

Shape coexistence, isospin symmetry breaking, stellar weak processes within beyond-mean-field approach

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Outline

- *complex EXCITED VAMPIR beyond-mean-field model*
- **shape-coexistence and isospin-symmetry-breaking effects in A=70 isovector triplet**
superallowed Fermi β -decay of ^{70}Kr
- **shape-coexistence effects on terrestrial and stellar weak interaction rates**
 - *Z=N+2 isotope ^{70}Kr*
 - *rp-process waiting point ^{72}Kr*

*A~70 proton-rich nuclei exhibit
drastic changes in structure with number of nucleons, spin, excitation energy
generated by*

- *shape coexistence and shape mixing*
- *competing T=0 and T=1 pairing correlations*
- *isospin-symmetry-breaking interactions*

Challenges for theory

- *realistic effective Hamiltonians in adequate model spaces, beyond-mean-field methods*
- *comprehensive understanding of structure phenomena and β -decay properties*
- *reliable predictions on stellar weak interaction rates*

based on

self-consistent description of experimentally accessible properties

complex VAMPIR model family

- the **model space** is defined by a finite dimensional set of **spherical single particle states**
 - the effective many-body Hamiltonian is represented as a sum of **one- and two-body terms**
 - the basic building blocks are **Hartree-Fock-Bogoliubov (HFB) vacua**
 - the **HFB transformations** are essentially *complex* and allow for proton-neutron, parity and angular momentum mixing being restricted by time-reversal and axial symmetry
($T=1$ and $T=0$ neutron-proton pairing correlations already included at the mean-field level)
 - the broken symmetries ($s=N, Z, I, p$) are restored by projection before variation
- * *The models allow to use rather large model spaces and realistic effective interactions*

Beyond-mean-field variational procedure

complex Vampir

$$E^s[F_1^s] = \frac{\langle F_1^s | \hat{H} \hat{\Theta}_{00}^s | F_1^s \rangle}{\langle F_1^s | \hat{\Theta}_{00}^s | F_1^s \rangle}$$

$\hat{\Theta}_{00}^s$ - symmetry projector
 $|F_1^s\rangle$ - HFB vacuum

$$|\psi(F_1^s); sM\rangle = \frac{\hat{\Theta}_{M0}^s |F_1^s\rangle}{\sqrt{\langle F_1^s | \hat{\Theta}_{00}^s | F_1^s \rangle}}$$

complex Excited Vampir

$$|\psi(F_i^s); sM\rangle = \sum_{j=1}^i |\phi(F_j^s)\rangle \alpha_j^i \quad \text{for } i = 1, \dots, n-1$$

$$|\phi(F_i^s); sM\rangle = \hat{\Theta}_{M0}^s |F_i^s\rangle$$

$$|\psi(F_n^s); sM\rangle = \sum_{j=1}^{n-1} |\phi(F_j^s)\rangle \alpha_j^n + |\phi(F_n^s)\rangle \alpha_n^n$$

$$(H - E^{(n)} N) f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi_\alpha^{(n)}; sM\rangle = \sum_{i=1}^n |\psi_i; sM\rangle f_{i\alpha}^{(n)}, \quad \alpha = 1, \dots, n$$

A ~ 70 mass region

^{40}Ca - core

model space for protons and neutrons

$1p_{1/2} \ 1p_{3/2} \ 0f_{5/2} \ 0f_{7/2} \ 1d_{5/2} \ 0g_{9/2}$

(charge-symmetric basis + Coulomb contributions to the π -spe from the core)

$1p_{1/2} \ 1p_{3/2} \ 0f_{5/2} \ 0f_{7/2} \ 2s_{1/2} \ 1d_{3/2} \ 1d_{5/2} \ 0g_{7/2} \ 0g_{9/2} \ 0h_{11/2}$ (*ext-model space*)

renormalized G-matrix (OBEP- Bonn A/ CD)

- *pairing properties enhanced by short range Gaussians for:*

T = 1 : pp (-35 MeV), np (-20 MeV), nn (-35 MeV)

T = 0: np (-35 MeV)

- *onset of deformation influenced by monopole shifts:*

$\langle 0g_{9/2} \ 0f; T=0 | G | 0g_{9/2} \ 0f; T=0 \rangle \quad (0f_{5/2}, \ 0f_{7/2})$

$\langle 1d_{5/2} \ 1p; T=0 | G | 1d_{5/2} \ 1p; T=0 \rangle \quad (1p_{1/2}, \ 1p_{3/2})$

- *Coulomb interaction between valence protons added*

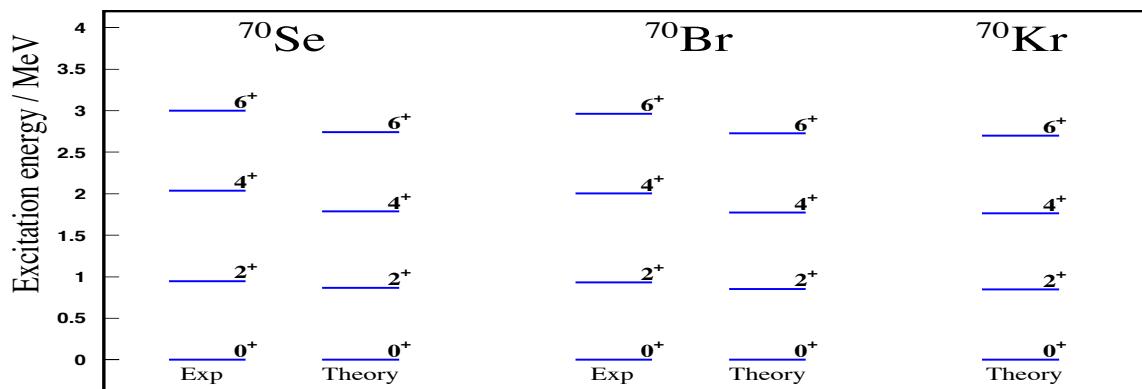
Isospin-symmetry-breaking and shape-coexistence effects in A=70 isovector triplet

$^{36}\text{Kr}_{34} - ^{35}\text{Br}_{35} - ^{34}\text{Se}_{36}$

Coulomb Energy Differences: A = 70 exotic case (anomaly)

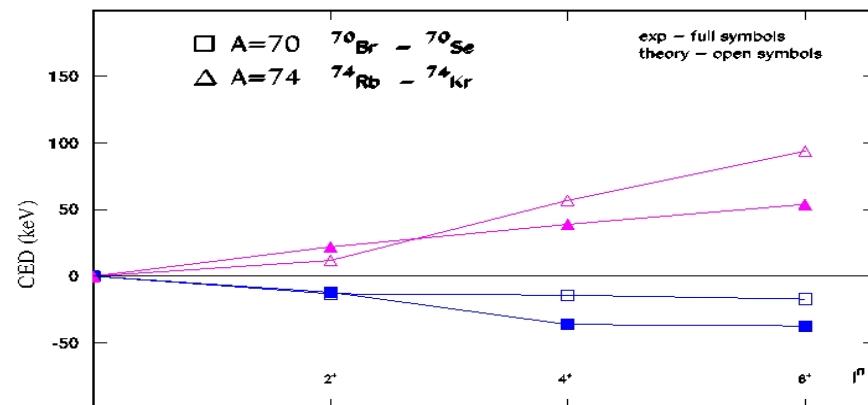
Isospin-symmetry violation induced by Coulomb interaction and strong force

A. Petrovici, Phys. Rev. C 91, 014302 (2015)



Coulomb Energy Differences

$$CED_{J,T=I} = E^*_{J,T=I,Tz=0} - E^*_{J,T=I,Tz=+1}$$



Shape mixing manifested in the structure of wave functions A = 70

- ♦ strong oblate-prolate mixing decreasing with increasing spin

^{70}Se	$I(\hbar)$	Prolate content	Oblate content
0^+	$41(4)(1)(1) \%$	$51(1) \%$	
2^+	$56(2) \%$	$39(2) \%$	
4^+	$52(2) \%$	$43(2) \%$	
6^+	$76(3)(1)(1) \%$	$17(1) \%$	

- ♦ oblate components dominate the ground state in ^{70}Se , but prolate ones in ^{70}Br

^{70}Br	$I(\hbar)$	Prolate content	Oblate content
0^+		$68(1) \%$	$26(2)(1) \%$
2^+		$66(2) \%$	$29(1) \%$
4^+		$68(2)(1) \%$	$26(1) \%$
6^+		$81(4)(2)(1)(1) \%$	$10(1) \%$

