Detail studies of Holstein-Primakoff boson representation applied to the particle-rotor model

Application of particle-rotor model with rigid MoI to $11/2^-$ band in ^{135}Pr

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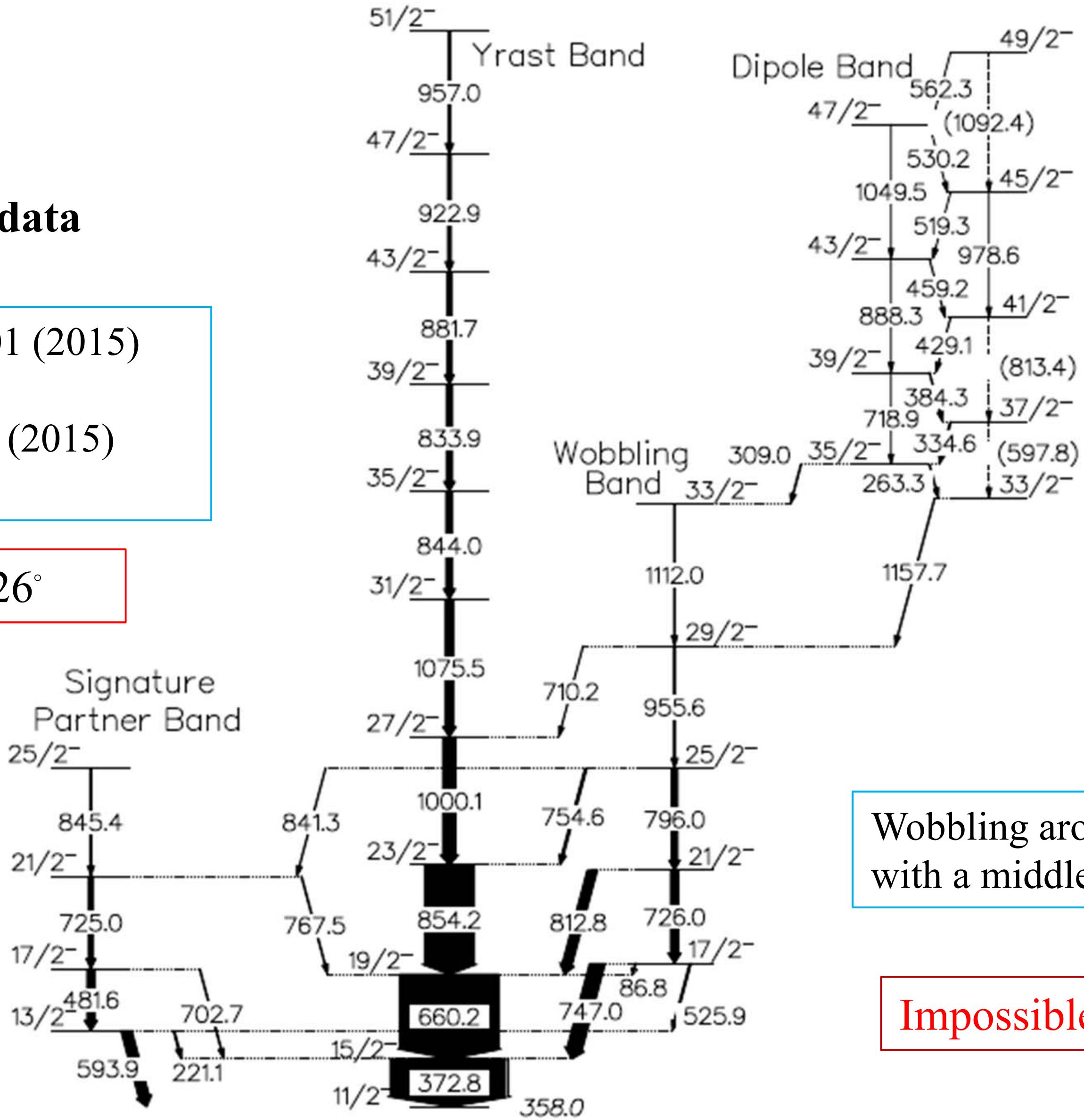
Motivation

^{135}Pr exp. data

PRL 114, 082501 (2015)
J.T. Matta *et al.*
PR C92, 054325 (2015)
R. Garg *et al.*

β_2 =0.18, γ= 26°



Wobbling around the axis with a middle MoI

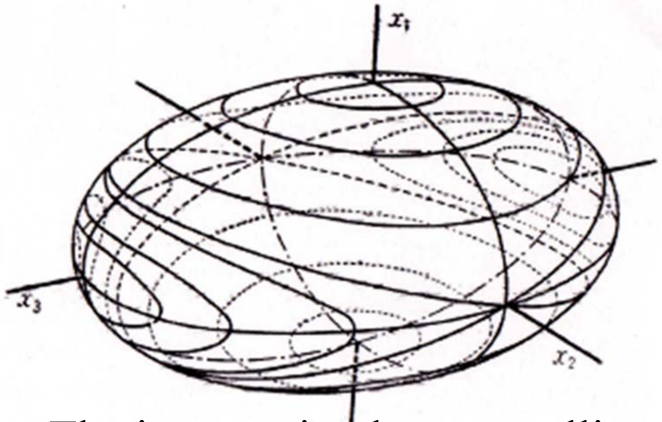
Impossible !

Motivation and Purpose

- Recent exp. on ^{135}Pr (PRL 114, 082501 ('15), J.T. Matta, U. Garg et al) use **transverse wobbling** (the wobbling motion around the axis with middle moment of Inertia (MoI)) for analyzing their data.
- **Wobbling motion** is discussed in the **Classical Mechanics.** It is shown there exist two wobbling motions **around the axis with the maximum MoI and the minimum MoI, but never around the axis with middle MoI.** Quantum mechanically, Bohr-Mottelson show the same result in **Nuclear Structure** (1967).
- We discussed if it really exists in odd-A nucleus by adopting the hydrodynamical MoI, and found there is **no transverse wobbling** in an **odd-A nucleus even with hydrodynamical MoI**.
- Then, is it possible to explain the experimental data by the particle rotor model with the **rigid MoI which includes the Corioli-anti-pairing effect?**

Irrespective of rig or hyd. MoI

Wobbling motion （Precession)



Landau and Lifschits (1958)：
Goldstein(2002)

$$\mathcal{J}_1 < \mathcal{J}_2 < \mathcal{J}_3$$

$$2E = \frac{L_1^2}{\mathcal{J}_1} + \frac{L_2^2}{\mathcal{J}_2} + \frac{L_3^2}{\mathcal{J}_3} \text{ (ellipsoid)}$$

$$L^2 = L_1^2 + L_2^2 + L_3^2 \text{ (sphere)}$$

$$2E\mathcal{J}_1 < L^2 < 2E\mathcal{J}_3$$

The intersection between ellipsoid and sphere is the orbit for given L and E. In the plane perpendicular to x_1 or x_3 axis the orbit is **ellipse**, while to x_2 axis it becomes **hyperbola**. There is **no stable rotation around the x_2 axis with middle MoI**.

Bohr-Mottelson (1967)

$$\mathcal{J}_1 > \mathcal{J}_2 > \mathcal{J}_3 \qquad A_k = 1/(2\mathcal{J}_k) \; (k = 1, 2, 3 \text{ or } x, y, z)$$

$$I_+ = \sqrt{2I}c^+, \; I_- = \sqrt{2I}c, \; I_1 = I$$

$$H_{\mathrm{rot}} = A_1 I(I+1) + (n + \frac{1}{2})\hbar\omega$$

$$\hbar\omega = 2I\sqrt{(A_2 - A_1)(A_3 - A_1)}$$

There is no wobbling around 2-axis with middle MoI, because wobbling energy is **imaginary**.

Top-on-top model (particle-rotor model)

H=Hrot+Hsp

$$H_{\mathrm{rot}} = \sum_{k=x,y,z} A_k (I_k - j_k)^2, \qquad A_k = 1/(2\mathcal{J}_k)$$

- Single-particle *H*

$$H_{\mathrm{sp}} = \frac{V}{j(j+1)}\left[\cos\gamma\left(3j_z^2 - \vec{j}^2\right) - \sqrt{3}\sin\gamma\left(j_x^2 - j_y^2\right)\right],$$

- Rigid-body *MoI*

$$\mathcal{J}_k^{\mathrm{rig}} = \frac{\mathcal{J}_0}{1 + \left(\frac{5}{16\pi}\right)^{1/2}\beta_2}\left[1 - \left(\frac{5}{4\pi}\right)^{1/2}\beta_2\cos\left(\gamma + \frac{2}{3}\pi k\right)\right]$$

- Hydrodynamical *MoI*

$$\mathcal{J}_k^{\mathrm{hyd}} = \frac{4}{3}\mathcal{J}_0\sin^2\left(\gamma + \frac{2}{3}\pi k\right)$$

- D2-invariance

$$\left\{\sqrt{\frac{2I+1}{16\pi^2}}\left[\mathcal{D}^I_{MK'}(\theta_i)\phi^j_{\Omega'} + (-1)^{I-j}\mathcal{D}^I_{M-K'}(\theta_i)\phi^j_{-\Omega'}\right]; \quad |K' - \Omega'| = \text{even}, \quad \Omega' > 0\right\},$$

The potential strength is determined by Wigner-Eckart theorem

K.S.-T., K.T. & N.Yoshinaga ,
PTEP 2014,063D01(2004)

$$H_{sp} = H_0 + H_\delta,$$
$$H_\delta = -\hbar\omega_0(\delta)\beta_2 r^2\left(\cos\gamma Y_{20} - \sin\gamma\frac{1}{2}(Y_{22}+Y_{2-2})\right)$$

$$\langle jm|r^2(\cos\gamma Y_{20} - \sin\gamma\frac{1}{2}(Y_{22}+Y_{2-2})|jm\rangle$$
$$= -\ \langle jm|r^2\frac{1}{8j(j+1)}\sqrt{\frac{5}{\pi}}(\cos\gamma(3j_z^2-\vec{j}^2) - \sqrt{3}\sin\gamma(j_x^2-j_y^2))|jm\rangle$$

$$\boxed{V = \frac{1}{8}\sqrt{\frac{5}{\pi}}\hbar\omega_0(\delta)\beta_2\langle r^2\rangle}$$

$$\langle r^2\rangle = N + \frac{3}{2}$$

$h_{11/2}$ at β_2 =0.18 → V=1.5 MeV

The competition between Coriolis term ***I.j***
and single-particle pot. V/j(j+1)

