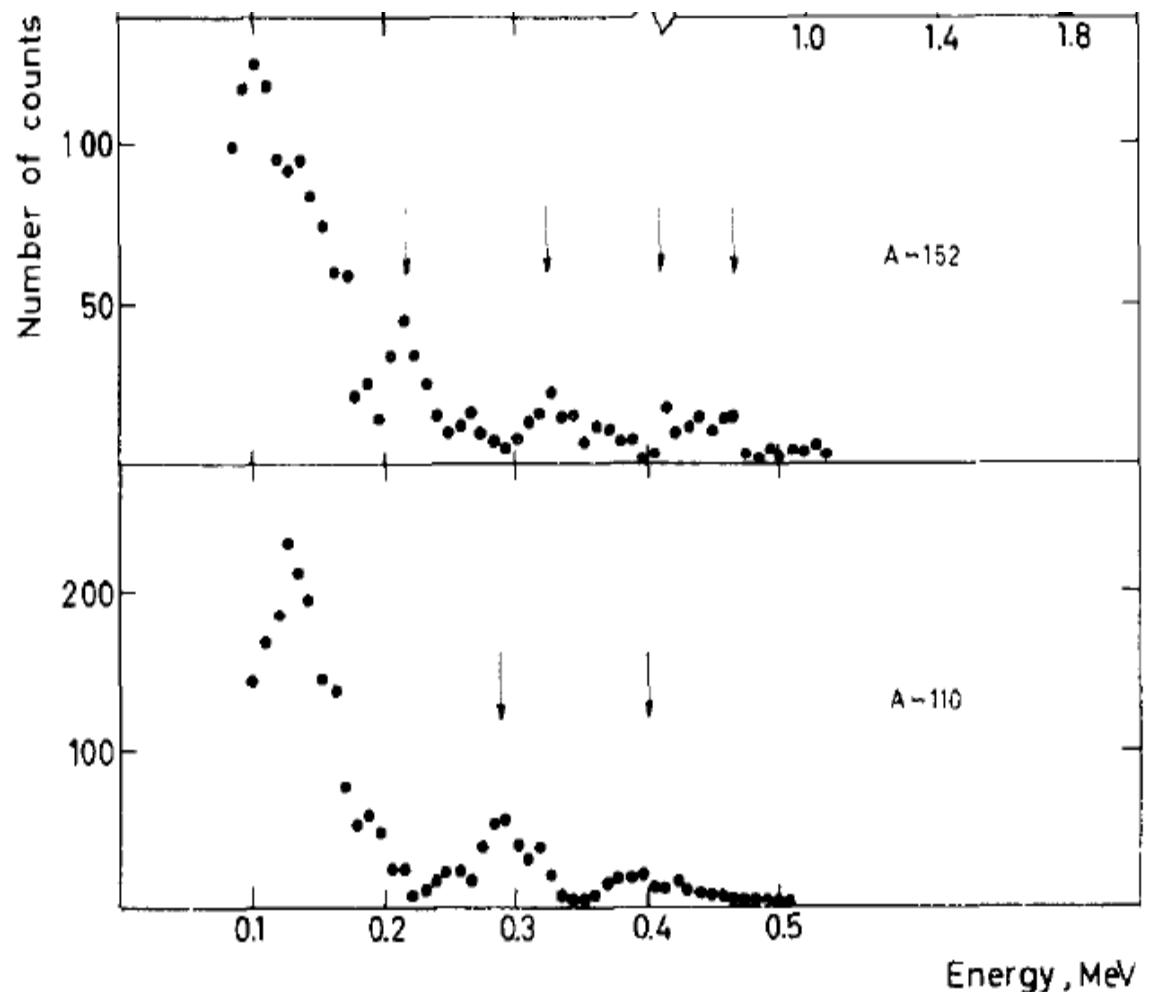
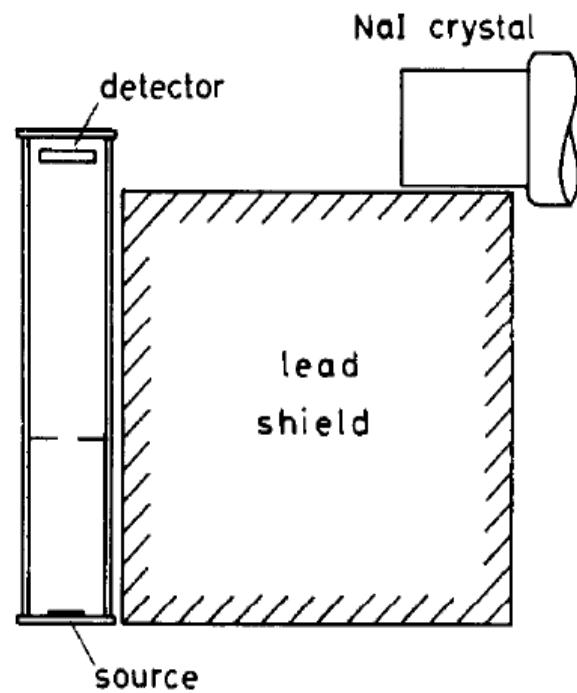


# **Shape coexistence at the 40 sub-shell closure and neutron number 60**

E.Clément-GANIL  
IS451 Collaboration

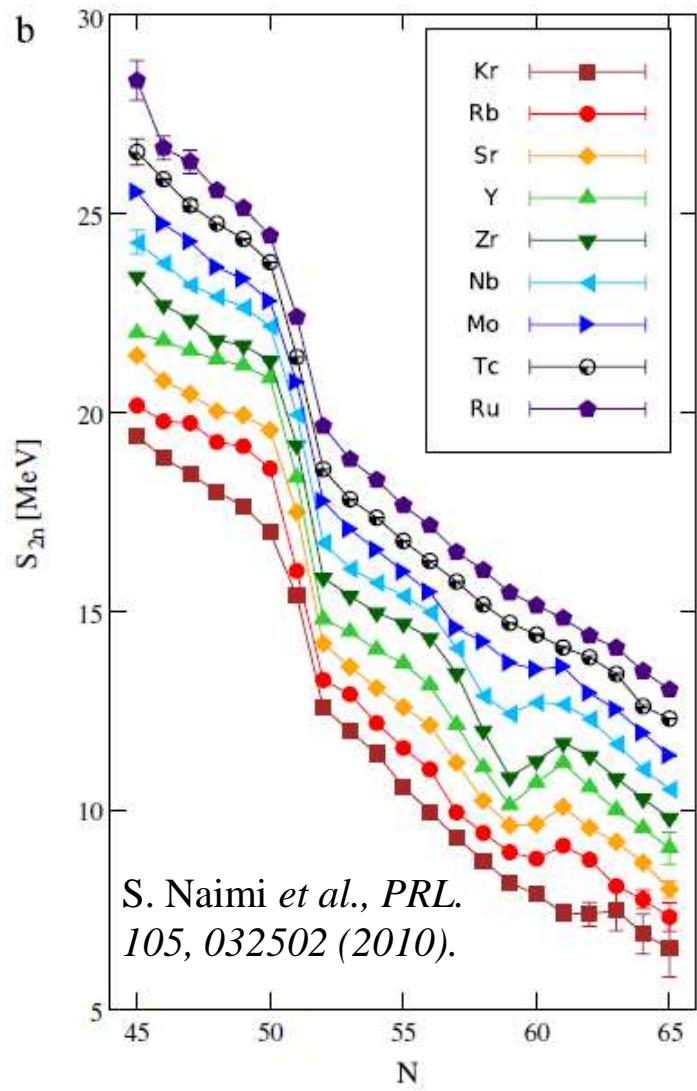
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It has been established in the 60's that elements with  $A \sim 110$   $Z \sim 40$  belong to a new island of stable deformation similar to the rare earth region

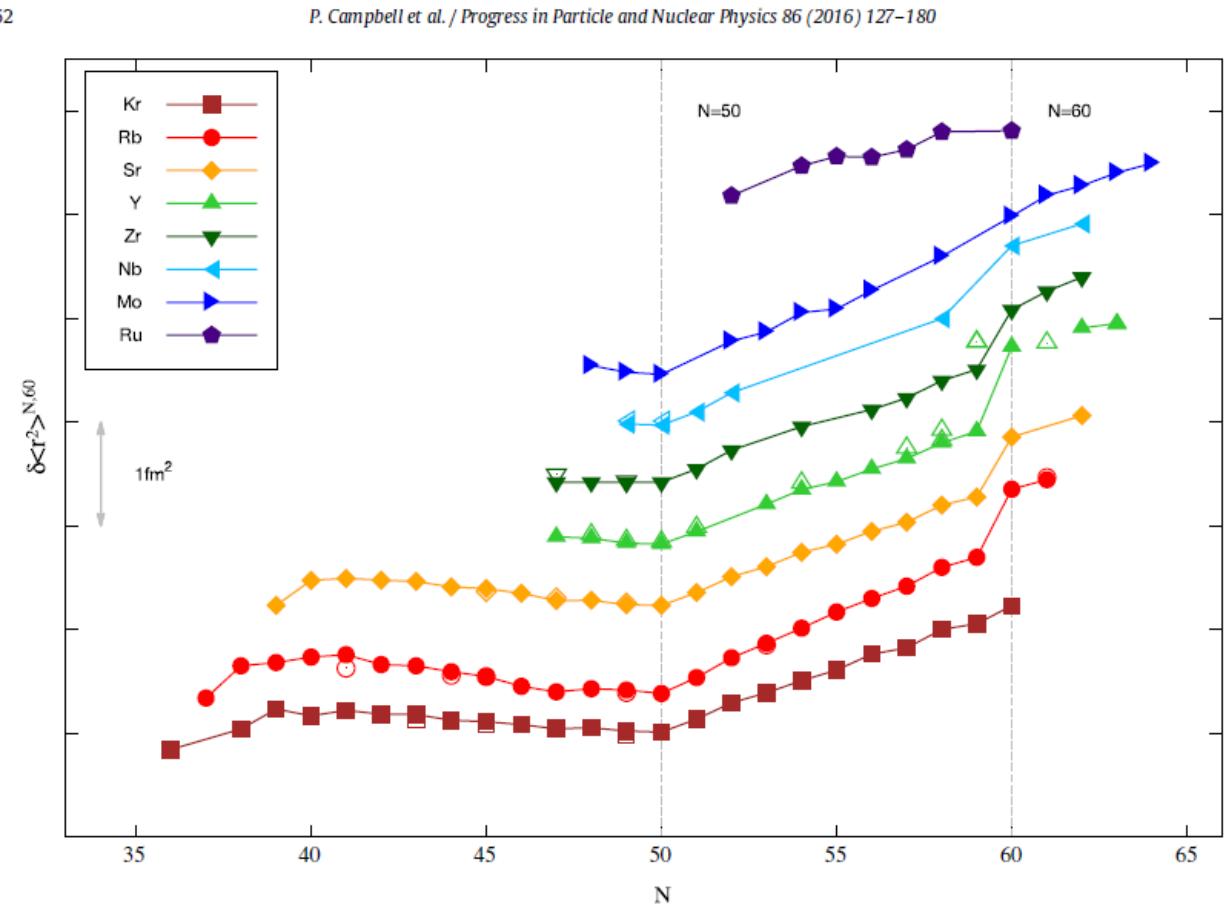


S. A. E. Johansson, Nucl. Phys. **64** (1965) 147

P. Campbell, I.D. Moore, M.R. Pearson  
*Progress in Particle and Nuclear Physics* 86  
 (2016) 127–180

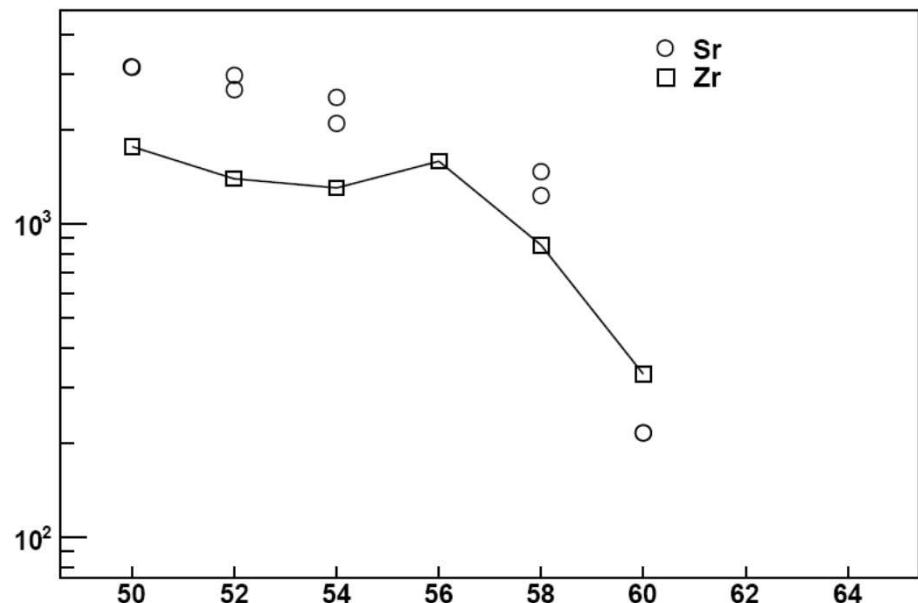
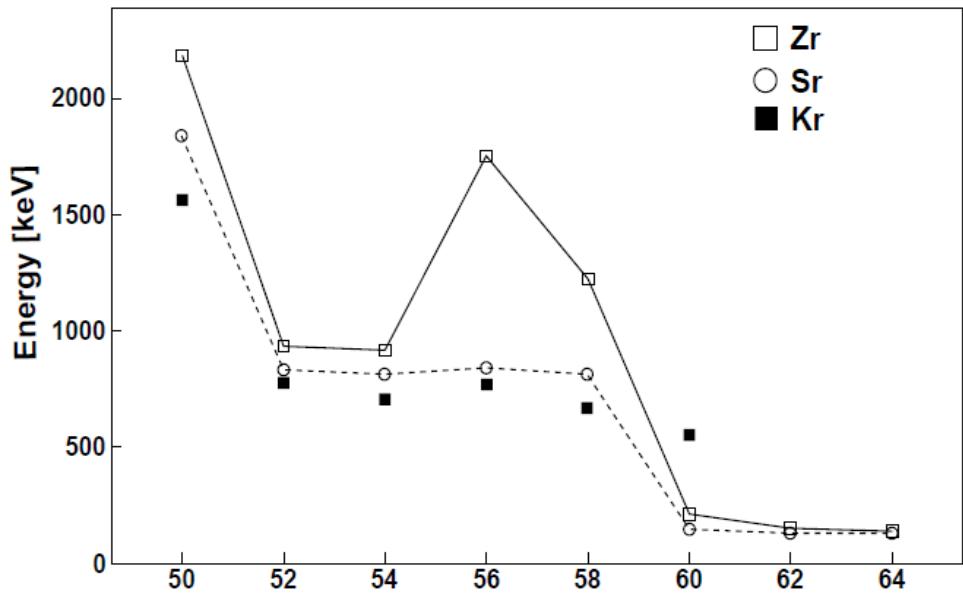


- The n-rich nuclei between  $Z=37$  and  $Z=41$  present at  $N=60$  one of the most impressive deformation change in the nuclear chart
- Localized within the  $Z$  degree of freedom  
 → Point to a specific  $\pi$ - $v$  interaction



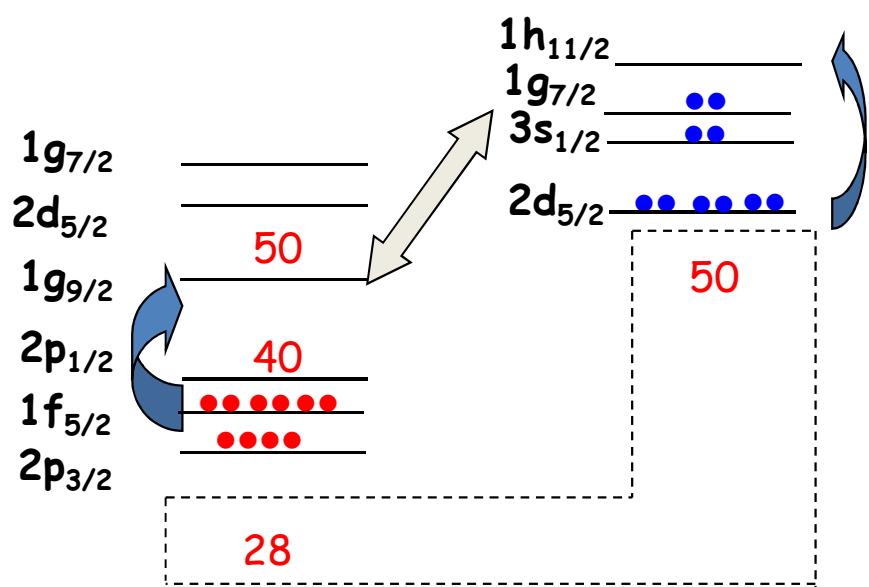
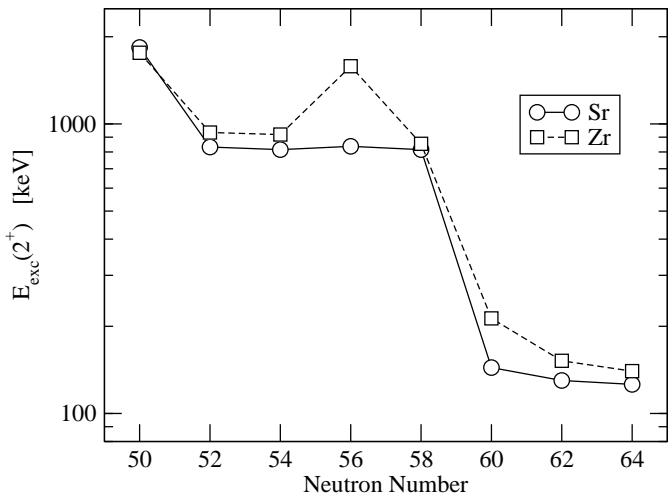
# Shape Transition at N=60

