Partial Dynamical Symmetries and Shape Coexistence in Nuclei

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Shape Coexistence in Nuclei

Exp:	prolate-oblate	Kr, Se, Hg neutron-deficient isotopes
	spherical-prolate-oblate	^{186}Pb
	spherical-prolate	Sr neutron-rich isotopes

- Shell model approach

  - Multiparticle-multihole intruder excitations across shell gaps
  - Drastic truncation of large SM spaces

- Mean-field approach (EDF)

  - Coexisting shapes associated with different minima of an energy surface
  - Beyond MF methods: restoration of broken symmetries

- Symmetry-based approach

  - Dynamical symmetries ↔ phases
  - Geometry: coherent (intrinsic) states

    Global min: equilibrium shape (β_0,γ_0)

$$E_N(\beta,\gamma) = \langle \beta,\gamma;N|\hat{H}|\beta,\gamma;N\rangle$$

$$|\beta,\gamma;N\rangle = (N!)^{-1/2}(b_c^\dagger)^N\,|0\,\rangle$$

$$b_c^\dagger \propto \beta\cos\gamma d_0^\dagger + \beta\sin\gamma(d_2^\dagger + d_{-2}^\dagger)/\sqrt{2} + s^\dagger$$

Dynamical Symmetry

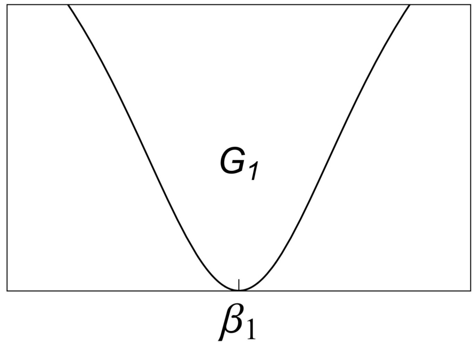
$$\hat{H} = \sum_G a_G \hat{C}_G$$

$U(6) \supset U(5) \supset SO(5) \supset SO(3)$ $\quad |[N]\, n_d\, \tau\, n_\Delta\, L\rangle$ $\quad$ Spherical vibrator $\quad \beta=0$

$U(6) \supset SU(3) \supset SO(3)$ $\quad |[N]\, (\lambda, \mu)\, K\, L\rangle$ $\quad$ Axial rotor $\quad \beta = \pm\sqrt{2},\ \gamma=0$

$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$ $\quad |[N]\, \sigma\, \tau\, n_\Delta\, L\rangle$ $\quad$ γ-unstable rotor $\quad \beta=1,\ \gamma$ arbitrary

- Complete solvability
- Good quantum numbers for all states
- DS: benchmark for a single shape [G_1 = U(5), SU(3), SO(6)]



Dynamical Symmetry

$$\hat{H} = \sum_G a_G \hat{C}_G$$

$U(6) \supset U(5) \supset SO(5) \supset SO(3)$	$\lvert [N]\ n_d\ \tau\ n_\Delta\ L \rangle$	Spherical vibrator	$\beta=0$
$U(6) \supset SU(3) \supset SO(3)$	$\lvert [N]\ (\lambda, \mu)\ K\ L \rangle$	Axial rotor	$\beta = \pm\sqrt{2},\ \gamma=0$
$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$	$\lvert [N]\ \sigma\ \tau\ n_\Delta\ L \rangle$	γ-unstable rotor	$\beta = 1$, γ arbitrary

- Complete solvability
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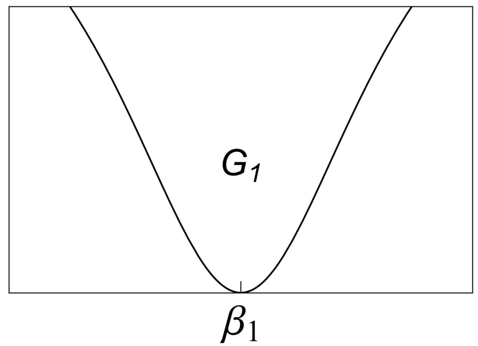
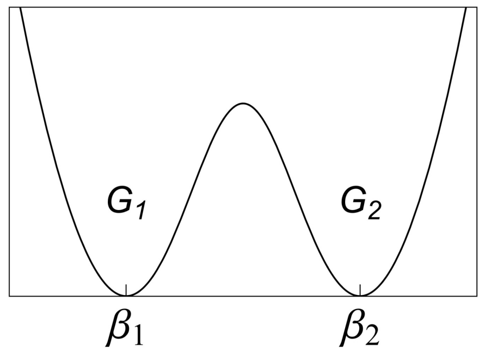
- DS: benchmark for a single shape [G_1 = U(5), SU(3), SO(6)]



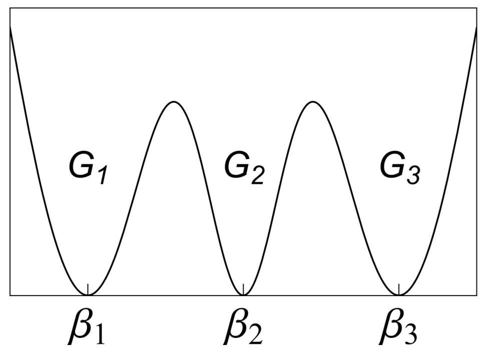
Partial Dynamical Symmetry

- Some states solvable and/or with good quantum numbers
- G_1, G_2 incompatible (non-commuting) symmetries
- PDS: benchmark for shape coexistence



$G_1 = U(5)$	$G_2 = SU(3)$	spherical - prolate
$G_1 = SU(3)$	$G_2 = \overline{SU(3)}$	prolate - oblate
$G_1 = U(5)$	$G_2 = SO(6)$	spherical - γ-unstable

$G_1 = U(5)$ $G_2 = SU(3)$ $G_3 = \overline{SU(3)}$ spherical-prolate-oblate



Dynamical Symmetry

$U(6) \supset U(5) \supset SO(5) \supset SO(3)$ $\quad |[N]\, n_d\, \tau\, n_\Delta\, L\rangle$ Spherical vibrator $\beta=0$

$U(6) \supset SU(3) \supset SO(3)$ $\quad |[N]\, (\lambda, \mu)\, K\, L\rangle$ Axial rotor $\beta = \pm\sqrt{2}, \gamma=0$

$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$ $\quad |[N]\, \sigma\, \tau\, n_\Delta\, L\rangle$ γ-unstable rotor $\beta=1$, γ arbitrary

Partial Dynamical Symmetry

This talk:

- SU(3)-$\overline{\text{SU(3)}}$ PDS prolate-oblate
- U(5)-SU(3)-$\overline{\text{SU(3)}}$ PDS spherical-prolate-oblate

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- U(5)-SO(6) PDS spherical - γ-unstable

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SU(3) and $\overline{\mathrm{SU(3)}}$ Dynamical Symmetries

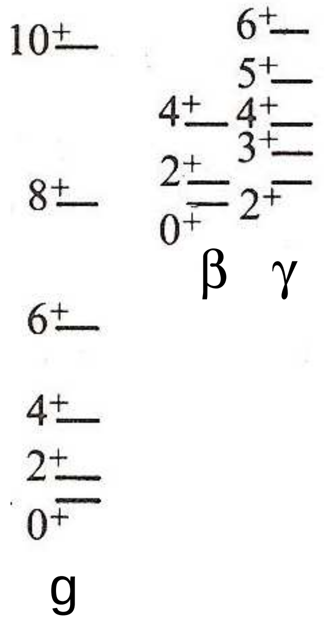
U(6) ⊃ SU(3) ⊃ SO(3) |[N] (λ ,μ) K L⟩ Prolate-deformed rotor

U(6) ⊃ $\overline{\mathrm{SU(3)}}$ ⊃ SO(3) |[N] ($\bar{\lambda}$,$\bar{\mu}$) K L⟩ Oblate-deformed rotor