→V=1.6MeV

D2-symmetry+Bohr symmetry = D2 invariance

$$\left\{\sqrt{\frac{2I+1}{16\pi^2}}\left[\mathcal{D}^I_{MK'}(\theta_i)\phi^j_{\Omega'} + (-1)^{I-j}\mathcal{D}^I_{M-K'}(\theta_i)\phi^j_{-\Omega'}\right];\right.$$
$$\left.|K'-\Omega'| = \text{even}, \quad K'>0\right\},$$

$$\mathcal{R}_k(\pi) = \exp(-i\pi R_k) \text{ with } R_k = I_k - j_k \ (k=1,2,3)$$

r_k stands for an eigenvalue of $\mathcal{R}_k(\pi)$

Bohr symmetry requires $(r_1, r_2, r_3) = (+1, +1, +1)$

$$\vec{R} = \vec{I} + (-\vec{j}) \qquad R = |I-j|, |I-j|+1, \cdots, I+j-1, \text{ or } I+j,$$

$$R = I - j + n_{\beta'} \qquad n_{\beta'} = 0, 1, 2, \cdots, 2j-1, \text{or } 2j$$

As R_x runs from R to $-R$, and $R_x = I_x - j_x$

$$R_x = R - n_{\alpha'}$$

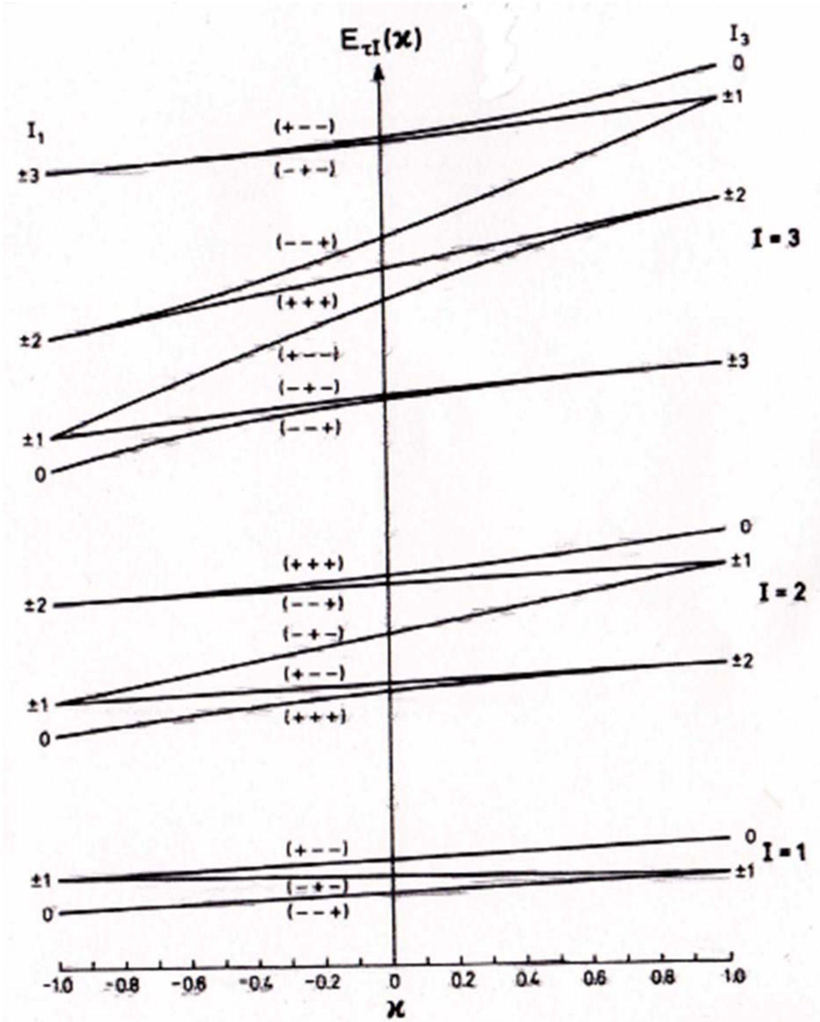
$$\begin{aligned} n_{\alpha'} &= 0, 2, 4, \cdots, R, && \text{for} \quad R = \text{even}, \\ n_{\alpha'} &= 1, 3, 5, \cdots, R, && \text{for} \quad R = \text{odd}. \end{aligned}$$

If Bohr-symmetry is destroyed

BM: Nuclear Structure vol II
King, Hainer, Cross, J.Chem. Phys. 11, 27 (1943)

Rx

Rz



Oblate ($A_2=A_3$)

Prolate ($A_1=A_2$)

Rotor Hamiltonian is invariant under the rotation of $R_k=\exp(-i\pi I_k)$, whose eigenvalue is r_k . D_2-invariance demands $(r_1,r_2,r_3)=(+1,+1,+1)$.

If D_2 invariance is not required , or Bohr symmetry is not required then $I_z=\pm 2$ levels in I=3 are splitted into (+,+,+) and (+,-,-) at $0<\kappa<1.0$, and $I_x=\pm 2$ in I=2 and 3 are splitted into (+,+,+) and (-,+,+) at $-1.0<\kappa<0$. ,

$$E_{\rm rot} = \frac{1}{2}(A_1+A_3)I(I+1)+\frac{1}{2}(A_1-A_3)E_{\tau I}(\kappa)$$

$$\kappa = \frac{2A_2-A_1-A_3}{A_1-A_3}$$

$$A_1 \le A_2 \le A_3$$

HP boson representation at V=0

$$\boxed{[I_i, I_j] = -\mathrm{i}I_{i\times j},\ [j_i, j_j] = \mathrm{i}j_{i\times j}}$$

$$\boxed{\mathcal{J}_x^{\mathrm{rig}} \geq \mathcal{J}_y^{\mathrm{rig}} \geq \mathcal{J}_z^{\mathrm{rig}}}$$

$$\begin{aligned} I_+ &= I_-^\dagger = I_y + iI_z = -\hat{a}^\dagger\sqrt{2I-\hat{n}_a}, \\ I_x &= I - \hat{n}_a \quad \text{with} \quad \hat{n}_a = \hat{a}^\dagger\hat{a}; \\ j_+ &= j_-^\dagger = j_y + ij_z = \sqrt{2j-\hat{n}_b}\,\hat{b}, \\ j_x &= j - \hat{n}_b \quad \text{with} \quad \hat{n}_b = \hat{b}^\dagger\hat{b} \end{aligned}$$

$$R = I - j + n_\beta, \quad R_x = R - n_\alpha;$$

$$p = A_y^{\mathrm{rig}} - A_x^{\mathrm{rig}}, \quad q = A_z^{\mathrm{rig}} - A_x^{\mathrm{rig}}.$$

$$\omega_{(-)} = (I-j)\sqrt{pq} = \omega_\alpha \text{ and } \omega_{(+)} = (I-j)A_x^{\mathrm{rig}} = \omega_\beta,$$

$$\boxed{\mathcal{J}_y^{\mathrm{hyd}} > \mathcal{J}_x^{\mathrm{hyd}} > \mathcal{J}_z^{\mathrm{hyd}}}$$

$$\begin{aligned} I_+ &= I_-^\dagger = I_z + iI_x = -\hat{a}^\dagger\sqrt{2I-\hat{n}_a}, \\ I_y &= I - \hat{n}_a \quad \text{with} \quad \hat{n}_a = \hat{a}^\dagger\hat{a}; \\ j_+ &= j_-^\dagger = j_z + ij_x = \sqrt{2j-\hat{n}_b}\,\hat{b}, \\ j_y &= j - \hat{n}_b \quad \text{with} \quad \hat{n}_b = \hat{b}^\dagger\hat{b}. \end{aligned}$$

$$R = I - j + n_\beta, \quad R_y = R - n_\alpha;$$

$$p' = A_z^{\mathrm{hyd}} - A_y^{\mathrm{hyd}}, \quad q' = A_x^{\mathrm{hyd}} - A_y^{\mathrm{hyd}}.$$

$$\omega_{(-)} = \omega_\beta = (I-j)A_y^{\mathrm{hyd}}$$

$$\omega_{(+)} = \omega_\alpha = (I-j)\sqrt{p'q'}$$

$$\boxed{\gamma \sim 0 \text{ but } \neq 0}$$

K.T.& K.S.-T., P.L.B34,575(1971)

$$\begin{aligned} I_+ &= I_-^\dagger = I_x + iI_y = -\hat{a}^\dagger\sqrt{2I-\hat{n}_a}, \\ I_z &= I - \hat{n}_a \quad \text{with} \quad \hat{n}_a = \hat{a}^\dagger\hat{a}; \\ j_+ &= j_-^\dagger = j_x + ij_y = \sqrt{2j-\hat{n}_b}\,\hat{b}, \\ j_z &= j - \hat{n}_b \quad \text{with} \quad \hat{n}_b = \hat{b}^\dagger\hat{b}. \end{aligned}$$

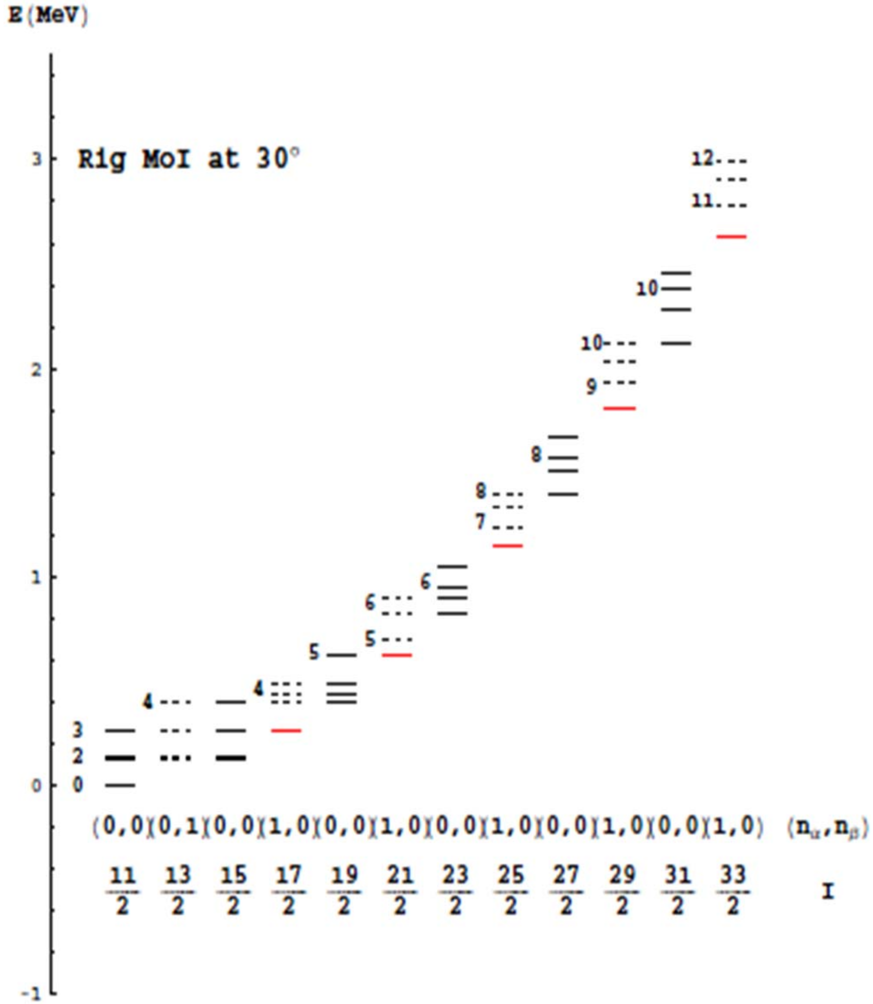
$$R = I - j + n_\beta, \quad R_z = R - n_\alpha;$$

$$p'' = A_z - A_x, \quad q'' = A_z - A_y.$$

$$\gamma = 0^\circ, \qquad E_{\mathrm{rot}} = A_x R(R+1) - (A_x - A_z)(R - n_\alpha)^2.$$

