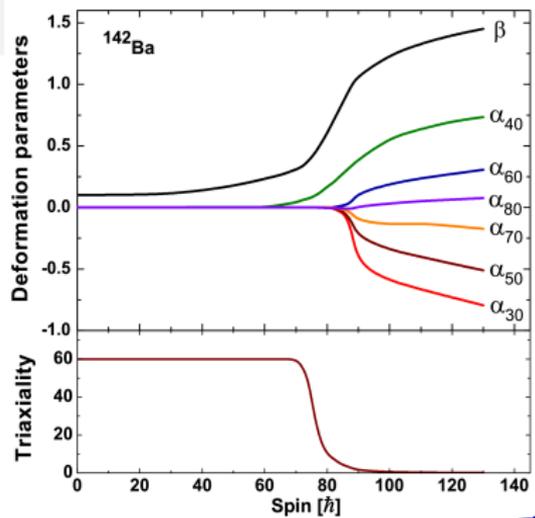


# **Dynamical description of the Poincaré and Jacobi shape transitions in nuclei: Rotation induced symmetry breaking phenomena**

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November 8, 2016



H.Poincare, Acta Math. 7 (1885) 259  
 A.Maj, K.M., J.Dudek, et al, Int. J. Mod. Phys. E 19 (2010) 532

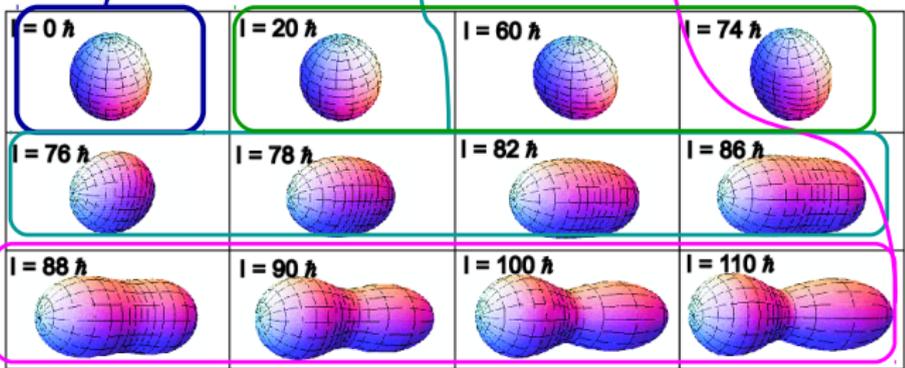
Oblate (MacLaurin)

Octupole, left- right asymmetric (Poincare)

Sphere

Elongated triaxial (Jacobi)

Theoretical prediction (for the first time) of the Poincare shape transition in atomic nuclei



## Method

- At high temperatures, the total nuclear energy can be approximated by the macroscopic energy expression only
- The angular momentum effects can be treated, to the first approximation classically

$$E_{\text{total}}(\{\text{def.}\}; I) = E_{\text{macro}}(\{\text{def.}\}) + \frac{\hbar^2}{2\mathcal{J}\{\text{def.}\}} \cdot I(I + 1)$$

- Conclusion:

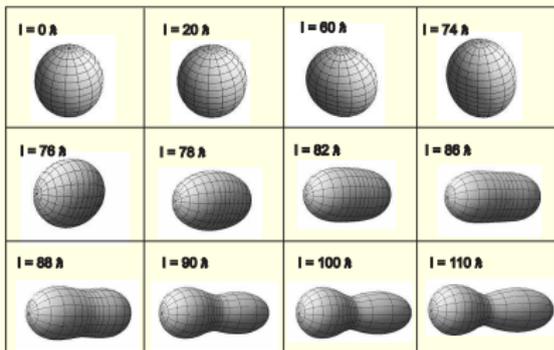
Using the macroscopic energy as optimal as possible will be of importance:  
in our case  $\rightarrow$

**Lublin - Strasbourg Drop with shape-dependent congruence energy:**

$$\begin{aligned} V = E_{\text{LSD-C}}(N, Z; \text{def}) &= ZM_{\text{H}} + NM_{\text{n}} - 0.00001433 Z^{2.39} \\ &+ E_{\text{vol.}}(N, Z) + E_{\text{surf.}}(N, Z; \text{def}) \\ &+ E_{\text{curv.}}(N, Z; \text{def}) + E_{\text{cong.}}(N, Z; \text{def}). \end{aligned}$$

