Int. Workshop on Shapes and Symmetries in Nuclei:
From Experiment to Theory (SSNET' 2016)
Gif sur Yvette
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Shell evolution, shape transition and shape coexistence with realistic nuclear forces

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Outline

- Introduction
  - shapes and phase transition

- Computational background : advanced Monte Carlo shell model (MCSM)
  - intrinsic shapes of shell-model wave functions (T-plot)

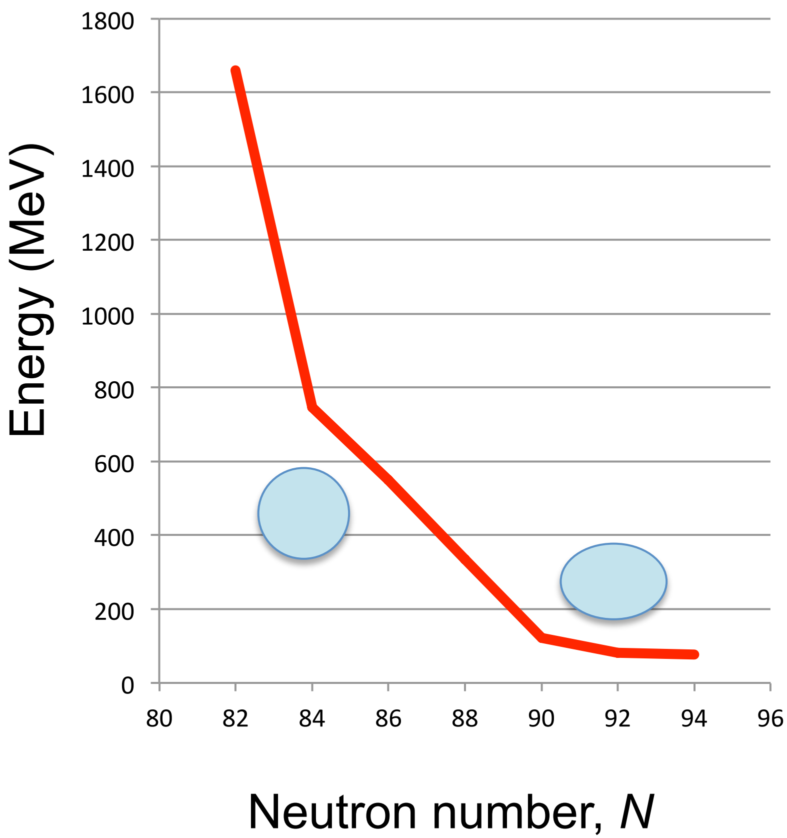
- Shape transition in Zr isotopes and Quantum Phase Transition

- Summary and Perspectives

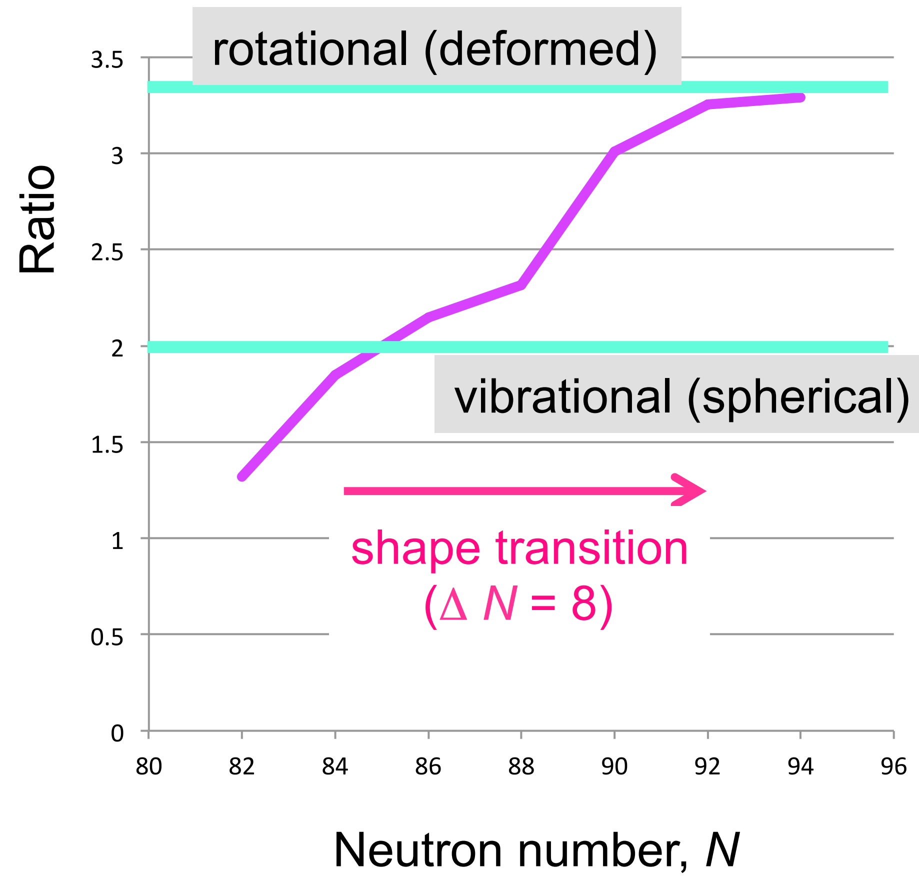
  Are symmetries in shapes favored in atomic nuclei ?

2^+ and 4^+ level properties of Sm isotopes

Ex (2^+) :
excitation energy of first 2^+ state



$R_{4/2}$ = Ex (4^+) / Ex(2^+)



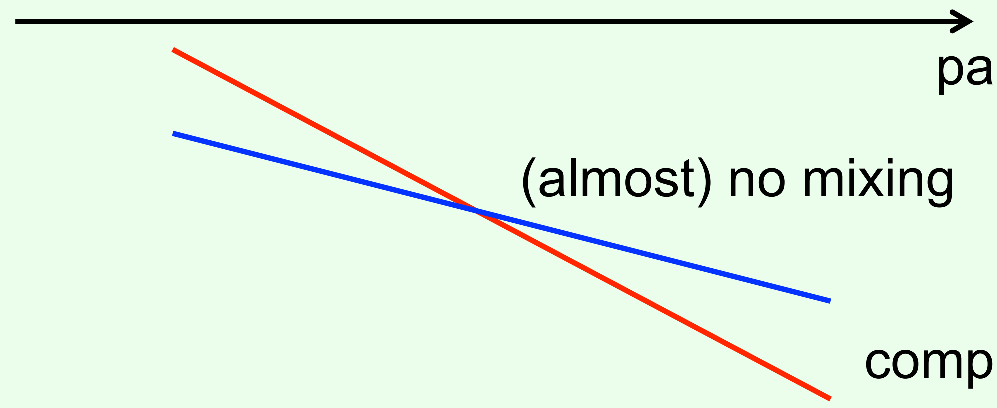
Can this be a kind of *Phase Transition* ?

Can the shape transition be a “Quantum Phase Transition” ?

The shape transition occurs rather gradually.

The definition of Quantum Phase Transition :
an abrupt change in the ground state of a many-body system
by varying a physical parameter at zero temperature

possible scenario



The usual shape transition may not fulfill the condition being *abrupt*.
Where can we see it ?
If it occurs in atomic nuclei, what is the underlying mechanism ?
Note that sizable mixing occurs usually in finite quantum systems.

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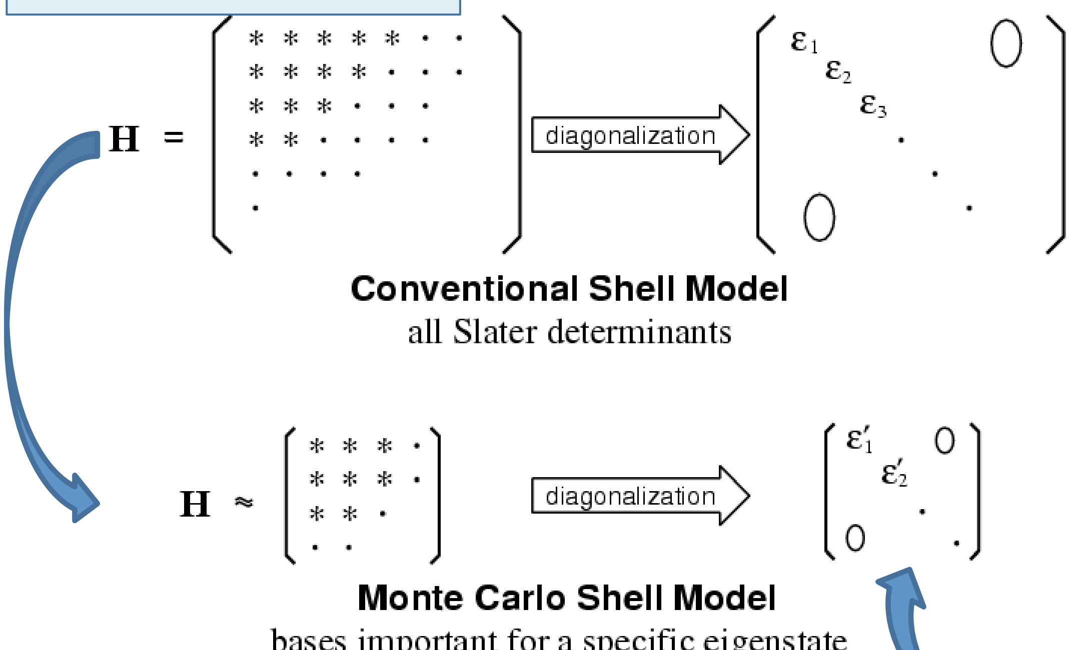
Advanced Monte Carlo shell model (MCSM)

Superposition of the projected Slater determinants
+ Extrapolation by energy variance

$$|\Psi\rangle = \sum_{k=1}^{N_{MCSM}} f_k P^{J,\pi} |\phi_k\rangle \qquad |\phi_k\rangle = \prod_{\alpha=1}^{N} \left(\sum_{i=1}^{N_{sp}} c_i^{\dagger} D_{i\alpha}^{(k)} \right) |-\rangle$$

MCSM basis, deformed Slater det.