$\boxed{\mathcal{J}_x^{\rm rig} \geq \mathcal{J}_y^{\rm rig} \geq \mathcal{J}_z^{\rm rig}}$ V=0 $p = A_y^{\rm rig} - A_x^{\rm rig}, \quad q = A_z^{\rm rig} - A_x^{\rm rig}$

$$\omega_- = (I-j)\sqrt{pq} = \omega_\alpha \text{ and } \omega_+ = (I-j)A_x^{\rm rig} = \omega_\beta,$$



In the leading order approx.
$\omega_\alpha = \omega_-$ becomes BM formula with
I-j except for *I*

n_α : wobbling q.n. with max. MoI
n_β : *j* precession q.n. with max,MoI

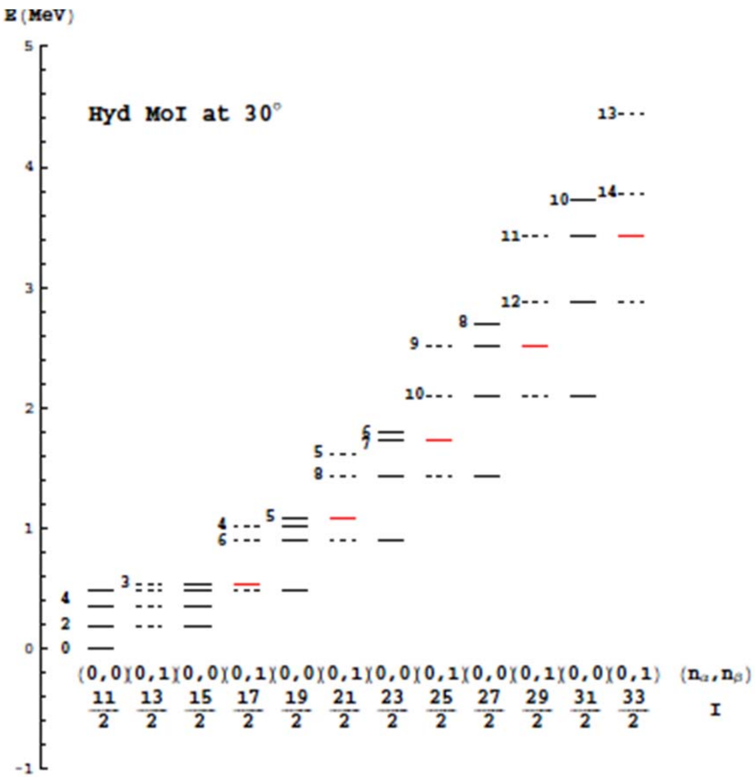
($n\alpha$, $n\beta$) are shown only for the yrast levels .

***I-j*=odd** has (1,0) except for *I*=13/2 where R=1 is not allowed from D2-invariance, and it comes from R=2 (0,1).
***I-j*=even** has (0,0).
I=13/2 is not the member of Band 2.

$\mathcal{J}_y^{hyd} > \mathcal{J}_x^{hyd} > \mathcal{J}_z^{hyd}$,

V=0

HP boson transformation where I_y and j_y are in the diagonal representation.



$$p' = A_z^{\rm hyd} - A_y^{\rm hyd}, \quad q' = A_x^{\rm hyd} - A_y^{\rm hyd}$$

$$R = I - j + n_\beta, \quad R_y = R - n_\alpha$$

$$\omega_- = \omega_\beta \text{ and } \omega_+ = \omega_\alpha$$

***I-j*=even** levels are (0,0), while ***I-j*=odd** levels are always (0,1). **I=13/2 level belongs to Band 2**.

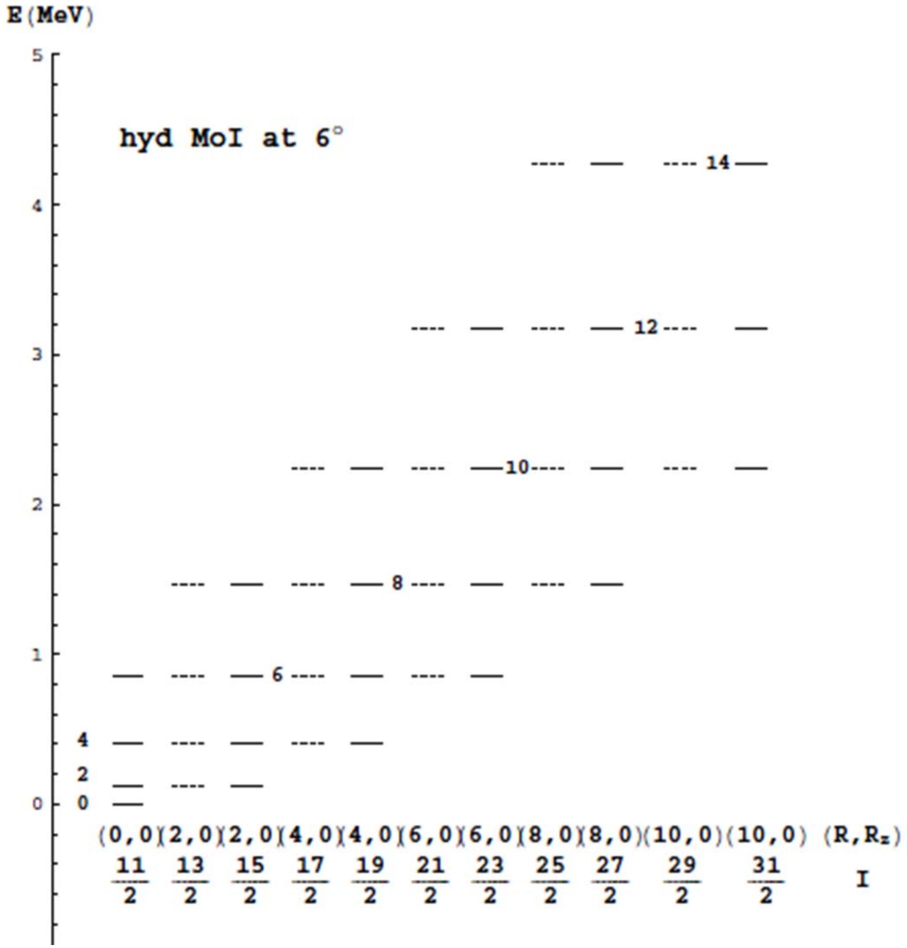
Wobbling level with (1,0) appears as a yrare level in *I-j*=odd band and not yrast.

***I-j*=odd** band is the **precession of *j* mode around y-axis with max. MoI**.

$\mathcal{J}_y^{hyd} > \mathcal{J}_x^{hyd} > \mathcal{J}_z^{hyd}$

Hyd. MoI with V=0 at $\gamma = 6^\circ$

K.T. & K. S.-T., P.L. B34,575(1971)



HP boson transformation where $\boldsymbol{I_z}$ and $\boldsymbol{j_z}$ are in the diagonal representation.

$$E_{\mathrm{rot}} \cong A_z R(R+1) + \frac{p'+q'}{2} n_\alpha^2 - \left(2R\sqrt{p'q'} + \sqrt{p'q'} - \frac{p'+q'}{2}\right)\left(n_\alpha + \frac{1}{2}\right)$$

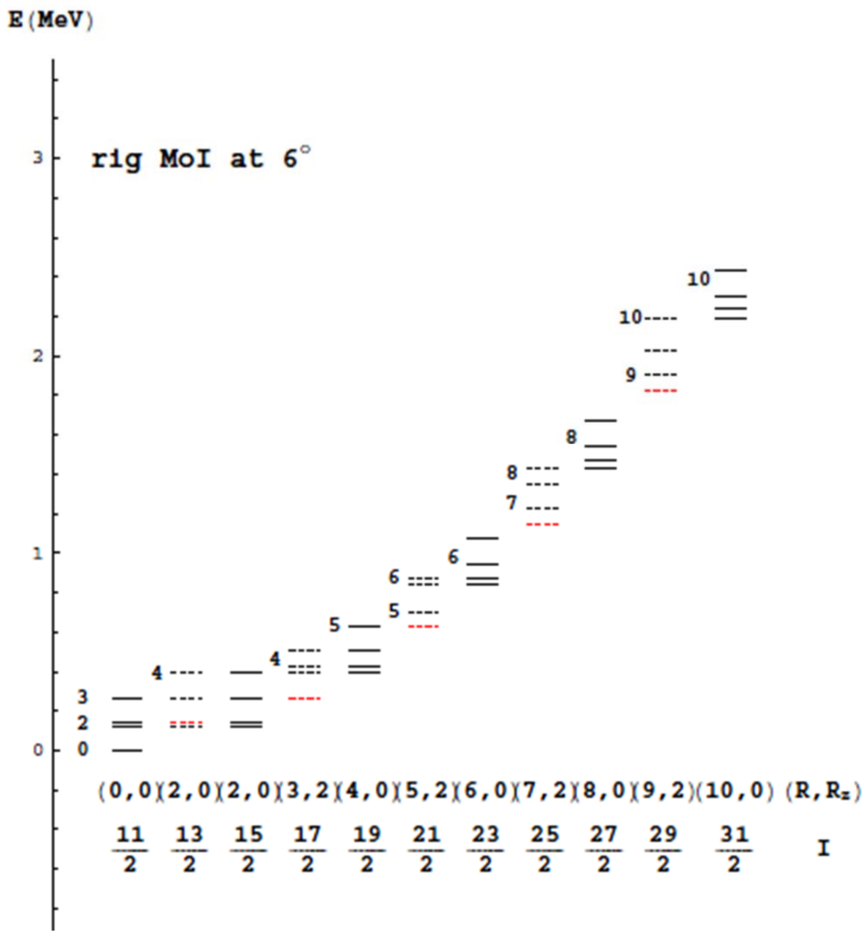
$$p' = A_z - A_y, \quad q' = A_z - A_x$$

$$R = I - j + n_\beta, \quad R_z = R - n_\alpha$$

Both ***I-j*=even** and ***I-j*=odd** levels have **Rz=0**

$\mathcal{J}_x^{\mathrm{rig}} > \mathcal{J}_y^{\mathrm{rig}} > \mathcal{J}_z^{\mathrm{rig}}$