SU(3)	$\overline{\mathrm{SU(3)}}$	SO(3)
$(\lambda,\mu)=(2N-4k-6m,2k)$	$(\bar{\lambda},\bar{\mu})=(2k,2N-4k-6m)$	
$\hat{C}_2[SU(3)] = 2Q^{(2)}\cdot Q^{(2)} + \frac{3}{4}L^{(1)}\cdot L^{(1)}$	$\hat{C}_2[\overline{\mathrm{SU(3)}}] = 2\bar{Q}^{(2)}\cdot \bar{Q}^{(2)} + \frac{3}{4}L^{(1)}\cdot L^{(1)}$	$\hat{C}_2[SO(3)] = L^{(1)}\cdot L^{(1)}$
$Q^{(2)} = d^\dagger s + s^\dagger\tilde{d} - \frac{1}{2}\sqrt{7}(d^\dagger\tilde{d})^{(2)}$	$\bar{Q}^{(2)} = d^\dagger s + s^\dagger\tilde{d} + \frac{1}{2}\sqrt{7}(d^\dagger\tilde{d})^{(2)}$	$L^{(1)} = \sqrt{10}(d^\dagger\tilde{d})^{(1)}$

$(2N,0)$ $g(K=0)$
$(2N-4,2)$ $\beta(K=0)$ $\gamma(K=2)$

$(0,2N)$ $g(\bar{K}=0)$
$(2,2N-4)$ $\beta(\bar{K}=0)$ $\gamma(\bar{K}=2)$



$$(s^\dagger,s)\to(-s^\dagger,-s) \qquad \mathcal{R}_s=\exp(i\pi\hat{n}_s) \qquad \hat{n}_s=s^\dagger s$$

$$\mathcal{R}_s|N,\,(\lambda,\mu),\,K,\,L\rangle = |N,\,(\bar{\lambda},\bar{\mu}),\,\bar{K},\,L\rangle$$

$$\mathcal{R}_s Q^{(2)}\mathcal{R}_s^{-1} = -\bar{Q}^{(2)}$$

SU(3) and $\overline{\mathrm{SU(3)}}$ DS spectra are identical
Quadrupole moments of corresponding states differ in sign

Construction of the Hamiltonian

A single number-conserving H which preserves relevant DSs for prolate and oblate ground bands and selected spherical states

Construction based on an intrinsic-collective resolution of H

Focus on the dynamics in the vicinity of the critical point where the corresponding multiple minima are near degenerate

Prolate-Oblate Critical-Point Hamiltonian

Intrinsic part of C.P. Hamiltonian

$$\begin{cases} \hat{H}|N,\, (\lambda,\mu) = (2N,0),\, K=0,\, L\rangle = 0 \\ \hat{H}|N,\, (\bar{\lambda},\bar{\mu}) = (0,2N),\, \bar{K}=0,\, L\rangle = 0 \end{cases}$$

$$\hat{H} = h_0\, P_0^\dagger \hat{n}_s P_0 + h_2\, P_0^\dagger \hat{n}_d P_0 + \eta_3\, G_3^\dagger \cdot \tilde{G}_3$$

$$P_0^\dagger = d^\dagger \cdot d^\dagger - 2(s^\dagger)^2 \qquad G_{3,\mu}^\dagger = \sqrt{7}[(d^\dagger d^\dagger)^{(2)} d^\dagger]^{(3)}_\mu$$

Energy Surface

$$\tilde{E}(\beta,\gamma) = (1+\beta^2)^{-3}\left\{(\beta^2-2)^2\left[h_0 + h_2\beta^2\right] + \eta_3\beta^6\sin^2(3\gamma)\right\}$$

$$= z_0 + (1+\beta^2)^{-3}[A\beta^6 + B\beta^6\Gamma^2 + D\beta^4 + F\beta^2] \qquad \Gamma = \cos 3\gamma$$

- Two degenerate P-O global minima

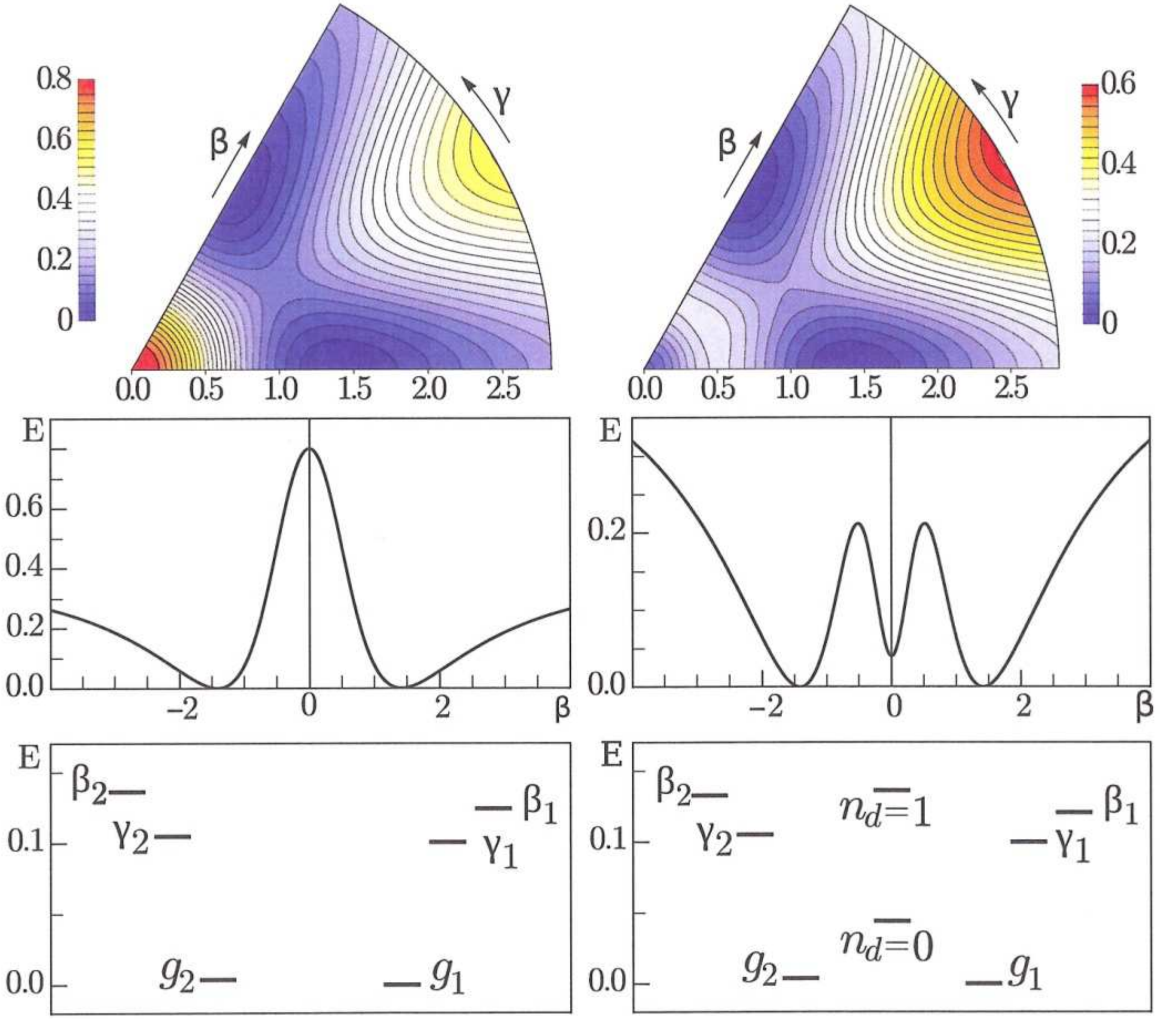
  ($\beta=\sqrt{2},\gamma = 0$) and ($\beta=\sqrt{2},\gamma = \pi/3$) [or equivalently ($\beta= -\sqrt{2},\gamma\ \ 0$)]

- $\beta=0$ always extremum: local min (max) for F>0 (F<0)
- Saddle points: [$\beta_1,\gamma = 0,\pi/3$] , [$\beta_{2,}\gamma = \pi/3$]
- Maximum: [$\beta_3,\gamma = \pi/3$]

$$\hat{H}(h_0=0)|N, n_d = \tau = L = 0\rangle = 0$$

- Three degenerate S-P-O global minima: $\beta=0$, ($\beta= \pm\sqrt{2},\gamma = 0$)

oblate-prolate oblate-spherical-prolate



E(β,γ)

