Multipole modes of deformed superfluid nuclei with the finite amplitude method in three-dimensional coordinate space

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Introduction: Shape fluctuation

Goal: Constrained HFB + Local QRPA

Method: Finite amplitude method

Result: Isoscalar quadrupole strength

Result: Isoscalar/vector monopole

Result: Triaxial nucleus

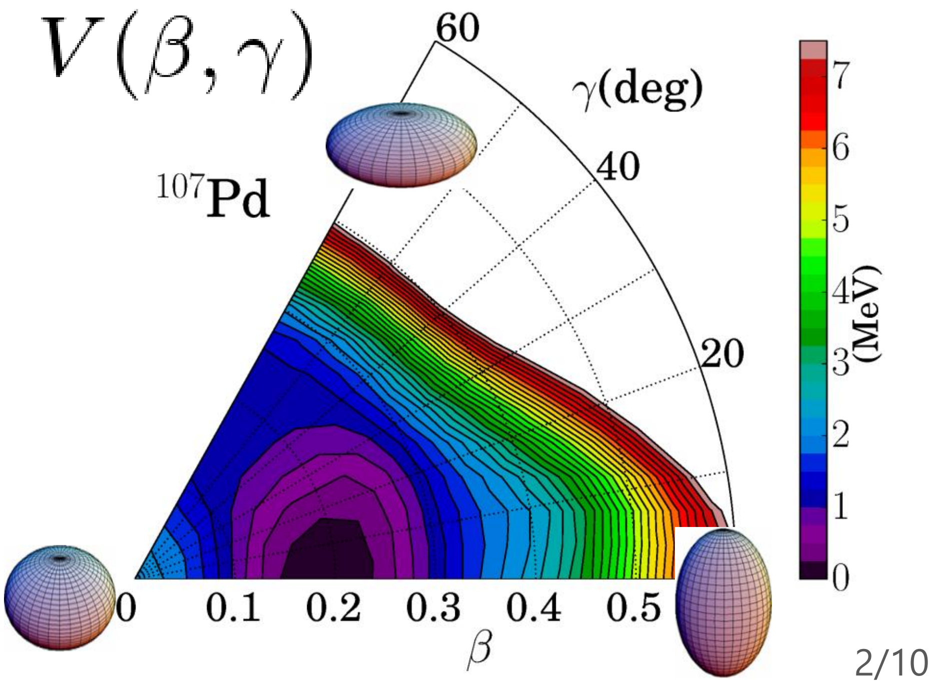
This work was funded by ImPACT Program of Council for Science, Technology and Innovation (Cabinet Office, Government of Japan)

Introduction: Shape fluctuation

A~100 nuclei

- Spherical ⟶ deformed, soft, transitional
- Excited states

Description of shape fluctuations is necessary



Goal: 5D collective (Bohr) Hamiltonian

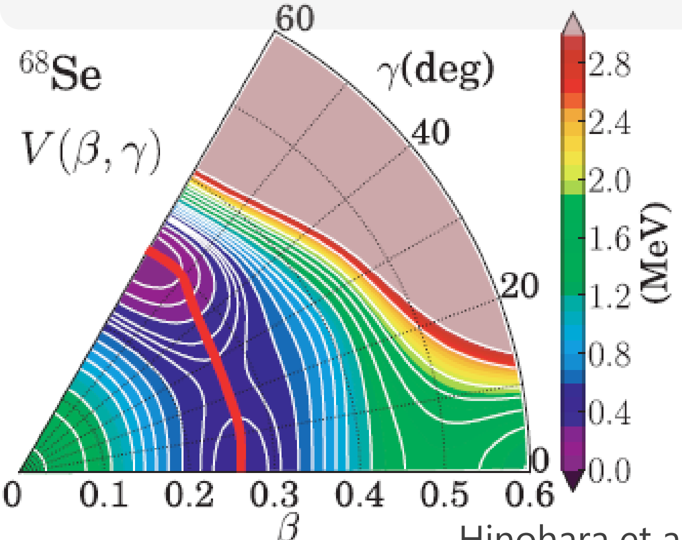
5D quadrupole collective Hamiltonian

$$\mathcal{H} = T_{\rm vib} + T_{\rm rot} + V(\beta,\gamma)$$

$$T_{\rm vib} = \frac{1}{2}D_{\beta\beta}(\beta,\gamma)\dot{\beta}^2 + D_{\beta\gamma}(\beta,\gamma)\dot{\beta}\dot{\gamma} + \frac{1}{2}D_{\gamma\gamma}(\beta,\gamma)\dot{\gamma}^2$$