M. Albers *et al.*, Phys. Rev. Lett. 108, 062701 (2012)



- ❖ First spectroscopy (GS and  $2^+_1$ ) indicated a shape change from  $\beta \sim 0.1$  to  $\beta \sim 0.4$
- ❖  $0^+_2$  states are indication of shape coexistence  
→ Shape inversion ?
- ❖ Kr isotopes behave differently : smooth  $2^+$  change, no onset in  $S_{2n}$ , no low lying  $0^+_2$

# Shape Transition at N=60



The sharp transition and magnitude of the deformation remain still a challenge for theories (> 100 theoretical papers since the 70's)

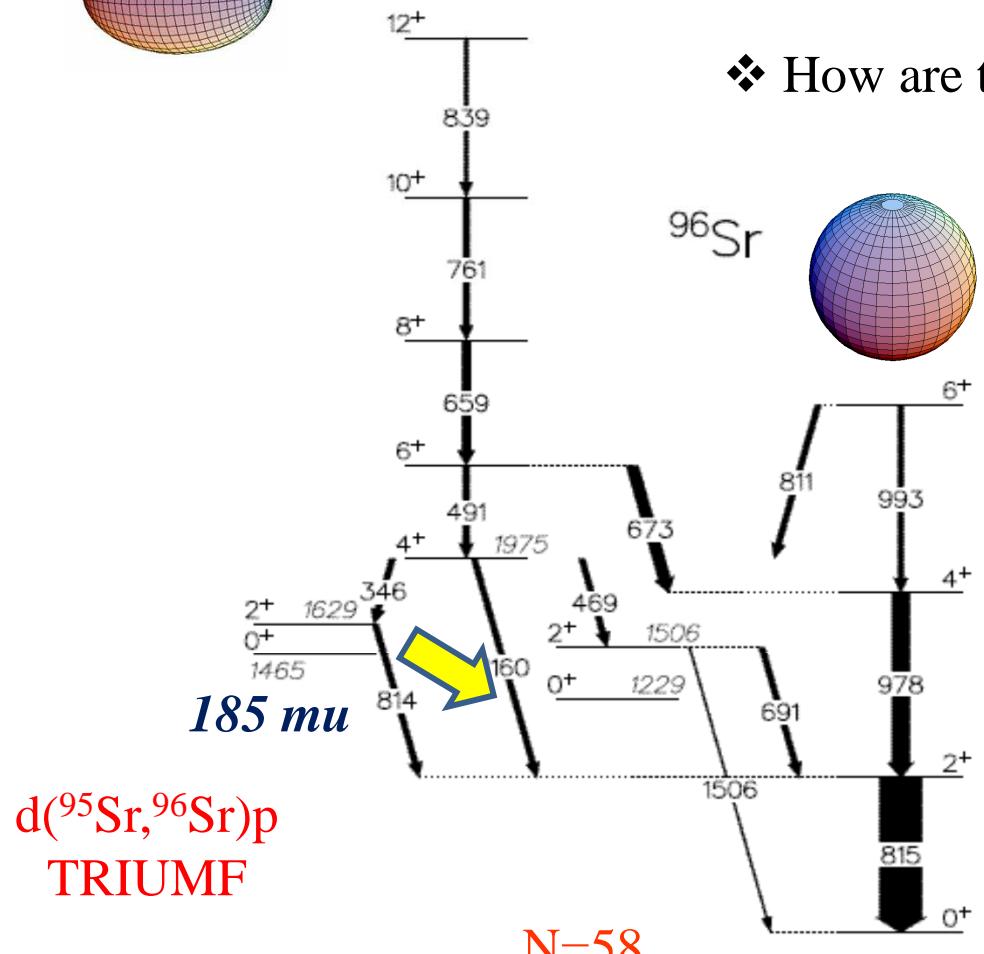
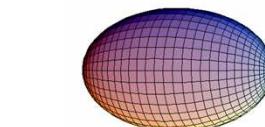
- ✓ HFB + the generator coordinate method (GCM)
- ✓ the macroscopic-microscopic method
- ✓ the shell model
- ✓ the Monte Carlo shell model
- ✓ the interacting boson model (IBM) approximation
- ✓ the VAMPIR model
- ✓ covariant density functional (DF) theory (PC-PK1).

- ❑ 0<sup>+</sup><sub>2</sub> state created by (2p-2h/4p-4h) excitation across Z=40
- ❑ Beyond N=60, g<sub>7/2</sub> and h<sub>11/2</sub> are populated, the π-ν interaction and the participates to the lowering 0<sup>+</sup><sub>2</sub> state and to the high collectivity of 2<sup>+</sup><sub>1</sub> state.
- ❑ In BMF calculations, two minima appear in the PES

Mainly GS and level scheme are known and limit the comparison with theoretical models

- K. Sieja et al PRC 79, 064310 (2009)  
 A. Petrovici PRC 85, 034337 (2012)  
 T. Togashi et al, Phys.Rev.Lett. 117, 172502 (2016)  
 C. Kremer et al, Phys.Rev.Lett. 117, 172503 (2016)

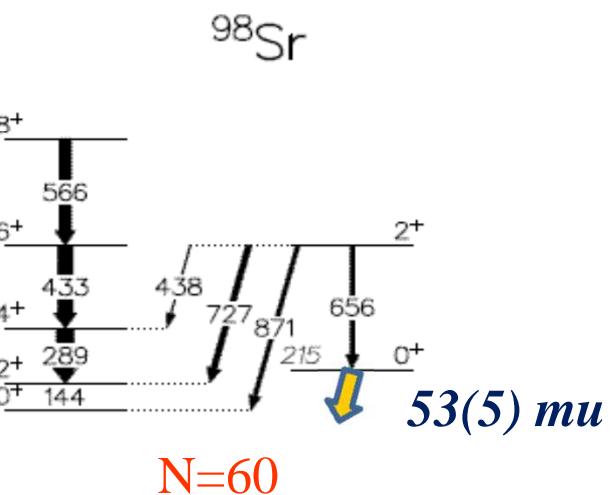
# Shape Transition at N=60



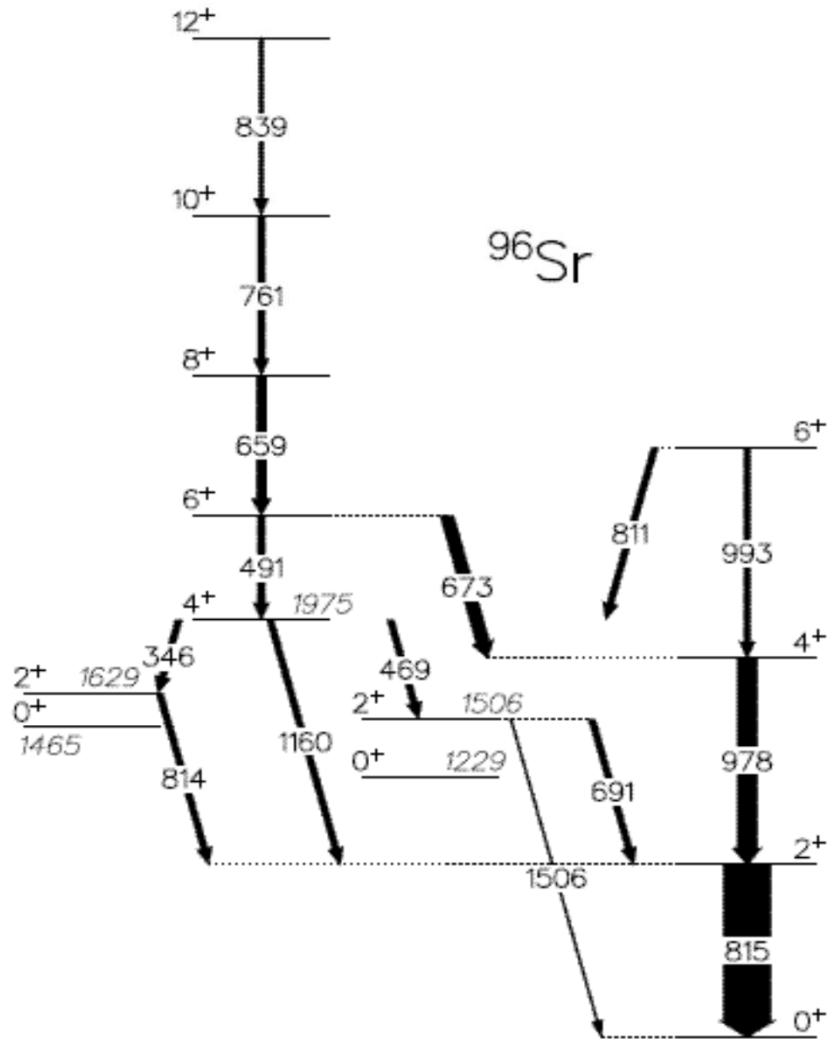
❖ What is the shape of the corresponding 2<sup>+</sup> states ?

❖ How are the different configurations connected ?

→ Coulomb excitation of RIB

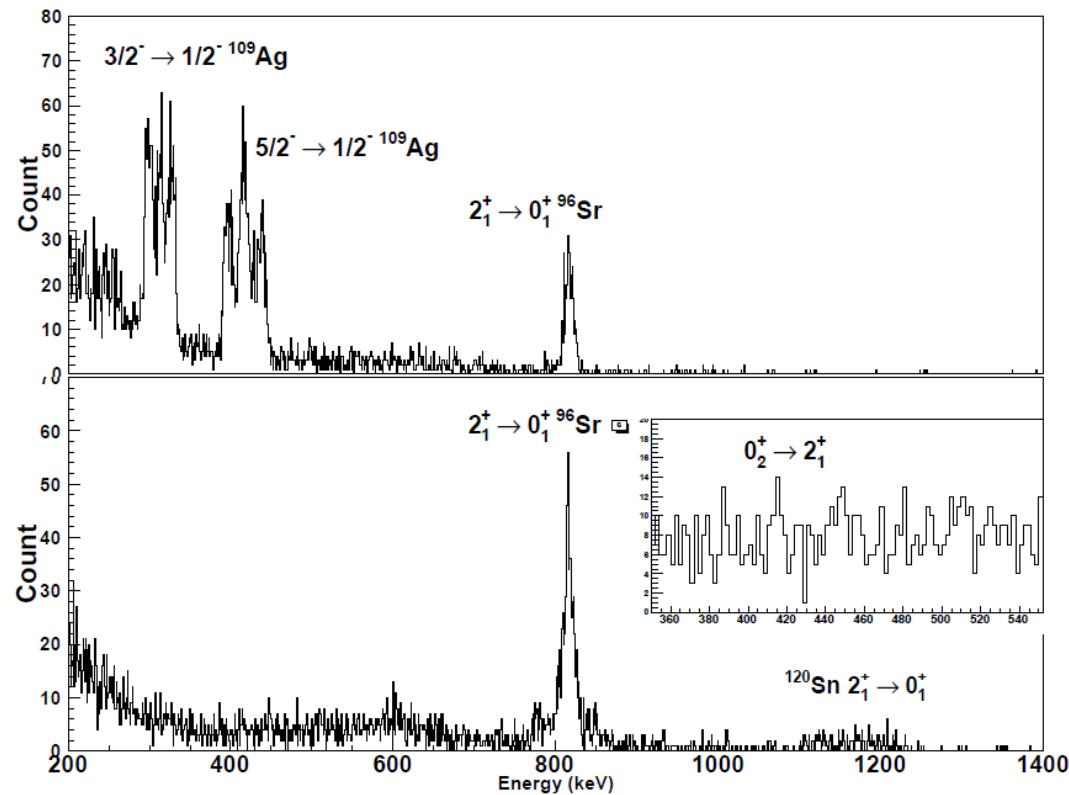


# Safe Coulomb excitation of $^{96,98}\text{Sr}$ beams at REX-ISOLDE using the MINIBALL array at CERN

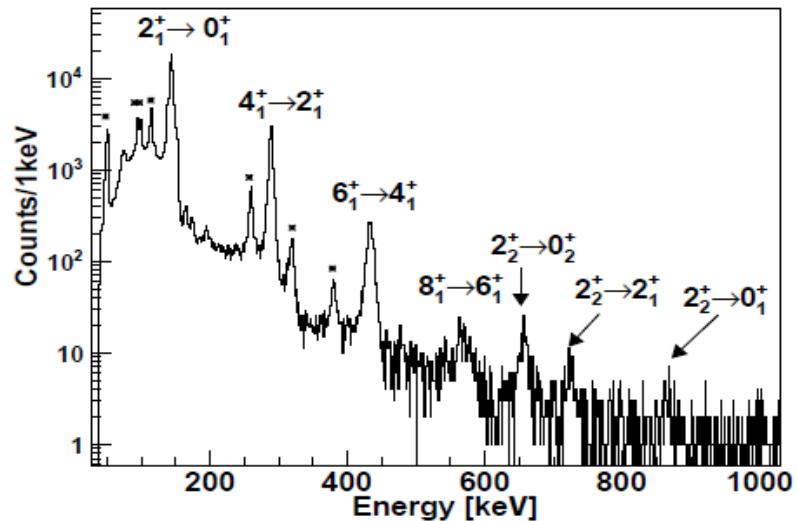
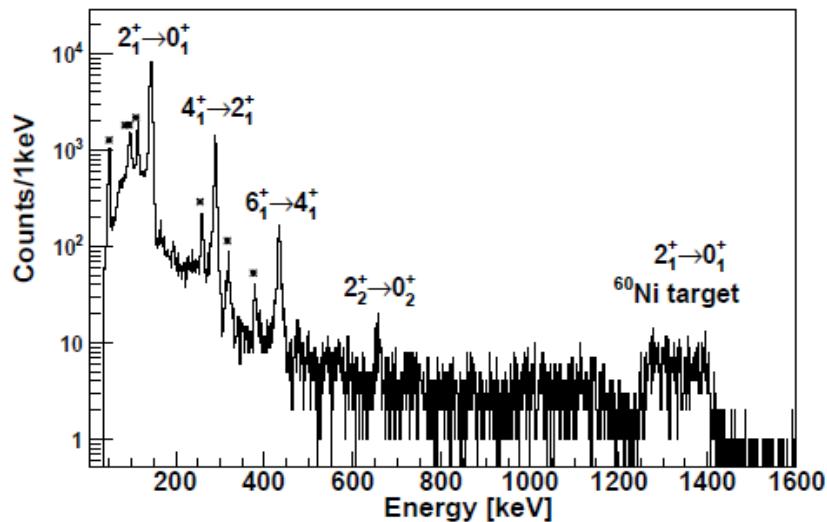


$^{96}\text{Sr} T_{1/2} = 1.07 \text{ sec. } 7000 \text{ pps at 275 MeV}$

$^{98}\text{Sr} T_{1/2} = 0.65 \text{ sec. } 60000 \text{ pps at 276 MeV}$



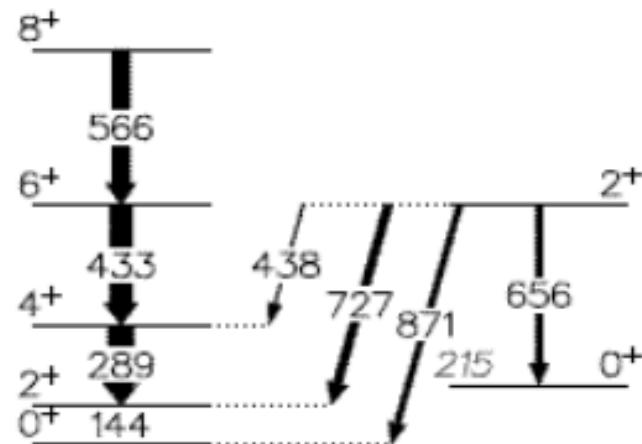
# Safe Coulomb excitation of $^{96,98}\text{Sr}$ beams at REX-ISOLDE using the MINIBALL array at CERN



$^{96}\text{Sr} T_{1/2} = 1.07$  sec. 7000 pps at 275 MeV  
 $^{98}\text{Sr} T_{1/2} = 0.65$  sec. 60000 pps at 276 MeV

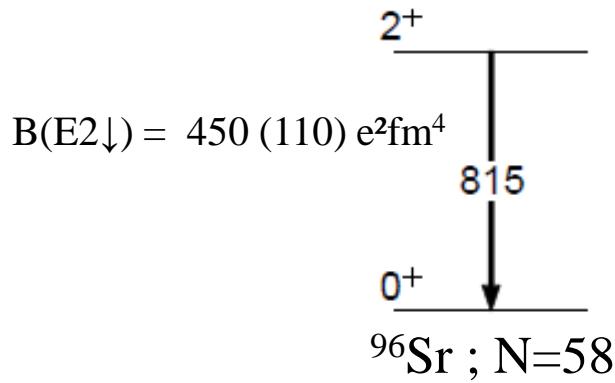
The Coulomb excitation cross section is analyzed using the least-squares fitting code GOSIA  
 T. Czosnyka, D. Cline, and C. Y. Wu, Bull. Am. Phys. Soc. **28**, 745 (1983).