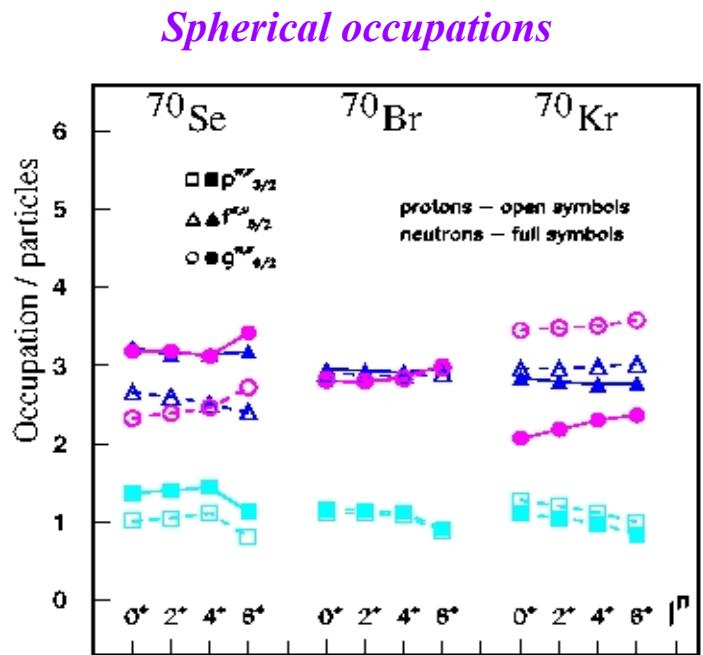
- ♦ similar structure for ^{70}Br and ^{70}Kr

^{70}Kr	$I(\hbar)$	Prolate content	Oblate content
0^+		$69(3) \%$	$24(3) \%$
2^+		$70(3) \%$	$24(1) \%$
4^+		$75(3) \%$	$19(2) \%$
6^+		$86(3)(2) \%$	$7(2) \%$

Shape coexistence and deformation revealed by spectroscopic quadrupole moments

- *yrast states dominated by prolate deformed configurations in the intrinsic system*
- *yrare states manifest oblate dominated content*

$I(\hbar)$	^{70}Se	^{70}Br	^{70}Kr
2_1^+	-7	-18	-25
2_2^+	4	16	18
4_1^+	-7	-30	-42
4_2^+	0	25	33
6_1^+	-49	-59	-65
6_2^+	38	51	53



Precise quadrupole moments for low spin states could clarify the open problem concerning the oblate-prolate coexistence and mixing

Self-consistent terrestrial and stellar weak interaction rates

Fermi transition probabilities

$$B_{if}(F) = \frac{1}{2J_i + 1} \frac{g_V^2}{4\pi} |M_F|^2$$

$$M_F \equiv (\xi_f J_f || \hat{1} || \xi_i J_i)$$

$$= \delta_{J_i J_f} \sum_{ab} M_F(ab) (\xi_f J_f || [c_a^\dagger \tilde{c}_b]_0 || \xi_i J_i)$$

$$M_F(ab) = (a || \hat{1} || b)$$

Gamow-Teller transition probabilities

$$B_{if}(GT) = \frac{1}{2J_i + 1} \frac{g_A^2}{4\pi} |M_{GT}|^2$$

$$M_{GT} \equiv (\xi_f J_f || \hat{\sigma} || \xi_i J_i)$$

$$= \sum_{ab} M_{GT}(ab) (\xi_f J_f || [c_a^\dagger \tilde{c}_b]_1 || \xi_i J_i)$$

$$M_{GT}(ab) = 1/\sqrt{3}(a || \hat{\sigma} || b)$$

Independent chains of variational calculations for the parent and daughter nuclei

Weak interaction rates and shape coexistence for the Z=N+2 isotope ^{70}Kr

Isospin-symmetry-breaking and shape-coexistence effects on superallowed Fermi β -decay

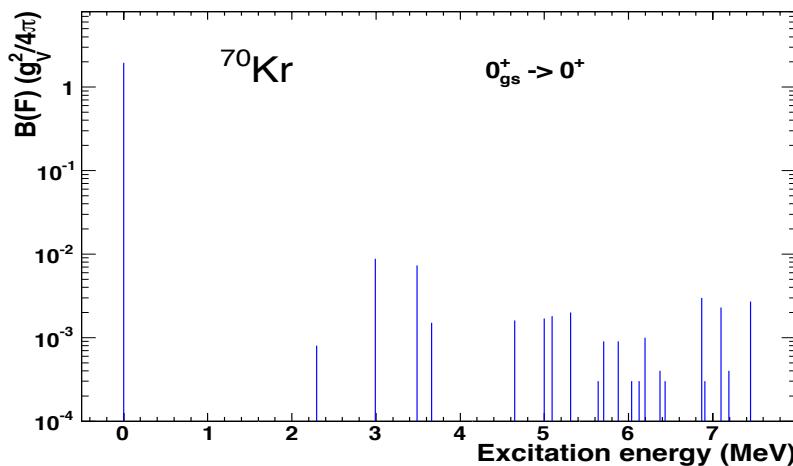
A. Petrovici, J. Phys.: Conf. Series 724 (2016) 012038

*test of the CVC hypothesis
test of the unitarity of CKM matrix*

$$ft(1 + \delta_R)(1 - \delta_c) = \frac{K}{2G_v^2(1 + \Delta_R^v)}$$

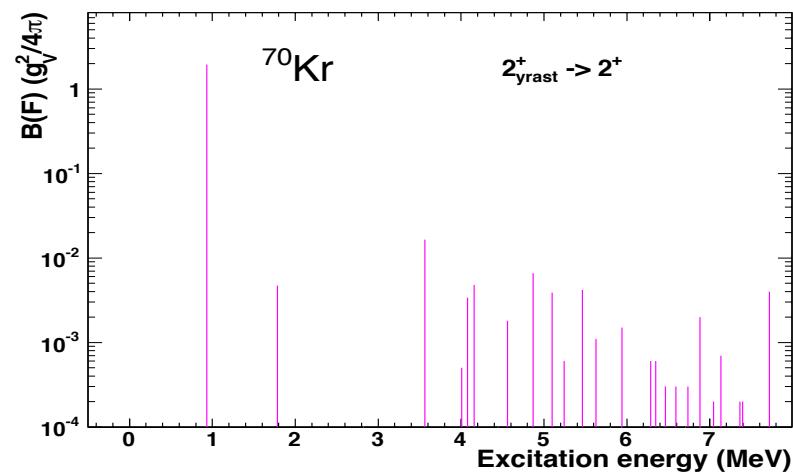
δc – isospin-symmetry-breaking correction

^{70}Kr $Q_{EC} = 10.480 \text{ MeV}$



$1\% \leq \delta_c \leq 2\%$

Nonanalog branches:
 $0_{IV}^+, 0_{V}^+ \leq 0.4\%$



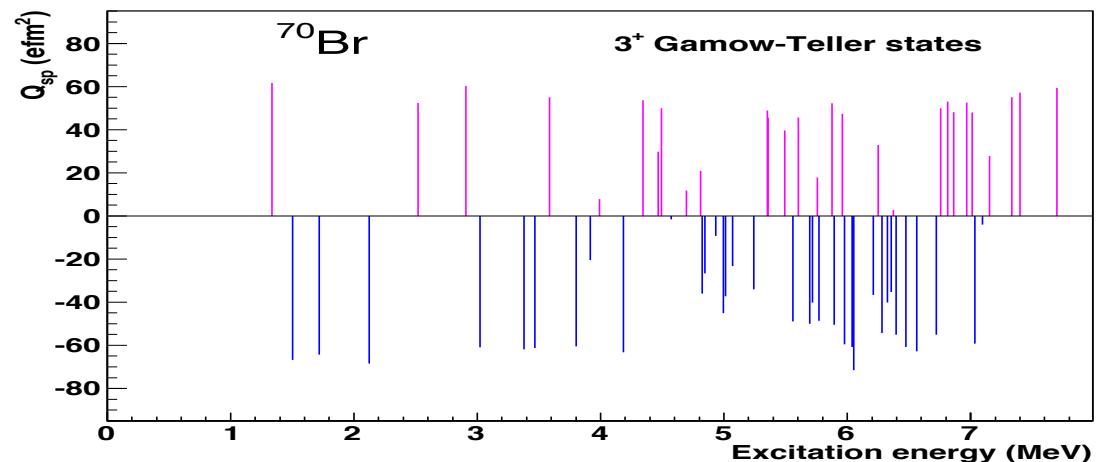
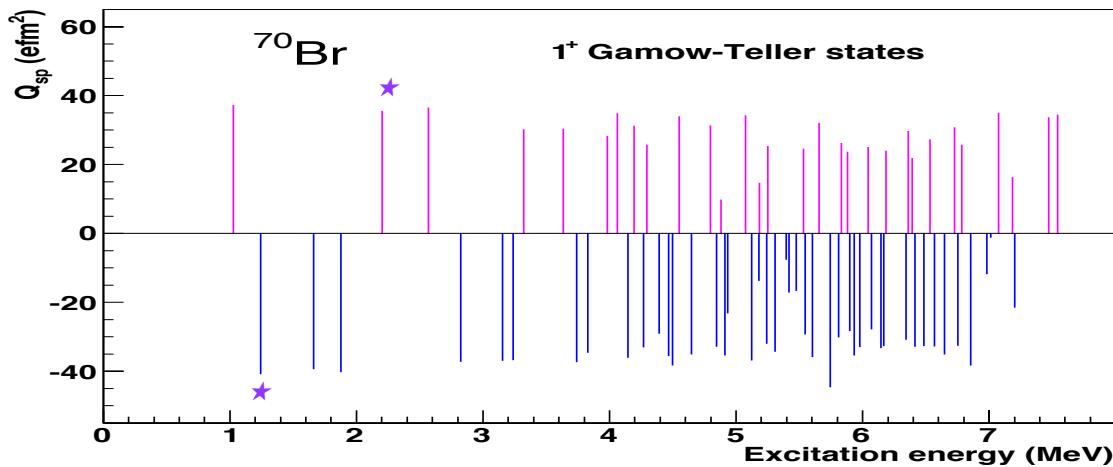
$1\% \leq \delta_c \leq 3\%$