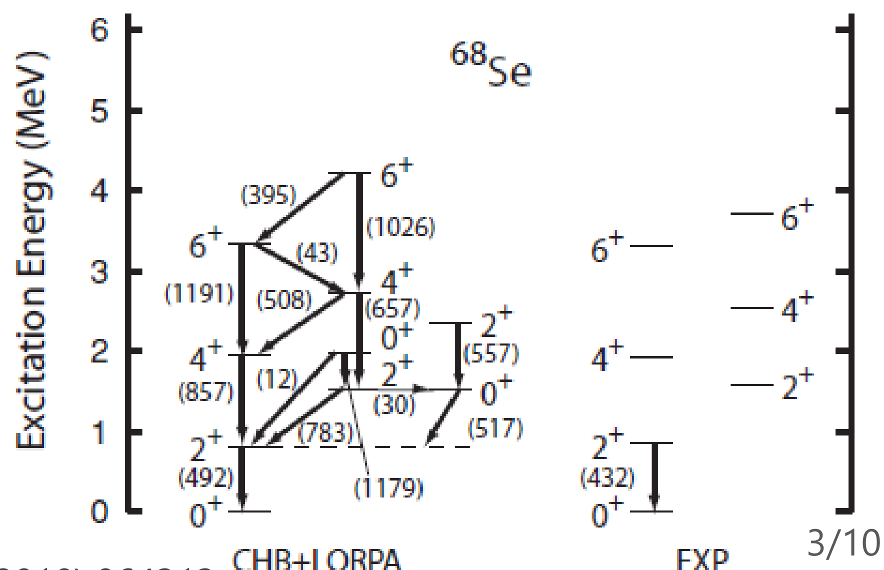
$$T_{\rm rot} = \frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k(\beta,\gamma)\omega_k^2$$

$V(\beta,\gamma)$, $D_{\mu\nu}(\beta,\gamma)$, $\mathcal{J}_k(\beta,\gamma)$



Quantization

Energy spectra



Hinohara et al., PRC82 (2010) 064313

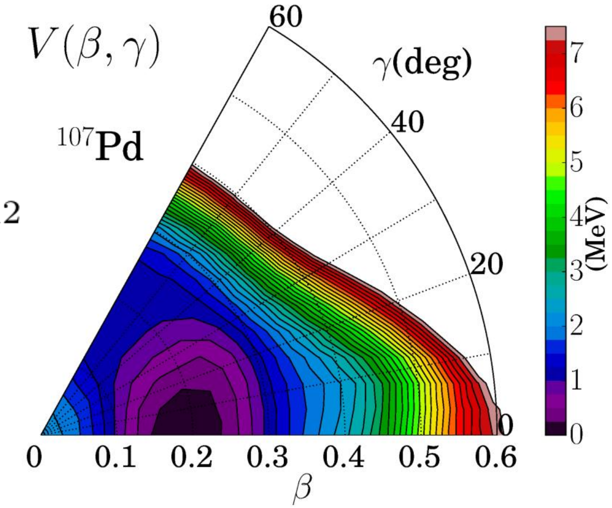
Goal: Constrained HFB + Local QRPA

5D quadrupole collective Hamiltonian

$$\mathcal{H} = T_{\text{vib}} + T_{\text{rot}} + V(\beta,\gamma)$$

$$T_{\text{vib}} = \frac{1}{2}D_{\beta\beta}(\beta,\gamma)\dot{\beta}^2 + D_{\beta\gamma}(\beta,\gamma)\dot{\beta}\dot{\gamma} + \frac{1}{2}D_{\gamma\gamma}(\beta,\gamma)\dot{\gamma}^2$$

$$T_{\text{rot}} = \frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k(\beta,\gamma)\omega_k^2$$