$^{98}\text{Sr}$



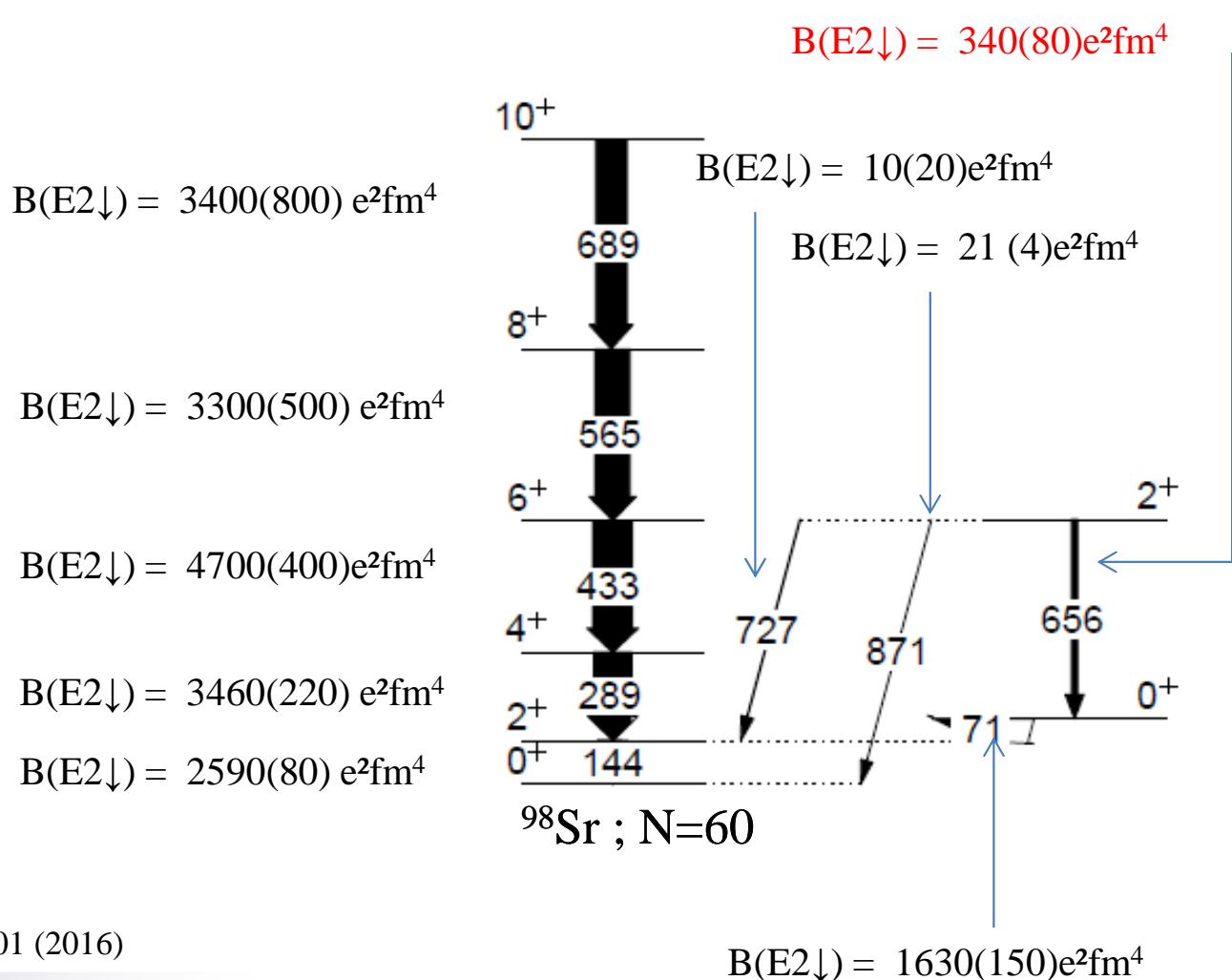
# Shape Coexistence in $^{96,98}\text{Sr}$

$$Q_s = -22(31) \text{ efm}^2$$



E. C. , M. Zielinska et al, Phys.Rev.Lett. 116, 022701 (2016)

E.Clément



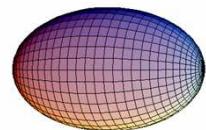
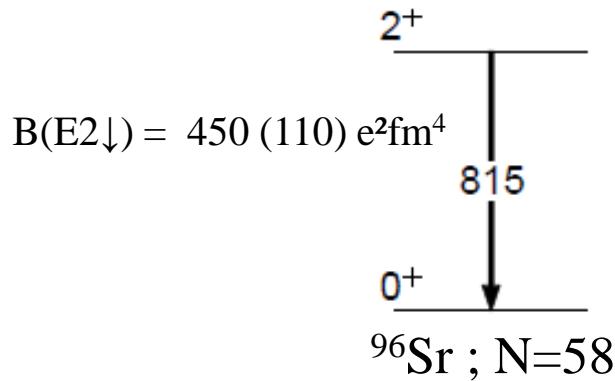
# Shape Coexistence in $^{96,98}\text{Sr}$

The  $2^+_1$  in  $^{96}\text{Sr}$  is weakly deformed

The ground state band in  $^{98}\text{Sr}$  has a large prolate deformation and the  $2^+_2$  is similar to the ground state in  $^{96}\text{Sr}$

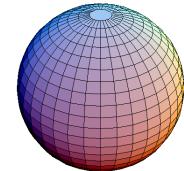
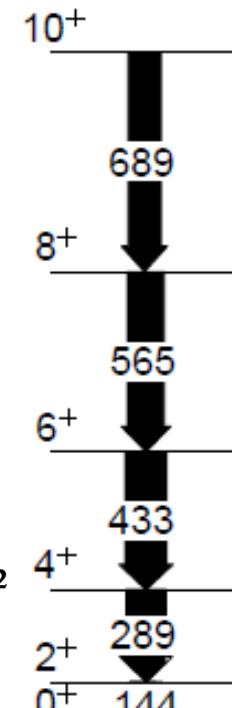
→ Shape coexistence in  $^{98}\text{Sr}$   
Shape inversion at N=60

$$Q_s = -22(31) \text{ efm}^2$$



$$\beta > 0.4$$

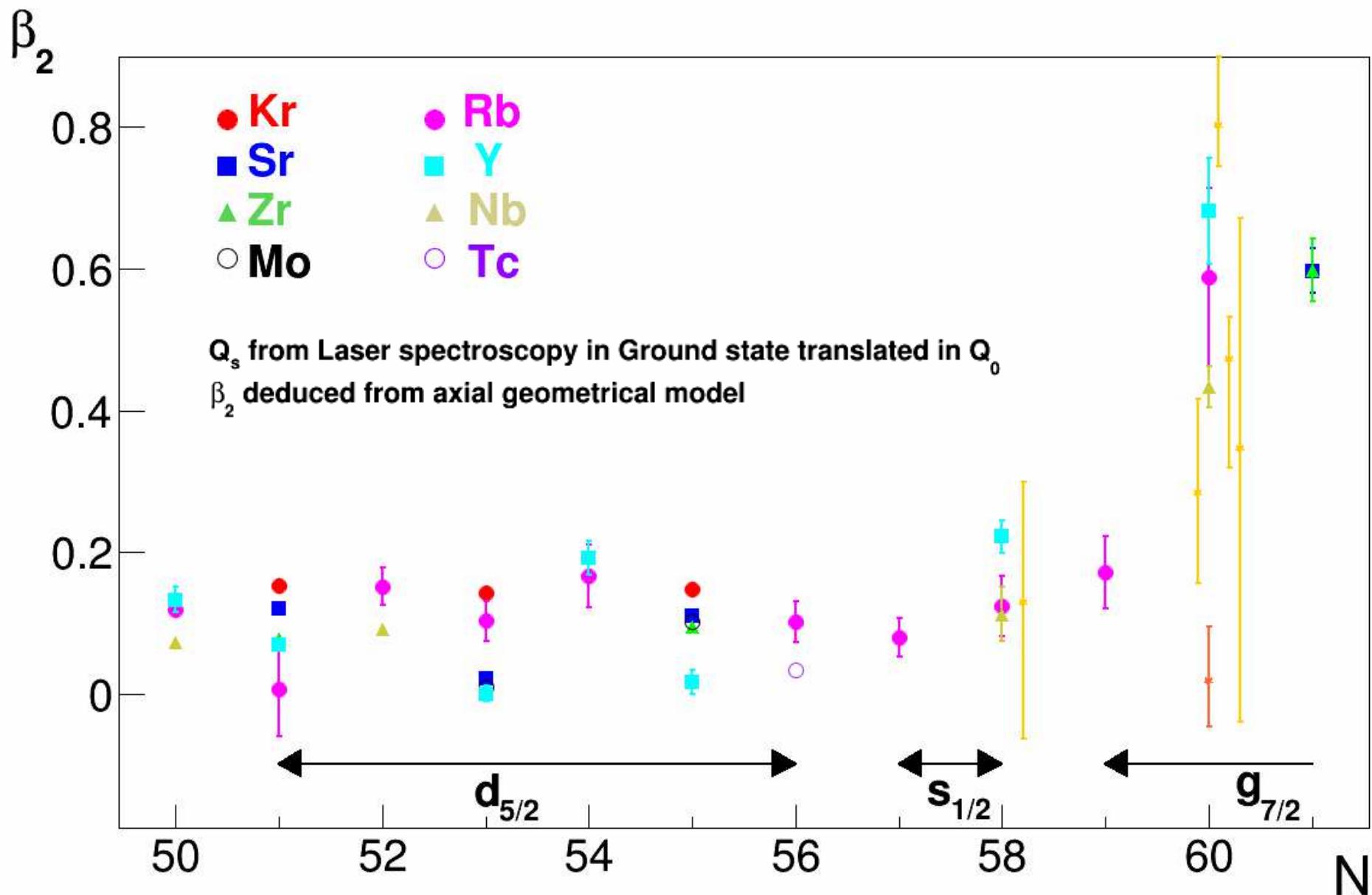
$$Q_s = -90(80) \text{ efm}^2$$



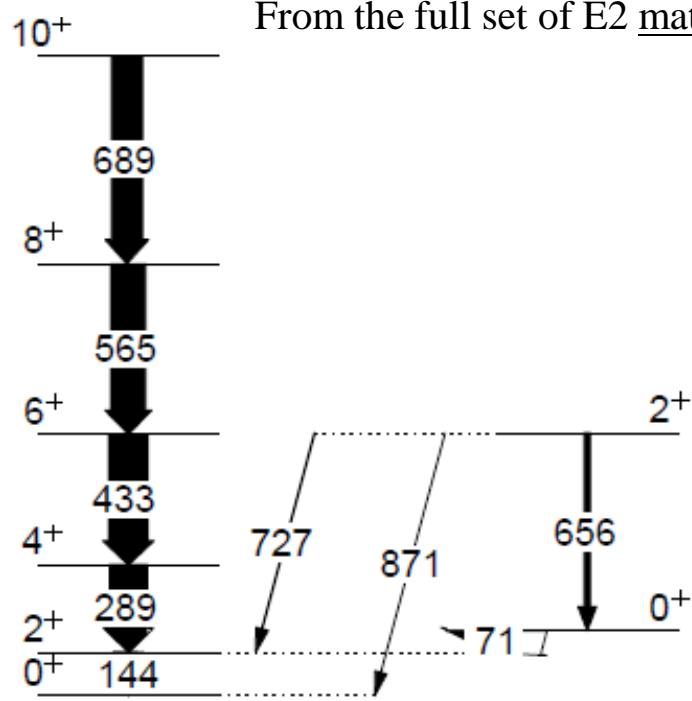
$$|\beta| < 0.2$$

# Shape Coexistence at N=60

*Comparison between Ground and Excited state Quadrupole moments*



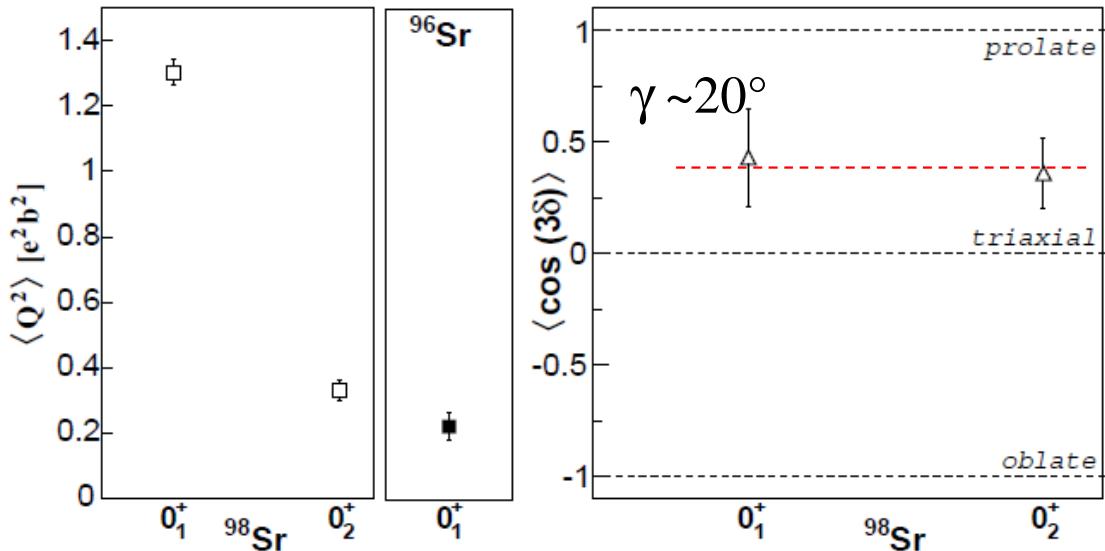
# Shape Coexistence in $^{96,98}\text{Sr}$



From the full set of E2 matrix elements,  $0^+$  states deformation can be probed using the QSR formalism

D. Cline, Ann. Rev. of Nucl. and Part. Sc. 36, 683 (1986)

J. Srebrny *et al.*, Int. J. Mod. Phys. E 20, 422 (2011).



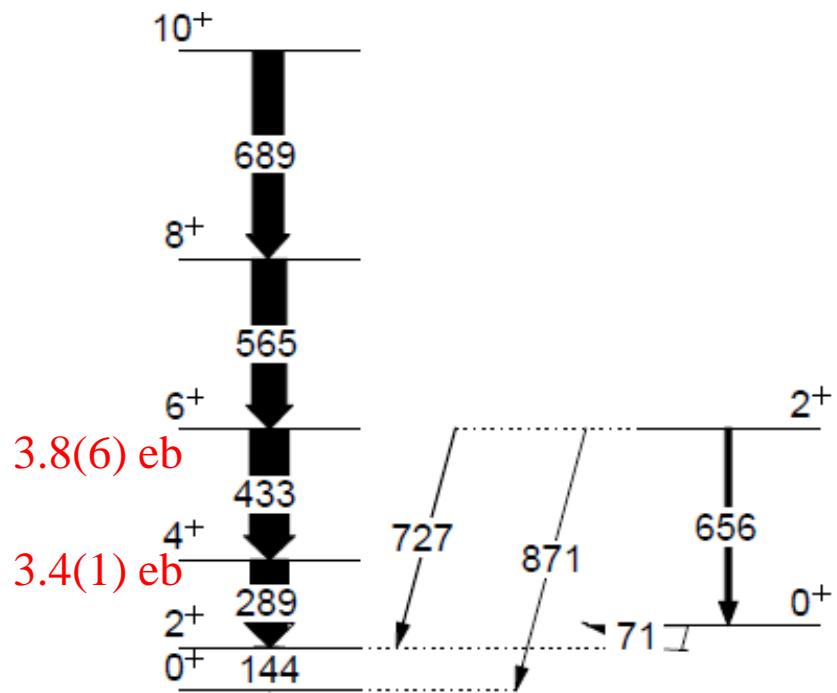
$Q^2$  : global deformation

$$\langle s | [E2 \times E2]_0 | s \rangle = \frac{1}{\sqrt{5}} Q^2 = \frac{(-1)^{2s}}{\sqrt{2s+1}} \sum_t \underbrace{\langle s | |E2| |t \rangle \langle t | |E2| |s \rangle}_{\text{red bracket}} \left\{ \begin{array}{ccc} 2 & 2 & 0 \\ s & s & t \end{array} \right\}$$

$\cos 3\delta$  : triaxiality parameter

$$\langle s | [[E2 \times E2]_2 \times E2]_0 | s \rangle = -\sqrt{\frac{2}{35}} Q^3 \cos(3\delta) = \frac{1}{2s+1} \sum_{t \neq u} \langle s | |E2| |t \rangle \langle t | |E2| |u \rangle \langle u | |E2| |s \rangle \left\{ \begin{array}{ccc} 2 & 2 & 2 \\ s & t & u \end{array} \right\}$$

# Shape Coexistence in $^{96,98}\text{Sr}$



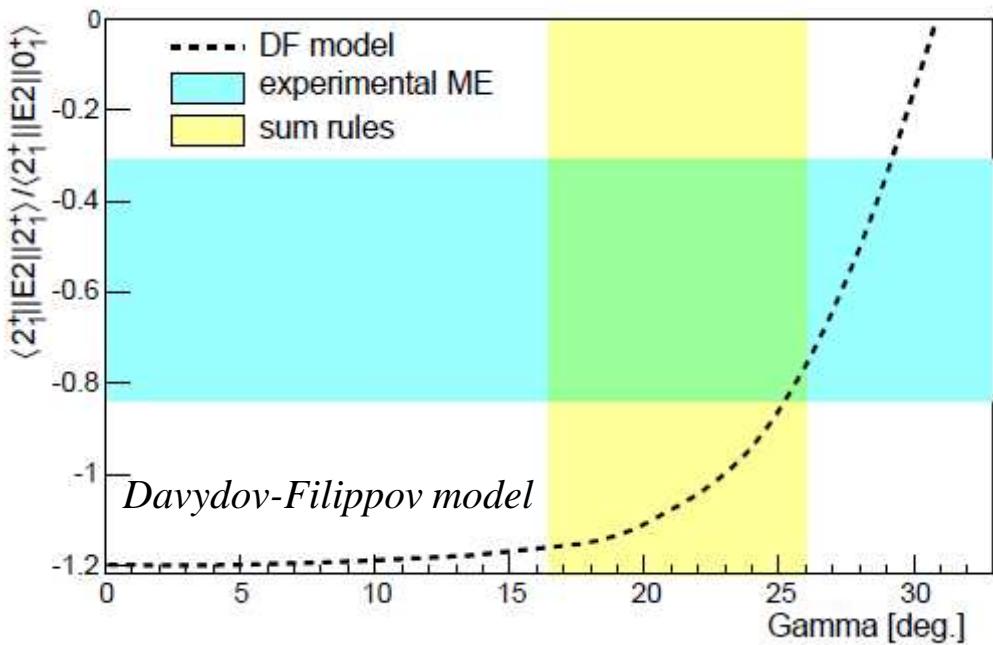
Shape coexistence in a two-state mixing model

$$\begin{aligned} |I_1\rangle &= + \cos \theta_I |I_{\text{pr}}\rangle + \sin \theta_I |I_{\text{ob}}\rangle \\ |I_2\rangle &= - \sin \theta_I |I_{\text{pr}}\rangle + \cos \theta_I |I_{\text{ob}}\rangle \end{aligned}$$