Nonanalog branches:
 $2_{IV}^+ \leq 1.3\%$

Gamow-Teller β decay and shape coexistence for ^{70}Kr

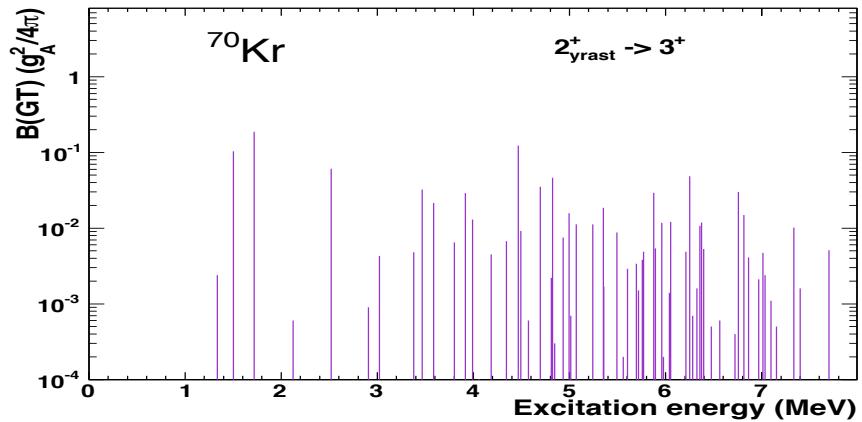
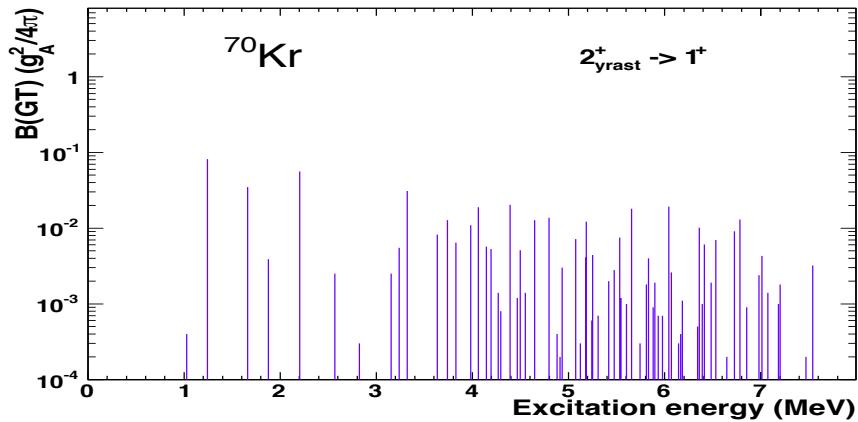
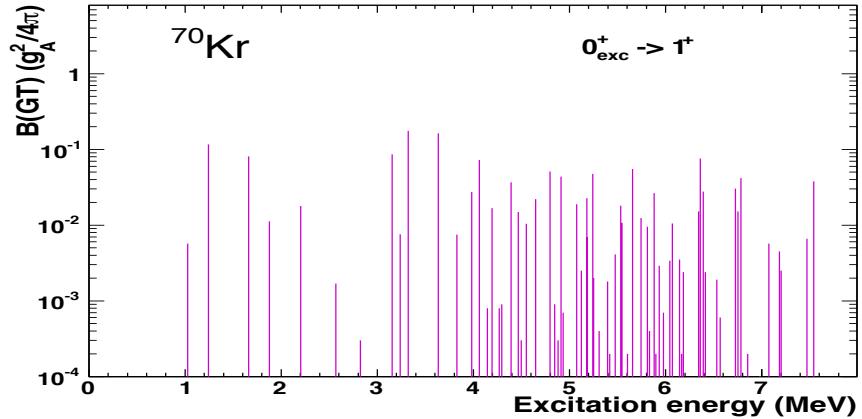
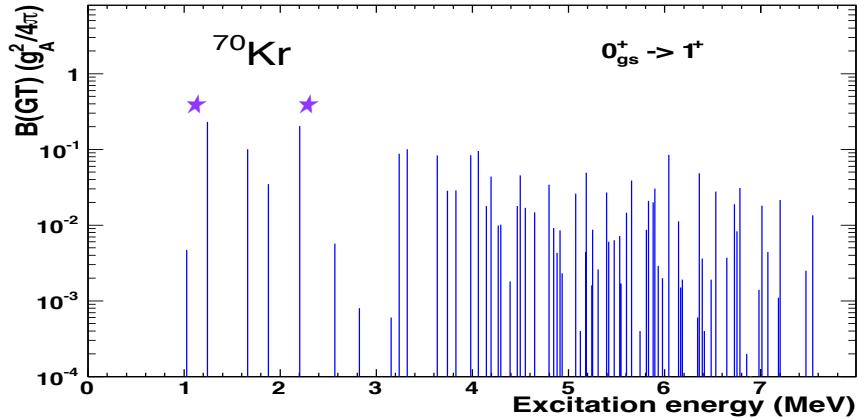
A. Petrovici and O. Andrei, Phys. Rev. C92, 064305 (2015)

Large variety of deformations in daughter states revealed by spectroscopic quadrupole moments

