

Studies of super-deformed states in atomic nuclei using the Coulomb excitation method

Kasia Hadyńska-Klęk

INFN Laboratori Nazionali di Legnaro, Italy

SSNET Workshop, Orsay, France
07.11 – 11.11.2016

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- The story of ^{42}Ca
- COULEX results
- Theory
- Definition of **superdefomation?**

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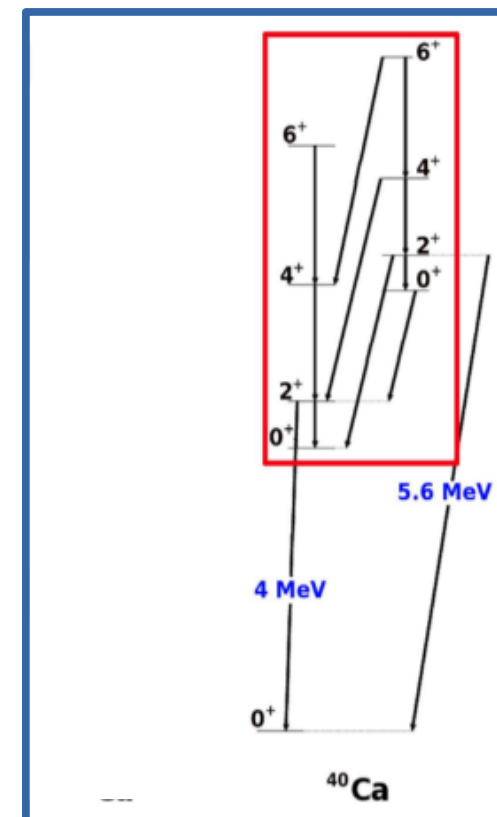
Motivation – SD in ^{42}Ca ?

→ **Superdeformed band in ^{40}Ca** (DSAM, ANL)

$$B(E2; 4^+ \rightarrow 2^+) = 170 \text{ Wu}$$

$$Q_t = 1.80(+10.39, -0.29) \text{ eb} \rightarrow \beta_2 = 0.59(+0.11, -0.07)$$

(E. Ideguchi et al., PRL 87, 222501 (2001), C.J. Chiara et al., PRC 67, 041303R (2003))



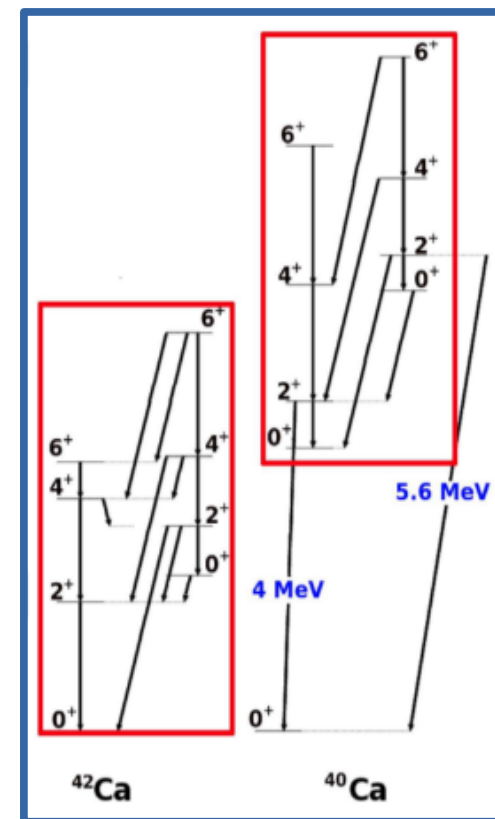
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→ **Superdeformed bands in A~40 mass region:**

^{36}Ar , ^{38}Ar , ^{40}Ar , ^{44}Ti

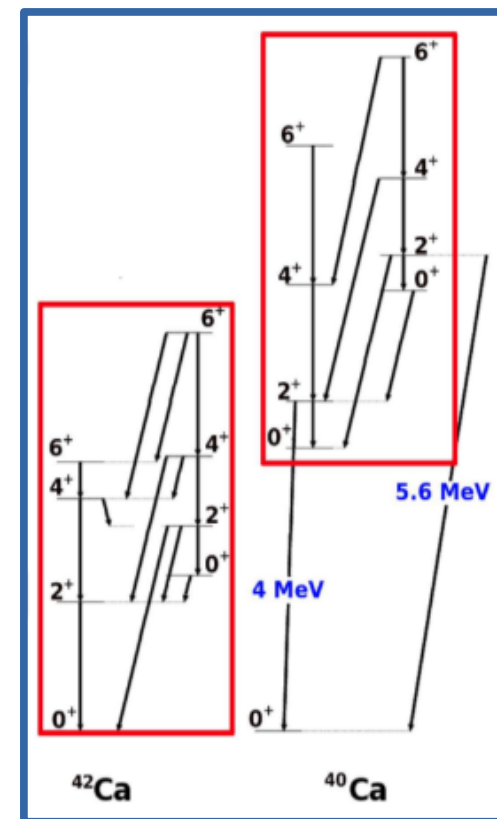
(in all cases β_2 between 0.4-0.6)

C.E. Svensson et al., PRL 85 (2000) 2693

D. Rudolph et al., PRC 65 (2002) 034305

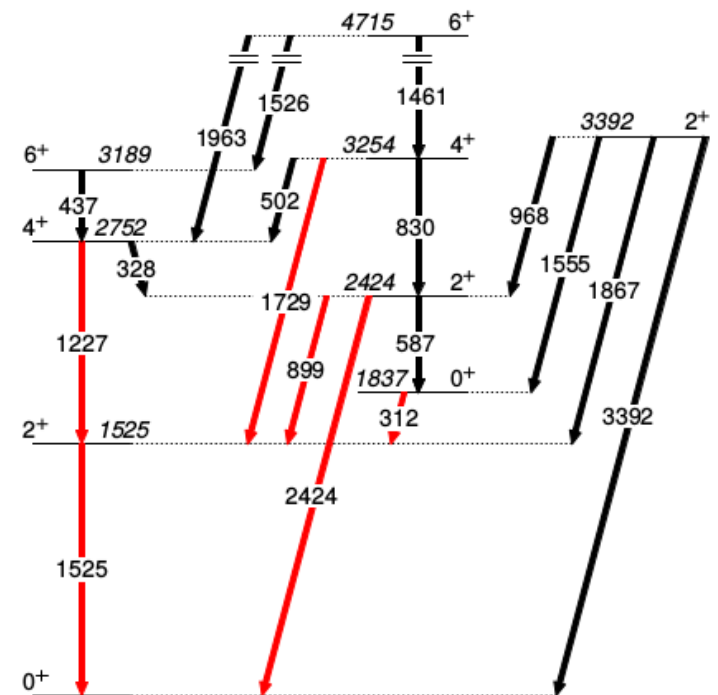
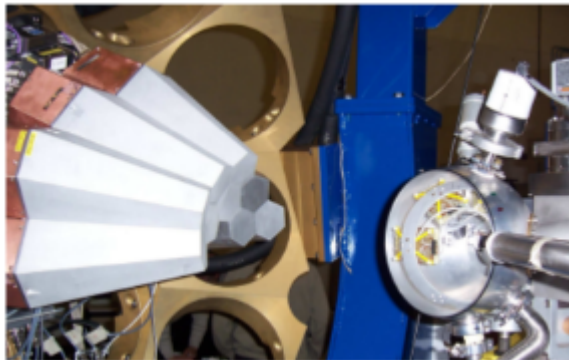
E. Ideguchi et al., PLB 686 (2010) 18