Perturbed states

Pure states

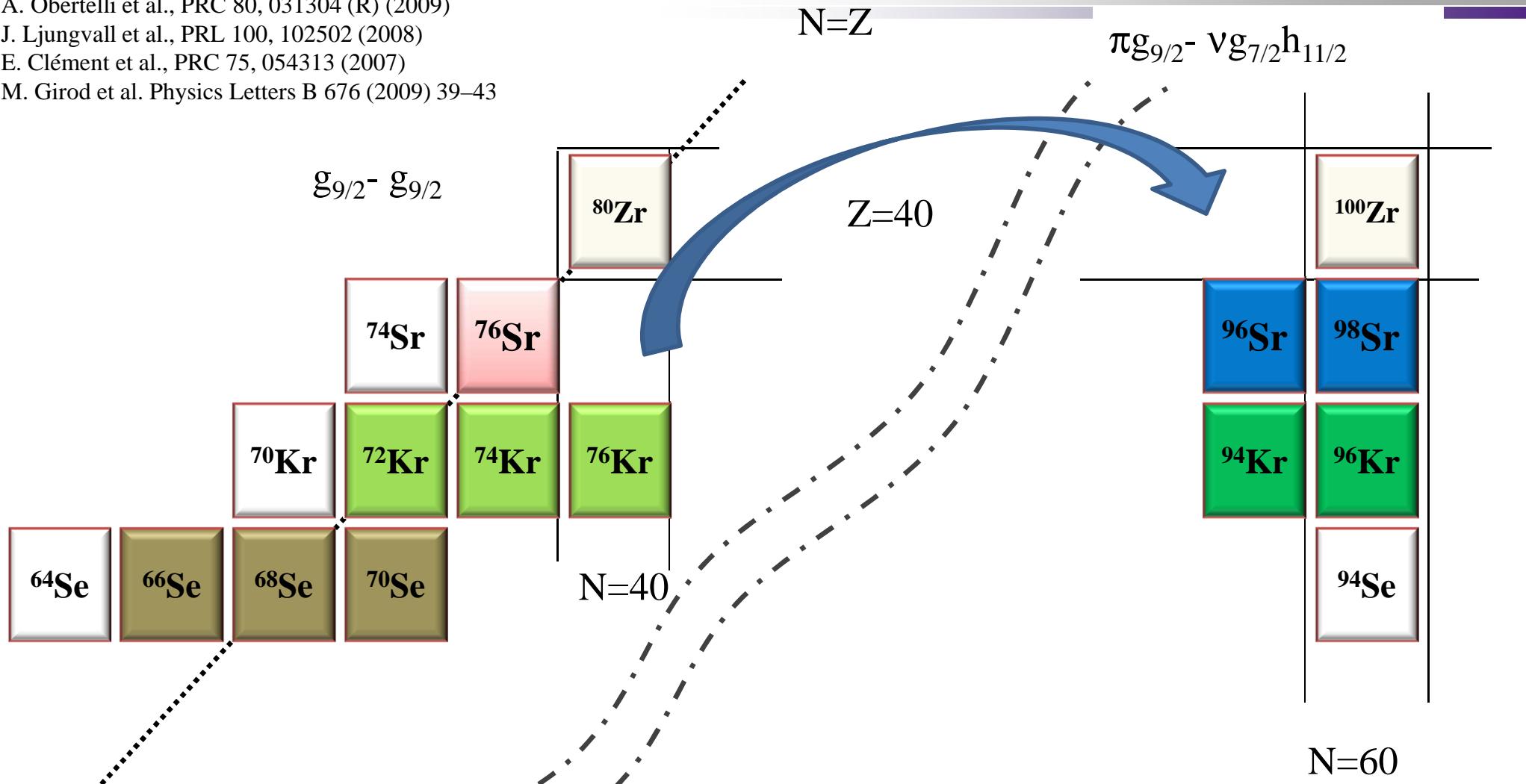
The sharp transition is related to the very weak mixing between competing configurations in contrast to the N~Z cases in Kr and Hg isotopes



$\cos^2 \theta_0$	$0.87(1)$	Un-perturbated
$\cos^2 \theta_2$	$0.99(1)$	
$Q_0^{\text{pr}}$	$+3.85(6)$ eb	exp
$Q_0^{\text{sph}}$	-0.5(3) eb	
$\langle 2_p^+    E2    2_p^+ \rangle$	-1.45(2) eb	$-0.63^{+0.32}_{-0.28}$
$\langle 2_s^+    E2    2_s^+ \rangle$	+0.18(10) eb	
		$+0.04^{+0.32}_{-0.20}$

Mean-field calculations using the Gogny interaction and the 5DCH approach have shown remarkable agreement with the experimental data in the N~Z~40

- A. Obertelli et al., PRC 80, 031304 (R) (2009)  
 J. Ljungvall et al., PRL 100, 102502 (2008)  
 E. Clément et al., PRC 75, 054313 (2007)  
 M. Girod et al. Physics Letters B 676 (2009) 39–43

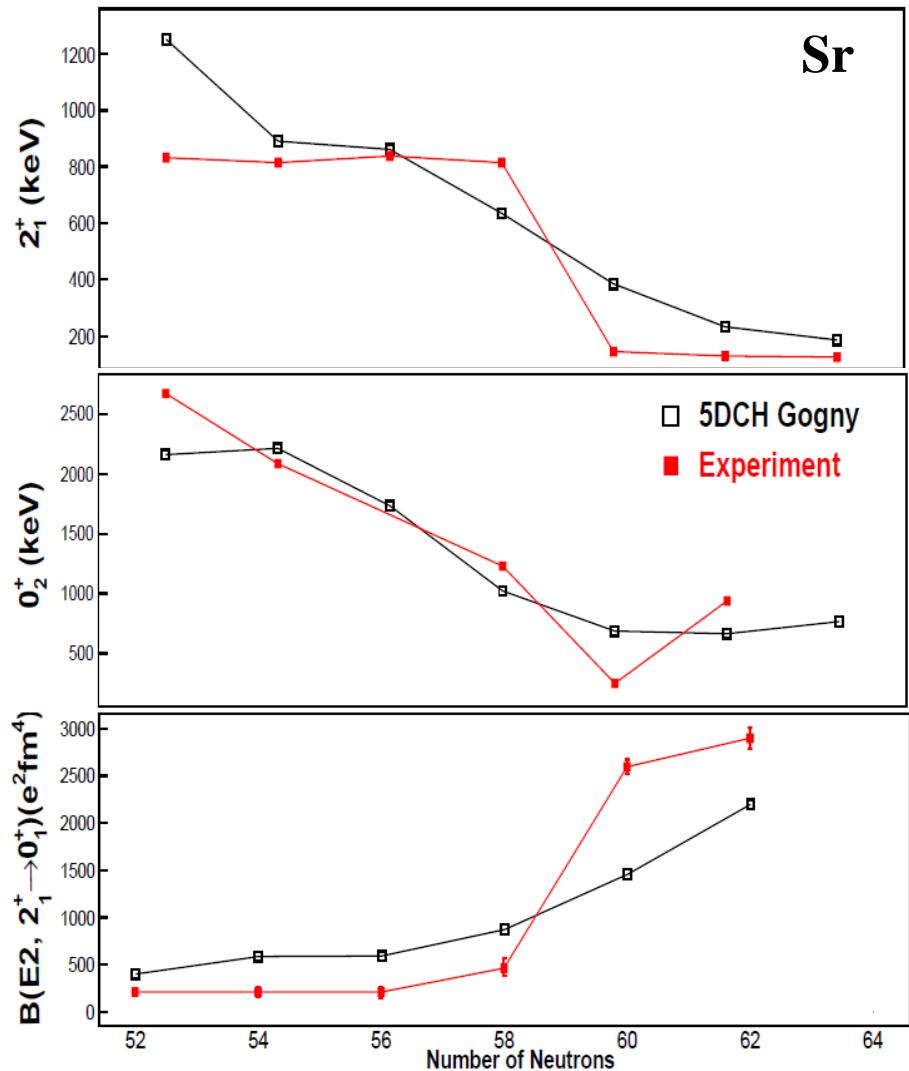


- In a simplified picture, we play with the  $\pi-\nu$  interaction by keeping the proton number equal and including beyond the  $vd_{5/2}$  up to the  $vh_{11/2}$  orbits

# Collectivity around N=60

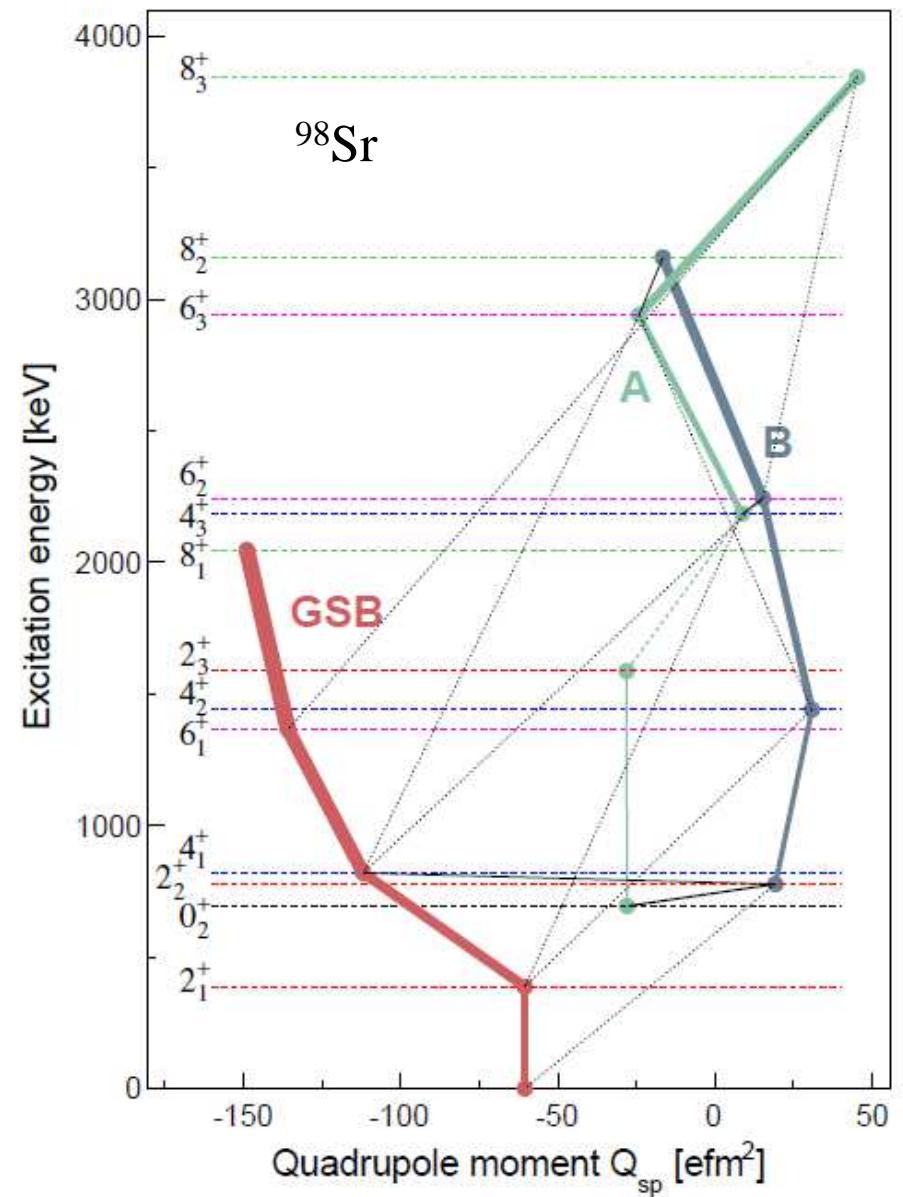
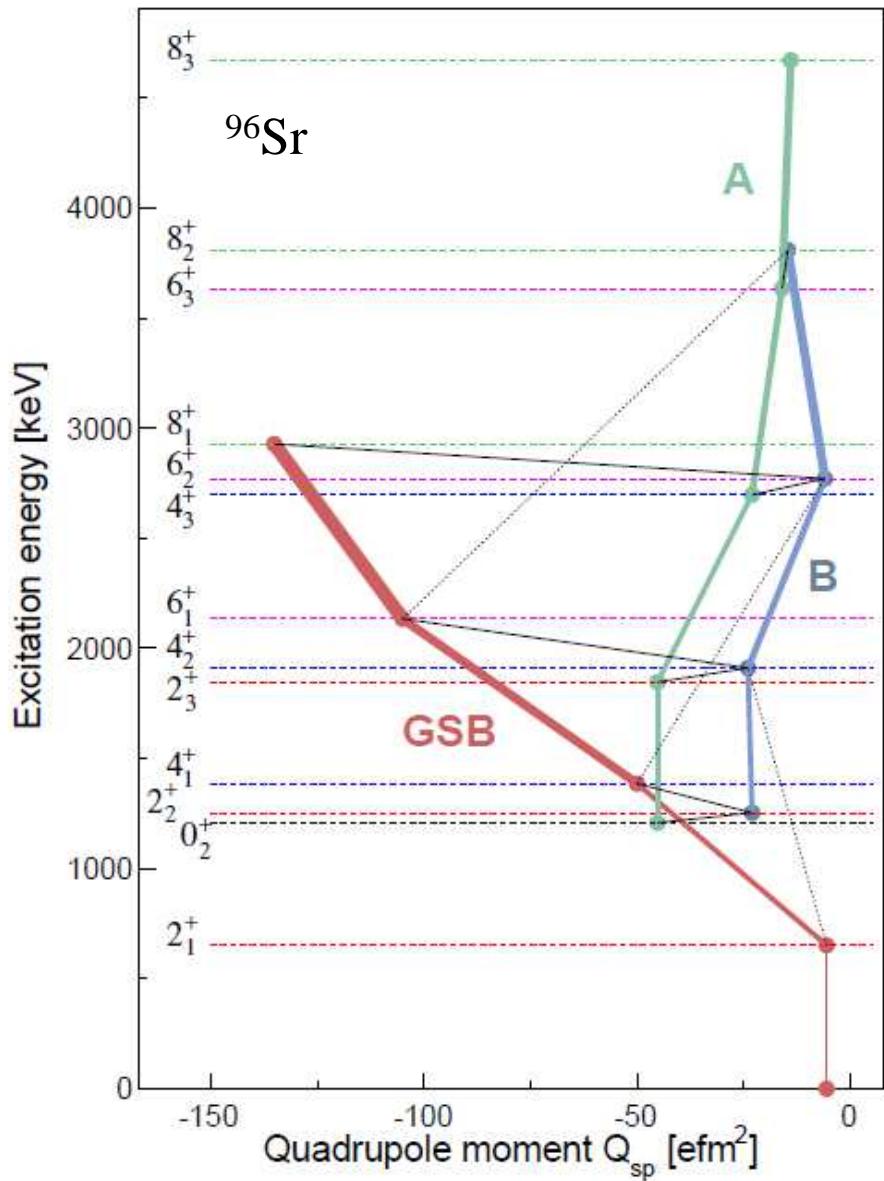
From a theoretical point of view