Saddle points support a barrier separating the various minima

E(β,γ=0)

bandhead spectrum

Normal modes:

$$\epsilon_{\beta 1} = \epsilon_{\beta 2} = \frac{8}{3}(h_0 + 2h_2)N^2 \qquad \epsilon = 4h_2 N^2$$

$$\epsilon_{\gamma 1} = \epsilon_{\gamma 2} = 4\eta_3 N^2$$

$$\hat{H} = h_0\, P_0^\dagger \hat{n}_s P_0 + h_2\, P_0^\dagger \hat{n}_d P_0 + \eta_3\, G_3^\dagger \cdot \tilde{G}_3 \qquad P_0^\dagger = d^\dagger \cdot d^\dagger - 2(s^\dagger)^2 \qquad G^\dagger_{3,\mu} = \sqrt{7}[(d^\dagger d^\dagger)^{(2)} d^\dagger]^{(3)}_\mu$$

$$\tilde{E}(\beta,\gamma) = z_0 + (1+\beta^2)^{-3}[A\beta^6 + B\beta^6\Gamma^2 + D\beta^4 + F\beta^2] \qquad \Gamma = \cos 3\gamma$$

$\hat{H}$ Is invariant under $(s^\dagger, s) \to (-s^\dagger, -s)$ $[\ \hat{H}\,,\ \mathcal{R}_s\ ] = 0$

⇒ All non-degenerate e.s. have well-defined s-parity

⇒ Vanishing quadrupole moments for $\mathcal{R}_s T(E2) \mathcal{R}_s^{-1} = -T(E2)$

$$\alpha\, \hat{\theta}_2 = \alpha\, [-\hat{C}_2[SU(3)] + 2\hat{N}(2\hat{N}+3)]$$

$$\tilde{\alpha}(1+\beta^2)^{-2}[(\beta^2-2)^2 + 2\beta^2(2-2\sqrt{2}\beta\Gamma+\beta^2)] \qquad \tilde{\alpha} = \alpha/(N-2)$$

Linear Γ dependence distinguishes the two deformed minima and slightly lifts their degeneracy, as well as that of the normal modes

Complete Hamiltonian

$$\hat{H}' = \hat{H}(h_0, h_2, \eta_3) + \alpha\, \hat{\theta}_2 + \rho\, \hat{C}_2[\mathrm{SO}(3)]$$

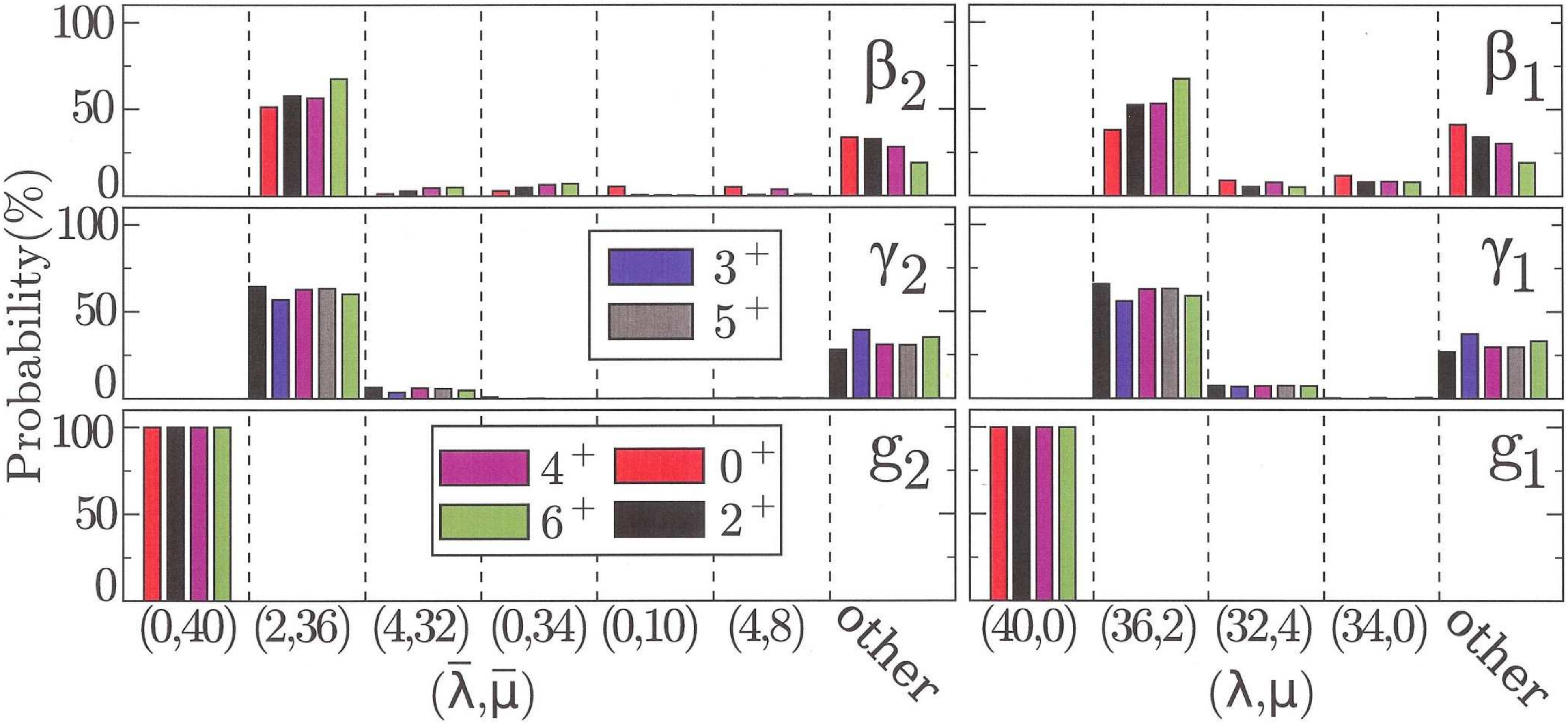
g_1 band: solvable with $E_{g1}(L) = \rho L(L+1)$

g_2 band: slight shift ~ αN^2; rigid-rotor spectrum

Prolate-Oblate Coexistence

$\overline{SU(3)}$ decomposition

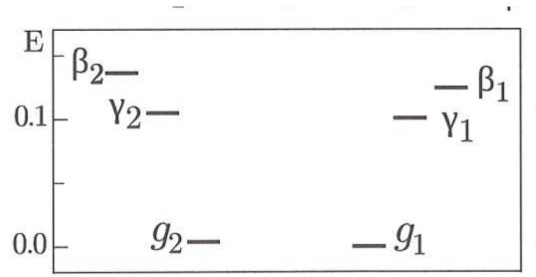
SU(3) decomposition



Ground g_1 band: pure SU(3)-DS states (2N,0)

Ground g_2 band: pure $\overline{SU(3)}$-DS states (0,2N)