D.C. O'Leary et al., PRC 61 (2000) 064314

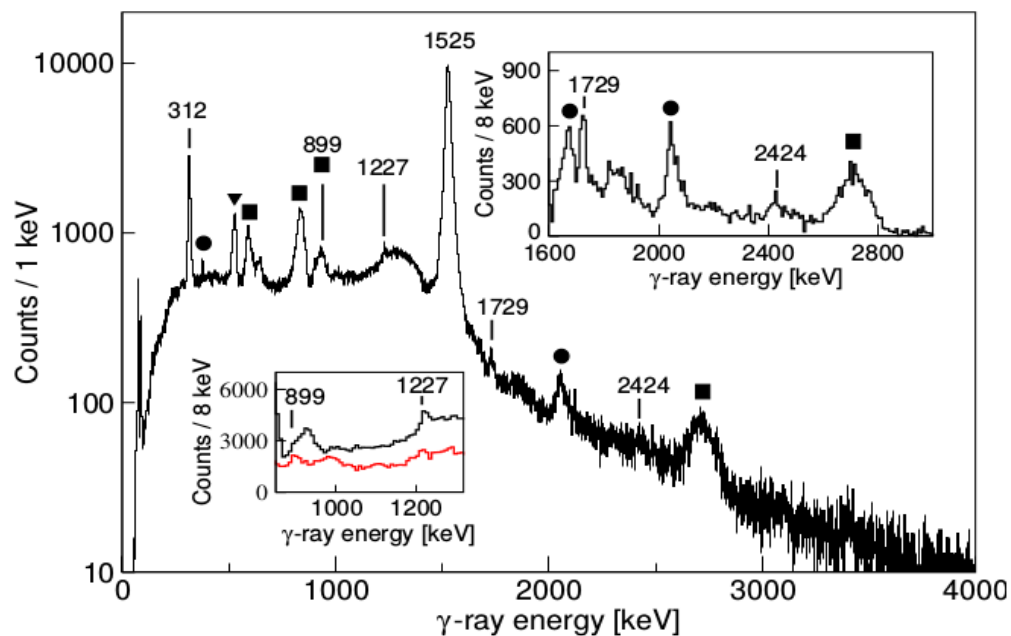


Coulomb excitation of ^{42}Ca

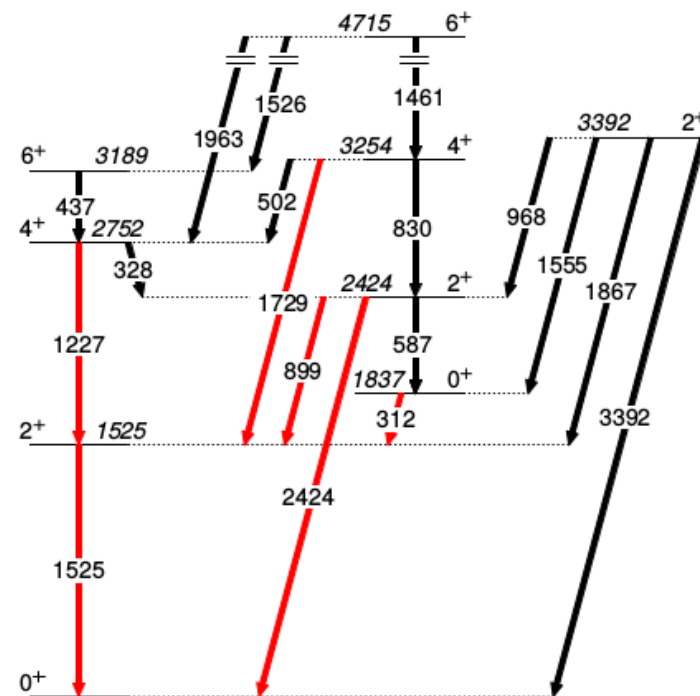
- INFN LNL
- Beam: ^{42}Ca , 170 MeV
- Targets:
 - ^{208}Pb , 1 mg/cm²
 - ^{197}Au , 1 mg/cm²
- AGATA: 3 triple clusters, 143.8 mm from the target
- DANTE: 3 MCP detectors, 100-144°