Rig. MoI with V=0 at $\gamma = 6^\circ$



HP boson transformation where both I_z and j_z are in the diagonal representation.

$$E_{\mathrm{rot}} \cong A_z R(R+1) + \frac{p'+q'}{2} n_\alpha^2 - \left(2R\sqrt{p'q'} + \sqrt{p'q'} - \frac{p'+q'}{2}\right)\left(n_\alpha + \frac{1}{2}\right)$$

$$p' = A_z - A_y, \quad q' = A_z - A_x$$

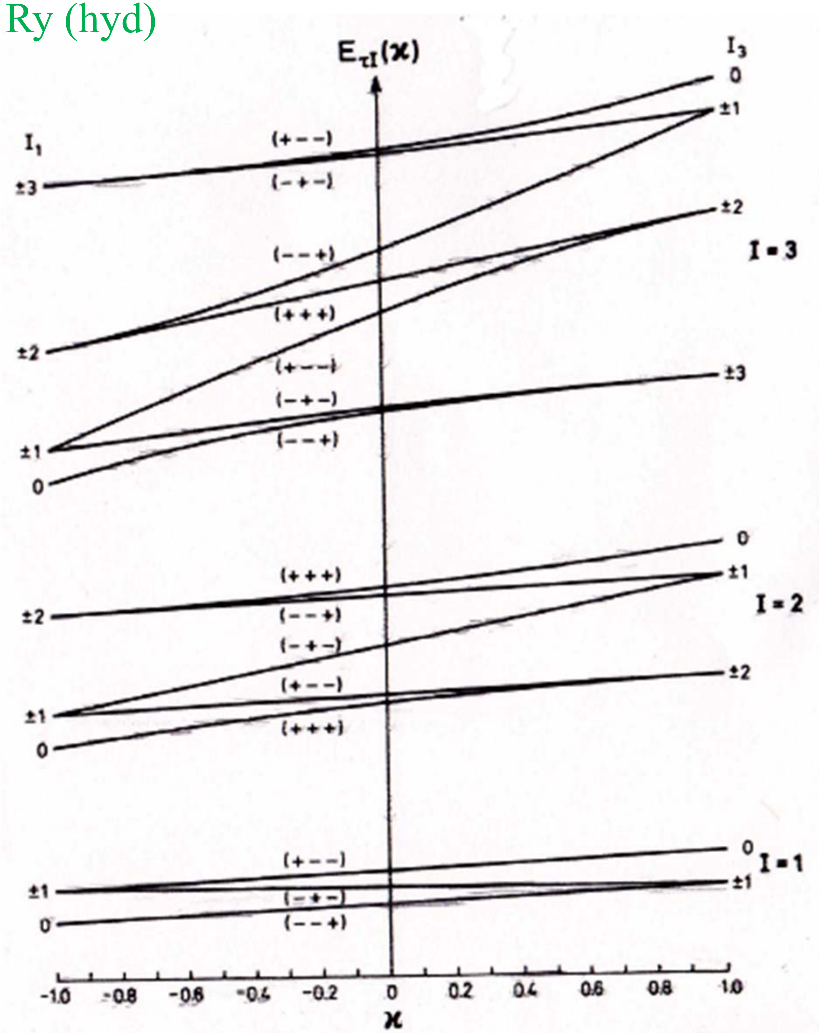
$$R = I - j + n_\beta, \quad R_z = R - n_\alpha$$

***I-j*=even** levels have **Rz=0**, while
***I-j*=odd** levels have **Rz=2**,
except for I=13/2 (**Rz=0**).

Rx (rig)

Ry (hyd)

Rz



60° rig 30° 0°

30° hyd 15° 0°

For small γ

I=2 → R=even Rz=0 (+ + +) no splitting.

I=3→ R=odd Rz=±2 (+ + +) splits to

(+ + +) and (+ - -). rig

Rz=0 (+ - -) not Bohr-sym. hyd

For large γ

I=2 (R=even) Rx=Ry=±2 (nα=0) splits to

(+ + +) and (- - +) . rig and hyd

Band1 may have signature partner band.

I=3 (R=odd) Ry=±3 (nα=0) no (+ + +). hyd

Rx=±2 (nα=1) (+ + +) splits to

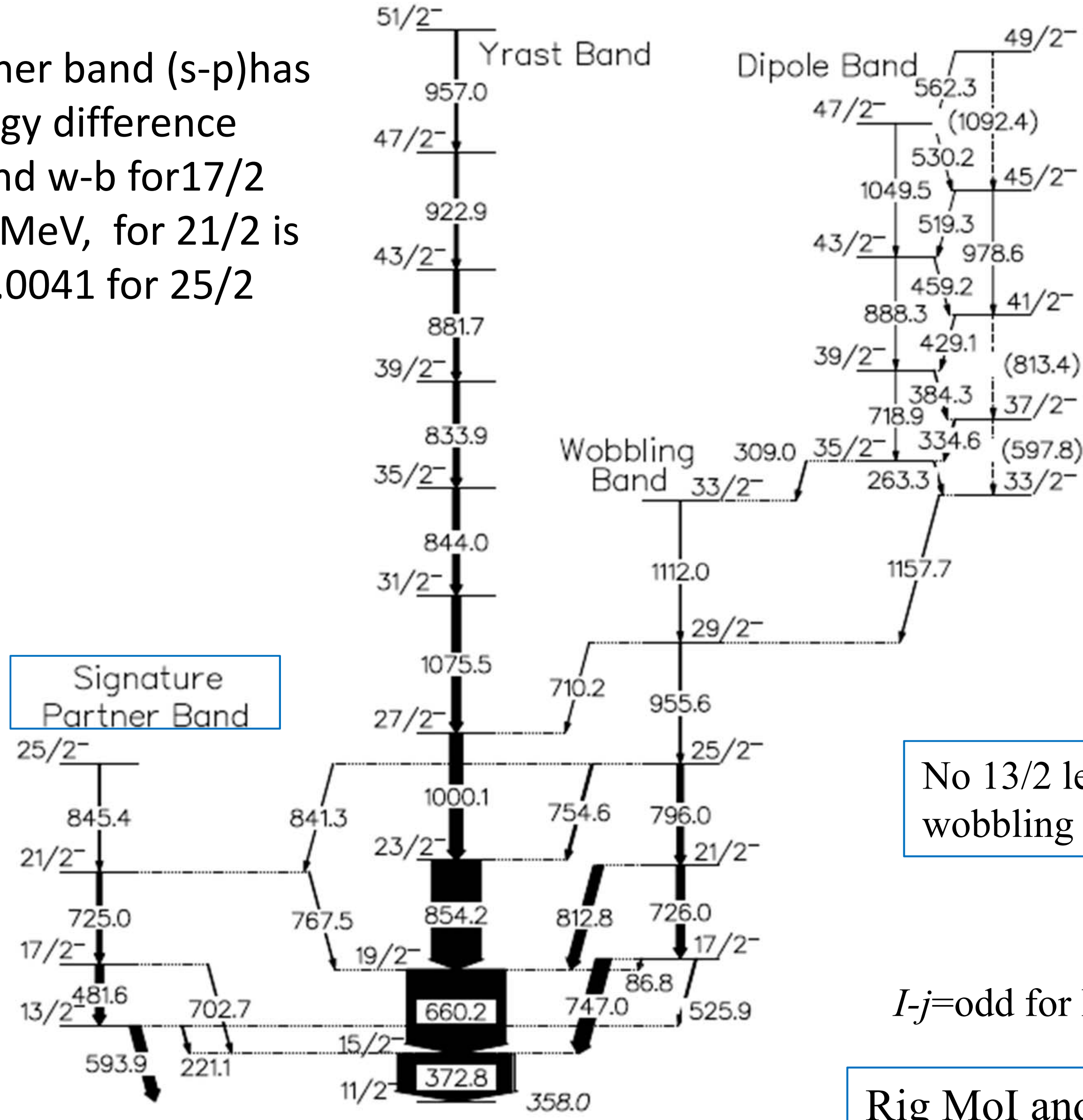
(+ + +) and (- - +). rig

$$E_{rot} = \frac{1}{2}(A_1 + A_3)I(I+1) + \frac{1}{2}(A_1 - A_3)E_{\tau I}(\kappa)$$

$$\kappa = \frac{2A_2 - A_1 - A_3}{A_1 - A_3}$$

$$A_1 \le A_2 \le A_3$$

Signature partner band (s-p)has 13/2. The energy difference between s-p and w-b for17/2 levels is -0.044MeV, for 21/2 is -0.0453, and 0.0041 for 25/2 level.



Band 2 Band 1 Band 4

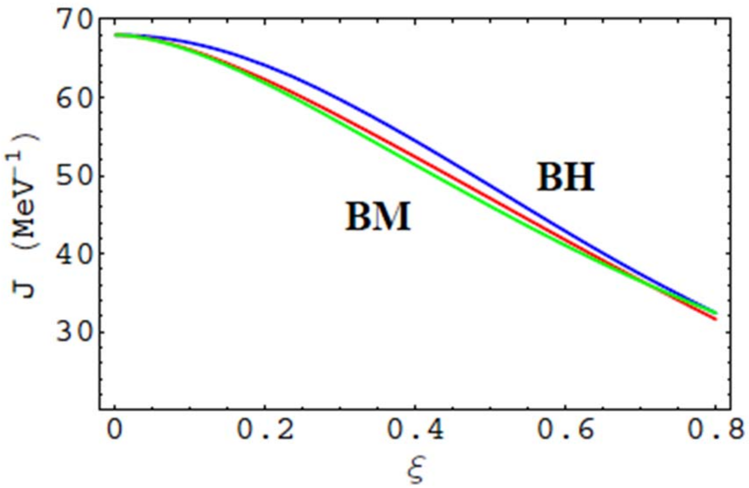
No 13/2 level in wobbling band (w-b)

I-j=odd for Band 2 and 4

Rig MoI and $0^\circ < \gamma < 30^\circ$ seems reasonable.