# Jacobi transition $\rightarrow$ OBLATE-TRIAxIAL-PROLATE Axial Symmetry Breaking

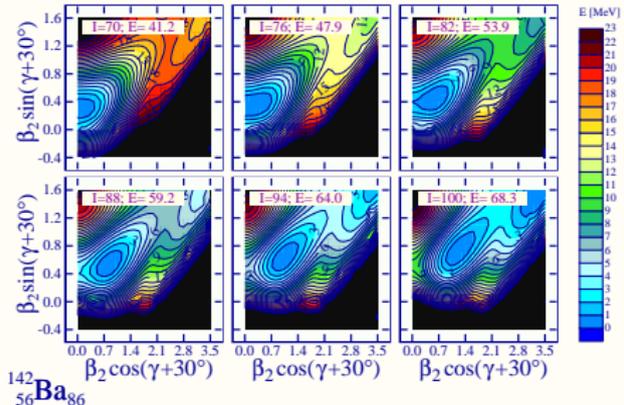
The  $^{142}\text{Ba}$  with spin  $I = 0 - 110 \hbar$ . A. Maj et al. *Int.J.Mod.Phys.E* 19 (2010).



The shape evolution of the minimum macroscopic energy

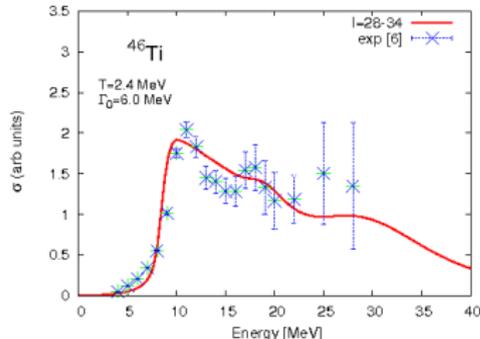
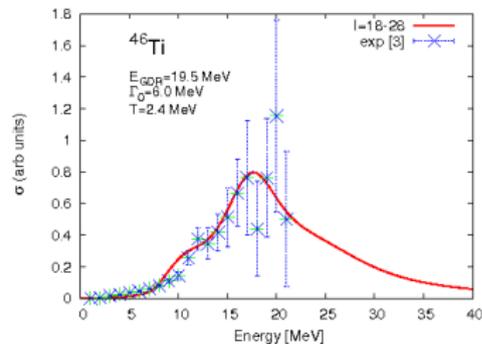
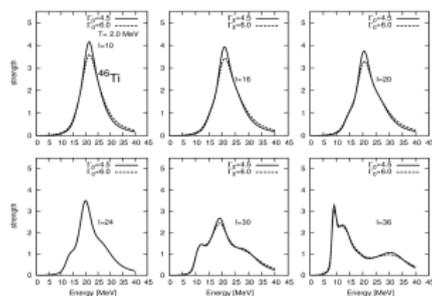
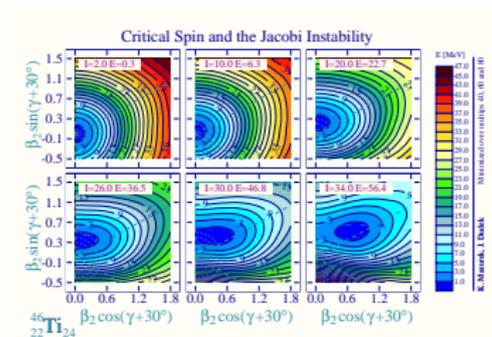
Nuclear surface parametrization:

$$\mathcal{R}(\vartheta, \varphi) = R_0 c(\{\alpha\}) \sum_{\lambda, \mu} [1 + \alpha_{\lambda, \pm \mu} Y_{\lambda, \pm \mu}(\vartheta, \varphi)]$$