Results – spectrum of ^{42}Ca

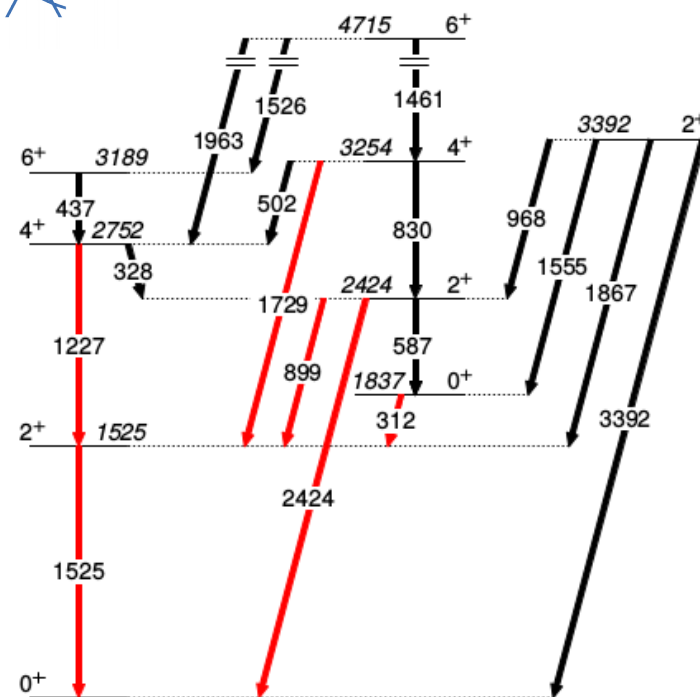


- Pb (208, 207, 206, 204)
- ▼ 511 keV
- ^{43}Ca



Results – transition probabilities

$I_i^+ \rightarrow I_f^+$	$\langle I_i E2 I_f \rangle [e \text{ fm}^2]$	$B(E2 \downarrow; I_i^+ \rightarrow I_f^+) [\text{W.u.}]$	
	Present	Present	Previous
$2_1^+ \rightarrow 0_1^+$	$20.5^{+0.6}_{-0.6}$	$9.7^{+0.6}_{-0.6}$	9.3 ± 1 [36] 11 ± 2 [28] 9 ± 3 [27] 8.5 ± 1.9 [45]
$4_1^+ \rightarrow 2_1^+$	$24.3^{+1.2}_{-1.2}$	$7.6^{+0.7}_{-0.7}$	50 ± 15 [28] 11 ± 3 [27] 10^{+10}_{-8} [45]
$6_1^+ \rightarrow 4_1^+$	$9.3^{+0.2}_{-0.2}$	$0.77^{+0.03}_{-0.03}$	0.7 ± 0.3 [27]
$0_2^+ \rightarrow 2_1^+$	$22.2^{+1.1}_{-1.1}$	57^{+6}_{-6}	64 ± 4 [27] 100 ± 6 [28] 55 ± 1 [42] 64 ± 4 [45]
$2_2^+ \rightarrow 0_1^+$	$-6.4^{+0.3}_{-0.3}$	$1.0^{+0.1}_{-0.1}$	2.2 ± 0.6 [28] 1.5 ± 0.5 [27] 1.2 ± 0.3 [45]
$2_2^+ \rightarrow 2_1^+$	$-23.7^{+2.3}_{-2.7}$	$12.9^{+2.5}_{-2.5}$	17 ± 11 [28] 19^{+22}_{-14} [27] 14^{+35}_{-9} [45]
$4_2^+ \rightarrow 2_1^+$	42^{+3}_{-4}	23^{+3}_{-4}	30 ± 11 [28] 16 ± 5 [27] 12^{+7}_{-4} [45]
$2_2^+ \rightarrow 0_2^+$	26^{+5}_{-3}	15^{+6}_{-4}	< 61 [27] < 46 [45]
$4_2^+ \rightarrow 2_2^+$	46^{+3}_{-6}	27^{+4}_{-6}	60 ± 30 [27] 60 ± 20 [28] 40^{+40}_{-30} [45]
$\langle I_i E2 I_f \rangle [e \text{ fm}^2]$		$Q_{sp} [e \text{ fm}^2]$	
$2_1^+ \rightarrow 2_1^+$	-16^{+9}_{-3}	-12^{+7}_{-2}	-19 ± 8 [36]
$2_2^+ \rightarrow 2_2^+$	-55^{+15}_{-15}	-42^{+12}_{-12}	

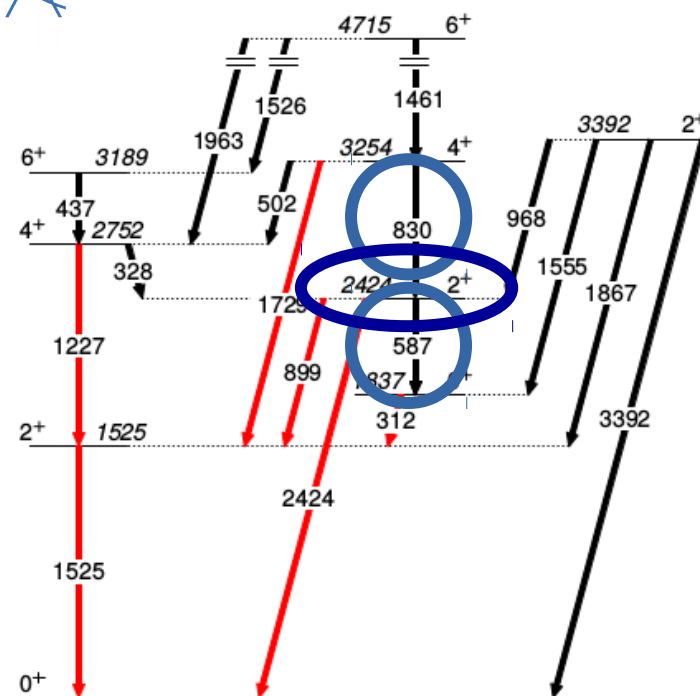


K. Hadyńska-Klęk *et al.*, PRL 117, 062501 (2016)

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K. Hadyńska-Klęk *et al.*, PRL 117, 062501 (2016)