J. P. Delaroche *et al.*, Phys. Rev. C 81, 014303 (2010).



# Collectivity around N=60

From a theoretical point of view

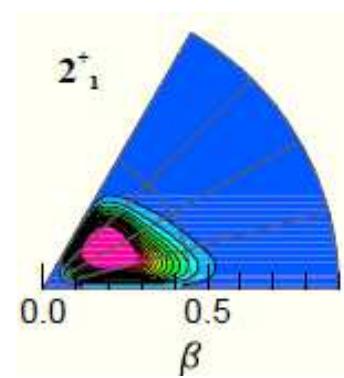
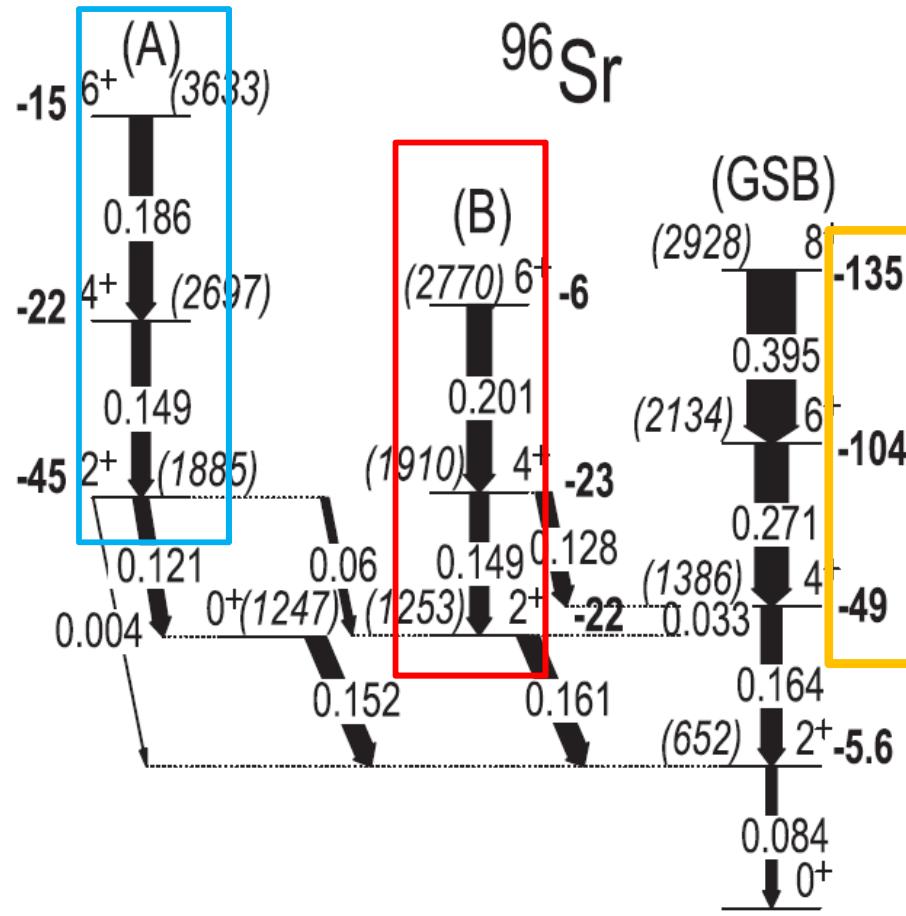
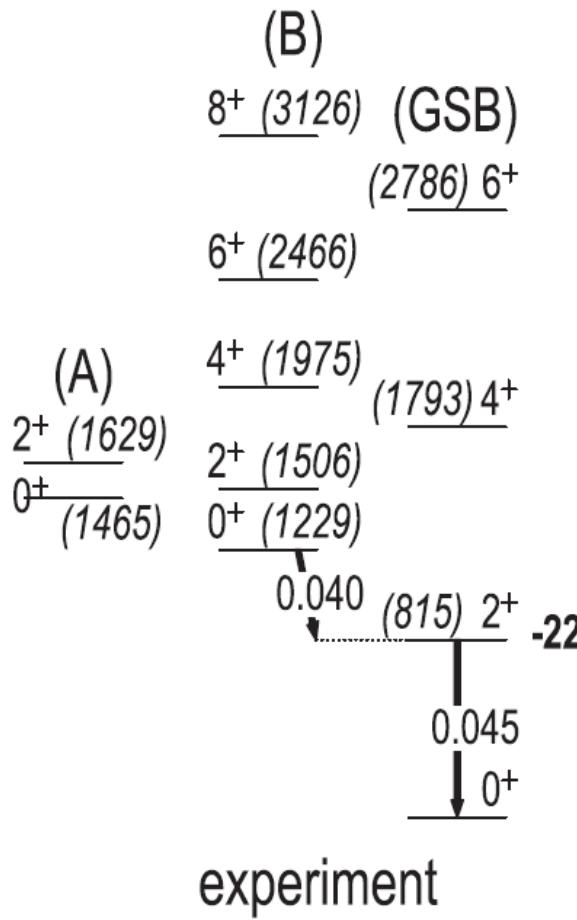


# Collectivity around N=60

From a theoretical point of view

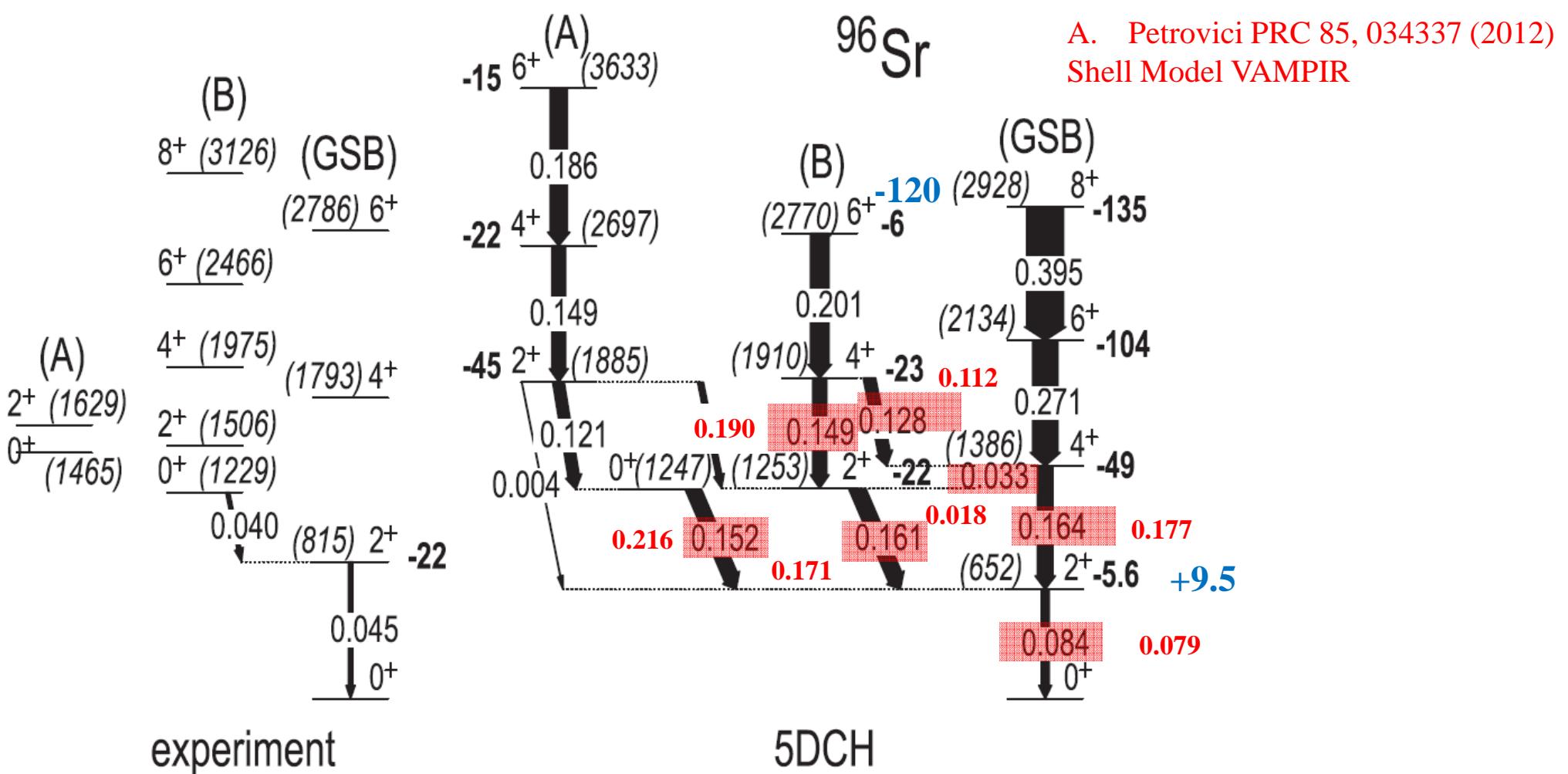
Large B(E2), higher Qs K=0 ~70%

Large B(E2), low Qs K=0 < 50%



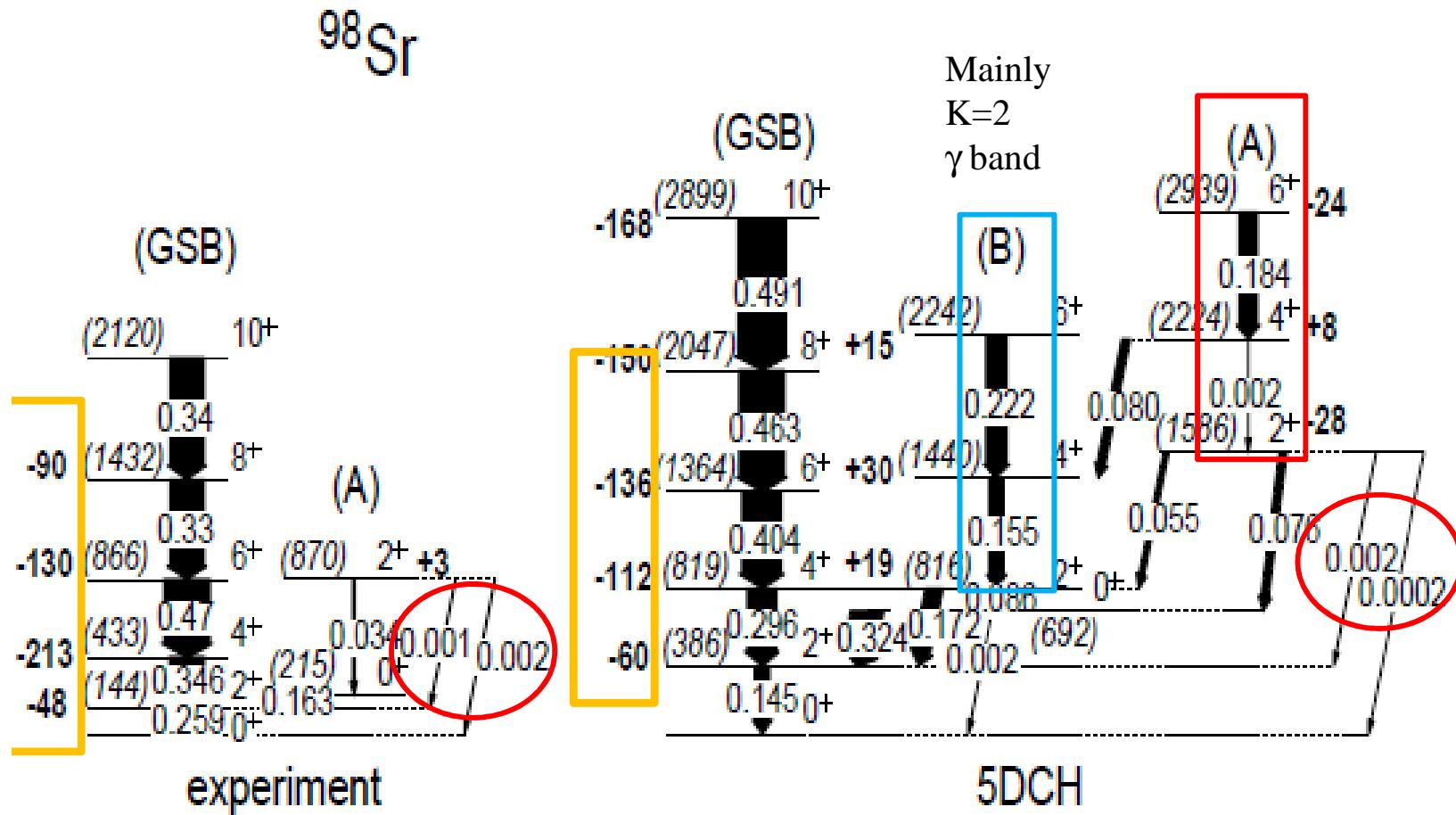
# Collectivity around N=60

*From a theoretical point of view*



# Collectivity around N=60

From a theoretical point of view



J. Xiang et al . Phys. Rev. C 93, 054324 (2016)

E. Clément, et PRL. 116, 022701 (2016)

E. Clément, M. Zielinska , S. Péru et al, Phys.Rev.C accepted (2016)

## <sup>96</sup>Sr

$I_1^\pi$	$I_2^\pi$	B( $E2, I_1 \rightarrow I_2$ ) (W.u.)		
		experiment	5DCH (Gogny)	Excited VAMPIR
2 <sub>1</sub> <sup>+</sup>	0 <sub>1</sub> <sup>+</sup>	17.3 <sup>+4.0</sup> <sub>-3.2</sub>	32	30
4 <sub>1</sub> <sup>+</sup>	2 <sub>1</sub> <sup>+</sup>		63	68
0 <sub>2</sub> <sup>+</sup>	2 <sub>1</sub> <sup>+</sup>	15.3(16) [10]	58	83
0 <sub>3</sub> <sup>+</sup>	2 <sub>1</sub> <sup>+</sup>	0.028(11) [11]		
2 <sub>2</sub> <sup>+</sup>	2 <sub>1</sub> <sup>+</sup>	> 8.9 [10]	62	65
4 <sub>1</sub> <sup>+</sup>	2 <sub>2</sub> <sup>+</sup>		13	7
4 <sub>2</sub> <sup>+</sup>	2 <sub>2</sub> <sup>+</sup>		57	73
4 <sub>2</sub> <sup>+</sup>	4 <sub>1</sub> <sup>+</sup>		49	47
$\rho^2(E0) (10^{-3})$				
0 <sub>2</sub> <sup>+</sup>	0 <sub>1</sub> <sup>+</sup>	106		66
0 <sub>3</sub> <sup>+</sup>	0 <sub>1</sub> <sup>+</sup>	22		
0 <sub>3</sub> <sup>+</sup>	0 <sub>2</sub> <sup>+</sup>	185(50)[13]	95	9

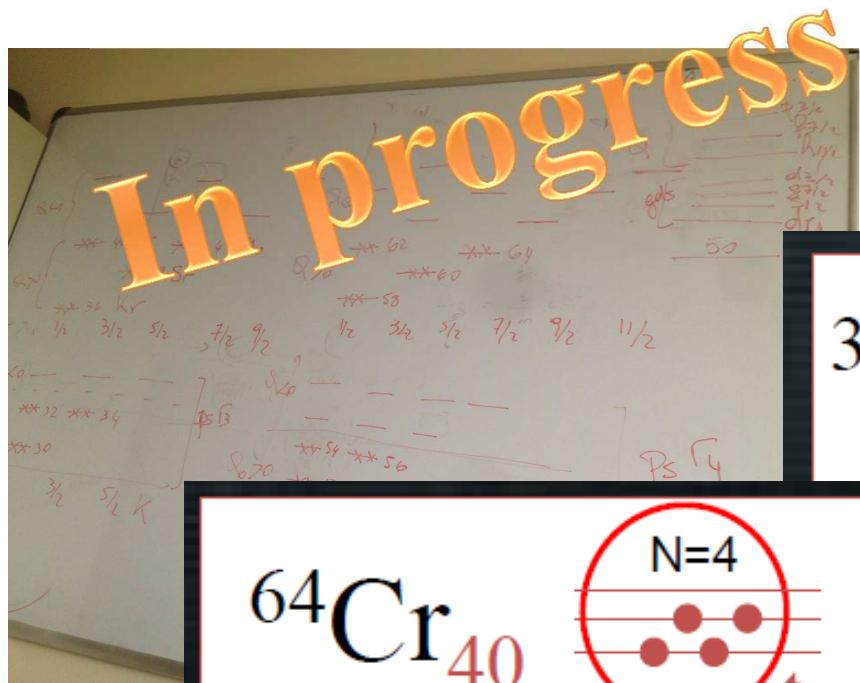
## <sup>98</sup>Sr

$I_1^\pi$	$I_2^\pi$	B( $E2, I_1 \rightarrow I_2$ ) (W.u.)		
		experiment	5DCH (Gogny)	5DCH (PC-PK1)
2 <sub>1</sub> <sup>+</sup>	0 <sub>1</sub> <sup>+</sup>	96 (3)	54	73.5
4 <sub>1</sub> <sup>+</sup>	2 <sub>1</sub> <sup>+</sup>	129 <sup>+8</sup> <sub>-7</sub>	110	162
6 <sub>1</sub> <sup>+</sup>	4 <sub>1</sub> <sup>+</sup>	175 <sup>+17</sup> <sub>-14</sub>	150	196
8 <sub>1</sub> <sup>+</sup>	6 <sub>1</sub> <sup>+</sup>	123 <sup>+19</sup> <sub>-14</sub>	173	211
2 <sub>2</sub> <sup>+</sup>	0 <sub>2</sub> <sup>+</sup>	13 (2)	28	39.2
0 <sub>2</sub> <sup>+</sup>	2 <sub>1</sub> <sup>+</sup>	61 (5)	120	195 <sup>a</sup>
2 <sub>2</sub> <sup>+</sup>	0 <sub>1</sub> <sup>+</sup>	0.77 (13)	0.07	
2 <sub>2</sub> <sup>+</sup>	2 <sub>1</sub> <sup>+</sup>	0.61 <sup>+0.22</sup> <sub>-0.30</sub>	0.78	
2 <sub>2</sub> <sup>+</sup>	4 <sub>1</sub> <sup>+</sup>	4 <sup>+4</sup> <sub>-2</sub>	19.4	
$\rho^2(E0) (10^{-3})$				
0 <sub>2</sub> <sup>+</sup>	0 <sub>1</sub> <sup>+</sup>	53(5)[21]	179	117
0 <sub>3</sub> <sup>+</sup>	0 <sub>1</sub> <sup>+</sup>		40	
0 <sub>3</sub> <sup>+</sup>	0 <sub>2</sub> <sup>+</sup>		75	