$>10^{10}$ basis vectors



$\sim 10^{2}$ basis vectors

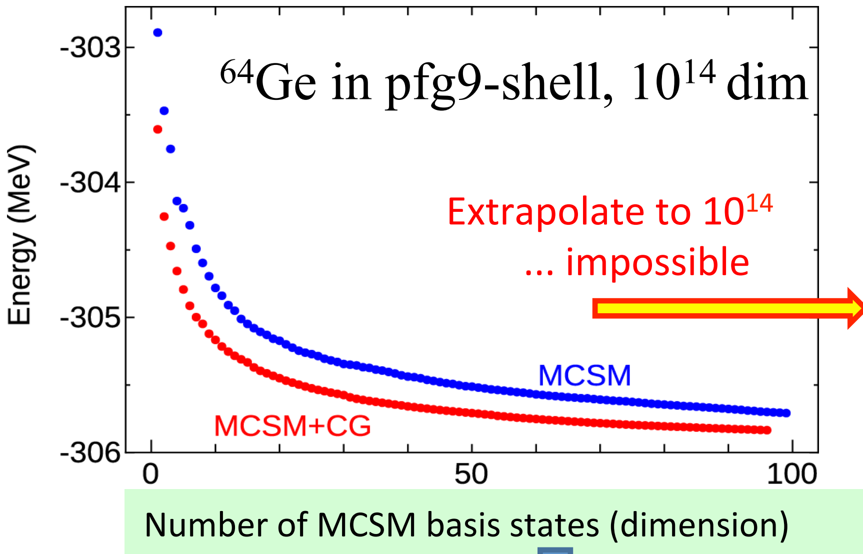
Step 1

- Select D stochastically from many candidates generated by the auxiliary-field Monte Carlo technique

Step 2

- optimize D variationally by the conjugate gradient method so as to minimize the energy eigenvalue of this small matrix

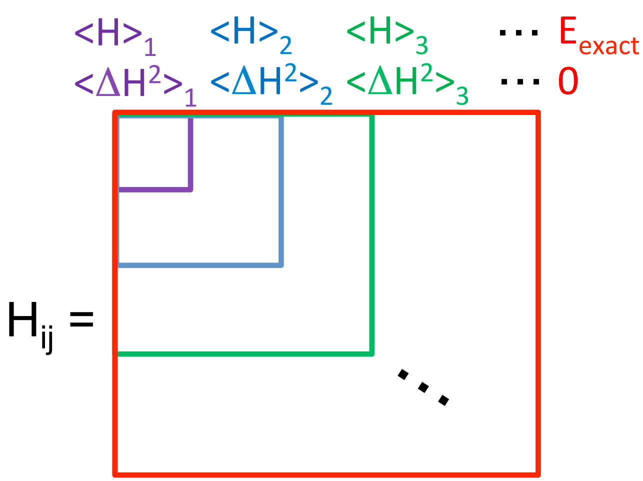
Step 3 : Energy variance extrapolation

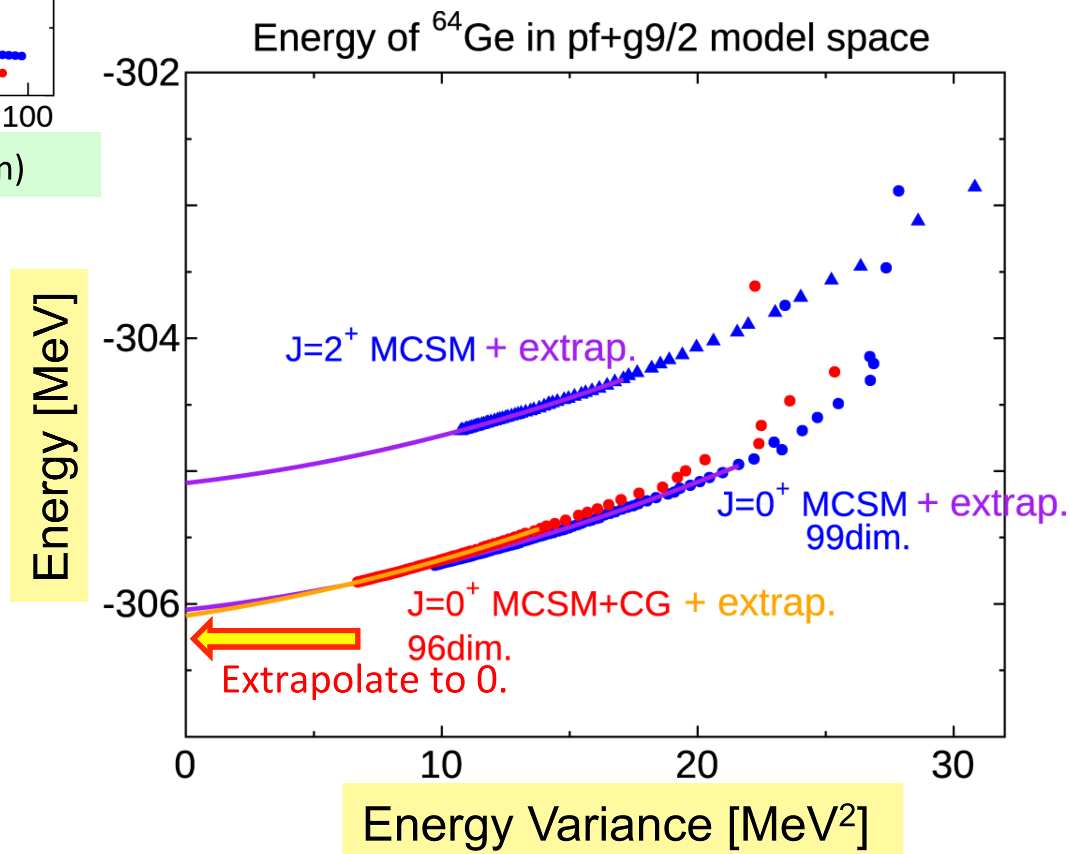


Energy variance : $\langle \Delta H^2 \rangle = \langle H^2 \rangle - \langle H \rangle^2$

As the number of basis vectors increases, the approximated w.f. approaches the exact one and the energy variance approaches zero.

Extrapolate towards $\langle \Delta H^2 \rangle \to 0$



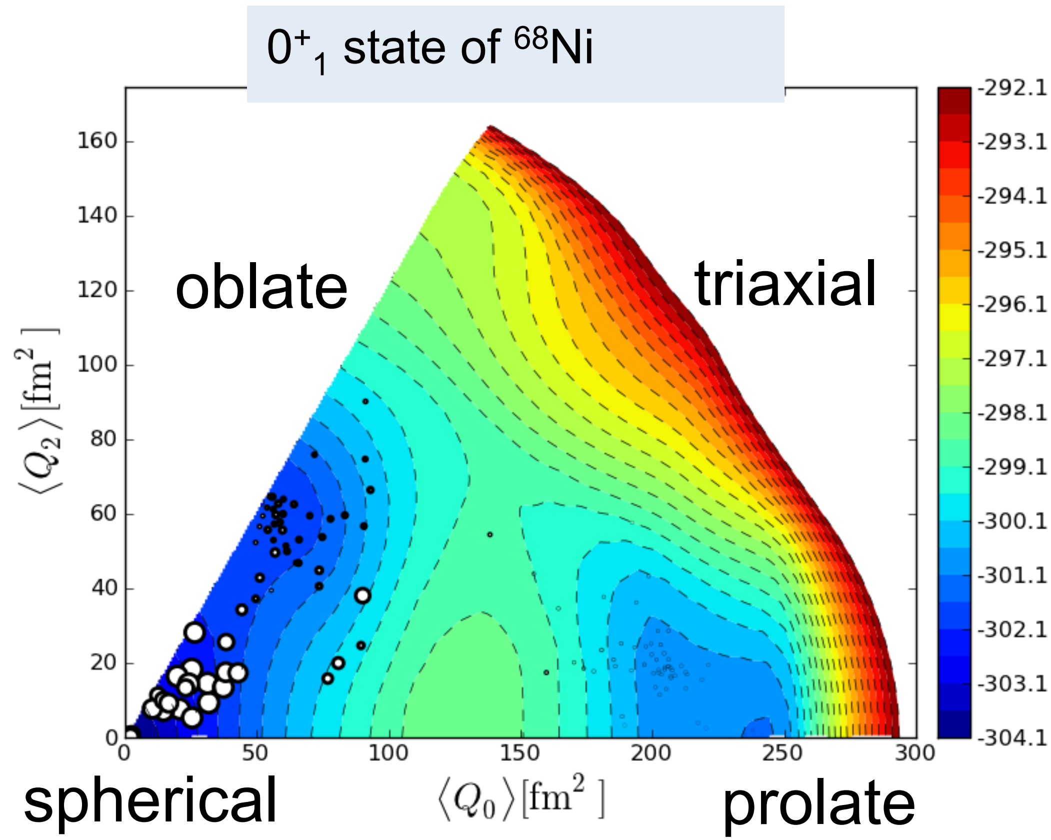


MCSM basis vectors on Potential Energy Surface (*T-plot*)

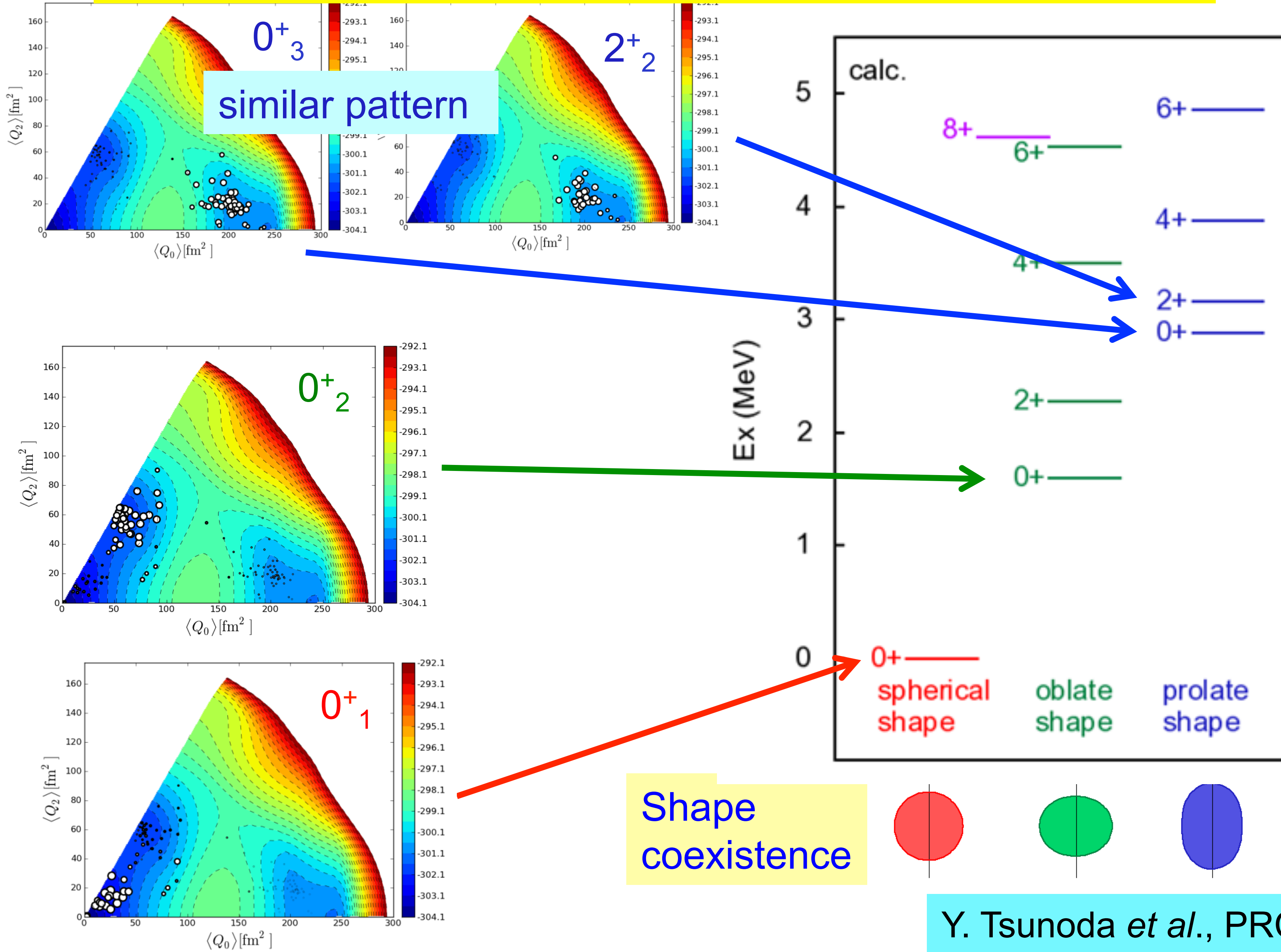
eigenstate $\Psi = \sum_i c_i P[J^\pi] \Phi_i$