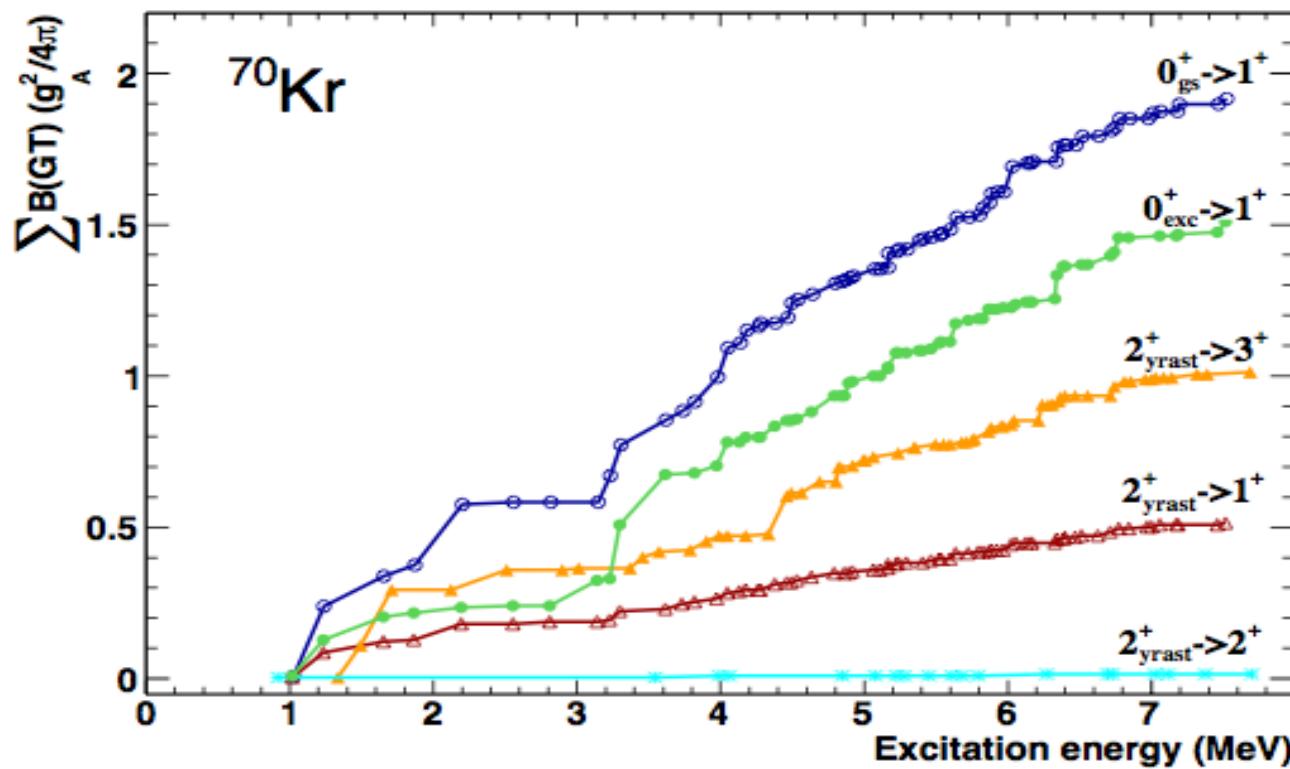


Gamow-Teller strength distributions for the decay of 0^+ and 2^+ states in ^{70}Kr

Specific shape mixing for each parent and daughter state influences the strength distributions



Contributions from $p_{1/2}^{\nu(\pi)} p_{3/2}^{\pi(\nu)}$, $p_{3/2}^\nu p_{3/2}^\pi$, $f_{5/2}^\nu f_{5/2}^\pi$, $f_{5/2}^{\nu(\pi)} f_{7/2}^{\pi(\nu)}$, $g_{9/2}^\nu g_{9/2}^\pi$ matrix elements
(coherent / cancelling effect)



Terrestrial half-lives

$$\frac{1}{T_{1/2}} = \frac{1}{D} \sum_{0 < E_f < Q_{EC}} f(Z, E_f) [B_{if}(GT) + B_{if}(F)]$$

$$T_{1/2}^{GT} = 258 \text{ ms} \quad T_{1/2}^F = 63 \text{ ms}$$

$$T_{1/2}^{\exp} = 52(17) \text{ ms} \quad T_{1/2}^{\text{EXVAM}} = 51 \text{ ms}$$

Weak interaction rates in X-ray burst astrophysical environment

In the X-ray burst stellar environment at densities ($\sim 10^6 \text{ mol/cm}^3$) and temperatures ($\sim 10^9 \text{ K}$) typical for the rp-process the contribution of thermally populated low-lying 0^+ and 2^+ states may be relevant.

(H. Schatz et al., Phys. Rep. 294, 167 (1998))

$$\lambda^\alpha = \frac{\ln 2}{K} \sum_i \frac{(2J_i + 1)e^{-E_i/(kT)}}{G(Z, A, T)} \sum_j B_{ij} \phi_{ij}^\alpha$$

$$G(Z, A, T) = \sum_i (2J_i + 1) \exp(-E_i/(kT))$$

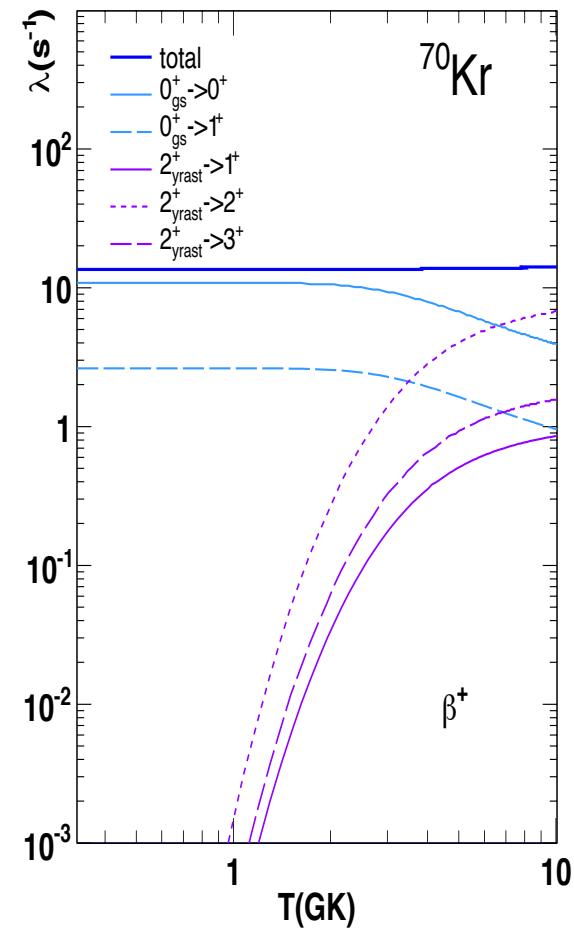
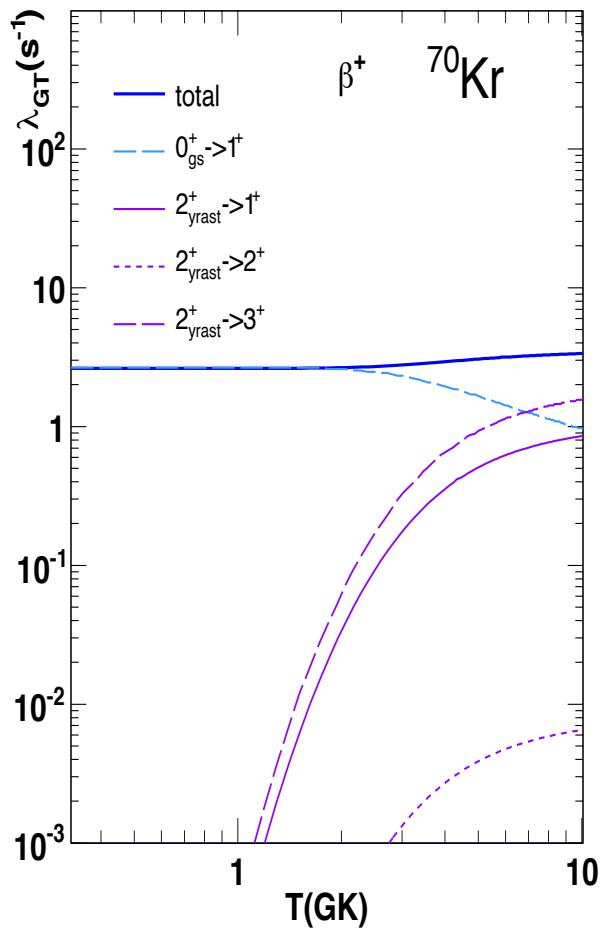
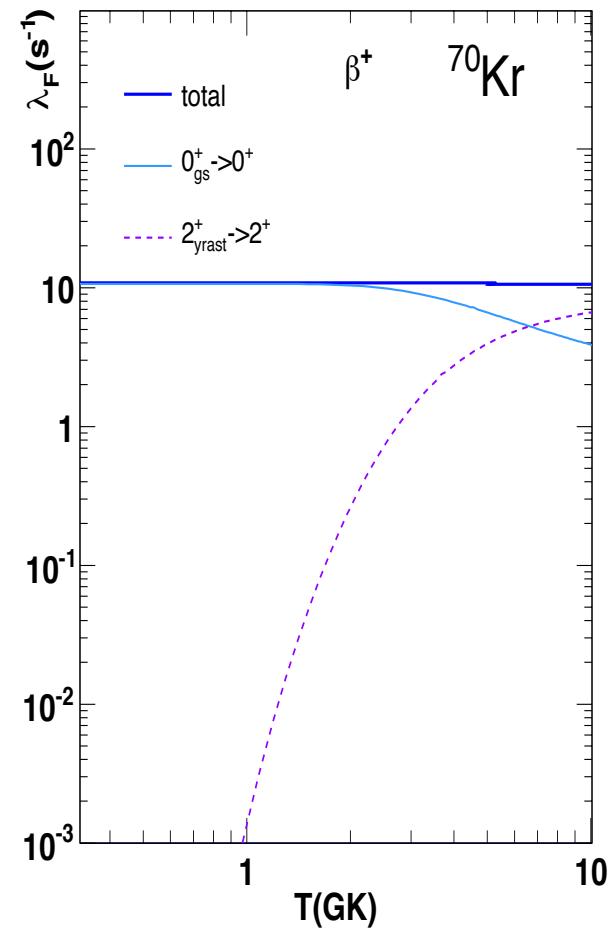
$$B_{ij} = B_{ij}(F) + B_{ij}(GT)$$

$$\phi_{ij}^{ec} = \int_{w_l}^{\infty} wp(Q_{ij} + w)^2 F(Z, w) S_e(w) (1 - S_\nu(Q_{ij} + w)) dw$$