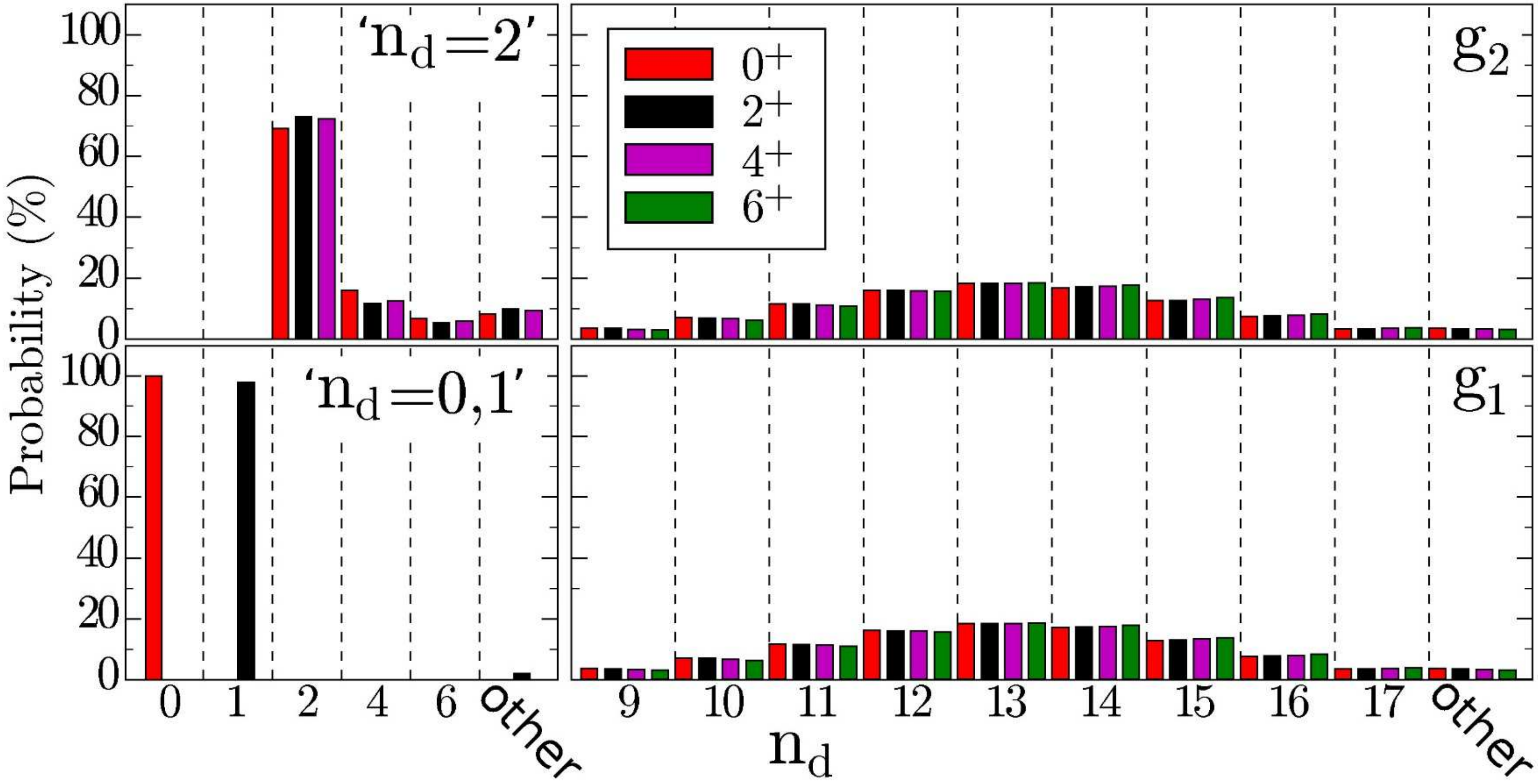
Excited β and γ bands: considerable mixing



oblate prolate

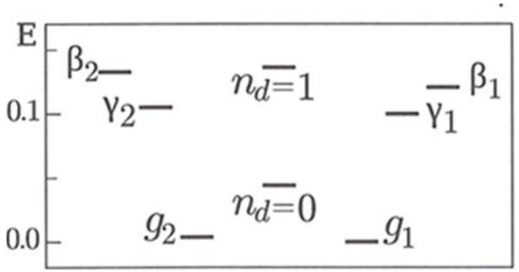
Triple Spherical-Prolate-Oblate Coexistence

U(5) decompostion



P-O bands show similar behavior

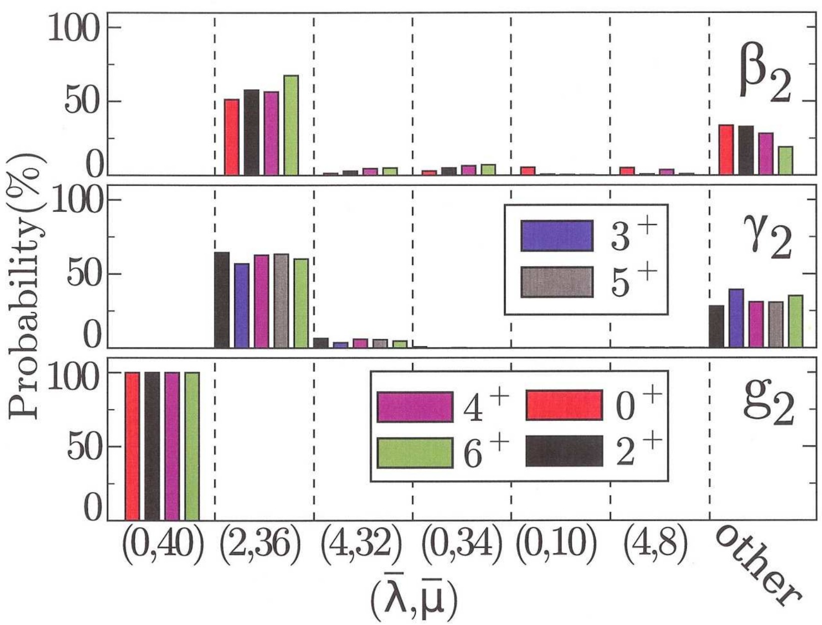
New aspect: occurrence of spherical type of states
(n_d=L=0) and (n_d=1,L=2) pure U(5)-DS
Higher spherical states: pronounced (~70%) n_d=2



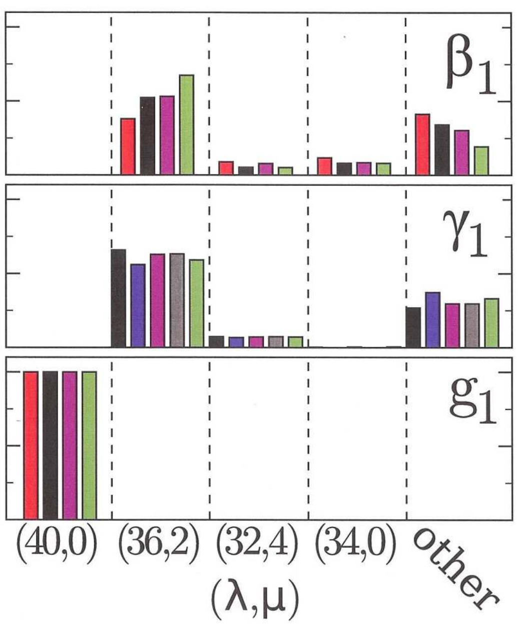
prolate spherical oblate

Partial Dynamical Symmetries

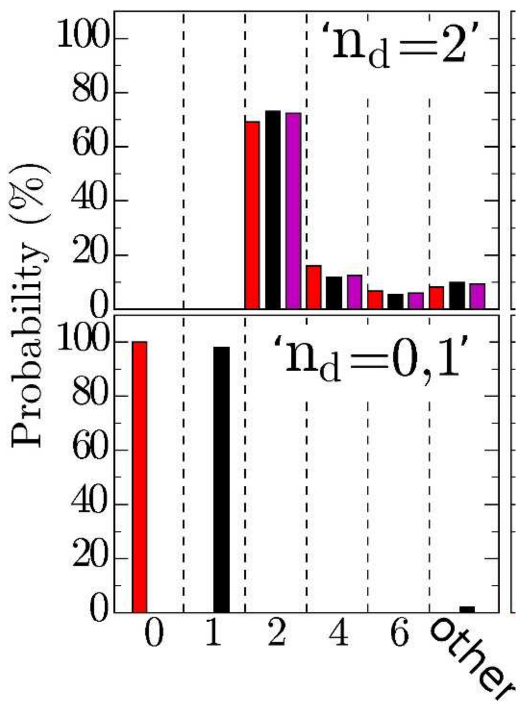
$\overline{\mathrm{SU(3)}}$ decomposition



SU(3) decompostion



U(5) decompostion



The purity of selected sets of states with respect to SU(3), $\overline{\mathrm{SU(3)}}$, and U(5), in the presence of other mixed states, are the hallmarks of a Partial Dynamical Symmetry