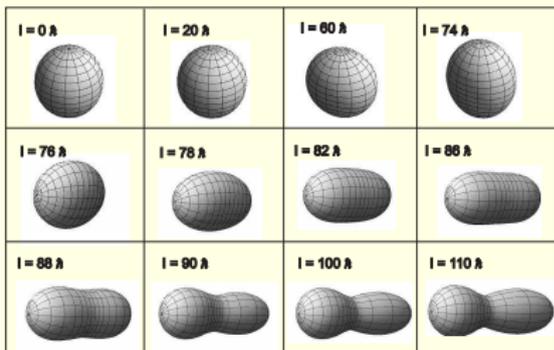
# Jacobi Shape Transition

## Giant Dipole Resonances

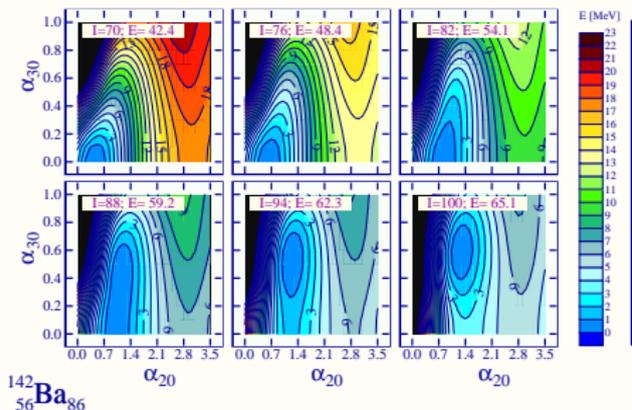


# Poincaré transition Left-Right Symmetry Breaking

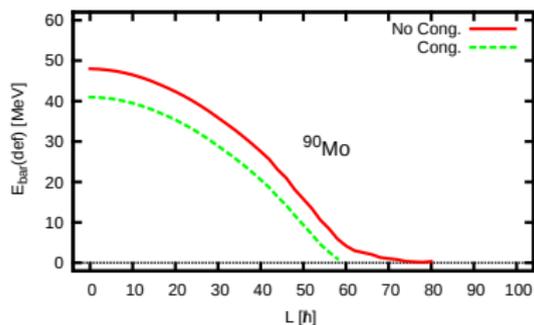
The  $^{142}\text{Ba}$  with spin  $I = 0 - 110 \hbar$ . A. Maj et al. *Int.J.Mod.Phys.E* 19 (2010).



The shape evolution of the minimum macroscopic energy



# Fission barriers



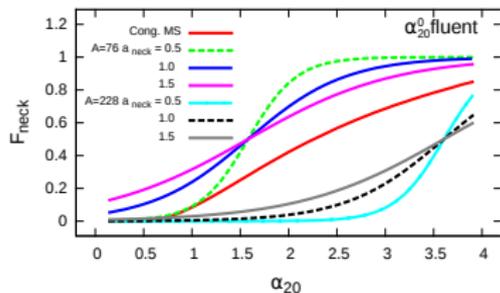
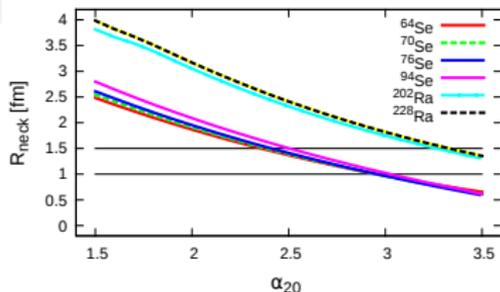
## Shape-dependent congruence energy

$$E_{\text{cong}}(\alpha_{20}) = W_0(Z, N) \cdot F_{\text{neck}}(\alpha_{20})$$

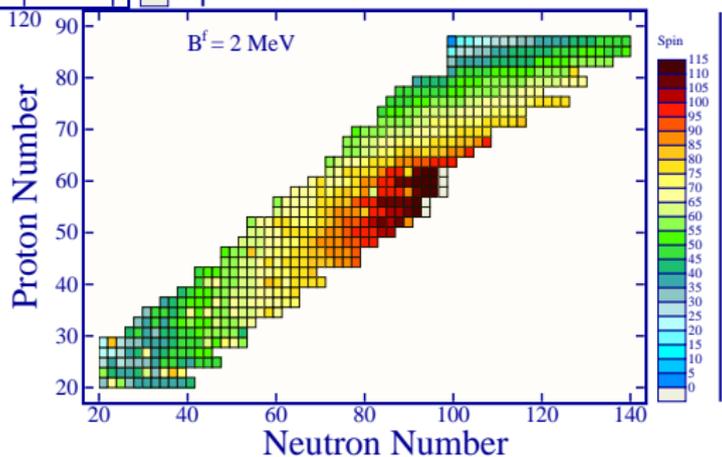
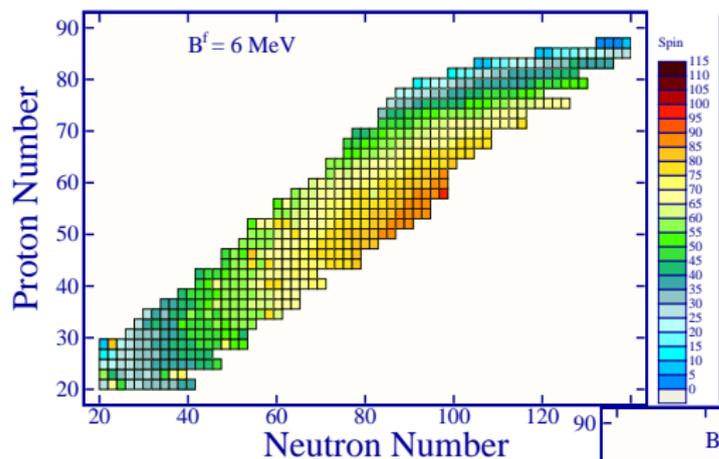
$$F_{\text{neck}}(\alpha_{20}) = 1 + \frac{1}{2} \left\{ 1 + \tanh \left[ (\alpha_{20} - \alpha_{20}^0) / a_{\text{neck}} \right] \right\}$$