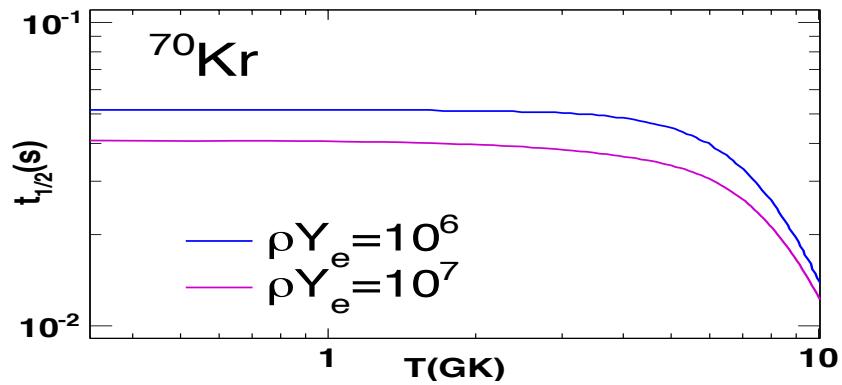
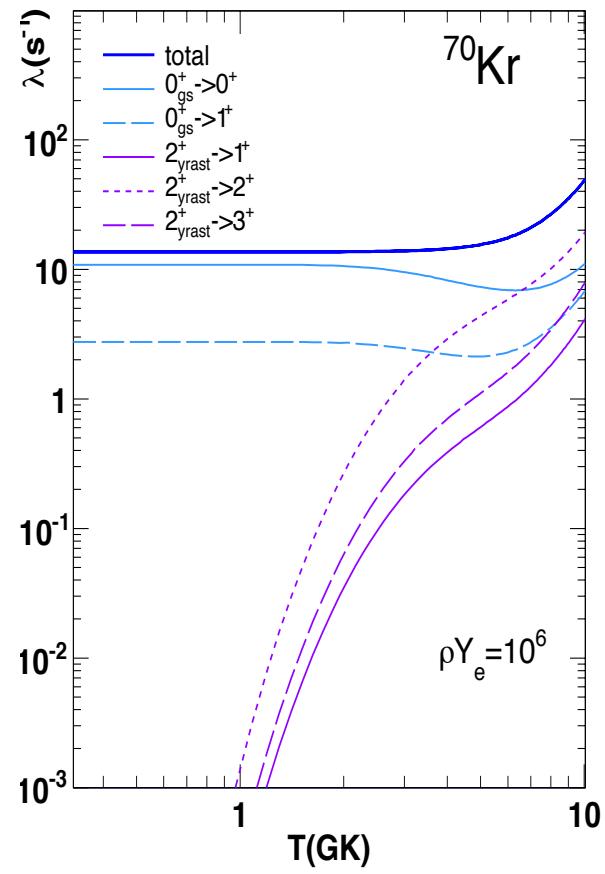
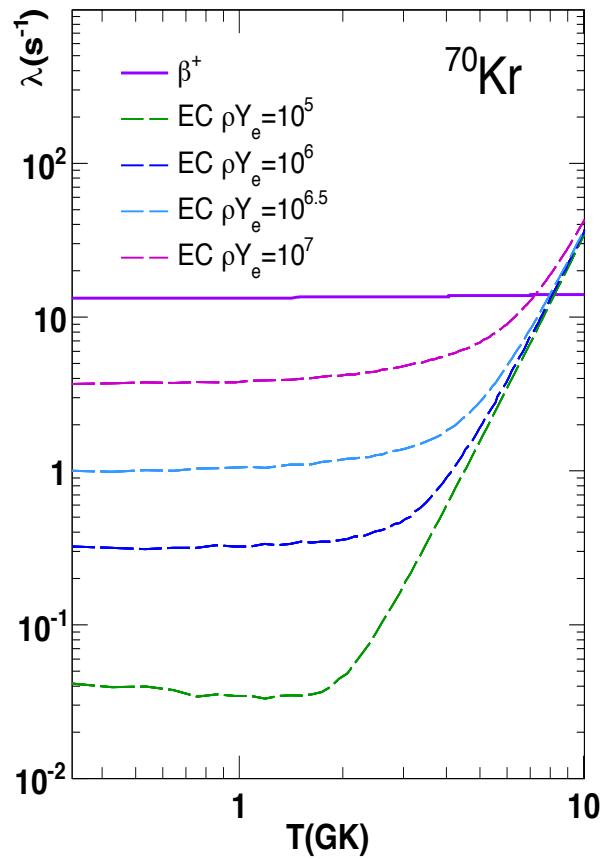
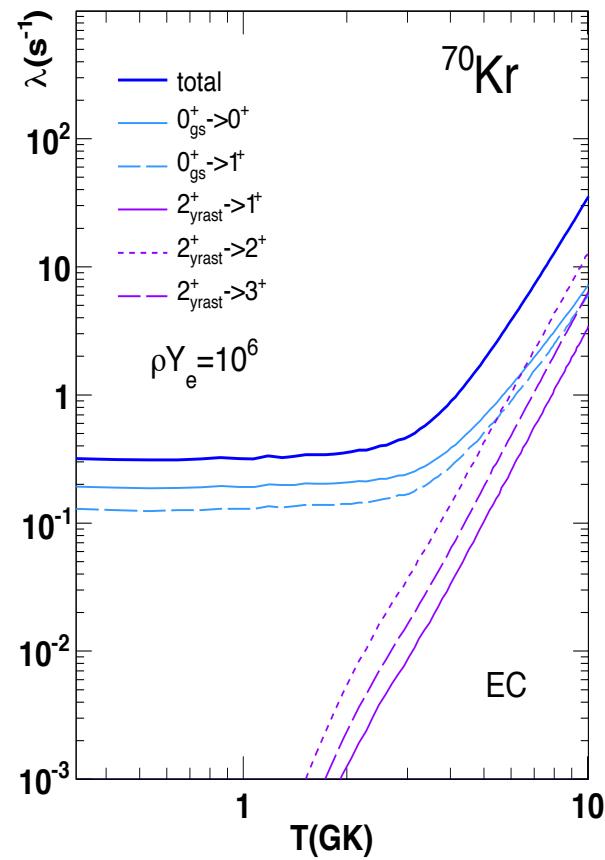
$$\phi_{ij}^{\beta^+} = \int_1^{Q_{ij}} wp(Q_{ij} - w)^2 F(-Z + 1, w) (1 - S_p(w)) (1 - S_\nu(Q_{ij} - w)) dw$$