Slater determinant
→ intrinsic deformation

- PES is calculated by CHF
- Location of circle : quadrupole deformation of unprojected MCSM basis vectors
- Area of circle : overlap probability between each projected basis and eigen wave function



T-plot analysis of band structure of ^{68}Ni



Y. Tsunoda *et al*., PRC89, 031301(R) (2014)

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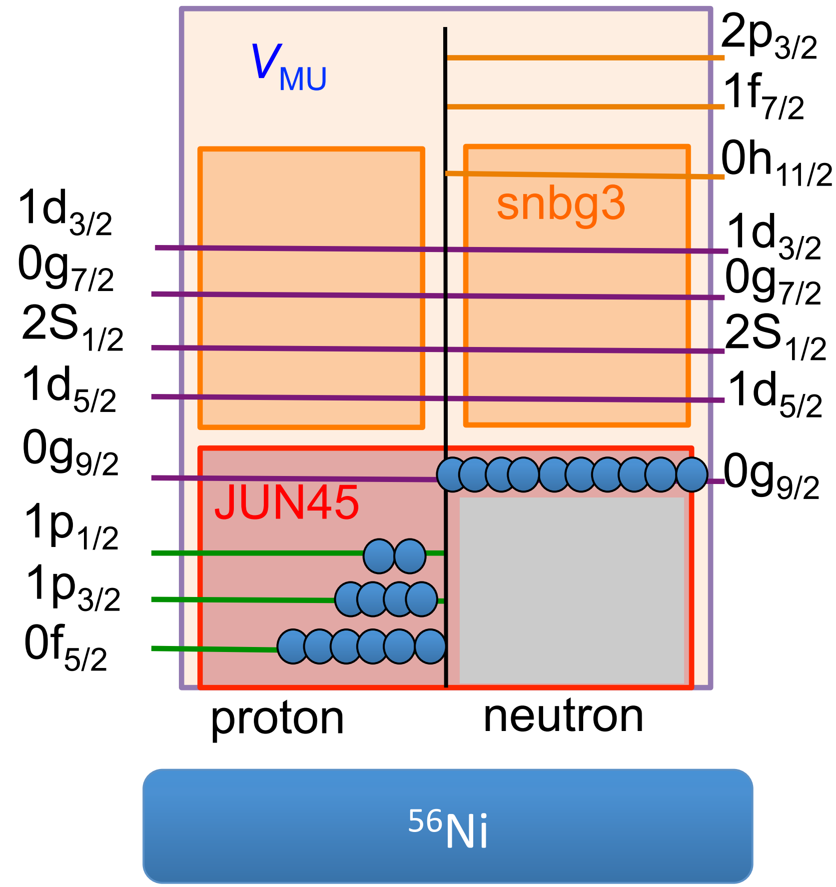
Present work : model space and effective interaction

- Effective interaction:
  JUN45 + snbg3 + V_{MU}

  known effective interactions

  + minor fit for a part of
  T=1 TBME's

  Nucleons are excited fully
  within this model space
  (no truncation)



We performed Monte Carlo Shell Model (MCSM) calculations, where the largest case corresponds to the diagonalization of 3.7 x 10 ^{23} dimension matrix.

Togashi, Tsunoda, TO *et al*. PRL 117, 172502 (2016)

From earlier shell-model works …

PHYSICAL REVIEW C VOLUME 20, NUMBER 2 AUGUST 1979

Unified shell-model description of nuclear deformation

P. Federman
Instituto de Fisica, Universidad Nacional Autonoma de Mexico, Apartado Postal 20-364, Mexico 20, D. F.

S. Pittel

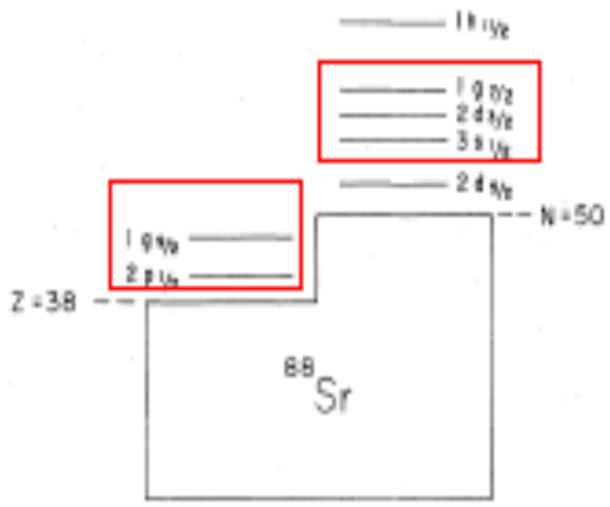


FIG. 3. Single-particle levels appropriate to a description of nuclei in the Zr-Mo region. An ^{88}Sr core is assumed.

PHYSICAL REVIEW C **79**, 064310 (2009)

Shell model description of zirconium isotopes

K. Sieja,[1,2] F. Nowacki,[3] K. Langanke,[2,4] and G. Martínez-Pinedo[1]

In this paper, we perform for the first time a SM study of Zr isotopes in an extended model space ($1f_{5/2}$, $2p_{1/2}$, $2p_{3/2}$, $1g_{9/2}$) for protons and ($2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, $1g_{7/2}$, $1h_{11/2}$) for neutrons, dubbed hereafter $\pi(r3-g)$, $\nu(r4-h)$.

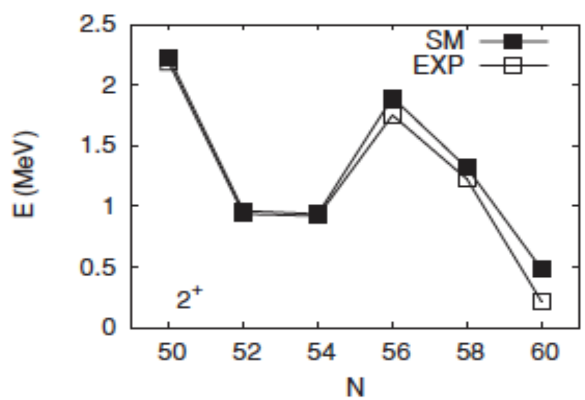
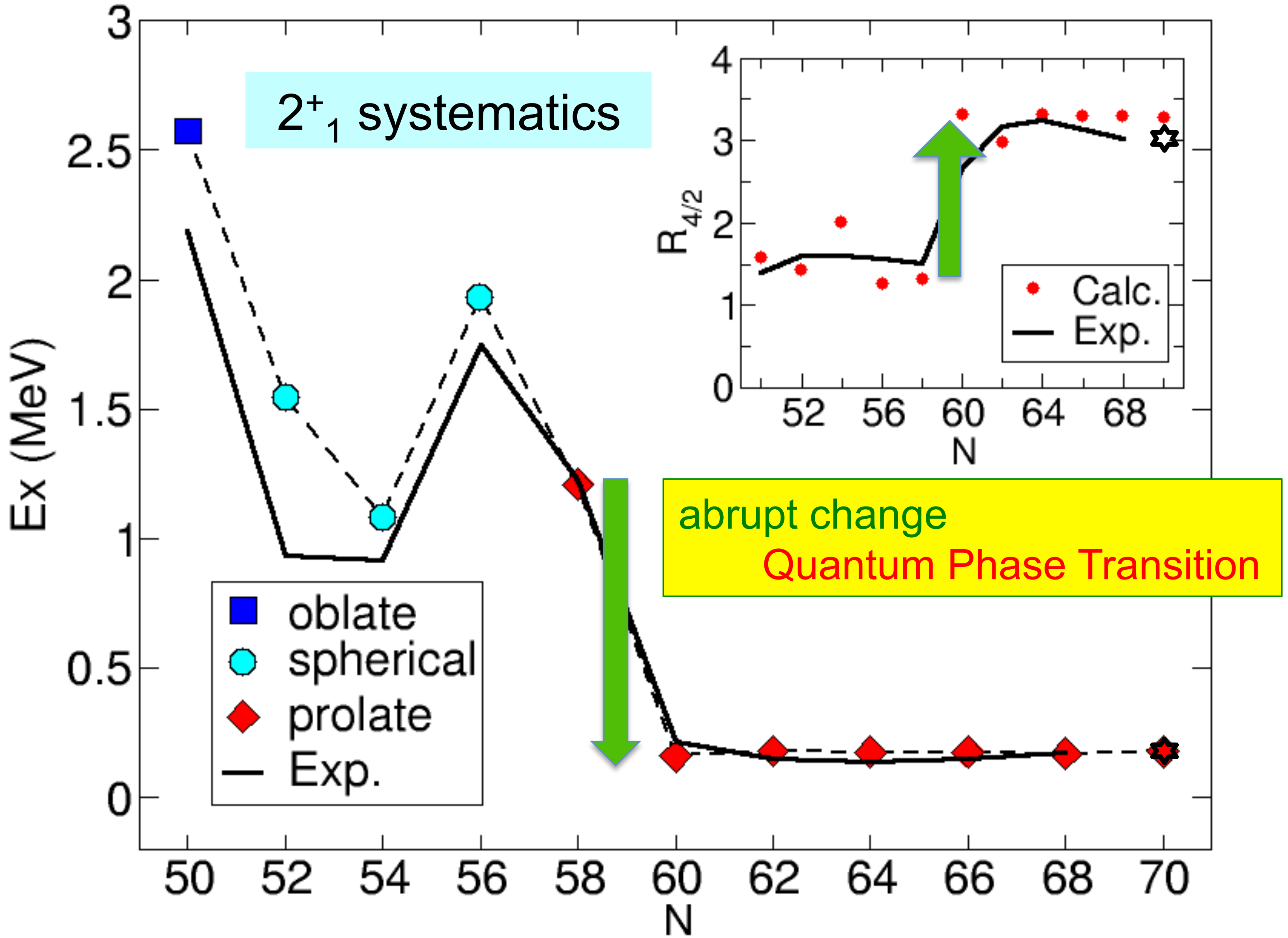
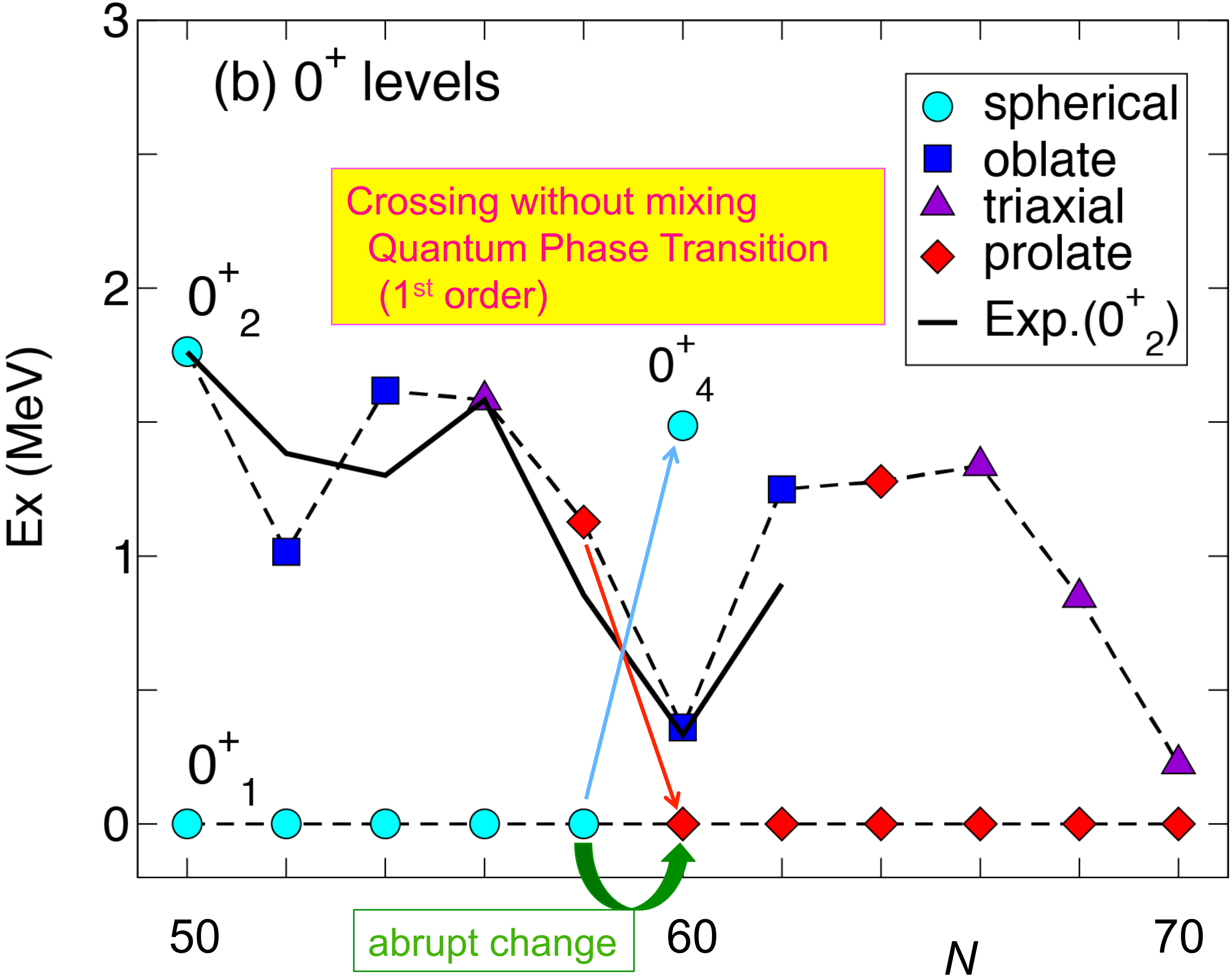