$$W_0(Z, N) = -C_0 \exp(-W|I|/C_0) \text{ with } I \equiv (N - Z)/A$$

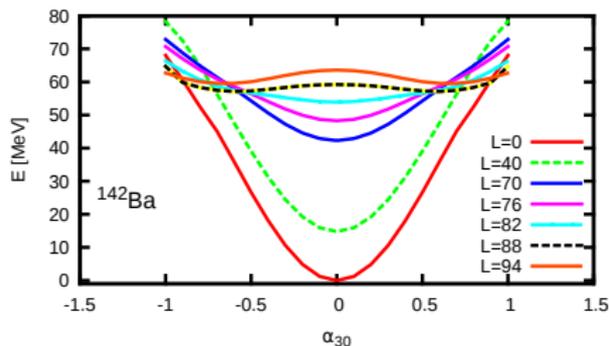
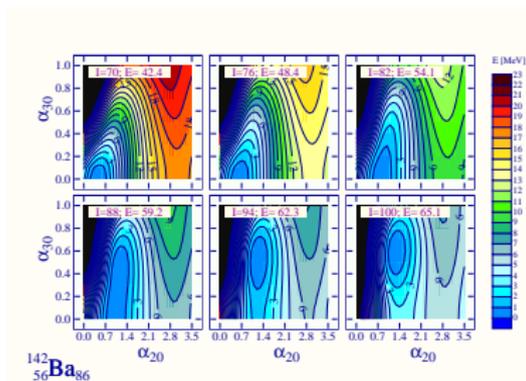
Wigner E, Phys. Rev. 51 (1937) 106,947; Myers W D, Swiatecki W J, Nucl. Phys. A 81 (1966) 1; Nucl. Phys. A 612 (1997) 249



# Fission barriers



# Collective Hamiltonian



## Octupole vibration

(<http://oer.physics.manchester.ac.uk/NP/Collective/I3.html>)

$$\hat{H} = \frac{\hbar^2}{2B} \frac{\delta^2}{\delta \alpha_{30}^2} + \frac{1}{2} C \alpha_{30}^2$$

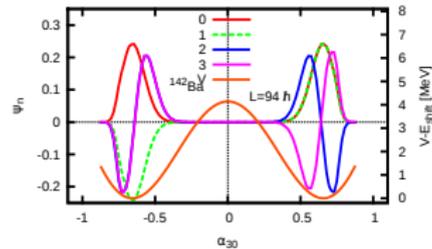
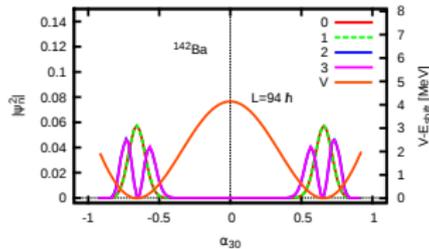
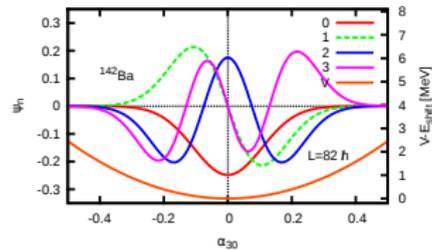
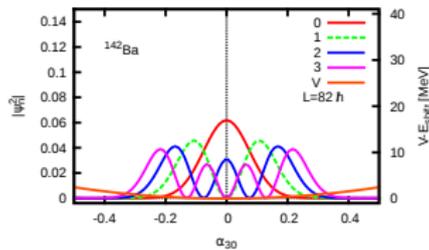
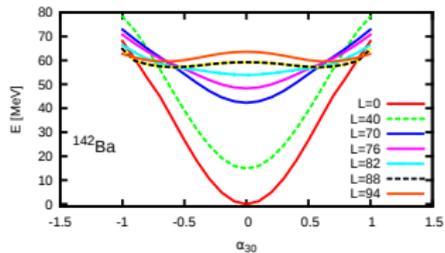
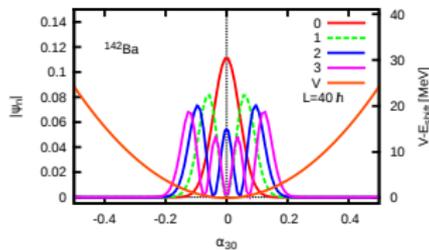
where  $B$  is the mass parameter and  $C$  the stiffness of the potential.