E2 rates and Quadrupole Moments

$$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$$

Quadrupole moments in the g_1 and g_2 bands

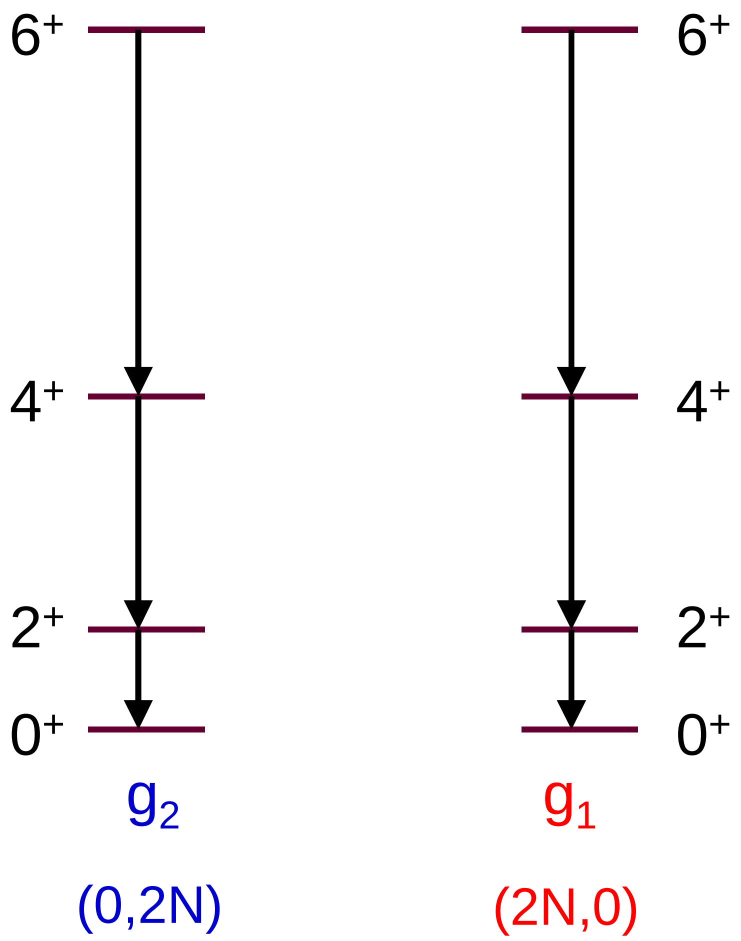
$$Q_L = \mp e_B \sqrt{\frac{16\pi}{40}} \frac{L}{2L+3} \frac{4(2N-L)(2N+L+1)}{3(2N-1)}$$

Intraband ($g_1 \rightarrow g_1$, $g_2 \rightarrow g_2$) E2 transitions

$$B(E2; g_i,\, L+2 \rightarrow g_i,\, L) =$$

$$e_B^2 \frac{3(L+1)(L+2)}{2(2L+3)(2L+5)} \frac{(4N-1)^2(2N-L)(2N+L+3)}{18(2N-1)^2}$$

$$\mathcal{R}_s T(E2) \mathcal{R}_s^{-1} = -T(E2)$$



prolate-oblate coexistence

E2 rates and Quadrupole Moments

$$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$$

Deformed states ($g_1 \to g_1$, $g_2 \to g_2$)

$$Q_L = \mp e_B \sqrt{\frac{16\pi}{40}} \frac{L}{2L+3} \frac{4(2N-L)(2N+L+1)}{3(2N-1)}$$

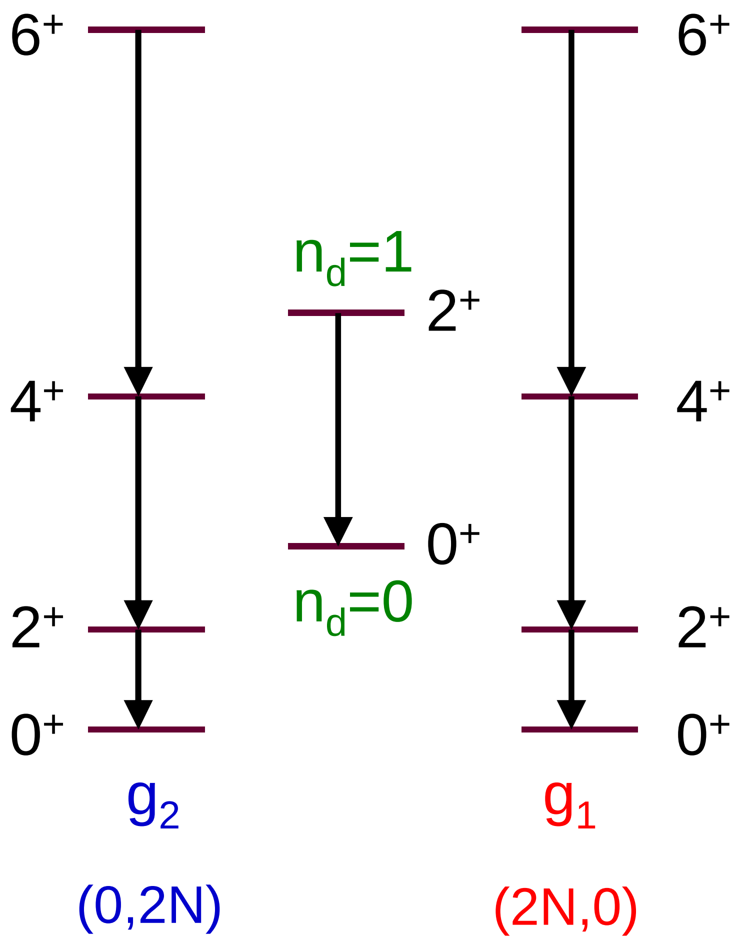
$$B(E2; g_i,\, L+2 \to g_i,\, L) =$$

$$e_B^2 \frac{3(L+1)(L+2)}{2(2L+3)(2L+5)} \frac{(4N-1)^2(2N-L)(2N+L+3)}{18(2N-1)^2}$$

Spherical U(5)-DS states ($n_d=1 \to n_d=0$)

$Q(n_d=1, L=2) = 0$

$$B(E2; n_d=1, L=2 \to n_d=0, L=0) = e_B^2 N$$



spherical-prolate-oblate coexistence

Selection Rules and Isomeric States

$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$ (2,2) tensor

(2,2) $\otimes$ (2N,0) $\rightarrow$ (2N,0) $\oplus$ (2N-4,2)

g1 exhausts the (2N,0) irrep of SU(3)

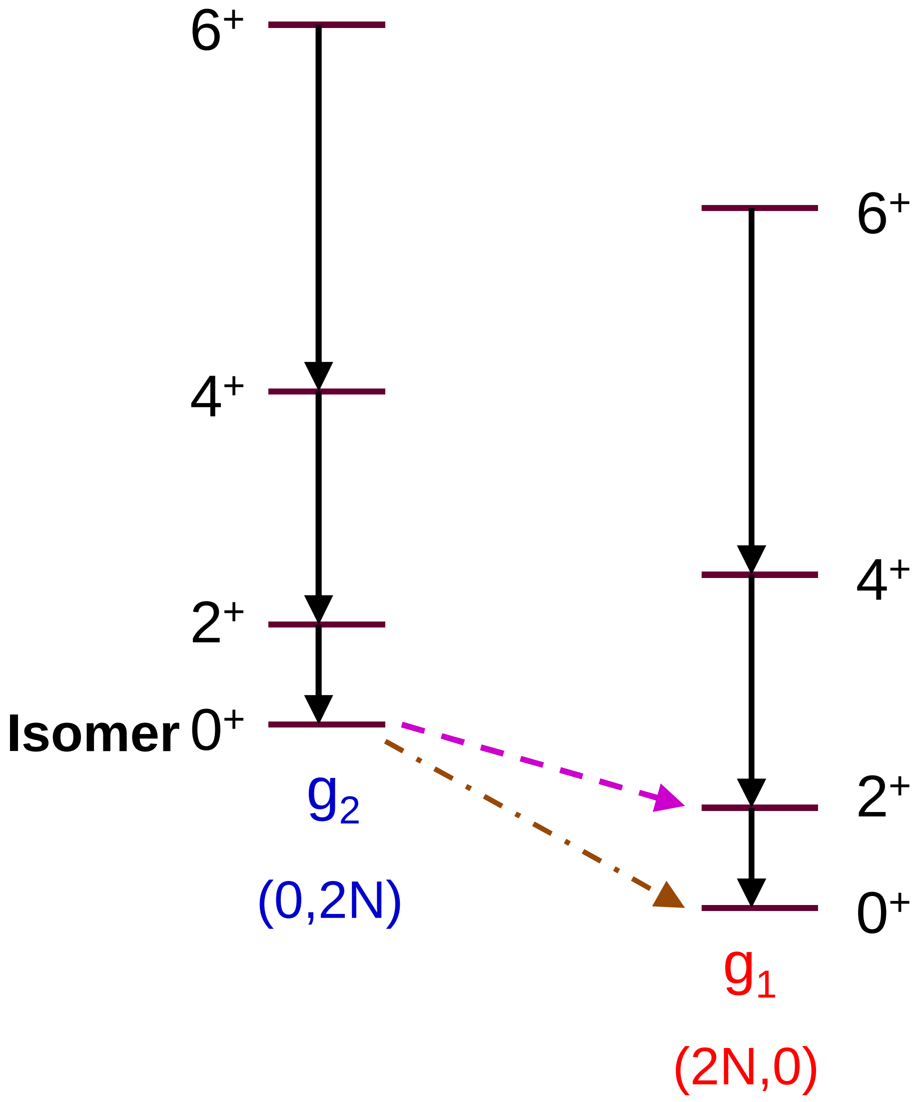
(2N-4,2) component of g2 is vanishingly small