Discussion: Quadrupole Shape Invariants

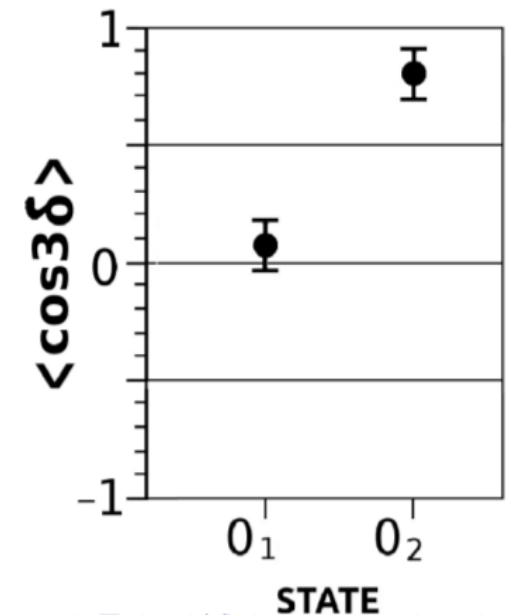
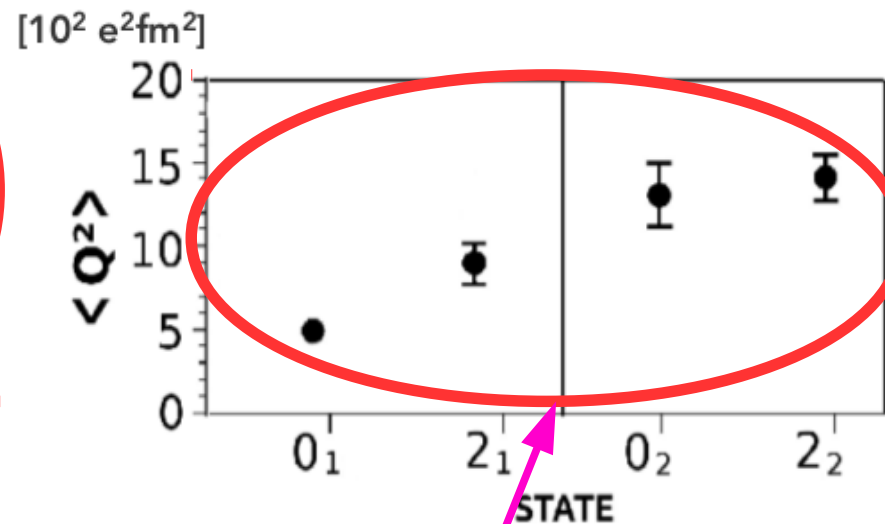
$$\frac{1}{\sqrt{5}} \langle Q^2 \rangle = \frac{1}{\sqrt{2I_i + 1}} \sum_t \langle i || E2 || t \rangle \langle t || E2 || f \rangle \begin{Bmatrix} 2 & 2 & 0 \\ I_i & I_f & I_t \end{Bmatrix}$$

$$\langle Q^3 \cos(3\delta) \rangle = \mp \frac{\sqrt{35}}{\sqrt{2}} \frac{1}{\sqrt{2I_i + 1}} \sum_{tu} \langle s || E2 || u \rangle \langle u || E2 || t \rangle \langle t || E2 || s \rangle \begin{Bmatrix} 2 & 2 & 2 \\ I_s & I_t & I_u \end{Bmatrix}$$

K. Kumar, PRL 28, 249 (1972)

state	$\langle Q^2 \rangle_{exp}$
0_1^+	500 (20)
2_1^+	900 (100)
0_2^+	1300 (230)
2_2^+	1400 (250)

state	$\langle \cos(3\delta) \rangle_{exp}$
0_1^+	0.06 (10)
0_2^+	0.79 (13)



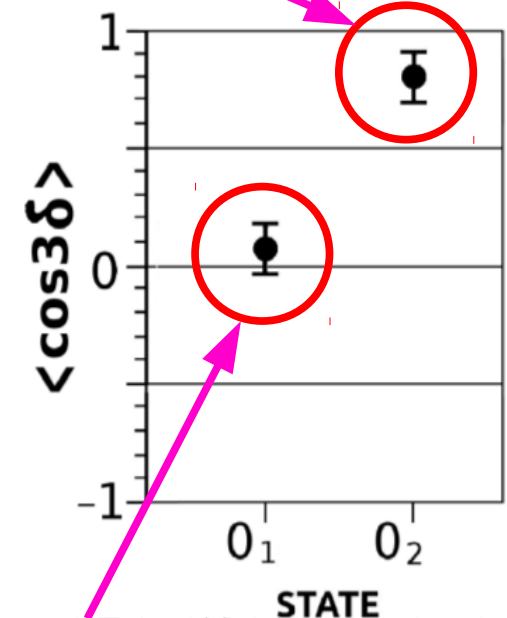
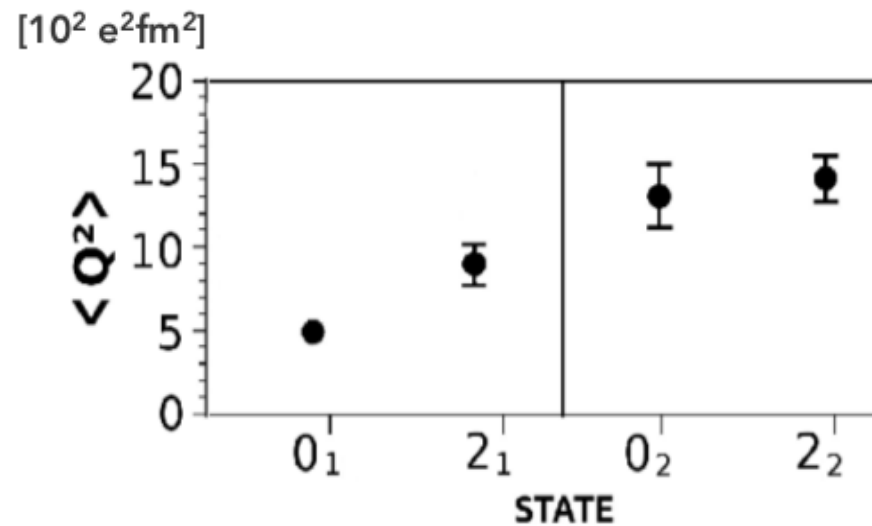
Increasing deformation in GSB and stable in the side band