$$\langle \psi_n | \hat{H} | \psi_m \rangle = e_n \delta_{nm}$$

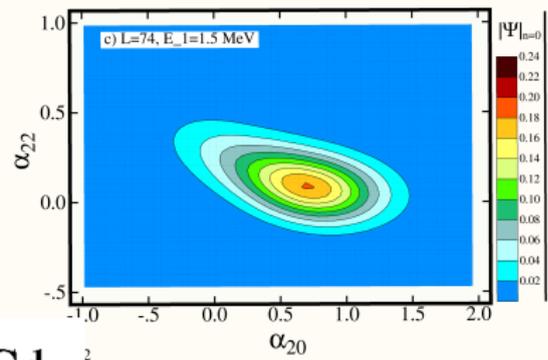
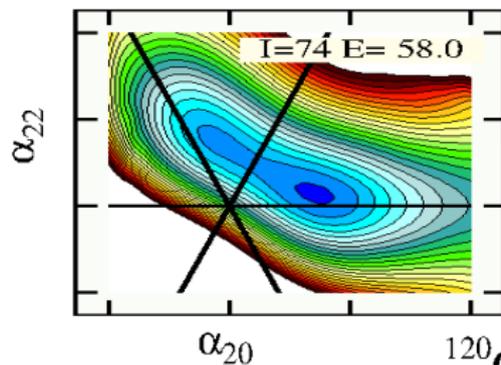
$$\hat{H} = \frac{\hbar^2}{2B} \frac{\delta^2}{\delta \alpha_{30}^2} + V(\alpha_{30})$$

A. Bohr, B. Mottelson, Nuclear Structure vol. II

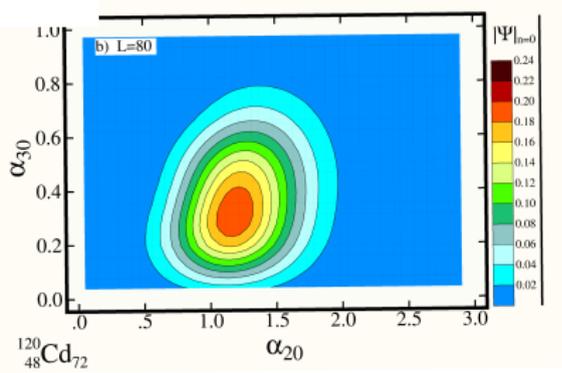
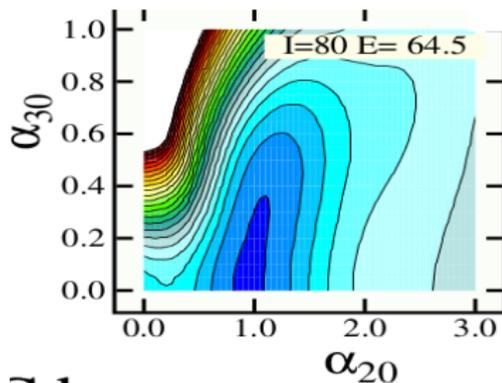
# Collective hamiltonian – 1D Wave functions