FIG. 12. Systematics of the experimental and theoretical first excited 2^+ states along the zirconium chain.

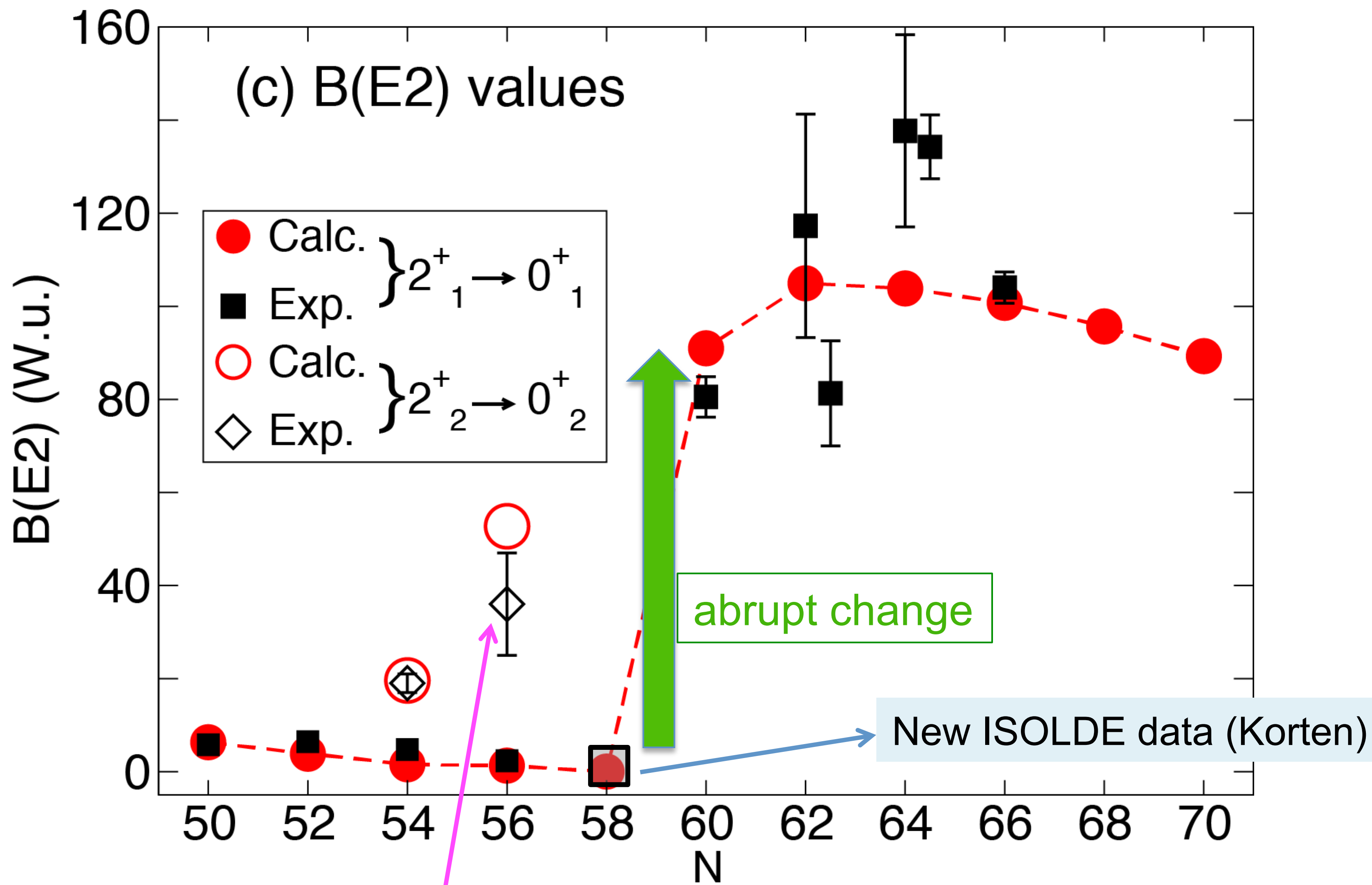


2+1 systematics
abrupt change
Quantum Phase Transition
oblate
spherical
prolate
Exp.
Calc.
Exp.
Ex (MeV)
N
R4/2
N

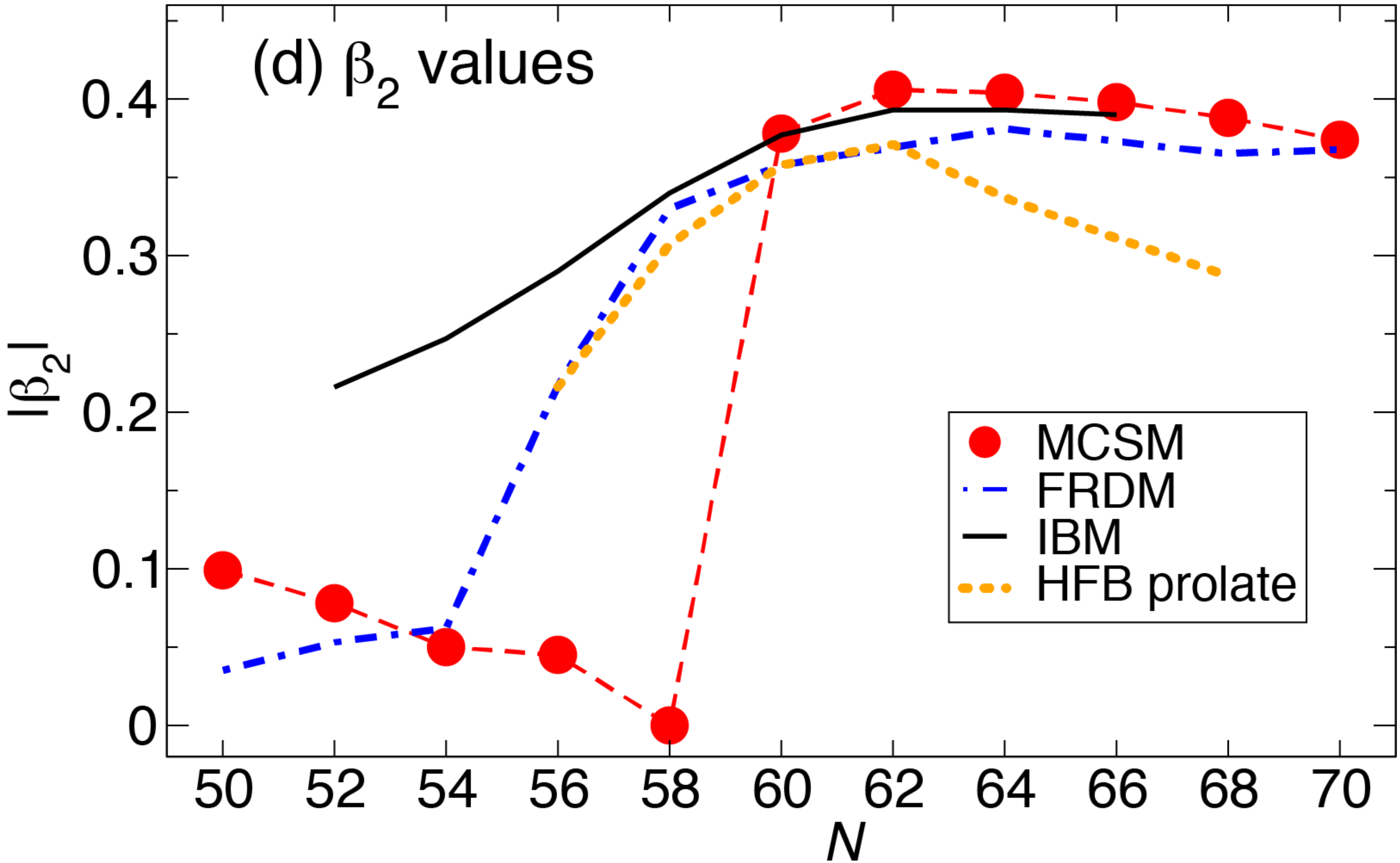


(b) 0+ levels
Crossing without mixing
Quantum Phase Transition
(1st order)
spherical
oblate
triaxial
prolate
Exp.(0+2)
0+2
0+4
0+1
abrupt change
Ex (MeV)
N
0
1
2
3
50
60
70