Stellar rates for ^{70}Kr : β^+ - decay

0^+_{gs} and 2^+_{yrast} - parent states



β^+ and electron capture rates for ^{70}Kr



Weak interaction rates and shape coexistence for ^{72}Kr waiting point

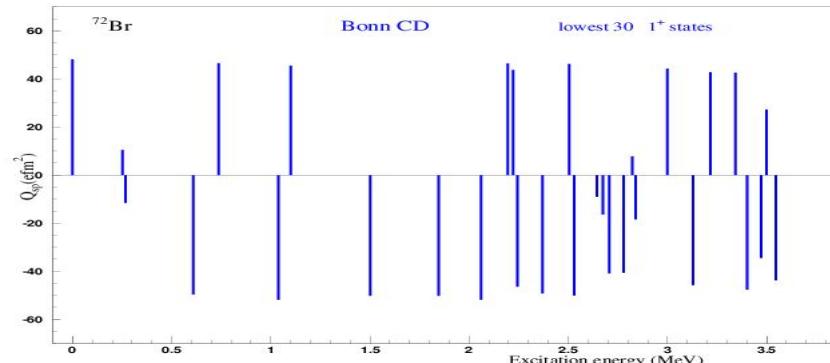
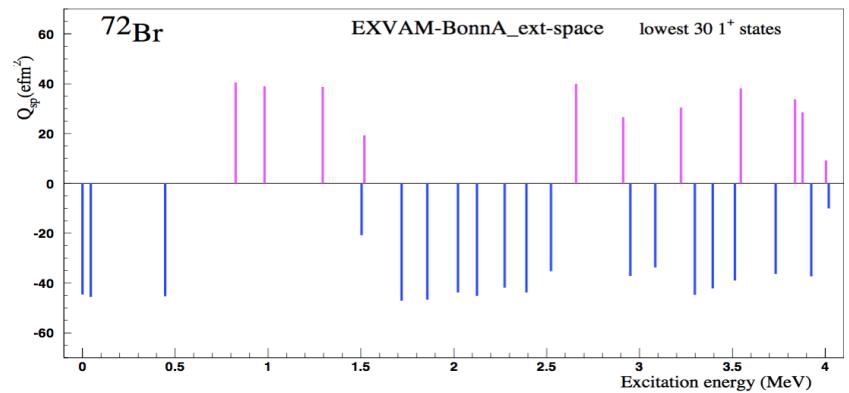
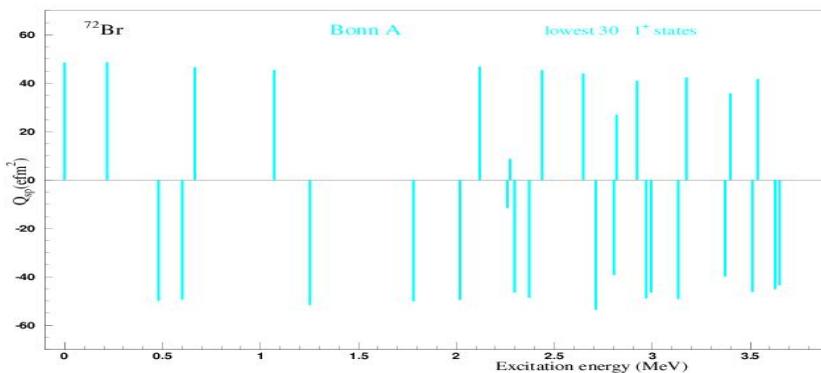
A. Petrovici and O. Andrei, Eur. Phys. J. A51, 133 (2015)

Shape coexistence and mixing in parent and daughter nuclei

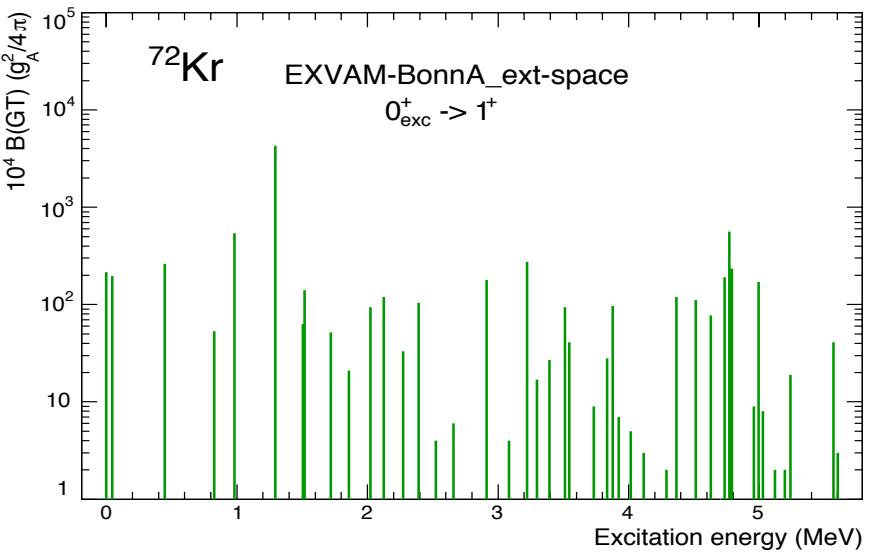
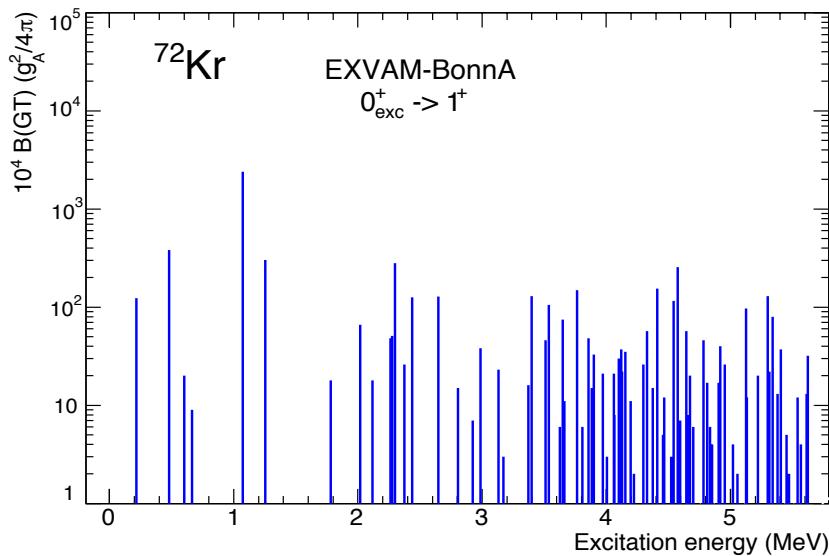
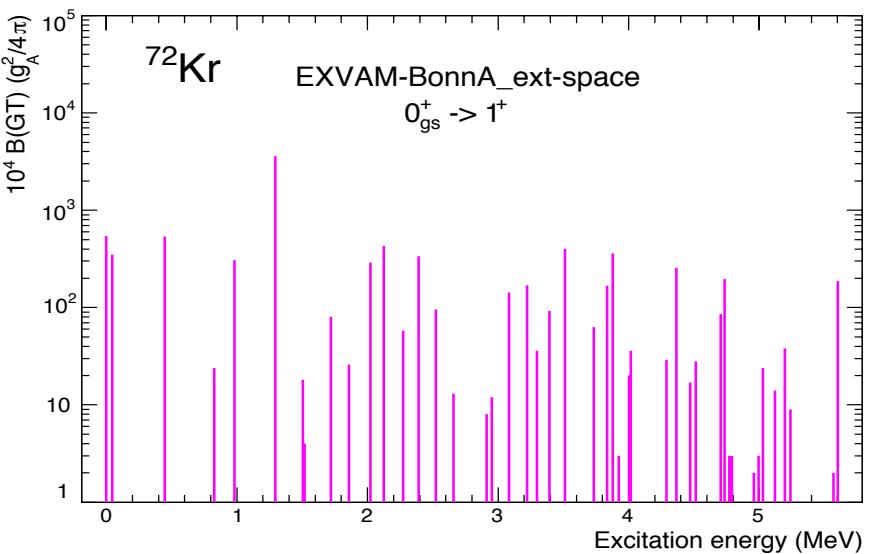
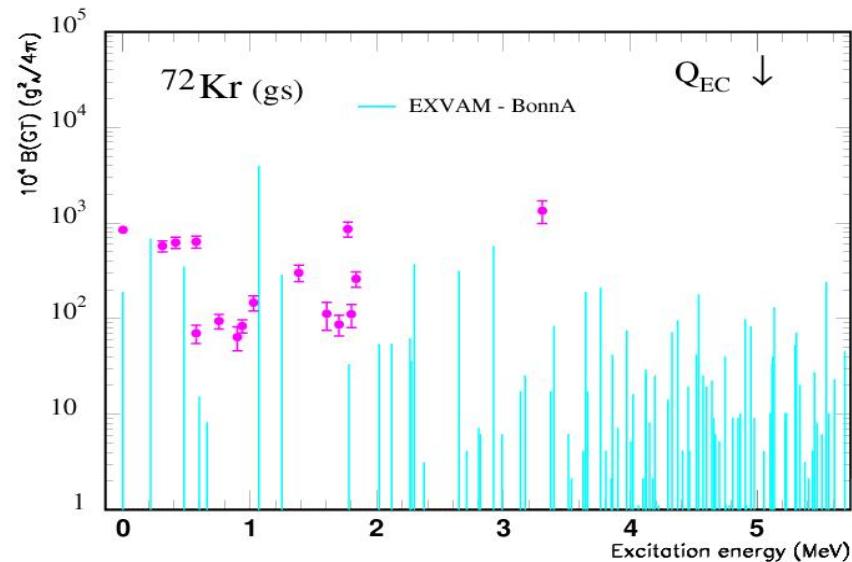
^{72}Kr :

$E_{0^+_{\text{gs}}}$	= 0.0 MeV	\Rightarrow	p/o mixing	60/40% (BonnA-ext-space)	34/66% (BonnA-standard space)
$E_{0^+_{\text{exc}}}$	= 0.671 MeV	\Rightarrow	p/o mixing	38/62% (BonnA-ext-space)	63/37% (BonnA-standard space)
$E_{2^+_{\text{yrast}}}$	= 0.710 MeV	\Rightarrow	p/o mixing	41/59% (BonnA-ext-space)	7/93% (BonnA-standard space)