$V(\beta,\gamma)$ Constrained HFB with Skyrme energy density functional

Three-dimension in β-γ plane

$D_{\mu\nu}(\beta,\gamma)$
$\mathcal{J}_k(\beta,\gamma)$ Local QRPA: Finite Amplitude Method

Efficient method with a reasonable computational cost

3D QRPA is necessary for β-γ dynamics

Method: Quasi-particle RPA (QRPA)

QRPA equation

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ -Y \end{pmatrix}$$

1. Construct A and B matrix

   $$A_{minj} = (\varepsilon_m - \varepsilon_i)\delta_{mn}\delta_{ij} + \frac{\partial h_{mi}}{\partial \rho_{nj}} \qquad B_{minj} = \frac{\partial h_{mi}}{\partial \rho_{jn}}$$ (for RPA)

2. Diagonalize A B matrix to obtain ω and (X,Y) amplitude

- Time-consuming computation $\frac{\delta h}{\delta \rho}$ (residual interaction)
- Diagonalization of big matrix A B (~$10^{5\text{-}6}$)

Method: Finite amplitude method

QRPA equation

$$(E_\mu + E_\nu - \omega)X_{\mu\nu} + \delta H^{20}(\omega) = -F^{20}_{\mu\nu}$$
$$(E_\mu + E_\nu + \omega)Y_{\mu\nu} + \delta H^{02}(\omega) = -F^{02}_{\mu\nu}$$

Nakatsukasa et al., PRC76 (2007) 024318
Avogadro & Nakatsukasa, PRC84(2011)014314
Stoitsov et al., PRC84 (2011) 041305
Liang et al., PRC87 (2013) 054310
Niksic et al., PRC88 (2013) 044327
Pei et al., PRC90 (2014) 051304
Kortelainen et al., PRC92(2015)051302

Finite amplitude method (FAM)

$$\delta h = \frac{\delta h}{\delta \rho}\delta\rho \longrightarrow \qquad \delta h(\omega) = \frac{h[\rho_0 + \delta\rho] - h[\rho_0]}{\eta}$$

Advantages:

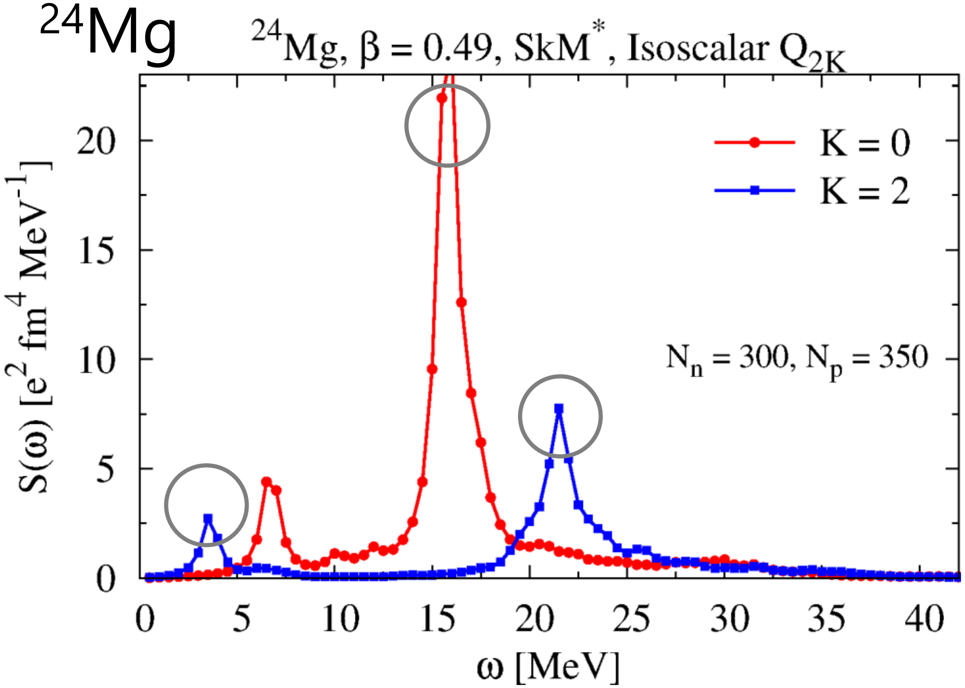
- Avoid computing $\frac{\delta h}{\delta \rho}$
- δh can be computed by static HFB codes
- Avoid diagonalizing A, B: Iterative method