B(E2; 2^+ -> 0^+) systematics



New data from Darmstadt, Kremer *et al*. PRL 117, 172503 (2016)

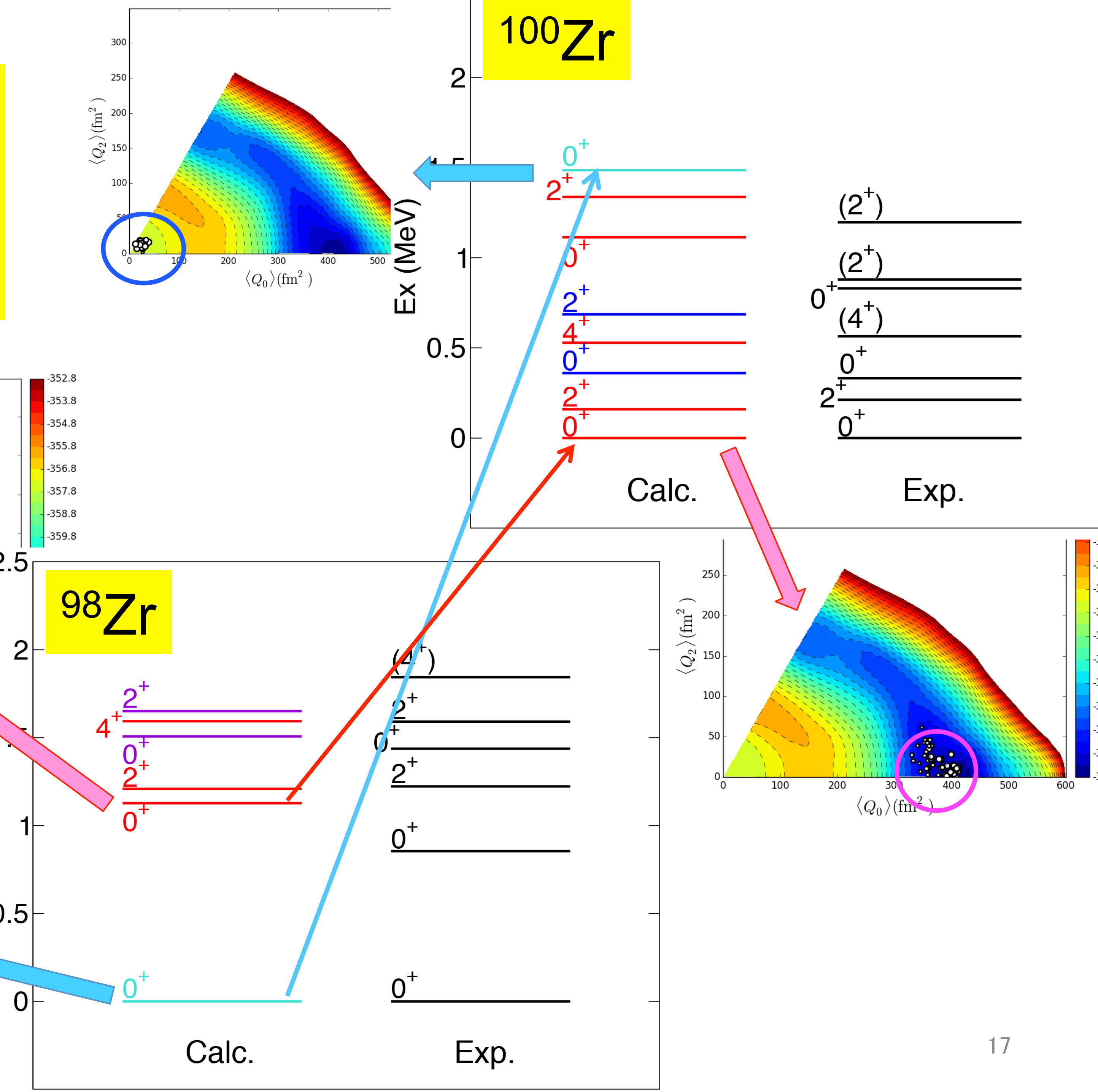


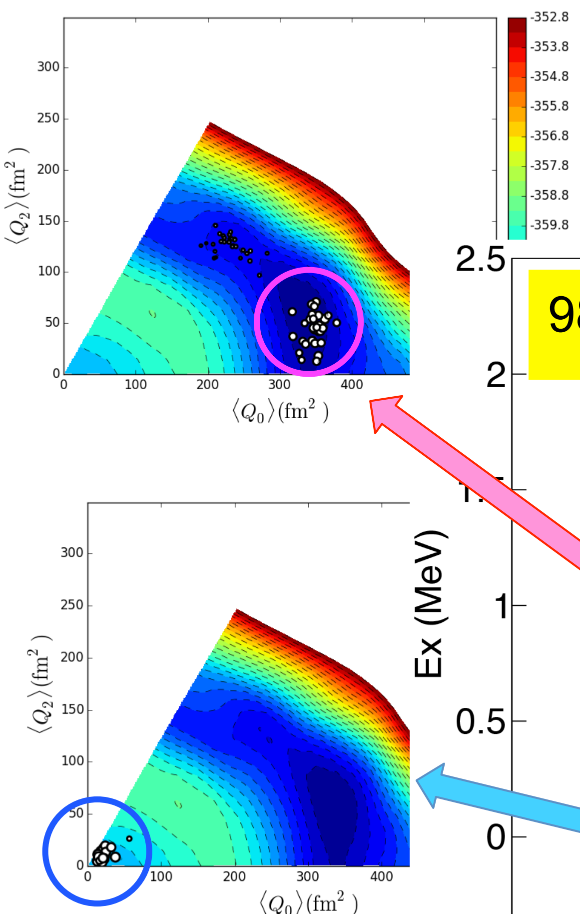
FRDM: S. Moeller et al. At. Data Nucl. Data Tables 59, 185 (1995).

IBM: M. Boyukata et al. J. Phys. G 37, 105102 (2010).

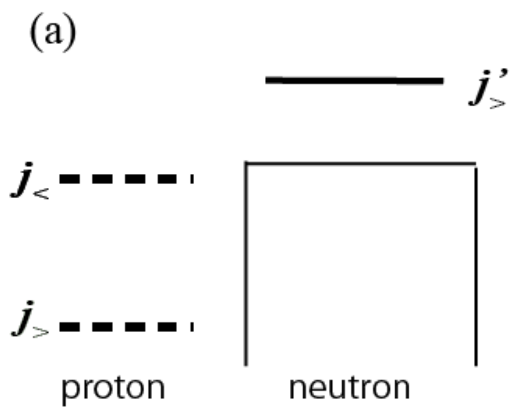
HFB: R. Rodriuez-Guzman et al. Phys. Lett. B 691, 202 (2010).

Quantum Phase Transition (1st order) due to crossing without mixing

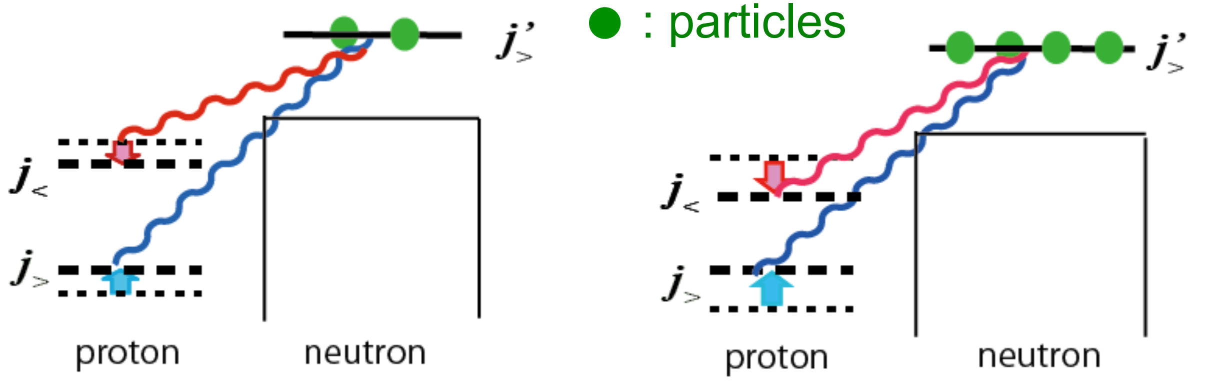




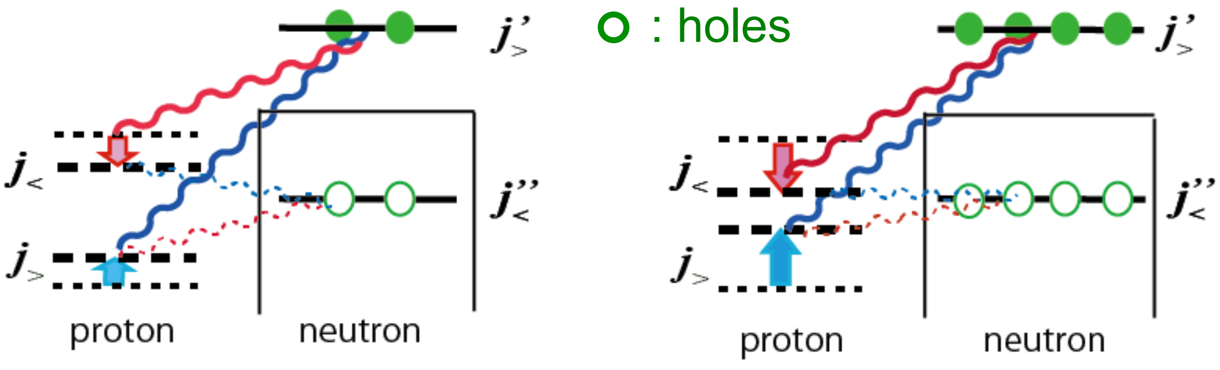
Shell Evolution - from Type I to Type II -