Pairing Effect on MoI

Pairing effect is related to rig MoI through the cranking formula , but not to hyd MoI.



$$\xi = \frac{2\Delta}{\delta}$$

δ is the energy distance between two levels connected by j_x:

$$\mathcal{J}_x = \mathcal{J}_x^{\mathrm{rig}} \langle g \rangle_{\mathrm{av}}.$$

$$\langle g \rangle_{\mathrm{av}} = \frac{1}{(1+\xi^2)^{3/2}}.$$

Bengtsson and Helgessen Oak Ridge summer school

$$\langle g \rangle_{\mathrm{av}} = 1 - \xi^2 \ln\left(\frac{1+\sqrt{1+\xi^2}}{\xi}\right) + \frac{19}{90}\xi^2$$

K. T. & K.S.-T., PR C91 034328 (2015)

$$\langle g \rangle_{\mathrm{av}} = 1 - \frac{\xi^2}{\sqrt{1+\xi^2}} \ln\left(\frac{1+\sqrt{1+\xi^2}}{\xi}\right)$$

Bohr and Mottelson, Nuclear Structure vol.II

instead of integration an asymptotic series expansion is applied

Perturbation treatment of **ω･I** in self-consistent constrained HFB equation for odd-A nucleus

CAP effect is taken into account as the second order perturbation to the BCS basis together with the blocking effect.

$$\frac{4}{G} = -\frac{2}{E_{\tilde{\ell}}} + \sum_{\alpha} \frac{1}{E_\alpha}\left[1 - \Omega_x^2 \sum_{\beta} \frac{(j_x)^2_{\alpha\beta}}{E_\alpha + E_\beta}\left(\frac{E_\alpha E_\beta - (\varepsilon_\alpha - \lambda)(\varepsilon_\beta - \lambda) - \Delta^2}{E_\alpha E_\beta (E_\alpha + E_\beta)} + \frac{(\varepsilon_\alpha - \lambda)(\varepsilon_\alpha - \varepsilon_\beta)}{E_\alpha^2 E_\beta}\right)\right]$$
$$- 2\Omega_x^2 \sum_{\alpha>0,\alpha\neq\ell} \frac{(j_x)^2_{\alpha\ell}}{E_{\tilde{\ell}}(E_{\tilde{\ell}}^2 - E_\alpha^2)}\left[3 - \frac{(\varepsilon_\alpha - \lambda)(\varepsilon_\ell - \lambda) + \Delta^2}{E_{\tilde{\ell}}^2}\right].$$

$$\mathcal{J}_x = \sum_{\varepsilon_\alpha<\varepsilon_\beta} \frac{(j_x)^2_{\alpha\beta}}{E_\alpha + E_\beta}\left(1 - \frac{(\varepsilon_\alpha - \lambda)(\varepsilon_\beta - \lambda) + \Delta^2}{E_\alpha E_\beta}\right)$$
$$+ 2 \sum_{\alpha>0,\alpha\neq\ell} \frac{(j_x)^2_{\alpha\ell}}{E_\alpha^2 - E_{\tilde{\ell}}^2}\left(E_{\tilde{\ell}} + \frac{(\varepsilon_\alpha - \lambda)(\varepsilon_\ell - \lambda) + \Delta^2}{E_\ell}\right).$$

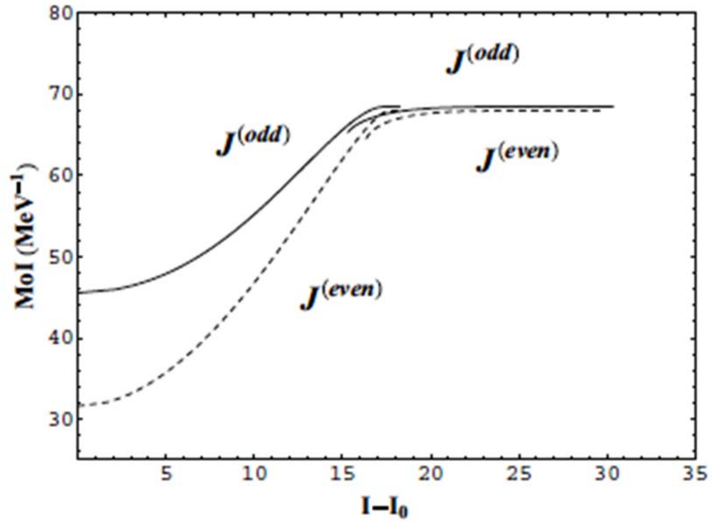
K.S.,PTP 35(1966)44.

K.T. & K. S.-T., Phys. Rev. C91 034328 (2015)

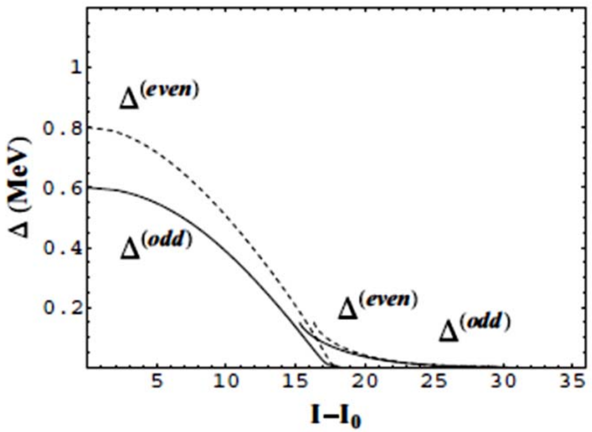
$$N = \frac{\varepsilon_\ell - \lambda}{E_{\tilde{\ell}}} + \sum_{\alpha} v_\alpha^2 + \frac{\Omega_x^2}{2} \sum_{\alpha,\beta} \frac{(j_x)^2_{\alpha\beta}}{E_\alpha + E_\beta}\left[\frac{\varepsilon_\alpha - \lambda}{E_\alpha(E_\alpha + E_\beta)}\right.$$
$$\left.\times\left(1 - \frac{(\varepsilon_\alpha - \lambda)(\varepsilon_\beta - \lambda) + \Delta^2}{E_\alpha E_\beta}\right) - \frac{\Delta^2(\varepsilon_\alpha - \varepsilon_\beta)}{E_\alpha^3 E_\beta}\right]$$
$$+ \frac{\Omega_x^2 \Delta^2}{E_{\tilde{\ell}}^3} \sum_{\alpha>0,\alpha\neq\ell} \frac{(j_x)^2_{\alpha\ell}(\varepsilon_\alpha - \varepsilon_\ell)(5E_{\tilde{\ell}}^2 - E_\alpha^2)}{(E_\alpha^2 - E_{\tilde{\ell}}^2)^2}.$$

We apply the same technique to these equations as used for MoI. Instead of integrations an asymptotic series expansion method is applied.

I-dependence of MoI through Δ

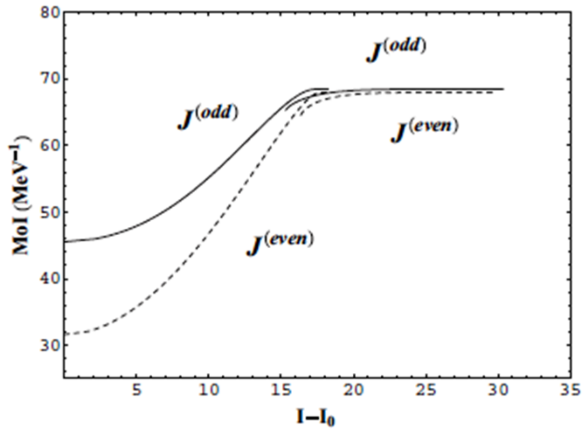


2 kinds of asymptotic series expansion for Δ≥d/2 and Δ<d case. d is the average single-particle level distance.

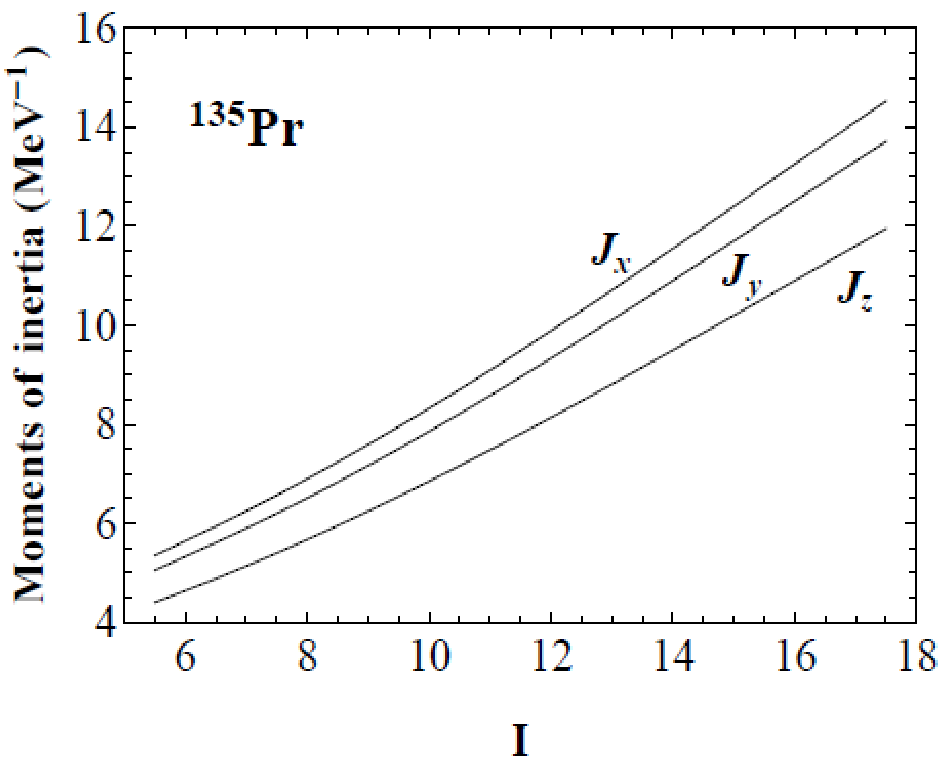


Δ keeps a small but finite values even for high spin states, indicating no sharp phase transition in nucleus.

I-dependence of MoI for low-spin state



From this figure we assume the *I*-dependence of MoI in low-spin and low-excited levels.