Setups:

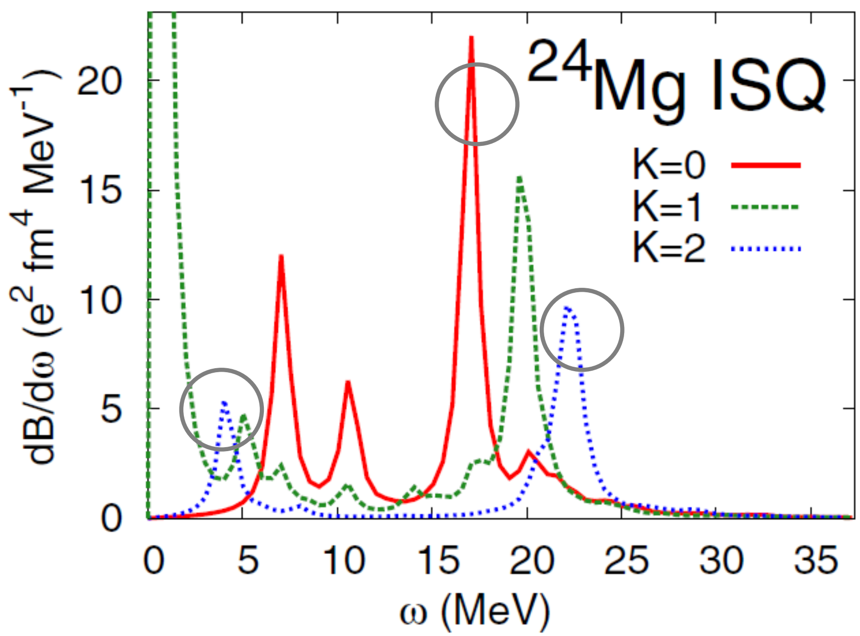
- Computer code based on evb8 (HFB in 3 dimension)
- Hartree-Fock basis and quasiparticle basis

Benchmark: Isoscalar quadrupole strength

^{24}Mg



Kortelainen et al., PRC92(2015)051302



K=0: Giant resonance is reproduced.
A peak at E~10MeV is not present.

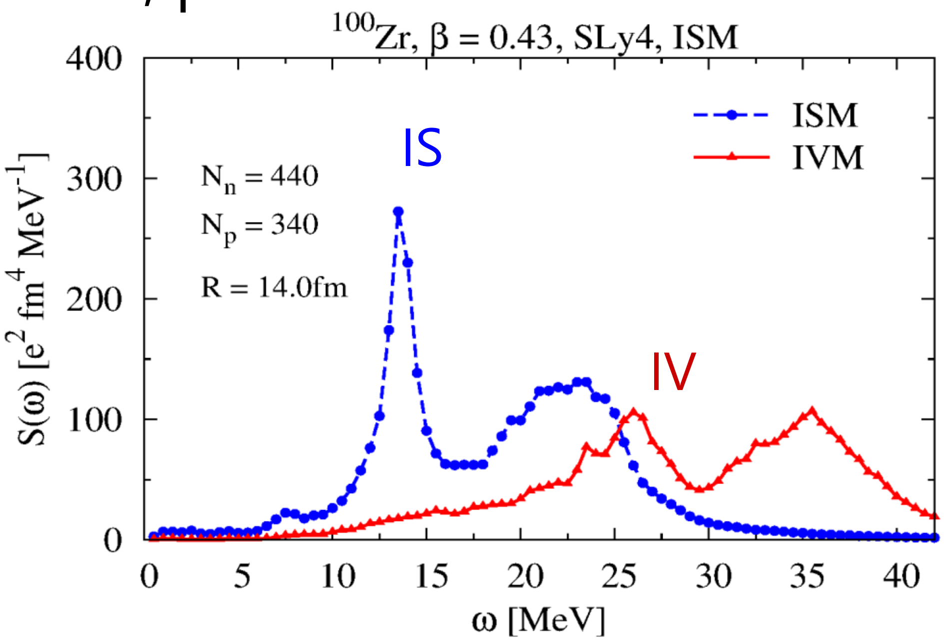
K=2: Height of the peaks is underestimated.