Type I Shell Evolution : different isotopes

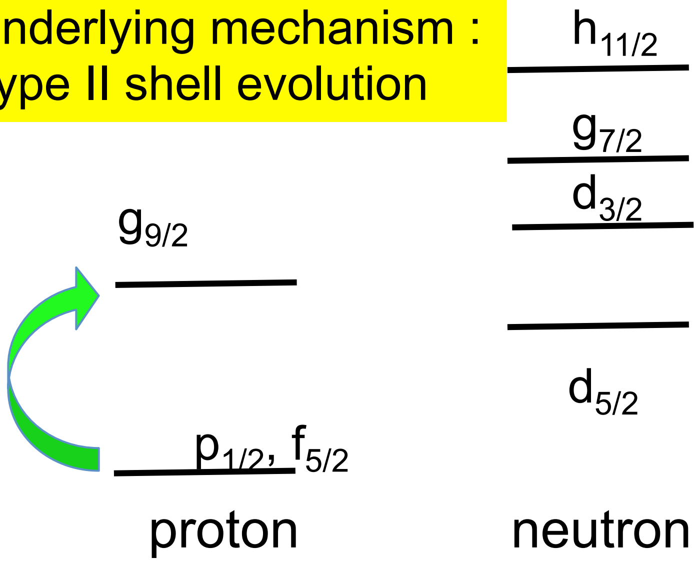


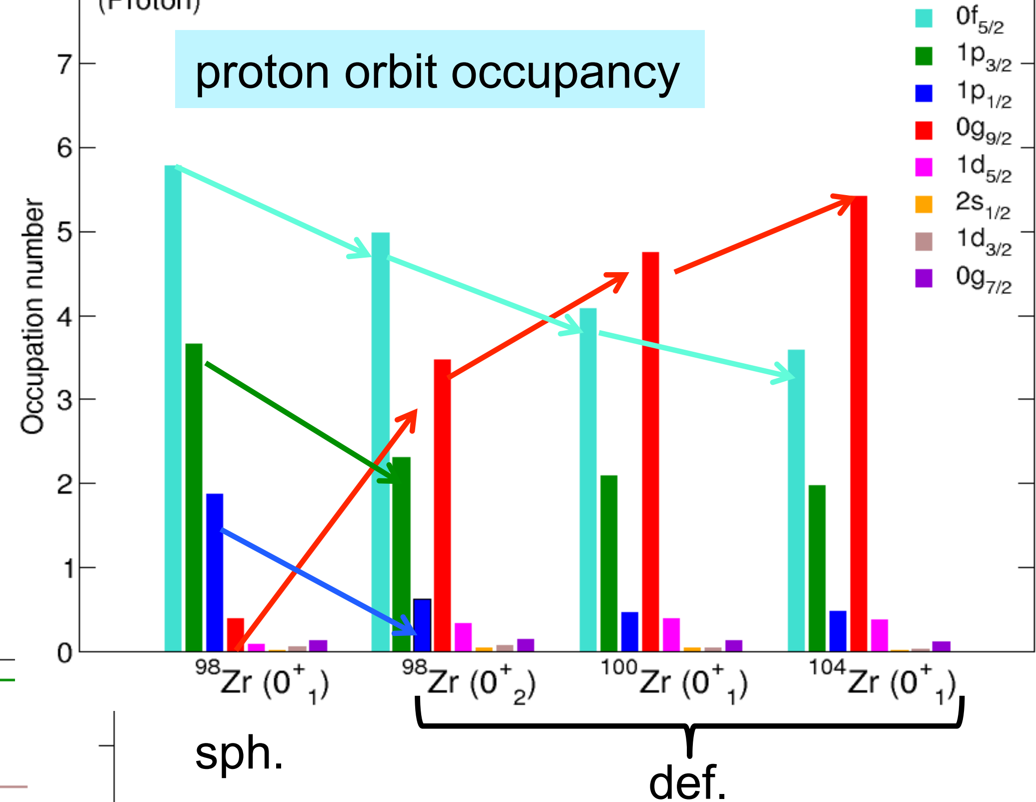
Type II Shell Evolution : within the same nucleus

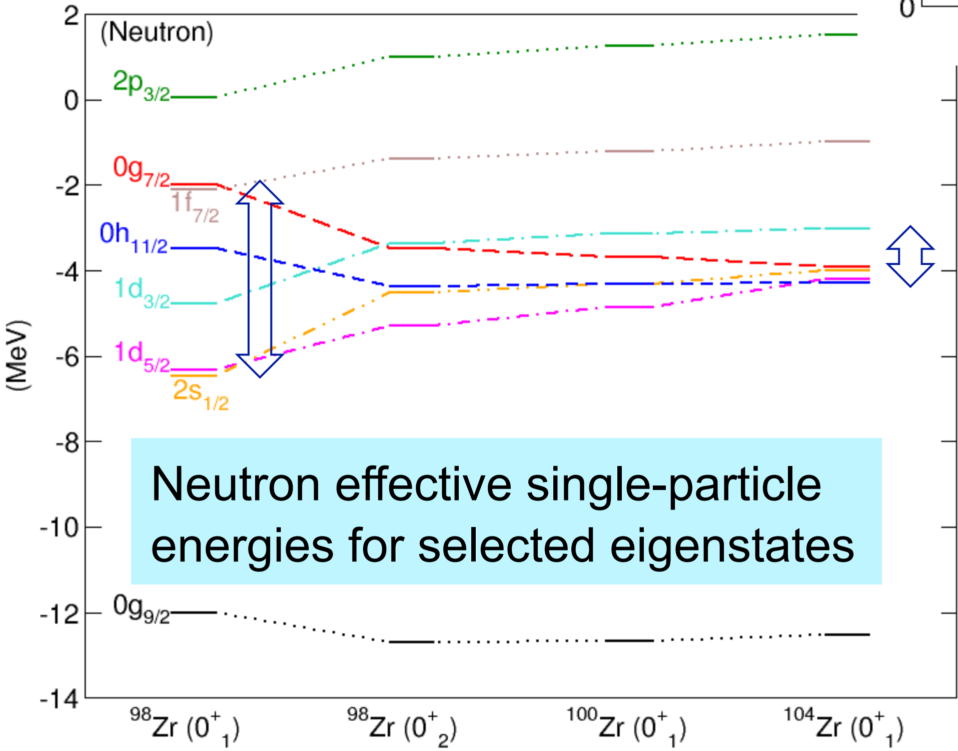


TO and Tsunoda, J. Phys. G: Nucl. Part. Phys. 43 (2016) 024009

Underlying mechanism : Type II shell evolution







Neutron effective single-particle energies are self-reorganized by nuclear forces (tensor and central) and certain configurations, so as to reduce resistance power against deformation.

- a case of type II shell evolution -

Intuitively speaking,

$$\text{deformation} = \frac{\text{quadrupole force}}{\text{resistance power}}$$

resitance power ← pairing force

↖ single-particle energies

Atomic nuclei can "organize" their single-particle energies by taking particular configurations of protons and neutrons, thanks to orbit-dependences of nuclear forces (*e.g.*, tensor force).

Quantum Self Organization

Note : spherical single-particle energies are often treated being constant

Nilsson model Hamiltonian

"*Nuclear structure II*" by Bohr and Mottelson

deformed nuclei, is obtained by a simple modification of the harmonic oscillator (Nilsson, 1955; Gustafson *et al.*, 1967),

$$H = \frac{\mathbf{p}^2}{2M} + \frac{1}{2}M\left(\omega_3^2 x_3^2 + \omega_\perp^2 (x_1^2 + x_2^2)\right) + v_{ll}\hbar\omega_0(\mathbf{l}^2 - \langle \mathbf{l}^2 \rangle_N) + v_{ls}\hbar\omega_0(\mathbf{l}\cdot\mathbf{s}) \qquad (5\text{-}10)$$

quadrupole deformed field

spherical field

$$\langle \mathbf{l}^2 \rangle_N = \frac{1}{2}N(N+3)$$

Figure	Region	$-v_{ls}$	$-v_{ll}$
5-1	N and $Z < 20$	0.16	0
5-2	$50 < Z < 82$	0.127	0.0382
5-3	$82 < N < 126$	0.127	0.0268
5-4	$82 < Z < 126$	0.115	0.0375
5-5	$126 < N$	0.127	0.0206

Table 5-1 Parameters used in the single-particle potentials of Figs. 5-1 to 5-5.