Wood-Saxon type

$$\frac{\mathcal{J}_0}{1+\exp(-(I-\bar{b})/\bar{a})}$$

$\bar{a} = 7.5$ and $\bar{b} = 15.5$ $\gamma = 18°$

$\mathcal{J}_0 = 25$ MeV^{-1}, $V = 1.6$ MeV, $\beta_2 = 0.18$

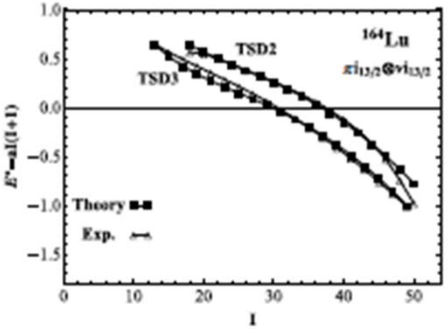
Cf. **High spin highly excited band case**

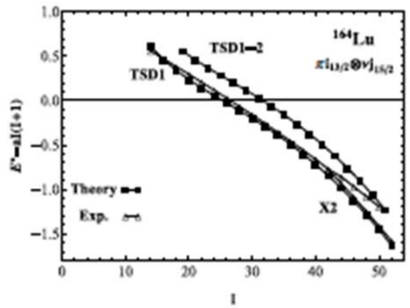
$$\mathcal{J}_0 \rightarrow \mathcal{J}_0 \frac{I - c_1}{I + c_2}.$$

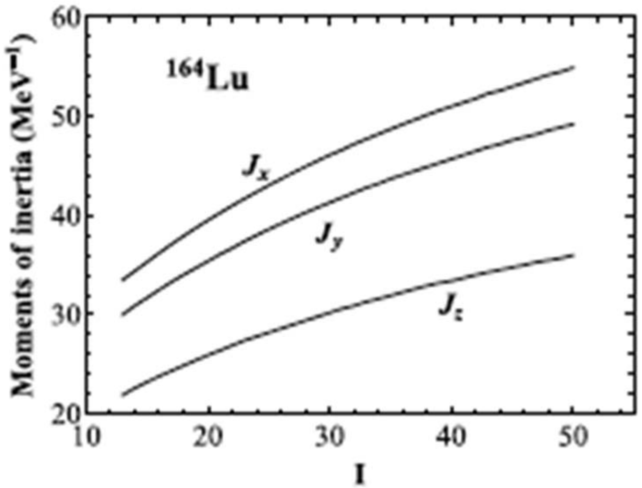
PRC 77, 064318(2008) ^{161,163,165}Lu: c1=0.69, c2=23.5

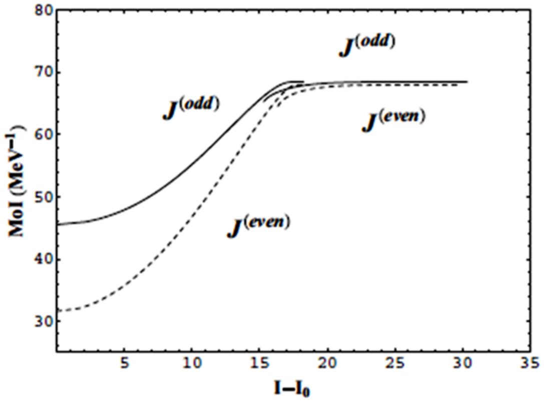
PRC 82, 051303 (R) (2010) ^{167}Lu,^{167}Ta: c1=4,c2=27.8

PTEP 2014,063D01 (2014) ^{164}Lu: c1=8,c2=41.0





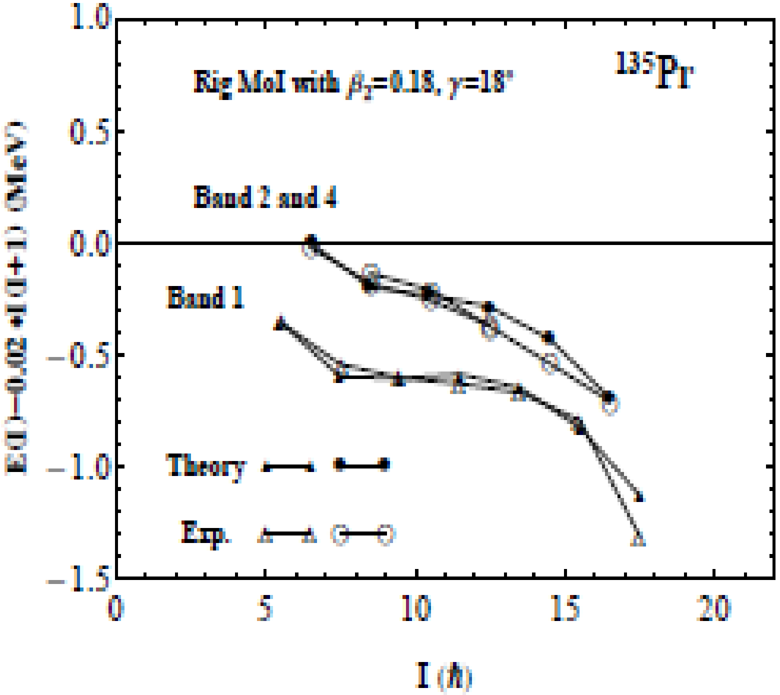


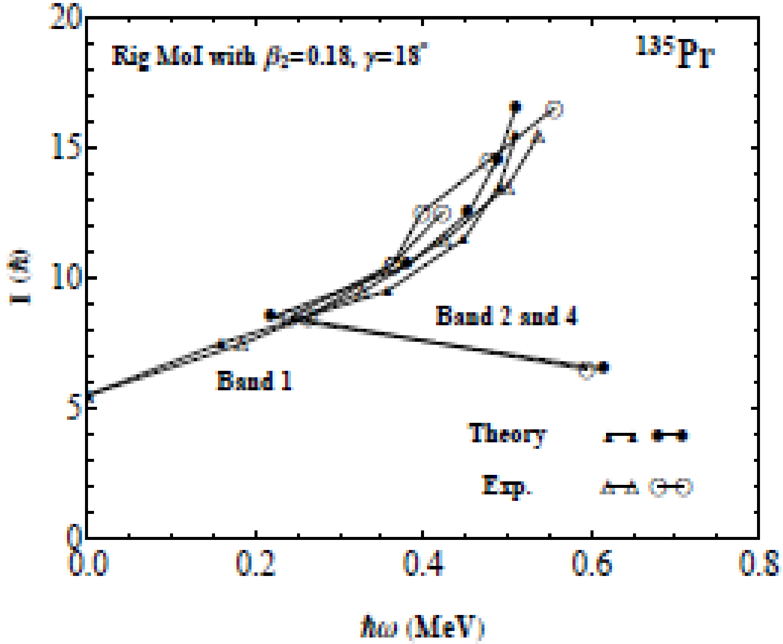


E(I)-0.02I(I+1) versus *I* plot
I versus hω/2π plot

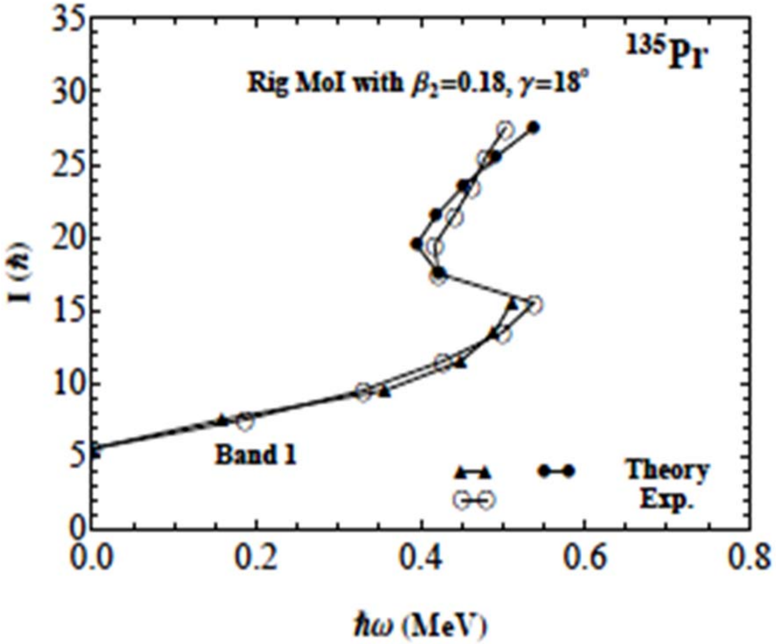
$$\hbar\omega = E_\gamma/2$$

$\hbar\omega$ for the $I = 13/2$ is E_γ to the $I = 11/2$





Backbending plot of Band 1



Exp.: E.S.Paul et al,P.R.C84, 047302 (2011)

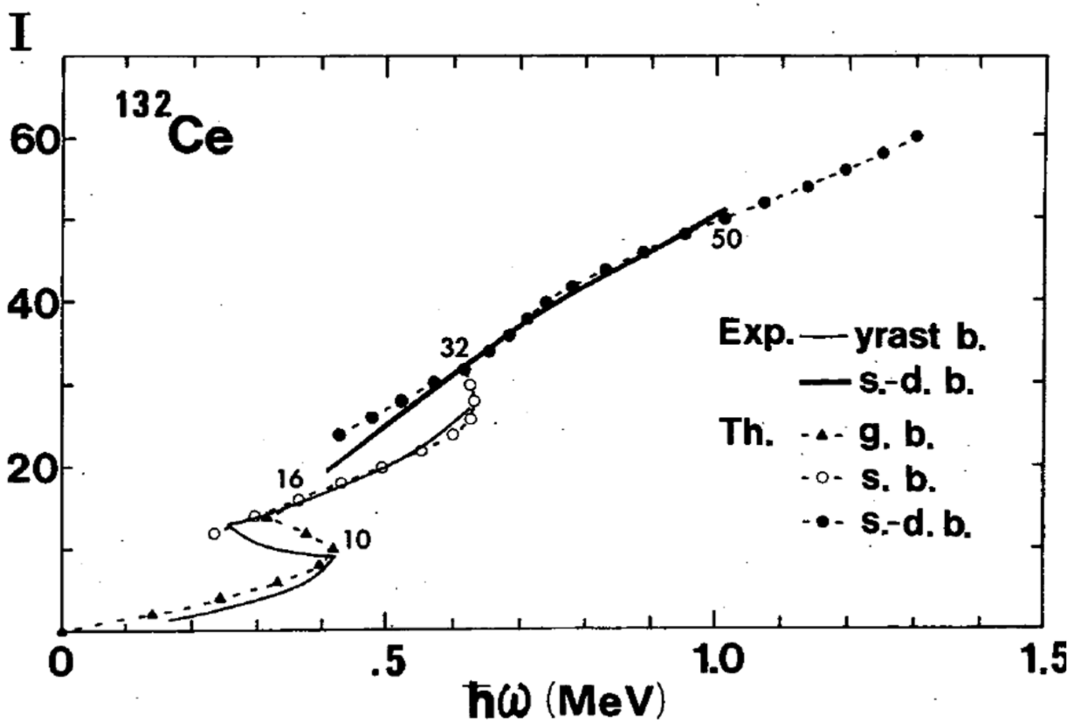
11/2 33/2
35/2 57/2

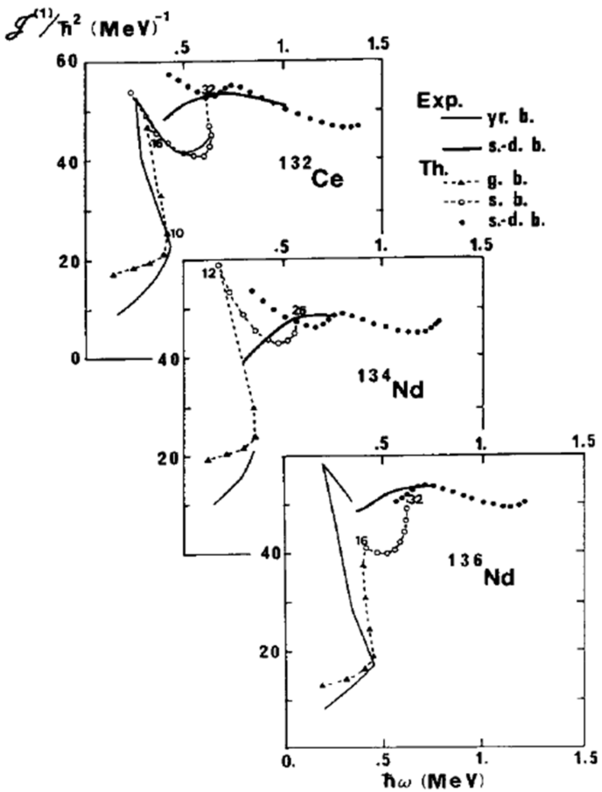
$\mathcal{J}_0$=25 MeV^{-1}

$\mathcal{J}_0$ (=35MeV^{-1})

the new bands where the lowest two i13/2 levels in neutron shell are decoupled,

Self-consistent microscopic HFB calculation
with constrains on Number and Angular momentum