# Collective hamiltonian – 2D Wave functions



$^{120}_{48}\text{Cd}_{72}$

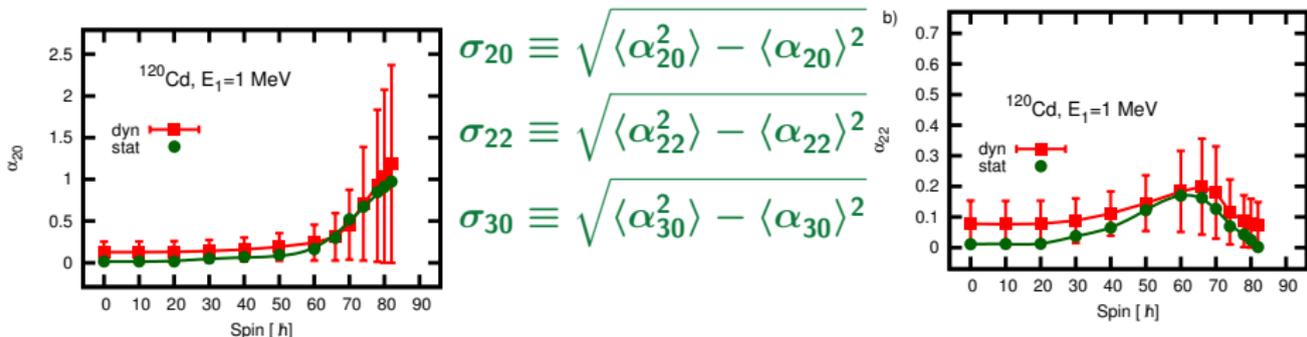


$^{120}_{48}\text{Cd}_{72}$

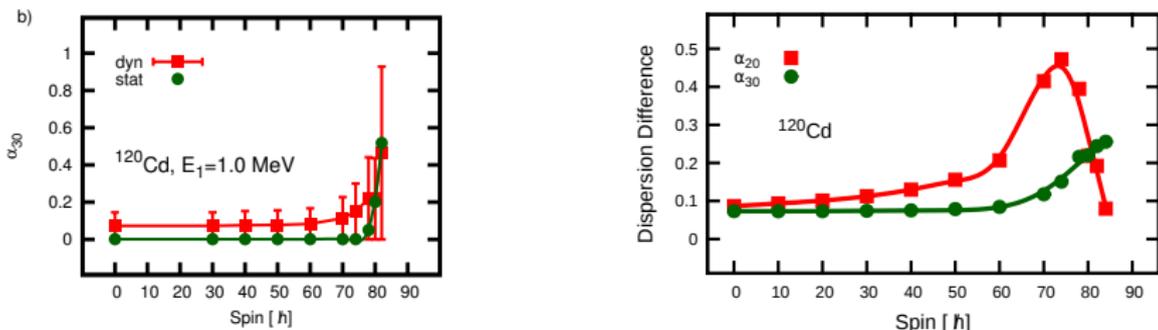
# Collective hamiltonian – deformation dispersion

The mean values of the dynamical quadrupole deformation

K. M., J. Dudek, A. Maj, PRC 91, 034301

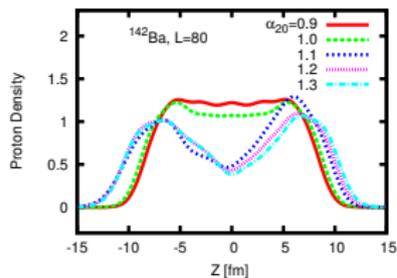
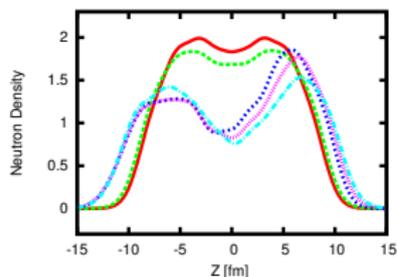


The mean values of the dynamical octupole deformation

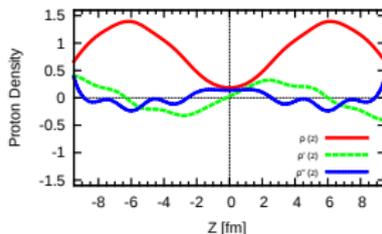


# Fission fragment shape estimation

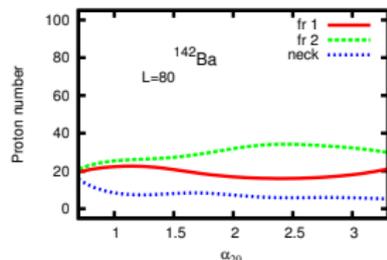
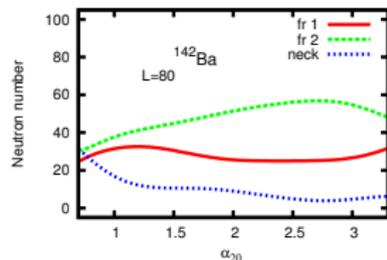
Nuclear matter densities got with Woods-Saxon single particle potential.