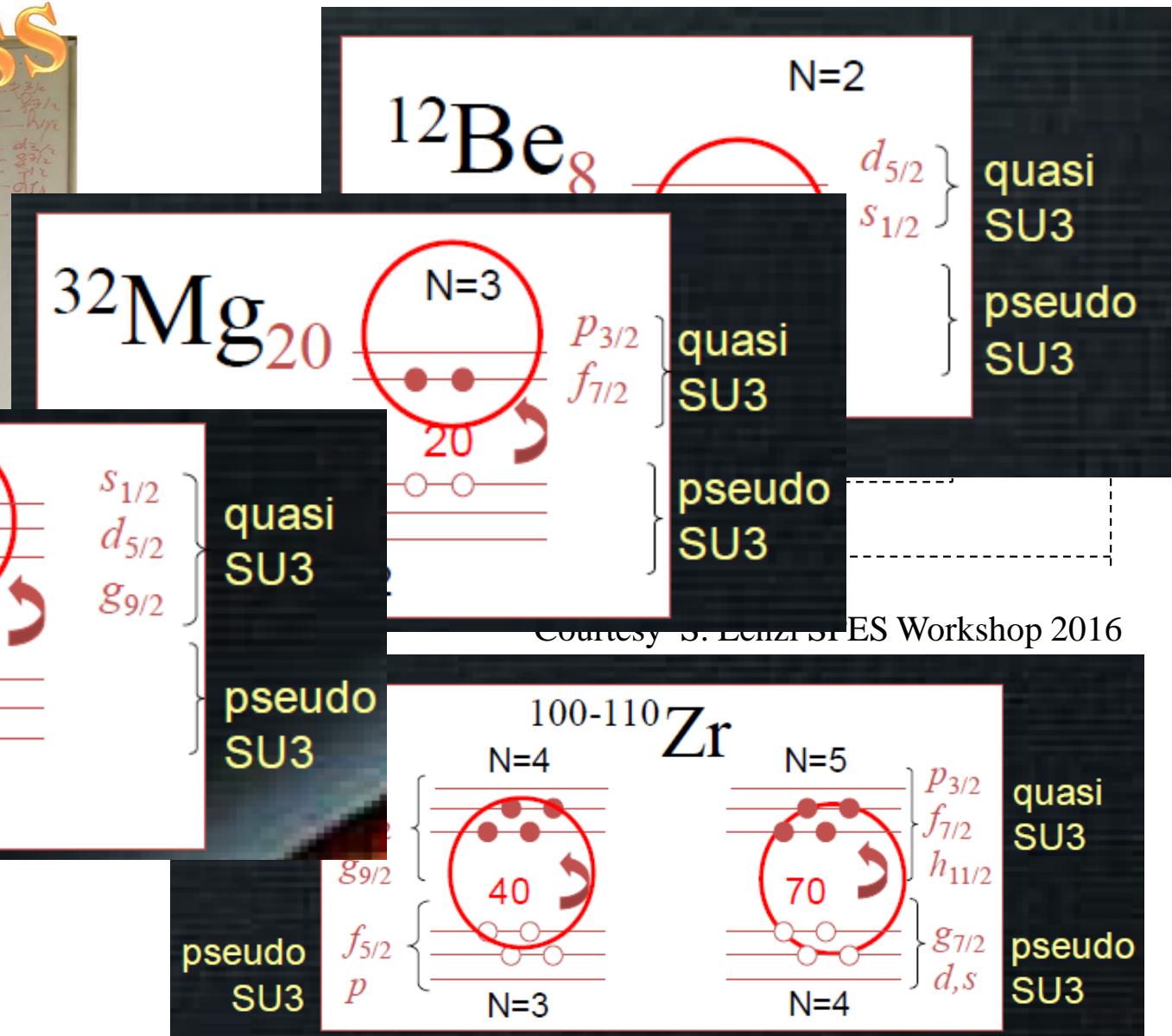
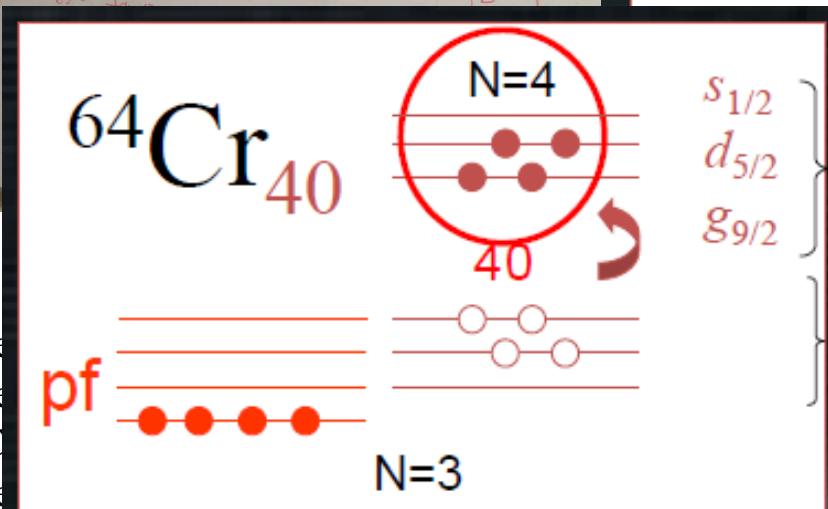
## Theory

<sup>96</sup> Sr, $\rho^2(E0) (10^{-3})$														
0 <sub>1</sub> <sup>+</sup>	0 <sub>2</sub> <sup>+</sup>	2 <sub>1</sub> <sup>+</sup>	2 <sub>2</sub> <sup>+</sup>	4 <sub>1</sub> <sup>+</sup>	4 <sub>2</sub> <sup>+</sup>	6 <sub>1</sub> <sup>+</sup>	6 <sub>2</sub> <sup>+</sup>	8 <sub>1</sub> <sup>+</sup>	8 <sub>2</sub> <sup>+</sup>					
0 <sub>2</sub> <sup>+</sup>	106	-	2 <sub>2</sub> <sup>+</sup>	28	-	4 <sub>2</sub> <sup>+</sup>	83	-	6 <sub>2</sub> <sup>+</sup>	82	-	8 <sub>2</sub> <sup>+</sup>	60	-
0 <sub>3</sub> <sup>+</sup>	22	95	2 <sub>3</sub> <sup>+</sup>	117	65	4 <sub>3</sub> <sup>+</sup>	113	43	6 <sub>3</sub> <sup>+</sup>	89	32	8 <sub>3</sub> <sup>+</sup>	44	44
<sup>98</sup> Sr, $\rho^2(E0) (10^{-3})$														
0 <sub>1</sub> <sup>+</sup>	0 <sub>2</sub> <sup>+</sup>	2 <sub>1</sub> <sup>+</sup>	2 <sub>2</sub> <sup>+</sup>	4 <sub>1</sub> <sup>+</sup>	4 <sub>2</sub> <sup>+</sup>	6 <sub>1</sub> <sup>+</sup>	6 <sub>2</sub> <sup>+</sup>	8 <sub>1</sub> <sup>+</sup>	8 <sub>2</sub> <sup>+</sup>					
0 <sub>2</sub> <sup>+</sup>	179	-	2 <sub>2</sub> <sup>+</sup>	81	-	4 <sub>2</sub> <sup>+</sup>	43	-	6 <sub>2</sub> <sup>+</sup>	27	-	8 <sub>2</sub> <sup>+</sup>	20	-
0 <sub>3</sub> <sup>+</sup>	40	75	2 <sub>3</sub> <sup>+</sup>	120	1	4 <sub>3</sub> <sup>+</sup>	0	16	6 <sub>3</sub> <sup>+</sup>	0	27	8 <sub>3</sub> <sup>+</sup>	2	32

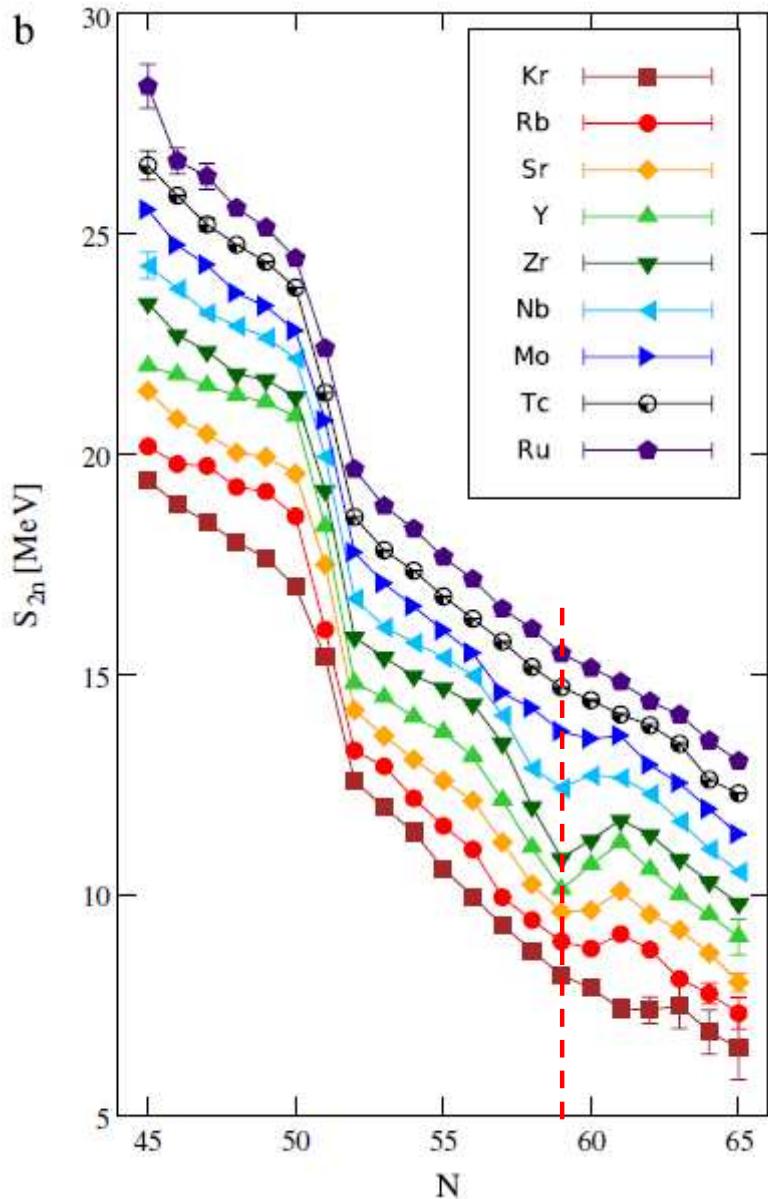
E. Clément, M. Zielinska, S. Péru et al, Phys.Rev.C accepted (2016)



A.P. Zuker  
 A.P. Zuker  
 PRC  
 A.P. Zuker  
 92, 024320 (2015)



# Conclusions



- We investigated the collectivity and the deformation in  $^{96,98}\text{Sr}$  at the shape transition using RIB and the Coulomb excitation technique at REX-ISOLDE, CERN
- First levels in  $^{98}\text{Rb}$
- E2 matrix elements have been extracted and establish shape coexistence between small and large prolate deformations that do not mix and give rise to a sharp transition at  $N=60$  where triaxiality plays a role
- *HFB+GCM Gogny force D1S* calculations reproduce the trend
- Large set of E0 transition strengths calculated
- Shell Model calculations (VAMPIR) show a nice agreement with BMF for  $B(E2)$  between low lying states
- The MCSM calculations are very promising

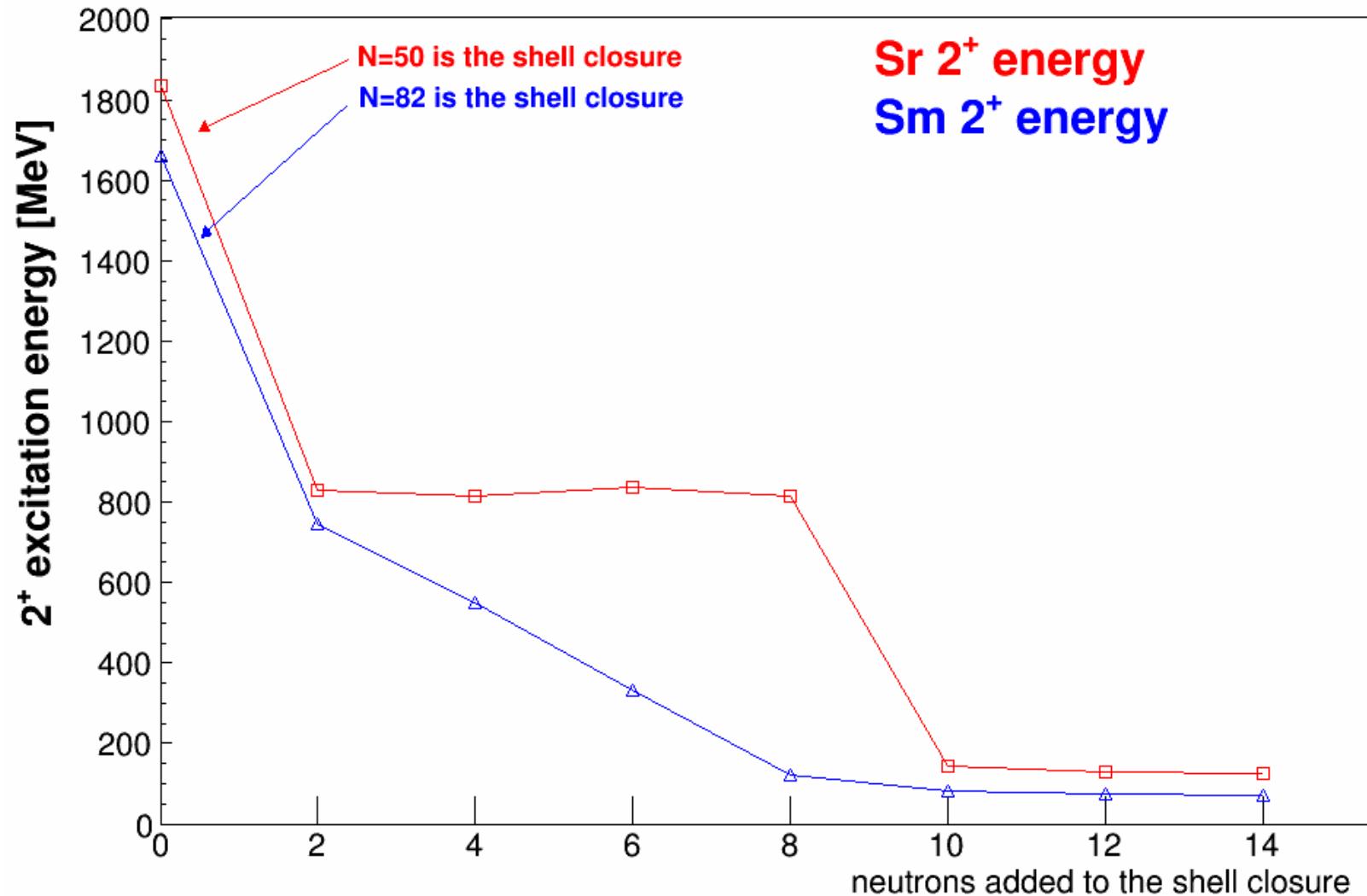
→ Why Kr behave differently ?

Fission runs at AGATA@GANIL (spectroscopy,  $T_{1/2}$ )  
ISOL facilities beams

→ Confusing predictions for  $^{96}\text{Sr}$  beyond the  $2^+_1$  ?

→ Pseudo and quasi -SU(3) approach is valid in this case, similarly to an island of inversion

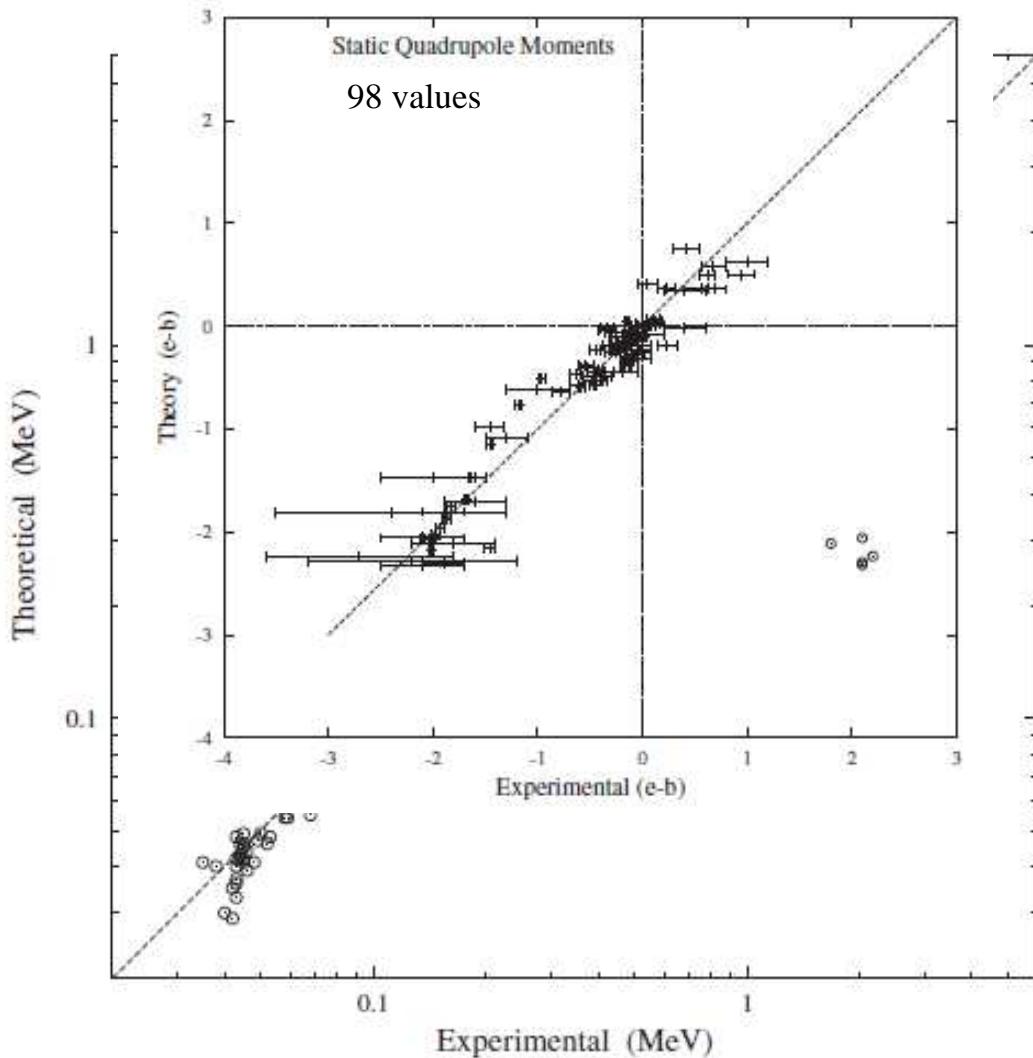
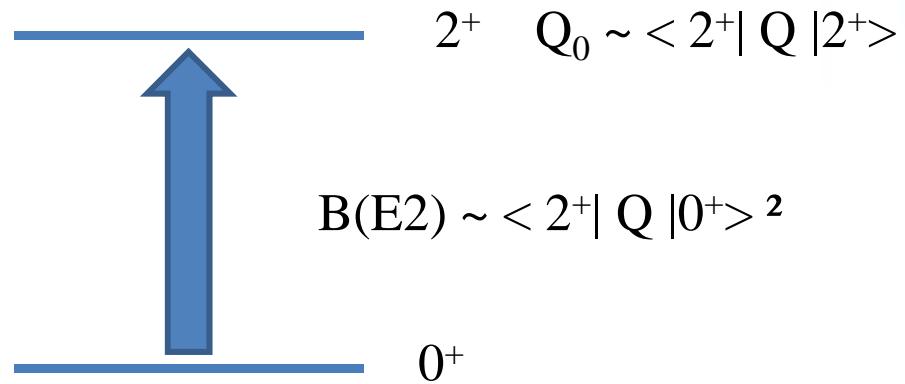