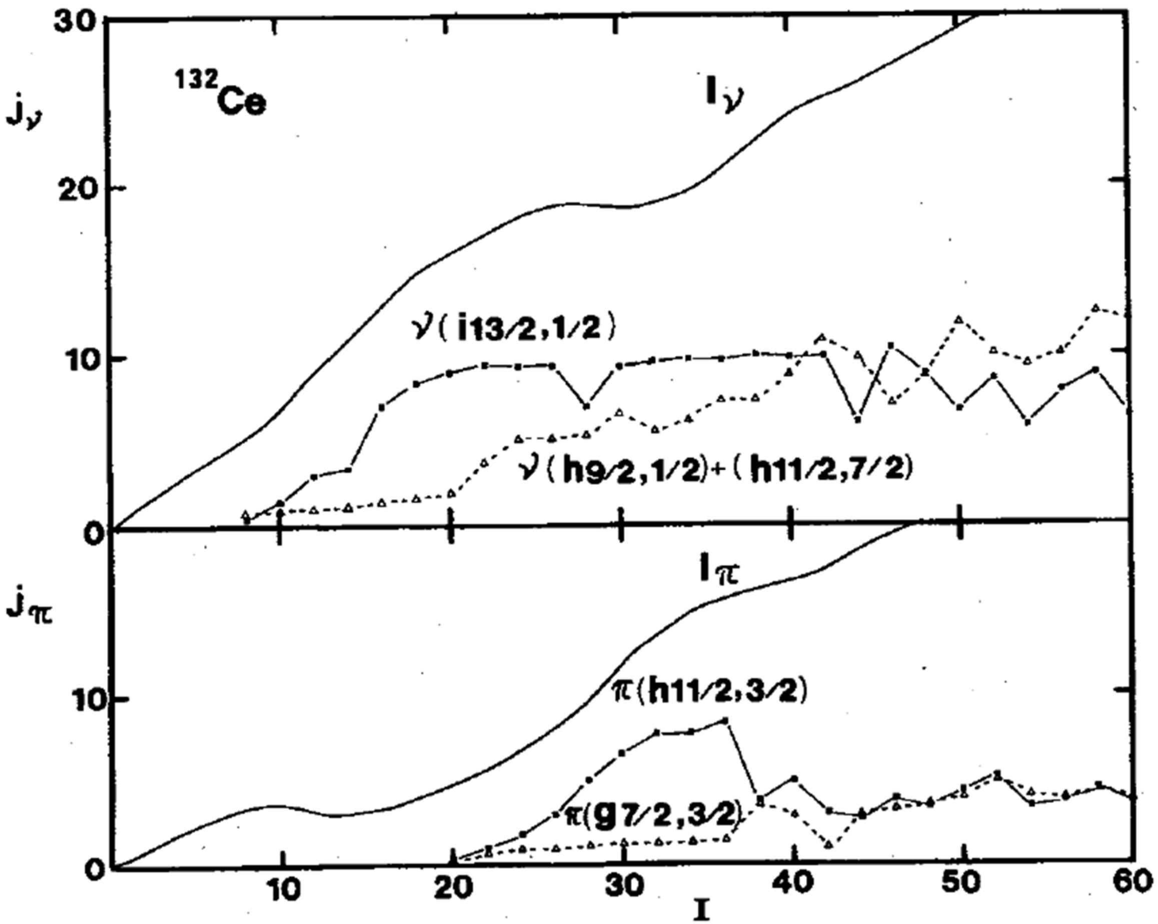




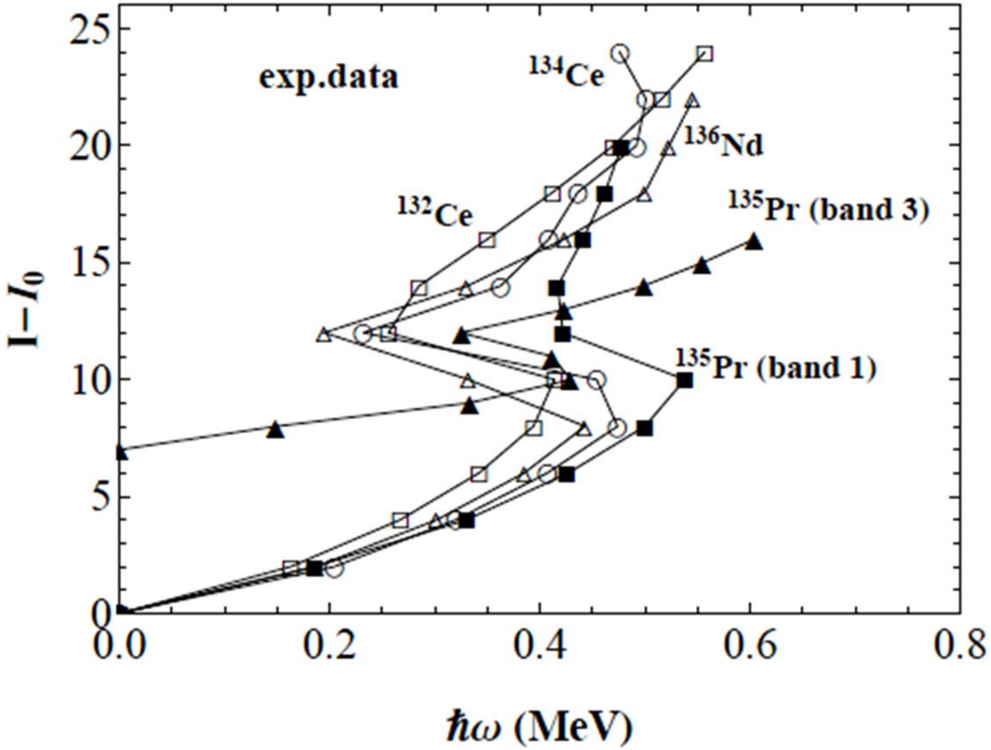
K.T. & K. S.-T., PTP 83,1148(1990)

K.T. &K.S.-T., PL 247B , 12(1991)

Gapless superconductor (alignment only in high-*j* and small *jz* orbital)



Backbending plot of exp. data

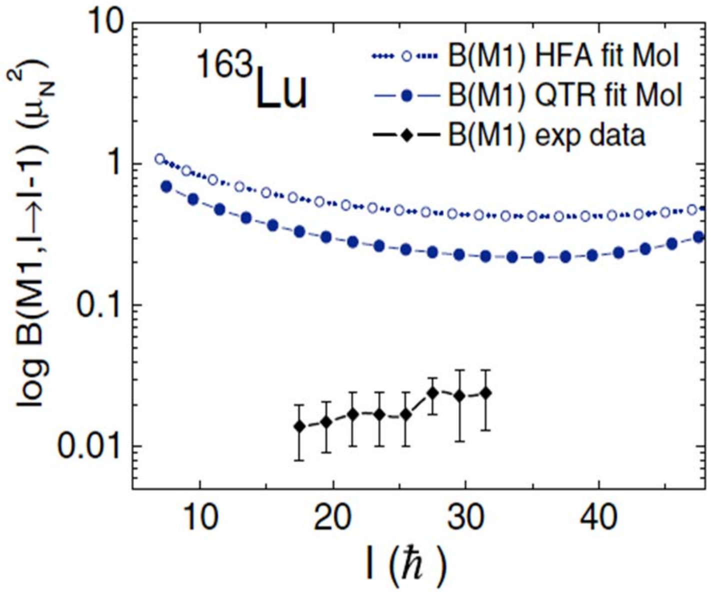


All data show a sharp backbending lump around I-I_0 ~ 12, indicating neutron i13/2 level pair is melted.

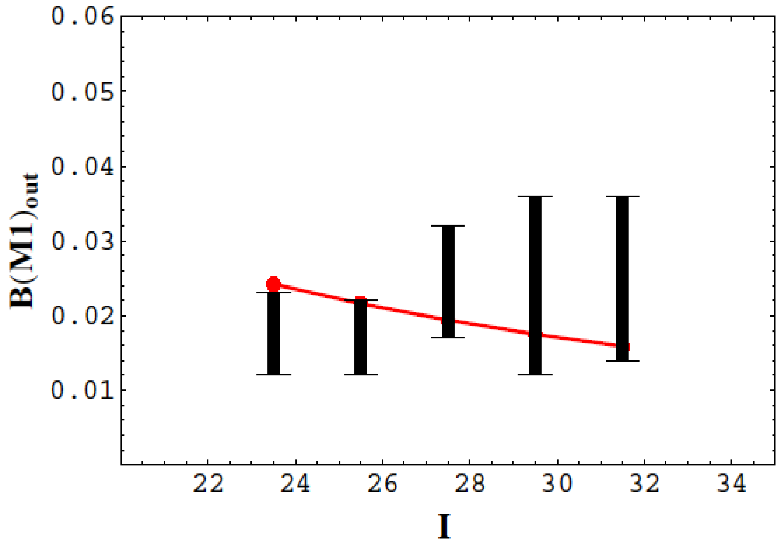
Another shortcoming of hyd MoI: larger B(M1) and smaller B(E2)

$B(M1)_{out}$ in ^{163}Lu exp: A.Görgen et.al. , Phys. Rev. C69, (2004) 031301(R)

Frauendorf and Doenau
P.R.C.89,014322(2014)



K.T. and K. S.-T., P.R.C73, 034305(2006)



$B(M1) \sim |\langle I\text{-}1\, \boldsymbol{j}\, I\rangle|^2$ hyd MoI $|\langle 01\ \boldsymbol{j}\ 00\rangle|^2$ large,

rig MoI $|\langle 10\ \boldsymbol{j}\ 00\rangle|^2$ small.