Discussion: Quadrupole Shape Invariants

$\cos(3\delta) \sim 0.8$ – slightly triaxial - prolate 0_2

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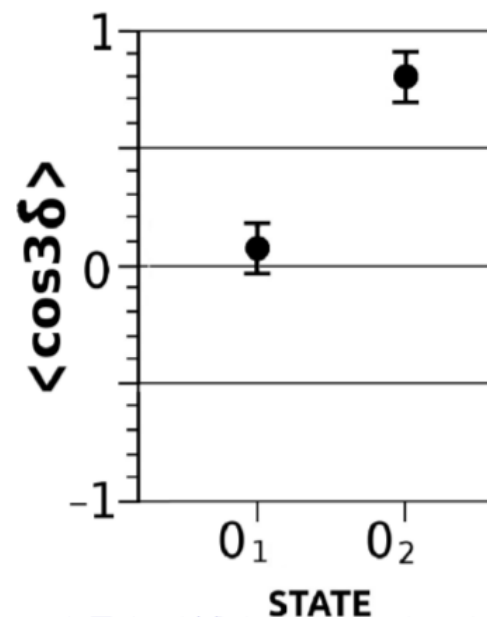
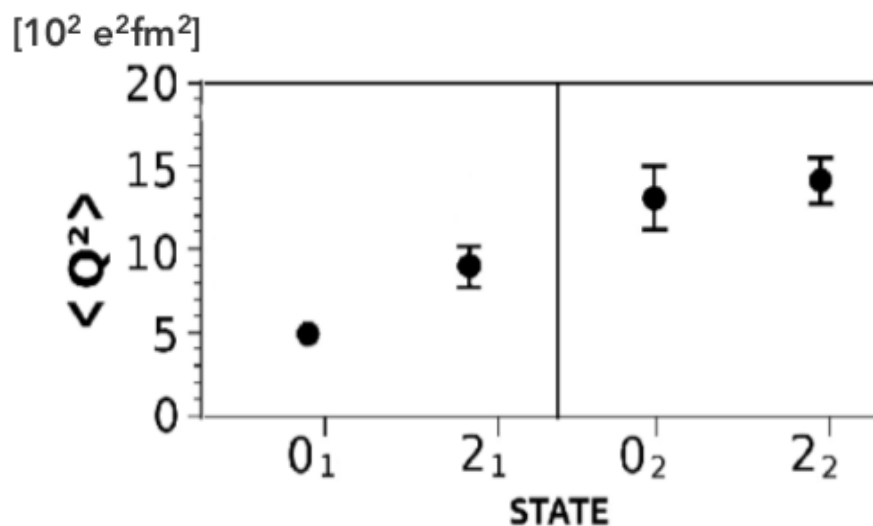


$\cos(3\delta) \sim 0$ – triaxial GS

Discussion: Quadrupole Shape Invariants

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0_1 $\beta=0.26(2)$ and $\gamma=29(2)^\circ$
 0_2 $\beta=0.43(2)$ and $\gamma=13(6)^\circ$

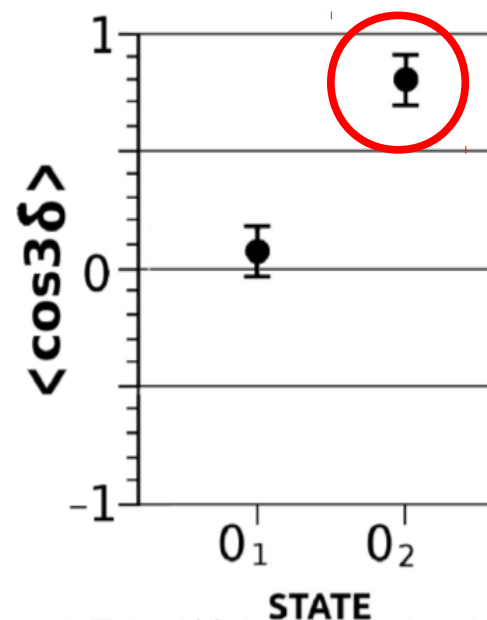
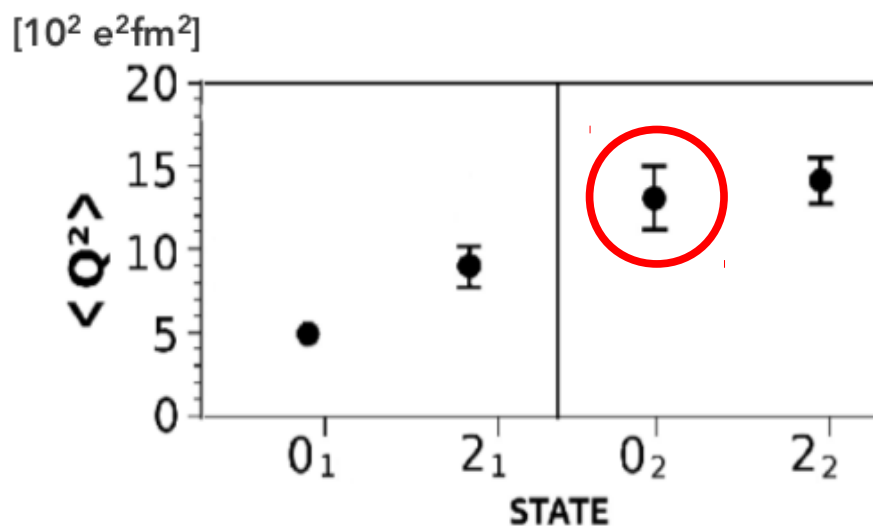
$$\beta = \sqrt{\langle \beta^2 \rangle} = \sqrt{\frac{\langle Q^2 \rangle}{q_0^2}},$$

$$\gamma = \arccos \langle \cos(3\delta) \rangle,$$

Discussion: Quadrupole Shape Invariants

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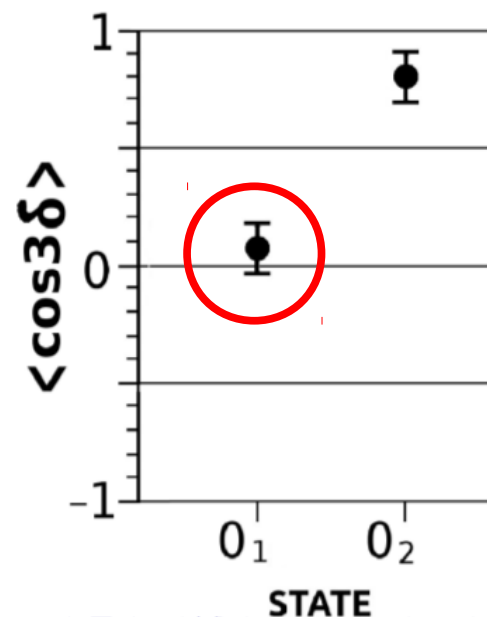
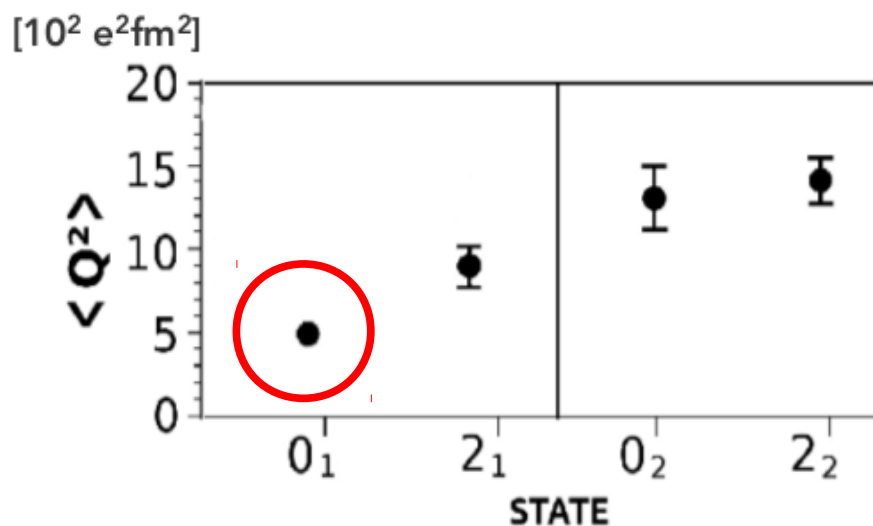
0_2 $\beta=0.43(2)$ and $\gamma=13(6)^\circ$

SUPERDEFORMED
slightly triaxial
SIDE BAND

Discussion: Quadrupole Shape Invariants

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Non-zero triaxial deformation of the ground state?