Best examples of the interplay between single-particle and collective modes of excitation in the nuclear matter

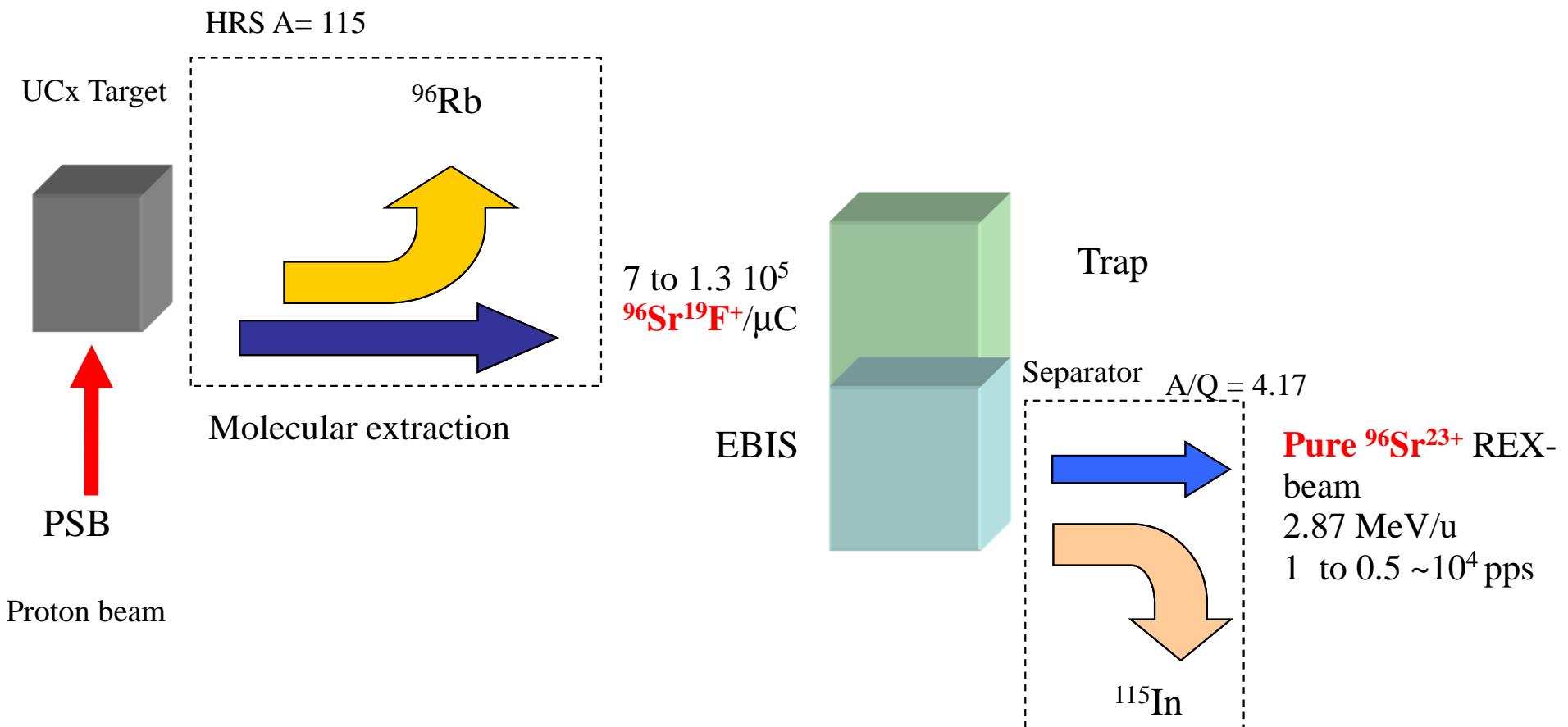
Important constraints for the nuclear models

Motivate the need for more spectroscopic data

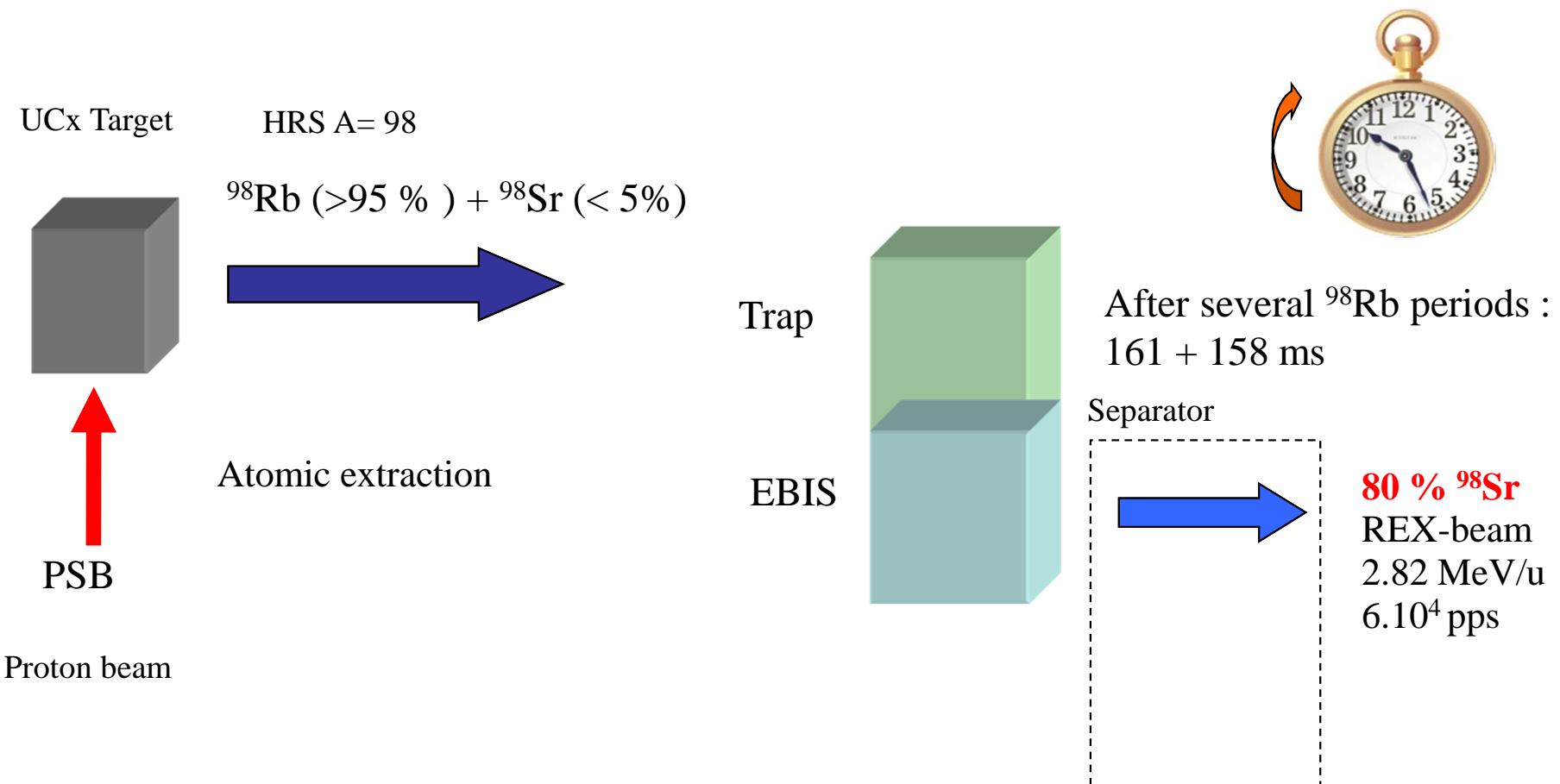


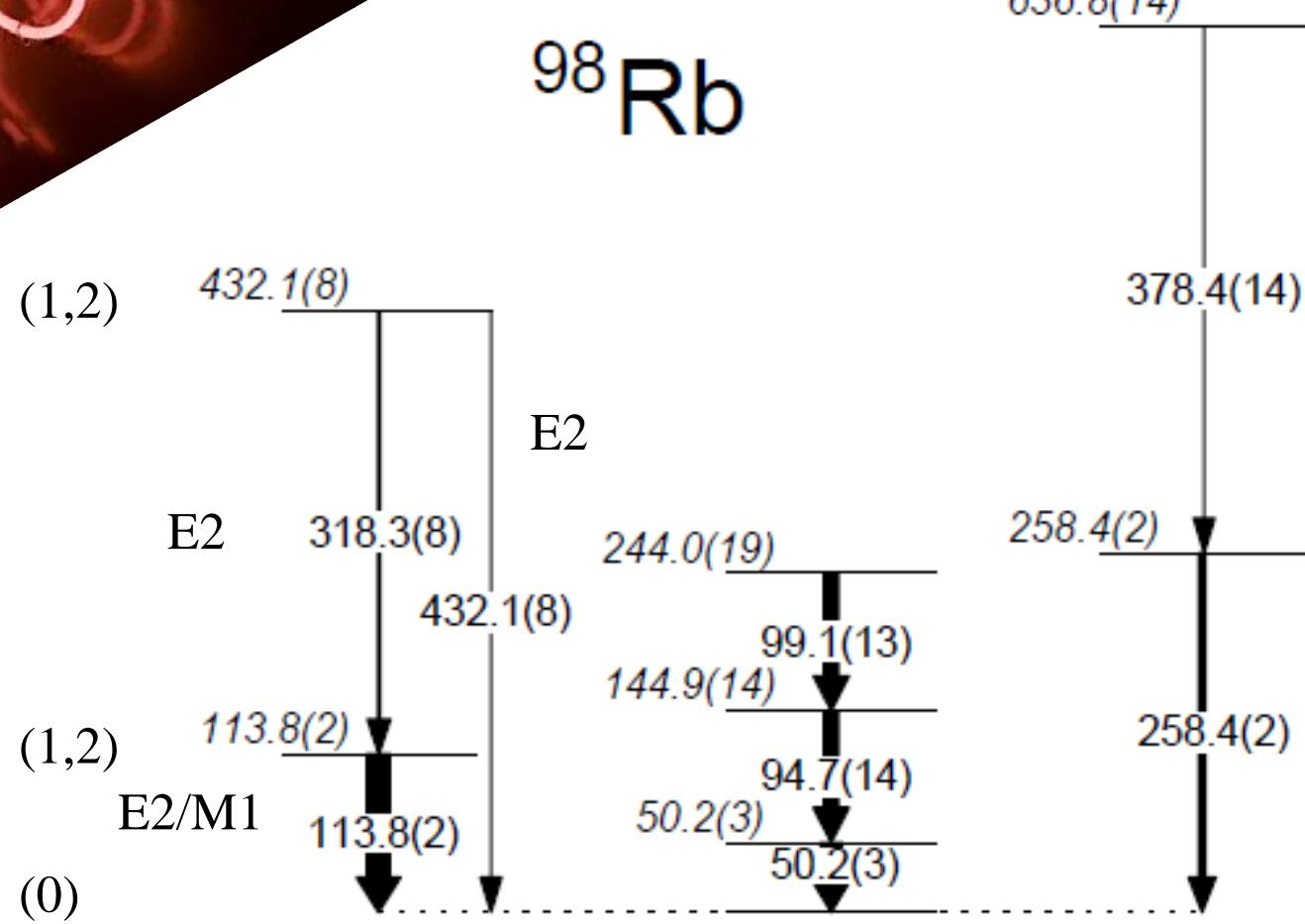
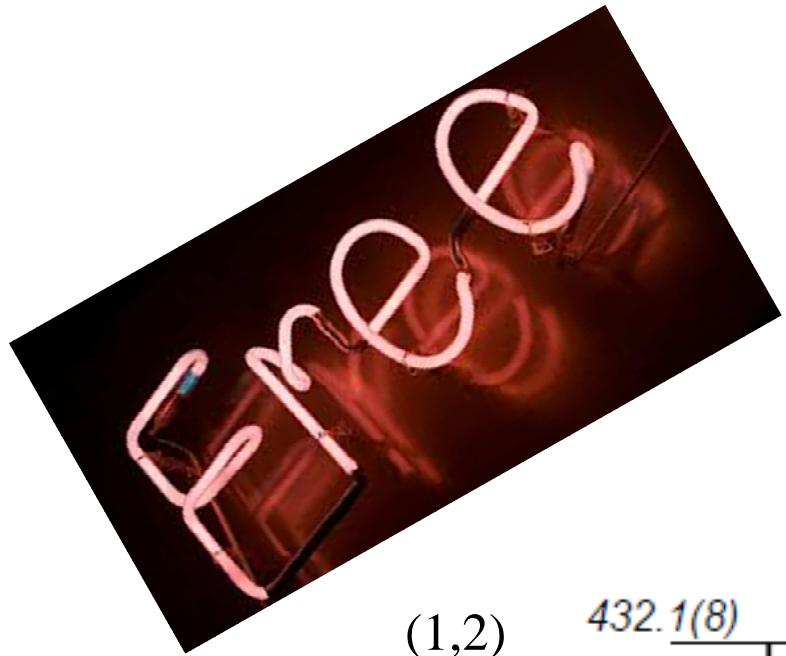
G. F. Bertsch, M. Girod, S. Hilaire, J.-P. Delaroche, H. Goutte, and S. Péru , PRL 99, 032502 (2007)

★ Post-accelerated radioactive  $^{96,98}\text{Sr}$  beam at REX-ISOLDE (2.8 MeV/u)



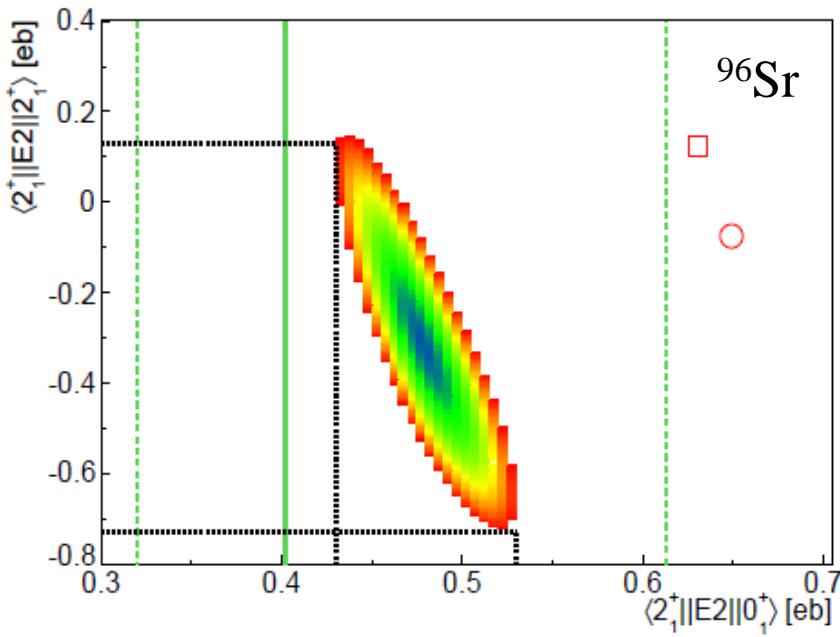
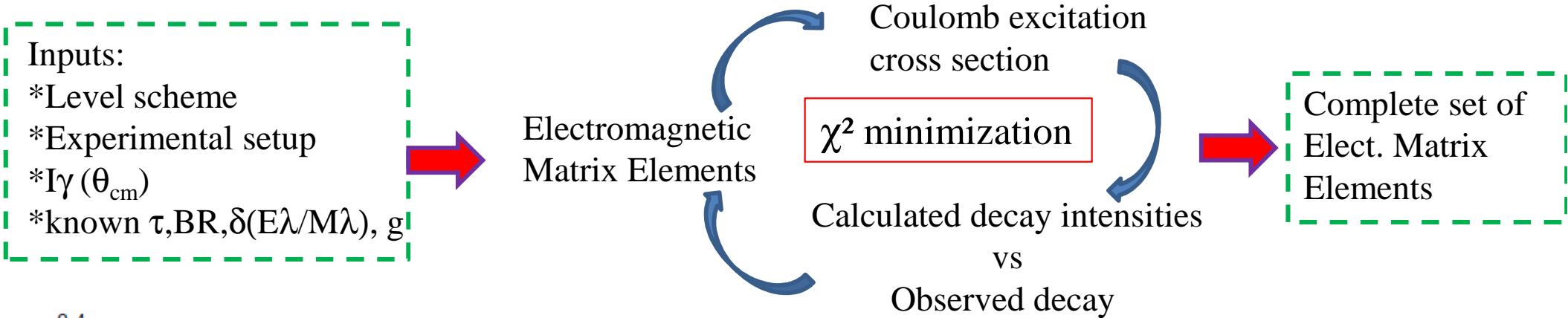
★ Post-accelerated radioactive  $^{96,98}\text{Sr}$  beam at REX-ISOLDE (2.8 MeV/u)