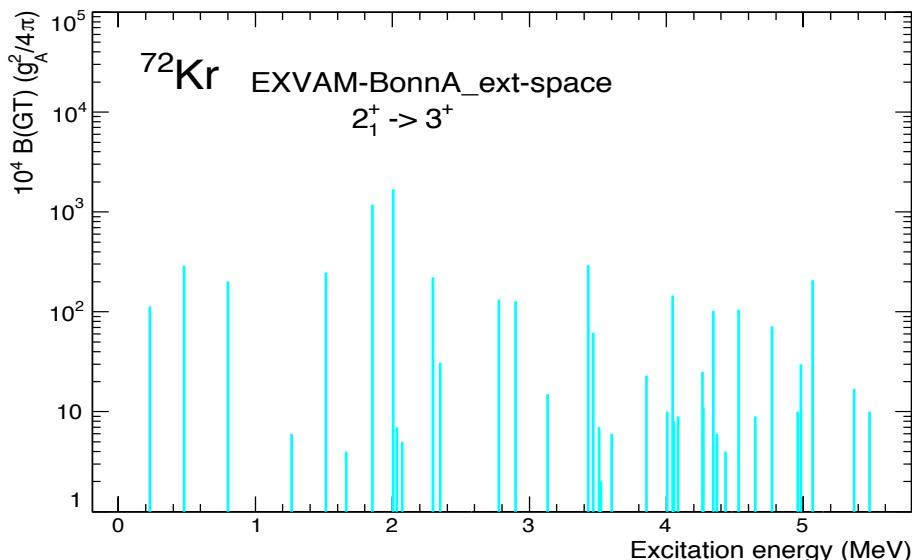
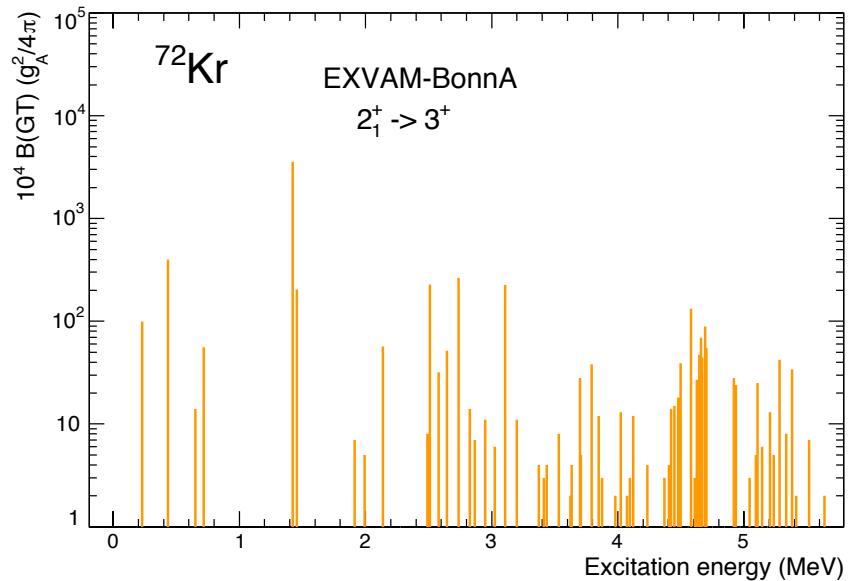
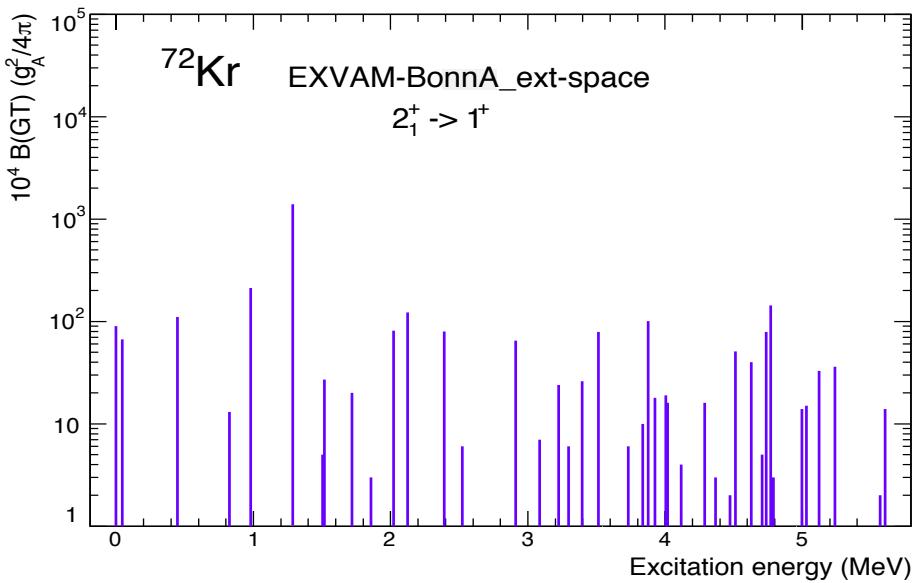
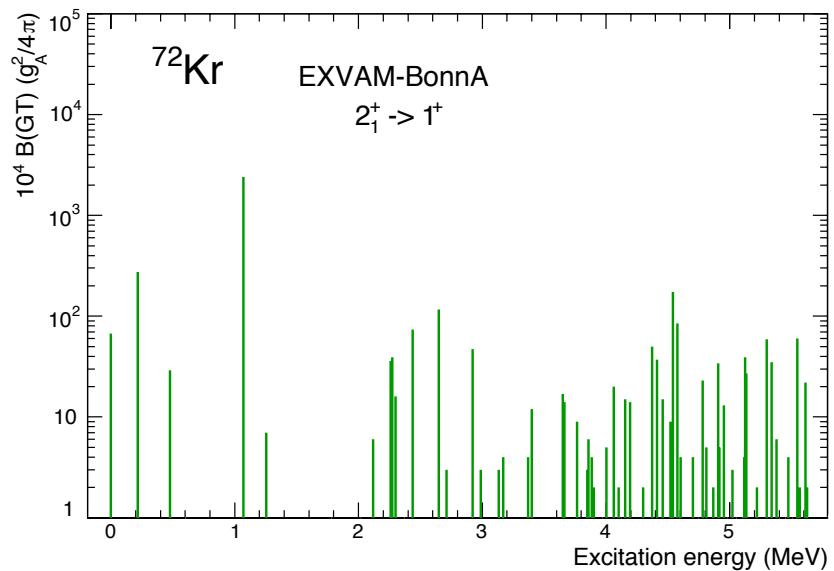
$$B(E2; 2^+ \rightarrow 0^+) = 853/670 \text{ e}^2\text{fm}^4 \text{ (BonnA-extended space/BonnA-standard space)} \quad \text{Exp.: } 810 \text{ (150) e}^2\text{fm}^4$$