Discussion:

Shape parameters

Why non-zero deformation of the ground state?

- **fluctuations** about the spherical shape
- maximum triaxiality - effect of averaging over all possible quadrupole shapes
- **the dispersion of Q^2 , $\sigma(Q^2)$, should be comparable to Q^2 value**

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Insufficient experimental data

Theoretical predictions and the full set of ME from:

Large Scale Shell Model

F.Nowacki, H.Naïdja, B.Bounthong
Université de Strasbourg, France

Beyond Mean Field

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Universidad Autónoma de Madrid, Spain

Discussion: Shape parameters

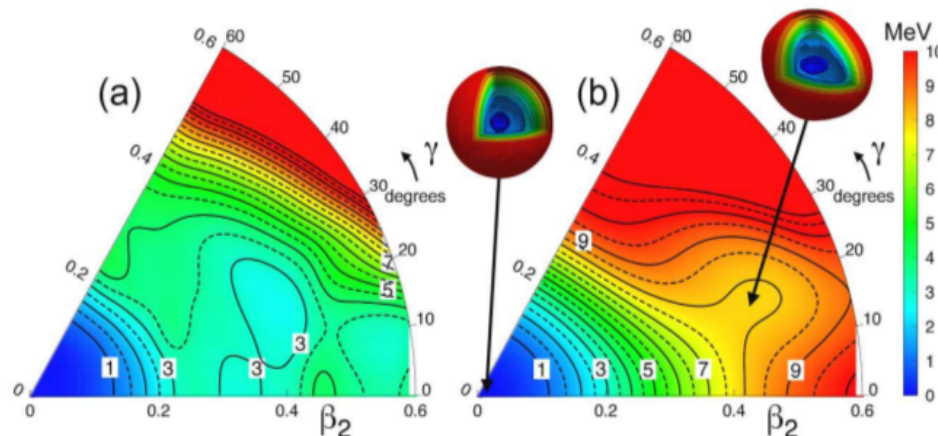
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Insufficient experimental data

Theoretical predictions and the full set of ME from the SM and BMF calculations



Both approaches predict:

- 0_1 - SPHERICAL
- 0_2 - TRIAXIAL/PROLATE

Discussion:

Shape parameters

We use the theoretical predictions and the full set of ME from the calculations:

- Large Scale Shell Model (F.Nowacki, H.Naïdja, B.Bounthong - Strasbourg)
- Beyond Mean Field (T. R. Rodríguez - Madrid)

state	$\langle Q^2 \rangle_{exp}$	$\langle Q^2 \rangle_{SM}$	$\sigma(Q^2)_{SM}$	$\langle Q^2 \rangle_{BMF}$	$\sigma(Q^2)_{BMF}$
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0_2^+	1300 (230)	1200	500	1900	520
2_2^+	1400 (250)	1130	500	1900	300

state	$\langle \cos(3\delta) \rangle_{exp}$	$\langle \cos(3\delta) \rangle_{SM}$	$\langle \cos(3\delta) \rangle_{BMF}$
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0_1 - SPHERICAL with large fluctuations around minimum

0_2 - SUPERDEFORMED, SLIGHTLY TRIAXIAL/PROLATE shape

What does it mean SUPERDEFORMED?

- a quantitative definition of superdeformation does not seem to exist
- authors use various subjective criteria, rarely clearly defined

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- deformation corresponding to axes ratio between **3:2:1** and **2:1:1** → corresponding β parameter: 0.4-0.6: A~130 mass region, e.g. ¹⁵²Dy, and some in the A~40 region:
 - $\beta_2=0.46\pm0.03$ (³⁶Ar),
 $\beta_2=0.48\pm 0.05$ (⁴⁰Ar)
 - P. Nolan and P. Twin, Annu. Rev. Nucl. Part. Sci. 38, 533 (1988)
 - R.V.F. Janssens, T.L. Khoo, Annu. Rev. Nucl. Part. Sci. 41, 321 (1991)
 - C.E. Svensson, et al., Phys. Rev. Lett. 85, 2693 (2000)
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- normal deformation: 1.3:1:1 ($\beta \sim 0.3$)

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$$\beta_2 = 0.46 \pm 0.03 \text{ } (^{36}\text{Ar}),$$

$$\beta_2 = 0.48 \pm 0.05 \text{ } (^{40}\text{Ar})$$

- the deformation parameters determined from quadrupole invariants in ^{42}Ca :
 $\beta_2 = 0.43(2)$ for 0_2 and $\beta_2 = 0.45(2)$ for 2_2 state

What does it mean SUPERDEFORMED?

Shell configuration in A~40 region

→ complex particle-hole configuration

^{40}Ca : 8p-8h, $\beta_2 \approx 0.6$

TABLE IV. Percentage of np - nh components and energy of the first three 0^+ states (GS, ND, and SD) of ^{40}Ca .