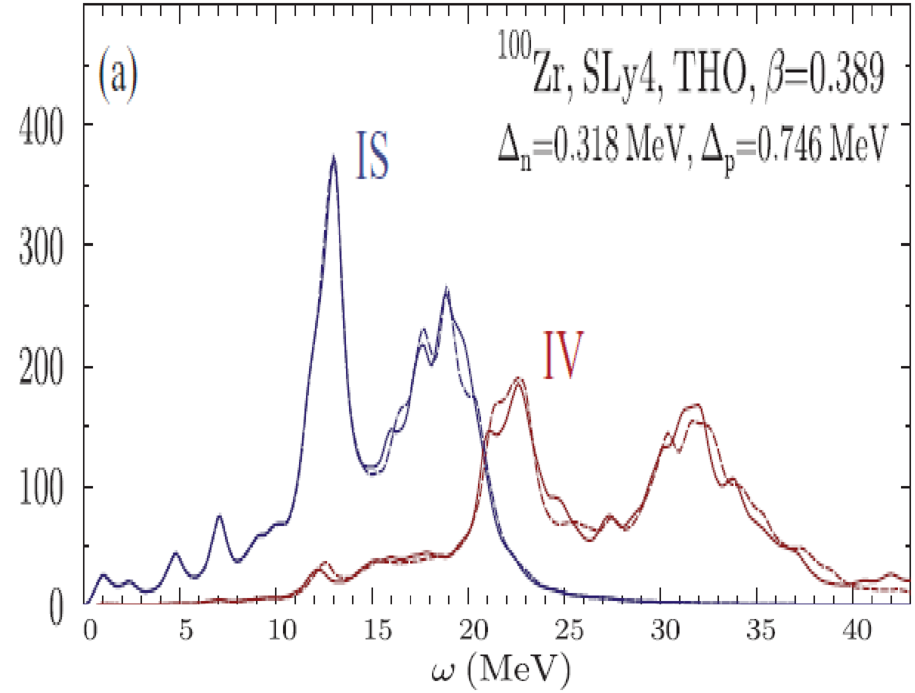
Energy weighted sum rule
FAM/HFB = 111% → Overestimate