E2 selection rule: $g_1 \not\leftrightarrow g_2$

Valid also for $T(E2) = \alpha\,\hat{Q} + \theta\,(d^\dagger s + \tilde{d} s)$

$T(E0) \propto \hat{n}_d$ (0,0) $\oplus$ (2,2) tensor

E0 selection rule: $g_1 \not\leftrightarrow g_2$



prolate-oblate coexistence

Selection Rules and Isomeric States

$$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$$

$\Delta n_d = \pm 1$

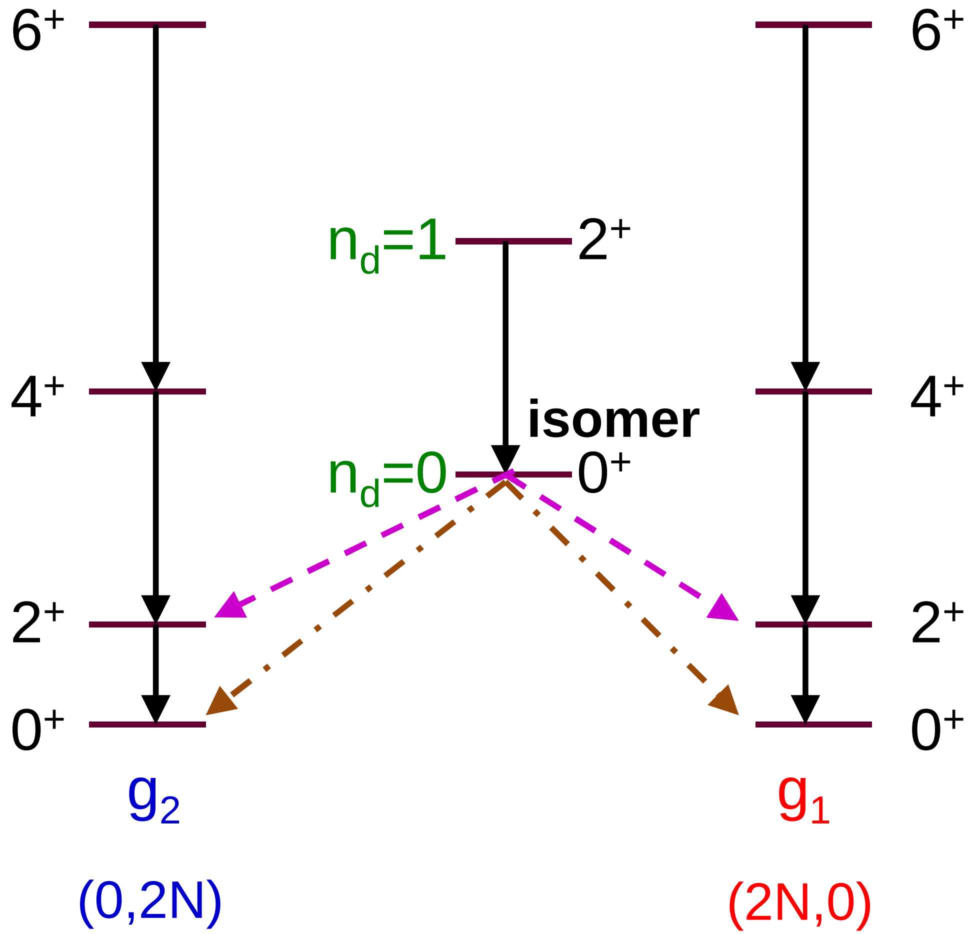
The spherical states exhaust the (n_d=0,1) irreps of U(5)

The n_d=2 component in the (L=0,2,4) states of the g_1 and g_2 bands is extremely small

Spherical $\rightarrow$ deformed
E2 rates very weak

$T(E0) \propto \hat{n}_d$ diagnal in n_d

No E0 transitions involving these spherical states

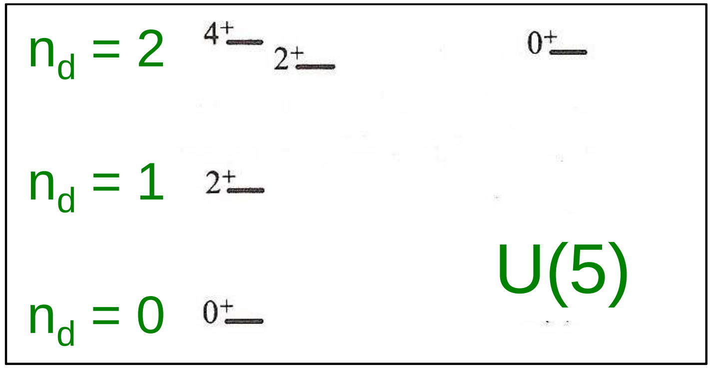


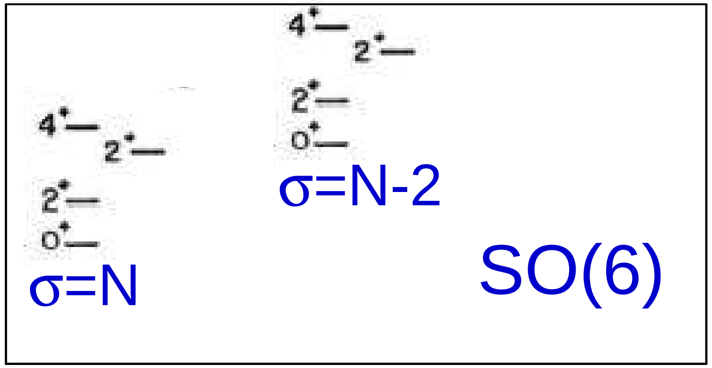
prolate-spherical-oblate coexistence

U(5) and SO(6) Dynamical Symmetries

$U(6) \supset U(5) \supset SO(5) \supset SO(3)$ $\quad |[N]\, n_d\, \tau\, n_\Delta\, L\rangle$ Spherical vibrator

$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$ $\quad |[N]\, \sigma\, \tau\, n_\Delta\, L\rangle$ γ-unstable rotor





Spherical and γ-unstable Critical-Point Hamiltonian

Intrinsic part of C.P. Hamiltonian

$$\begin{cases} \hat{H}|N,\, \sigma = N,\, \tau,\, L\rangle = 0 \\ \hat{H}|N, n_d = \tau = L = 0\rangle = 0 \end{cases}$$