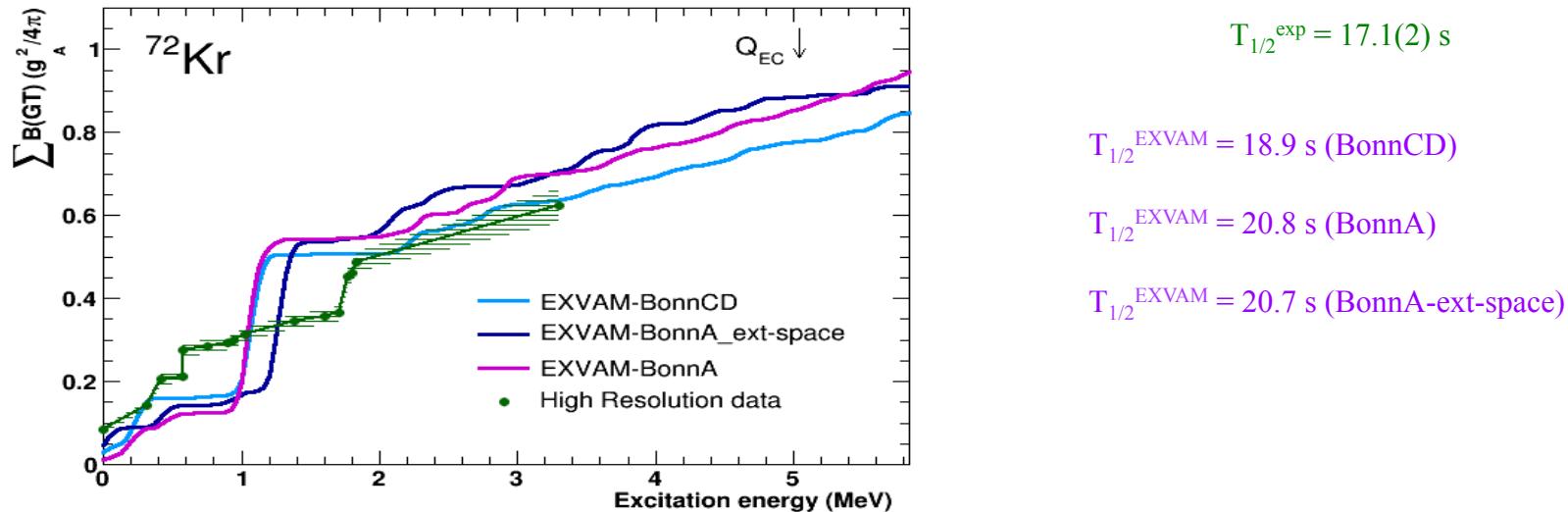


Gamow-Teller strength distributions depend strongly on parent structure

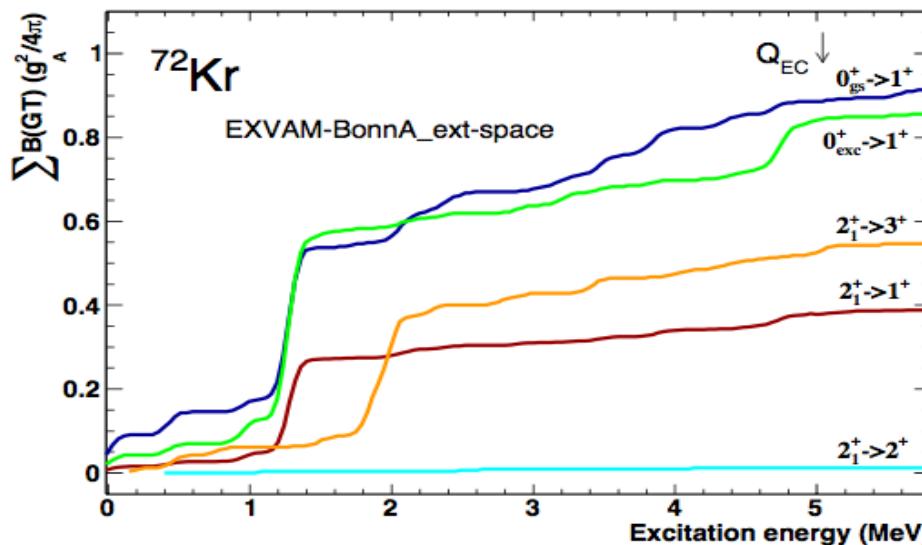


Gamow-Teller strength distributions depend strongly on daughter structure



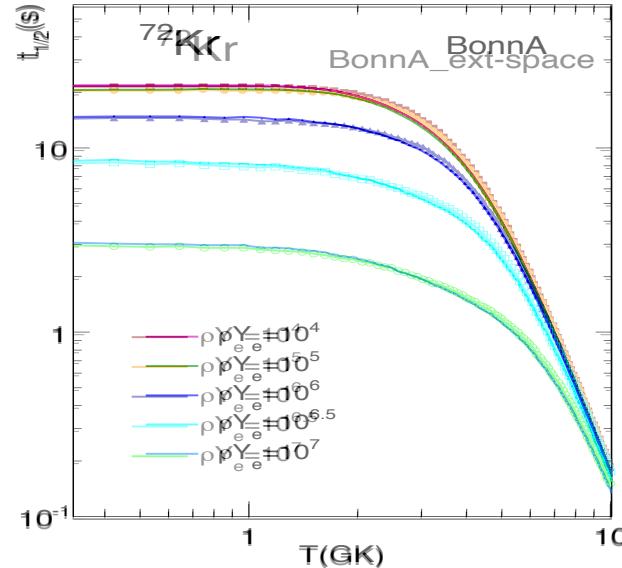
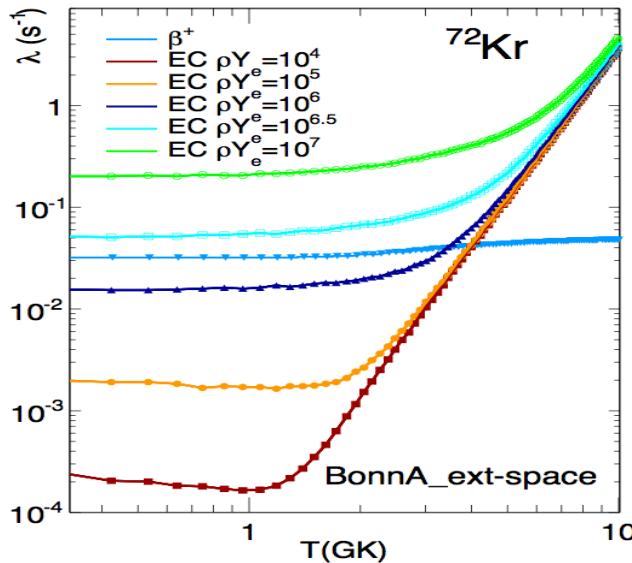
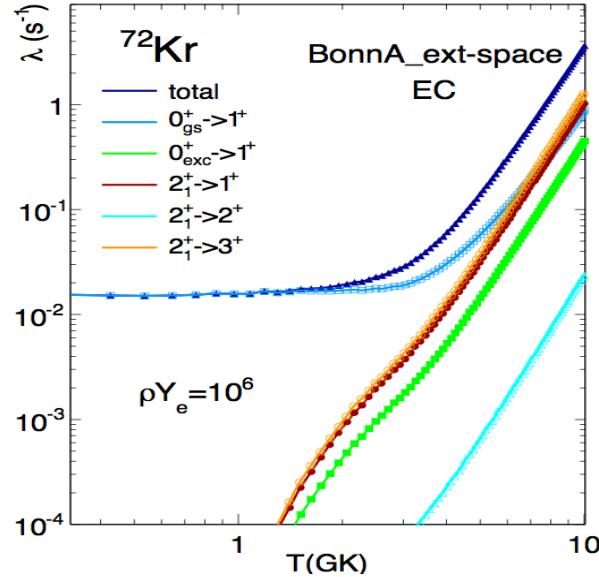
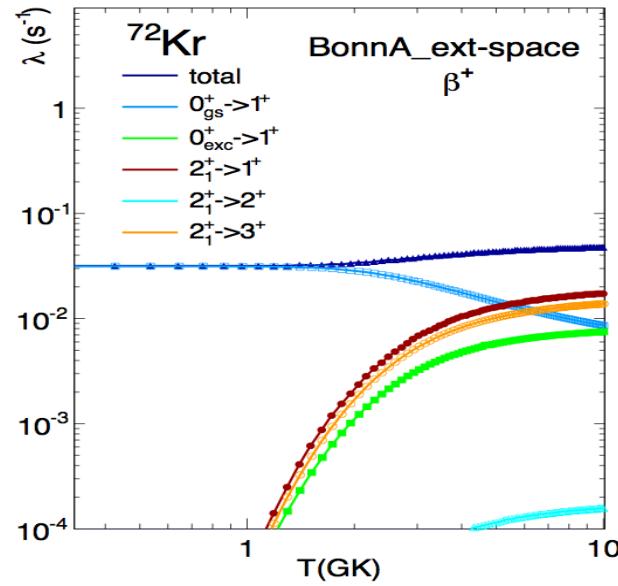


Contributions: - $p^{v(\pi)}_{1/2} p^{\pi(v)}_{3/2}$, $p^v_{3/2} p^\pi_{3/2}$, $f^v_{5/2} f^\pi_{5/2}$, $f^{v(\pi)}_{5/2} f^{\pi(v)}_{7/2}$, $g^v_{9/2} g^\pi_{9/2}$ matrix elements (decay to 1^+ states)
- $p^v_{3/2} p^\pi_{1/2}$, $p^v_{3/2} p^\pi_{3/2}$, $f^v_{5/2} f^\pi_{7/2}$ matrix elements (decay to 3^+ states)



Stellar rates for ^{72}Kr : β^+ and continuum electron capture

Significant continuum electron capture contribution



Summary

*complex EXCITED VAMPIR beyond-mean-field model self-consistently describes
shape-coexistence effects on*

- *isospin-related phenomena in the $A=70$ isovector triplet*
- *terrestrial and stellar weak interaction rates for $A \sim 70$ proton-rich nuclei*