In boson model, transition matrix elements are calculated from overlap integral between $|n_\alpha, n_\beta, I, j>$ and $|n_a, n_b, I, j>$.

K.T. & K.S.-T.,
P.R.C73,034305 (2006)

$$|n_\alpha n_\beta, Ij\rangle = \frac{1}{\sqrt{n_\alpha! n_\beta!}}(\alpha^\dagger)^{n_\alpha}(\beta^\dagger)^{n_\beta}|0\rangle_\alpha.$$

$$\mathcal{M}(E2,\mu) = \sqrt{\frac{5}{16\pi}}e\left[Q_0\mathcal{D}^2_{\mu 0} + Q_2\left(\mathcal{D}^2_{\mu 2} + \mathcal{D}^2_{\mu -2}\right)\right],$$

$$\mathcal{M}(M1,\mu) = \sqrt{\frac{3}{4\pi}}\mu_\mathrm{N}\sum_{\nu=0,\pm1}[(g_\ell - g_R)j_\nu + (g_s - g_\ell)s_\nu + g_R I_\nu]\mathcal{D}^1_{\mu\nu},$$

$$\begin{aligned}G^{Ij}_{n_a,n_b;n_\alpha,n_\beta} &\equiv \frac{{}_a\langle 0|\hat{a}^{n_a}\hat{b}^{n_b}(\alpha^\dagger)^{n_\alpha}(\beta^\dagger)^{n_\beta}|0\rangle_\alpha}{(n_a!n_b!n_\alpha!n_\beta!)^{1/2}} \\ &= \frac{{}_a\langle 0|0\rangle_\alpha}{(n_a!n_b!n_\alpha!n_\beta!)^{1/2}}\,{}_a\langle 0|\hat{a}^{n_a} \\ &\quad\times \hat{b}^{n_b}(\hat{O})(\alpha^\dagger)^{n_\alpha}(\beta^\dagger)^{n_\beta}|0\rangle_\alpha.\end{aligned}$$

$$\begin{aligned}&B(E2; I_i n^i_\alpha n^i_\beta \to I_f n^f_\alpha n^f_\beta) \\ &= \frac{5e^2}{16\pi}\Bigg|\sum_{\Omega(\Omega>0)}\sum_{K(K>0,|K-\Omega|=\mathrm{even})}\Big[G^{I_f j}_{I_f-K,j-\Omega;n^f_\alpha,n^f_\beta} \\ &\quad\times Q'_0\langle I_i K 20|I_f K\rangle + G^{I_f j}_{I_f-K-2,j-\Omega;n^f_\alpha,n^f_\beta} \\ &\quad\times Q'_2\langle I_i K 22|I_f K+2\rangle + G^{I_f j}_{I_f-K+2,j-\Omega;n^f_\alpha,n^f_\beta} \\ &\quad\times Q'_2\langle I_i K 2-2|I_f K-2\rangle\Big]G^{I_i j}_{I_i-K,j-\Omega;n^i_\alpha,n^i_\beta}\Bigg|^2.\end{aligned}$$

$$\begin{aligned}&B(M1; I_i n^i_\alpha n^i_\beta \to I_f n^f_\alpha n^f_\beta) \\ &= \frac{3(\mu_\mathrm{N} g_\mathrm{eff})^2}{16\pi}\Bigg|\sum_{\Omega(\Omega>0)}\sum_{K(K>0,|K-\Omega|=\mathrm{even})}\Big[G^{I_f j}_{I_f-K-1,j-\Omega-1;n^f_\alpha,n^f_\beta} \\ &\quad\times\sqrt{2(j-\Omega)(j+\Omega+1)}\langle I_i K 11|I_f K+1\rangle \\ &\quad- G^{I_f j}_{I_f-K+1,j-\Omega+1;n^f_\alpha,n^f_\beta}\sqrt{2(j+\Omega)(j-\Omega+1)} \\ &\quad\times\langle I_i K 1-1|I_f K-1\rangle - 2G^{I_f j}_{I_f-K,j-\Omega;n^i_\alpha,n^i_\beta} \\ &\quad\times\Omega\langle I_i K 10|I_f K\rangle\Big]G^{I_i j}_{I_i-K,j-\Omega;n^i_\alpha,n^i_\beta}\Bigg|^2. \qquad (58)\end{aligned}$$

K.T. & K.S.-T.P.R.C77, 064318(2008)

Final $I-2$	(0, 0)	(1, 0)	(0, 1)
(0, 0)	$(Q'_2 G_{0000}{}^2)^2$	–	–
(1, 0)	–	$(Q'_2 G_{1010}{}^2)^2$	$[Q'_2(G_{0101}G_{0110} + G_{1010}G_{1001})]^2$
(2, 0)	h.o.	–	–
(3, 0)	–	h.o.	h.o.
(0, 1)	–	$[Q'_2(G_{0101}G_{0110} + G_{1010}G_{1001})]^2$	$(Q'_2 G_{0101}{}^2)^2$

B(E2)in: 00→00,10→10 , 01→01 no difference

Final $I-1$	(0, 0)	(1, 0)	(0, 1)
(0, 0)	–	$\frac{3}{7}(Q'_0 G_{0000} G_{1010})^2$	$\frac{3}{7}(Q'_0 G_{0000} G_{1001})^2$
(1, 0)	$\frac{2}{7}(Q'_2 G_{0000} G_{1010})^2$	–	–
(2, 0)	–	$\frac{4}{7}(Q'_2 G_{2020} G_{1010})^2$	$\frac{2}{7}[Q'_2(\sqrt{2}G_{2020}G_{1001} + G_{0101}G_{1120})]^2$
(3, 0)	h.o.	–	–
(0, 1)	$\frac{2}{7}(Q'_2 G_{0000} G_{1001})^2$	–	–

B(E2)out: 00→10 > 00 →01

Final $I-1$	(0, 0)	(1, 0)	(0, 1)
(0, 0)	–	$\frac{4j^2}{I}(G_{0000}G_{1010})^2$	$\frac{4j^2}{I}(G_{0000}G_{1001})^2$
(1, 0)	$4j(G_{0000}G_{0110})^2$	–	–
(2, 0)	–	$4j(G_{1120}G_{1010})^2$	$8j(G_{0220}G_{0101})^2$
(3, 0)	h.o.	–	–
(0, 1)	$4j(G_{0000}G_{0101})^2$	–	–

B(M1)out: 00→10 < 00→01

$$G^{Ij}_{n_a,n_b;n_\alpha,n_\beta} \equiv \frac{{}_a\langle 0|\hat{a}^{n_a}\hat{b}^{n_b}(\alpha^\dagger)^{n_\alpha}(\beta^\dagger)^{n_\beta}|0\rangle_\alpha}{(n_a!n_b!n_\alpha!n_\beta!)^{1/2}}$$

$$= \frac{{}_a\langle 0|0\rangle_\alpha}{(n_a!n_b!n_\alpha!n_\beta!)^{1/2}}\,{}_a\langle 0|\hat{a}^{n_a}\hat{b}^{n_b}(\hat{O})(\alpha^\dagger)^{n_\alpha}(\beta^\dagger)^{n_\beta}|0\rangle_\alpha$$

Electromagnetic transition rates and the mixing ratio δ

$$Q_0 = \frac{Z}{5}(2R_z^2 - R_x^2 - R_y^2) = \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta_2 \cos\gamma,$$

$Q_0 = 3$ b for ^{135}Pr, with β_2=0.18 and $\gamma = 18°$.

$g_{\rm eff} = 0.414$,
bare value of $g_\ell - g_R + (g_s - g_\ell)/(2j)$ multiplied by the quenching factor 0.5.

	$B(E2)_{\rm out}/B(E2)_{\rm in}$		$B(M1)_{\rm out}/B(E2)_{\rm in}$		δ	
I	exp.	theory	exp.	theory	exp.	theory
17/2	⋯	0.648	⋯	0.192	-1.24 ± 0.13	−1.13
21/2	0.843 ± 0.032	0.542	0.164 ± 0.014	0.130	-1.54 ± 0.09	−1.34
25/2	0.500 ± 0.025	0.463	0.035 ± 0.009	0.0987	-2.38 ± 0.37	−1.44
29/2	$\geq 0.261 \pm 0.014$	0.402	$\leq 0.016 \pm 0.004$	0.0791	⋯	−1.49

$$B(M1)_{\rm out}/B(E2)_{\rm in} \propto (g_{\rm eff}/Q_0)^2$$

$$\delta \propto g_{\rm eff}/Q_0$$

$$B(E2)_{\rm out}/B(E2)_{\rm in} \propto \tan^2\gamma$$

Comparison between hyd MoI and rig MoI

(1) When V=0, in both γ=30° and 6° cases hyd MoI shows I=13/2 belongs to Band 2, while **rig MoI shows *I*=13/2 does not belong to Band 2.**

(2) In 0° <γ <15° in hyd MoI, R=odd R_z=0 is not (+ + +), while 0° < γ < 30° in rig MoI, R=odd R_z=±2 splits to (+ + +) and (- - +) (Band 2 and 4). In 15° <γ <30° in hyd MoI, R=odd R_y=±3 (n_α=0) is not (+ + +), while 30° < γ < 60° in rig MoI, R=odd R_x=±2 (n_α=1) splits to (+ + +) and (+ - -) (Band 2 and 4). However both MoI give R=even R_y=R_x=±2 (n_α=0) splits to (+ + +) and (- - +), indicating Band 1 has its signature partner, which is not observed. Thus , **rig MoI in 0° < γ < 30° is preferable.**

(3) Angular-momentum dependence of MoI (Coriolis anti-pairing effect) **can be included into rig MoI,** but not into hyd MoI.

(4) Hyd MoI gives **larger B(M1) and smaller B(E2) than rig MoI**.

Not only from **the stability equation**, but also from the **above results**, **we employ rig MoI.**

Conclusion

(1) We apply particle-rotor model to 11/2- band in ^{135}Pr.
Our model uses I-dependent rigid MoI, which is obtained from the perturbation treatment of the Coriolis-anti-pairing effect microscopically.

(2) This model attains quite good reproduction of experimental data not only in the energy scheme, but also in the electro-magnetic transition rates $B(E2)_{out}/B(E2)_{in}$, $B(M1)_{out}/B(E2)_{in}$ and the mixing ratio δ.

(3) As for the signature partner-band, rigid MoI can explain that the small breaking of Bohr-symmetry induces the splitting of D_2-invariant state (+ + +) into (+ + +) and (+ - -) states in $0^\circ < \gamma < 30^\circ$.