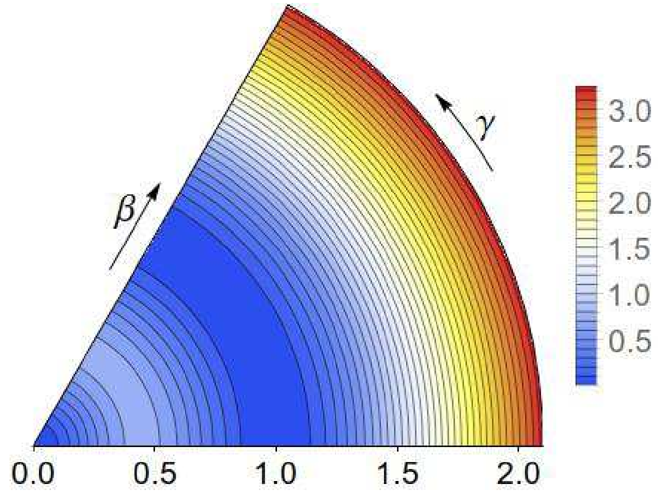
$$\hat{H} = h_2\, P_0^\dagger \hat{n}_d P_0 \qquad P_0^\dagger = d^\dagger \cdot d^\dagger - (s^\dagger)^2$$

Energy Surface
$$\tilde{E}(\beta) = (1+\beta^2)^{-3} h_2\, \beta^2 (\beta^2 - 1)^2 (1+\beta^2)^{-3}$$
$$= (1+\beta^2)^{-3} [\, A\beta^6 + D\beta^4 + F\beta^2\,]$$

- Two degenerate spherical and γ-unstable deformed global minima: $\beta=0$ and $\beta=1$

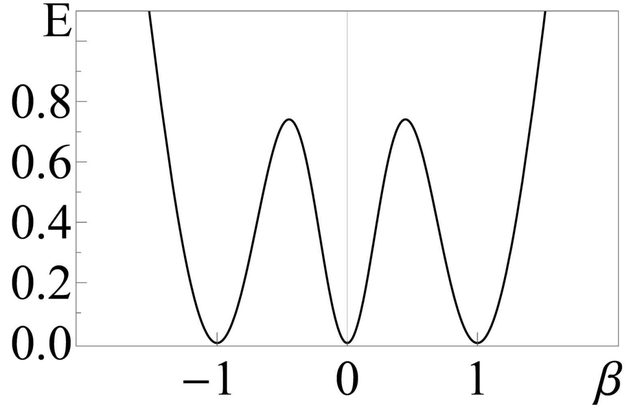
Spherical & γ-unstable deformed

Energy surface independent of γ
SO(5) symmetry



E(β,γ)

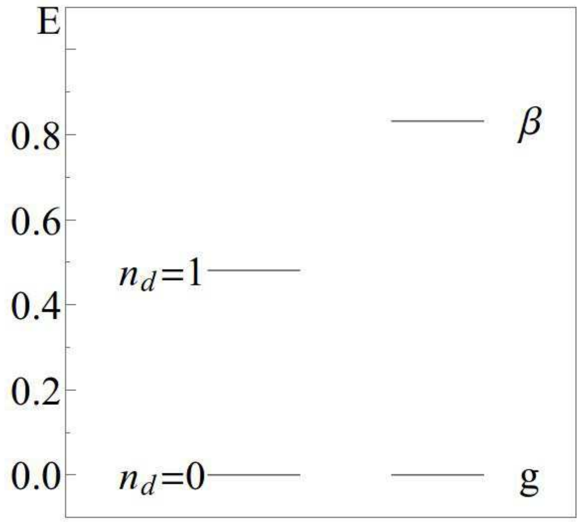
a barrier separates the spherical and γ-unstable deformed minima



E(β,γ=0)

Normal modes:

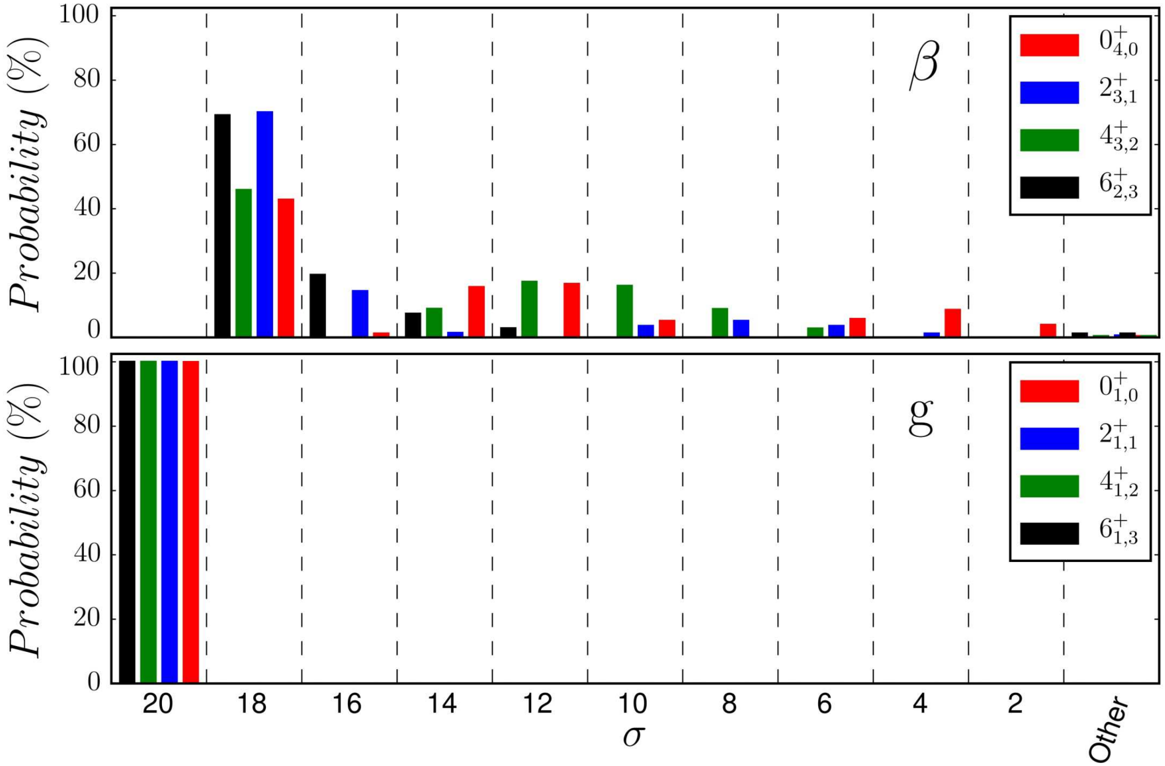
$$\epsilon_\beta = 2h_2 N^2$$
$$\epsilon = h_2 N^2$$



bandhead spectrum

Complete Hamiltonian

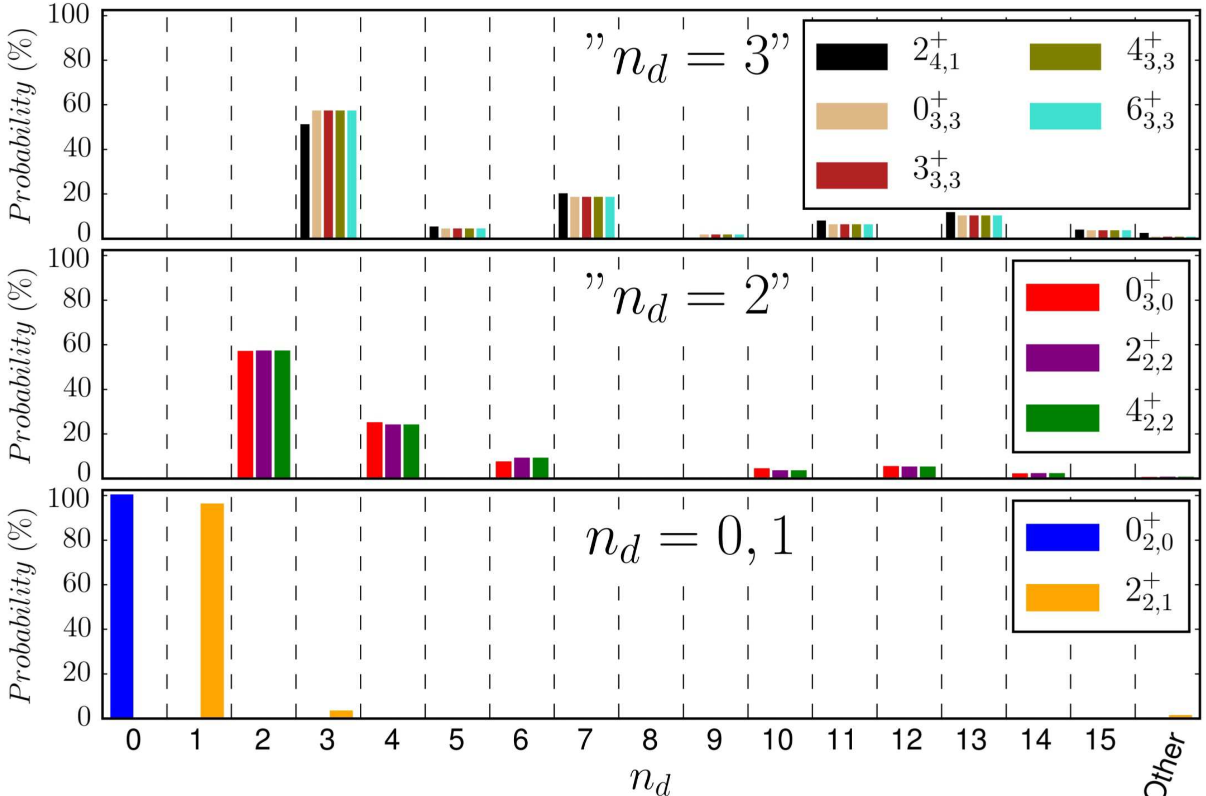
$$\hat{H}' = h_2\, P_0^\dagger \hat{n}_d P_0 + B\, \hat{C}_2[\mathrm{SO}(5)]$$