	0p-0h	2p-2h	4p-4h	6p-6h	8p-8h	$E(\text{th})$	$E(\text{exp})$
0_{GS}^+	65	29	5	—	—	0	0
0_{ND}^+	1	1	64	25	9	3.49	3.35
0_{SD}^+	—	—	9	4	87	4.80	5.21

^{42}Ca , $\beta_2=0.43(2)$ (0_2) and $\beta_2=0.45(2)$ (2_2)

$J_i^{\pi_i}$	2p0h	4p2h	6p4h	8p6h
0_1^+	40%	40%	17%	3%
2_1^+	45%	36%	16%	3%
4_1^+	55%	35%	9%	1%
6_1^+	55%	35%	9%	1%
0_2^+	10%	18%	49%	23%
2_2^+	12%	13%	50%	24%
2_3^+	0%	14%	59%	26%
3_1^+	0%	4%	66%	30%
4_2^+	1%	15%	62%	22%
6_2^+	1%	24%	61%	14%

4p-4h: $^{36,40}\text{Ar}$, $\beta_2=0.46$ and $\beta_2=0.48$

3p-3h: ^{35}Cl : $\beta_2 \approx 0.37$

Summary and outlook

- the properties of low-lying states in ^{42}Ca were studied using low-energy Coulomb excitation – 0^+ , 2^+ and 4^+ states observed in both bands
- the quadrupole deformation parameters of the 0^+ and 2^+ states in GSB and SDB in ^{42}Ca were determined from the measured reduced matrix elements
- the results were compared with SM and BMF calculations
- the non-zero deformation of the ground state has been attributed to the fluctuations around the spherical shape
- a large static deformation of $\beta=0.43(2)$ and $\beta=0.45(2)$, for 0_2^+ and 2_2^+ was observed, proving the **superdeformed** character of the side band
- the $\cos(3\delta)$ parameter measured for 0_2 brings the first experimental evidence for **non-axial** character of SD bands in the $A\sim 40$ mass region
- COULEX of SD bands in other $A\sim 40$ nuclei: ^{40}Ca , $^{36-40}\text{Ar}$, ^{44}Ti – projects in preparation/ongoing

Many thanks to:

PRL 117, 062501 (2016)

PHYSICAL REVIEW LETTERS

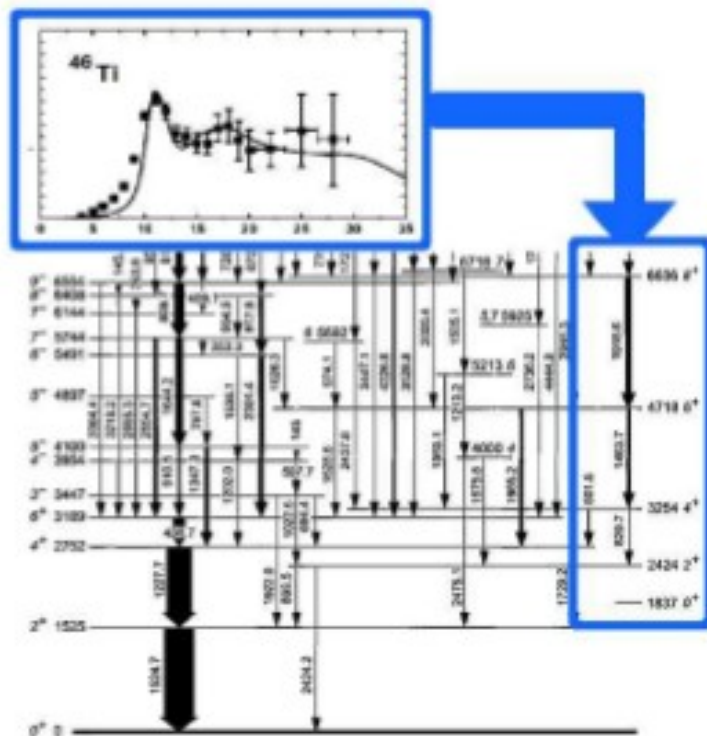
week ending
5 AUGUST 2016

Superdeformed and Triaxial States in ^{42}Ca

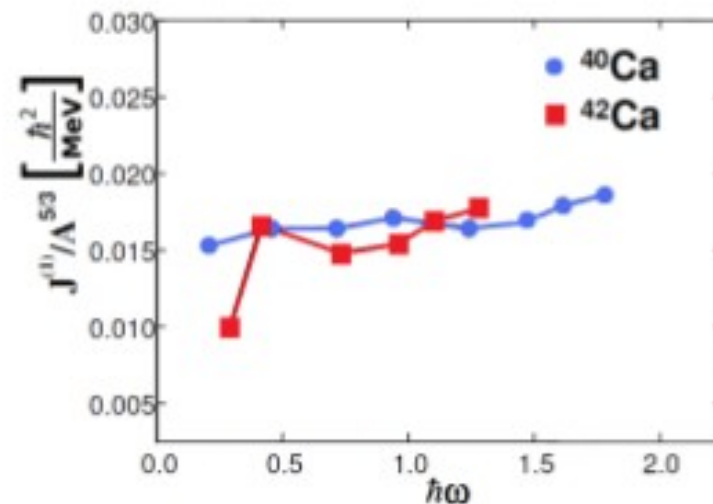
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Motivation – SD in ^{42}Ca ?

Low energy branch of ^{46}Ti Giant Dipole Resonance decay feeding the states in the side band in ^{42}Ca (M.Kmiecik *et al.*, Acta Phys. Pol. B36, 1169(2005))



Moments of inertia of states in the side band in ^{42}Ca look very similar to states in SD-band in ^{40}Ca , (A.Maj *et al.* Key Topics in Nuclear Structure, page 417, (2005), M.Lach *et al.* EPJ A 16, 3, 309-311 (2003))



Theory

Large Scale Shell Model

F.Nowacki, H.Naïdja, B.Bounthong
Université de Strasbourg, France

ANTOINE code

6 particle-hole excitations from $s_{1/2}$ and $d_{3/2}$ orbitals to pf orbitals

Effective charges: 1.5e (protons) and 0.5e (neutrons)

Same method as the one used for SD in ^{40}Ca :
E.Caurier, J.Menendez, F.Nowacki and A.Poves,
Phys. Rev. C 75, 054317 (2007)

Beyond Mean Field

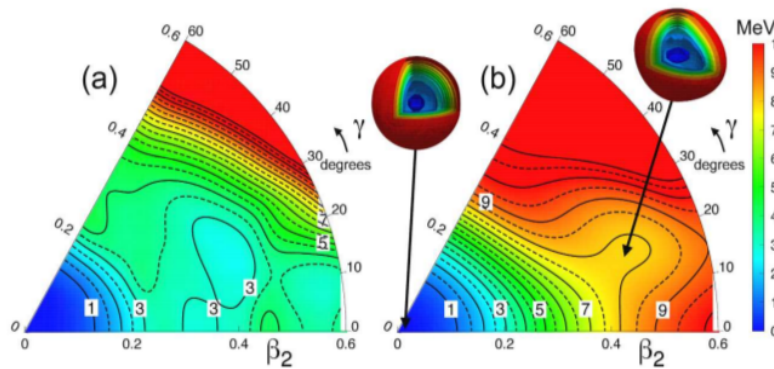
T. R. Rodríguez
Universidad Autónoma de Madrid, Spain