Matter density obtained in adiabatic scheme.

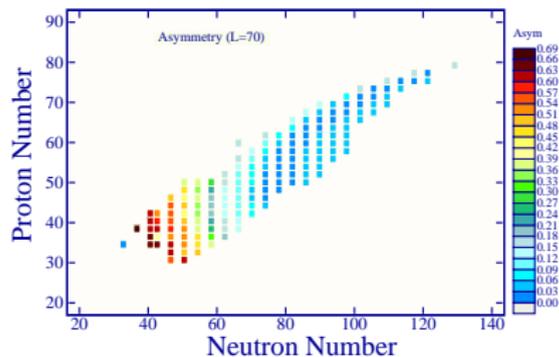
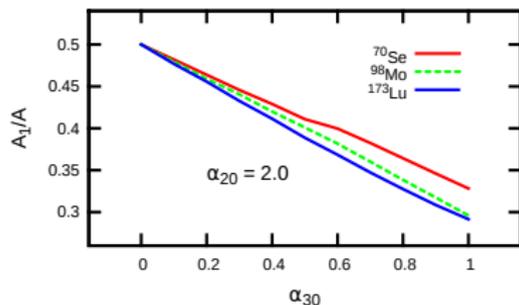
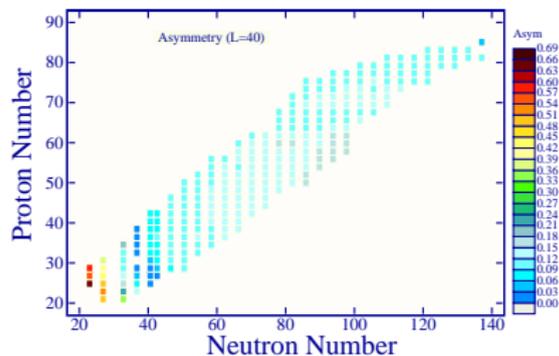
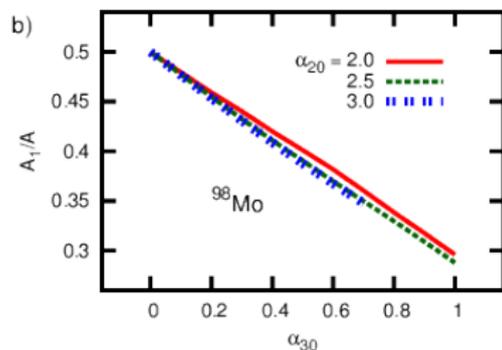


Shape of  $^{142}\text{Ba}$  for spin  $80 \hbar$  energy minimum. Derivatives of the microscopic densities helps with estimation of the neck region.



Proton and neutron number evolution in FF and neck with increasing the elongation.