SO(6) decompostion

- g-band: pure SO(6)-DS (σ=N)
- Excited β bands: mixed

$\Rightarrow$ SO(6)-PDS



U(5) decompostion

- Spherical states: pure U(5)-DS with (n_d=L=0) and (n_d=1,L=2)
- Higher spherical states: pronounced & coherent mixing

$\Rightarrow$ U(5)-PDS

E2 rates and Quadrupole Moments

$$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$$

Deformed states ($g \to g$)

$$Q(\sigma=N,\tau) = 0$$

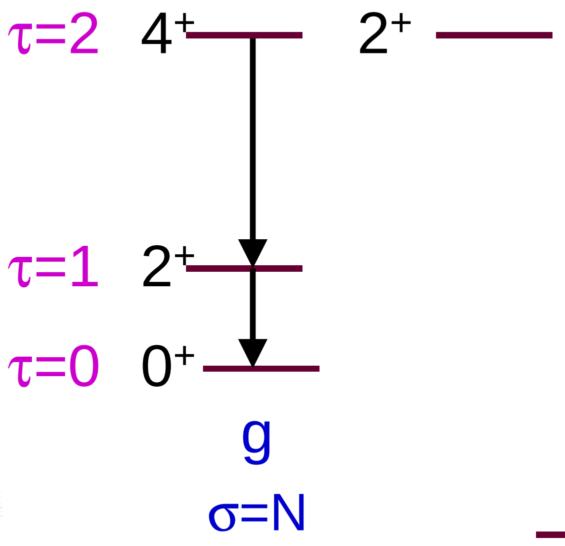
$$B(E2; g; \tau+1, L' = 2\tau+2 \to g; \tau, L = 2\tau) =$$

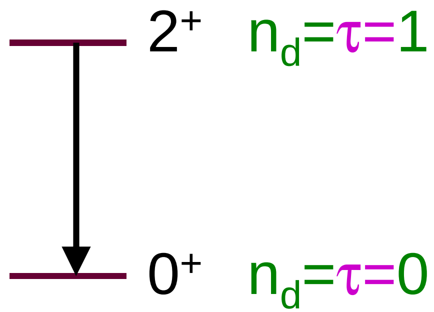
$$e_B^2 \,\frac{\tau+1}{2\tau+5}(N-\tau)(N+\tau+4)$$

Spherical U(5)-DS states ($n_d=1 \to n_d=0$)

$$Q(n_d=1,L=2) = 0$$

$$B(E2; n_d=1, L=2 \to n_d=0, L=0) = e_B^2 N$$





Spherical γ-unstable def. coexistence

Selection Rules and Isomeric States

$T(E2) = e_B(d^\dagger s + s^\dagger \tilde{d})$ SO(6) generator

$\Delta\sigma=0$, $\Delta n_d = \pm 1$, $\Delta\tau = \pm 1$

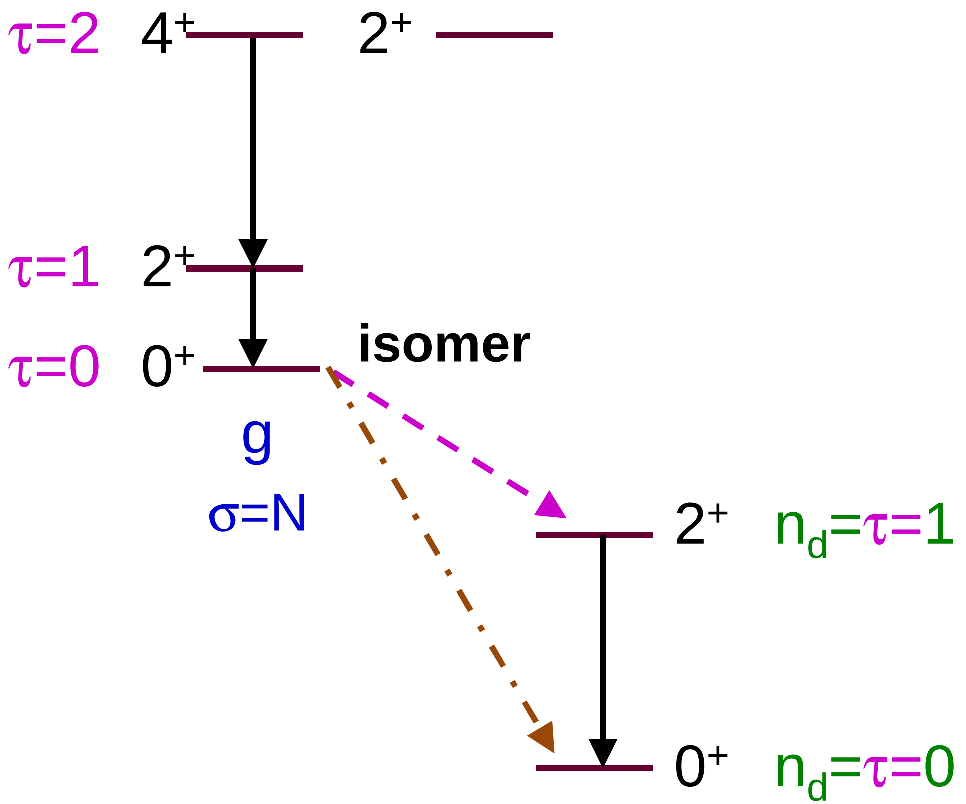
g-band exhausts the σ=N irrep of SO(6)

E2 selection rule: g $\not\leftrightarrow$ other states

In particular,
deformed $\rightarrow$ spherical
E2 rates very weak

$T(E0) \propto \hat{n}_d$ diagnal in n_d

No E0 transitions involving these spherical states



Spherical γ-unstable def. coexistence

Concluding Remarks

- A number-conserving rotational-invariant Hamiltonian which captures essential features of shape-coexistence in nuclei
  Symmetry-based benchmark

- H conserves the dynamical symmetry for selected bands
  Partial Dynamical Symmetries relevant for shape-coexistence

SU(3) and $\overline{\text{SU(3)}}$ PDS	prolate-oblate
U(5), SU(3) and $\overline{\text{SU(3)}}$ PDS	spherical-prolate-oblate
U5) and SO(6) PDS	spherical - γ-unstable deformed

- Closed expressions for quadrupole moments and B(E2) values;
  selection rules to E2 & E0 transitions and isomeric states

- PDS: solvable bands are unmixed
  Band mixing can be incorporated but, if strong, may destroy the PDS

- Study of shape-coexistence provides a fertile ground for the development of
  generalized notions of symmetries

Thank you