RVAMPIRE code

T.R.Rodríguez and J.L.Egido,
Phys. Rev. C 81, 064323 (2010)

HFB, Gogny D1S interaction to define the energy density functional

→ Particle number and angular momentum symmetry restoration
→ Quadrupole (axial and non-axial) shape mixing within generator coordinate method



Both approaches predict:

0_1 - SPHERICAL
 0_2 - TRIAXIAL/PROLATE

$I_i^+ \rightarrow I_f^+$	$\langle I_i E2 I_f \rangle [e \text{ fm}^2]$			$B(E2 \downarrow; I_i^+ \rightarrow I_f^+) [\text{W.u.}]$	
	Present	SM	BMF	Present	Previous
$2_1^+ \rightarrow 0_1^+$	$20.5^{+0.6}_{-0.6}$	11.5	9.14	$9.7^{+0.6}_{-0.6}$	9.3 ± 1 [36] 11 ± 2 [28] 9 ± 3 [27] 8.5 ± 1.9 [45]
$4_1^+ \rightarrow 2_1^+$	$24.3^{+1.2}_{-1.2}$	11.3	12.2	$7.6^{+0.7}_{-0.7}$	50 ± 15 [28] 11 ± 3 [27] 10^{+10}_{-8} [45]
$6_1^+ \rightarrow 4_1^+$	$9.3^{+0.2}_{-0.2}$	8.2	14.3	$0.77^{+0.03}_{-0.03}$	0.7 ± 0.3 [27]
$0_2^+ \rightarrow 2_1^+$	$22.2^{+1.1}_{-1.1}$	11.9	6.1	57^{+6}_{-6}	64 ± 4 [27] 100 ± 6 [28] 55 ± 1 [42] 64 ± 4 [45]
$2_2^+ \rightarrow 0_1^+$	$-6.4^{+0.3}_{-0.3}$	9.4	4.4	$1.0^{+0.1}_{-0.1}$	2.2 ± 0.6 [28] 1.5 ± 0.5 [27] 1.2 ± 0.3 [45]
$2_2^+ \rightarrow 2_1^+$	$-23.7^{+2.3}_{-2.7}$	-13.6	-7.7	$12.9^{+2.5}_{-2.5}$	17 ± 11 [28] 19^{+22}_{-14} [27] 14^{+35}_{-9} [45]
$4_2^+ \rightarrow 2_1^+$	42^{+3}_{-4}	21.9	10.1	23^{+3}_{-4}	30 ± 11 [28] 16 ± 5 [27] 12^{+7}_{-4} [45]
$2_2^+ \rightarrow 0_2^+$	26^{+5}_{-3}	32	42	15^{+6}_{-4}	< 61 [27] < 46 [45]
$4_2^+ \rightarrow 2_2^+$	46^{+3}_{-6}	52	70	27^{+4}_{-6}	60 ± 30 [27] 60 ± 20 [28] 40^{+40}_{-30} [45]
$\langle I_i E2 I_f \rangle [e \text{ fm}^2]$			$Q_{sp} [e \text{ fm}^2]$		
$2_1^+ \rightarrow 2_1^+$	-16^{+9}_{-3}	-4.3	0.1	-12^{+7}_{-2}	-19 ± 8 [36]
$2_2^+ \rightarrow 2_2^+$	-55^{+15}_{-15}	-31	-42	-42^{+12}_{-12}	

			$6^+ \text{ ————— } 7435$
			$3^+ \text{ ————— } 5770$
			$4^+ \text{ ————— } 5446$
			$2^+ \text{ ————— } 5029$
$6^+ \text{ ————— } 4715$	$6^+ \text{ ————— } 4807$		
		$6^+ \text{ ————— } 4275$	
		$3^+ \text{ ————— } 4045$	
$2^+ \text{ ————— } 3392$	$4^+ \text{ ————— } 3421$	$4^+ \text{ ————— } 3509$	
$4^+ \text{ ————— } 3254$	$2^+ \text{ ————— } 3322$	$0^+ \text{ ————— } 3352$	
$6^+ \text{ ————— } 3189$	$6^+ \text{ ————— } 3220$		
$4^+ \text{ ————— } 2752$			
$2^+ \text{ ————— } 2424$	$4^+ \text{ ————— } 2468$	$2^+ \text{ ————— } 2481$	
	$2^+ \text{ ————— } 2362$		
$0^+ \text{ ————— } 1837$			
$2^+ \text{ ————— } 1525$	$0^+ \text{ ————— } 1666$		
	$2^+ \text{ ————— } 1531$		
$0^+ \text{ ————— } 0$	$0^+ \text{ ————— } 0$	$0^+ \text{ ————— } 0$	
COULEX	SM	BMF	

What does it mean SUPERDEFORMED?

Shell configuration in A~40 region

→ complex particle-hole configuration

^{40}Ca : 8p-8h, $\beta_2 \approx 0.6$

TABLE IV. Percentage of np - nh components and energy of the first three 0^+ states (GS, ND, and SD) of ^{40}Ca .

	0p-0h	2p-2h	4p-4h	6p-6h	8p-8h	$E(\text{th})$	$E(\text{exp})$
0_{GS}^+	65	29	5	—	—	0	0
0_{ND}^+	1	1	64	25	9	3.49	3.35
0_{SD}^+	—	—	9	4	87	4.80	5.21

^{42}Ca , $\beta_2=0.43(2)$ (0_2) and $\beta_2=0.45(2)$ (2_2)

J_{π}^i	2p0h	4p2h	6p4h	8p6h
0_1^+	40%	40%	17%	3%
2_1^+	45%	36%	16%	3%
4_1^+	55%	35%	9%	1%
6_1^+	55%	35%	9%	1%
0_2^+	10%	18%	49%	23%
2_2^+	12%	13%	50%	24%
2_3^+	0%	14%	59%	26%
3_1^+	0%	4%	66%	30%
4_2^+	1%	15%	62%	22%
6_2^+	1%	24%	61%	14%

4p-4h: $^{36,40}\text{Ar}$, $\beta_2=0.46$ and $\beta_2=0.48$

3p-3h: ^{35}Cl : $\beta_2 \approx 0.37$

Conclusion:

a quantitative definition of superdeformation does not seem to exist