Spin-orbit force

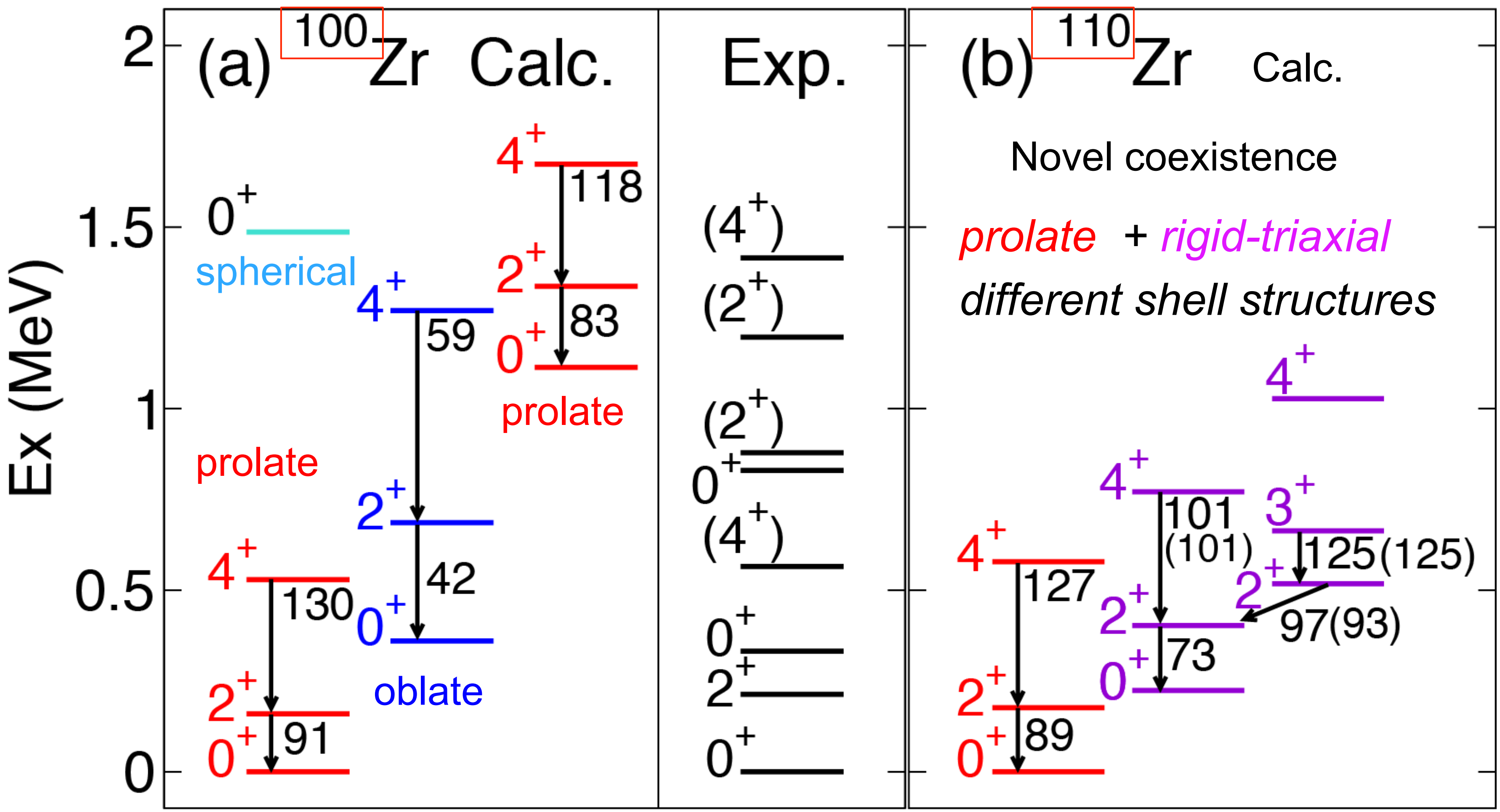
A= 68 1.28

A=100 1.12

A=186 0.91

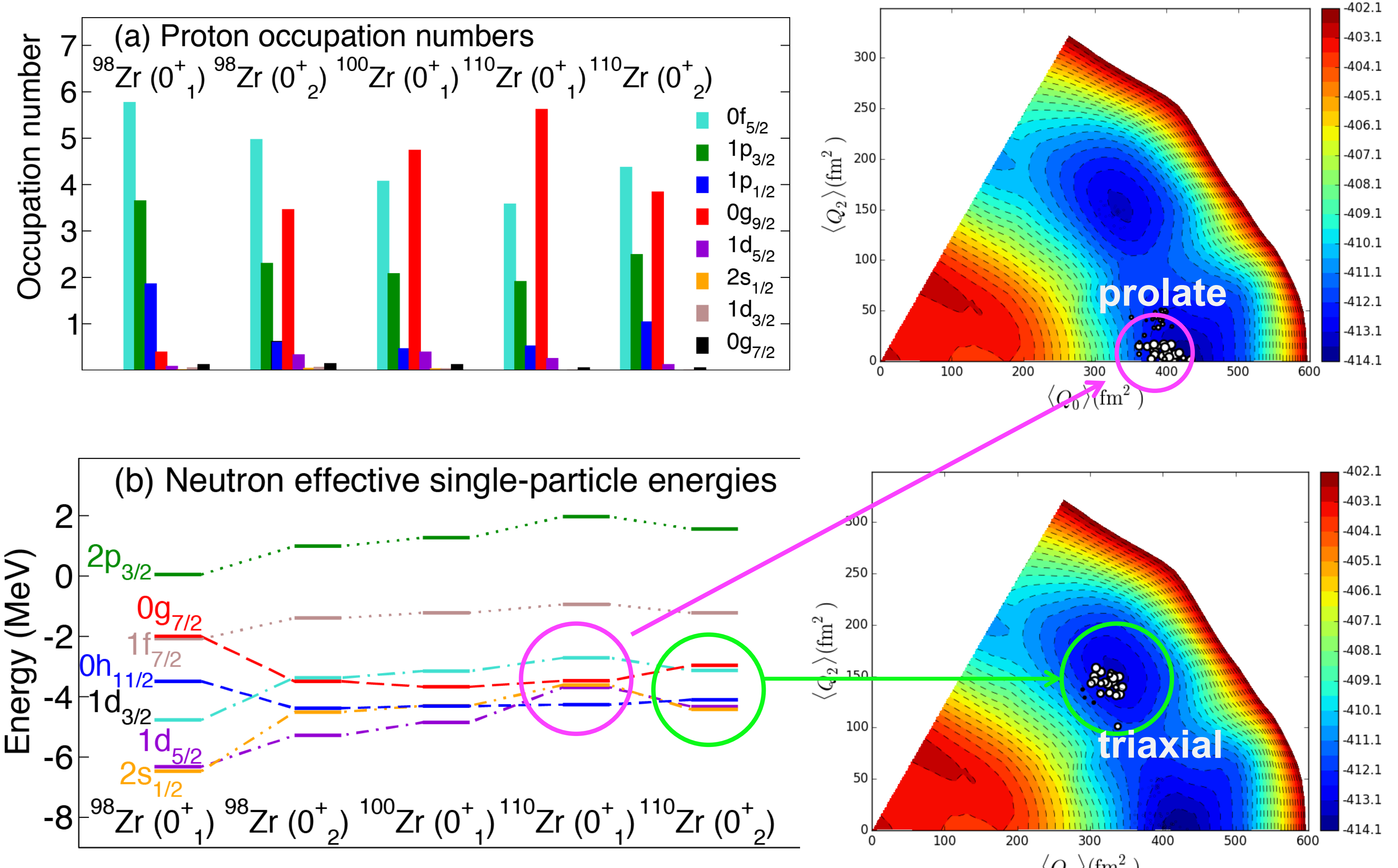
$(\mathbf{l}\cdot\mathbf{s})$

Prolate – rigid-triaxial shape coexistence



( ) : Rigid-triaxial rotor with gamma=28 degrees normalized at 2^+_2 -> 0^+_2

different shell structures ~ like "different nuclei"



Summary and Perspectives

- The shape transition in Zr is described with a standard SM interaction.
- The quantum phase transition (QPM) is identified between ^{98}Zr and ^{100}Zr.

  (abrupt qualitative change in the ground state as a function of N)
- Type II shell evolution reduces, in general, the resistance against deformation, with very different configurations for different shapes.

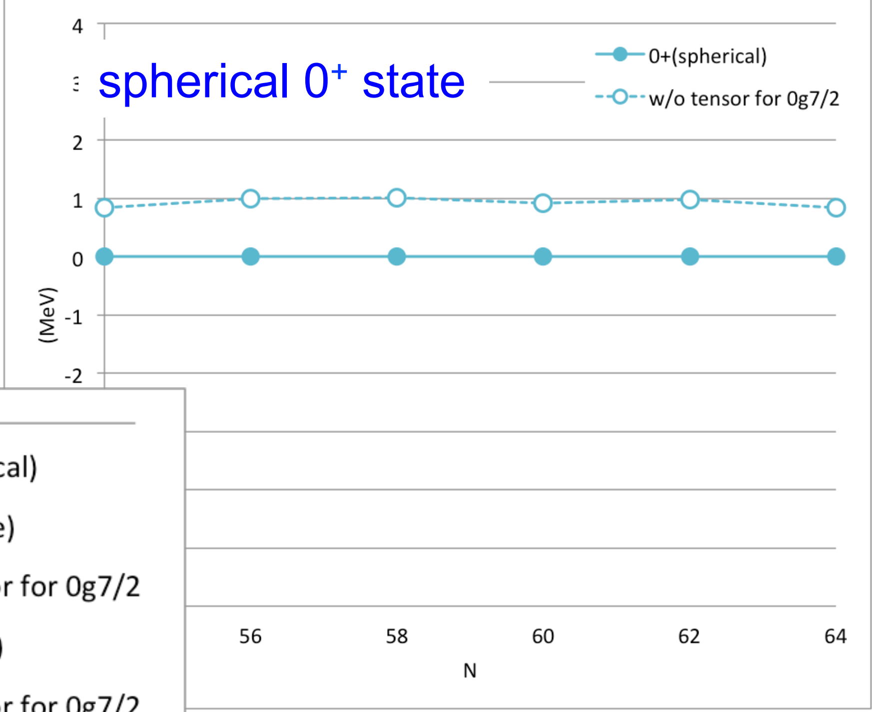
  → shape coexistence, QPM in shapes, *super D, octupole D, fission*, *etc*.
- The abrupt change (1st order QPM) is a consequence of level crossing with (almost) no mixing due to completely different configurations, despite that the nucleus is a finite quantum system.
- In more general terms, single particle energies can be self-organized (*quantum self-organization*), with the following condition:

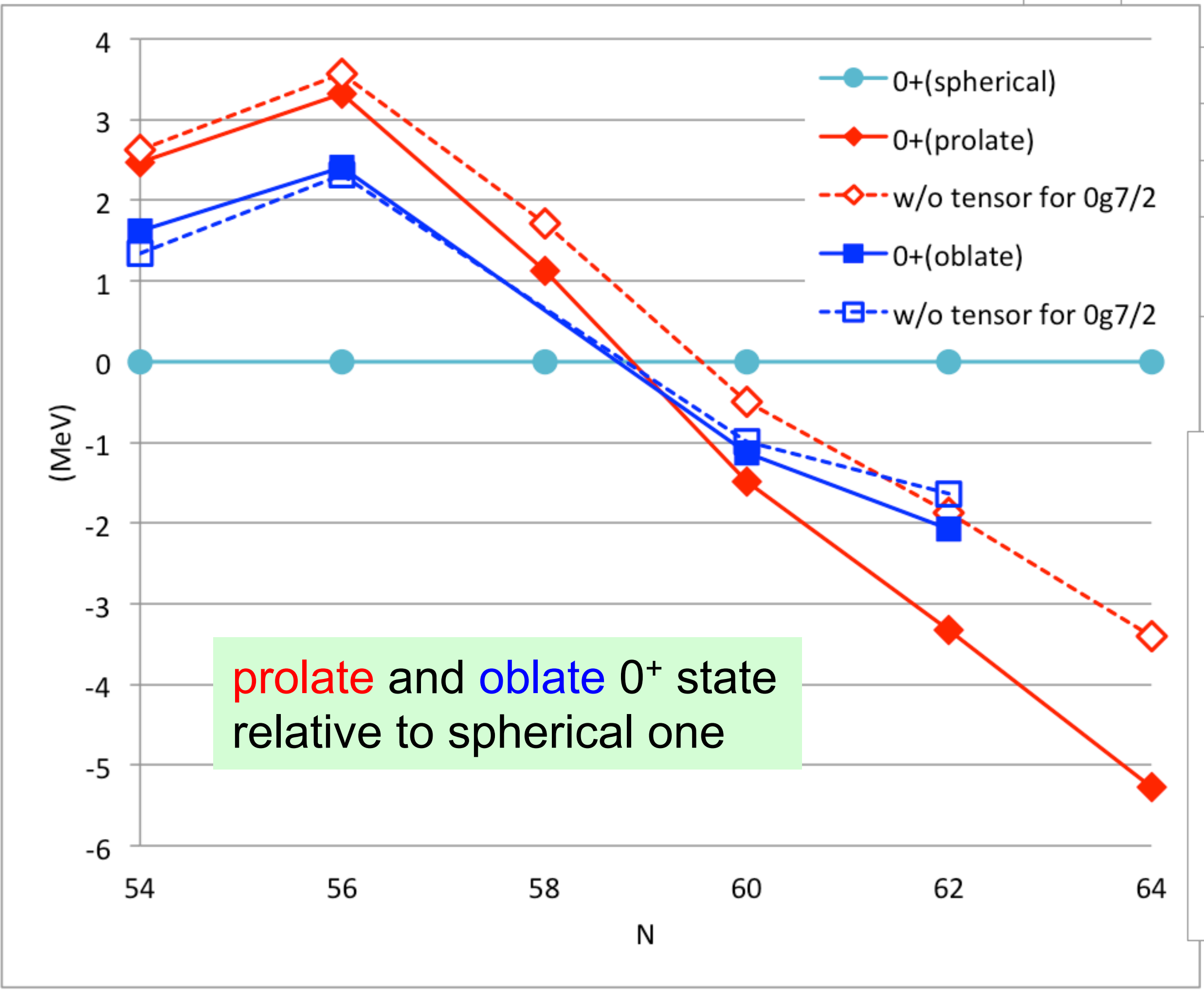
  (i) two quantum liquids (protons and neutrons),

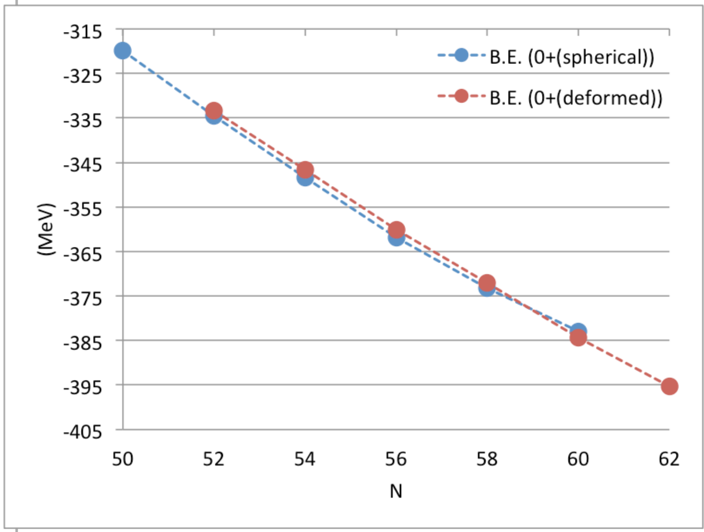
  (ii) two major force components : *e.g.*, quadrupole interaction to drive deformation and monopole interaction to control resistance.