Pairing collapse in the ground state
Smearing width = 0.5 MeV

Benchmark: Monopole strength

^{100}Zr, prolate





Stoitsov et al., PRC84(2011)041305

Energy weighted sum rule (FAM/HFB) = 132%

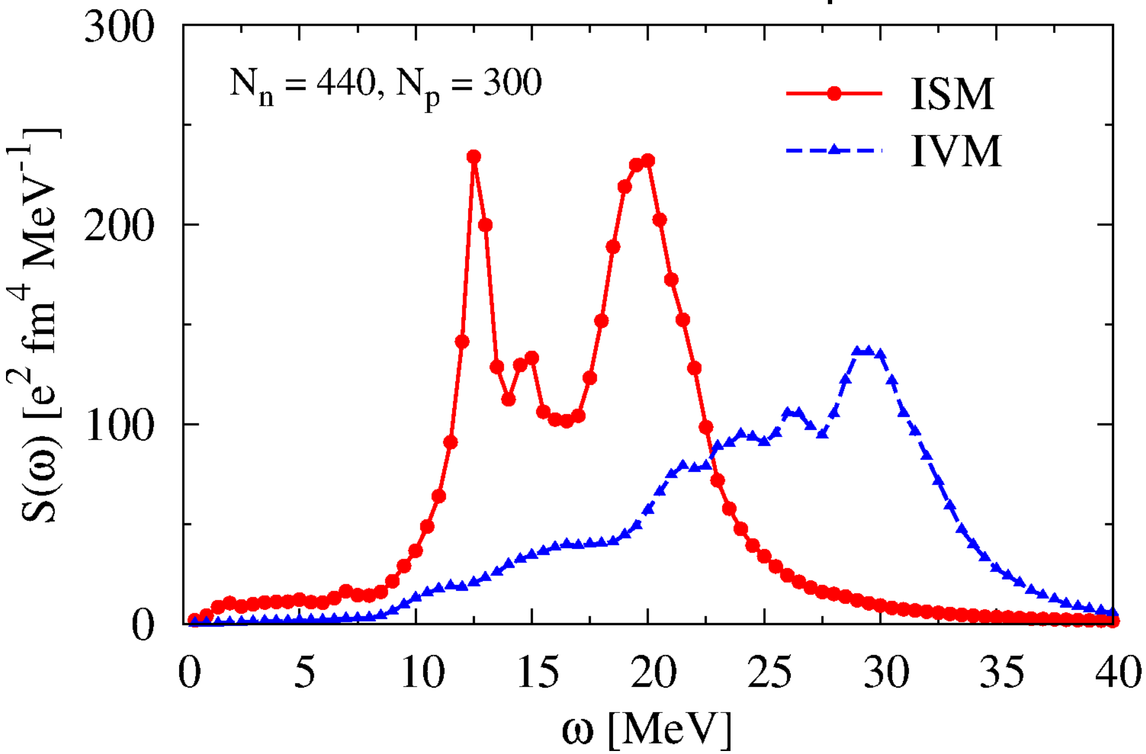
Two-peak structure

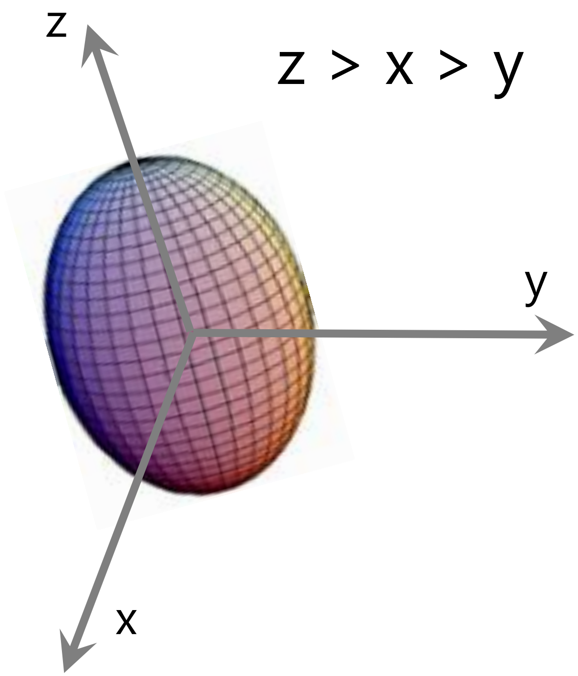
Difference in height of the peaks

Result: Triaxial nucleus

^{110}Ru, β = 0.31, γ = 20°

Isoscalar & isovector monopole





Finite pairing in the ground state

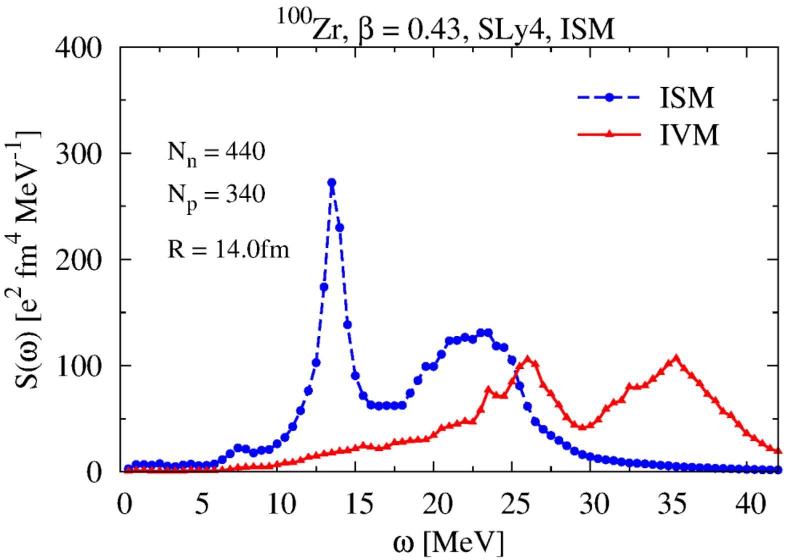
Smearing width = 0.5 MeV

Summary

3D FAM+QRPA is almost ready

Benchmark

Triaxial nuclei



Future plan

FAM+Local QRPA → Mass inertia

Bohr Hamiltonian