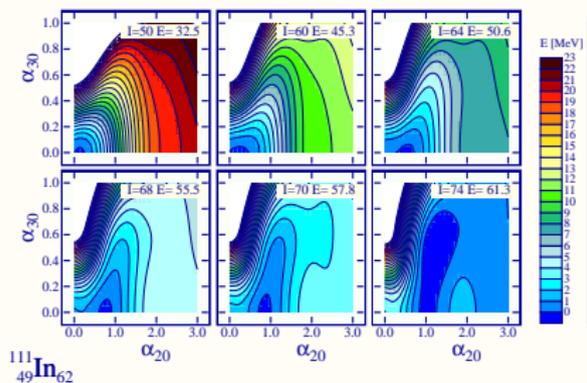
# Fission fragment shape estimation



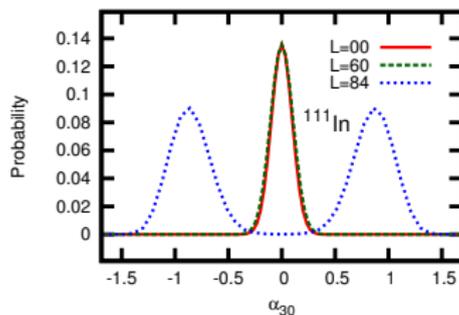
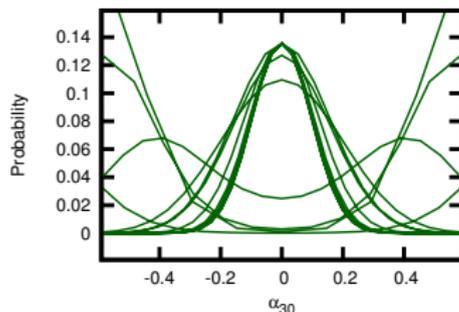
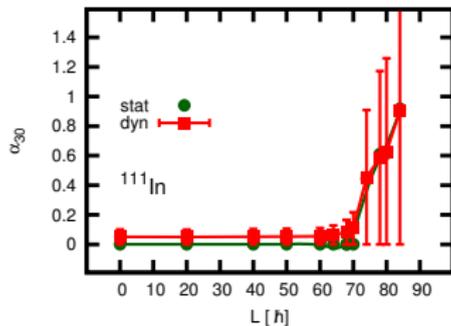
Mass asymmetry of light fission fragment ( $A_1/A$ ) for three nuclear elongations:  $\alpha_{20} = 2.0$ ; 2.5 i 3.0 for  $^{98}\text{Mo}$ . ( $L=0$ , minimization over  $\alpha_{30}$ -  $\lambda < 12$ )

# Probability of octupole deformation

The wave function are obtained by the collective Hamiltonian

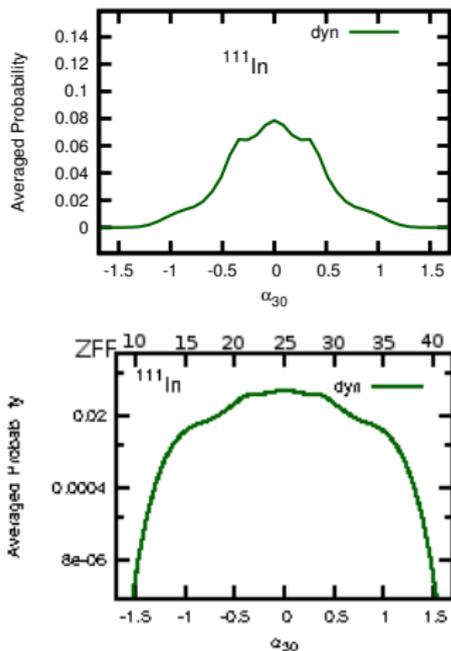


$^{111}_{49}\text{In}_{62}$



The probability for various spin got within octupole vibrations ( $n=0$ ).

# Comparison with existing experimental data: $^{111}\text{In}$



The probability integrated with triangular spin distribution assumed.

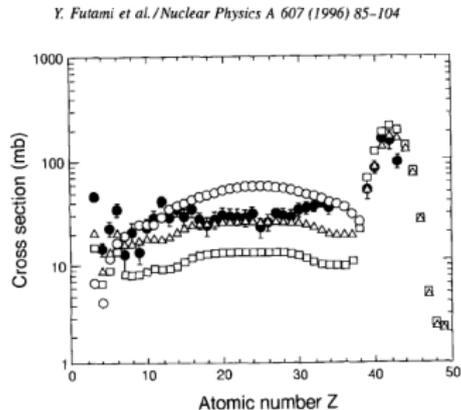


Fig. 6. The obtained Z-distribution of the reaction  $^{84}\text{Kr} + ^{27}\text{Al}$  at  $E/A = 10.6$  MeV/nucleon is shown by closed circles. The calculated results of Z-distribution by the EHPM are also indicated. Open circles, squares and triangles correspond to the cases of the full excitation energy,  $E_{\text{cut-off}} = 60$  MeV and  $E_{\text{cut-off}} = 80$  MeV, respectively.

The full charge distribution from reaction:



Y. Futami et al., Nucl. Phys. **A 607**, 85 (1996).

# Summary

- The rotation induced shape transitions are the wonderful example of the symmetry breaking in nuclei.
- The oblate–triaxial–prolate shape changes break the axial symmetry - Jacobi transition.
- The Poincaré transition is defined as changing of nuclear shape from prolate to octupoly deformed with increasing spin.
- The asymmetry of the nascent nuclei coming from fission is calculated microscopically.
- The experimental evidence of the Poincaré transition could be the evolution of the shape of the GDR strength function or/and widening of the fission fragment mass/charge distributions with increasing spin.
- The experimental data for fission of hot medium mass nuclei in well established spin range are necessary for further discussion.