→ ***Nuclei favor more distinct shapes, i.e., enhanced symmetries !***

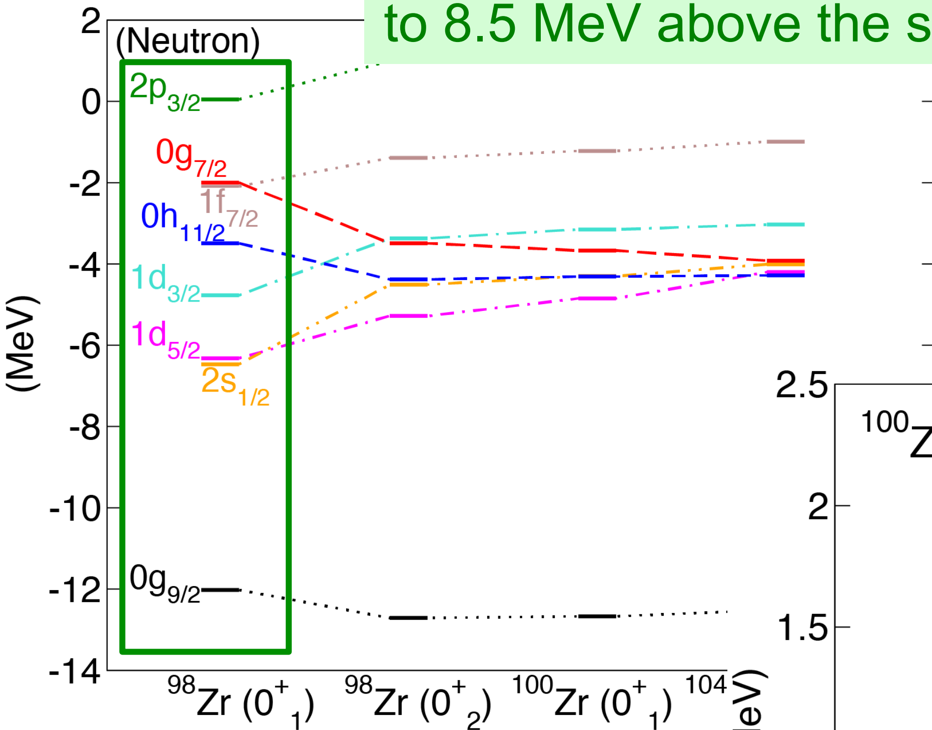
Effect of tensor force for $g_{7/2}$

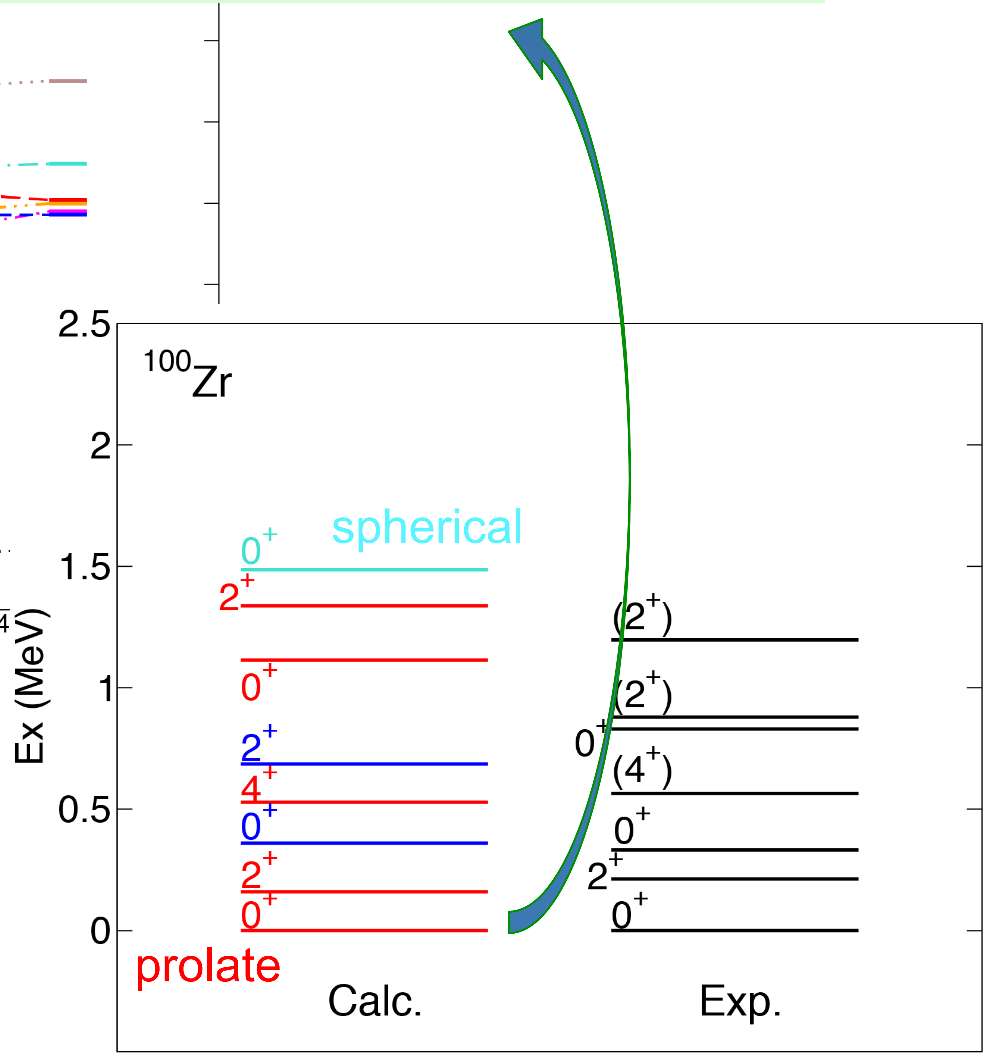






If these s.p.e.'s are used and the monopole interaction is eliminated, the prolate 0^+ is raised to 8.5 MeV above the spherical one.

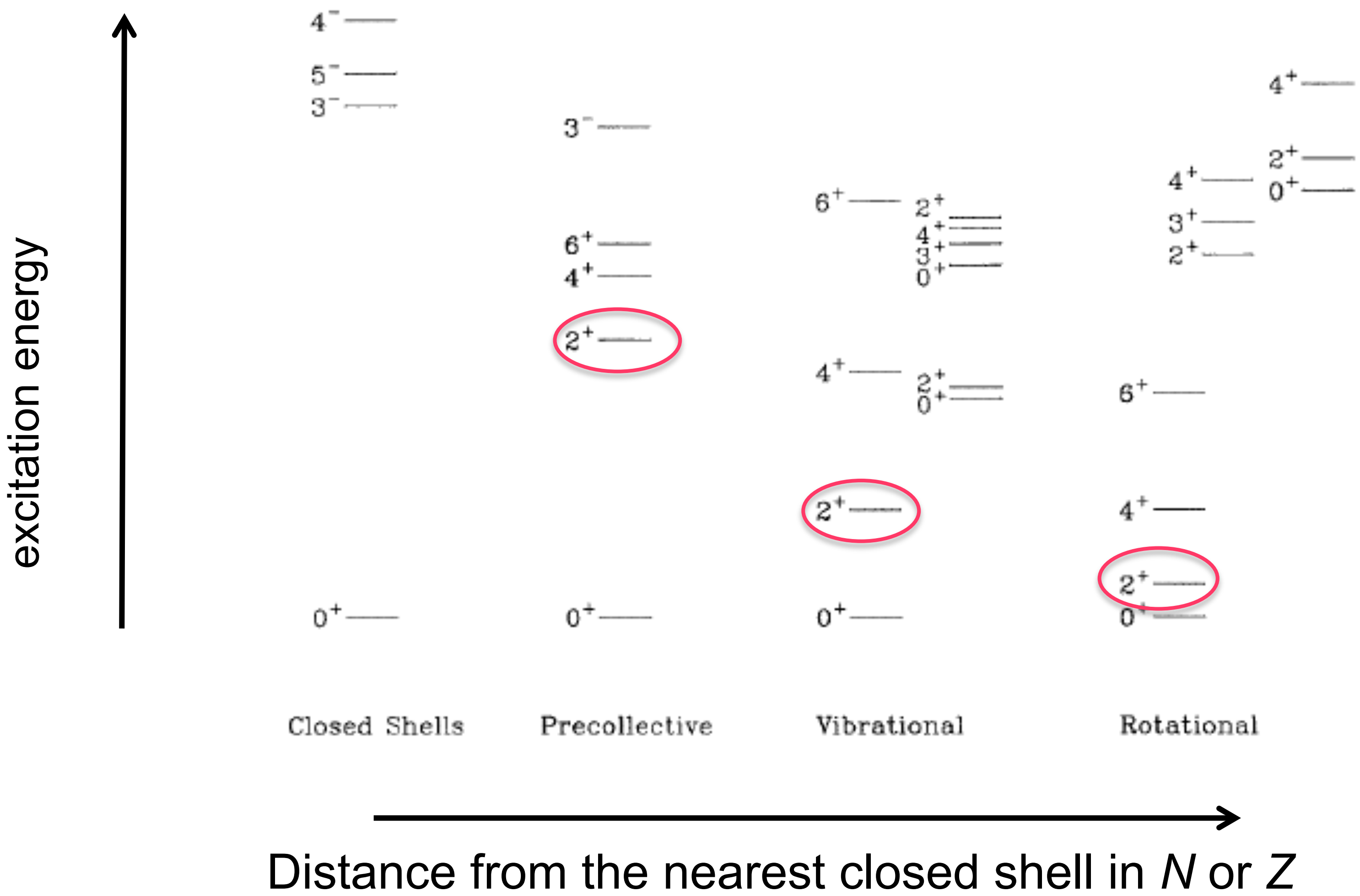




Schematic picture of shape evolution (sphere to ellipsoid)

- monotonic pattern throughout the nuclear chart –

one "shape" per one nucleus in many stable nuclei



From "Nuclear Structure from a Simple Perspective", R.F. Casten (2001)