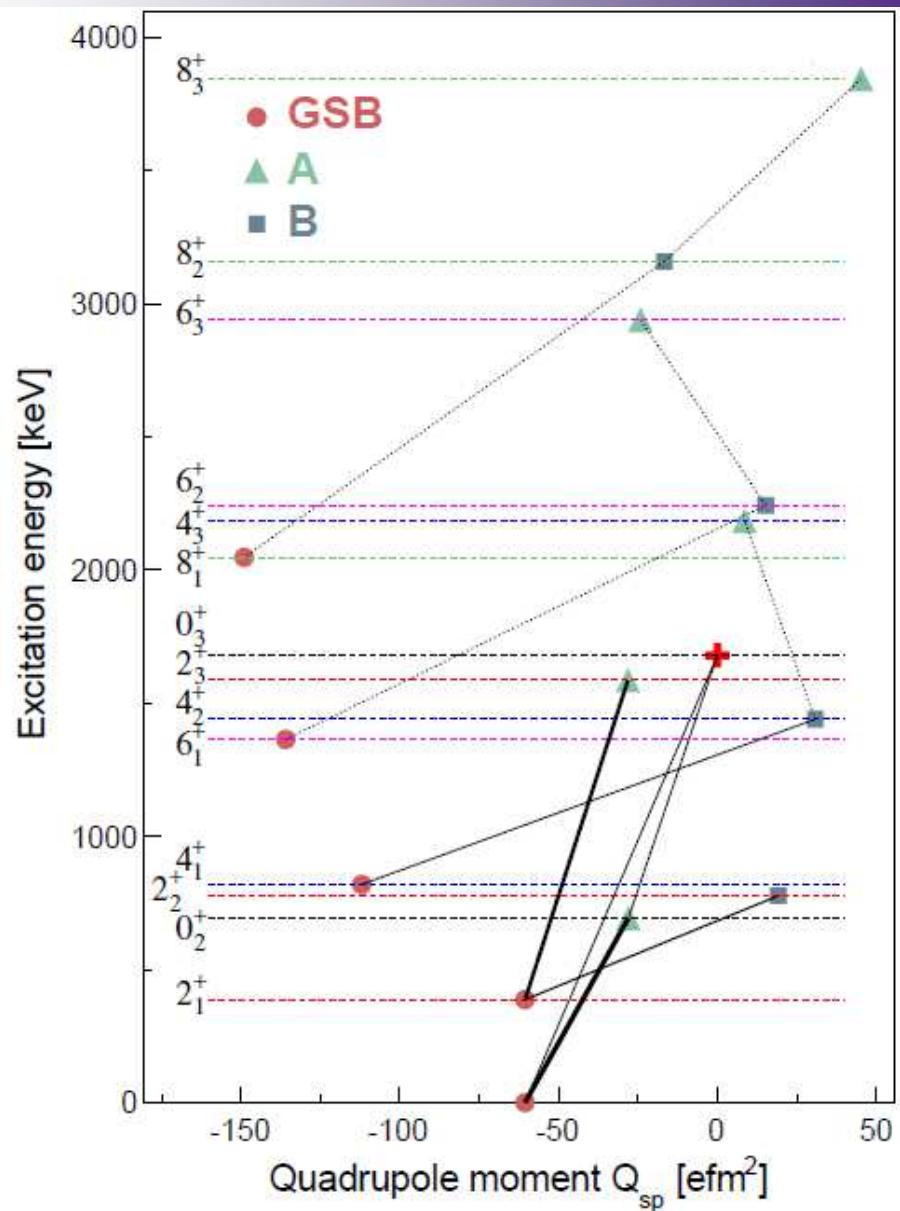
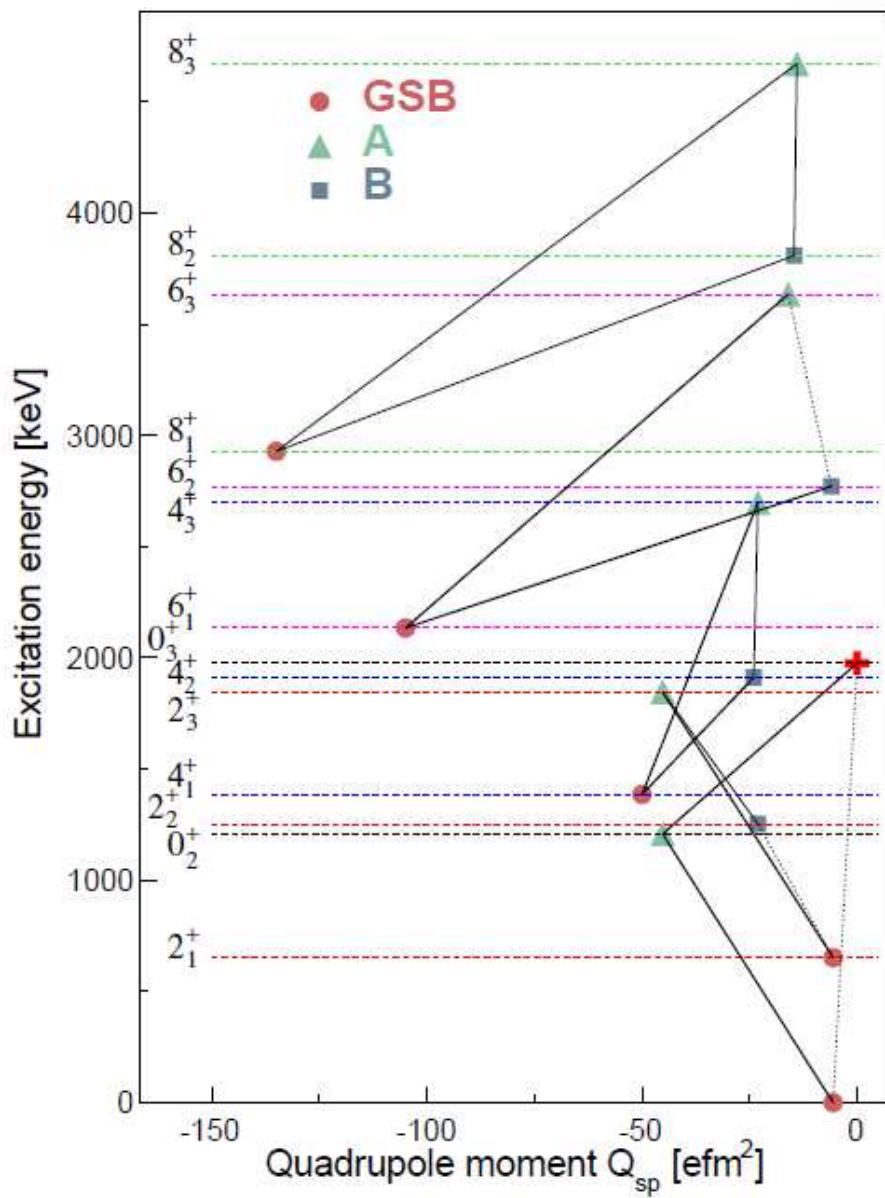


The Coulomb excitation cross section is analyzed using the least-squares fitting code GOSIA

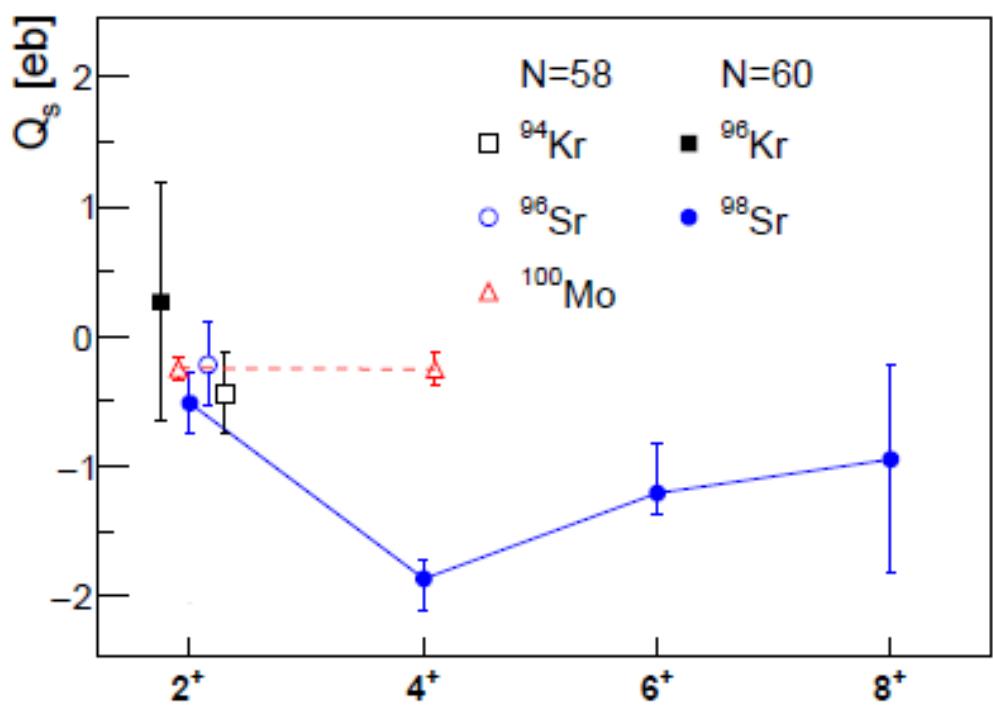
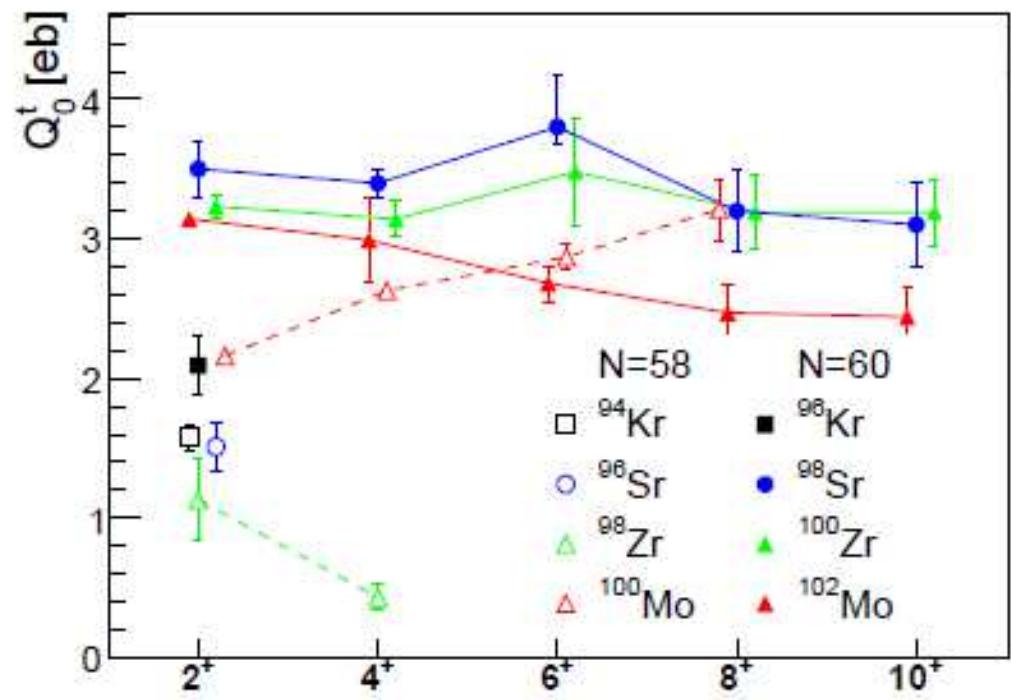
T. Czosnyka, D. Cline, and C. Y. Wu, Bull. Am. Phys. Soc. **28**, 745 (1983).



M. Zielinska et al Eur. Phys. J. A (2016) 52: 99

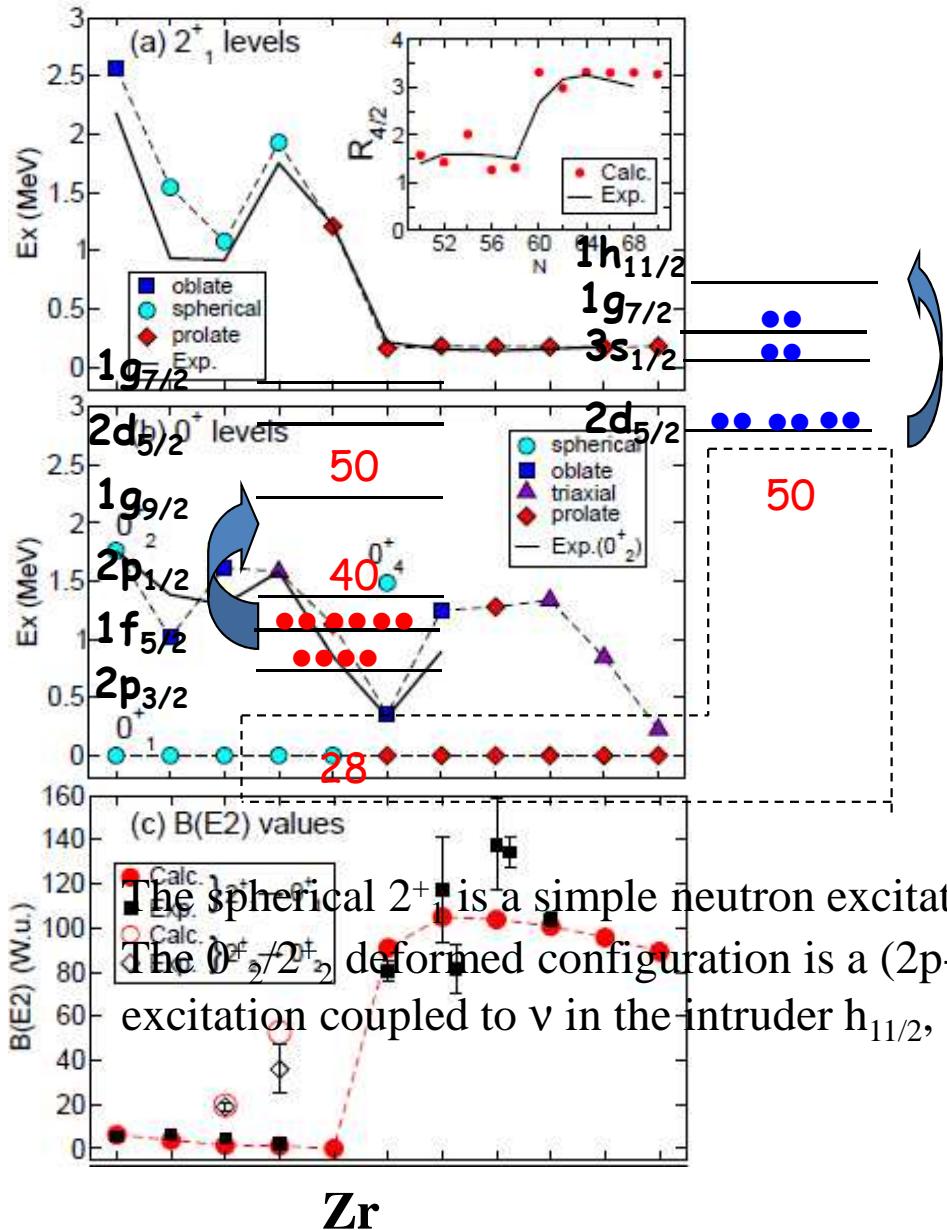


# Collectivity around N=60



# Collectivity around N=60

From a theoretical point of view



	$0^+_1$	$2^+_1$	$0^+_2$	$2^+_2$				
	$\pi$	$\nu$	$\pi$	$\nu$	$\pi$	$\nu$	$\pi$	$\nu$
$2f_{7/2}$	-	0.09	-	0.06	-	0.12	-	0.08
$3p_{3/2}$	-	0.01	-	0.01	-	0.01	-	0.01
$1h_{11/2}$	-	0.68	-	0.40	-	2.04	-	1.96
$1g_{7/2}$	0.14	0.09	0.11	0.06	0.15	1.38	0.15	1.37
$2d_{3/2}$	0.06	0.10	0.04	0.16	0.05	0.71	0.05	0.73
$3s_{1/2}$	0.01	0.20	0.01	1.11	0.03	0.41	0.03	0.38
$2d_{5/2}$	0.08	5.04	0.07	4.38	0.22	1.69	0.21	1.84
$1g_{9/2}$	0.51	9.79	0.42	9.81	3.49	9.64	3.40	9.63
$2p_{1/2}$	1.85	-	1.87	-	0.80	-	0.88	-
$2p_{3/2}$	3.59	-	3.69	-	2.54	-	2.60	-
$1f_{5/2}$	5.76	-	5.79	-	4.72	-	4.68	-

T. Togashi et al, Phys.Rev.Lett. 117, 172502 (2016)  
C. Kremer et al, Phys.Rev.Lett. 117, 172503 (2016)